Computer Algebra
Independent Integration Tests

Summer 2024

3-Logarithms/173-3.5

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3 Listing of integrals **115**

3.1 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 126
3.2 $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 137
3.3 $\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 149
3.4 $\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 160
3.5 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$ 168

3.6 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$ 177
3.7 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$ 185
3.8 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$ 194
3.9 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$ 205
3.10 $\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 216
3.11 $\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 228
3.12 $\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 239
3.13 $\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 252
3.14 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$ 261
3.15 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$ 273
3.16 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$ 282
3.17 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$ 293
3.18 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$ 302
3.19 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$ 313
3.20 $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 324
3.21 $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 336
3.22 $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 348
3.23 $\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 359
3.24 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$ 369
3.25 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$ 382
3.26 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$ 392
3.27 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$ 402
3.28 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$ 415
3.29 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$ 424
3.30 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx$ 434

3.31 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$ 445
3.32 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$ 455
3.33 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$ 464
3.34 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx$ 473
3.35 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx$ 481
3.36 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx$ 488
3.37 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx$ 498
3.38 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx$ 509
3.39 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$ 520
3.40 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$ 531
3.41 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$ 541
3.42 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx$ 550
3.43 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$ 557
3.44 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$ 566
3.45 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx$ 577
3.46 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx$ 588
3.47 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$ 599
3.48 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$ 610
3.49 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$ 620
3.50 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx$ 629
3.51 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx$ 637
3.52 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$ 648
3.53 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx$ 658
3.54 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx$ 669
3.55 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$ 681

3.56 $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$ 695
3.57 $\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$ 707
3.58 $\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$ 719
3.59 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx} dx$ 728
3.60 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2} dx$ 739
3.61 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3} dx$ 748
3.62 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4} dx$ 759
3.63 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5} dx$ 771
3.64 $\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$ 783
3.65 $\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$ 802
3.66 $\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$ 821
3.67 $\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$ 836
3.68 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx} dx$ 849
3.69 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2} dx$ 864
3.70 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3} dx$ 873
3.71 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4} dx$ 886
3.72 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5} dx$ 896
3.73 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6} dx$ 908
3.74 $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$ 919
3.75 $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$ 943
3.76 $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$ 965
3.77 $\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$ 987
3.78 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx} dx$ 1002
3.79 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2} dx$ 1026

3.80 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$ 1035
3.81 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$ 1045
3.82 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx$ 1055
3.83 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx$ 1066
3.84 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$ 1078
3.85 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$ 1088
3.86 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$ 1097
3.87 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$ 1105
3.88 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$ 1113
3.89 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$ 1121
3.90 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$ 1131
3.91 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx$ 1142
3.92 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$ 1154
3.93 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$ 1163
3.94 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$ 1172
3.95 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$ 1179
3.96 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$ 1188
3.97 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx$ 1199
3.98 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx$ 1210
3.99 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx$ 1222
3.100 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$ 1234
3.101 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$ 1244
3.102 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$ 1253

3.103 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$ 1264
3.104 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$ 1275
3.105 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx$ 1287
3.106 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx$ 1298
3.107 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx$ 1310
3.108 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1322
3.109 $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1332
3.110 $\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1343
3.111 $\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1353
3.112 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$ 1360
3.113 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$ 1368
3.114 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$ 1375
3.115 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$ 1382
3.116 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$ 1391
3.117 $\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1401
3.118 $\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1412
3.119 $\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1423
3.120 $\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1434
3.121 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$ 1443
3.122 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$ 1454
3.123 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$ 1462
3.124 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$ 1471
3.125 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$ 1479
3.126 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx$ 1489
3.127 $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1499
3.128 $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1510
3.129 $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1521
3.130 $\int (ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1531

3.131 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$ 1541
3.132 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$ 1553
3.133 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$ 1561
3.134 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$ 1569
3.135 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$ 1581
3.136 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$ 1589
3.137 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$ 1598
3.138 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx$ 1606
3.139 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx$ 1613
3.140 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx$ 1619
3.141 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx$ 1627
3.142 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx$ 1636
3.143 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$ 1647
3.144 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$ 1656
3.145 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$ 1664
3.146 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx$ 1671
3.147 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx$ 1678
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3.154 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx$ 1741

3.155 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$ 1748
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3.167 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$ 1880
3.168 $\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$ 1891
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CHAPTER 1

Introduction

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**261**]. This is test number [173].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (261)	0.00 (0)
Mathematica	94.64 (247)	5.36 (14)
Maxima	68.20 (178)	31.80 (83)
Maple	60.92 (159)	39.08 (102)
Fricas	59.00 (154)	41.00 (107)
Reduce	50.19 (131)	49.81 (130)
Mupad	47.89 (125)	52.11 (136)
Giac	45.21 (118)	54.79 (143)
Sympy	20.69 (54)	79.31 (207)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

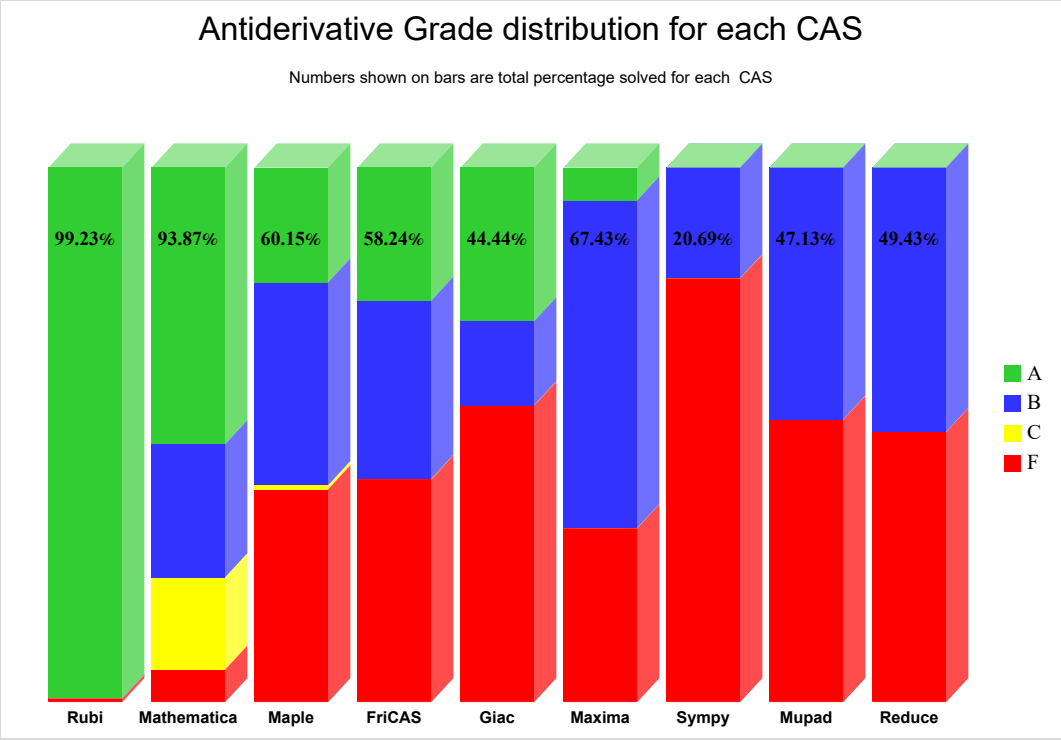
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

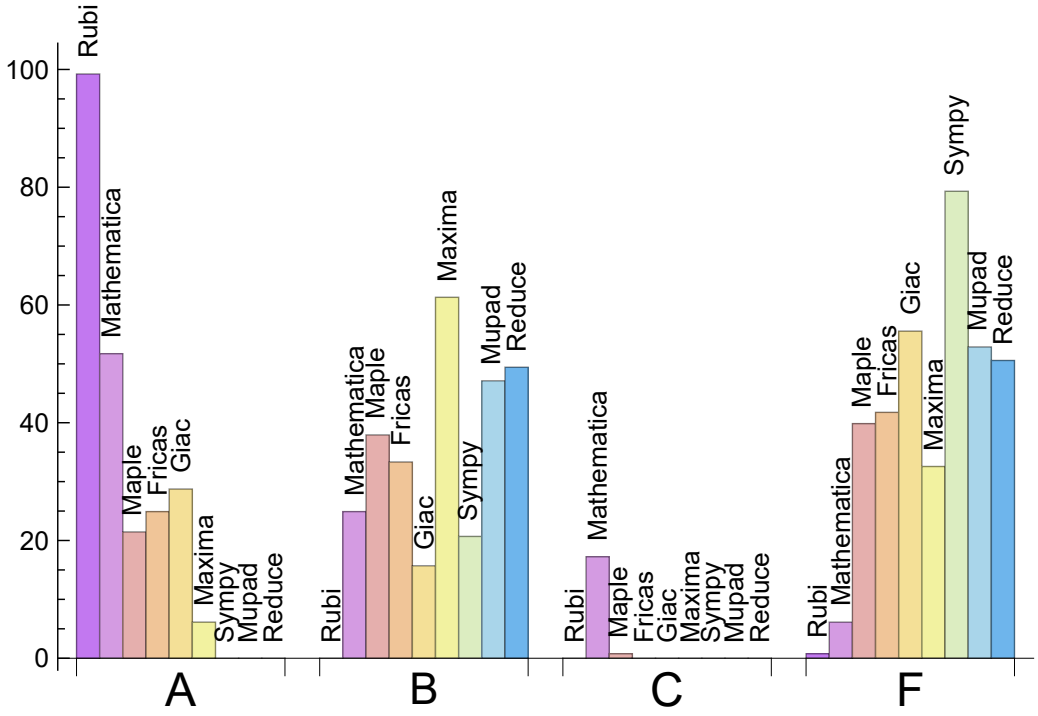
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.234	0.000	0.000	0.766
Mathematica	51.724	24.904	17.241	6.130
Giac	28.736	15.709	0.000	55.556
Fricas	24.904	33.333	0.000	41.762
Maple	21.456	37.931	0.766	39.847
Maxima	6.130	61.303	0.000	32.567
Mupad	0.000	47.126	0.000	52.874
Reduce	0.000	49.425	0.000	50.575
Sympy	0.000	20.690	0.000	79.310

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	14	100.00	0.00	0.00
Maxima	83	100.00	0.00	0.00
Maple	102	100.00	0.00	0.00
Fricas	107	100.00	0.00	0.00
Reduce	130	100.00	0.00	0.00
Mupad	136	0.00	100.00	0.00
Giac	143	82.52	16.08	1.40
Sympy	207	28.02	67.63	4.35

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.11
Reduce	0.22
Maxima	0.24
Rubi	0.72
Mathematica	1.37
Sympy	17.47
Maple	17.72
Giac	17.82
Mupad	28.79

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	283.10	0.90	229.00	0.88
Fricas	667.80	2.44	447.00	2.09
Maple	727.47	2.68	522.00	2.20
Mathematica	907.77	2.65	374.00	1.27
Giac	919.63	3.97	397.50	1.58
Mupad	1064.22	3.40	647.00	2.81
Sympy	1071.65	4.28	869.50	4.35
Reduce	1362.80	5.07	801.00	3.59
Maxima	2313.22	7.51	1433.50	4.95

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

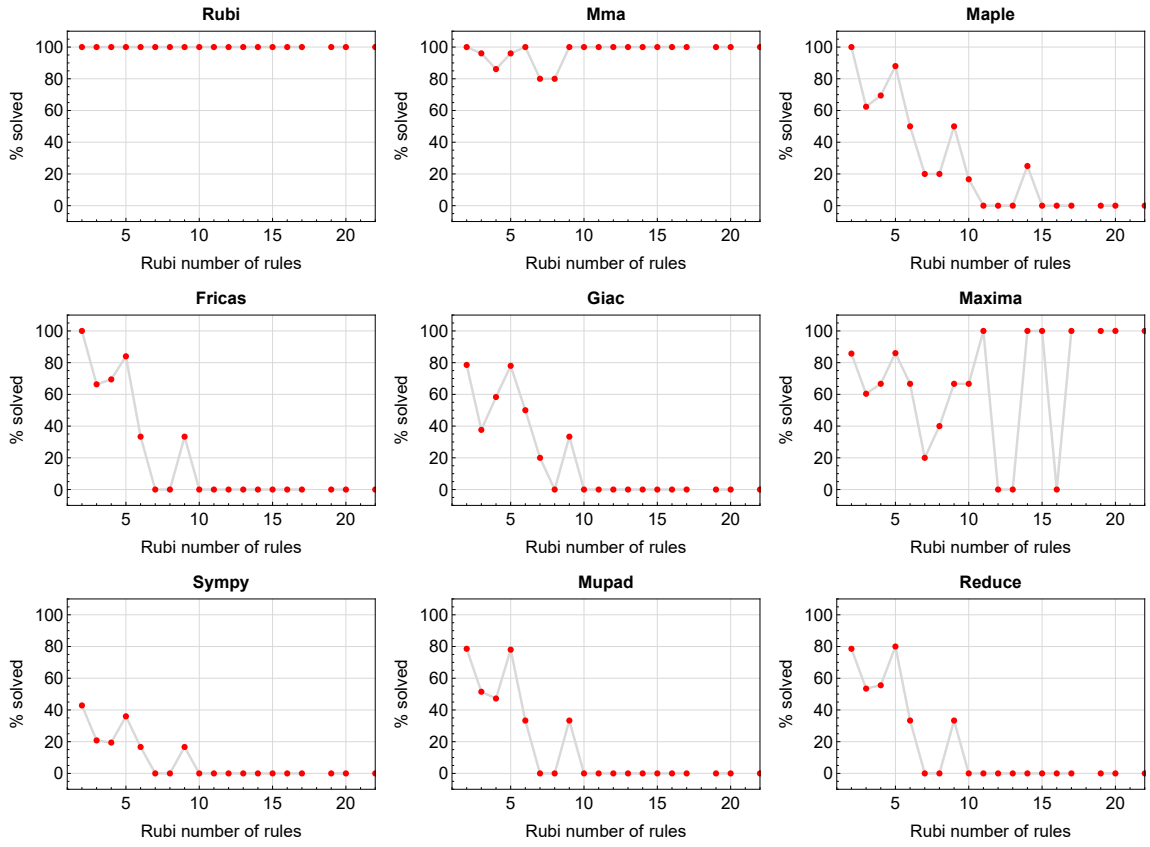


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

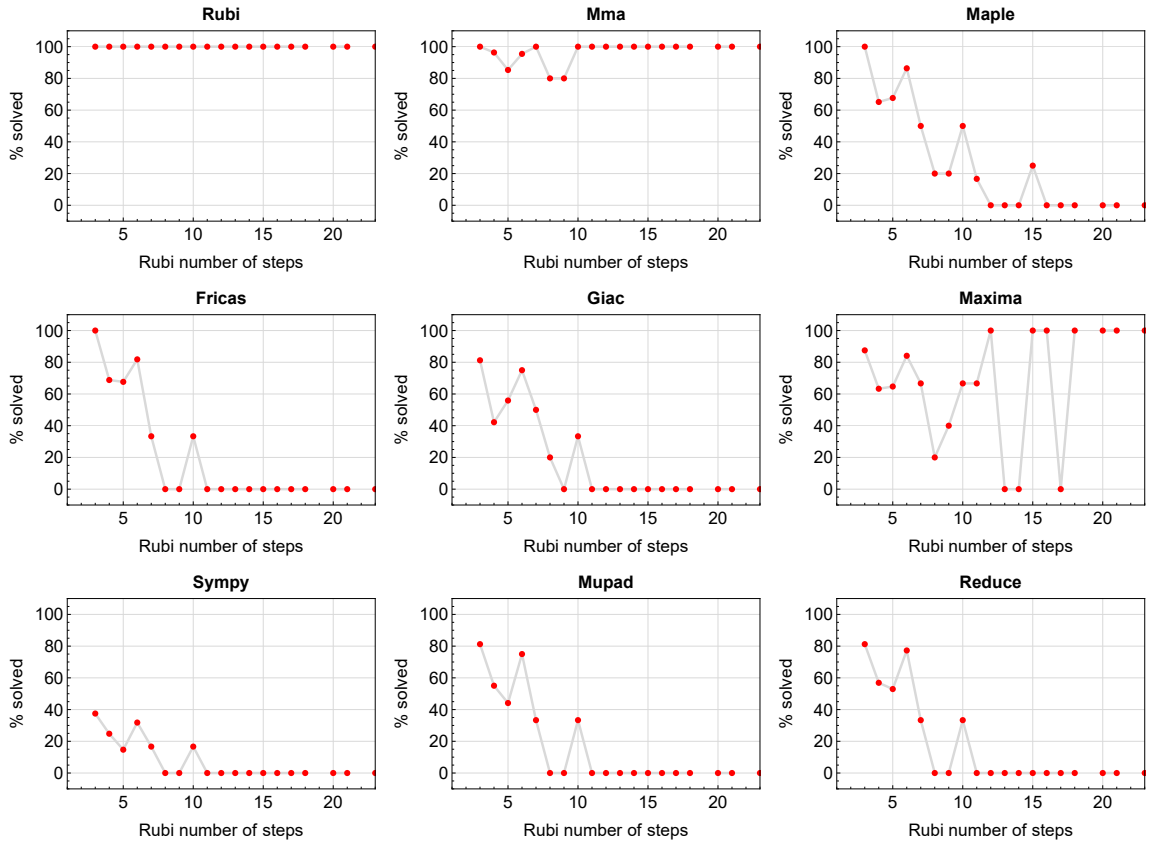


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

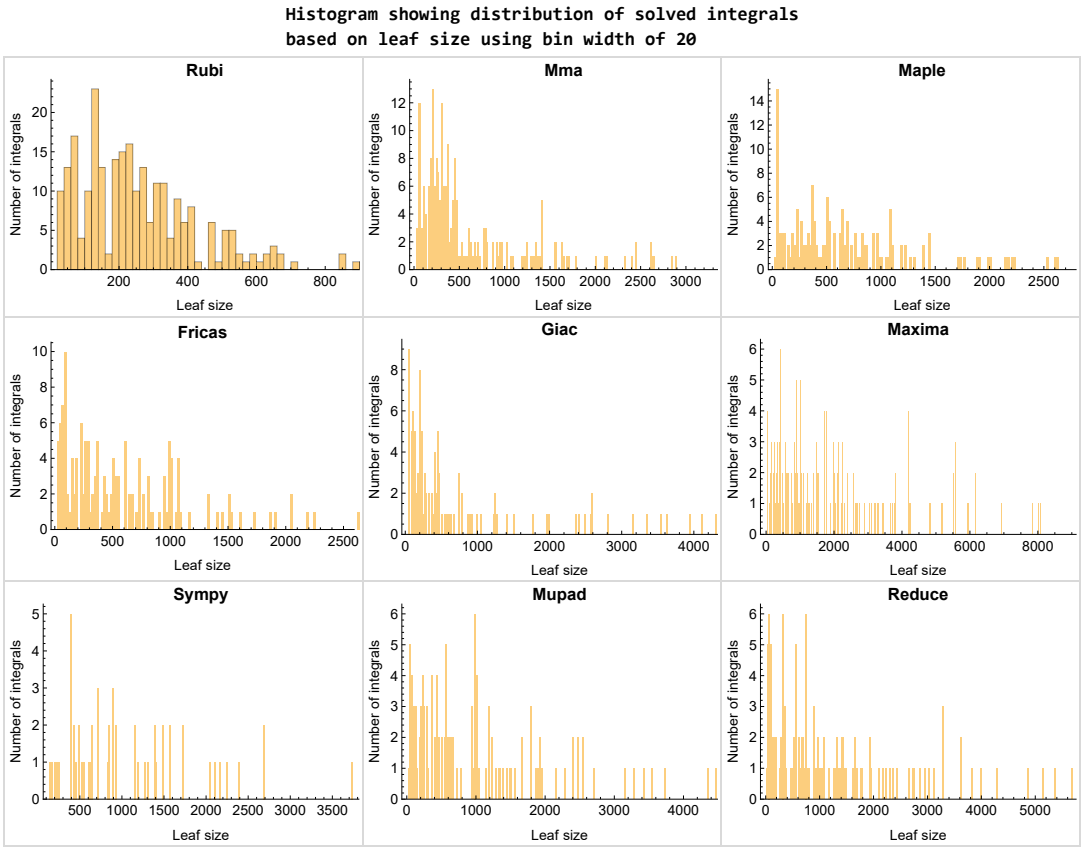


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

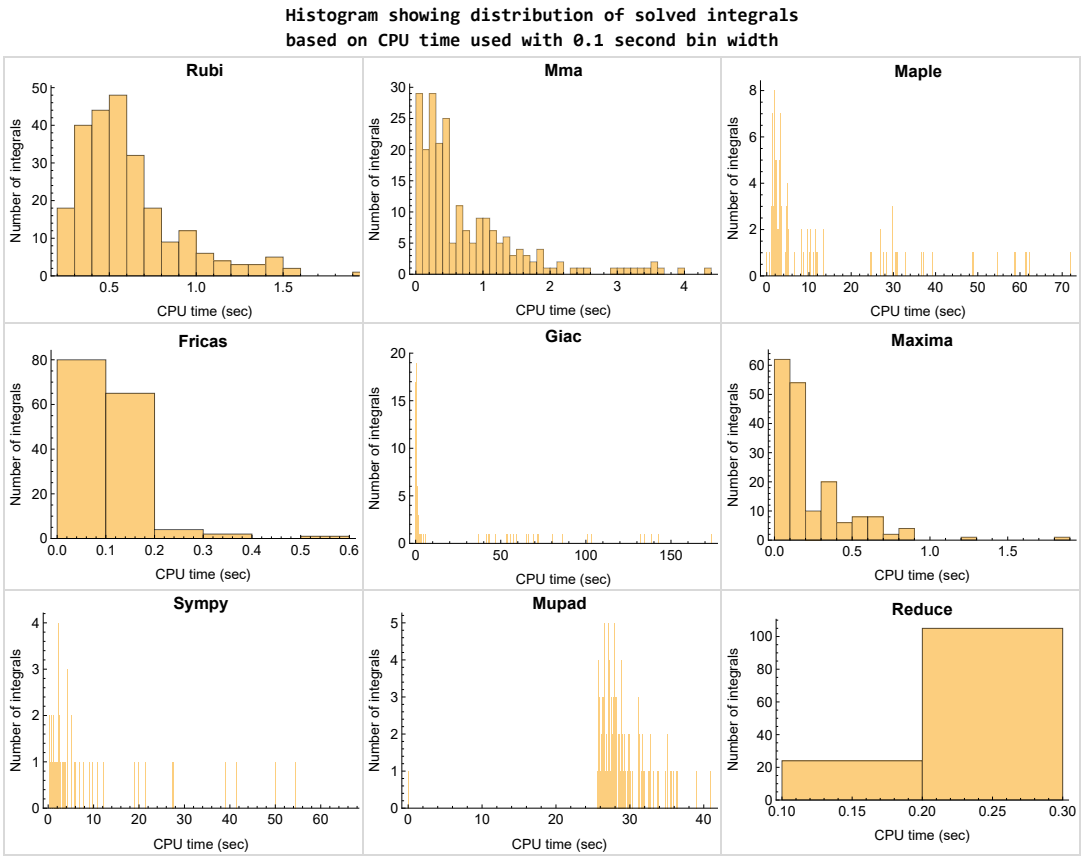


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

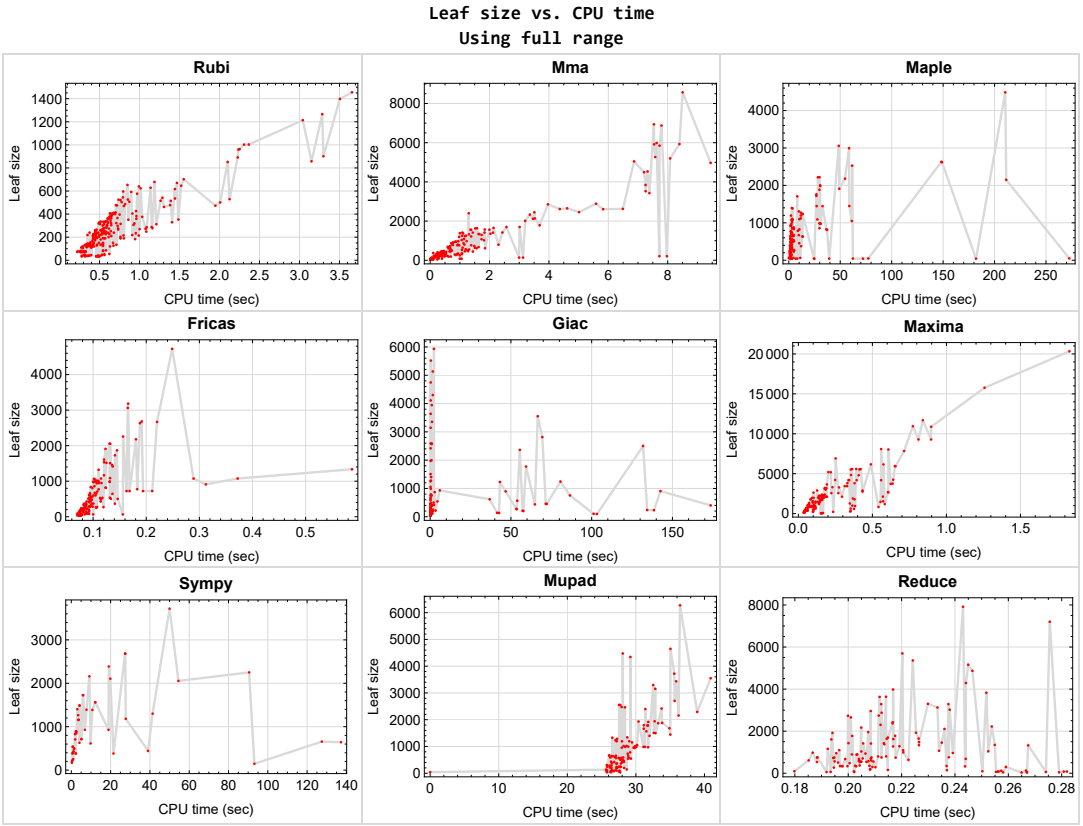


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{253, 254}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {88, 96, 104, 190, 198, 206, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 249, 250, 251, 252, 259, 260, 261}

Mathematica {}

Maple {252, 261}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

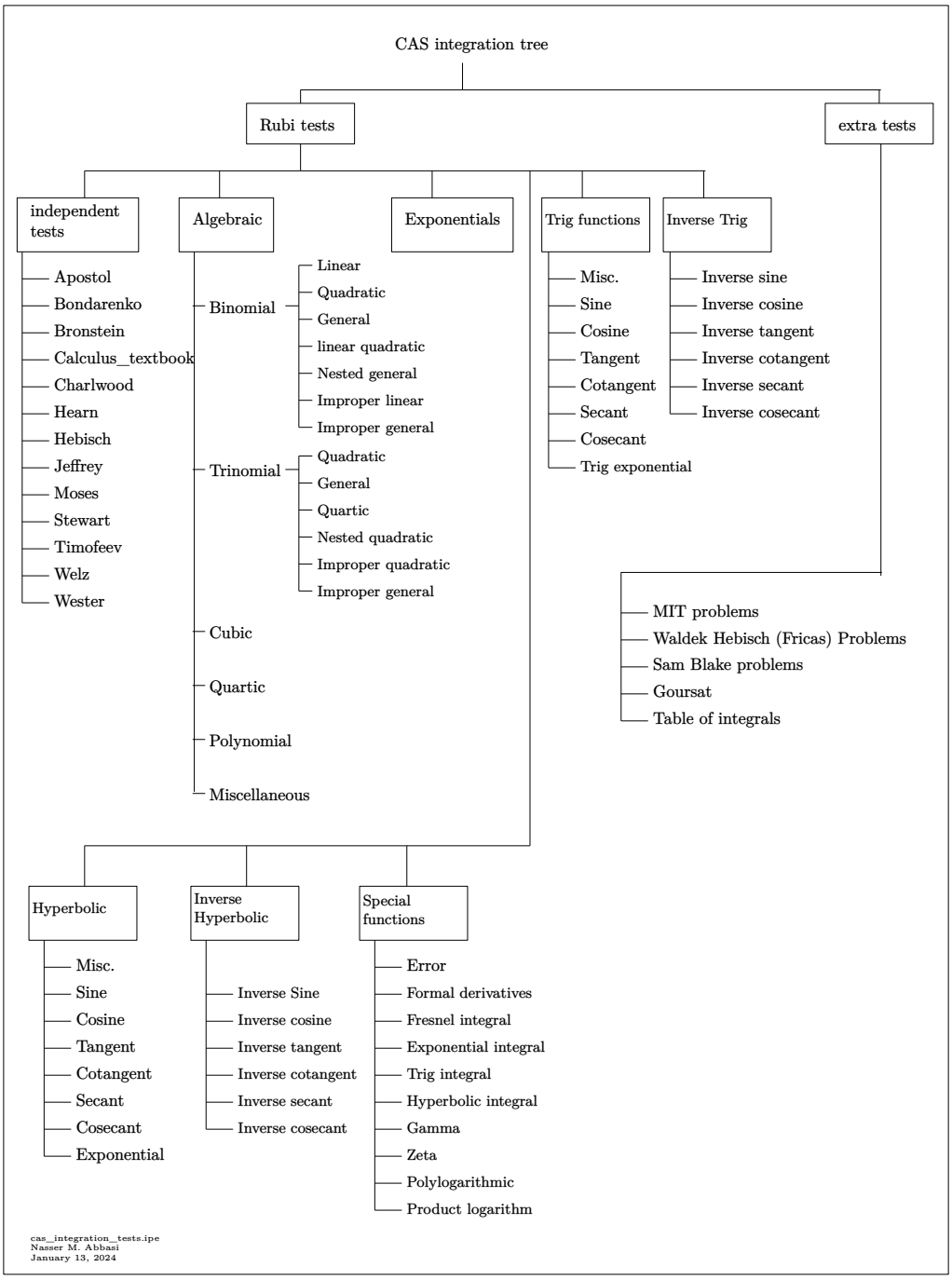
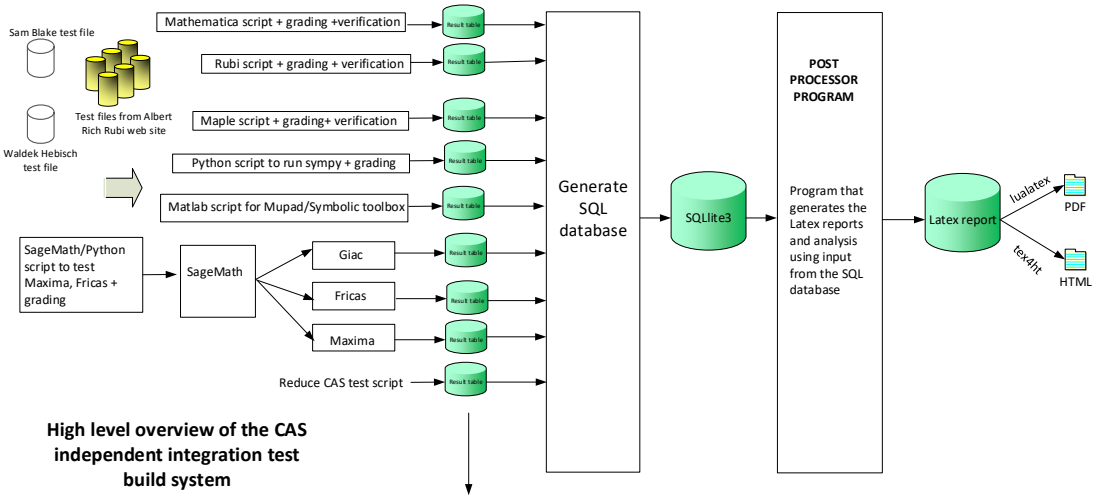


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
January 13, 2024
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 34
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2.1 List of integrals sorted by grade for each CAS

Rubi 34
Mma 35
Maple 35
Fricas 36
Maxima 36
Giac 37
Mupad 38
Sympy 38
Reduce 39

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 255, 256, 257, 258, 259, 260, 261 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 39, 40, 41, 42, 47, 48, 50, 55, 56, 58, 65, 66, 67, 75, 76, 77, 84, 85, 87, 88, 89, 90, 91, 96, 97, 98, 99, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 145, 146, 151, 152, 154, 159, 160, 162, 169, 170, 171, 179, 180, 181, 186, 187, 189, 190, 212, 213, 214, 218, 219, 220, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248 }

B grade { 7, 17, 18, 28, 29, 30, 49, 57, 59, 60, 64, 68, 69, 70, 74, 78, 79, 80, 86, 92, 93, 94, 100, 101, 114, 124, 125, 153, 161, 163, 164, 168, 172, 173, 174, 178, 182, 183, 184, 185, 188, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 251, 252, 255, 256, 257, 258, 260, 261 }

C grade { 35, 36, 37, 38, 43, 44, 45, 46, 51, 52, 53, 54, 61, 62, 63, 71, 72, 73, 81, 82, 83, 95, 102, 103, 139, 140, 141, 142, 147, 148, 149, 150, 155, 156, 157, 158, 165, 166, 167, 175, 176, 177, 197, 204, 205 }

F normal fail { 210, 211, 215, 216, 217, 221, 222, 223, 226, 227, 240, 249, 250, 259 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 3, 4, 7, 8, 9, 11, 12, 13, 18, 19, 30, 34, 35, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 95, 96, 103, 139, 140, 146, 147, 154, 197, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 245, 255, 256, 257, 258 }

B grade { 1, 2, 5, 6, 10, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 39, 40, 49, 61, 62, 63, 71, 72, 73, 81, 82, 83, 87, 88, 89, 90, 91, 97, 98, 99, 102, 104, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 124, 125, 126, 127, 128, 129, 130, 141, 142, 148, 149, 150, 153, 155, 156, 157, 158, 165, 166, 167, 175, 176, 177, 190, 191, 192, 193, 198, 199, 200, 201, 204, 205, 206, 207, 208, 209, 213, 214, 219, 220 }

C grade { 252, 261 }

F normal fail { 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 84, 85, 86, 92, 93, 94, 100, 101, 112, 113, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 145, 151, 152, 159, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 194, 195, 196, 202, 203, 210, 211, 212,

215, 216, 217, 218, 221, 222, 223, 226, 227, 236, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 259, 260 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 3, 4, 10, 11, 12, 22, 35, 36, 37, 38, 42, 43, 44, 45, 50, 51, 52, 89, 90, 91, 95, 96, 97, 98, 103, 104, 105, 139, 140, 141, 146, 147, 148, 154, 155, 197, 198, 215, 216, 220, 221, 222, 224, 225, 230, 231, 232, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248, 255, 256, 257, 258 }

B grade { 1, 2, 7, 8, 9, 13, 17, 18, 19, 20, 21, 23, 28, 29, 30, 46, 49, 53, 54, 61, 62, 63, 71, 72, 73, 81, 82, 83, 88, 99, 102, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 124, 125, 126, 127, 128, 129, 130, 142, 149, 150, 153, 156, 157, 158, 165, 166, 167, 175, 176, 177, 190, 191, 192, 193, 199, 200, 201, 204, 205, 206, 207, 208, 209, 212, 213, 214, 217, 218, 219, 223, 228, 229, 233 }

C grade { }

F normal fail { 5, 6, 14, 15, 16, 24, 25, 26, 27, 31, 32, 33, 34, 39, 40, 41, 47, 48, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 84, 85, 86, 87, 92, 93, 94, 100, 101, 112, 113, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 145, 151, 152, 159, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 194, 195, 196, 202, 203, 210, 211, 226, 227, 240, 249, 250, 251, 252, 259, 260, 261 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 4, 5, 14, 33, 42, 50, 111, 112, 146, 154, 231, 232, 237, 238, 239, 245 }

B grade { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 61, 62, 63, 64, 65, 66, 67, 71, 72, 73, 74, 75, 76, 77, 81, 82, 83, 88, 89, 90, 91, 95, 96, 97, 98, 99, 102, 103, 104, 105, 106, 107, 108, 109, 110, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 153, 155, 156, 157, 158, 159, 160, 161, 162, 165, 166, 167, 168, 169, 170, 171, 175,

176, 177, 178, 179, 180, 181, 190, 191, 192, 193, 197, 198, 199, 200, 201, 204, 205, 206, 207, 208, 209, 228, 229, 230, 233, 255, 256, 257, 258 }

C grade { }

F normal fail { 6, 16, 27, 34, 41, 48, 59, 60, 68, 69, 70, 78, 79, 80, 84, 85, 86, 87, 92, 93, 94, 100, 101, 113, 123, 134, 138, 145, 152, 163, 164, 172, 173, 174, 182, 183, 184, 185, 186, 187, 188, 189, 194, 195, 196, 202, 203, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 259, 260, 261 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 7, 8, 9, 17, 18, 19, 28, 29, 30, 37, 38, 42, 43, 44, 45, 46, 49, 50, 51, 61, 62, 63, 71, 72, 73, 81, 82, 83, 90, 91, 95, 96, 97, 98, 99, 102, 103, 104, 114, 115, 116, 124, 125, 126, 139, 141, 142, 146, 147, 148, 149, 150, 153, 154, 155, 165, 166, 167, 175, 176, 177, 197, 198, 204, 205, 206, 224, 225, 231, 234, 235, 236, 237, 238, 239 }

B grade { 1, 2, 3, 4, 10, 11, 12, 13, 20, 21, 22, 23, 31, 32, 33, 34, 35, 39, 40, 41, 88, 108, 109, 110, 111, 117, 118, 119, 120, 127, 128, 129, 130, 136, 137, 138, 145, 190, 232, 233, 245 }

C grade { }

F normal fail { 5, 6, 14, 15, 16, 24, 25, 26, 27, 36, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 84, 85, 86, 87, 89, 92, 93, 94, 100, 101, 105, 106, 107, 112, 113, 121, 122, 123, 131, 132, 133, 134, 140, 151, 152, 156, 157, 158, 161, 162, 163, 164, 171, 172, 173, 181, 182, 186, 187, 188, 189, 191, 195, 196, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 228, 229, 230, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 255, 256, 257, 258, 259, 260, 261 }

F(-1) timedout fail { 135, 143, 144, 159, 160, 168, 169, 170, 174, 178, 179, 180, 183, 184, 185, 192, 193, 194, 199, 200, 201, 202, 203 }

F(-2) exception fail { 226, 227 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 35, 36, 37, 38, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 61, 62, 63, 71, 72, 73, 81, 82, 83, 88, 89, 90, 91, 95, 96, 97, 98, 99, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 124, 125, 126, 127, 128, 129, 130, 139, 140, 141, 142, 146, 147, 148, 149, 150, 153, 154, 155, 156, 157, 158, 165, 166, 167, 175, 176, 177, 190, 191, 192, 193, 197, 198, 200, 201, 204, 205, 206, 207, 208, 209, 228, 229, 230, 231, 232, 233, 237, 238, 239, 245 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 14, 15, 16, 24, 25, 26, 27, 31, 32, 33, 34, 39, 40, 41, 47, 48, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 84, 85, 86, 87, 92, 93, 94, 100, 101, 112, 113, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 145, 151, 152, 159, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 194, 195, 196, 199, 202, 203, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 255, 256, 257, 258, 259, 260, 261 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 35, 36, 37, 38, 42, 43, 44, 45, 49, 50, 51, 52, 53, 61, 62, 63, 71, 72, 88, 89, 90, 91, 95, 96, 97, 98, 102, 103, 104, 105, 106, 110, 111, 120, 146, 245 }

C grade { }

F normal fail { 5, 14, 16, 24, 26, 27, 31, 32, 33, 34, 47, 48, 59, 68, 70, 78, 80, 84, 85, 86, 87, 100, 101, 112, 121, 123, 131, 133, 134, 135, 136, 137, 138, 139, 151, 152, 160, 161, 162, 163, 165, 166, 170, 171, 172, 174, 175, 181, 182, 186, 187, 188, 189, 190, 197, 203, 204, 205 }

F(-1) timedout fail { 6, 15, 25, 28, 29, 30, 39, 40, 41, 46, 54, 55, 56, 57, 58, 60, 64, 65, 66, 67, 69, 73, 74, 75, 76, 77, 79, 81, 82, 83, 92, 93, 94, 99, 107, 108, 109, 113, 114, 115, 116, 117, 118, 119, 122, 124, 125, 126, 127, 128, 129, 130, 132, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 153, 154, 155, 156, 157, 158, 159, 164, 167, 168, 169, 173, 176, 177, 178, 179, 180, 183, 184, 185, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 206, 207, 208, 209, 210,

211, 217, 218, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261 }

F(-2) exception fail { 212, 213, 214, 215, 216, 219, 220, 221, 222 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 35, 36, 37, 38, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 61, 62, 63, 71, 72, 73, 81, 82, 83, 88, 89, 90, 91, 95, 96, 97, 98, 99, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 124, 125, 126, 127, 128, 129, 130, 139, 140, 141, 142, 146, 147, 148, 149, 150, 153, 154, 155, 156, 157, 158, 165, 166, 167, 175, 176, 177, 190, 191, 192, 193, 197, 198, 199, 200, 201, 204, 205, 206, 207, 208, 209, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 245 }

C grade { }

F normal fail { 5, 6, 14, 15, 16, 24, 25, 26, 27, 31, 32, 33, 34, 39, 40, 41, 47, 48, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 84, 85, 86, 87, 92, 93, 94, 100, 101, 112, 113, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 145, 151, 152, 159, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 194, 195, 196, 202, 203, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 226, 227, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 255, 256, 257, 258, 259, 260, 261 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	189	261	574	1022	504	1158	3637	757	1195
N.S.	1	0.89	1.23	2.71	4.82	2.38	5.46	17.16	3.57	5.64
time (sec)	N/A	0.435	0.260	1.483	0.065	0.133	3.785	0.309	0.238	27.203

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	165	217	396	671	370	850	2418	541	638
N.S.	1	0.92	1.21	2.20	3.73	2.06	4.72	13.43	3.01	3.54
time (sec)	N/A	0.410	0.181	1.260	0.052	0.101	2.504	0.283	0.204	26.501

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	137	181	206	361	226	498	1254	326	282
N.S.	1	0.98	1.29	1.47	2.58	1.61	3.56	8.96	2.33	2.01
time (sec)	N/A	0.369	0.272	1.116	0.042	0.089	1.513	0.236	0.244	26.197

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	77	70	108	144	127	253	627	181	126
N.S.	1	0.95	0.86	1.33	1.78	1.57	3.12	7.74	2.23	1.56
time (sec)	N/A	0.255	0.037	1.027	0.036	0.082	0.943	0.199	0.208	26.502

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	126	164	417	241	0	0	0	101	0
N.S.	1	0.95	1.23	3.14	1.81	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.673	0.140	2.937	0.100	0.000	0.000	0.000	0.205	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	134	175	370	0	0	0	0	449	0
N.S.	1	0.94	1.23	2.61	0.00	0.00	0.00	0.00	3.16	0.00
time (sec)	N/A	0.512	0.179	2.986	0.000	0.000	0.000	0.000	0.204	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	72	208	153	570	177	384	134	335	197
N.S.	1	0.85	2.45	1.80	6.71	2.08	4.52	1.58	3.94	2.32
time (sec)	N/A	0.287	0.187	1.353	0.054	0.082	2.298	0.216	0.200	27.159

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	136	187	279	933	363	629	270	894	361
N.S.	1	0.79	1.08	1.61	5.39	2.10	3.64	1.56	5.17	2.09
time (sec)	N/A	0.350	0.407	1.501	0.070	0.083	4.150	0.268	0.201	27.821

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	201	210	390	1386	602	928	426	954	590
N.S.	1	0.75	0.78	1.45	5.15	2.24	3.45	1.58	3.55	2.19
time (sec)	N/A	0.423	0.489	2.093	0.097	0.092	6.895	0.336	0.196	28.155

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	375	429	925	1789	724	1727	4746	1036	2473
N.S.	1	0.89	1.01	2.19	4.23	1.71	4.08	11.22	2.45	5.85
time (sec)	N/A	0.622	0.400	1.764	0.084	0.211	6.069	0.355	0.205	27.930

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	301	362	636	1200	534	1266	3144	754	1287
N.S.	1	0.89	1.07	1.89	3.56	1.58	3.76	9.33	2.24	3.82
time (sec)	N/A	0.539	0.281	1.520	0.072	0.132	3.484	0.317	0.209	27.254

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	227	216	396	671	367	850	1975	545	636
N.S.	1	0.95	0.90	1.66	2.81	1.54	3.56	8.26	2.28	2.66
time (sec)	N/A	0.421	0.209	1.286	0.053	0.141	2.299	0.281	0.205	26.626

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	105	97	206	280	223	491	1056	268	290
N.S.	1	0.89	0.82	1.75	2.37	1.89	4.16	8.95	2.27	2.46
time (sec)	N/A	0.289	0.043	1.184	0.041	0.089	1.344	0.237	0.195	26.442

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	307	252	698	518	0	0	0	191	0
N.S.	1	1.11	0.91	2.53	1.88	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.925	0.215	3.183	0.106	0.000	0.000	0.000	0.189	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	223	221	568	992	0	0	0	753	0
N.S.	1	0.90	0.89	2.30	4.02	0.00	0.00	0.00	3.05	0.00
time (sec)	N/A	0.523	0.259	3.061	0.112	0.000	0.000	0.000	0.207	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	203	244	522	0	0	0	0	1318	0
N.S.	1	0.88	1.06	2.27	0.00	0.00	0.00	0.00	5.73	0.00
time (sec)	N/A	0.762	0.349	3.106	0.000	0.000	0.000	0.000	0.219	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	74	315	185	1515	271	614	140	534	423
N.S.	1	0.83	3.54	2.08	17.02	3.04	6.90	1.57	6.00	4.75
time (sec)	N/A	0.312	0.371	1.645	0.099	0.093	9.795	0.279	0.188	28.258

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	138	454	319	2218	510	928	282	1274	647
N.S.	1	0.76	2.51	1.76	12.25	2.82	5.13	1.56	7.04	3.57
time (sec)	N/A	0.369	0.418	2.211	0.138	0.091	18.930	0.350	0.217	28.885

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	205	344	430	3029	807	1300	444	1323	941
N.S.	1	0.73	1.22	1.53	10.78	2.87	4.63	1.58	4.71	3.35
time (sec)	N/A	0.427	0.935	2.311	0.184	0.099	41.371	0.376	0.267	29.824

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	401	586	1194	2637	912	2161	5520	1347	4347
N.S.	1	0.88	1.28	2.61	5.77	2.00	4.73	12.08	2.95	9.51
time (sec)	N/A	0.661	0.681	2.127	0.102	0.312	9.198	0.429	0.255	29.202

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	327	429	925	1789	722	1727	4110	1040	2465
N.S.	1	0.88	1.16	2.49	4.82	1.95	4.65	11.08	2.80	6.64
time (sec)	N/A	0.562	0.378	1.679	0.083	0.194	5.844	0.364	0.252	28.411

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	253	261	568	1022	502	1158	2589	703	1192
N.S.	1	0.93	0.96	2.10	3.77	1.85	4.27	9.55	2.59	4.40
time (sec)	N/A	0.445	0.232	1.441	0.062	0.146	3.535	0.303	0.205	27.976

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	129	120	314	439	322	706	1506	477	566
N.S.	1	0.87	0.81	2.11	2.95	2.16	4.74	10.11	3.20	3.80
time (sec)	N/A	0.311	0.065	1.244	0.044	0.096	1.976	0.270	0.201	27.080

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	462	352	1086	850	0	0	0	295	0
N.S.	1	1.30	0.99	3.05	2.39	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	1.306	0.309	3.314	0.113	0.000	0.000	0.000	0.216	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	326	374	863	1501	0	0	0	1084	0
N.S.	1	0.87	1.00	2.31	4.02	0.00	0.00	0.00	2.91	0.00
time (sec)	N/A	0.609	0.436	3.302	0.117	0.000	0.000	0.000	0.233	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	293	314	732	2302	0	0	0	0	0
N.S.	1	0.85	0.91	2.12	6.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.587	0.438	3.217	0.166	0.000	0.000	0.000	0.236	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	270	308	685	0	0	0	0	0	0
N.S.	1	0.87	0.99	2.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.001	0.493	3.256	0.000	0.000	0.000	0.000	0.378	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	74	427	185	3107	355	0	140	727	780
N.S.	1	0.83	4.80	2.08	34.91	3.99	0.00	1.57	8.17	8.76
time (sec)	N/A	0.320	0.538	2.145	0.177	0.086	0.000	0.393	0.194	29.163

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	138	608	357	4218	644	0	282	1647	1053
N.S.	1	0.76	3.36	1.97	23.30	3.56	0.00	1.56	9.10	5.82
time (sec)	N/A	0.373	0.616	2.264	0.249	0.136	0.000	0.394	0.226	30.184

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	205	642	481	5524	991	0	444	1696	1396
N.S.	1	0.73	2.28	1.71	19.66	3.53	0.00	1.58	6.04	4.97
time (sec)	N/A	0.433	1.093	2.948	0.354	0.104	0.000	0.443	0.215	31.986

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	298	354	1105	790	0	0	3552	298	0
N.S.	1	1.18	1.40	4.38	3.13	0.00	0.00	14.10	1.18	0.00
time (sec)	N/A	0.726	0.332	3.352	0.115	0.000	0.000	66.588	0.230	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	225	254	657	477	0	0	2364	195	0
N.S.	1	1.14	1.28	3.32	2.41	0.00	0.00	11.94	0.98	0.00
time (sec)	N/A	0.577	0.208	3.199	0.099	0.000	0.000	55.507	0.215	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	145	162	332	221	0	0	1230	105	0
N.S.	1	1.16	1.30	2.66	1.77	0.00	0.00	9.84	0.84	0.00
time (sec)	N/A	0.426	0.127	2.961	0.099	0.000	0.000	43.187	0.203	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	95	146	0	0	0	617	45	0
N.S.	1	1.00	1.17	1.80	0.00	0.00	0.00	7.62	0.56	0.00
time (sec)	N/A	0.542	0.047	2.846	0.000	0.000	0.000	36.838	0.186	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	207	72	172	60	170	104	56	69
N.S.	1	1.00	4.70	1.64	3.91	1.36	3.86	2.36	1.27	1.57
time (sec)	N/A	0.302	0.135	1.389	0.040	0.074	0.303	0.133	0.192	27.498

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	127	292	230	424	144	386	0	297	241
N.S.	1	0.73	1.69	1.33	2.45	0.83	2.23	0.00	1.72	1.39
time (sec)	N/A	0.412	0.335	1.776	0.057	0.087	0.606	0.000	0.209	27.533

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	194	418	365	885	349	889	139	679	545
N.S.	1	0.76	1.64	1.43	3.47	1.37	3.49	0.55	2.66	2.14
time (sec)	N/A	0.471	0.426	1.990	0.091	0.087	2.615	41.449	0.204	28.843

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	262	492	500	1469	611	1392	270	1284	970
N.S.	1	0.70	1.32	1.34	3.94	1.64	3.73	0.72	3.44	2.60
time (sec)	N/A	0.536	0.732	2.427	0.147	0.093	7.732	53.522	0.219	31.784

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	304	359	846	1341	0	0	2814	1103	0
N.S.	1	0.89	1.05	2.48	3.93	0.00	0.00	8.25	3.23	0.00
time (sec)	N/A	0.647	0.482	3.293	0.115	0.000	0.000	69.450	0.232	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	227	239	532	886	0	0	1774	773	0
N.S.	1	0.87	0.92	2.05	3.41	0.00	0.00	6.82	2.97	0.00
time (sec)	N/A	0.538	0.269	3.063	0.100	0.000	0.000	59.344	0.214	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	146	175	295	0	0	0	895	464	0
N.S.	1	0.91	1.09	1.84	0.00	0.00	0.00	5.59	2.90	0.00
time (sec)	N/A	0.376	0.198	2.923	0.000	0.000	0.000	46.757	0.200	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	73	104	82	134	88	231	115	144	106
N.S.	1	0.74	1.06	0.84	1.37	0.90	2.36	1.17	1.47	1.08
time (sec)	N/A	0.222	0.058	1.148	0.043	0.076	0.634	0.186	0.237	26.593

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	109	292	231	421	151	386	233	301	247
N.S.	1	0.70	1.87	1.48	2.70	0.97	2.47	1.49	1.93	1.58
time (sec)	N/A	0.396	0.262	1.710	0.062	0.074	0.597	0.199	0.259	27.667

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	182	324	366	859	334	828	133	894	415
N.S.	1	0.70	1.24	1.40	3.29	1.28	3.17	0.51	3.43	1.59
time (sec)	N/A	0.417	0.418	1.884	0.082	0.082	1.881	42.641	0.203	27.994

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	254	453	506	1721	664	1562	276	1775	984
N.S.	1	0.73	1.31	1.46	4.97	1.92	4.51	0.80	5.13	2.84
time (sec)	N/A	0.520	0.651	3.593	0.161	0.094	12.134	53.396	0.201	31.107

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	313	520	640	2560	1019	0	436	2879	1679
N.S.	1	0.68	1.14	1.40	5.60	2.23	0.00	0.95	6.30	3.67
time (sec)	N/A	0.507	1.229	3.399	0.270	0.105	0.000	64.853	0.213	34.899

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	300	317	681	2037	0	0	0	0	0
N.S.	1	0.83	0.88	1.89	5.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.595	0.428	3.198	0.162	0.000	0.000	0.000	0.244	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	217	245	463	0	0	0	0	1347	0
N.S.	1	0.86	0.98	1.84	0.00	0.00	0.00	0.00	5.37	0.00
time (sec)	N/A	0.455	0.352	3.118	0.000	0.000	0.000	0.000	0.239	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	72	207	174	567	185	382	132	339	198
N.S.	1	0.85	2.44	2.05	6.67	2.18	4.49	1.55	3.99	2.33
time (sec)	N/A	0.262	0.183	1.339	0.056	0.076	2.277	0.219	0.197	28.294

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	138	111	221	255	221	422	236	577	208
N.S.	1	0.96	0.77	1.53	1.77	1.53	2.93	1.64	4.01	1.44
time (sec)	N/A	0.336	0.122	1.354	0.045	0.081	1.103	0.210	0.204	27.255

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	182	418	367	885	355	889	442	674	545
N.S.	1	0.75	1.72	1.51	3.64	1.46	3.66	1.82	2.77	2.24
time (sec)	N/A	0.444	0.463	2.005	0.091	0.084	2.569	0.226	0.199	29.274

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	253	452	506	1721	672	1562	0	1782	983
N.S.	1	0.69	1.24	1.39	4.72	1.84	4.28	0.00	4.88	2.69
time (sec)	N/A	0.452	0.687	3.543	0.157	0.103	12.232	0.000	0.218	31.329

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	319	533	649	2380	1011	2106	0	2732	1443
N.S.	1	0.69	1.15	1.40	5.14	2.18	4.55	0.00	5.90	3.12
time (sec)	N/A	0.488	1.070	2.819	0.177	0.109	19.848	0.000	0.200	35.022

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	388	637	787	3816	1509	0	0	3982	2291
N.S.	1	0.69	1.13	1.40	6.78	2.68	0.00	0.00	7.07	4.07
time (sec)	N/A	0.638	1.514	8.757	0.379	0.132	0.000	0.000	0.217	38.946

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	539	617	905	0	3186	0	0	0	0	0
N.S.	1	1.14	1.68	0.00	5.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.442	0.766	0.000	0.190	0.000	0.000	0.000	0.262	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	450	516	680	0	2243	0	0	0	1657	0
N.S.	1	1.15	1.51	0.00	4.98	0.00	0.00	0.00	3.68	0.00
time (sec)	N/A	1.263	0.597	0.000	0.155	0.000	0.000	0.000	0.245	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	343	405	869	0	1252	0	0	0	940	0
N.S.	1	1.18	2.53	0.00	3.65	0.00	0.00	0.00	2.74	0.00
time (sec)	N/A	0.912	0.708	0.000	0.138	0.000	0.000	0.000	0.233	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	203	227	205	0	633	0	0	0	578	0
N.S.	1	1.12	1.01	0.00	3.12	0.00	0.00	0.00	2.85	0.00
time (sec)	N/A	0.625	0.218	0.000	0.117	0.000	0.000	0.000	0.261	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	286	272	1214	0	0	0	0	0	178	0
N.S.	1	0.95	4.24	0.00	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	1.076	1.383	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	241	218	1407	0	0	0	0	0	859	0
N.S.	1	0.90	5.84	0.00	0.00	0.00	0.00	0.00	3.56	0.00
time (sec)	N/A	0.909	2.116	0.000	0.000	0.000	0.000	0.000	0.264	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	114	765	330	1987	289	714	206	684	469
N.S.	1	0.81	5.43	2.34	14.09	2.05	5.06	1.46	4.85	3.33
time (sec)	N/A	0.341	0.981	1.468	0.172	0.092	5.145	0.264	0.244	28.198

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	218	1032	643	3282	601	1387	455	1481	955
N.S.	1	0.76	3.60	2.24	11.44	2.09	4.83	1.59	5.16	3.33
time (sec)	N/A	0.446	1.064	1.655	0.275	0.095	10.724	0.334	0.227	29.658

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	334	1255	926	4808	985	2251	744	2224	1870
N.S.	1	0.75	2.82	2.08	10.80	2.21	5.06	1.67	5.00	4.20
time (sec)	N/A	0.548	1.209	2.359	0.408	0.100	90.492	0.389	0.254	33.251

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	711	963	1559	0	5178	0	0	0	0	0
N.S.	1	1.35	2.19	0.00	7.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.251	1.360	0.000	0.219	0.000	0.000	0.000	0.253	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	761	852	1194	0	3656	0	0	0	0	0
N.S.	1	1.12	1.57	0.00	4.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.103	0.976	0.000	0.193	0.000	0.000	0.000	0.228	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	589	670	677	0	2259	0	0	0	1663	0
N.S.	1	1.14	1.15	0.00	3.84	0.00	0.00	0.00	2.82	0.00
time (sec)	N/A	1.468	0.597	0.000	0.169	0.000	0.000	0.000	0.221	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	334	360	287	0	1202	0	0	0	962	0
N.S.	1	1.08	0.86	0.00	3.60	0.00	0.00	0.00	2.88	0.00
time (sec)	N/A	0.960	0.243	0.000	0.140	0.000	0.000	0.000	0.224	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	535	501	2615	0	0	0	0	0	314	0
N.S.	1	0.94	4.89	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	2.008	4.366	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	442	366	2649	0	0	0	0	0	1397	0
N.S.	1	0.83	5.99	0.00	0.00	0.00	0.00	0.00	3.16	0.00
time (sec)	N/A	0.724	4.615	0.000	0.000	0.000	0.000	0.000	0.263	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	387	329	3426	0	0	0	0	0	0	0
N.S.	1	0.85	8.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.407	7.384	0.000	0.000	0.000	0.000	0.000	0.314	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	119	1352	367	5532	444	1182	219	1067	1153
N.S.	1	0.81	9.20	2.50	37.63	3.02	8.04	1.49	7.26	7.84
time (sec)	N/A	0.392	2.028	1.723	0.412	0.094	27.695	0.343	0.234	29.440

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	220	1703	685	8031	837	2055	479	2107	1940
N.S.	1	0.74	5.70	2.29	26.86	2.80	6.87	1.60	7.05	6.49
time (sec)	N/A	0.497	3.013	2.428	0.608	0.096	54.500	0.404	0.236	32.738

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	336	2010	968	10880	1323	0	780	3022	3434
N.S.	1	0.73	4.34	2.09	23.50	2.86	0.00	1.68	6.53	7.42
time (sec)	N/A	0.579	3.200	2.506	0.897	0.110	0.000	0.429	0.238	35.876

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1089	1398	2330	0	6921	0	0	0	0	0
N.S.	1	1.28	2.14	0.00	6.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.506	3.359	0.000	0.252	0.000	0.000	0.000	5.235	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	908	1213	1555	0	5196	0	0	0	0	0
N.S.	1	1.34	1.71	0.00	5.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.042	1.381	0.000	0.202	0.000	0.000	0.000	0.753	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	730	959	901	0	3218	0	0	0	0	0
N.S.	1	1.31	1.23	0.00	4.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.236	0.765	0.000	0.179	0.000	0.000	0.000	0.231	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	420	515	389	0	1789	0	0	0	1533	0
N.S.	1	1.23	0.93	0.00	4.26	0.00	0.00	0.00	3.65	0.00
time (sec)	N/A	1.391	0.361	0.000	0.145	0.000	0.000	0.000	0.219	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	712	857	5044	0	0	0	0	0	464	0
N.S.	1	1.20	7.08	0.00	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	3.150	6.868	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	692	577	5266	0	0	0	0	0	0	0
N.S.	1	0.83	7.61	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.970	7.582	0.000	0.000	0.000	0.000	0.000	0.248	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	604	486	5989	0	0	0	0	0	0	0
N.S.	1	0.80	9.92	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.820	7.628	0.000	0.000	0.000	0.000	0.000	0.348	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	119	2401	367	11688	559	0	218	1432	1565
N.S.	1	0.81	16.33	2.50	79.51	3.80	0.00	1.48	9.74	10.65
time (sec)	N/A	0.385	1.299	2.362	0.841	0.099	0.000	0.454	0.200	31.532

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	220	2456	722	15765	1045	0	479	2741	3720
N.S.	1	0.74	8.21	2.41	52.73	3.49	0.00	1.60	9.17	12.44
time (sec)	N/A	0.504	5.015	2.503	1.258	0.104	0.000	0.430	0.211	35.595

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	336	2606	1019	20330	1610	0	780	3828	6275
N.S.	1	0.73	5.63	2.20	43.91	3.48	0.00	1.68	8.27	13.55
time (sec)	N/A	0.598	5.826	3.197	1.831	0.121	0.000	0.501	0.252	36.440

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	718	629	1020	0	0	0	0	0	471	0
N.S.	1	0.88	1.42	0.00	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	1.138	1.812	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	536	470	727	0	0	0	0	0	323	0
N.S.	1	0.88	1.36	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.804	1.485	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	283	260	1227	0	0	0	0	0	188	0
N.S.	1	0.92	4.34	0.00	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.550	1.172	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	120	252	338	0	0	0	0	84	0
N.S.	1	0.94	1.98	2.66	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.432	0.996	3.110	0.000	0.000	0.000	0.000	0.213	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	37	79	106	397	87	206	132	85	96
N.S.	1	0.84	1.80	2.41	9.02	1.98	4.68	3.00	1.93	2.18
time (sec)	N/A	0.357	0.400	1.497	0.057	0.071	0.351	0.147	0.202	27.621

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	136	186	400	1008	231	541	0	537	419
N.S.	1	0.74	1.02	2.19	5.51	1.26	2.96	0.00	2.93	2.29
time (sec)	N/A	0.470	0.695	1.795	0.104	0.079	0.719	0.000	0.207	28.036

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	248	318	767	2115	540	1488	211	1426	981
N.S.	1	0.72	0.93	2.24	6.17	1.57	4.34	0.62	4.16	2.86
time (sec)	N/A	0.578	1.075	2.230	0.193	0.095	4.166	57.212	0.213	29.849

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	507	364	442	1076	3434	940	2388	451	2371	1882
N.S.	1	0.72	0.87	2.12	6.77	1.85	4.71	0.89	4.68	3.71
time (sec)	N/A	0.647	1.438	2.638	0.329	0.107	19.092	72.167	0.217	33.826

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	722	592	5199	0	0	0	0	0	0	0
N.S.	1	0.82	7.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.897	8.082	0.000	0.000	0.000	0.000	0.000	0.251	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	469	380	2622	0	0	0	0	0	1446	0
N.S.	1	0.81	5.59	0.00	0.00	0.00	0.00	0.00	3.08	0.00
time (sec)	N/A	0.642	6.491	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	261	239	1412	0	0	0	0	0	905	0
N.S.	1	0.92	5.41	0.00	0.00	0.00	0.00	0.00	3.47	0.00
time (sec)	N/A	0.485	1.831	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	111	315	182	416	155	432	173	332	222
N.S.	1	0.73	2.07	1.20	2.74	1.02	2.84	1.14	2.18	1.46
time (sec)	N/A	0.272	0.447	1.200	0.058	0.076	1.149	0.221	0.203	27.362

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	140	187	398	1004	236	539	339	565	423
N.S.	1	0.65	0.87	1.86	4.69	1.10	2.52	1.58	2.64	1.98
time (sec)	N/A	0.534	0.619	1.783	0.106	0.084	0.720	0.241	0.195	28.060

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	254	307	766	1995	515	1404	203	1957	731
N.S.	1	0.70	0.84	2.10	5.47	1.41	3.85	0.56	5.36	2.00
time (sec)	N/A	0.558	0.910	2.005	0.166	0.092	3.063	57.730	0.207	28.806

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	523	364	466	1080	4187	1005	2683	460	3290	1497
N.S.	1	0.70	0.89	2.07	8.01	1.92	5.13	0.88	6.29	2.86
time (sec)	N/A	0.624	1.275	3.477	0.394	0.108	27.334	71.573	0.238	32.857

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	682	475	613	1386	6160	1534	0	754	5157	2701
N.S.	1	0.70	0.90	2.03	9.03	2.25	0.00	1.11	7.56	3.96
time (sec)	N/A	0.718	1.849	3.591	0.575	0.121	0.000	86.562	0.245	35.648

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	635	504	5929	0	0	0	0	0	0	0
N.S.	1	0.79	9.34	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.776	8.396	0.000	0.000	0.000	0.000	0.000	0.287	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	410	358	3840	0	0	0	0	0	0	0
N.S.	1	0.87	9.37	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.604	7.256	0.000	0.000	0.000	0.000	0.000	0.285	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	115	767	344	1966	295	712	198	695	474
N.S.	1	0.82	5.44	2.44	13.94	2.09	5.05	1.40	4.93	3.36
time (sec)	N/A	0.339	0.915	1.468	0.162	0.086	5.174	0.254	0.208	28.153

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	219	444	483	848	373	892	371	886	507
N.S.	1	0.74	1.50	1.63	2.86	1.26	3.01	1.25	2.99	1.71
time (sec)	N/A	0.350	0.426	1.477	0.092	0.086	2.284	0.253	0.203	27.985

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	351	290	769	2116	545	1488	672	1457	984
N.S.	1	0.94	0.77	2.05	5.64	1.45	3.97	1.79	3.89	2.62
time (sec)	N/A	0.860	0.829	2.089	0.295	0.095	4.170	0.281	0.235	30.064

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	525	366	453	1080	4188	1008	2683	0	3298	1505
N.S.	1	0.70	0.86	2.06	7.98	1.92	5.11	0.00	6.28	2.87
time (sec)	N/A	0.625	1.166	3.407	0.559	0.105	27.487	0.000	0.230	32.616

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	685	478	611	1396	5583	1517	3720	0	4870	2155
N.S.	1	0.70	0.89	2.04	8.15	2.21	5.43	0.00	7.11	3.15
time (sec)	N/A	0.705	1.564	3.035	0.421	0.125	50.009	0.000	0.247	36.224

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	851	596	793	1708	9282	2257	0	0	7204	3550
N.S.	1	0.70	0.93	2.01	10.91	2.65	0.00	0.00	8.47	4.17
time (sec)	N/A	0.796	2.301	8.223	0.897	0.156	0.000	0.000	0.276	40.901

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	197	269	1195	1118	720	0	3945	801	1237
N.S.	1	0.88	1.21	5.36	5.01	3.23	0.00	17.69	3.59	5.55
time (sec)	N/A	0.468	0.284	11.737	0.072	0.169	0.000	0.992	0.214	27.083

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	173	225	876	740	529	0	2571	573	663
N.S.	1	0.91	1.18	4.61	3.89	2.78	0.00	13.53	3.02	3.49
time (sec)	N/A	0.444	0.205	4.976	0.057	0.113	0.000	0.845	0.208	26.142

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	145	189	507	393	309	643	1276	346	295
N.S.	1	0.97	1.27	3.40	2.64	2.07	4.32	8.56	2.32	1.98
time (sec)	N/A	0.382	0.307	1.871	0.049	0.095	137.269	0.486	0.237	25.837

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	81	74	250	156	162	382	580	194	134
N.S.	1	0.94	0.86	2.91	1.81	1.88	4.44	6.74	2.26	1.56
time (sec)	N/A	0.269	0.027	0.795	0.038	0.093	21.461	0.244	0.208	25.638

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	134	172	0	276	0	0	0	105	0
N.S.	1	0.95	1.22	0.00	1.96	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.711	0.144	0.000	0.351	0.000	0.000	0.000	0.244	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	142	189	0	0	0	0	0	467	0
N.S.	1	0.95	1.26	0.00	0.00	0.00	0.00	0.00	3.11	0.00
time (sec)	N/A	0.545	0.187	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	76	216	237	582	250	0	100	353	204
N.S.	1	0.85	2.43	2.66	6.54	2.81	0.00	1.12	3.97	2.29
time (sec)	N/A	0.268	0.201	4.732	0.058	0.086	0.000	0.631	0.208	26.435

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	143	196	455	945	478	0	234	937	374
N.S.	1	0.79	1.08	2.51	5.22	2.64	0.00	1.29	5.18	2.07
time (sec)	N/A	0.351	0.478	10.158	0.086	0.098	0.000	0.883	0.206	26.502

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	211	220	986	1398	773	0	394	994	610
N.S.	1	0.75	0.78	3.51	4.98	2.75	0.00	1.40	3.54	2.17
time (sec)	N/A	0.415	0.577	29.720	0.100	0.110	0.000	1.175	0.221	26.915

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	388	441	1761	1978	1074	0	5133	1093	2555
N.S.	1	0.88	1.00	3.98	4.48	2.43	0.00	11.61	2.47	5.78
time (sec)	N/A	0.654	0.446	26.818	0.103	0.289	0.000	1.629	0.221	27.557

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	311	374	1274	1336	774	0	3352	796	1328
N.S.	1	0.88	1.06	3.62	3.80	2.20	0.00	9.52	2.26	3.77
time (sec)	N/A	0.555	0.298	11.690	0.072	0.183	0.000	1.132	0.211	26.555

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	234	224	837	740	530	0	1997	577	661
N.S.	1	0.94	0.90	3.35	2.96	2.12	0.00	7.99	2.31	2.64
time (sec)	N/A	0.442	0.224	4.967	0.066	0.118	0.000	0.846	0.204	26.092

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	109	101	463	309	297	656	990	285	303
N.S.	1	0.88	0.81	3.73	2.49	2.40	5.29	7.98	2.30	2.44
time (sec)	N/A	0.291	0.032	2.013	0.043	0.095	127.477	0.401	0.210	25.767

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	289	319	264	0	580	0	0	0	197	0
N.S.	1	1.10	0.91	0.00	2.01	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.936	0.228	0.000	0.376	0.000	0.000	0.000	0.214	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	235	233	0	1190	0	0	0	779	0
N.S.	1	0.91	0.90	0.00	4.59	0.00	0.00	0.00	3.01	0.00
time (sec)	N/A	0.553	0.263	0.000	0.583	0.000	0.000	0.000	0.214	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	242	215	258	0	0	0	0	0	1359	0
N.S.	1	0.89	1.07	0.00	0.00	0.00	0.00	0.00	5.62	0.00
time (sec)	N/A	0.800	0.392	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	78	329	376	1544	409	0	108	560	421
N.S.	1	0.84	3.54	4.04	16.60	4.40	0.00	1.16	6.02	4.53
time (sec)	N/A	0.311	0.397	11.940	0.097	0.093	0.000	1.327	0.209	26.830

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	145	474	855	2247	710	0	250	1333	652
N.S.	1	0.77	2.51	4.52	11.89	3.76	0.00	1.32	7.05	3.45
time (sec)	N/A	0.386	0.459	29.830	0.144	0.108	0.000	1.815	0.227	27.083

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	215	357	1049	3058	1087	0	418	1378	954
N.S.	1	0.73	1.22	3.58	10.44	3.71	0.00	1.43	4.70	3.26
time (sec)	N/A	0.455	1.067	61.252	0.183	0.132	0.000	2.420	0.214	27.943

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	477	414	631	2179	2901	1336	0	5934	1419	4476
N.S.	1	0.87	1.32	4.57	6.08	2.80	0.00	12.44	2.97	9.38
time (sec)	N/A	0.702	0.659	54.626	0.127	0.587	0.000	2.349	0.206	28.086

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	337	441	1721	1978	1075	0	4300	1097	2547
N.S.	1	0.87	1.14	4.45	5.11	2.78	0.00	11.11	2.83	6.58
time (sec)	N/A	0.572	0.381	26.806	0.092	0.372	0.000	1.638	0.197	27.724

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	260	269	1119	1118	721	0	2584	743	1234
N.S.	1	0.92	0.95	3.95	3.95	2.55	0.00	9.13	2.63	4.36
time (sec)	N/A	0.445	0.241	11.686	0.068	0.163	0.000	1.016	0.218	27.036

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	133	124	652	479	429	0	1402	507	588
N.S.	1	0.85	0.79	4.18	3.07	2.75	0.00	8.99	3.25	3.77
time (sec)	N/A	0.323	0.044	4.886	0.056	0.114	0.000	0.603	0.196	26.269

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	373	478	368	0	935	0	0	0	303	0
N.S.	1	1.28	0.99	0.00	2.51	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	1.383	0.327	0.000	0.383	0.000	0.000	0.000	0.229	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	390	343	394	0	1785	0	0	0	1118	0
N.S.	1	0.88	1.01	0.00	4.58	0.00	0.00	0.00	2.87	0.00
time (sec)	N/A	0.638	0.454	0.000	0.380	0.000	0.000	0.000	0.220	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	361	309	331	0	2746	0	0	0	0	0
N.S.	1	0.86	0.92	0.00	7.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.595	0.448	0.000	0.433	0.000	0.000	0.000	0.253	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	326	286	326	0	0	0	0	0	0	0
N.S.	1	0.88	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.090	0.533	0.000	0.000	0.000	0.000	0.000	0.258	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	269	315	370	0	1003	0	0	0	306	0
N.S.	1	1.17	1.38	0.00	3.73	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	0.764	0.342	0.000	0.364	0.000	0.000	0.000	0.203	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	238	266	0	627	0	0	2499	201	0
N.S.	1	1.13	1.26	0.00	2.97	0.00	0.00	11.84	0.95	0.00
time (sec)	N/A	0.599	0.206	0.000	0.353	0.000	0.000	131.852	0.202	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	154	170	0	306	0	0	1242	109	0
N.S.	1	1.15	1.27	0.00	2.28	0.00	0.00	9.27	0.81	0.00
time (sec)	N/A	0.427	0.141	0.000	0.353	0.000	0.000	80.683	0.227	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	101	0	0	0	0	566	47	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	6.66	0.55	0.00
time (sec)	N/A	0.546	0.045	0.000	0.000	0.000	0.000	53.919	0.202	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	219	84	175	74	0	92	63	76
N.S.	1	1.00	4.38	1.68	3.50	1.48	0.00	1.84	1.26	1.52
time (sec)	N/A	0.314	0.123	1.728	0.039	0.079	0.000	0.245	0.202	26.801

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	134	304	269	427	194	0	0	334	239
N.S.	1	0.74	1.68	1.49	2.36	1.07	0.00	0.00	1.85	1.32
time (sec)	N/A	0.419	0.300	5.068	0.057	0.076	0.000	0.000	0.198	27.180

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	204	434	633	888	483	0	104	747	573
N.S.	1	0.77	1.63	2.38	3.34	1.82	0.00	0.39	2.81	2.15
time (sec)	N/A	0.495	0.376	13.374	0.084	0.094	0.000	101.187	0.195	27.497

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	275	518	1090	1472	859	0	232	1414	986
N.S.	1	0.71	1.33	2.80	3.78	2.21	0.00	0.60	3.63	2.53
time (sec)	N/A	0.547	0.708	27.757	0.131	0.102	0.000	138.569	0.208	28.441

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	359	322	375	0	1892	0	0	0	1137	0
N.S.	1	0.90	1.04	0.00	5.27	0.00	0.00	0.00	3.17	0.00
time (sec)	N/A	0.709	0.462	0.000	0.368	0.000	0.000	0.000	0.219	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	275	242	252	0	1273	0	0	0	799	0
N.S.	1	0.88	0.92	0.00	4.63	0.00	0.00	0.00	2.91	0.00
time (sec)	N/A	0.552	0.262	0.000	0.348	0.000	0.000	0.000	0.213	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	154	183	0	0	0	0	906	482	0
N.S.	1	0.92	1.09	0.00	0.00	0.00	0.00	5.39	2.87	0.00
time (sec)	N/A	0.383	0.216	0.000	0.000	0.000	0.000	142.584	0.209	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	77	114	81	136	105	444	91	154	113
N.S.	1	0.75	1.12	0.79	1.33	1.03	4.35	0.89	1.51	1.11
time (sec)	N/A	0.232	0.054	2.280	0.037	0.076	39.033	0.364	0.215	25.804

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	119	304	269	424	196	0	208	338	241
N.S.	1	0.72	1.83	1.62	2.55	1.18	0.00	1.25	2.04	1.45
time (sec)	N/A	0.415	0.289	5.109	0.059	0.083	0.000	0.865	0.198	25.834

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	192	342	671	862	450	0	96	961	432
N.S.	1	0.70	1.25	2.46	3.16	1.65	0.00	0.35	3.52	1.58
time (sec)	N/A	0.434	0.437	9.756	0.077	0.095	0.000	103.275	0.239	26.374

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	267	478	977	1724	946	0	238	1921	1016
N.S.	1	0.74	1.32	2.71	4.78	2.62	0.00	0.66	5.32	2.81
time (sec)	N/A	0.533	0.690	30.930	0.153	0.110	0.000	134.380	0.194	28.206

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	477	329	549	1910	2563	1458	0	401	3127	1665
N.S.	1	0.69	1.15	4.00	5.37	3.06	0.00	0.84	6.56	3.49
time (sec)	N/A	0.498	1.314	49.065	0.223	0.136	0.000	173.654	0.233	31.144

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	382	321	334	0	2894	0	0	0	0	0
N.S.	1	0.84	0.87	0.00	7.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.645	0.453	0.000	0.433	0.000	0.000	0.000	0.246	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	263	229	259	0	0	0	0	0	1388	0
N.S.	1	0.87	0.98	0.00	0.00	0.00	0.00	0.00	5.28	0.00
time (sec)	N/A	0.458	0.361	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	76	215	237	578	250	0	101	357	205
N.S.	1	0.85	2.42	2.66	6.49	2.81	0.00	1.13	4.01	2.30
time (sec)	N/A	0.268	0.181	4.892	0.050	0.106	0.000	0.626	0.203	26.326

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	142	115	272	259	266	0	207	606	221
N.S.	1	0.94	0.76	1.80	1.72	1.76	0.00	1.37	4.01	1.46
time (sec)	N/A	0.349	0.134	4.753	0.040	0.106	0.000	0.567	0.209	26.213

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	192	434	633	888	487	0	418	742	573
N.S.	1	0.76	1.71	2.49	3.50	1.92	0.00	1.65	2.92	2.26
time (sec)	N/A	0.438	0.377	13.384	0.085	0.115	0.000	1.296	0.188	27.877

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	266	477	977	1724	949	0	0	1928	1018
N.S.	1	0.70	1.25	2.56	4.52	2.49	0.00	0.00	5.06	2.67
time (sec)	N/A	0.463	0.692	30.639	0.145	0.116	0.000	0.000	0.225	28.638

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	483	335	561	1450	2383	1416	0	0	2958	1341
N.S.	1	0.69	1.16	3.00	4.93	2.93	0.00	0.00	6.12	2.78
time (sec)	N/A	0.516	1.195	58.839	0.160	0.137	0.000	0.000	0.208	28.742

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	587	407	671	2149	3819	2181	0	0	4291	2400
N.S.	1	0.69	1.14	3.66	6.51	3.72	0.00	0.00	7.31	4.09
time (sec)	N/A	0.629	1.632	211.336	0.331	0.181	0.000	0.000	0.244	31.382

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	584	646	949	0	3764	0	0	0	0	0
N.S.	1	1.11	1.62	0.00	6.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.521	0.870	0.000	0.637	0.000	0.000	0.000	0.235	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	487	541	716	0	2691	0	0	0	0	0
N.S.	1	1.11	1.47	0.00	5.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.286	0.672	0.000	0.605	0.000	0.000	0.000	0.223	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	372	426	937	0	1542	0	0	0	1011	0
N.S.	1	1.15	2.52	0.00	4.15	0.00	0.00	0.00	2.72	0.00
time (sec)	N/A	0.957	0.839	0.000	0.573	0.000	0.000	0.000	0.216	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	239	216	0	825	0	0	0	627	0
N.S.	1	1.09	0.98	0.00	3.75	0.00	0.00	0.00	2.85	0.00
time (sec)	N/A	0.679	0.146	0.000	0.541	0.000	0.000	0.000	0.216	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	306	288	1372	0	0	0	0	0	186	0
N.S.	1	0.94	4.48	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	1.152	1.111	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	261	234	1564	0	0	0	0	0	913	0
N.S.	1	0.90	5.99	0.00	0.00	0.00	0.00	0.00	3.50	0.00
time (sec)	N/A	0.953	1.995	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	122	801	530	2017	600	0	195	750	561
N.S.	1	0.81	5.30	3.51	13.36	3.97	0.00	1.29	4.97	3.72
time (sec)	N/A	0.355	0.935	4.812	0.139	0.105	0.000	1.513	0.212	27.515

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	238	1082	1099	3312	1167	0	501	1624	993
N.S.	1	0.78	3.52	3.58	10.79	3.80	0.00	1.63	5.29	3.23
time (sec)	N/A	0.469	1.141	10.207	0.230	0.121	0.000	2.152	0.217	28.893

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	475	364	1319	2220	4838	1868	0	871	2426	1794
N.S.	1	0.77	2.78	4.67	10.19	3.93	0.00	1.83	5.11	3.78
time (sec)	N/A	0.602	1.306	29.838	0.355	0.145	0.000	2.841	0.217	31.163

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	766	1003	1634	0	5952	0	0	0	0	0
N.S.	1	1.31	2.13	0.00	7.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.366	1.578	0.000	0.652	0.000	0.000	0.000	0.227	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	819	891	1254	0	4247	0	0	0	0	0
N.S.	1	1.09	1.53	0.00	5.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.228	1.076	0.000	0.641	0.000	0.000	0.000	0.224	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	635	702	713	0	2662	0	0	0	0	0
N.S.	1	1.11	1.12	0.00	4.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.553	0.671	0.000	0.607	0.000	0.000	0.000	0.217	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	361	376	303	0	1473	0	0	0	1046	0
N.S.	1	1.04	0.84	0.00	4.08	0.00	0.00	0.00	2.90	0.00
time (sec)	N/A	1.034	0.195	0.000	0.555	0.000	0.000	0.000	0.232	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	572	529	2852	0	0	0	0	0	326	0
N.S.	1	0.92	4.99	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	2.126	3.968	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	472	396	2885	0	0	0	0	0	1467	0
N.S.	1	0.84	6.11	0.00	0.00	0.00	0.00	0.00	3.11	0.00
time (sec)	N/A	0.751	5.588	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	417	353	4535	0	0	0	0	0	0	0
N.S.	1	0.85	10.88	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.486	7.323	0.000	0.000	0.000	0.000	0.000	0.302	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	127	1418	825	5588	975	0	216	1161	1195
N.S.	1	0.81	9.03	5.25	35.59	6.21	0.00	1.38	7.39	7.61
time (sec)	N/A	0.402	2.435	10.297	0.363	0.101	0.000	3.371	0.192	29.032

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	240	1787	1884	8087	1729	0	541	2302	1934
N.S.	1	0.75	5.60	5.91	25.35	5.42	0.00	1.70	7.22	6.06
time (sec)	N/A	0.531	3.687	29.841	0.559	0.141	0.000	4.493	0.212	30.382

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	366	2112	2528	10936	2633	0	931	3287	3296
N.S.	1	0.74	4.28	5.13	22.18	5.34	0.00	1.89	6.67	6.69
time (sec)	N/A	0.630	3.464	61.605	0.773	0.188	0.000	5.969	0.212	32.527

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1172	1455	2448	0	7845	0	0	0	0	0
N.S.	1	1.24	2.09	0.00	6.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.657	3.510	0.000	0.714	0.000	0.000	0.000	0.280	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	976	1266	1627	0	5931	0	0	0	0	0
N.S.	1	1.30	1.67	0.00	6.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.285	1.520	0.000	0.656	0.000	0.000	0.000	0.238	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	786	1002	945	0	3724	0	0	0	0	0
N.S.	1	1.27	1.20	0.00	4.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.306	0.856	0.000	0.624	0.000	0.000	0.000	0.238	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	454	535	409	0	2129	0	0	0	0	0
N.S.	1	1.18	0.90	0.00	4.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.444	0.246	0.000	0.571	0.000	0.000	0.000	0.208	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	762	901	4969	0	0	0	0	0	480	0
N.S.	1	1.18	6.52	0.00	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	3.300	9.447	0.000	0.000	0.000	0.000	0.000	0.238	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	739	624	5917	0	0	0	0	0	0	0
N.S.	1	0.84	8.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.013	7.544	0.000	0.000	0.000	0.000	0.000	0.244	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	644	526	6938	0	0	0	0	0	0	0
N.S.	1	0.82	10.77	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.854	7.537	0.000	0.000	0.000	0.000	0.000	0.285	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	561	473	8570	0	0	0	0	0	0	0
N.S.	1	0.84	15.28	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.948	8.504	0.000	0.000	0.000	0.000	0.000	0.402	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	768	679	1072	0	0	0	0	0	487	0
N.S.	1	0.88	1.40	0.00	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	1.187	1.729	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	573	507	763	0	0	0	0	0	335	0
N.S.	1	0.88	1.33	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.873	1.381	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	303	280	1385	0	0	0	0	0	196	0
N.S.	1	0.92	4.57	0.00	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.557	1.000	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	128	268	0	0	0	0	0	88	0
N.S.	1	0.93	1.96	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.466	0.874	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	40	90	118	407	149	0	167	94	122
N.S.	1	0.80	1.80	2.36	8.14	2.98	0.00	3.34	1.88	2.44
time (sec)	N/A	0.370	0.287	1.747	0.060	0.080	0.000	0.403	0.180	27.087

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	152	793	509	1018	428	0	0	612	361
N.S.	1	0.76	3.98	2.56	5.12	2.15	0.00	0.00	3.08	1.81
time (sec)	N/A	0.493	0.763	5.098	0.102	0.088	0.000	0.000	0.185	27.162

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	274	975	1236	2126	1068	0	0	1609	1011
N.S.	1	0.74	2.64	3.35	5.76	2.89	0.00	0.00	4.36	2.74
time (sec)	N/A	0.618	1.227	13.404	0.187	0.102	0.000	0.000	0.194	29.929

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	400	1295	2216	3445	1910	0	0	2656	1921
N.S.	1	0.74	2.38	4.08	6.34	3.52	0.00	0.00	4.89	3.54
time (sec)	N/A	0.694	1.734	28.330	0.313	0.123	0.000	0.000	0.201	31.810

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	770	640	5850	0	0	0	0	0	0	0
N.S.	1	0.83	7.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.993	7.726	0.000	0.000	0.000	0.000	0.000	0.269	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	500	411	3522	0	0	0	0	0	1516	0
N.S.	1	0.82	7.04	0.00	0.00	0.00	0.00	0.00	3.03	0.00
time (sec)	N/A	0.679	7.258	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	282	260	1570	0	0	0	0	0	959	0
N.S.	1	0.92	5.57	0.00	0.00	0.00	0.00	0.00	3.40	0.00
time (sec)	N/A	0.511	1.841	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	119	331	294	428	263	0	175	370	237
N.S.	1	0.73	2.03	1.80	2.63	1.61	0.00	1.07	2.27	1.45
time (sec)	N/A	0.286	0.432	2.219	0.049	0.083	0.000	0.805	0.204	27.824

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	A	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	151	789	509	1014	432	0	368	640	365
N.S.	1	0.65	3.42	2.20	4.39	1.87	0.00	1.59	2.77	1.58
time (sec)	N/A	0.544	0.762	4.602	0.104	0.093	0.000	0.694	0.223	27.228

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	281	870	1307	2006	983	0	0	2172	0
N.S.	1	0.72	2.22	3.33	5.12	2.51	0.00	0.00	5.54	0.00
time (sec)	N/A	0.586	1.123	9.774	0.156	0.111	0.000	0.000	0.205	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	401	1340	1986	4198	2052	0	0	3630	1784
N.S.	1	0.72	2.39	3.55	7.50	3.66	0.00	0.00	6.48	3.19
time (sec)	N/A	0.665	1.658	30.547	0.338	0.132	0.000	0.000	0.212	31.672

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	729	522	1695	3056	6171	3183	0	0	5699	3157
N.S.	1	0.72	2.33	4.19	8.47	4.37	0.00	0.00	7.82	4.33
time (sec)	N/A	0.762	2.569	48.602	0.490	0.166	0.000	0.000	0.220	32.825

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	676	545	6878	0	0	0	0	0	0	0
N.S.	1	0.81	10.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.793	7.791	0.000	0.000	0.000	0.000	0.000	0.309	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	441	389	4493	0	0	0	0	0	0	0
N.S.	1	0.88	10.19	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.648	7.206	0.000	0.000	0.000	0.000	0.000	0.282	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	123	803	530	1995	600	0	195	761	565
N.S.	1	0.81	5.32	3.51	13.21	3.97	0.00	1.29	5.04	3.74
time (sec)	N/A	0.343	1.153	4.747	0.133	0.101	0.000	1.616	0.188	29.011

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	240	464	672	861	654	0	407	977	505
N.S.	1	0.76	1.46	2.12	2.72	2.06	0.00	1.28	3.08	1.59
time (sec)	N/A	0.375	0.459	4.808	0.080	0.106	0.000	0.823	0.187	27.726

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	383	971	1236	2126	1076	0	747	1640	1007
N.S.	1	0.95	2.42	3.07	5.29	2.68	0.00	1.86	4.08	2.50
time (sec)	N/A	0.961	0.948	13.411	0.183	0.115	0.000	0.901	0.214	29.963

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	562	403	1334	2000	4199	2057	0	0	3638	1785
N.S.	1	0.72	2.37	3.56	7.47	3.66	0.00	0.00	6.47	3.18
time (sec)	N/A	0.659	1.631	30.478	0.343	0.131	0.000	0.000	0.214	31.764

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	732	525	1653	2993	5594	3062	0	0	5363	2419
N.S.	1	0.72	2.26	4.09	7.64	4.18	0.00	0.00	7.33	3.30
time (sec)	N/A	0.737	2.135	58.612	0.391	0.165	0.000	0.000	0.224	33.752

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	908	653	2138	4486	9293	4725	0	0	7917	4649
N.S.	1	0.72	2.35	4.94	10.23	5.20	0.00	0.00	8.72	5.12
time (sec)	N/A	0.850	3.524	210.380	0.811	0.249	0.000	0.000	0.243	35.042

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	0	0	0	0	0	0	89	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.558	0.000	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	0	0	0	0	0	0	91	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.572	0.000	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	292	237	206	0	0	2680	0	0	332	0
N.S.	1	0.81	0.71	0.00	0.00	9.18	0.00	0.00	1.14	0.00
time (sec)	N/A	0.607	7.720	0.000	0.000	0.191	0.000	0.000	0.216	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	180	134	2629	0	991	0	0	241	0
N.S.	1	0.86	0.64	12.52	0.00	4.72	0.00	0.00	1.15	0.00
time (sec)	N/A	0.515	3.127	148.283	0.000	0.131	0.000	0.000	0.187	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	123	78	826	0	274	0	0	150	0
N.S.	1	0.96	0.61	6.45	0.00	2.14	0.00	0.00	1.17	0.00
time (sec)	N/A	0.390	1.056	36.541	0.000	0.107	0.000	0.000	0.202	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	98	0	0	173	0
N.S.	1	1.00	0.00	0.00	0.00	0.78	0.00	0.00	1.38	0.00
time (sec)	N/A	0.542	0.000	0.000	0.000	0.087	0.000	0.000	0.197	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	201	0	0	0	293	0	0	302	0
N.S.	1	0.98	0.00	0.00	0.00	1.42	0.00	0.00	1.47	0.00
time (sec)	N/A	0.613	0.000	0.000	0.000	0.085	0.000	0.000	0.214	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	295	266	0	0	0	818	0	0	431	0
N.S.	1	0.90	0.00	0.00	0.00	2.77	0.00	0.00	1.46	0.00
time (sec)	N/A	0.733	0.000	0.000	0.000	0.124	0.000	0.000	0.221	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	309	252	206	0	0	2669	0	0	334	0
N.S.	1	0.82	0.67	0.00	0.00	8.64	0.00	0.00	1.08	0.00
time (sec)	N/A	0.640	7.973	0.000	0.000	0.220	0.000	0.000	0.214	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	192	134	2619	0	983	0	0	243	0
N.S.	1	0.86	0.60	11.74	0.00	4.41	0.00	0.00	1.09	0.00
time (sec)	N/A	0.543	2.997	148.895	0.000	0.131	0.000	0.000	0.207	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	132	78	812	0	269	0	0	152	0
N.S.	1	0.96	0.57	5.93	0.00	1.96	0.00	0.00	1.11	0.00
time (sec)	N/A	0.391	1.001	37.200	0.000	0.120	0.000	0.000	0.223	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	0	0	0	93	0	0	175	0
N.S.	1	1.00	0.00	0.00	0.00	0.73	0.00	0.00	1.37	0.00
time (sec)	N/A	0.543	0.000	0.000	0.000	0.091	0.000	0.000	0.194	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	214	209	0	0	0	284	0	0	296	0
N.S.	1	0.98	0.00	0.00	0.00	1.33	0.00	0.00	1.38	0.00
time (sec)	N/A	0.647	0.000	0.000	0.000	0.099	0.000	0.000	0.214	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	306	276	0	0	0	815	0	0	417	0
N.S.	1	0.90	0.00	0.00	0.00	2.66	0.00	0.00	1.36	0.00
time (sec)	N/A	0.733	0.000	0.000	0.000	0.124	0.000	0.000	0.244	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	40	43	0	65	0	41	64	0
N.S.	1	1.00	0.98	1.05	0.00	1.59	0.00	1.00	1.56	0.00
time (sec)	N/A	0.314	0.040	8.248	0.000	0.084	0.000	0.114	0.193	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	40	63	0	65	0	41	64	0
N.S.	1	1.00	0.98	1.54	0.00	1.59	0.00	1.00	1.56	0.00
time (sec)	N/A	0.376	0.004	11.125	0.000	0.156	0.000	0.121	0.255	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	189	0	0	0	0	0	0	89	0
N.S.	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.762	0.000	0.000	0.000	0.000	0.000	0.000	0.281	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	190	0	0	0	0	0	0	91	0
N.S.	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.805	0.000	0.000	0.000	0.000	0.000	0.000	0.269	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	34	43	44	766	375	0	0	120	141
N.S.	1	0.76	0.96	0.98	17.02	8.33	0.00	0.00	2.67	3.13
time (sec)	N/A	0.459	0.028	39.288	0.102	0.097	0.000	0.000	0.257	28.668

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	34	43	44	387	188	0	0	91	100
N.S.	1	0.76	0.96	0.98	8.60	4.18	0.00	0.00	2.02	2.22
time (sec)	N/A	0.458	0.021	6.621	0.063	0.092	0.000	0.000	0.250	27.150

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	44	43	62	151	72	0	0	60	71
N.S.	1	0.98	0.96	1.38	3.36	1.60	0.00	0.00	1.33	1.58
time (sec)	N/A	0.397	0.018	4.460	0.041	0.074	0.000	0.000	0.274	26.302

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	40	39	43	49	45	0	41	42	40
N.S.	1	0.98	0.95	1.05	1.20	1.10	0.00	1.00	1.02	0.98
time (sec)	N/A	0.479	0.100	24.734	0.157	0.074	0.000	0.112	0.258	25.790

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	30	41	43	81	86	0	99	88	42
N.S.	1	0.70	0.95	1.00	1.88	2.00	0.00	2.30	2.05	0.98
time (sec)	N/A	0.485	0.027	72.098	0.167	0.073	0.000	0.111	0.256	25.788

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	34	43	44	220	238	0	321	129	72
N.S.	1	0.76	0.96	0.98	4.89	5.29	0.00	7.13	2.87	1.60
time (sec)	N/A	0.504	0.031	181.952	0.235	0.078	0.000	0.129	0.267	25.732

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	48	47	51	0	81	0	49	75	0
N.S.	1	0.98	0.96	1.04	0.00	1.65	0.00	1.00	1.53	0.00
time (sec)	N/A	0.531	0.035	272.589	0.000	0.092	0.000	0.122	0.256	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	54	51	57	0	85	0	53	81	0
N.S.	1	0.98	0.93	1.04	0.00	1.55	0.00	0.96	1.47	0.00
time (sec)	N/A	0.623	0.057	0.034	0.000	0.092	0.000	0.123	0.282	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	51	50	0	0	83	0	51	78	0
N.S.	1	0.98	0.96	0.00	0.00	1.60	0.00	0.98	1.50	0.00
time (sec)	N/A	0.561	0.005	0.000	0.000	0.090	0.000	0.119	0.281	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	40	39	43	49	45	0	41	42	40
N.S.	1	0.98	0.95	1.05	1.20	1.10	0.00	1.00	1.02	0.98
time (sec)	N/A	0.491	0.011	24.596	0.152	0.074	0.000	0.115	0.267	0.002

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	46	43	49	53	51	0	45	48	44
N.S.	1	0.98	0.91	1.04	1.13	1.09	0.00	0.96	1.02	0.94
time (sec)	N/A	0.562	0.236	77.312	0.154	0.074	0.000	0.114	0.279	26.411

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	43	42	46	51	48	0	43	45	42
N.S.	1	0.98	0.95	1.05	1.16	1.09	0.00	0.98	1.02	0.95
time (sec)	N/A	0.529	0.013	62.267	0.156	0.079	0.000	0.120	0.267	26.284

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	86	0	0	0	0	0	0	107	0
N.S.	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	1.22	0.00
time (sec)	N/A	0.583	0.000	0.000	0.000	0.000	0.000	0.000	0.282	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	110	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.47	0.00	0.00	0.00	0.00
time (sec)	N/A	0.328	0.039	0.000	0.000	0.088	0.000	0.000	0.406	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	88	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.17	0.00	0.00	0.00	0.00
time (sec)	N/A	0.323	0.033	0.000	0.000	0.071	0.000	0.000	0.320	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	66	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.88	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	0.026	0.000	0.000	0.100	0.000	0.000	0.333	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	40	0	0	544	0
N.S.	1	1.00	1.00	0.00	0.00	0.56	0.00	0.00	7.56	0.00
time (sec)	N/A	0.263	0.100	0.000	0.000	0.073	0.000	0.000	0.265	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	34	35	37	34	146	88	35	33
N.S.	1	1.00	1.03	1.06	1.12	1.03	4.42	2.67	1.06	1.00
time (sec)	N/A	0.277	0.074	1.467	0.159	0.073	93.173	0.135	0.265	27.716

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	40	0	0	546	0
N.S.	1	1.00	1.00	0.00	0.00	0.56	0.00	0.00	7.69	0.00
time (sec)	N/A	0.276	0.090	0.000	0.000	0.078	0.000	0.000	0.275	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	66	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.88	0.00	0.00	0.00	0.00
time (sec)	N/A	0.295	0.020	0.000	0.000	0.077	0.000	0.000	0.305	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	88	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.17	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	0.026	0.000	0.000	0.073	0.000	0.000	0.334	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	361	312	0	0	0	0	0	0	0	0
N.S.	1	0.86	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.211	0.000	0.000	0.000	0.000	0.000	0.000	0.350	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	282	248	0	0	0	0	0	0	0	0
N.S.	1	0.88	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.085	0.000	0.000	0.000	0.000	0.000	0.000	0.370	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	203	183	1415	0	0	0	0	0	1241	0
N.S.	1	0.90	6.97	0.00	0.00	0.00	0.00	0.00	6.11	0.00
time (sec)	N/A	0.945	1.436	0.000	0.000	0.000	0.000	0.000	0.406	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	123	120	304	1447	0	0	0	0	651	0
N.S.	1	0.98	2.47	11.76	0.00	0.00	0.00	0.00	5.29	0.00
time (sec)	N/A	0.631	0.455	27.073	0.000	0.000	0.000	0.000	0.312	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	45	43	45	76	0	45	1185	45
N.S.	1	1.00	1.05	1.00	1.05	1.77	0.00	1.05	27.56	1.05
time (sec)	N/A	0.482	0.301	0.648	0.199	0.080	0.000	0.127	0.336	28.121

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	45	43	506	145	0	45	4555	45
N.S.	1	1.00	1.05	1.00	11.77	3.37	0.00	1.05	105.93	1.05
time (sec)	N/A	0.473	0.571	0.042	0.244	0.079	0.000	0.146	0.336	26.954

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	298	42	357	32	0	0	1201	0
N.S.	1	1.00	9.03	1.27	10.82	0.97	0.00	0.00	36.39	0.00
time (sec)	N/A	0.318	0.235	3.013	0.048	0.076	0.000	0.000	0.832	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	38	375	46	344	38	0	0	463	0
N.S.	1	1.41	13.89	1.70	12.74	1.41	0.00	0.00	17.15	0.00
time (sec)	N/A	0.279	0.383	3.071	0.047	0.070	0.000	0.000	0.268	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	38	375	46	336	38	0	0	463	0
N.S.	1	1.41	13.89	1.70	12.44	1.41	0.00	0.00	17.15	0.00
time (sec)	N/A	0.347	0.354	2.787	0.046	0.075	0.000	0.000	0.286	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	38	375	46	343	38	0	0	463	0
N.S.	1	1.41	13.89	1.70	12.70	1.41	0.00	0.00	17.15	0.00
time (sec)	N/A	0.338	0.102	2.689	0.048	0.076	0.000	0.000	0.422	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	282	279	0	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.161	0.000	0.000	0.000	0.000	0.000	0.000	0.344	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	203	214	1415	0	0	0	0	0	1241	0
N.S.	1	1.05	6.97	0.00	0.00	0.00	0.00	0.00	6.11	0.00
time (sec)	N/A	0.990	1.085	0.000	0.000	0.000	0.000	0.000	0.321	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	123	151	303	1447	0	0	0	0	651	0
N.S.	1	1.23	2.46	11.76	0.00	0.00	0.00	0.00	5.29	0.00
time (sec)	N/A	0.669	0.335	32.815	0.000	0.000	0.000	0.000	0.307	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [74] had the largest ratio of [.523809999999999998]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	0.89	38	0.132
2	A	5	5	0.92	38	0.132
3	A	5	5	0.98	36	0.139
4	A	4	4	0.95	28	0.143
5	A	9	8	0.95	38	0.211
6	A	6	5	0.94	38	0.132
7	A	3	2	0.85	38	0.053
8	A	6	5	0.79	38	0.132
9	A	6	5	0.75	38	0.132
10	A	6	5	0.89	40	0.125
11	A	6	5	0.89	40	0.125
12	A	6	5	0.95	38	0.132
13	A	4	4	0.89	30	0.133
14	A	11	10	1.11	40	0.250
15	A	4	3	0.90	40	0.075
16	A	8	7	0.88	40	0.175
17	A	3	2	0.83	40	0.050
18	A	6	5	0.76	40	0.125
19	A	6	5	0.73	40	0.125
20	A	6	5	0.88	40	0.125
21	A	6	5	0.88	40	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	6	5	0.93	38	0.132
23	A	4	4	0.87	30	0.133
24	A	15	14	1.30	40	0.350
25	A	4	3	0.87	40	0.075
26	A	4	3	0.85	40	0.075
27	A	10	9	0.87	40	0.225
28	A	3	2	0.83	40	0.050
29	A	6	5	0.76	40	0.125
30	A	6	5	0.73	40	0.125
31	A	7	6	1.18	40	0.150
32	A	6	5	1.14	40	0.125
33	A	5	4	1.16	38	0.105
34	A	8	7	1.00	30	0.233
35	A	3	2	1.00	40	0.050
36	A	6	5	0.73	40	0.125
37	A	6	5	0.76	40	0.125
38	A	6	5	0.70	40	0.125
39	A	6	5	0.89	40	0.125
40	A	5	4	0.87	40	0.100
41	A	4	3	0.91	38	0.079
42	A	3	2	0.74	30	0.067
43	A	5	4	0.70	40	0.100
44	A	4	3	0.70	40	0.075
45	A	6	5	0.73	40	0.125
46	A	4	3	0.68	40	0.075
47	A	5	4	0.83	40	0.100
48	A	4	3	0.86	40	0.075
49	A	3	2	0.85	38	0.053
50	A	4	4	0.96	30	0.133
51	A	4	3	0.75	40	0.075
52	A	4	3	0.69	40	0.075
53	A	4	3	0.69	40	0.075

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	6	5	0.69	40	0.125
55	A	12	11	1.14	40	0.275
56	A	11	10	1.15	40	0.250
57	A	10	9	1.18	38	0.237
58	A	8	7	1.12	30	0.233
59	A	9	8	0.95	40	0.200
60	A	8	7	0.90	40	0.175
61	A	4	3	0.81	40	0.075
62	A	4	3	0.76	40	0.075
63	A	4	3	0.75	40	0.075
64	A	18	17	1.35	42	0.405
65	A	16	15	1.12	42	0.357
66	A	15	14	1.14	40	0.350
67	A	12	11	1.08	32	0.344
68	A	13	12	0.94	42	0.286
69	A	4	3	0.83	42	0.071
70	A	11	10	0.85	42	0.238
71	A	4	3	0.81	42	0.071
72	A	4	3	0.74	42	0.071
73	A	4	3	0.73	42	0.071
74	A	23	22	1.28	42	0.524
75	A	21	20	1.34	42	0.476
76	A	20	19	1.31	40	0.475
77	A	16	15	1.23	32	0.469
78	A	17	16	1.20	42	0.381
79	A	4	3	0.83	42	0.071
80	A	4	3	0.80	42	0.071
81	A	4	3	0.81	42	0.071
82	A	4	3	0.74	42	0.071
83	A	4	3	0.73	42	0.071
84	A	4	3	0.88	42	0.071
85	A	4	3	0.88	42	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	3	0.92	40	0.075
87	A	5	4	0.94	32	0.125
88	A	4	3	0.84	42	0.071
89	A	4	3	0.74	42	0.071
90	A	4	3	0.72	42	0.071
91	A	4	3	0.72	42	0.071
92	A	4	3	0.82	42	0.071
93	A	4	3	0.81	42	0.071
94	A	4	3	0.92	40	0.075
95	A	4	3	0.73	32	0.094
96	A	7	6	0.65	42	0.143
97	A	4	3	0.70	42	0.071
98	A	4	3	0.70	42	0.071
99	A	4	3	0.70	42	0.071
100	A	4	3	0.79	42	0.071
101	A	4	3	0.87	42	0.071
102	A	4	3	0.82	40	0.075
103	A	4	3	0.74	32	0.094
104	A	10	9	0.94	42	0.214
105	A	4	3	0.70	42	0.071
106	A	4	3	0.70	42	0.071
107	A	4	3	0.70	42	0.071
108	A	5	5	0.88	41	0.122
109	A	5	5	0.91	41	0.122
110	A	5	5	0.97	39	0.128
111	A	4	4	0.94	31	0.129
112	A	9	8	0.95	41	0.195
113	A	6	5	0.95	41	0.122
114	A	3	2	0.85	41	0.049
115	A	6	5	0.79	41	0.122
116	A	6	5	0.75	41	0.122
117	A	6	5	0.88	43	0.116

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	6	5	0.88	43	0.116
119	A	6	5	0.94	41	0.122
120	A	4	4	0.88	33	0.121
121	A	11	10	1.10	43	0.233
122	A	4	3	0.91	43	0.070
123	A	8	7	0.89	43	0.163
124	A	3	2	0.84	43	0.047
125	A	6	5	0.77	43	0.116
126	A	6	5	0.73	43	0.116
127	A	6	5	0.87	43	0.116
128	A	6	5	0.87	43	0.116
129	A	6	5	0.92	41	0.122
130	A	4	4	0.85	33	0.121
131	A	15	14	1.28	43	0.326
132	A	4	3	0.88	43	0.070
133	A	4	3	0.86	43	0.070
134	A	10	9	0.88	43	0.209
135	A	7	6	1.17	43	0.140
136	A	6	5	1.13	43	0.116
137	A	5	4	1.15	41	0.098
138	A	8	7	1.00	33	0.212
139	A	3	2	1.00	43	0.047
140	A	6	5	0.74	43	0.116
141	A	6	5	0.77	43	0.116
142	A	6	5	0.71	43	0.116
143	A	6	5	0.90	43	0.116
144	A	5	4	0.88	43	0.093
145	A	4	3	0.92	41	0.073
146	A	3	2	0.75	33	0.061
147	A	5	4	0.72	43	0.093
148	A	4	3	0.70	43	0.070
149	A	6	5	0.74	43	0.116

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	4	3	0.69	43	0.070
151	A	5	4	0.84	43	0.093
152	A	4	3	0.87	43	0.070
153	A	3	2	0.85	41	0.049
154	A	4	4	0.94	33	0.121
155	A	4	3	0.76	43	0.070
156	A	4	3	0.70	43	0.070
157	A	4	3	0.69	43	0.070
158	A	6	5	0.69	43	0.116
159	A	12	11	1.11	43	0.256
160	A	11	10	1.11	43	0.233
161	A	10	9	1.15	41	0.220
162	A	8	7	1.09	33	0.212
163	A	9	8	0.94	43	0.186
164	A	8	7	0.90	43	0.163
165	A	4	3	0.81	43	0.070
166	A	4	3	0.78	43	0.070
167	A	4	3	0.77	43	0.070
168	A	18	17	1.31	45	0.378
169	A	16	15	1.09	45	0.333
170	A	15	14	1.11	43	0.326
171	A	12	11	1.04	35	0.314
172	A	13	12	0.92	45	0.267
173	A	4	3	0.84	45	0.067
174	A	11	10	0.85	45	0.222
175	A	4	3	0.81	45	0.067
176	A	4	3	0.75	45	0.067
177	A	4	3	0.74	45	0.067
178	A	23	22	1.24	45	0.489
179	A	21	20	1.30	45	0.444
180	A	20	19	1.27	43	0.442
181	A	16	15	1.18	35	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	17	16	1.18	45	0.356
183	A	4	3	0.84	45	0.067
184	A	4	3	0.82	45	0.067
185	A	14	13	0.84	45	0.289
186	A	4	3	0.88	45	0.067
187	A	4	3	0.88	45	0.067
188	A	4	3	0.92	43	0.070
189	A	5	4	0.93	35	0.114
190	A	4	3	0.80	45	0.067
191	A	4	3	0.76	45	0.067
192	A	4	3	0.74	45	0.067
193	A	4	3	0.74	45	0.067
194	A	4	3	0.83	45	0.067
195	A	4	3	0.82	45	0.067
196	A	4	3	0.92	43	0.070
197	A	4	3	0.73	35	0.086
198	A	7	6	0.65	45	0.133
199	A	4	3	0.72	45	0.067
200	A	4	3	0.72	45	0.067
201	A	4	3	0.72	45	0.067
202	A	4	3	0.81	45	0.067
203	A	4	3	0.88	45	0.067
204	A	4	3	0.81	43	0.070
205	A	4	3	0.76	35	0.086
206	A	10	9	0.95	45	0.200
207	A	4	3	0.72	45	0.067
208	A	4	3	0.72	45	0.067
209	A	4	3	0.72	45	0.067
210	A	4	3	1.00	49	0.061
211	A	4	3	1.00	49	0.061
212	A	5	4	0.81	49	0.082
213	A	4	3	0.86	49	0.061

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	3	2	0.96	47	0.043
215	A	4	3	1.00	49	0.061
216	A	5	4	0.98	49	0.082
217	A	6	5	0.90	49	0.102
218	A	5	4	0.82	49	0.082
219	A	4	3	0.86	49	0.061
220	A	3	2	0.96	47	0.043
221	A	4	3	1.00	49	0.061
222	A	5	4	0.98	49	0.082
223	A	6	5	0.90	49	0.102
224	A	4	3	1.00	35	0.086
225	A	5	4	1.00	42	0.095
226	A	5	4	0.98	50	0.080
227	A	5	4	0.98	50	0.080
228	A	5	4	0.76	40	0.100
229	A	5	4	0.76	40	0.100
230	A	4	3	0.98	38	0.079
231	A	5	4	0.98	40	0.100
232	A	5	4	0.70	40	0.100
233	A	5	4	0.76	40	0.100
234	A	5	4	0.98	40	0.100
235	A	5	4	0.98	46	0.087
236	A	6	5	0.98	50	0.100
237	A	5	4	0.98	40	0.100
238	A	5	4	0.98	46	0.087
239	A	6	5	0.98	50	0.100
240	A	5	4	0.98	40	0.100
241	A	4	3	1.00	35	0.086
242	A	4	3	1.00	35	0.086
243	A	4	3	1.00	33	0.091
244	A	4	3	1.00	28	0.107
245	A	4	3	1.00	35	0.086

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	4	3	1.00	28	0.107
247	A	4	3	1.00	33	0.091
248	A	4	3	1.00	35	0.086
249	A	9	8	0.86	43	0.186
250	A	8	7	0.88	43	0.163
251	A	7	6	0.90	43	0.140
252	A	6	5	0.98	41	0.122
253	N/A	3	0	1.00	43	0.000
254	N/A	3	0	1.00	43	0.000
255	A	5	4	1.00	40	0.100
256	A	3	2	1.41	38	0.053
257	A	4	3	1.41	33	0.091
258	A	4	3	1.41	38	0.079
259	A	8	7	0.99	51	0.137
260	A	7	6	1.05	51	0.118
261	A	6	5	1.23	49	0.102

CHAPTER 3

Listing of integrals

3.1 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 126

3.2 $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 137

3.3 $\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 149

3.4 $\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 160

3.5 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$ 168

3.6 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$ 177

3.7 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$ 185

3.8 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$ 194

3.9 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$ 205

3.10 $\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 216

3.11 $\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 228

3.12 $\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 239

3.13 $\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ 252

3.14 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$ 261

3.15 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$ 273

3.16 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$ 282

3.17 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$ 293

3.18 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$ 302

3.19 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$ . . . 313

3.20 $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ . . . 324

3.21 $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ . . . 336

3.22 $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ . . . 348

3.23 $\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$ . . . 359

3.24 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$ . . . 369

3.25 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$ . . . 382

3.26 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$ . . . 392

3.27 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$ . . . 402

3.28 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$ . . . 415

3.29 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$ . . . 424

3.30 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx$ . . . 434

3.31 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$ . . . 445

3.32 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$ . . . 455

3.33 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$ . . . 464

3.34 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx$ . . . 473

3.35 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx$ . . . 481

3.36 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx$ . . . 488

3.37 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx$ . . . 498

3.38 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx$ . . . 509

3.39 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$ . . . 520

3.40 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$ . . . 531

3.41 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$ . . . 541

3.42 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx$ . . . 550

3.43 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$ . . . 557

3.44 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$ 566

3.45 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx$ 577

3.46 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx$ 588

3.47 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$ 599

3.48 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$ 610

3.49 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$ 620

3.50 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx$ 629

3.51 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx$ 637

3.52 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$ 648

3.53 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx$ 658

3.54 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx$ 669

3.55 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$ 681

3.56 $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$ 695

3.57 $\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$ 707

3.58 $\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$ 719

3.59 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$ 728

3.60 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$ 739

3.61 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$ 748

3.62 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$ 759

3.63 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$ 771

3.64 $\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$ 783

3.65 $\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$ 802

3.66 $\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$ 821

3.67 $\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$ 836

3.68 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$ 849
3.69 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$ 864
3.70 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$ 873
3.71 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$ 886
3.72 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$ 896
3.73 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx$ 908
3.74 $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$ 919
3.75 $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$ 943
3.76 $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$ 965
3.77 $\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$ 987
3.78 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$ 1002
3.79 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$ 1026
3.80 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$ 1035
3.81 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$ 1045
3.82 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx$ 1055
3.83 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx$ 1066
3.84 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$ 1078
3.85 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$ 1088
3.86 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$ 1097
3.87 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$ 1105
3.88 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$ 1113
3.89 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$ 1121
3.90 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$ 1131
3.91 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx$ 1142

3.92 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$ 1154
3.93 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$ 1163
3.94 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$ 1172
3.95 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$ 1179
3.96 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$ 1188
3.97 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx$ 1199
3.98 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx$ 1210
3.99 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx$ 1222
3.100 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$ 1234
3.101 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$ 1244
3.102 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$ 1253
3.103 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$ 1264
3.104 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$ 1275
3.105 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx$ 1287
3.106 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx$ 1298
3.107 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx$ 1310
3.108 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1322
3.109 $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1332
3.110 $\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1343
3.111 $\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1353
3.112 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$ 1360
3.113 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$ 1368
3.114 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$ 1375
3.115 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$ 1382
3.116 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$ 1391

3.117 $\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1401
3.118 $\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1412
3.119 $\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1423
3.120 $\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1434
3.121 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$ 1443
3.122 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$ 1454
3.123 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$ 1462
3.124 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$ 1471
3.125 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$ 1479
3.126 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx$ 1489
3.127 $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1499
3.128 $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1510
3.129 $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1521
3.130 $\int (ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 1531
3.131 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$ 1541
3.132 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$ 1553
3.133 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$ 1561
3.134 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$ 1569
3.135 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$ 1581
3.136 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$ 1589
3.137 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$ 1598
3.138 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx$ 1606
3.139 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx$ 1613
3.140 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx$ 1619
3.141 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx$ 1627
3.142 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx$ 1636
3.143 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$ 1647

3.144 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$ 1656

3.145 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$ 1664

3.146 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx$ 1671

3.147 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx$ 1678

3.148 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$ 1686

3.149 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx$ 1695

3.150 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx$ 1706

3.151 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$ 1717

3.152 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$ 1726

3.153 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$ 1734

3.154 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx$ 1741

3.155 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$ 1748

3.156 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$ 1757

3.157 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx$ 1767

3.158 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx$ 1777

3.159 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$ 1789

3.160 $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$ 1802

3.161 $\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$ 1816

3.162 $\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$ 1828

3.163 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$ 1837

3.164 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$ 1848

3.165 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$ 1858

3.166 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$ 1868

3.167 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$ 1880

3.168 $\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$ 1891

3.169 $\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$ 1909

3.170 $\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$ . . . 1928
3.171 $\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$ . . . 1945
3.172 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$ . . . 1959
3.173 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$ . . . 1974
3.174 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$ . . . 1983
3.175 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$ . . . 1995
3.176 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$ . . . 2005
3.177 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx$ . . . 2016
3.178 $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$ . . . 2027
3.179 $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$ . . . 2050
3.180 $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$ . . . 2071
3.181 $\int (ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$ . . . 2093
3.182 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$ . . . 2109
3.183 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$ . . . 2133
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3.191 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$ . . . 2206
3.192 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$ . . . 2215
3.193 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx$ . . . 2226
3.194 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$ . . . 2237

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3.201 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx$ 2301
3.202 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$ 2312
3.203 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$ 2321
3.204 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$ 2330
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3.208 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx$ 2373
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3.210 $\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$ 2395
3.211 $\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$ 2401
3.212 $\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx$ 2407
3.213 $\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$ 2415
3.214 $\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 2424
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3.216 $\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$ 2437
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3.218 $\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx$ 2452
3.219 $\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$ 2460
3.220 $\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$ 2468

3.221 $\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$ . . . 2475
3.222 $\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$ . . . 2481
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3.224 $\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$ . . . 2497
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3.226 $\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$ . . . 2508
3.227 $\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$ . . . 2514
3.228 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)(c+dx)}\,dx$ . . . 2520
3.229 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)(c+dx)}\,dx$ . . . 2527
3.230 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx$ . . . 2534
3.231 $\int \frac{1}{(a+bx)(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$ . . . 2540
3.232 $\int \frac{1}{(a+bx)(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}\,dx$ . . . 2546
3.233 $\int \frac{1}{(a+bx)(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}\,dx$ . . . 2552
3.234 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p}{(a+bx)(c+dx)}\,dx$ . . . 2559
3.235 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p}{(af+bfx)(cg+dgx)}\,dx$ . . . 2565
3.236 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p}{acf+(bc+ad)fx+bdfx^2}\,dx$ . . . 2571
3.237 $\int \frac{1}{(a+bx)(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$ . . . 2577
3.238 $\int \frac{1}{(af+bfx)(cg+dgx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$ . . . 2583
3.239 $\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$ . . . 2589
3.240 $\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log(e(a+bx)^n(c+dx)^{-n})}\,dx$ . . . 2595
3.241 $\int \frac{(a+bx)^3}{(c+dx)^5\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$ . . . 2601
3.242 $\int \frac{(a+bx)^2}{(c+dx)^4\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$ . . . 2607
3.243 $\int \frac{a+bx}{(c+dx)^3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$ . . . 2613
3.244 $\int \frac{1}{(c+dx)^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$ . . . 2619
3.245 $\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$ . . . 2624
3.246 $\int \frac{1}{(a+bx)^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$ . . . 2630
3.247 $\int \frac{c+dx}{(a+bx)^3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$ . . . 2635
3.248 $\int \frac{(c+dx)^2}{(a+bx)^4\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$ . . . 2641
3.249 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^4}{(f+gx)(ah+bhx)}\,dx$ . . . 2647

3.250 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(f+gx)(ah+bhx)}\,dx$ 2655
3.251 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(f+gx)(ah+bhx)}\,dx$ 2663
3.252 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx$ 2671
3.253 $\int \frac{1}{(f+gx)(ah+bhx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$ 2679
3.254 $\int \frac{1}{(f+gx)(ah+bhx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}\,dx$ 2685
3.255 $\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx$ 2692
3.256 $\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$ 2699
3.257 $\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$ 2706
3.258 $\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$ 2713
3.259 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{afh+bghx^2+h(bfx+agx)}\,dx$ 2720
3.260 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{afh+bghx^2+h(bfx+agx)}\,dx$ 2728
3.261 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh+bghx^2+h(bfx+agx)}\,dx$ 2736

3.1 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

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Optimal result

Integrand size = 38, antiderivative size = 212

$$\begin{aligned}
&\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx \\
&= -\frac{B(bc-ad)^4g^3ix}{20bd^3}+\frac{B(bc-ad)^3g^3i(a+bx)^2}{40b^2d^2} \\
&\quad -\frac{B(bc-ad)^2g^3i(a+bx)^3}{60b^2d}+\frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b} \\
&\quad +\frac{(bc-ad)g^3i(a+bx)^4\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^2}+\frac{B(bc-ad)^5g^3i\log(c+dx)}{20b^2d^4}
\end{aligned}$$

output

```
-1/20*B*(-a*d+b*c)^4*g^3*i*x/b/d^3+1/40*B*(-a*d+b*c)^3*g^3*i*(b*x+a)^2/b^2
/d^2-1/60*B*(-a*d+b*c)^2*g^3*i*(b*x+a)^3/b^2/d+1/5*g^3*i*(b*x+a)^4*(d*x+c)
*(A+B*ln(e*(b*x+a)/(d*x+c)))/b+1/20*(-a*d+b*c)*g^3*i*(b*x+a)^4*(A-B+B*ln(e
*(b*x+a)/(d*x+c)))/b^2+1/20*B*(-a*d+b*c)^5*g^3*i*ln(d*x+c)/b^2/d^4
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.23

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{g^3 i\Big(30(bc-ad)(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+24d(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-\frac{5B(bc-ad)^2(6bd($$

input
```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)
]),x]
```

output
```
(g^3*i*(30*(b*c - a*d)*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) +
24*d*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (5*B*(b*c - a*d)^2
*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*
x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/d^4 + (2*B*(b*c - a*d)*(12*b*d*(b*c
- a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c - a*d)*(a + b*x)
^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]))/d^4))/(120*b^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2960, 27, 2948, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$
$$\downarrow \text{2960}$$
$$\frac{i(bc-ad)\int g^3(a+bx)^3\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx}{5b}+$$
$$\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}$$

$$\downarrow 27$$

$$\frac{g^3 i(bc-ad)\int (a+bx)^3\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx}{5b}+\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}$$

$$\downarrow 2948$$

$$\frac{g^3 i(bc-ad)\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{4b}-\frac{B(bc-ad)\int\frac{(a+bx)^3}{c+dx}dx}{4b}\right)}{5b}+\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}$$

$$\downarrow 49$$

$$\frac{g^3 i(bc-ad)\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{4b}-\frac{B(bc-ad)\int\left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{4b}\right)}{5b}+\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}$$

$$\downarrow 2009$$

$$\frac{g^3 i(bc-ad)\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{4b}-\frac{B(bc-ad)\left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{4b}\right)}{5b}+\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g^3*i*(a + b*x)^4*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*b) +
 ((b*c - a*d)*g^3*i*(((a + b*x)^4*(A - B + B*Log[(e*(a + b*x))/(c + d*x)])
)/(4*b) - (B*(b*c - a*d)*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)
^2)/(2*d^2) + (a + b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4))/(4*b)
))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

rule 2960
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Sim
p[(f + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*
(m + 2))), x] + Simp[i*((b*c - a*d)/(b*d*(m + 2)))   Int[(f + g*x)^m*(A - B
*n + B*Log[e*((a + b*x)^n/(c + d*x)^n)]), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, i, A, B, m, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IGtQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(200) = 400.

Time = 1.48 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.71

method	result
risch	$\frac{i g^3 bB \ln(-dx-c) a^2 c^3}{2d^2} - \frac{i g^3 b^2 B \ln(-dx-c) a c^4}{4d^3} + \frac{i g^3 B a^3 cx}{4} - \frac{i g^3 bB a^2 c^2 x}{2d} + \frac{i g^3 b^2 Ba c^3 x}{4d^2} + i g^3 b^2 Aac$
parallelrisch	Expression too large to display
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURN
VERBOSE)
```

output
```
1/2*i*g^3*b/d^2*B*ln(-d*x-c)*a^2*c^3-1/4*i*g^3*b^2/d^3*B*ln(-d*x-c)*a*c^4+
1/4*i*g^3*B*a^3*c*x-1/2*i*g^3*b/d*B*a^2*c^2*x+1/4*i*g^3*b^2/d^2*B*a*c^3*x+
i*g^3*b^2*A*a*c*x^3-1/6*i*g^3*b^2*B*a*c*x^3+3/2*i*g^3*b*A*a^2*c*x^2-1/8*i*
g^3*b*B*a^2*c*x^2-1/8*i*g^3*b^2/d*B*a*c^2*x^2+i*g^3*A*a^3*c*x+1/40*i*g^3*b
^3/d^2*B*c^3*x^2+1/20*i*g^3/b*d*B*a^4*x-1/20*i*g^3*b^3/d^3*B*c^4*x+1/4*i*g
^3/b*B*ln(b*x+a)*a^4*c-1/2*i*g^3/d*B*ln(-d*x-c)*a^3*c^2-1/20*i*g^3/b^2*d*B
*ln(b*x+a)*a^5+1/20*i*g^3*b^3/d^4*B*ln(-d*x-c)*c^5+3/4*i*g^3*b^2*d*A*a*x^4
+1/4*i*g^3*b^3*A*c*x^4+1/20*i*g^3*b^2*d*B*a*x^4-1/20*i*g^3*b^3*B*c*x^4+i*g
^3*b*d*A*a^2*x^3+11/60*i*g^3*b*d*B*a^2*x^3-1/60*i*g^3*b^3/d*B*c^2*x^3+1/2*
i*g^3*d*A*a^3*x^2+9/40*i*g^3*d*B*a^3*x^2+1/5*i*g^3*b^3*d*A*x^5+1/20*i*g^3*
B*x*(4*b^3*d*x^4+15*a*b^2*d*x^3+5*b^3*c*x^3+20*a^2*b*d*x^2+20*a*b^2*c*x^2+
10*a^3*d*x+30*a^2*b*c*x+20*a^3*c)*ln(e*(b*x+a)/(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. 2(200) = 400.

Time = 0.13 (sec) , antiderivative size = 504, normalized size of antiderivative = 2.38

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$= \frac{24\,Ab^5d^5g^3ix^5+6\left((5\,A-B)b^5cd^4+(15\,A+B)ab^4d^5\right)g^3ix^4-2\left(Bb^5c^2d^3-10\,(6\,A-B)ab^4cd^4-(60\,A$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="fricas")
```

output
```
1/120*(24*A*b^5*d^5*g^3*i*x^5 + 6*((5*A - B)*b^5*c*d^4 + (15*A + B)*a*b^4*
d^5)*g^3*i*x^4 - 2*(B*b^5*c^2*d^3 - 10*(6*A - B)*a*b^4*c*d^4 - (60*A + 11*
B)*a^2*b^3*d^5)*g^3*i*x^3 + 3*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 5*(12*A
 - B)*a^2*b^3*c*d^4 + (20*A + 9*B)*a^3*b^2*d^5)*g^3*i*x^2 - 6*(B*b^5*c^4*d
 - 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 5*(4*A + B)*a^3*b^2*c*d^4 -
B*a^4*b*d^5)*g^3*i*x + 6*(5*B*a^4*b*c*d^4 - B*a^5*d^5)*g^3*i*log(b*x + a)
+ 6*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2
*d^3)*g^3*i*log(d*x + c) + 6*(4*B*b^5*d^5*g^3*i*x^5 + 20*B*a^3*b^2*c*d^4*g
^3*i*x + 5*(B*b^5*c*d^4 + 3*B*a*b^4*d^5)*g^3*i*x^4 + 20*(B*a*b^4*c*d^4 + B
*a^2*b^3*d^5)*g^3*i*x^3 + 10*(3*B*a^2*b^3*c*d^4 + B*a^3*b^2*d^5)*g^3*i*x^2
)*log((b*e*x + a*e)/(d*x + c)))/(b^2*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1158 vs. $2(194) = 388$.

Time $= 3.79$ (sec) , antiderivative size $= 1158$, normalized size of antiderivative $= 5.46$

$$\int (ag + bgx)^3 (ci + dix) \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right) dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output

```
A*b**3*d*g**3*i*x**5/5 - B*a**4*g**3*i*(a*d - 5*b*c)*log(x + (B*a**5*c*d**
4*g**3*i + B*a**5*d**4*g**3*i*(a*d - 5*b*c)/b - 15*B*a**4*b*c**2*d**3*g**3
*i - B*a**4*c*d**3*g**3*i*(a*d - 5*b*c) + 10*B*a**3*b**2*c**3*d**2*g**3*i
- 5*B*a**2*b**3*c**4*d*g**3*i + B*a*b**4*c**5*g**3*i)/(B*a**5*d**5*g**3*i
- 5*B*a**4*b*c*d**4*g**3*i - 10*B*a**3*b**2*c**2*d**3*g**3*i + 10*B*a**2*b
**3*c**3*d**2*g**3*i - 5*B*a*b**4*c**4*d*g**3*i + B*b**5*c**5*g**3*i))/(20
*b**2) - B*c**2*g**3*i*(10*a**3*d**3 - 10*a**2*b*c*d**2 + 5*a*b**2*c**2*d
- b**3*c**3)*log(x + (B*a**5*c*d**4*g**3*i - 15*B*a**4*b*c**2*d**3*g**3*i
+ 10*B*a**3*b**2*c**3*d**2*g**3*i - 5*B*a**2*b**3*c**4*d*g**3*i + B*a*b**4
*c**5*g**3*i + B*a*b*c**2*g**3*i*(10*a**3*d**3 - 10*a**2*b*c*d**2 + 5*a*b*
*2*c**2*d - b**3*c**3) - B*b**2*c**3*g**3*i*(10*a**3*d**3 - 10*a**2*b*c*d*
*2 + 5*a*b**2*c**2*d - b**3*c**3)/d)/(B*a**5*d**5*g**3*i - 5*B*a**4*b*c*d*
*4*g**3*i - 10*B*a**3*b**2*c**2*d**3*g**3*i + 10*B*a**2*b**3*c**3*d**2*g**
3*i - 5*B*a*b**4*c**4*d*g**3*i + B*b**5*c**5*g**3*i))/(20*d**4) + x**4*(3*
A*a*b**2*d*g**3*i/4 + A*b**3*c*g**3*i/4 + B*a*b**2*d*g**3*i/20 - B*b**3*c*
g**3*i/20) + x**3*(A*a**2*b*d*g**3*i + A*a*b**2*c*g**3*i + 11*B*a**2*b*d*g
**3*i/60 - B*a*b**2*c*g**3*i/6 - B*b**3*c**2*g**3*i/(60*d)) + x**2*(A*a**3
*d*g**3*i/2 + 3*A*a**2*b*c*g**3*i/2 + 9*B*a**3*d*g**3*i/40 - B*a**2*b*c*g*
*3*i/8 - B*a*b**2*c**2*g**3*i/(8*d) + B*b**3*c**3*g**3*i/(40*d**2)) + x*(A
*a**3*c*g**3*i + B*a**4*d*g**3*i/(20*b) + B*a**3*c*g**3*i/4 - B*a**2*b*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1022 vs. $2(200) = 400$.

Time $= 0.07$ (sec) , antiderivative size $= 1022$, normalized size of antiderivative $= 4.82$

$$\int (ag + bgx)^3 (ci + dix) \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="maxima")
```

output

```
1/5*A*b^3*d*g^3*i*x^5 + 1/4*A*b^3*c*g^3*i*x^4 + 3/4*A*a*b^2*d*g^3*i*x^4 +
A*a*b^2*c*g^3*i*x^3 + A*a^2*b*d*g^3*i*x^3 + 3/2*A*a^2*b*c*g^3*i*x^2 + 1/2*
A*a^3*d*g^3*i*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x +
a)/b - c*log(d*x + c)/d)*B*a^3*c*g^3*i + 3/2*(x^2*log(b*e*x/(d*x + c) + a*
e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x
/(b*d))*B*a^2*b*c*g^3*i + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))
+ 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x
^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b^2*c*g^3*i + 1/24*(6*x^4*log
(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x
 + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2
 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*c*g^3*i + 1/2*(x^2*log(b*e*x/
(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 -
 (b*c - a*d)*x/(b*d))*B*a^3*d*g^3*i + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e
/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d
- a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^2*b*d*g^3*i + 1/8
*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*
c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2
*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*a*b^2*d*g^3*i + 1/60*(
12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12
*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3637 vs. $2(200) = 400$.

Time $= 0.31$ (sec) , antiderivative size $= 3637$, normalized size of antiderivative $= 17.16$

$$\int (ag + bgx)^3 (ci + dix) \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="giac")
```

output

```
-1/120*(6*(B*b^9*c^6*e^6*g^3*i - 6*B*a*b^8*c^5*d*e^6*g^3*i + 15*B*a^2*b^7*
c^4*d^2*e^6*g^3*i - 20*B*a^3*b^6*c^3*d^3*e^6*g^3*i + 15*B*a^4*b^5*c^2*d^4*
e^6*g^3*i - 6*B*a^5*b^4*c*d^5*e^6*g^3*i + B*a^6*b^3*d^6*e^6*g^3*i - 5*(b*e
*x + a*e)*B*b^8*c^6*d*e^5*g^3*i/(d*x + c) + 30*(b*e*x + a*e)*B*a*b^7*c^5*d
^2*e^5*g^3*i/(d*x + c) - 75*(b*e*x + a*e)*B*a^2*b^6*c^4*d^3*e^5*g^3*i/(d*x
 + c) + 100*(b*e*x + a*e)*B*a^3*b^5*c^3*d^4*e^5*g^3*i/(d*x + c) - 75*(b*e*
x + a*e)*B*a^4*b^4*c^2*d^5*e^5*g^3*i/(d*x + c) + 30*(b*e*x + a*e)*B*a^5*b^
3*c*d^6*e^5*g^3*i/(d*x + c) - 5*(b*e*x + a*e)*B*a^6*b^2*d^7*e^5*g^3*i/(d*x
 + c) + 10*(b*e*x + a*e)^2*B*b^7*c^6*d^2*e^4*g^3*i/(d*x + c)^2 - 60*(b*e*x
 + a*e)^2*B*a*b^6*c^5*d^3*e^4*g^3*i/(d*x + c)^2 + 150*(b*e*x + a*e)^2*B*a^
2*b^5*c^4*d^4*e^4*g^3*i/(d*x + c)^2 - 200*(b*e*x + a*e)^2*B*a^3*b^4*c^3*d^
5*e^4*g^3*i/(d*x + c)^2 + 150*(b*e*x + a*e)^2*B*a^4*b^3*c^2*d^6*e^4*g^3*i/
(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a^5*b^2*c*d^7*e^4*g^3*i/(d*x + c)^2 + 1
0*(b*e*x + a*e)^2*B*a^6*b*d^8*e^4*g^3*i/(d*x + c)^2 - 10*(b*e*x + a*e)^3*B
*b^6*c^6*d^3*e^3*g^3*i/(d*x + c)^3 + 60*(b*e*x + a*e)^3*B*a*b^5*c^5*d^4*e^
3*g^3*i/(d*x + c)^3 - 150*(b*e*x + a*e)^3*B*a^2*b^4*c^4*d^5*e^3*g^3*i/(d*x
 + c)^3 + 200*(b*e*x + a*e)^3*B*a^3*b^3*c^3*d^6*e^3*g^3*i/(d*x + c)^3 - 15
0*(b*e*x + a*e)^3*B*a^4*b^2*c^2*d^7*e^3*g^3*i/(d*x + c)^3 + 60*(b*e*x + a*
e)^3*B*a^5*b*c*d^8*e^3*g^3*i/(d*x + c)^3 - 10*(b*e*x + a*e)^3*B*a^6*d^9*e^
3*g^3*i/(d*x + c)^3)*log((b*e*x + a*e)/(d*x + c))/(b^5*d^4*e^5 - 5*(b*e...
```

Mupad [B] (verification not implemented)

Time = 27.20 (sec) , antiderivative size = 1195, normalized size of antiderivative = 5.64

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \text{Too large to display}$$

input

```
int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output

```
x*((a*c*(((20*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d - B*b
*c))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(20*b*d) - (b*g^3*i*(24*A*a^
2*d^2 + 4*A*b^2*c^2 + 3*B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d - 2*B*a*b*c*d
))/(4*d) + A*a*b^2*c*g^3*i))/(b*d) - ((20*a*d + 20*b*c)*(((20*a*d + 20*b*c
)*(((20*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d - B*b*c))/5
 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(20*b*d) - (b*g^3*i*(24*A*a^2*d^2
+ 4*A*b^2*c^2 + 3*B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d - 2*B*a*b*c*d))/(4*
d) + A*a*b^2*c*g^3*i))/(20*b*d) - (a*c*((b^2*g^3*i*(20*A*a*d + 10*A*b*c +
B*a*d - B*b*c))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(b*d) + (a*g^3*i*
(4*A*a^2*d^2 + 4*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 12*A*a*b*c*d))/d))/(2
0*b*d) + (a^2*g^3*i*(2*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2 - 3*B*b^2*c^2
+ 16*A*a*b*c*d + 2*B*a*b*c*d))/(2*b*d)) + x^4*((b^2*g^3*i*(20*A*a*d + 10*A
*b*c + B*a*d - B*b*c))/20 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/80) - x^3*(((2
0*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d - B*b*c))/5 - (A*
b^2*g^3*i*(20*a*d + 20*b*c))/20))/(60*b*d) - (b*g^3*i*(24*A*a^2*d^2 + 4*A*
b^2*c^2 + 3*B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d - 2*B*a*b*c*d))/(12*d) +
(A*a*b^2*c*g^3*i)/3) + x^2*(((20*a*d + 20*b*c)*(((20*a*d + 20*b*c)*((b^2*g
^3*i*(20*A*a*d + 10*A*b*c + B*a*d - B*b*c))/5 - (A*b^2*g^3*i*(20*a*d + 20*
b*c))/20))/(20*b*d) - (b*g^3*i*(24*A*a^2*d^2 + 4*A*b^2*c^2 + 3*B*a^2*d^2 -
 B*b^2*c^2 + 32*A*a*b*c*d - 2*B*a*b*c*d))/(4*d) + A*a*b^2*c*g^3*i))/(40...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 757, normalized size of antiderivative = 3.57

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{g^3 i\left(24\log\left(\frac{bex+ae}{dx+c}\right)b^5d^5x^5+6a^4b\,d^5x+27a^3b^2d^5x^2+22a^2b^3d^5x^3+6a\,b^4d^5x^4-6b^5c^4dx+3b^5c^3d^2x^2-2\right.}{}$$

input

```
int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output

```
(g**3*i*( - 6*log(a + b*x)*a**5*d**5 + 30*log(a + b*x)*a**4*b*c*d**4 - 60*
log(a + b*x)*a**3*b**2*c**2*d**3 + 60*log(a + b*x)*a**2*b**3*c**3*d**2 - 3
0*log(a + b*x)*a*b**4*c**4*d + 6*log(a + b*x)*b**5*c**5 + 60*log((a*e + b*
e*x)/(c + d*x))*a**3*b**2*c**2*d**3 + 120*log((a*e + b*e*x)/(c + d*x))*a**
3*b**2*c*d**4*x + 60*log((a*e + b*e*x)/(c + d*x))*a**3*b**2*d**5*x**2 - 60
*log((a*e + b*e*x)/(c + d*x))*a**2*b**3*c**3*d**2 + 180*log((a*e + b*e*x)/
(c + d*x))*a**2*b**3*c*d**4*x**2 + 120*log((a*e + b*e*x)/(c + d*x))*a**2*b
**3*d**5*x**3 + 30*log((a*e + b*e*x)/(c + d*x))*a*b**4*c**4*d + 120*log((a
*e + b*e*x)/(c + d*x))*a*b**4*c*d**4*x**3 + 90*log((a*e + b*e*x)/(c + d*x)
)*a*b**4*d**5*x**4 - 6*log((a*e + b*e*x)/(c + d*x))*b**5*c**5 + 30*log((a*
e + b*e*x)/(c + d*x))*b**5*c*d**4*x**4 + 24*log((a*e + b*e*x)/(c + d*x))*b
**5*d**5*x**5 + 120*a**4*b*c*d**4*x + 60*a**4*b*d**5*x**2 + 6*a**4*b*d**5*
x + 180*a**3*b**2*c*d**4*x**2 + 30*a**3*b**2*c*d**4*x + 120*a**3*b**2*d**5
*x**3 + 27*a**3*b**2*d**5*x**2 - 60*a**2*b**3*c**2*d**3*x + 120*a**2*b**3*
c*d**4*x**3 - 15*a**2*b**3*c*d**4*x**2 + 90*a**2*b**3*d**5*x**4 + 22*a**2*
b**3*d**5*x**3 + 30*a*b**4*c**3*d**2*x - 15*a*b**4*c**2*d**3*x**2 + 30*a*b
**4*c*d**4*x**4 - 20*a*b**4*c*d**4*x**3 + 24*a*b**4*d**5*x**5 + 6*a*b**4*d
**5*x**4 - 6*b**5*c**4*d*x + 3*b**5*c**3*d**2*x**2 - 2*b**5*c**2*d**3*x**3
 - 6*b**5*c*d**4*x**4))/(120*b*d**4)
```

3.2 $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

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Optimal result

Integrand size = 38, antiderivative size = 180

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx \\
&= \frac{B(bc-ad)^3g^2ix}{12bd^2} - \frac{B(bc-ad)^2g^2i(a+bx)^2}{24b^2d} \\
&\quad + \frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b} \\
&\quad + \frac{(bc-ad)g^2i(a+bx)^3\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2} - \frac{B(bc-ad)^4g^2i\log(c+dx)}{12b^2d^3}
\end{aligned}$$

output

```
1/12*B*(-a*d+b*c)^3*g^2*i*x/b/d^2-1/24*B*(-a*d+b*c)^2*g^2*i*(b*x+a)^2/b^2/
d+1/4*g^2*i*(b*x+a)^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b+1/12*(-a*d+b*c
)*g^2*i*(b*x+a)^3*(A-B+B*ln(e*(b*x+a)/(d*x+c)))/b^2-1/12*B*(-a*d+b*c)^4*g^
2*i*ln(d*x+c)/b^2/d^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.21

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{g^2 i\left(8(bc-ad)(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+6d(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{4B(bc-ad)^2(2bd(bc-}{24b^2}$$

input
```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)
]),x]
```

output
```
(g^2*i*(8*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6
*d*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (4*B*(b*c - a*d)^2*(
2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]))/d^3
 - (B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^
2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/d^3))/(24*b^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2960, 27, 2948, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$
$$\downarrow 2960$$
$$\frac{i(bc-ad)\int g^2(a+bx)^2\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx}{4b}+$$
$$\frac{g^2 i(a+bx)^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}$$
$$\downarrow 27$$

$$\frac{g^2 i(bc-ad)\int (a+bx)^2\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx}{4b}+\frac{g^2 i(a+bx)^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}$$

$$\downarrow 2948$$

$$\frac{g^2 i(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{3b}-\frac{B(bc-ad)\int\frac{(a+bx)^2}{c+dx}dx}{3b}\right)}{4b}+\frac{g^2 i(a+bx)^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}$$

$$\downarrow 49$$

$$\frac{g^2 i(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{3b}-\frac{B(bc-ad)\int\left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{3b}\right)}{4b}+\frac{g^2 i(a+bx)^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}$$

$$\downarrow 2009$$

$$\frac{g^2 i(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{3b}-\frac{B(bc-ad)\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{3b}\right)}{4b}+\frac{g^2 i(a+bx)^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}$$

input

```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output

```
(g^2*i*(a + b*x)^3*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*b) +
 ((b*c - a*d)*g^2*i*(((a + b*x)^3*(A - B + B*Log[(e*(a + b*x))/(c + d*x)])
)/(3*b) - (B*(b*c - a*d)*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + (
(b*c - a*d)^2*Log[c + d*x])/d^3))/(3*b)))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

rule 2960
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Sim
p[(f + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*
(m + 2))), x] + Simp[i*((b*c - a*d)/(b*d*(m + 2)))   Int[(f + g*x)^m*(A - B
*n + B*Log[e*((a + b*x)^n/(c + d*x)^n)]), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, i, A, B, m, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IGtQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(170) = 340.

Time = 1.26 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.20

method	result
risch	$\frac{i g^2 Bx(3b^2dx^3+8abdx^2+4b^2cx^2+6a^2dx+12abcx+12a^2c)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{12}+\frac{ig^2b^2dAx^4}{4}+\frac{2ig^2bdAax^3}{3}+\frac{ig^2b^2}{3}$
parallelrisch	$\frac{16Bx^3\ln\left(\frac{e(bx+a)}{dx+c}\right)ab^3d^4g^2i+8Ba^2b^2c^2d^2g^2i+7Bab^3c^3dg^2i-11Ba^3bcd^3g^2i+12Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2b^2d^4g^2i+24A$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURN
VERBOSE)
```

output
```
1/12*i*g^2*B*x*(3*b^2*d*x^3+8*a*b*d*x^2+4*b^2*c*x^2+6*a^2*d*x+12*a*b*c*x+1
2*a^2*c)*ln(e*(b*x+a)/(d*x+c))+1/4*i*g^2*b^2*d*A*x^4+2/3*i*g^2*b*d*A*a*x^3
+1/3*i*g^2*b^2*A*c*x^3+1/12*i*g^2*b*d*B*a*x^3-1/12*i*g^2*b^2*B*c*x^3+1/2*i
*g^2*d*A*a^2*x^2+i*g^2*b*A*a*c*x^2+5/24*i*g^2*d*B*a^2*x^2-1/6*i*g^2*b*B*a*
c*x^2-1/24*i*g^2*b^2/d*B*c^2*x^2+i*g^2*A*a^2*c*x-1/2*i*g^2/d*B*ln(-d*x-c)*
a^2*c^2+1/3*i*g^2*b/d^2*B*ln(-d*x-c)*a*c^3-1/12*i*g^2*b^2/d^3*B*ln(-d*x-c)
*c^4-1/12*i*g^2/b^2*d*B*ln(b*x+a)*a^4+1/3*i*g^2/b*B*ln(b*x+a)*a^3*c+1/12*i
*g^2/b*d*B*a^3*x+1/6*i*g^2*B*a^2*c*x-1/3*i*g^2*b/d*B*a*c^2*x+1/12*i*g^2*b^
2/d^2*B*c^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(170) = 340.

Time = 0.10 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.06

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{6Ab^4d^4g^2ix^4+2\left((4A-B)b^4cd^3+(8A+B)ab^3d^4\right)g^2ix^3-\left(Bb^4c^2d^2-4(6A-B)ab^3cd^3-(12A+5\right.}{}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="fricas")
```

output
```
1/24*(6*A*b^4*d^4*g^2*i*x^4 + 2*((4*A - B)*b^4*c*d^3 + (8*A + B)*a*b^3*d^4
)*g^2*i*x^3 - (B*b^4*c^2*d^2 - 4*(6*A - B)*a*b^3*c*d^3 - (12*A + 5*B)*a^2*
b^2*d^4)*g^2*i*x^2 + 2*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 2*(6*A + B)*a^2*
b^2*c*d^3 + B*a^3*b*d^4)*g^2*i*x + 2*(4*B*a^3*b*c*d^3 - B*a^4*d^4)*g^2*i*l
og(b*x + a) - 2*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2)*g^2*i*
log(d*x + c) + 2*(3*B*b^4*d^4*g^2*i*x^4 + 12*B*a^2*b^2*c*d^3*g^2*i*x + 4*(
B*b^4*c*d^3 + 2*B*a*b^3*d^4)*g^2*i*x^3 + 6*(2*B*a*b^3*c*d^3 + B*a^2*b^2*d^
4)*g^2*i*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^2*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(163) = 326$.

Time = 2.50 (sec) , antiderivative size = 850, normalized size of antiderivative = 4.72

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{Ab^2dg^2ix^4}{4}$$
$$-\frac{Ba^3g^2i(ad-4bc)\log\left(x+\frac{\frac{Ba^4cd^3g^2i+\frac{Ba^4d^3g^2i(ad-4bc)}{b}-10Ba^3bc^2d^2g^2i-Ba^3cd^2g^2i(ad-4bc)+4Ba^2b^2c^3dg^2i-Bab^3c^4g^2i}{Ba^4d^4g^2i-4Ba^3bcd^3g^2i-6Ba^2b^2c^2d^2g^2i+4Bab^3c^3dg^2i-Bb^4c^4g^2i}\right)}{12b^2}$$
$$-\frac{Bc^2g^2i(6a^2d^2-4abcd+b^2c^2)\log\left(x+\frac{Ba^4cd^3g^2i-10Ba^3bc^2d^2g^2i+4Ba^2b^2c^3dg^2i-Bab^3c^4g^2i+Babc^2g^2i(6a^2d^2-4abcd+}{Ba^4d^4g^2i-4Ba^3bcd^3g^2i-6Ba^2b^2c^2d^2g^2i+4Bab^3c^3dg^2i-}\right)}{12d^3}$$
$$+x^3\cdot\left(\frac{2Aabdg^2i}{3}+\frac{Ab^2cg^2i}{3}+\frac{Babdg^2i}{12}-\frac{Bb^2cg^2i}{12}\right)$$
$$+x^2\left(\frac{Aa^2dg^2i}{2}+Aabcg^2i+\frac{5Ba^2dg^2i}{24}-\frac{Babcg^2i}{6}-\frac{Bb^2c^2g^2i}{24d}\right)$$
$$+x\left(Aa^2cg^2i+\frac{Ba^3dg^2i}{12b}+\frac{Ba^2cg^2i}{6}-\frac{Babc^2g^2i}{3d}+\frac{Bb^2c^3g^2i}{12d^2}\right)+\left(Ba^2cg^2ix\right.$$
$$\left.+\frac{Ba^2dg^2ix^2}{2}+Babcg^2ix^2+\frac{2Babdg^2ix^3}{3}+\frac{Bb^2cg^2ix^3}{3}+\frac{Bb^2dg^2ix^4}{4}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output

```
A*b**2*d*g**2*i*x**4/4 - B*a**3*g**2*i*(a*d - 4*b*c)*log(x + (B*a**4*c*d**
3*g**2*i + B*a**4*d**3*g**2*i*(a*d - 4*b*c)/b - 10*B*a**3*b*c**2*d**2*g**2
*i - B*a**3*c*d**2*g**2*i*(a*d - 4*b*c) + 4*B*a**2*b**2*c**3*d*g**2*i - B*
a*b**3*c**4*g**2*i)/(B*a**4*d**4*g**2*i - 4*B*a**3*b*c*d**3*g**2*i - 6*B*a
**2*b**2*c**2*d**2*g**2*i + 4*B*a*b**3*c**3*d*g**2*i - B*b**4*c**4*g**2*i)
)/(12*b**2) - B*c**2*g**2*i*(6*a**2*d**2 - 4*a*b*c*d + b**2*c**2)*log(x +
(B*a**4*c*d**3*g**2*i - 10*B*a**3*b*c**2*d**2*g**2*i + 4*B*a**2*b**2*c**3*
d*g**2*i - B*a*b**3*c**4*g**2*i + B*a*b*c**2*g**2*i*(6*a**2*d**2 - 4*a*b*c
*d + b**2*c**2) - B*b**2*c**3*g**2*i*(6*a**2*d**2 - 4*a*b*c*d + b**2*c**2)
/d)/(B*a**4*d**4*g**2*i - 4*B*a**3*b*c*d**3*g**2*i - 6*B*a**2*b**2*c**2*d*
*2*g**2*i + 4*B*a*b**3*c**3*d*g**2*i - B*b**4*c**4*g**2*i))/(12*d**3) + x*
*3*(2*A*a*b*d*g**2*i/3 + A*b**2*c*g**2*i/3 + B*a*b*d*g**2*i/12 - B*b**2*c*
g**2*i/12) + x**2*(A*a**2*d*g**2*i/2 + A*a*b*c*g**2*i + 5*B*a**2*d*g**2*i/
24 - B*a*b*c*g**2*i/6 - B*b**2*c**2*g**2*i/(24*d)) + x*(A*a**2*c*g**2*i +
B*a**3*d*g**2*i/(12*b) + B*a**2*c*g**2*i/6 - B*a*b*c**2*g**2*i/(3*d) + B*b
**2*c**3*g**2*i/(12*d**2)) + (B*a**2*c*g**2*i*x + B*a**2*d*g**2*i*x**2/2 +
 B*a*b*c*g**2*i*x**2 + 2*B*a*b*d*g**2*i*x**3/3 + B*b**2*c*g**2*i*x**3/3 +
B*b**2*d*g**2*i*x**4/4)*log(e*(a + b*x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(170) = 340$.

Time = 0.05 (sec) , antiderivative size = 671, normalized size of antiderivative = 3.73

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{1}{4}Ab^2dg^2ix^4+\frac{1}{3}Ab^2cg^2ix^3+\frac{2}{3}Aabdg^2ix^3+Aabcg^2ix^2+\frac{1}{2}Aa^2dg^2ix^2$$
$$+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Ba^2cg^2i$$
$$+\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Babcg^2i$$
$$+\frac{1}{6}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-}{b^2d^2}\right.$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Ba^2dg^2i$$
$$+\frac{1}{3}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-}{b^2d^2}\right.$$
$$+\frac{1}{24}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log(dx+c)}{d^4}-\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3}{}\right.$$
$$+Aa^2cg^2ix$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="maxima")
```

output
```
1/4*A*b^2*d*g^2*i*x^4 + 1/3*A*b^2*c*g^2*i*x^3 + 2/3*A*a*b*d*g^2*i*x^3 + A*
a*b*c*g^2*i*x^2 + 1/2*A*a^2*d*g^2*i*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a^2*c*g^2*i + (x^2*log(b*
e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d
^2 - (b*c - a*d)*x/(b*d))*B*a*b*c*g^2*i + 1/6*(2*x^3*log(b*e*x/(d*x + c) +
 a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*
c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*c*g^2*i + 1
/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*
log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a^2*d*g^2*i + 1/3*(2*x^3*log(b*e
*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c
)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a
*b*d*g^2*i + 1/24*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(
b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^2*d
*g^2*i + A*a^2*c*g^2*i*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2418 vs. $2(170) = 340$.

Time $= 0.28$ (sec) , antiderivative size $= 2418$, normalized size of antiderivative $= 13.43$

$$\int (ag + bgx)^2(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="giac")
```

output

```
1/24*(2*(B*b^7*c^5*e^5*g^2*i - 5*B*a*b^6*c^4*d*e^5*g^2*i + 10*B*a^2*b^5*c^
3*d^2*e^5*g^2*i - 10*B*a^3*b^4*c^2*d^3*e^5*g^2*i + 5*B*a^4*b^3*c*d^4*e^5*g
^2*i - B*a^5*b^2*d^5*e^5*g^2*i - 4*(b*e*x + a*e)*B*b^6*c^5*d*e^4*g^2*i/(d*
x + c) + 20*(b*e*x + a*e)*B*a*b^5*c^4*d^2*e^4*g^2*i/(d*x + c) - 40*(b*e*x
+ a*e)*B*a^2*b^4*c^3*d^3*e^4*g^2*i/(d*x + c) + 40*(b*e*x + a*e)*B*a^3*b^3*
c^2*d^4*e^4*g^2*i/(d*x + c) - 20*(b*e*x + a*e)*B*a^4*b^2*c*d^5*e^4*g^2*i/(
d*x + c) + 4*(b*e*x + a*e)*B*a^5*b*d^6*e^4*g^2*i/(d*x + c) + 6*(b*e*x + a*
e)^2*B*b^5*c^5*d^2*e^3*g^2*i/(d*x + c)^2 - 30*(b*e*x + a*e)^2*B*a*b^4*c^4*
d^3*e^3*g^2*i/(d*x + c)^2 + 60*(b*e*x + a*e)^2*B*a^2*b^3*c^3*d^4*e^3*g^2*i
/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a^3*b^2*c^2*d^5*e^3*g^2*i/(d*x + c)^2
+ 30*(b*e*x + a*e)^2*B*a^4*b*c*d^6*e^3*g^2*i/(d*x + c)^2 - 6*(b*e*x + a*e)
^2*B*a^5*d^7*e^3*g^2*i/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/(b^4*d^3*
e^4 - 4*(b*e*x + a*e)*b^3*d^4*e^3/(d*x + c) + 6*(b*e*x + a*e)^2*b^2*d^5*e^
2/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b*d^6*e/(d*x + c)^3 + (b*e*x + a*e)^4*d^
7/(d*x + c)^4) + (2*A*b^8*c^5*e^5*g^2*i + B*b^8*c^5*e^5*g^2*i - 10*A*a*b^7
*c^4*d*e^5*g^2*i - 5*B*a*b^7*c^4*d*e^5*g^2*i + 20*A*a^2*b^6*c^3*d^2*e^5*g^
2*i + 10*B*a^2*b^6*c^3*d^2*e^5*g^2*i - 20*A*a^3*b^5*c^2*d^3*e^5*g^2*i - 10
*B*a^3*b^5*c^2*d^3*e^5*g^2*i + 10*A*a^4*b^4*c*d^4*e^5*g^2*i + 5*B*a^4*b^4*
c*d^4*e^5*g^2*i - 2*A*a^5*b^3*d^5*e^5*g^2*i - B*a^5*b^3*d^5*e^5*g^2*i - 8*
(b*e*x + a*e)*A*b^7*c^5*d*e^4*g^2*i/(d*x + c) - 2*(b*e*x + a*e)*B*b^7*c...
```

Mupad [B] (verification not implemented)

Time = 26.50 (sec) , antiderivative size = 638, normalized size of antiderivative = 3.54

$$
\begin{aligned}
&\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx \\
&= x^3\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d-B\,b\,c)}{12}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{36}\right) \\
&\quad -x^2\left(\frac{\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{12}\right)\,(12\,a\,d+12\,b\,c)}{24\,b\,d}\right. \\
&\qquad \left.-\frac{g^2\,i\,(9\,A\,a^2\,d^2+3\,A\,b^2\,c^2+2\,B\,a^2\,d^2-B\,b^2\,c^2+18\,A\,a\,b\,c\,d-B\,a\,b\,c\,d)}{6\,d}\right. \\
&\qquad \left.+\frac{A\,a\,b\,c\,g^2\,i}{2}\right)+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(B\,a^2\,c\,g^2\,i\,x+\frac{B\,a\,g^2\,i\,x^2\,(a\,d+2\,b\,c)}{2}\right. \\
&\qquad \left.+\frac{B\,b\,g^2\,i\,x^3\,(2\,a\,d+b\,c)}{3}+\frac{B\,b^2\,d\,g^2\,i\,x^4}{4}\right) \\
&\quad +x\left(\frac{(12\,a\,d+12\,b\,c)\left(\frac{\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{12}\right)\,(12\,a\,d+12\,b\,c)}{12\,b\,d}-\frac{g^2\,i\,(9\,A\,a^2\,d^2+3\,A\,b^2\,c^2+2\,B\,a}{12\,b\,d}\right. \\
&\qquad \left.-\frac{a\,c\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{12}\right)}{b\,d}\right. \\
&\qquad \left.+\frac{a\,g^2\,i\,(2\,A\,a^2\,d^2+6\,A\,b^2\,c^2+B\,a^2\,d^2-2\,B\,b^2\,c^2+12\,A\,a\,b\,c\,d+B\,a\,b\,c\,d)}{2\,b\,d}\right) \\
&\quad -\frac{\ln(c+d\,x)\,(6\,B\,i\,a^2\,c^2\,d^2\,g^2-4\,B\,i\,a\,b\,c^3\,d\,g^2+B\,i\,b^2\,c^4\,g^2)}{12\,d^3} \\
&\quad -\frac{\ln(a+b\,x)\,(B\,a^4\,d\,g^2\,i-4\,B\,a^3\,b\,c\,g^2\,i)}{12\,b^2}+\frac{A\,b^2\,d\,g^2\,i\,x^4}{4}
\end{aligned}
$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output

```
x^3*((b*g^2*i*(12*A*a*d + 8*A*b*c + B*a*d - B*b*c))/12 - (A*b*g^2*i*(12*a*
d + 12*b*c))/36) - x^2*((((b*g^2*i*(12*A*a*d + 8*A*b*c + B*a*d - B*b*c))/4
 - (A*b*g^2*i*(12*a*d + 12*b*c))/12)*(12*a*d + 12*b*c))/(24*b*d) - (g^2*i*
(9*A*a^2*d^2 + 3*A*b^2*c^2 + 2*B*a^2*d^2 - B*b^2*c^2 + 18*A*a*b*c*d - B*a*
b*c*d))/(6*d) + (A*a*b*c*g^2*i)/2) + log((e*(a + b*x))/(c + d*x))*(B*a^2*c
*g^2*i*x + (B*a*g^2*i*x^2*(a*d + 2*b*c))/2 + (B*b*g^2*i*x^3*(2*a*d + b*c))
/3 + (B*b^2*d*g^2*i*x^4)/4) + x*(((12*a*d + 12*b*c)*((((b*g^2*i*(12*A*a*d
+ 8*A*b*c + B*a*d - B*b*c))/4 - (A*b*g^2*i*(12*a*d + 12*b*c))/12)*(12*a*d
+ 12*b*c))/(12*b*d) - (g^2*i*(9*A*a^2*d^2 + 3*A*b^2*c^2 + 2*B*a^2*d^2 - B*
b^2*c^2 + 18*A*a*b*c*d - B*a*b*c*d))/(3*d) + A*a*b*c*g^2*i))/(12*b*d) - (a
*c*((b*g^2*i*(12*A*a*d + 8*A*b*c + B*a*d - B*b*c))/4 - (A*b*g^2*i*(12*a*d
+ 12*b*c))/12))/(b*d) + (a*g^2*i*(2*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2 -
2*B*b^2*c^2 + 12*A*a*b*c*d + B*a*b*c*d))/(2*b*d)) - (log(c + d*x)*(B*b^2*c
^4*g^2*i + 6*B*a^2*c^2*d^2*g^2*i - 4*B*a*b*c^3*d*g^2*i))/(12*d^3) - (log(a
 + b*x)*(B*a^4*d*g^2*i - 4*B*a^3*b*c*g^2*i))/(12*b^2) + (A*b^2*d*g^2*i*x^4
)/4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 541, normalized size of antiderivative = 3.01

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{g^2i\left(2a^3b\,d^4x+5a^2b^2d^4x^2+2a\,b^3d^4x^3+2b^4c^3dx-b^4c^2d^2x^2-2b^4c\,d^3x^3+4a^2b^2c\,d^3x-8a\,b^3c^2d^2x-4a\right.}{}$$

input

```
int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output

```
(g**2*i*( - 2*log(a + b*x)*a**4*d**4 + 8*log(a + b*x)*a**3*b*c*d**3 - 12*l
og(a + b*x)*a**2*b**2*c**2*d**2 + 8*log(a + b*x)*a*b**3*c**3*d - 2*log(a +
 b*x)*b**4*c**4 + 12*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*c**2*d**2 + 24
*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*c*d**3*x + 12*log((a*e + b*e*x)/(c
 + d*x))*a**2*b**2*d**4*x**2 - 8*log((a*e + b*e*x)/(c + d*x))*a*b**3*c**3*
d + 24*log((a*e + b*e*x)/(c + d*x))*a*b**3*c*d**3*x**2 + 16*log((a*e + b*e
*x)/(c + d*x))*a*b**3*d**4*x**3 + 2*log((a*e + b*e*x)/(c + d*x))*b**4*c**4
 + 8*log((a*e + b*e*x)/(c + d*x))*b**4*c*d**3*x**3 + 6*log((a*e + b*e*x)/(
c + d*x))*b**4*d**4*x**4 + 24*a**3*b*c*d**3*x + 12*a**3*b*d**4*x**2 + 2*a*
*3*b*d**4*x + 24*a**2*b**2*c*d**3*x**2 + 4*a**2*b**2*c*d**3*x + 16*a**2*b*
*2*d**4*x**3 + 5*a**2*b**2*d**4*x**2 - 8*a*b**3*c**2*d**2*x + 8*a*b**3*c*d
**3*x**3 - 4*a*b**3*c*d**3*x**2 + 6*a*b**3*d**4*x**4 + 2*a*b**3*d**4*x**3
+ 2*b**4*c**3*d*x - b**4*c**2*d**2*x**2 - 2*b**4*c*d**3*x**3))/(24*b*d**3)
```

3.3 $\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$

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Optimal result

Integrand size = 36, antiderivative size = 140

$$\begin{aligned}&\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx\\&=-\frac{B(bc-ad)^2gix}{6bd}+\frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b}\\&\quad+\frac{(bc-ad)gi(a+bx)^2\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^2}+\frac{B(bc-ad)^3gi\log(c+dx)}{6b^2d^2}\end{aligned}$$

output

```
-1/6*B*(-a*d+b*c)^2*g*i*x/b/d+1/3*g*i*(b*x+a)^2*(d*x+c)*(A+B*ln(e*(b*x+a)/
(d*x+c)))/b+1/6*(-a*d+b*c)*g*i*(b*x+a)^2*(A-B+B*ln(e*(b*x+a)/(d*x+c)))/b^2
+1/6*B*(-a*d+b*c)^3*g*i*ln(d*x+c)/b^2/d^2
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.29

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{gi\Big(-a^2Bd^2(3bc+ad)\log(a+bx)+b\Big(dx(a^2Bd^2-b^2Bc(c+dx)+Ab^2dx(3c+2dx)+abd(6Ac+3Ad$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])
,x]
```

output
```
(g*i*(-(a^2*B*d^2*(3*b*c + a*d)*Log[a + b*x]) + b*(d*x*(a^2*B*d^2 - b^2*B*
c*(c + d*x) + A*b^2*d*x*(3*c + 2*d*x) + a*b*d*(6*A*c + 3*A*d*x + B*d*x)) +
 B*d^2*(6*a^2*c + 3*a*b*x*(2*c + d*x) + b^2*x^2*(3*c + 2*d*x))*Log[(e*(a +
 b*x))/(c + d*x)] + B*c*(b^2*c^2 - 3*a*b*c*d + 6*a^2*d^2)*Log[c + d*x])))/
(6*b^2*d^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2960, 27, 2948, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$
$$\downarrow 2960$$
$$\frac{i(bc-ad)\int g(a+bx)\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx}{3b}+$$
$$\frac{gi(a+bx)^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}$$
$$\downarrow 27$$

$$\frac{gi(bc-ad)\int(a+bx)\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}$$

$$\downarrow 2948$$

$$\frac{gi(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{2b}-\frac{B(bc-ad)\int\frac{a+bx}{c+dx}dx}{2b}\right)}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}$$

$$\downarrow 49$$

$$\frac{gi(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{2b}-\frac{B(bc-ad)\int\left(\frac{b}{d}+\frac{ad-bc}{d(c+dx)}\right)dx}{2b}\right)}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}$$

$$\downarrow 2009$$

$$\frac{gi(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{2b}-\frac{B(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\right)}{2b}\right)}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}$$

input
```
Int[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g*i*(a + b*x)^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b) + (
(b*c - a*d)*g*i*(((a + b*x)^2*(A - B + B*Log[(e*(a + b*x))/(c + d*x)]))/(2
*b) - (B*(b*c - a*d)*((b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2))/(2*b)))/(
3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

rule 2960
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Sim
p[(f + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*
(m + 2))), x] + Simp[i*((b*c - a*d)/(b*d*(m + 2)))   Int[(f + g*x)^m*(A - B
*n + B*Log[e*((a + b*x)^n/(c + d*x)^n)]), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, i, A, B, m, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IGtQ[m, -2]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.47

method	result
risch	$\frac{giBx\left(2bd\,x^2+3xda+3bcx+6ac\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{6}+\frac{igbdA\,x^3}{3}+\frac{igdAa\,x^2}{2}+\frac{igbAc\,x^2}{2}+\frac{igdBa\,x^2}{6}-\frac{igbBc\,x^2}{6}+$
parallelrisch	$\frac{3B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3cd^2gi+6Axa\,b^2cd^2gi+3B\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^2c^2dgi+3B\ln(bx+a)a^2bcd^2gi-3B\ln(bx+a)a\,b^2c^2dg}$
parts	$Agi\left(acx+\frac{(da+bc)x^2}{2}+\frac{bdx^3}{3}\right)-\frac{Bgi(da-bc)^2e^2\left(\left(-\frac{1}{2bed\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}-\frac{\ln\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-b\right.}{2b^2e^2d}\right.\right.}{}$
derivativedivides	$-\frac{e(da-bc)\left(Ad^2egi\left(a^2d^2-2acdb+c^2b^2\right)\left(-\frac{1}{2d^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^2}+\frac{be}{3d^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^3}\right)+Bd^2egi(\right.}{}$
default	$-\frac{e(da-bc)\left(Ad^2egi\left(a^2d^2-2acdb+c^2b^2\right)\left(-\frac{1}{2d^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^2}+\frac{be}{3d^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^3}\right)+Bd^2egi(\right.}{}$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVE
RBOSE)
```

output
```
1/6*g*i*B*x*(2*b*d*x^2+3*a*d*x+3*b*c*x+6*a*c)*ln(e*(b*x+a)/(d*x+c))+1/3*i*
g*b*d*A*x^3+1/2*i*g*d*A*a*x^2+1/2*i*g*b*A*c*x^2+1/6*i*g*d*B*a*x^2-1/6*i*g*
b*B*c*x^2+i*g*A*a*c*x-1/2*i*g/d*B*ln(-d*x-c)*a*c^2+1/6*i*g*b/d^2*B*ln(-d*x
-c)*c^3-1/6*i*g/b^2*d*B*ln(b*x+a)*a^3+1/2*i*g/b*B*ln(b*x+a)*a^2*c+1/6*i*g/
b*d*B*a^2*x-1/6*i*g*b/d*B*c^2*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.61

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{2\,Ab^3d^3gix^3+((3\,A-B)b^3cd^2+(3\,A+B)ab^2d^3)gix^2-(Bb^3c^2d-6\,Aab^2cd^2-Ba^2bd^3)gix+(3\,Ba^2bc$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
1/6*(2*A*b^3*d^3*g*i*x^3 + ((3*A - B)*b^3*c*d^2 + (3*A + B)*a*b^2*d^3)*g*i*x^2 - (B*b^3*c^2*d - 6*A*a*b^2*c*d^2 - B*a^2*b*d^3)*g*i*x + (3*B*a^2*b*c*d^2 - B*a^3*d^3)*g*i*log(b*x + a) + (B*b^3*c^3 - 3*B*a*b^2*c^2*d)*g*i*log(d*x + c) + (2*B*b^3*d^3*g*i*x^3 + 6*B*a*b^2*c*d^2*g*i*x + 3*(B*b^3*c*d^2 + B*a*b^2*d^3)*g*i*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^2*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(126) = 252.

Time = 1.51 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.56

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{Abdgix^3}{3}$$
$$-\frac{Ba^2gi(ad-3bc)\log\left(x+\frac{Ba^3cd^2gi+\frac{Ba^3d^2gi(ad-3bc)}{b}-6Ba^2bc^2dgi-Ba^2cdgi(ad-3bc)+Bab^2c^3gi}{Ba^3d^3gi-3Ba^2bcd^2gi-3Bab^2c^2dgi+Bb^3c^3gi}\right)}{6b^2}$$
$$-\frac{Bc^2gi(3ad-bc)\log\left(x+\frac{Ba^3cd^2gi-6Ba^2bc^2dgi+Bab^2c^3gi+Babc^2gi(3ad-bc)-\frac{Bb^2c^3gi(3ad-bc)}{d}}{Ba^3d^3gi-3Ba^2bcd^2gi-3Bab^2c^2dgi+Bb^3c^3gi}\right)}{6d^2}$$
$$+x^2\left(\frac{Aadgi}{2}+\frac{Abcgi}{2}+\frac{Badgi}{6}-\frac{Bbcgi}{6}\right)+x\left(Aacgi+\frac{Ba^2dgi}{6b}-\frac{Bbc^2gi}{6d}\right)$$
$$+\left(Bacgix+\frac{Badgix^2}{2}+\frac{Bbcgix^2}{2}+\frac{Bbdgix^3}{3}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b*d*g*i*x**3/3 - B*a**2*g*i*(a*d - 3*b*c)*log(x + (B*a**3*c*d**2*g*i + B
*a**3*d**2*g*i*(a*d - 3*b*c)/b - 6*B*a**2*b*c**2*d*g*i - B*a**2*c*d*g*i*(a
*d - 3*b*c) + B*a*b**2*c**3*g*i)/(B*a**3*d**3*g*i - 3*B*a**2*b*c*d**2*g*i
- 3*B*a*b**2*c**2*d*g*i + B*b**3*c**3*g*i))/(6*b**2) - B*c**2*g*i*(3*a*d -
 b*c)*log(x + (B*a**3*c*d**2*g*i - 6*B*a**2*b*c**2*d*g*i + B*a*b**2*c**3*g
*i + B*a*b*c**2*g*i*(3*a*d - b*c) - B*b**2*c**3*g*i*(3*a*d - b*c)/d)/(B*a*
*3*d**3*g*i - 3*B*a**2*b*c*d**2*g*i - 3*B*a*b**2*c**2*d*g*i + B*b**3*c**3*
g*i))/(6*d**2) + x**2*(A*a*d*g*i/2 + A*b*c*g*i/2 + B*a*d*g*i/6 - B*b*c*g*i
/6) + x*(A*a*c*g*i + B*a**2*d*g*i/(6*b) - B*b*c**2*g*i/(6*d)) + (B*a*c*g*i
*x + B*a*d*g*i*x**2/2 + B*b*c*g*i*x**2/2 + B*b*d*g*i*x**3/3)*log(e*(a + b*
x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(132) = 264$.

Time $= 0.04$ (sec) , antiderivative size $= 361$, normalized size of antiderivative $= 2.58$

$$\begin{aligned}\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx &= \frac{1}{3}Abdgix^3+\frac{1}{2}Abcgix^2\\ &+\frac{1}{2}Aadgix^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bacgi\\ &+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bbcgi\\ &+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Badgi\\ &+\frac{1}{6}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-}{b^2d^2}\right.\\ &+Aacgix\end{aligned}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorith
m="maxima")
```

output

```
1/3*A*b*d*g*i*x^3 + 1/2*A*b*c*g*i*x^2 + 1/2*A*a*d*g*i*x^2 + (x*log(b*e*x/(
d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a*c*g*i
 + 1/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 +
c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*b*c*g*i + 1/2*(x^2*log(b*e*x
/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2
- (b*c - a*d)*x/(b*d))*B*a*d*g*i + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d
*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a
*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b*d*g*i + A*a*c*g*i*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1254 vs. $2(132) = 264$.

Time $= 0.24$ (sec) , antiderivative size $= 1254$, normalized size of antiderivative $= 8.96$

$$\int (ag + bgx)(ci + dix)\left(A + B\log\left(\frac{e(a + bx)}{c + dx}\right)\right)\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorith
m="giac")
```

output

```
-1/6*((B*b^5*c^4*e^4*g*i - 4*B*a*b^4*c^3*d*e^4*g*i + 6*B*a^2*b^3*c^2*d^2*e
^4*g*i - 4*B*a^3*b^2*c*d^3*e^4*g*i + B*a^4*b*d^4*e^4*g*i - 3*(b*e*x + a*e)
*B*b^4*c^4*d*e^3*g*i/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^3*c^3*d^2*e^3*g*i/
(d*x + c) - 18*(b*e*x + a*e)*B*a^2*b^2*c^2*d^3*e^3*g*i/(d*x + c) + 12*(b*e
*x + a*e)*B*a^3*b*c*d^4*e^3*g*i/(d*x + c) - 3*(b*e*x + a*e)*B*a^4*d^5*e^3*
g*i/(d*x + c))*log((b*e*x + a*e)/(d*x + c))/(b^3*d^2*e^3 - 3*(b*e*x + a*e)
*b^2*d^3*e^2/(d*x + c) + 3*(b*e*x + a*e)^2*b*d^4*e/(d*x + c)^2 - (b*e*x +
a*e)^3*d^5/(d*x + c)^3) + (A*b^6*c^4*e^4*g*i - 4*A*a*b^5*c^3*d*e^4*g*i + 6
*A*a^2*b^4*c^2*d^2*e^4*g*i - 4*A*a^3*b^3*c*d^3*e^4*g*i + A*a^4*b^2*d^4*e^4
*g*i - 3*(b*e*x + a*e)*A*b^5*c^4*d*e^3*g*i/(d*x + c) + (b*e*x + a*e)*B*b^5
*c^4*d*e^3*g*i/(d*x + c) + 12*(b*e*x + a*e)*A*a*b^4*c^3*d^2*e^3*g*i/(d*x +
 c) - 4*(b*e*x + a*e)*B*a*b^4*c^3*d^2*e^3*g*i/(d*x + c) - 18*(b*e*x + a*e)
*A*a^2*b^3*c^2*d^3*e^3*g*i/(d*x + c) + 6*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e
^3*g*i/(d*x + c) + 12*(b*e*x + a*e)*A*a^3*b^2*c*d^4*e^3*g*i/(d*x + c) - 4*
(b*e*x + a*e)*B*a^3*b^2*c*d^4*e^3*g*i/(d*x + c) - 3*(b*e*x + a*e)*A*a^4*b*
d^5*e^3*g*i/(d*x + c) + (b*e*x + a*e)*B*a^4*b*d^5*e^3*g*i/(d*x + c) - (b*e
*x + a*e)^2*B*b^4*c^4*d^2*e^2*g*i/(d*x + c)^2 + 4*(b*e*x + a*e)^2*B*a*b^3*
c^3*d^3*e^2*g*i/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^2*b^2*c^2*d^4*e^2*g*i/
(d*x + c)^2 + 4*(b*e*x + a*e)^2*B*a^3*b*c*d^5*e^2*g*i/(d*x + c)^2 - (b*e*x
 + a*e)^2*B*a^4*d^6*e^2*g*i/(d*x + c)^2)/(b^4*d^2*e^3 - 3*(b*e*x + a*e)...
```

Mupad [B] (verification not implemented)

Time = 26.20 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.01

$$\begin{aligned}
&\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx \\
&= x^2\left(\frac{gi\,(6\,A\,a\,d+6\,A\,b\,c+B\,a\,d-B\,b\,c)}{6}-\frac{A\,g\,i\,(6\,a\,d+6\,b\,c)}{12}\right) \\
&\quad - x\left(\frac{\left(\frac{gi\,(6\,A\,a\,d+6\,A\,b\,c+B\,a\,d-B\,b\,c)}{3}-\frac{A\,g\,i\,(6\,a\,d+6\,b\,c)}{6}\right)(6\,a\,d+6\,b\,c)}{6\,b\,d}+A\,a\,c\,g\,i\right. \\
&\qquad\qquad \left.-\frac{gi\,(2\,A\,a^2\,d^2+2\,A\,b^2\,c^2+B\,a^2\,d^2-B\,b^2\,c^2+8\,A\,a\,b\,c\,d)}{2\,b\,d}\right) \\
&\quad + \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{B\,b\,d\,g\,i\,x^3}{3}+\frac{B\,g\,i\,(a\,d+b\,c)\,x^2}{2}+B\,a\,c\,g\,i\,x\right) \\
&\quad - \frac{\ln(a+b\,x)\,(B\,a^3\,d\,g\,i-3\,B\,a^2\,b\,c\,g\,i)}{6\,b^2} \\
&\quad + \frac{\ln(c+d\,x)\,(B\,b\,c^3\,g\,i-3\,B\,a\,c^2\,d\,g\,i)}{6\,d^2}+\frac{A\,b\,d\,g\,i\,x^3}{3}
\end{aligned}$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x^2*((g*i*(6*A*a*d + 6*A*b*c + B*a*d - B*b*c))/6 - (A*g*i*(6*a*d + 6*b*c))
/12) - x*((((g*i*(6*A*a*d + 6*A*b*c + B*a*d - B*b*c))/3 - (A*g*i*(6*a*d +
6*b*c))/6)*(6*a*d + 6*b*c))/(6*b*d) + A*a*c*g*i - (g*i*(2*A*a^2*d^2 + 2*A*
b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 8*A*a*b*c*d))/(2*b*d)) + log((e*(a + b*x
))/(c + d*x))*((B*g*i*x^2*(a*d + b*c))/2 + (B*b*d*g*i*x^3)/3 + B*a*c*g*i*x
) - (log(a + b*x)*(B*a^3*d*g*i - 3*B*a^2*b*c*g*i))/(6*b^2) + (log(c + d*x)
*(B*b*c^3*g*i - 3*B*a*c^2*d*g*i))/(6*d^2) + (A*b*d*g*i*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.33

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{gi\left(-\log(bx+a)\,a^3d^3+3\log(bx+a)\,a^2bc\,d^2-3\log(bx+a)\,a\,b^2c^2d+\log(bx+a)\,b^3c^3+3\log\left(\frac{bex+ae}{dx+c}\right)a\right.}{}$$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output
```
(g*i*( - log(a + b*x)*a**3*d**3 + 3*log(a + b*x)*a**2*b*c*d**2 - 3*log(a +
 b*x)*a*b**2*c**2*d + log(a + b*x)*b**3*c**3 + 3*log((a*e + b*e*x)/(c + d*
x))*a*b**2*c**2*d + 6*log((a*e + b*e*x)/(c + d*x))*a*b**2*c*d**2*x + 3*log
((a*e + b*e*x)/(c + d*x))*a*b**2*d**3*x**2 - log((a*e + b*e*x)/(c + d*x))*
b**3*c**3 + 3*log((a*e + b*e*x)/(c + d*x))*b**3*c*d**2*x**2 + 2*log((a*e +
 b*e*x)/(c + d*x))*b**3*d**3*x**3 + 6*a**2*b*c*d**2*x + 3*a**2*b*d**3*x**2
 + a**2*b*d**3*x + 3*a*b**2*c*d**2*x**2 + 2*a*b**2*d**3*x**3 + a*b**2*d**3
*x**2 - b**3*c**2*d*x - b**3*c*d**2*x**2))/(6*b*d**2)
```

3.4 $\int (ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

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Optimal result

Integrand size = 28, antiderivative size = 81

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = -\frac{B(bc-ad)ix}{2b} - \frac{B(bc-ad)^2 i\log(a+bx)}{2b^2 d} + \frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d}$$

output
```
-1/2*B*(-a*d+b*c)*i*x/b-1/2*B*(-a*d+b*c)^2*i*ln(b*x+a)/b^2/d+1/2*i*(d*x+c)
^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int (ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{i\left(-\frac{B(bc-ad)(bdx+(bc-ad)\log(a+bx))}{b^2} + (c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\right)}{2d}$$

input
```
Integrate[(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(i*(-((B*(b*c - a*d)*(b*d*x + (b*c - a*d)*Log[a + b*x]))/b^2) + (c + d*x)^
2*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/(2*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci + dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)\,dx$$

$$\downarrow 2948$$

$$\frac{i(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d} - \frac{B(bc-ad)\int \frac{i^2(c+dx)}{a+bx}dx}{2di}$$

$$\downarrow 27$$

$$\frac{i(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d} - \frac{Bi(bc-ad)\int \frac{c+dx}{a+bx}dx}{2d}$$

$$\downarrow 49$$

$$\frac{i(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d} - \frac{Bi(bc-ad)\int \left(\frac{d}{b} + \frac{bc-ad}{b(a+bx)}\right)dx}{2d}$$

$$\downarrow 2009$$

$$\frac{i(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d} - \frac{Bi(bc-ad)\left(\frac{(bc-ad)\log(a+bx)}{b^2} + \frac{dx}{b}\right)}{2d}$$

input
```
Int[(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
-1/2*(B*(b*c - a*d)*i*((d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2))/d + (i*(
c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.33

method	result
risch	$\frac{iBx(dx+2c)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2}+\frac{idAx^2}{2}+iAcx-\frac{Bc^2i\ln(-dx-c)}{2d}-\frac{idB\ln(bx+a)a^2}{2b^2}+\frac{iB\ln(bx+a)ac}{b}+\frac{idB}{2}$
parallelrisch	$\frac{Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2d^2i+Ax^2b^2d^2i+2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2cdi+2Axb^2cdi-B\ln(bx+a)a^2d^2i+2B\ln(bx+a)abcdi-B\ln(}{2b^2d}$
parts	$Ai\left(\frac{1}{2}dx^2+xc\right)-Bi(da-bc)^2e^2\left(-\frac{1}{2bed\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}-\frac{\ln\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}{2b^2e^2d}+\frac{\ln}{}\right.$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{Adei(da-bc)}{2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^2}-Bd^2ei(da-bc)\left(-\frac{1}{2bed\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}+\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{2b^2e^2d}\right.\right.}{d^2}$
default	$-\frac{e(da-bc)\left(-\frac{Adei(da-bc)}{2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^2}-Bd^2ei(da-bc)\left(-\frac{1}{2bed\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}+\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{2b^2e^2d}\right.\right.}{d^2}$

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/2*i*B*x*(d*x+2*c)*ln(e*(b*x+a)/(d*x+c))+1/2*i*d*A*x^2+i*A*c*x-1/2*B*c^2*
i/d*ln(-d*x-c)-1/2*i/b^2*d*B*ln(b*x+a)*a^2+i/b*B*ln(b*x+a)*a*c+1/2*i/b*d*B
*a*x-1/2*i*B*c*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.57

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{Ab^2d^2ix^2-Bb^2c^2i\log(dx+c)+((2A-B)b^2cd+Babd^2)ix+(2Babcd-Ba^2d^2)i\log(bx+a)+(Bb^2$$
$$2b^2d$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
1/2*(A*b^2*d^2*i*x^2 - B*b^2*c^2*i*log(d*x + c) + ((2*A - B)*b^2*c*d + B*a
*b*d^2)*i*x + (2*B*a*b*c*d - B*a^2*d^2)*i*log(b*x + a) + (B*b^2*d^2*i*x^2
+ 2*B*b^2*c*d*i*x)*log((b*e*x + a*e)/(d*x + c)))/(b^2*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(68) = 136.

Time = 0.94 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.12

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{Adix^2}{2}-\frac{Bai(ad-2bc)\log\left(x+\frac{Ba^2cdi+\frac{Ba^2di(ad-2bc)}{b}-3Babc^2i-Baci(ad-2bc)}{Ba^2d^2i-2Babcdi-Bb^2c^2i}\right)}{2b^2}$$
$$-\frac{Bc^2i\log\left(x+\frac{Ba^2cdi-2Babc^2i-\frac{Bb^2c^3i}{d}}{Ba^2d^2i-2Babcdi-Bb^2c^2i}\right)}{2d}+x\left(Aci+\frac{Badi}{2b}-\frac{Bci}{2}\right)$$
$$+\left(Bcix+\frac{Bdix^2}{2}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*d*i*x**2/2 - B*a*i*(a*d - 2*b*c)*log(x + (B*a**2*c*d*i + B*a**2*d*i*(a*d
 - 2*b*c)/b - 3*B*a*b*c**2*i - B*a*c*i*(a*d - 2*b*c))/(B*a**2*d**2*i - 2*B
*a*b*c*d*i - B*b**2*c**2*i))/(2*b**2) - B*c**2*i*log(x + (B*a**2*c*d*i - 2
*B*a*b*c**2*i - B*b**2*c**3*i/d)/(B*a**2*d**2*i - 2*B*a*b*c*d*i - B*b**2*c
**2*i))/(2*d) + x*(A*c*i + B*a*d*i/(2*b) - B*c*i/2) + (B*c*i*x + B*d*i*x**
2/2)*log(e*(a + b*x)/(c + d*x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.78

$$\begin{aligned}
&\int (ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx \\
&= \frac{1}{2}Adix^2 + \left(x\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) + \frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right)Bci \\
&\quad + \frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) - \frac{a^2\log(bx+a)}{b^2} + \frac{c^2\log(dx+c)}{d^2} - \frac{(bc-ad)x}{bd}\right)Bdi \\
&\quad + Acix
\end{aligned}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
1/2*A*d*i*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b
 - c*log(d*x + c)/d)*B*c*i + 1/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))
 - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*d*
i + A*c*i*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(75) = 150$.

Time $= 0.20$ (sec) , antiderivative size $= 627$, normalized size of antiderivative $= 7.74$

$$\int (ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$

$$= \frac{1}{2}\left(\frac{(Bb^3c^3e^3i - 3\,Bab^2c^2de^3i + 3\,Ba^2bcd^2e^3i - Ba^3d^3e^3i)\log\left(\frac{bex+ae}{dx+c}\right)}{b^2de^2 - \frac{2\,(bex+ae)bd^2e}{dx+c} + \frac{(bex+ae)^2d^3}{(dx+c)^2}} + \frac{Ab^4c^3e^3i - Bb^4c^3e^3i - 3\,Aab^3\right.$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
1/2*((B*b^3*c^3*e^3*i - 3*B*a*b^2*c^2*d*e^3*i + 3*B*a^2*b*c*d^2*e^3*i - B*
a^3*d^3*e^3*i)*log((b*e*x + a*e)/(d*x + c))/(b^2*d*e^2 - 2*(b*e*x + a*e)*b
*d^2*e/(d*x + c) + (b*e*x + a*e)^2*d^3/(d*x + c)^2) + (A*b^4*c^3*e^3*i - B
*b^4*c^3*e^3*i - 3*A*a*b^3*c^2*d*e^3*i + 3*B*a*b^3*c^2*d*e^3*i + 3*A*a^2*b
^2*c*d^2*e^3*i - 3*B*a^2*b^2*c*d^2*e^3*i - A*a^3*b*d^3*e^3*i + B*a^3*b*d^3
*e^3*i + (b*e*x + a*e)*B*b^3*c^3*d*e^2*i/(d*x + c) - 3*(b*e*x + a*e)*B*a*b
^2*c^2*d^2*e^2*i/(d*x + c) + 3*(b*e*x + a*e)*B*a^2*b*c*d^3*e^2*i/(d*x + c)
 - (b*e*x + a*e)*B*a^3*d^4*e^2*i/(d*x + c))/(b^3*d*e^2 - 2*(b*e*x + a*e)*b
^2*d^2*e/(d*x + c) + (b*e*x + a*e)^2*b*d^3/(d*x + c)^2) + (B*b^3*c^3*e*i -
 3*B*a*b^2*c^2*d*e*i + 3*B*a^2*b*c*d^2*e*i - B*a^3*d^3*e*i)*log(-b*e + (b*
e*x + a*e)*d/(d*x + c))/(b^2*d) - (B*b^3*c^3*e*i - 3*B*a*b^2*c^2*d*e*i + 3
*B*a^2*b*c*d^2*e*i - B*a^3*d^3*e*i)*log((b*e*x + a*e)/(d*x + c))/(b^2*d))*
(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 26.50 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = x\left(\frac{i\,(2\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,b} - \frac{A\,i\,(2\,a\,d+2\,b\,c)}{2\,b}\right) + \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{B\,d\,i\,x^2}{2}+B\,c\,i\,x\right) - \frac{\ln(a+b\,x)\,(B\,a^2\,d\,i-2\,B\,a\,b\,c\,i)}{2\,b^2} + \frac{A\,d\,i\,x^2}{2} - \frac{B\,c^2\,i\,\ln(c+d\,x)}{2\,d}$$

input
```
int((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x*((i*(2*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(2*b) - (A*i*(2*a*d + 2*b*c))/(
2*b)) + log((e*(a + b*x))/(c + d*x))*((B*d*i*x^2)/2 + B*c*i*x) - (log(a +
b*x)*(B*a^2*d*i - 2*B*a*b*c*i))/(2*b^2) + (A*d*i*x^2)/2 - (B*c^2*i*log(c +
 d*x))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.23

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$= \frac{i\left(-\log(dx+c)\,a^2d^2+2\log(dx+c)\,abcd-\log(dx+c)\,b^2c^2-\log\left(\frac{bex+ae}{dx+c}\right)a^2d^2+2\log\left(\frac{bex+ae}{dx+c}\right)abcd+2\right.}{2bd}$$

input
```
int((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output

```
(i*( - log(c + d*x)*a**2*d**2 + 2*log(c + d*x)*a*b*c*d - log(c + d*x)*b**2
*c**2 - log((a*e + b*e*x)/(c + d*x))*a**2*d**2 + 2*log((a*e + b*e*x)/(c +
d*x))*a*b*c*d + 2*log((a*e + b*e*x)/(c + d*x))*b**2*c*d*x + log((a*e + b*e
*x)/(c + d*x))*b**2*d**2*x**2 + 2*a*b*c*d*x + a*b*d**2*x**2 + a*b*d**2*x -
 b**2*c*d*x))/(2*b*d)
```

3.5 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

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Optimal result

Integrand size = 38, antiderivative size = 133

$$\begin{aligned}
&\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx \\
&= \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg} \\
&\quad - \frac{(bc-ad)i\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g} \\
&\quad + \frac{B(bc-ad)i\,\mathrm{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{b^2g}
\end{aligned}$$

output

```
i*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g-(-a*d+b*c)*i*ln(-(-a*d+b*c)/d/(b
*x+a))*(A-B+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g+B*(-a*d+b*c)*i*polylog(2,1+(-a*
d+b*c)/d/(b*x+a))/b^2/g
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.23

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i\Big((-bBc+aBd)\log^2(a+bx)+2\Big(Abdx+Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+(-bBc+aBd)\log(c+dx)\Big)+2}{2b^2g}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*
x),x]
```

output
```
(i*((-(b*B*c) + a*B*d)*Log[a + b*x]^2 + 2*(A*b*d*x + B*d*(a + b*x)*Log[(e*
(a + b*x))/(c + d*x)] + (-(b*B*c) + a*B*d)*Log[c + d*x]) + 2*(b*c - a*d)*L
og[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)] + B*Log[(b*(c + d*x))/(b*c
 - a*d)]) + 2*B*(b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/(2*
b^2*g)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2960, 27, 2942, 2858, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{ag+bgx}\,dx$$
$$\downarrow 2960$$
$$\frac{i(bc-ad)\int \frac{A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{g(a+bx)}dx}{b} + \frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$
$$\downarrow 27$$

$$\frac{i(bc-ad)\int \frac{A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx}dx}{bg}+\frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\downarrow 2942$$

$$\frac{i(bc-ad)\left(\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}dx}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\downarrow 2858$$

$$\frac{i(bc-ad)\left(\frac{B(bc-ad)\int \frac{b\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)\left(b\left(c-\frac{ad}{b}\right)+d(a+bx)\right)}d(a+bx)}{b^2}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\downarrow 27$$

$$\frac{i(bc-ad)\left(\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(bc-ad+d(a+bx))}d(a+bx)}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\downarrow 2778$$

$$\frac{i(bc-ad)\left(-\frac{B(bc-ad)\int \frac{(a+bx)\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{bc-ad+d(a+bx)}d\frac{1}{a+bx}}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\downarrow 2005$$

$$\frac{i(bc-ad)\left(-\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{d+\frac{bc-ad}{a+bx}}d\frac{1}{a+bx}}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\downarrow 2752$$

$$\frac{i(bc-ad)\left(\frac{B\,\mathrm{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x),x]
```

output
```
(i*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b*g) + ((b*c - a*d)*i*
(-((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A - B + B*Log[(e*(a + b*x))/(c + d*
x)]))/b) + (B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/b))/(b*g)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2942
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
 - a*d)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*
c - a*d, 0] && EqQ[b*f - a*g, 0]
```

rule 2960
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Sim
p[(f + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*
(m + 2))), x] + Simp[i*((b*c - a*d)/(b*d*(m + 2)))   Int[(f + g*x)^m*(A - B
*n + B*Log[e*((a + b*x)^n/(c + d*x)^n)]), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, i, A, B, m, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IGtQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(133) = 266$.

Time = 2.94 (sec) , antiderivative size = 417, normalized size of antiderivative = 3.14

method	result
parts	$\frac{iA\left(\frac{xd}{b}+\frac{(-da+bc)\ln(bx+a)}{b^2}\right)}{g} - \frac{iB(da-bc)^2e^2\left(\frac{\left(\frac{\ln\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}{bed}-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}\right)d^4}{(da-bc)be}-$
derivativedivides	$-\frac{e(da-bc)\left(\frac{i\,d^2eA\left(\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{b^2e^2}+\frac{1}{be\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{b^2e^2}\right)}{g}+\frac{i\,d^2eB\left(-\frac{\left(\mathrm{dilog}\left(-\right.\right.\right.}{}\right.$
default	$-\frac{e(da-bc)\left(\frac{i\,d^2eA\left(\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{b^2e^2}+\frac{1}{be\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{b^2e^2}\right)}{g}+\frac{i\,d^2eB\left(-\frac{\left(\mathrm{dilog}\left(-\right.\right.\right.}{}\right.$
risch	Expression too large to display

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x,method=_RETURNVE
RBOSE)
```

output
```
i*A/g*(x*d/b+(-a*d+b*c)/b^2*ln(b*x+a))-i*B/g/d^3*(a*d-b*c)^2*e^2*((1/b/e*l
n((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/d-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(
b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e))/(a
*d-b*c)*d^4/b/e+1/2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/(a*d-b*c)*d^3/b^2/e^
2-(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)/(a*d-b*c)*d
^4/b^2/e^2)
```

Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorith
m="fricas")
```

output
```
integral((A*d*i*x + A*c*i + (B*d*i*x + B*c*i)*log((b*e*x + a*e)/(d*x + c))
)/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \frac{i\left(\int \frac{Ac}{a+bx}\,dx + \int \frac{Adx}{a+bx}\,dx + \int \frac{Bc\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx + \int \frac{Bdx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx\right)}{g}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
```

output
```
i*(Integral(A*c/(a + b*x), x) + Integral(A*d*x/(a + b*x), x) + Integral(B*
c*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Integral(B*d*x*log(
a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x))/g
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.81

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= Adi\left(\frac{x}{bg}-\frac{a\log(bx+a)}{b^2g}\right)+\frac{Aci\log(bgx+ag)}{bg}-\frac{Bci\log(dx+c)}{bg}$$
$$+\frac{(bci-adi)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^2g}$$
$$+\frac{2\,Bbdix\log(e)+(bci-adi)B\log(bx+a)^2+2\left(Bbdix+(bci\log(e)-(i\log(e)-i)ad)B\right)\log(bx+}{2\,b^2g}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorith
m="maxima")
```

output
```
A*d*i*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + A*c*i*log(b*g*x + a*g)/(b*g) -
B*c*i*log(d*x + c)/(b*g) + (b*c*i - a*d*i)*(log(b*x + a)*log((b*d*x + a*d)
/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^2*g) + 1/2*(2*
B*b*d*i*x*log(e) + (b*c*i - a*d*i)*B*log(b*x + a)^2 + 2*(B*b*d*i*x + (b*c*
i*log(e) - (i*log(e) - i)*a*d)*B)*log(b*x + a) - 2*(B*b*d*i*x + (b*c*i - a
*d*i)*B*log(b*x + a))*log(d*x + c))/(b^2*g)
```

Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx=\int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorith
m="giac")
```

output
```
integrate((d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x),x)
```

output
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x), x)
```

Reduce [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i\left(\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{bx+a}dx\right)b^3c+\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{bx+a}dx\right)b^3d-\log(bx+a)\,a^2d+\log(bx+a)\,abc+abdx\right)}{b^2g}$$

input
```
int((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
```

output
```
(i*(int(log((a*e + b*e*x)/(c + d*x))/(a + b*x),x)*b**3*c + int((log((a*e +
 b*e*x)/(c + d*x))*x)/(a + b*x),x)*b**3*d - log(a + b*x)*a**2*d + log(a +
b*x)*a*b*c + a*b*d*x))/(b**2*g)
```

3.6 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$

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Optimal result

Integrand size = 38, antiderivative size = 142

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = -\frac{Bi(c+dx)}{bg^2(a+bx)} - \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{Bdi\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}$$

output

```
-B*i*(d*x+c)/b/g^2/(b*x+a)-i*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g^2/(b*
x+a)-d*i*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^2/g^2+B*d
*i*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/g^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.23

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$$
$$= \frac{i\Big(-2(A+B)(bc-ad)-Bd(a+bx)\log^2(a+bx)+2(-bBc+aBd)\log\left(\frac{e(a+bx)}{c+dx}\right)+2Bd(a+bx)\log(c$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*
x)^2,x]
```

output
```
(i*(-2*(A + B)*(b*c - a*d) - B*d*(a + b*x)*Log[a + b*x]^2 + 2*(-(b*B*c) +
a*B*d)*Log[(e*(a + b*x))/(c + d*x)] + 2*B*d*(a + b*x)*Log[c + d*x] + 2*d*(
a + b*x)*Log[a + b*x]*(A - B + B*Log[(e*(a + b*x))/(c + d*x)] + B*Log[(b*(
c + d*x))/(b*c - a*d)]) + 2*B*d*(a + b*x)*PolyLog[2, (d*(a + b*x))/(-(b*c)
 + a*d)]))/(2*b^2*g^2*(a + b*x))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2962, 2780, 2741, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^2}\,dx$$
$$\downarrow 2962$$
$$\frac{i\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g^2}$$
$$\downarrow 2780$$

$$\frac{i\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^2}$$

$$\downarrow 2741$$

$$\frac{i\left(\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}}{b}\right)}{g^2}$$

$$\downarrow 2779$$

$$\frac{i\left(\frac{d\left(\frac{B\int \frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}\right)}{b}+\frac{-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}}{b}\right)}{g^2}$$

$$\downarrow 2838$$

$$\frac{i\left(\frac{d\left(\frac{B\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}\right)}{b}+\frac{-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}}{b}\right)}{g^2}$$

input

```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^2,x
]
```

output

```
(i*((-((B*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[(e*(a + b*x))/(c +
 d*x)]))/(a + b*x))/b + (d*(-(((A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1
 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(d*(a + b
 *x))])/b))/b))/g^2
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(142) = 284$.

Time = 2.99 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.61

method	result
parts	$\frac{iA\left(\frac{d\ln(bx+a)}{b^2}-\frac{-da+bc}{b^2(bx+a)}\right)}{g^2}-\frac{iB(da-bc)^2e^2\left(-\frac{\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)d^3}{(da-bc)^2be}-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2d^4}{2(da-bc)^2b^2e^2}+\frac{}{}\right.}{g^2}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{i\,d^2eA\left(-\frac{1}{be\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{d\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{b^2e^2}-\frac{d\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{b^2e^2}\right)}{(da-bc)g^2}-\frac{i\,d^2eB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(d}{d(}\right.\right.}{\frac{be}{d}+\frac{(da}{d(}}\right.}{}$
default	$-\frac{e(da-bc)\left(-\frac{i\,d^2eA\left(-\frac{1}{be\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{d\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{b^2e^2}-\frac{d\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{b^2e^2}\right)}{(da-bc)g^2}-\frac{i\,d^2eB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(d}{d(}\right.\right.}{\frac{be}{d}+\frac{(da}{d(}}\right.}{}$
risch	Expression too large to display

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x,method=_RETURN
VERBOSE)
```

output
```
i*A/g^2*(d/b^2*ln(b*x+a)-(-a*d+b*c)/b^2/(b*x+a))-i*B/g^2/d^3*(a*d-b*c)^2*e
^2*(-(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(
b*e/d+(a*d-b*c)*e/d/(d*x+c)))/(a*d-b*c)^2*d^3/b/e-1/2*ln(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^2/(a*d-b*c)^2*d^4/b^2/e^2+(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c
))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d
/(d*x+c))*d-b*e)/b/e)/d)/(a*d-b*c)^2*d^5/b^2/e^2)
```

Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algori
thm="fricas")
```

output
```
integral((A*d*i*x + A*c*i + (B*d*i*x + B*c*i)*log((b*e*x + a*e)/(d*x + c))
)/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algori
thm="maxima")
```

output
```
-B*d*i*(((b*x + a)*log(b*x + a) + a)*log(d*x + c)/(b^3*g^2*x + a*b^2*g^2)
- integrate((b^2*d*x^2*log(e) + a^2*d + (b^2*c*log(e) + a*b*d)*x + (2*b^2*
d*x^2 + a^2*d + (b^2*c + 2*a*b*d)*x)*log(b*x + a))/(b^4*d*g^2*x^3 + a^2*b^
2*c*g^2 + (b^4*c*g^2 + 2*a*b^3*d*g^2)*x^2 + (2*a*b^3*c*g^2 + a^2*b^2*d*g^2
)*x), x)) + A*d*i*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - B
*c*i*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1/(b^2*
g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/(
(b^2*c - a*b*d)*g^2)) - A*c*i/(b^2*g^2*x + a*b*g^2)
```

Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algori
thm="giac")
```

output
```
integrate((d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)^2
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(a\,g+b\,g\,x)^2}\,dx$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2,x
)
```

output
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2,
x)
```

Reduce [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$$
$$= \frac{i\left(\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{b^2x^2+2abx+a^2}dx\right)a^3b^3d^2 - \left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{b^2x^2+2abx+a^2}dx\right)a^2b^4cd + \left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{b^2x^2+2abx+a^2}dx\right)a^2b^4d^2x - \left(\int \frac{\log\left(\frac{bex+}{dx+}\right.\right.\right.}{\left.\left.\left.b^2x^2+2ab\right.\right.\right.}$$

input
```
int((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x)
```

output
```
(i*(int((log((a*e + b*e*x)/(c + d*x))*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a
**3*b**3*d**2 - int((log((a*e + b*e*x)/(c + d*x))*x)/(a**2 + 2*a*b*x + b**
2*x**2),x)*a**2*b**4*c*d + int((log((a*e + b*e*x)/(c + d*x))*x)/(a**2 + 2*
a*b*x + b**2*x**2),x)*a**2*b**4*d**2*x - int((log((a*e + b*e*x)/(c + d*x))
*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a*b**5*c*d*x + log(a + b*x)*a**4*d**2
 - log(a + b*x)*a**3*b*c*d + log(a + b*x)*a**3*b*d**2*x - log(a + b*x)*a**2
*b**2*c*d*x + log(a + b*x)*a*b**3*c**2 + log(a + b*x)*b**4*c**2*x - log(c
 + d*x)*a*b**3*c**2 - log(c + d*x)*b**4*c**2*x + log((a*e + b*e*x)/(c + d*x
))*a*b**3*c*d*x - log((a*e + b*e*x)/(c + d*x))*b**4*c**2*x - a**3*b*d**2*x
 + 2*a**2*b**2*c*d*x - a*b**3*c**2*x + a*b**3*c*d*x - b**4*c**2*x))/(a*b**
2*g**2*(a**2*d - a*b*c + a*b*d*x - b**2*c*x))
```

3.7 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$

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Optimal result

Integrand size = 38, antiderivative size = 85

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = -\frac{Bi(c+dx)^2}{4(bc-ad)g^3(a+bx)^2} - \frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)g^3(a+bx)^2}$$

output

```
-1/4*B*i*(d*x+c)^2/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*i*(d*x+c)^2*(A+B*ln(e*(b*x
+a)/(d*x+c)))/(-a*d+b*c)/g^3/(b*x+a)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 208 vs. 2(85) = 170.

Time = 0.19 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.45

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \frac{i\left(-\frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2(a+bx)^2} - \frac{d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2(a+bx)} - \frac{Bd\left(\frac{1}{a+bx}+\frac{d\log(a+bx)}{bc-ad}-\frac{d\log(c+dx)}{bc-ad}\right)}{b^2} - \frac{B\left(\frac{bc-ad}{(a+bx)^2}-\frac{2d}{a+bx}-\frac{2d^2\log(a+}{bc-ad}\right.\right.}{4b^2}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*
x)^3,x]
```

output
```
(i*(-1/2*((b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*(a + b*x)
^2) - (d*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*(a + b*x)) - (B*d*((a
+ b*x)^(-1) + (d*Log[a + b*x])/(b*c - a*d) - (d*Log[c + d*x])/(b*c - a*d))
)/b^2 - (B*((b*c - a*d)/(a + b*x)^2 - (2*d)/(a + b*x) - (2*d^2*Log[a + b*x
])/(b*c - a*d) + (2*d^2*Log[c + d*x])/(b*c - a*d)))/(4*b^2)))/g^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2962, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{i\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{i\left(-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}\right)}{g^3(bc-ad)}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^3,x
]
```

output
```
(i*(-1/4*(B*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)]))/(2*(a + b*x)^2)))/((b*c - a*d)*g^3)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.80

method	result
parts	$\frac{iA\left(-\frac{-da+bc}{2b^2(bx+a)^2}-\frac{d}{b^2(bx+a)}\right)}{g^3}-\frac{iBe^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}\right)}{g^3(da-bc)}$
norman	$\frac{\frac{Bcdix\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(da-bc)}-\frac{2Aadi+2Abci+Badi+Bbci}{4gb^2}-\frac{(2Adi+Bdi)x}{2gb}+\frac{Bic^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2g(da-bc)}+\frac{Bd^2ix^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2(da-bc)g}}{(bx+a)^2g^2}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{id^2eA}{2(da-bc)^2g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{id^2eB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}\right)}{(da-bc)^2g^3}\right)}{d^2}$
default	$-\frac{e(da-bc)\left(-\frac{id^2eA}{2(da-bc)^2g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{id^2eB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}\right)}{(da-bc)^2g^3}\right)}{d^2}$
parallelrisch	$-\frac{2Aa^2b^2d^3i+Ba^2b^2d^3i-Bb^4c^2di-4Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4cd^2i-4Axb^4cd^2i-2Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4d^3i+4Axab^3d^3i-2}{4g^3(bx+a)^2b^4d(da-bc)}$
risch	$-\frac{Bi(2bdx+da+bc)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2g^3(bx+a)^2b^2}-\frac{i\left(2B\ln(dx+c)b^2d^2x^2-2B\ln(-bx-a)b^2d^2x^2+4B\ln(dx+c)abd^2x-4B\ln(-bx-\right.}{}$
orering	$-\frac{(bx+a)(2d^2b^2x^3-3x^2abd^2+3x^2b^2cd-6xacdb+a^2cd-5ab c^2)(dix+ci)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{4a(da-bc)b(dx+c)(bgx+ag)^3}+\frac{(-bdx^2+ac)(bx+}{}$

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x,method=_RETURN
VERBOSE)
```

output
```
i*A/g^3*(-1/2*(-a*d+b*c)/b^2/(b*x+a)^2-d/b^2/(b*x+a))-i*B/g^3/(a*d-b*c)*e^
2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/
4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(81) = 162.

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.08

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = -\frac{2\left((2A+B)b^2cd-(2A+B)abd^2\right)ix+\left((2A+B)b^2c^2-(2A+B)a^2d^2\right)i+2\left(Bb^2d^2ix^2+2Bb^2cdix\right.}{4\left((b^5c-ab^4d)g^3x^2+2\left(ab^4c-a^2b^3d\right)g^3x+\left(a^2b^3c-a^3b^2d\right)g^3\right)}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algori
thm="fricas")
```

output
```
-1/4*(2*((2*A + B)*b^2*c*d - (2*A + B)*a*b*d^2)*i*x + ((2*A + B)*b^2*c^2 -
 (2*A + B)*a^2*d^2)*i + 2*(B*b^2*d^2*i*x^2 + 2*B*b^2*c*d*i*x + B*b^2*c^2*i
)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c -
a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(73) = 146.

Time = 2.30 (sec) , antiderivative size = 384, normalized size of antiderivative = 4.52

$$\begin{aligned}
&\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx \\
&= -\frac{Bd^2i\log\left(x+\frac{-\frac{Ba^2d^4i}{ad-bc}+\frac{2Babcd^3i}{ad-bc}+Bad^3i-\frac{Bb^2c^2d^2i}{ad-bc}+Bbcd^2i}{2Bbd^3i}\right)}{2b^2g^3(ad-bc)} \\
&\quad + \frac{Bd^2i\log\left(x+\frac{\frac{Ba^2d^4i}{ad-bc}-\frac{2Babcd^3i}{ad-bc}+Bad^3i+\frac{Bb^2c^2d^2i}{ad-bc}+Bbcd^2i}{2Bbd^3i}\right)}{2b^2g^3(ad-bc)} \\
&\quad + \frac{-2Aadi-2Abci-Badi-Bbci+x(-4Abdi-2Bbdi)}{4a^2b^2g^3+8ab^3g^3x+4b^4g^3x^2} \\
&\quad + \frac{(-Badi-Bbci-2Bbdix)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^2b^2g^3+4ab^3g^3x+2b^4g^3x^2}
\end{aligned}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3,x)
```

output
```
-B*d**2*i*log(x + (-B*a**2*d**4*i/(a*d - b*c) + 2*B*a*b*c*d**3*i/(a*d - b*
c) + B*a*d**3*i - B*b**2*c**2*d**2*i/(a*d - b*c) + B*b*c*d**2*i)/(2*B*b*d*
*3*i))/(2*b**2*g**3*(a*d - b*c)) + B*d**2*i*log(x + (B*a**2*d**4*i/(a*d -
b*c) - 2*B*a*b*c*d**3*i/(a*d - b*c) + B*a*d**3*i + B*b**2*c**2*d**2*i/(a*d
 - b*c) + B*b*c*d**2*i)/(2*B*b*d**3*i))/(2*b**2*g**3*(a*d - b*c)) + (-2*A*
a*d*i - 2*A*b*c*i - B*a*d*i - B*b*c*i + x*(-4*A*b*d*i - 2*B*b*d*i))/(4*a**
2*b**2*g**3 + 8*a*b**3*g**3*x + 4*b**4*g**3*x**2) + (-B*a*d*i - B*b*c*i -
2*B*b*d*i*x)*log(e*(a + b*x)/(c + d*x))/(2*a**2*b**2*g**3 + 4*a*b**3*g**3*
x + 2*b**4*g**3*x**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(81) = 162$.

Time $= 0.05$ (sec) , antiderivative size $= 570$, normalized size of antiderivative $= 6.71$

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx =$$
$$-\frac{1}{4}\,Bdi\left(\frac{2\,(2\,bx+a)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^4g^3x^2+2\,ab^3g^3x+a^2b^2g^3}+\frac{3\,abc-a^2d+2\,(2\,b^2c-abd)x}{(b^5c-ab^4d)g^3x^2+2\,(ab^4c-a^2b^3d)g^3x+(a^2b^3c-a^3b^2d)g^3}\right.$$
$$+\frac{1}{4}\,Bci\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}-\frac{2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}+\frac{}{(b^3}\right.$$
$$-\frac{(2\,bx+a)Adi}{2\,(b^4g^3x^2+2\,ab^3g^3x+a^2b^2g^3)}-\frac{Aci}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algori
thm="maxima")
```

output

```
-1/4*B*d*i*(2*(2*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g^3*x^
2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + (3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*
x)/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c
 - a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*
c*d + a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a
*b^3*c*d + a^2*b^2*d^2)*g^3)) + 1/4*B*c*i*((2*b*d*x - b*c + 3*a*d)/((b^4*c
 - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d
)*g^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^3*g^3*x^2 + 2*a*b^2*g^3
*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*
g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2
*(2*b*x + a)*A*d*i/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - 1/2*A*c*i
/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.58

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx =$$
$$-\frac{1}{4}\left(\frac{2\,(dx+c)^2Be^3i\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^2g^3}+\frac{(2\,Ae^3i+Be^3i)(dx+c)^2}{(bex+ae)^2g^3}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(}\right.$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algori
thm="giac")
```

output

```
-1/4*(2*(d*x + c)^2*B*e^3*i*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^2*
g^3) + (2*A*e^3*i + B*e^3*i)*(d*x + c)^2/((b*e*x + a*e)^2*g^3))*(b*c/((b*c
*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 27.16 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.32

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$$
$$= -\frac{x\,(2\,A\,b\,d\,i+B\,b\,d\,i)+A\,a\,d\,i+A\,b\,c\,i+\frac{B\,a\,d\,i}{2}+\frac{B\,b\,c\,i}{2}}{2\,a^2\,b^2\,g^3+4\,a\,b^3\,g^3\,x+2\,b^4\,g^3\,x^2}$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{B\,c\,i}{2\,b^2\,g^3}+\frac{B\,a\,d\,i}{2\,b^3\,g^3}+\frac{B\,d\,i\,x}{b^2\,g^3}\right)}{2\,a\,x+b\,x^2+\frac{a^2}{b}}-\frac{B\,d^2\,i\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,1\mathrm{i}}{b^2\,g^3\,(a\,d-b\,c)}$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^3,x
)
```

output
```
- (x*(2*A*b*d*i + B*b*d*i) + A*a*d*i + A*b*c*i + (B*a*d*i)/2 + (B*b*c*i)/2
)/(2*a^2*b^2*g^3 + 2*b^4*g^3*x^2 + 4*a*b^3*g^3*x) - (log((e*(a + b*x))/(c
+ d*x))*((B*c*i)/(2*b^2*g^3) + (B*a*d*i)/(2*b^3*g^3) + (B*d*i*x)/(b^2*g^3)
))/(2*a*x + b*x^2 + a^2/b) - (B*d^2*i*atan((b*c*2i + b*d*x*2i)/(a*d - b*c)
 + 1i)*1i)/(b^2*g^3*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 335, normalized size of antiderivative = 3.94

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$$
$$= \frac{i\big(2\log(bx+a)\,a^2bcd+4\log(bx+a)\,ab^2cdx+2\log(bx+a)\,b^3cd\,x^2-2\log(dx+c)\,a^2bcd-4\log(dx+c$$

input
```
int((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x)
```

output

```
(i*(2*log(a + b*x)*a**2*b*c*d + 4*log(a + b*x)*a*b**2*c*d*x + 2*log(a + b*
x)*b**3*c*d*x**2 - 2*log(c + d*x)*a**2*b*c*d - 4*log(c + d*x)*a*b**2*c*d*x
 - 2*log(c + d*x)*b**3*c*d*x**2 - 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*c*
d + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*c**2 + 2*log((a*e + b*e*x)/(c +
d*x))*a*b**2*d**2*x**2 - 2*log((a*e + b*e*x)/(c + d*x))*b**3*c*d*x**2 - 2*
a**3*c*d + 2*a**2*b*c**2 - a**2*b*c*d + 2*a**2*b*d**2*x**2 + a*b**2*c**2 -
 2*a*b**2*c*d*x**2 + a*b**2*d**2*x**2 - b**3*c*d*x**2))/(4*a*b*g**3*(a**3*
d - a**2*b*c + 2*a**2*b*d*x - 2*a*b**2*c*x + a*b**2*d*x**2 - b**3*c*x**2))
```

3.8 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$

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Optimal result

Integrand size = 38, antiderivative size = 173

$$\begin{aligned}\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx &= \frac{Bdi(c+dx)^2}{4(bc-ad)^2g^4(a+bx)^2}\\ &\quad-\frac{bBi(c+dx)^3}{9(bc-ad)^2g^4(a+bx)^3}\\ &\quad+\frac{di(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2g^4(a+bx)^2}\\ &\quad-\frac{bi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^2g^4(a+bx)^3}\end{aligned}$$

output

```
1/4*B*d*i*(d*x+c)^2/(-a*d+b*c)^2/g^4/(b*x+a)^2-1/9*b*B*i*(d*x+c)^3/(-a*d+b
*c)^2/g^4/(b*x+a)^3+1/2*d*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*
c)^2/g^4/(b*x+a)^2-1/3*b*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c
)^2/g^4/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.08

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{i\left(\frac{12Abc}{(a+bx)^3}+\frac{4bBc}{(a+bx)^3}-\frac{12aAd}{(a+bx)^3}-\frac{4aBd}{(a+bx)^3}+\frac{18Ad}{(a+bx)^2}+\frac{3Bd}{(a+bx)^2}-\frac{6Bd^2}{(bc-ad)(a+bx)}-\frac{6Bd^3\log(a+bx)}{(bc-ad)^2}+\frac{6B(2bc+ad+3b}{(a+}\right.}{36b^2g^4}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*
x)^4,x]
```

output
```
-1/36*(i*((12*A*b*c)/(a + b*x)^3 + (4*b*B*c)/(a + b*x)^3 - (12*a*A*d)/(a +
 b*x)^3 - (4*a*B*d)/(a + b*x)^3 + (18*A*d)/(a + b*x)^2 + (3*B*d)/(a + b*x)
^2 - (6*B*d^2)/((b*c - a*d)*(a + b*x)) - (6*B*d^3*Log[a + b*x])/(b*c - a*d
)^2 + (6*B*(2*b*c + a*d + 3*b*d*x)*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)
^3 + (6*B*d^3*Log[c + d*x])/(b*c - a*d)^2))/(b^2*g^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2962, 2772, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^4}\,dx$$

$$\downarrow 2962$$

$$\frac{i\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^4}\,d\frac{a+bx}{c+dx}}{g^4(bc-ad)^2}$$

$$\downarrow 2772$$

$$\frac{i\left(-B\int -\frac{(c+dx)^4\left(2b-\frac{3d(a+bx)}{c+dx}\right)}{6(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}\right)}{g^4(bc-ad)^2}$$

$$\downarrow 27$$

$$\frac{i\left(\frac{1}{6}B\int \frac{(c+dx)^4\left(2b-\frac{3d(a+bx)}{c+dx}\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}\right)}{g^4(bc-ad)^2}$$

$$\downarrow 53$$

$$\frac{i\left(\frac{1}{6}B\int \left(\frac{2b(c+dx)^4}{(a+bx)^4}-\frac{3d(c+dx)^3}{(a+bx)^3}\right)d\frac{a+bx}{c+dx}-\frac{b(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}\right)}{g^4(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{i\left(-\frac{b(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{1}{6}B\left(\frac{3d(c+dx)^2}{2(a+bx)^2}-\frac{2b(c+dx)^3}{3(a+bx)^3}\right)\right)}{g^4(bc-ad)^2}$$

input

```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^4,x
]
```

output

```
(i*((B*((3*d*(c + d*x)^2)/(2*(a + b*x)^2) - (2*b*(c + d*x)^3)/(3*(a + b*x)
^3)))/6 + (d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x
)^2) - (b*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(a + b*x)^3
)))/((b*c - a*d)^2*g^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.61

method	result
parts	$\frac{iA\left(-\frac{d}{2b^2(bx+a)^2}-\frac{-da+bc}{3b^2(bx+a)^3}\right)}{g^4}-\frac{iB(da-bc)^2e^2\left(\frac{d^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}\right)}{(da-bc)^4}-\frac{d^3be\left(-\frac{\ln\left(\frac{be}{d}+\right.\right.}{3\left(\frac{be}{d}+\frac{(}{d}\right.}\right.}{g^4d^3}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{i\,d^2e^2Ab}{3(da-bc)^3g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{i\,d^3eA}{2(da-bc)^3g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{i\,d^2e^2Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{}{9\left(\frac{be}{d}+\frac{(}{d}\right.}\right.}{(da-bc)^3g^4}\right.}{d^2}$
default	$-\frac{e(da-bc)\left(\frac{i\,d^2e^2Ab}{3(da-bc)^3g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{i\,d^3eA}{2(da-bc)^3g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{i\,d^2e^2Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{}{9\left(\frac{be}{d}+\frac{(d}{d}\right.}\right.}{(da-bc)^3g^4}\right.}{d^2}$
norman	$\frac{-\frac{6A\,a^2d^2i+6Aabcdi-12A\,b^2c^2i+3B\,a^2d^2i+5Babcdi-4B\,b^2c^2i}{36b^2g(da-bc)}-\frac{\left(6Aa\,d^2i-6Abcdi+3Ba\,d^2i-Bbcdi\right)x}{12g(da-bc)b}+\frac{B\,d^2ib\,x^3}{18a(da-bc)g}+\frac{Bi\,c^2(3}{6\left(a^{}\right.}}{(bx+a)^3g^3}$
risch	$-\frac{Bi(3bdx+da+2bc)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{6(bx+a)^3b^2g^4}-\frac{(-6B\ln(-bx-a)b^3d^3x^3+6B\ln(dx+c)b^3d^3x^3-18B\ln(-bx-a)a\,b^2d^3x^2+18B}{}$
orering	$-\frac{(bx+a)\left(-24b^3d^3x^3-57a\,b^2d^3x^2-15b^3c\,d^2x^2-114a\,b^2c\,d^2x+42b^3c^2dx+5a^3d^3-15a^2bc\,d^2-42a\,b^2c^2d+28b^3c^3\right)(dix}{36b^2(a^2d^2-2acdb+c^2b^2)(dx+c)(bgx+ag)^4}$
parallelrisch	$-\frac{-36Axa\,b^5c\,d^3i-18Bxa\,b^5c\,d^3i-18B\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^5c^2d^2i+4B\,b^6c^3di-6B\,x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)b^6d^4i+6A\,a^3b^3d^4i+12}{}$

input

```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x,method=_RETURN
VERBOSE)
```

output
```
i*A/g^4*(-1/2*d/b^2/(b*x+a)^2-1/3*(-a*d+b*c)/b^2/(b*x+a)^3)-i*B/g^4/d^3*(a
*d-b*c)^2*e^2*(d^4/(a*d-b*c)^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-d^3/(a*d-b
*c)^4*b*e*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(165) = 330$.

Time $= 0.08$ (sec) , antiderivative size $= 363$, normalized size of antiderivative $= 2.10$

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$$
$$= \frac{6\left(Bb^3cd^2-Bab^2d^3\right)ix^2-3\left((6A+B)b^3c^2d-6(2A+B)ab^2cd^2+(6A+5B)a^2bd^3\right)ix-\left(4(3A+B)\right.}{36\left(\left(b^7c^2-2ab^6cd+a^2b^5d^2\right)g^4x^3+3\left(ab^6c^2-2a^2\right.\right.}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algori
thm="fricas")
```

output
```
1/36*(6*(B*b^3*c*d^2 - B*a*b^2*d^3)*i*x^2 - 3*((6*A + B)*b^3*c^2*d - 6*(2*
A + B)*a*b^2*c*d^2 + (6*A + 5*B)*a^2*b*d^3)*i*x - (4*(3*A + B)*b^3*c^3 - 9
*(2*A + B)*a*b^2*c^2*d + (6*A + 5*B)*a^3*d^3)*i + 6*(B*b^3*d^3*i*x^3 + 3*B
*a*b^2*d^3*i*x^2 - 3*(B*b^3*c^2*d - 2*B*a*b^2*c*d^2)*i*x - (2*B*b^3*c^3 -
3*B*a*b^2*c^2*d)*i)*log((b*e*x + a*e)/(d*x + c)))/((b^7*c^2 - 2*a*b^6*c*d
+ a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x
^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 -
2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(158) = 316.

Time = 4.15 (sec) , antiderivative size = 629, normalized size of antiderivative = 3.64

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$$

$$= -\frac{Bd^3i\log\left(x+\frac{-\frac{Ba^3d^6i}{(ad-bc)^2}+\frac{3Ba^2bcd^5i}{(ad-bc)^2}-\frac{3Bab^2c^2d^4i}{(ad-bc)^2}+Bad^4i+\frac{Bb^3c^3d^3i}{(ad-bc)^2}+Bbcd^3i}{2Bbd^4i}\right)}{6b^2g^4(ad-bc)^2}$$

$$+\frac{Bd^3i\log\left(x+\frac{\frac{Ba^3d^6i}{(ad-bc)^2}-\frac{3Ba^2bcd^5i}{(ad-bc)^2}+\frac{3Bab^2c^2d^4i}{(ad-bc)^2}+Bad^4i-\frac{Bb^3c^3d^3i}{(ad-bc)^2}+Bbcd^3i}{2Bbd^4i}\right)}{6b^2g^4(ad-bc)^2}$$

$$+\frac{(-Badi-2Bbci-3Bbdix)\log\left(\frac{e(a+bx)}{c+dx}\right)}{6a^3b^2g^4+18a^2b^3g^4x+18ab^4g^4x^2+6b^5g^4x^3}$$

$$+\frac{-6Aa^2d^2i-6Aabcdi+12Ab^2c^2i-5Ba^2d^2i-5Babcdi+4Bb^2c^2i-6Bb^2d^2ix^2+x(-18Aabd^2i+18}{36a^4b^2dg^4-36a^3b^3cg^4+x^3\cdot(36ab^5dg^4-36b^6cg^4)+x^2\cdot(108a^2b^4dg^4-108ab^5cg^4)+x\,(108}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4,x)
```

output
```
-B*d**3*i*log(x + (-B*a**3*d**6*i/(a*d - b*c)**2 + 3*B*a**2*b*c*d**5*i/(a*
d - b*c)**2 - 3*B*a*b**2*c**2*d**4*i/(a*d - b*c)**2 + B*a*d**4*i + B*b**3*
c**3*d**3*i/(a*d - b*c)**2 + B*b*c*d**3*i)/(2*B*b*d**4*i))/(6*b**2*g**4*(a
*d - b*c)**2) + B*d**3*i*log(x + (B*a**3*d**6*i/(a*d - b*c)**2 - 3*B*a**2*
b*c*d**5*i/(a*d - b*c)**2 + 3*B*a*b**2*c**2*d**4*i/(a*d - b*c)**2 + B*a*d*
*4*i - B*b**3*c**3*d**3*i/(a*d - b*c)**2 + B*b*c*d**3*i)/(2*B*b*d**4*i))/(
6*b**2*g**4*(a*d - b*c)**2) + (-B*a*d*i - 2*B*b*c*i - 3*B*b*d*i*x)*log(e*(
a + b*x)/(c + d*x))/(6*a**3*b**2*g**4 + 18*a**2*b**3*g**4*x + 18*a*b**4*g*
*4*x**2 + 6*b**5*g**4*x**3) + (-6*A*a**2*d**2*i - 6*A*a*b*c*d*i + 12*A*b**
2*c**2*i - 5*B*a**2*d**2*i - 5*B*a*b*c*d*i + 4*B*b**2*c**2*i - 6*B*b**2*d*
*2*i*x**2 + x*(-18*A*a*b*d**2*i + 18*A*b**2*c*d*i - 15*B*a*b*d**2*i + 3*B*
b**2*c*d*i))/(36*a**4*b**2*d*g**4 - 36*a**3*b**3*c*g**4 + x**3*(36*a*b**5*
d*g**4 - 36*b**6*c*g**4) + x**2*(108*a**2*b**4*d*g**4 - 108*a*b**5*c*g**4)
 + x*(108*a**3*b**3*d*g**4 - 108*a**2*b**4*c*g**4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(165) = 330$.

Time = 0.07 (sec) , antiderivative size = 933, normalized size of antiderivative = 5.39

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag + bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algori
thm="maxima")
```

output
```
-1/36*B*d*i*(6*(3*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^5*g^4*x
^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) + (5*a*b^2*c^2 - 22*
a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*
a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3
 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 -
2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^
2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*d
 + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x + c
)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4)) - 1/18*
B*c*i*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^
2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d
+ a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*
log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^
2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d +
3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3
*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/6*(3*b*x + a)*A*d*i/(b^5*g
^4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/3*A*c*i/(b^4
*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.56

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{1}{36}\left(\frac{6\left(2\,Bbe^4i-\frac{3\,(bex+ae)Bde^3i}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^3bcg^4}{(dx+c)^3}-\frac{(bex+ae)^3adg^4}{(dx+c)^3}}+\frac{12\,Abe^4i+4\,Bbe^4i-\frac{18\,(bex+ae)Ade^3i}{dx+c}-\frac{9\,(bex+ae)Bde^3i}{dx+c}}{\frac{(bex+ae)^3bcg^4}{(dx+c)^3}-\frac{(bex+ae)^3adg^4}{(dx+c)^3}}\right)$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algori
thm="giac")
```

output
```
-1/36*(6*(2*B*b*e^4*i - 3*(b*e*x + a*e)*B*d*e^3*i/(d*x + c))*log((b*e*x + 
a*e)/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4/(d*x + c)^3 - (b*e*x + a*e)^3*a*d
*g^4/(d*x + c)^3) + (12*A*b*e^4*i + 4*B*b*e^4*i - 18*(b*e*x + a*e)*A*d*e^3
*i/(d*x + c) - 9*(b*e*x + a*e)*B*d*e^3*i/(d*x + c))/((b*e*x + a*e)^3*b*c*g
^4/(d*x + c)^3 - (b*e*x + a*e)^3*a*d*g^4/(d*x + c)^3))*(b*c/((b*c*e - a*d*
e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 27.82 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.09

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{\frac{6\,A\,a^2\,d^2\,i-12\,A\,b^2\,c^2\,i+5\,B\,a^2\,d^2\,i-4\,B\,b^2\,c^2\,i+6\,A\,a\,b\,c\,d\,i+5\,B\,a\,b\,c\,d\,i}{6\,(a\,d-b\,c)}+\frac{x\,(6\,A\,a\,b\,d^2\,i+5\,B\,a\,b\,d^2\,i-6\,A\,b^2\,c\,d\,i-B\,b^2\,c\,d\,i)}{2\,(a\,d-b\,c)}+\frac{B\,b^2\,d^2\,i\,x}{a\,d-b\,c}}{6\,a^3\,b^2\,g^4+18\,a^2\,b^3\,g^4\,x+18\,a\,b^4\,g^4\,x^2+6\,b^5\,g^4\,x^3}$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{B\,c\,i}{3\,b^2\,g^4}+\frac{B\,a\,d\,i}{6\,b^3\,g^4}+\frac{B\,d\,i\,x}{2\,b^2\,g^4}\right)}{3\,a^2\,x+\frac{a^3}{b}+b^2\,x^3+3\,a\,b\,x^2}-\frac{B\,d^3\,i\,\operatorname{atanh}\left(\frac{6\,b^4\,c^2\,g^4-6\,a^2\,b^2\,d^2\,g^4}{6\,b^2\,g^4\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{3\,b^2\,g^4\,(a\,d-b\,c)^2}$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^4,x
)
```

output

```
- ((6*A*a^2*d^2*i - 12*A*b^2*c^2*i + 5*B*a^2*d^2*i - 4*B*b^2*c^2*i + 6*A*a
*b*c*d*i + 5*B*a*b*c*d*i)/(6*(a*d - b*c)) + (x*(6*A*a*b*d^2*i + 5*B*a*b*d^
2*i - 6*A*b^2*c*d*i - B*b^2*c*d*i))/(2*(a*d - b*c)) + (B*b^2*d^2*i*x^2)/(a
*d - b*c))/(6*a^3*b^2*g^4 + 6*b^5*g^4*x^3 + 18*a^2*b^3*g^4*x + 18*a*b^4*g^
4*x^2) - (log((e*(a + b*x))/(c + d*x))*((B*c*i)/(3*b^2*g^4) + (B*a*d*i)/(6
*b^3*g^4) + (B*d*i*x)/(2*b^2*g^4)))/(3*a^2*x + a^3/b + b^2*x^3 + 3*a*b*x^2
) - (B*d^3*i*atanh((6*b^4*c^2*g^4 - 6*a^2*b^2*d^2*g^4)/(6*b^2*g^4*(a*d - b
*c)^2) - (2*b*d*x)/(a*d - b*c)))/(3*b^2*g^4*(a*d - b*c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 894, normalized size of antiderivative = 5.17

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x)
```

output

```
(i*(12*log(a + b*x)*a**4*b**2*c*d**2 - 6*log(a + b*x)*a**3*b**3*c**2*d + 3
6*log(a + b*x)*a**3*b**3*c*d**2*x - 18*log(a + b*x)*a**2*b**4*c**2*d*x + 3
6*log(a + b*x)*a**2*b**4*c*d**2*x**2 - 18*log(a + b*x)*a*b**5*c**2*d*x**2
+ 12*log(a + b*x)*a*b**5*c*d**2*x**3 - 6*log(a + b*x)*b**6*c**2*d*x**3 - 1
2*log(c + d*x)*a**4*b**2*c*d**2 + 6*log(c + d*x)*a**3*b**3*c**2*d - 36*log
(c + d*x)*a**3*b**3*c*d**2*x + 18*log(c + d*x)*a**2*b**4*c**2*d*x - 36*log
(c + d*x)*a**2*b**4*c*d**2*x**2 + 18*log(c + d*x)*a*b**5*c**2*d*x**2 - 12*
log(c + d*x)*a*b**5*c*d**2*x**3 + 6*log(c + d*x)*b**6*c**2*d*x**3 - 12*log
((a*e + b*e*x)/(c + d*x))*a**4*b**2*c*d**2 + 24*log((a*e + b*e*x)/(c + d*x
))*a**3*b**3*c**2*d + 18*log((a*e + b*e*x)/(c + d*x))*a**3*b**3*d**3*x**2
- 12*log((a*e + b*e*x)/(c + d*x))*a**2*b**4*c**3 - 36*log((a*e + b*e*x)/(c
 + d*x))*a**2*b**4*c*d**2*x**2 + 6*log((a*e + b*e*x)/(c + d*x))*a**2*b**4*
d**3*x**3 + 18*log((a*e + b*e*x)/(c + d*x))*a*b**5*c**2*d*x**2 - 12*log((a
*e + b*e*x)/(c + d*x))*a*b**5*c*d**2*x**3 + 6*log((a*e + b*e*x)/(c + d*x))
*b**6*c**2*d*x**3 - 6*a**6*d**3 - 18*a**5*b*d**3*x - 3*a**5*b*d**3 + 18*a*
*4*b**2*c**2*d + 36*a**4*b**2*c*d**2*x - 2*a**4*b**2*c*d**2 - 9*a**4*b**2*
d**3*x - 12*a**3*b**3*c**3 - 18*a**3*b**3*c**2*d*x + 9*a**3*b**3*c**2*d +
12*a**3*b**3*c*d**2*x - 4*a**2*b**4*c**3 - 3*a**2*b**4*c**2*d*x + 2*a**2*b
**4*d**3*x**3 - 2*a*b**5*c*d**2*x**3))/(36*a**2*b**2*g**4*(a**5*d**2 - 2*a
**4*b*c*d + 3*a**4*b*d**2*x + a**3*b**2*c**2 - 6*a**3*b**2*c*d*x + 3*a*...
```

3.9 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$

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Optimal result

Integrand size = 38, antiderivative size = 269

$$\begin{aligned}\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = &-\frac{Bd^2i(c+dx)^2}{4(bc-ad)^3g^5(a+bx)^2}\\ &+\frac{2bBdi(c+dx)^3}{9(bc-ad)^3g^5(a+bx)^3}\\ &-\frac{b^2Bi(c+dx)^4}{16(bc-ad)^3g^5(a+bx)^4}\\ &-\frac{d^2i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^5(a+bx)^2}\\ &+\frac{2bdi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^3g^5(a+bx)^3}\\ &-\frac{b^2i(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3g^5(a+bx)^4}\end{aligned}$$

output
```
-1/4*B*d^2*i*(d*x+c)^2/(-a*d+b*c)^3/g^5/(b*x+a)^2+2/9*b*B*d*i*(d*x+c)^3/(-
a*d+b*c)^3/g^5/(b*x+a)^3-1/16*b^2*B*i*(d*x+c)^4/(-a*d+b*c)^3/g^5/(b*x+a)^4
-1/2*d^2*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^5/(b*x+a)^
2+2/3*b*d*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^5/(b*x+a)
^3-1/4*b^2*i*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^5/(b*x+a
)^4
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.78

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{i\left(\frac{36Abc}{(a+bx)^4}+\frac{9bBc}{(a+bx)^4}-\frac{36aAd}{(a+bx)^4}-\frac{9aBd}{(a+bx)^4}+\frac{48Ad}{(a+bx)^3}+\frac{4Bd}{(a+bx)^3}-\frac{6Bd^2}{(bc-ad)(a+bx)^2}+\frac{12Bd^3}{(bc-ad)^2(a+bx)}+\frac{12Bd^4\log(a+b}{(bc-ad)^3}\right.}{144b^2g^5}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*
x)^5,x]
```

output
```
-1/144*(i*((36*A*b*c)/(a + b*x)^4 + (9*b*B*c)/(a + b*x)^4 - (36*a*A*d)/(a
+ b*x)^4 - (9*a*B*d)/(a + b*x)^4 + (48*A*d)/(a + b*x)^3 + (4*B*d)/(a + b*x
)^3 - (6*B*d^2)/((b*c - a*d)*(a + b*x)^2) + (12*B*d^3)/((b*c - a*d)^2*(a +
 b*x)) + (12*B*d^4*Log[a + b*x])/(b*c - a*d)^3 + (12*B*(3*b*c + a*d + 4*b*
d*x)*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^4 - (12*B*d^4*Log[c + d*x])/(
b*c - a*d)^3))/(b^2*g^5)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2962, 2772, 27, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^5}\,dx$$

$$\downarrow 2962$$

$$\frac{i\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^3}$$

$$\downarrow 2772$$

$$\frac{i\left(-B\int -\frac{(c+dx)^5\left(3b^2-\frac{8d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{12(a+bx)^5}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{2b}{}\right.}{g^5(bc-ad)^3}$$

$$\downarrow 27$$

$$\frac{i\left(\frac{1}{12}B\int \frac{(c+dx)^5\left(3b^2-\frac{8d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{2bd(}{}\right.}{g^5(bc-ad)^3}$$

$$\downarrow 1140$$

$$\frac{i\left(\frac{1}{12}B\int \left(\frac{3b^2(c+dx)^5}{(a+bx)^5}-\frac{8bd(c+dx)^4}{(a+bx)^4}+\frac{6d^2(c+dx)^3}{(a+bx)^3}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}\right.}{g^5(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{i\left(-\frac{b^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{2bd(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{1}{12}B\left(-\frac{3b^2(c+dx)^4}{4(a+bx)^4}-\right.\right.}{g^5(bc-ad)^3}$$

input

```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^5,x
]
```

output
```
(i*((B*((-3*d^2*(c + d*x)^2)/(a + b*x)^2 + (8*b*d*(c + d*x)^3)/(3*(a + b*x
)^3) - (3*b^2*(c + d*x)^4)/(4*(a + b*x)^4)))/12 - (d^2*(c + d*x)^2*(A + B*
Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^2) + (2*b*d*(c + d*x)^3*(A + B
*Log[(e*(a + b*x))/(c + d*x)]))/(3*(a + b*x)^3) - (b^2*(c + d*x)^4*(A + B*
Log[(e*(a + b*x))/(c + d*x)]))/(4*(a + b*x)^4)))/((b*c - a*d)^3*g^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.45

method	result
parts	$\frac{iA\left(-\frac{d}{3b^2(bx+a)^3}-\frac{-da+bc}{4b^2(bx+a)^4}\right)}{g^5}-\frac{iB(da-bc)^2e^2\left(\frac{d^5\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}\right)}{(da-bc)^5}-\frac{2d^4be\left(-\frac{\ln\left(\frac{be}{d}+\right.\right.}{3\left(\frac{be}{d}+\right.}\right.}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{i\,d^2e^3Ab^2}{4(da-bc)^4g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}+\frac{2i\,d^3e^2Ab}{3(da-bc)^4g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{i\,d^4eA}{2(da-bc)^4g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{i\,d^2e^3B}{}\right.}{}$
default	$-\frac{e(da-bc)\left(-\frac{i\,d^2e^3Ab^2}{4(da-bc)^4g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}+\frac{2i\,d^3e^2Ab}{3(da-bc)^4g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{i\,d^4eA}{2(da-bc)^4g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{i\,d^2e^3B}{}\right.}{}$
orering	$-\frac{(bx+a)(-60b^4d^4x^4-204ab^3d^4x^3-36b^4cd^3x^3-238a^2b^2d^4x^2-136ab^3cd^3x^2+14b^4c^2d^2x^2-476a^2b^2cd^3x+340ab^3c}{144b^2(a^3d^3-3a^2bcd^2+3ab^2c^2d-}$
risch	$-\frac{iB(4bdx+da+3bc)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{12(bx+a)^4b^2g^5}-\frac{(-36Ba^2b^2c^2d^2+6Bb^4c^2d^2x^2+12Baab^3d^4x^3-12Bb^4cd^3x^3+52Baa^3bd^4x-4}{}$
norman	$\frac{\left(12Aa^2cd^2i-24Aabc^2di+12Ab^2c^3i+6Ba^2cd^2i-8Babc^2di+3Bb^2c^3i\right)x}{12ga\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{\left(12Aa^3d^3i+12Aa^2bcd^2i-60Aab^2c^2di+36Ab^3c^3i+\right.}{24ga^2\left(a^2d^2-\right.}$
parallelrisch	$\frac{48Bx^3\ln\left(\frac{e(bx+a)}{dx+c}\right)a^7bcd^4i-144Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^7bc^3d^2i+12Bx^4\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6b^2cd^4i+72B\ln\left(\frac{e(bx+a)}{dx+c}\right)a^8c^3d^2}{}$

input

```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x,method=_RETURN
VERBOSE)
```

output

```
i*A/g^5*(-1/3*d/b^2/(b*x+a)^3-1/4*(-a*d+b*c)/b^2/(b*x+a)^4)-i*B/g^5/d^3*(a
*d-b*c)^2*e^2*(d^5/(a*d-b*c)^5*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2*d^4/(a*d
-b*c)^5*b*e*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)+d^3/(a*d-b*c)^5*b^2*e^2*(-1/4
/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e
/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(257) = 514$.

Time $= 0.09$ (sec) , antiderivative size $= 602$, normalized size of antiderivative $= 2.24$

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{12\,(Bb^4cd^3-Bab^3d^4)ix^3-6\,(Bb^4c^2d^2-8\,Bab^3cd^3+7\,Ba^2b^2d^4)ix^2+4\,((12\,A+B)b^4c^3d-6\,(6\,A+}{144\,((b^9c^3-3\,ab^8c^2d+3\,a}$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algori
thm="fricas")
```

output

```
-1/144*(12*(B*b^4*c*d^3 - B*a*b^3*d^4)*i*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*b^
3*c*d^3 + 7*B*a^2*b^2*d^4)*i*x^2 + 4*((12*A + B)*b^4*c^3*d - 6*(6*A + B)*a
*b^3*c^2*d^2 + 18*(2*A + B)*a^2*b^2*c*d^3 - (12*A + 13*B)*a^3*b*d^4)*i*x +
 (9*(4*A + B)*b^4*c^4 - 32*(3*A + B)*a*b^3*c^3*d + 36*(2*A + B)*a^2*b^2*c^
2*d^2 - (12*A + 13*B)*a^4*d^4)*i + 12*(B*b^4*d^4*i*x^4 + 4*B*a*b^3*d^4*i*x
^3 + 6*B*a^2*b^2*d^4*i*x^2 + 4*(B*b^4*c^3*d - 3*B*a*b^3*c^2*d^2 + 3*B*a^2*
b^2*c*d^3)*i*x + (3*B*b^4*c^4 - 8*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2)*i)*
log((b*e*x + a*e)/(d*x + c)))/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2
- a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2
- a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^
2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*
d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^
2 - a^7*b^2*d^3)*g^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(252) = 504$.

Time = 6.90 (sec) , antiderivative size = 928, normalized size of antiderivative = 3.45

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**5,x)
```

output

```
-B*d**4*i*log(x + (-B*a**4*d**8*i/(a*d - b*c)**3 + 4*B*a**3*b*c*d**7*i/(a*
d - b*c)**3 - 6*B*a**2*b**2*c**2*d**6*i/(a*d - b*c)**3 + 4*B*a*b**3*c**3*d
**5*i/(a*d - b*c)**3 + B*a*d**5*i - B*b**4*c**4*d**4*i/(a*d - b*c)**3 + B*
b*c*d**4*i)/(2*B*b*d**5*i))/(12*b**2*g**5*(a*d - b*c)**3) + B*d**4*i*log(x
 + (B*a**4*d**8*i/(a*d - b*c)**3 - 4*B*a**3*b*c*d**7*i/(a*d - b*c)**3 + 6*
B*a**2*b**2*c**2*d**6*i/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**5*i/(a*d - b*c
)**3 + B*a*d**5*i + B*b**4*c**4*d**4*i/(a*d - b*c)**3 + B*b*c*d**4*i)/(2*B
*b*d**5*i))/(12*b**2*g**5*(a*d - b*c)**3) + (-B*a*d*i - 3*B*b*c*i - 4*B*b*
d*i*x)*log(e*(a + b*x)/(c + d*x))/(12*a**4*b**2*g**5 + 48*a**3*b**3*g**5*x
 + 72*a**2*b**4*g**5*x**2 + 48*a*b**5*g**5*x**3 + 12*b**6*g**5*x**4) + (-1
2*A*a**3*d**3*i - 12*A*a**2*b*c*d**2*i + 60*A*a*b**2*c**2*d*i - 36*A*b**3*
c**3*i - 13*B*a**3*d**3*i - 13*B*a**2*b*c*d**2*i + 23*B*a*b**2*c**2*d*i -
9*B*b**3*c**3*i - 12*B*b**3*d**3*i*x**3 + x**2*(-42*B*a*b**2*d**3*i + 6*B*
b**3*c*d**2*i) + x*(-48*A*a**2*b*d**3*i + 96*A*a*b**2*c*d**2*i - 48*A*b**3
*c**2*d*i - 52*B*a**2*b*d**3*i + 20*B*a*b**2*c*d**2*i - 4*B*b**3*c**2*d*i)
)/(144*a**6*b**2*d**2*g**5 - 288*a**5*b**3*c*d*g**5 + 144*a**4*b**4*c**2*g
**5 + x**4*(144*a**2*b**6*d**2*g**5 - 288*a*b**7*c*d*g**5 + 144*b**8*c**2*
g**5) + x**3*(576*a**3*b**5*d**2*g**5 - 1152*a**2*b**6*c*d*g**5 + 576*a*b*
*7*c**2*g**5) + x**2*(864*a**4*b**4*d**2*g**5 - 1728*a**3*b**5*c*d*g**5 +
864*a**2*b**6*c**2*g**5) + x*(576*a**5*b**3*d**2*g**5 - 1152*a**4*b**4*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(257) = 514$.

Time $= 0.10$ (sec) , antiderivative size $= 1386$, normalized size of antiderivative $= 5.15$

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algori
thm="maxima")
```

output
```
-1/144*B*d*i*(12*(4*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^6*g^5
*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5
) + (7*a*b^3*c^3 - 33*a^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 13*a^4*d^3 + 12*(4*
b^4*c*d^2 - a*b^3*d^3)*x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c*d^2 + 7*a^2*b^2*d
^3)*x^2 + 4*(4*b^4*c^3 - 21*a*b^3*c^2*d + 57*a^2*b^2*c*d^2 - 13*a^3*b*d^3)
*x)/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4
*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6
*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 +
 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x +
 (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*b^2*d^3)*g^5) + 12
*(4*b*c*d^3 - a*d^4)*log(b*x + a)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^
2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5) - 12*(4*b*c*d^3 - a*d^4)*log(d
*x + c)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 +
a^4*b^2*d^4)*g^5)) + 1/48*B*c*i*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^
2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b
^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*
a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*
a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d +
3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d
+ 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d...
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.58

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{1}{144}\left(\frac{12\left(3\,Bb^2e^5i-\frac{8\,(bex+ae)Bbde^4i}{dx+c}+\frac{6\,(bex+ae)^2Bd^2e^3i}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^4b^2c^2g^5}{(dx+c)^4}-\frac{2\,(bex+ae)^4abcdg^5}{(dx+c)^4}+\frac{(bex+ae)^4a^2d^2g^5}{(dx+c)^4}}+\frac{36\,Ab^2e^5i+9\,Bb^2e^5i-\frac{96\,(bex+a}{dx+}}{\frac{(bex+ae)^4b}{(dx+c)}}\right.$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algori
thm="giac")
```

output
```
-1/144*(12*(3*B*b^2*e^5*i - 8*(b*e*x + a*e)*B*b*d*e^4*i/(d*x + c) + 6*(b*e
*x + a*e)^2*B*d^2*e^3*i/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x
 + a*e)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*e*x + a*e)^4*a*b*c*d*g^5/(d*x + c)
^4 + (b*e*x + a*e)^4*a^2*d^2*g^5/(d*x + c)^4) + (36*A*b^2*e^5*i + 9*B*b^2*
e^5*i - 96*(b*e*x + a*e)*A*b*d*e^4*i/(d*x + c) - 32*(b*e*x + a*e)*B*b*d*e^
4*i/(d*x + c) + 72*(b*e*x + a*e)^2*A*d^2*e^3*i/(d*x + c)^2 + 36*(b*e*x + a
*e)^2*B*d^2*e^3*i/(d*x + c)^2)/((b*e*x + a*e)^4*b^2*c^2*g^5/(d*x + c)^4 -
2*(b*e*x + a*e)^4*a*b*c*d*g^5/(d*x + c)^4 + (b*e*x + a*e)^4*a^2*d^2*g^5/(d
*x + c)^4))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c
 - a*d)))
```

Mupad [B] (verification not implemented)

Time = 28.15 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.19

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$$

$$=\frac{B\,d^4\,i\,\operatorname{atanh}\left(\frac{12\,a^3\,b^2\,d^3\,g^5-12\,a^2\,b^3\,c\,d^2\,g^5-12\,a\,b^4\,c^2\,d\,g^5+12\,b^5\,c^3\,g^5}{12\,b^2\,g^5\,(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{6\,b^2\,g^5\,(a\,d-b\,c)^3}$$

$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{B\,c\,i}{4\,b^2\,g^5}+\frac{B\,a\,d\,i}{12\,b^3\,g^5}+\frac{B\,d\,i\,x}{3\,b^2\,g^5}\right)}{4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3}$$

$$-\frac{\frac{12\,A\,a^3\,d^3\,i+36\,A\,b^3\,c^3\,i+13\,B\,a^3\,d^3\,i+9\,B\,b^3\,c^3\,i-60\,A\,a\,b^2\,c^2\,d\,i+12\,A\,a^2\,b\,c\,d^2\,i-23\,B\,a\,b^2\,c^2\,d\,i+13\,B\,a^2\,b\,c\,d^2\,i}{12\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{x\,(12\,A\,a^2\,b\,d^3\,i+13\,B}{}}{12\,a^4\,b^2\,g^5+48\,a^3\,b^3\,g^5\,x+72\,a^2\,b^4\,g^5\,x}$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^5,x
)
```

output
```
(B*d^4*i*atanh((12*b^5*c^3*g^5 + 12*a^3*b^2*d^3*g^5 - 12*a*b^4*c^2*d*g^5 -
 12*a^2*b^3*c*d^2*g^5)/(12*b^2*g^5*(a*d - b*c)^3) + (2*b*d*x*(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(6*b^2*g^5*(a*d - b*c)^3) - (log((e*(a
 + b*x))/(c + d*x))*((B*c*i)/(4*b^2*g^5) + (B*a*d*i)/(12*b^3*g^5) + (B*d*i
*x)/(3*b^2*g^5)))/(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)
 - ((12*A*a^3*d^3*i + 36*A*b^3*c^3*i + 13*B*a^3*d^3*i + 9*B*b^3*c^3*i - 60*
A*a*b^2*c^2*d*i + 12*A*a^2*b*c*d^2*i - 23*B*a*b^2*c^2*d*i + 13*B*a^2*b*c*d
^2*i)/(12*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(12*A*a^2*b*d^3*i + 13*B*a
^2*b*d^3*i + 12*A*b^3*c^2*d*i + B*b^3*c^2*d*i - 24*A*a*b^2*c*d^2*i - 5*B*a
*b^2*c*d^2*i))/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (d*x^2*(B*b^3*c*d*i -
 7*B*a*b^2*d^2*i))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B*b^3*d^3*i*x^3)
/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(12*a^4*b^2*g^5 + 12*b^6*g^5*x^4 + 48*a^
3*b^3*g^5*x + 48*a*b^5*g^5*x^3 + 72*a^2*b^4*g^5*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 954, normalized size of antiderivative = 3.55

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x)
```

output

```
(i*(12*log(a + b*x)*a**5*b*d**4 + 48*log(a + b*x)*a**4*b**2*d**4*x + 72*lo
g(a + b*x)*a**3*b**3*d**4*x**2 + 48*log(a + b*x)*a**2*b**4*d**4*x**3 + 12*
log(a + b*x)*a*b**5*d**4*x**4 - 12*log(c + d*x)*a**5*b*d**4 - 48*log(c + d
*x)*a**4*b**2*d**4*x - 72*log(c + d*x)*a**3*b**3*d**4*x**2 - 48*log(c + d*
x)*a**2*b**4*d**4*x**3 - 12*log(c + d*x)*a*b**5*d**4*x**4 - 12*log((a*e +
b*e*x)/(c + d*x))*a**5*b*d**4 - 48*log((a*e + b*e*x)/(c + d*x))*a**4*b**2*
d**4*x + 72*log((a*e + b*e*x)/(c + d*x))*a**3*b**3*c**2*d**2 + 144*log((a*
e + b*e*x)/(c + d*x))*a**3*b**3*c*d**3*x - 96*log((a*e + b*e*x)/(c + d*x))
*a**2*b**4*c**3*d - 144*log((a*e + b*e*x)/(c + d*x))*a**2*b**4*c**2*d**2*x
 + 36*log((a*e + b*e*x)/(c + d*x))*a*b**5*c**4 + 48*log((a*e + b*e*x)/(c +
 d*x))*a*b**5*c**3*d*x - 12*a**6*d**4 - 48*a**5*b*d**4*x - 10*a**5*b*d**4
+ 72*a**4*b**2*c**2*d**2 + 144*a**4*b**2*c*d**3*x - 3*a**4*b**2*c*d**3 - 4
0*a**4*b**2*d**4*x - 96*a**3*b**3*c**3*d - 144*a**3*b**3*c**2*d**2*x + 36*
a**3*b**3*c**2*d**2 + 60*a**3*b**3*c*d**3*x - 24*a**3*b**3*d**4*x**2 + 36*
a**2*b**4*c**4 + 48*a**2*b**4*c**3*d*x - 32*a**2*b**4*c**3*d - 24*a**2*b**
4*c**2*d**2*x + 30*a**2*b**4*c*d**3*x**2 + 9*a*b**5*c**4 + 4*a*b**5*c**3*d
*x - 6*a*b**5*c**2*d**2*x**2 + 3*a*b**5*d**4*x**4 - 3*b**6*c*d**3*x**4))/(
144*a*b**2*g**5*(a**7*d**3 - 3*a**6*b*c*d**2 + 4*a**6*b*d**3*x + 3*a**5*b*
*2*c**2*d - 12*a**5*b**2*c*d**2*x + 6*a**5*b**2*d**3*x**2 - a**4*b**3*c**3
 + 12*a**4*b**3*c**2*d*x - 18*a**4*b**3*c*d**2*x**2 + 4*a**4*b**3*d**3*...
```

3.10 $\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 423

$$
\begin{aligned}
&\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx \\
&= \frac{B(bc-ad)^5g^3i^2x}{60b^2d^3}+\frac{B(bc-ad)^4g^3i^2(c+dx)^2}{120bd^4}-\frac{19B(bc-ad)^3g^3i^2(c+dx)^3}{180d^4} \\
&\quad+\frac{13bB(bc-ad)^2g^3i^2(c+dx)^4}{120d^4}-\frac{b^2B(bc-ad)g^3i^2(c+dx)^5}{30d^4} \\
&\quad+\frac{B(bc-ad)^6g^3i^2\log\left(\frac{a+bx}{c+dx}\right)}{60b^3d^4}-\frac{(bc-ad)^3g^3i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^4} \\
&\quad+\frac{3b(bc-ad)^2g^3i^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^4} \\
&\quad-\frac{3b^2(bc-ad)g^3i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^4} \\
&\quad+\frac{b^3g^3i^2(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^4}+\frac{B(bc-ad)^6g^3i^2\log(c+dx)}{60b^3d^4}
\end{aligned}
$$

output
```
1/60*B*(-a*d+b*c)^5*g^3*i^2*x/b^2/d^3+1/120*B*(-a*d+b*c)^4*g^3*i^2*(d*x+c)
^2/b/d^4-19/180*B*(-a*d+b*c)^3*g^3*i^2*(d*x+c)^3/d^4+13/120*b*B*(-a*d+b*c)
^2*g^3*i^2*(d*x+c)^4/d^4-1/30*b^2*B*(-a*d+b*c)*g^3*i^2*(d*x+c)^5/d^4+1/60*
B*(-a*d+b*c)^6*g^3*i^2*ln((b*x+a)/(d*x+c))/b^3/d^4-1/3*(-a*d+b*c)^3*g^3*i^
2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4+3/4*b*(-a*d+b*c)^2*g^3*i^2*(d*
x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4-3/5*b^2*(-a*d+b*c)*g^3*i^2*(d*x+c)^
5*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4+1/6*b^3*g^3*i^2*(d*x+c)^6*(A+B*ln(e*(b*x
+a)/(d*x+c)))/d^4+1/60*B*(-a*d+b*c)^6*g^3*i^2*ln(d*x+c)/b^3/d^4
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.01

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{g^3i^2\Big(90d^4(bc-ad)^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+144d^5(bc-ad)(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)$$

input
```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)]),x]
```

output
```
(g^3*i^2*(90*d^4*(b*c - a*d)^2*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d
*x)]) + 144*d^5*(b*c - a*d)*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)
]) + 60*d^6*(a + b*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 15*B*(b*c -
 a*d)^3*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*
(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^2*(12*b*d*(
b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c - a*d)*(a +
b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]) - B*(b*c - a*d
)*(60*b*d*(b*c - a*d)^4*x + 30*d^2*(-(b*c) + a*d)^3*(a + b*x)^2 + 20*d^3*(
b*c - a*d)^2*(a + b*x)^3 + 15*d^4*(-(b*c) + a*d)*(a + b*x)^4 + 12*d^5*(a +
 b*x)^5 - 60*(b*c - a*d)^5*Log[c + d*x])))/(360*b^3*d^4)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2782, 27, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\downarrow 2962$$

$$g^3i^2(bc-ad)^6\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}$$

$$\downarrow 2782$$

$$g^3i^2(bc-ad)^6\left(-B\int-\frac{(c+dx)\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{15d^2(a+bx)^2b}{(c+dx)^2}-\frac{20d^3(a+bx)^3}{(c+dx)^3}\right)}{60d^4(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}+\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3b}{}\right.$$

$$\downarrow 27$$

$$g^3i^2(bc-ad)^6\left(\frac{B\int\frac{(c+dx)\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{15d^2(a+bx)^2b}{(c+dx)^2}-\frac{20d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{60d^4}+\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.$$

$$\downarrow 2123$$

$$g^3i^2(bc-ad)^6\left(\frac{B\int\left(-\frac{10db^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6}+\frac{26db}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{19d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3b}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2b^2}+\frac{c+dx}{(a+bx)b^3}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{60d^4}\right.$$

$$\downarrow 2009$$

$$g^3 i^2 (bc -$$

$$ad)^6 \left(\frac{b^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)}{6d^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^6} - \frac{3b^2 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)}{5d^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^5} + \frac{3b \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)}{4d^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^4} - \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) +}{3d^4 \left(b - \frac{d(a+bx)}{c+dx}\right)}$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x
]
```

output
```
(b*c - a*d)^6*g^3*i^2*((b^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(6*d^4*(
b - (d*(a + b*x))/(c + d*x))^6) - (3*b^2*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(5*d^4*(b - (d*(a + b*x))/(c + d*x))^5) + (3*b*(A + B*Log[(e*(a + b*x
))/(c + d*x)]))/(4*d^4*(b - (d*(a + b*x))/(c + d*x))^4) - (A + B*Log[(e*(a
 + b*x))/(c + d*x)])/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^3) + (B*((-2*b^2
)/(b - (d*(a + b*x))/(c + d*x))^5 + (13*b)/(2*(b - (d*(a + b*x))/(c + d*x)
)^4) - 19/(3*(b - (d*(a + b*x))/(c + d*x))^3) + 1/(2*b*(b - (d*(a + b*x))/
(c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c +
 d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(60*d^4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2782

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(401) = 802$.

Time = 1.76 (sec) , antiderivative size = 925, normalized size of antiderivative = 2.19

method	result
risch	$i^2g^3Aa^3c^2x + \frac{3i^2g^3b^2dAacx^4}{2} - \frac{i^2g^3b^2dBacx^4}{20} + 2i^2g^3bdAa^2cx^3 + i^2g^3b^2Aac^2x^3 + \frac{7i^2g^3bdE$
parallelrisch	Expression too large to display
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input

```
int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETU
RNVERBOSE)
```

output

```
i^2*g^3*A*a^3*c^2*x+3/2*i^2*g^3*b^2*d*A*a*c*x^4-1/20*i^2*g^3*b^2*d*B*a*c*x
^4+2*i^2*g^3*b*d*A*a^2*c*x^3+i^2*g^3*b^2*A*a*c^2*x^3+7/60*i^2*g^3*b*d*B*a^
2*c*x^3-13/60*i^2*g^3*b^2*B*a*c^2*x^3+i^2*g^3*d*A*a^3*c*x^2+3/2*i^2*g^3*b*
A*a^2*c^2*x^2+17/60*i^2*g^3*d*B*a^3*c*x^2-1/4*i^2*g^3*b*B*a^2*c^2*x^2-1/20
*i^2*g^3*b^2/d*B*a*c^3*x^2+1/10*i^2*g^3/b*d*B*a^4*c*x+1/12*i^2*g^3*B*a^3*c
^2*x-1/4*i^2*g^3*b/d*B*a^2*c^3*x+1/10*i^2*g^3*b^2/d^2*B*a*c^4*x-1/10*i^2*g
^3/b^2*d*B*ln(-b*x-a)*a^5*c+1/4*i^2*g^3*b/d^2*B*ln(d*x+c)*a^2*c^4-1/10*i^2
*g^3*b^2/d^3*B*ln(d*x+c)*a*c^5+3/5*i^2*g^3*b^2*d^2*A*a*x^5+2/5*i^2*g^3*b^3
*d*A*c*x^5+1/30*i^2*g^3*b^2*d^2*B*a*x^5-1/30*i^2*g^3*b^3*d*B*c*x^5+3/4*i^2
*g^3*b*d^2*A*a^2*x^4+1/4*i^2*g^3*b^3*A*c^2*x^4+13/120*i^2*g^3*b*d^2*B*a^2*
x^4-7/120*i^2*g^3*b^3*B*c^2*x^4+1/3*i^2*g^3*d^2*A*a^3*x^3+19/180*i^2*g^3*d
^2*B*a^3*x^3-1/180*i^2*g^3*b^3/d*B*c^3*x^3+1/120*i^2*g^3/b*d^2*B*a^4*x^2+1
/120*i^2*g^3*b^3/d^2*B*c^4*x^2-1/60*i^2*g^3/b^2*d^2*B*a^5*x-1/60*i^2*g^3*b
^3/d^3*B*c^5*x+1/4*i^2*g^3/b*B*ln(-b*x-a)*a^4*c^2-1/3*i^2*g^3/d*B*ln(d*x+c
)*a^3*c^3+1/60*i^2*g^3/b^3*d^2*B*ln(-b*x-a)*a^6+1/60*i^2*g^3*b^3/d^4*B*ln(
d*x+c)*c^6+1/6*i^2*g^3*b^3*d^2*A*x^6+1/60*i^2*g^3*B*x*(10*b^3*d^2*x^5+36*a
*b^2*d^2*x^4+24*b^3*c*d*x^4+45*a^2*b*d^2*x^3+90*a*b^2*c*d*x^3+15*b^3*c^2*x
^3+20*a^3*d^2*x^2+120*a^2*b*c*d*x^2+60*a*b^2*c^2*x^2+60*a^3*c*d*x+90*a^2*b
*c^2*x+60*a^3*c^2)*ln(e*(b*x+a)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.71

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{60\,Ab^6d^6g^3i^2x^6+12\left((12\,A-B)b^6cd^5+(18\,A+B)ab^5d^6\right)g^3i^2x^5+3\left((30\,A-7\,B)b^6c^2d^4+6\,(30\,A-B\right.}{}$$

input

```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="fricas")
```

output

```
1/360*(60*A*b^6*d^6*g^3*i^2*x^6 + 12*((12*A - B)*b^6*c*d^5 + (18*A + B)*a*
b^5*d^6)*g^3*i^2*x^5 + 3*((30*A - 7*B)*b^6*c^2*d^4 + 6*(30*A - B)*a*b^5*c*
d^5 + (90*A + 13*B)*a^2*b^4*d^6)*g^3*i^2*x^4 - 2*(B*b^6*c^3*d^3 - 3*(60*A
- 13*B)*a*b^5*c^2*d^4 - 3*(120*A + 7*B)*a^2*b^4*c*d^5 - (60*A + 19*B)*a^3*
b^3*d^6)*g^3*i^2*x^3 + 3*(B*b^6*c^4*d^2 - 6*B*a*b^5*c^3*d^3 + 30*(6*A - B)
*a^2*b^4*c^2*d^4 + 2*(60*A + 17*B)*a^3*b^3*c*d^5 + B*a^4*b^2*d^6)*g^3*i^2*
x^2 - 6*(B*b^6*c^5*d - 6*B*a*b^5*c^4*d^2 + 15*B*a^2*b^4*c^3*d^3 - 5*(12*A
+ B)*a^3*b^3*c^2*d^4 - 6*B*a^4*b^2*c*d^5 + B*a^5*b*d^6)*g^3*i^2*x + 6*(15*
B*a^4*b^2*c^2*d^4 - 6*B*a^5*b*c*d^5 + B*a^6*d^6)*g^3*i^2*log(b*x + a) + 6*
(B*b^6*c^6 - 6*B*a*b^5*c^5*d + 15*B*a^2*b^4*c^4*d^2 - 20*B*a^3*b^3*c^3*d^3
)*g^3*i^2*log(d*x + c) + 6*(10*B*b^6*d^6*g^3*i^2*x^6 + 60*B*a^3*b^3*c^2*d^
4*g^3*i^2*x + 12*(2*B*b^6*c*d^5 + 3*B*a*b^5*d^6)*g^3*i^2*x^5 + 15*(B*b^6*c
^2*d^4 + 6*B*a*b^5*c*d^5 + 3*B*a^2*b^4*d^6)*g^3*i^2*x^4 + 20*(3*B*a*b^5*c^
2*d^4 + 6*B*a^2*b^4*c*d^5 + B*a^3*b^3*d^6)*g^3*i^2*x^3 + 30*(3*B*a^2*b^4*c
^2*d^4 + 2*B*a^3*b^3*c*d^5)*g^3*i^2*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^
3*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1727 vs. $2(398) = 796$.

Time = 6.07 (sec) , antiderivative size = 1727, normalized size of antiderivative = 4.08

$$\int (ag + bgx)^3 (ci + dix)^2 \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output

```
A*b**3*d**2*g**3*i**2*x**6/6 + B*a**4*g**3*i**2*(a**2*d**2 - 6*a*b*c*d + 1
5*b**2*c**2)*log(x + (B*a**6*c*d**5*g**3*i**2 - 6*B*a**5*b*c**2*d**4*g**3*
i**2 + B*a**5*d**4*g**3*i**2*(a**2*d**2 - 6*a*b*c*d + 15*b**2*c**2)/b + 35
*B*a**4*b**2*c**3*d**3*g**3*i**2 - B*a**4*c*d**3*g**3*i**2*(a**2*d**2 - 6*
a*b*c*d + 15*b**2*c**2) - 15*B*a**3*b**3*c**4*d**2*g**3*i**2 + 6*B*a**2*b*
*4*c**5*d*g**3*i**2 - B*a*b**5*c**6*g**3*i**2)/(B*a**6*d**6*g**3*i**2 - 6*
B*a**5*b*c*d**5*g**3*i**2 + 15*B*a**4*b**2*c**2*d**4*g**3*i**2 + 20*B*a**3
*b**3*c**3*d**3*g**3*i**2 - 15*B*a**2*b**4*c**4*d**2*g**3*i**2 + 6*B*a*b**
5*c**5*d*g**3*i**2 - B*b**6*c**6*g**3*i**2))/(60*b**3) - B*c**3*g**3*i**2*
(20*a**3*d**3 - 15*a**2*b*c*d**2 + 6*a*b**2*c**2*d - b**3*c**3)*log(x + (B
*a**6*c*d**5*g**3*i**2 - 6*B*a**5*b*c**2*d**4*g**3*i**2 + 35*B*a**4*b**2*c
**3*d**3*g**3*i**2 - 15*B*a**3*b**3*c**4*d**2*g**3*i**2 + 6*B*a**2*b**4*c*
*5*d*g**3*i**2 - B*a*b**5*c**6*g**3*i**2 - B*a*b**2*c**3*g**3*i**2*(20*a**
3*d**3 - 15*a**2*b*c*d**2 + 6*a*b**2*c**2*d - b**3*c**3) + B*b**3*c**4*g**
3*i**2*(20*a**3*d**3 - 15*a**2*b*c*d**2 + 6*a*b**2*c**2*d - b**3*c**3)/d)/
(B*a**6*d**6*g**3*i**2 - 6*B*a**5*b*c*d**5*g**3*i**2 + 15*B*a**4*b**2*c**2
*d**4*g**3*i**2 + 20*B*a**3*b**3*c**3*d**3*g**3*i**2 - 15*B*a**2*b**4*c**4
*d**2*g**3*i**2 + 6*B*a*b**5*c**5*d*g**3*i**2 - B*b**6*c**6*g**3*i**2))/(6
0*d**4) + x**5*(3*A*a*b**2*d**2*g**3*i**2/5 + 2*A*b**3*c*d*g**3*i**2/5 + B
*a*b**2*d**2*g**3*i**2/30 - B*b**3*c*d*g**3*i**2/30) + x**4*(3*A*a**2*b...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1789 vs. $2(401) = 802$.

Time $= 0.08$ (sec) , antiderivative size $= 1789$, normalized size of antiderivative $= 4.23$

$$\int (ag + bgx)^3 (ci + dix)^2 \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="maxima")
```

output

```
1/6*A*b^3*d^2*g^3*i^2*x^6 + 2/5*A*b^3*c*d*g^3*i^2*x^5 + 3/5*A*a*b^2*d^2*g^
3*i^2*x^5 + 1/4*A*b^3*c^2*g^3*i^2*x^4 + 3/2*A*a*b^2*c*d*g^3*i^2*x^4 + 3/4*
A*a^2*b*d^2*g^3*i^2*x^4 + A*a*b^2*c^2*g^3*i^2*x^3 + 2*A*a^2*b*c*d*g^3*i^2*
x^3 + 1/3*A*a^3*d^2*g^3*i^2*x^3 + 3/2*A*a^2*b*c^2*g^3*i^2*x^2 + A*a^3*c*d*
g^3*i^2*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b -
 c*log(d*x + c)/d)*B*a^3*c^2*g^3*i^2 + 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/
(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(
b*d))*B*a^2*b*c^2*g^3*i^2 + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)
) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)
*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b^2*c^2*g^3*i^2 + 1/24*(6*x
^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*l
og(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^
3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*c^2*g^3*i^2 + (x^2*log(
b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)
/d^2 - (b*c - a*d)*x/(b*d))*B*a^3*c*d*g^3*i^2 + (2*x^3*log(b*e*x/(d*x + c)
 + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^
2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^2*b*c*d*g^3
*i^2 + 1/4*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a
)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c
^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*a*b^2*c*d...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4746 vs. $2(401) = 802$.

Time $= 0.36$ (sec) , antiderivative size $= 4746$, normalized size of antiderivative $= 11.22$

$$\int (ag + bgx)^3 (ci + dix)^2 \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="giac")
```

output

```
-1/360*(6*(B*b^10*c^7*e^7*g^3*i^2 - 7*B*a*b^9*c^6*d*e^7*g^3*i^2 + 21*B*a^2
*b^8*c^5*d^2*e^7*g^3*i^2 - 35*B*a^3*b^7*c^4*d^3*e^7*g^3*i^2 + 35*B*a^4*b^6
*c^3*d^4*e^7*g^3*i^2 - 21*B*a^5*b^5*c^2*d^5*e^7*g^3*i^2 + 7*B*a^6*b^4*c*d^
6*e^7*g^3*i^2 - B*a^7*b^3*d^7*e^7*g^3*i^2 - 6*(b*e*x + a*e)*B*b^9*c^7*d*e^
6*g^3*i^2/(d*x + c) + 42*(b*e*x + a*e)*B*a*b^8*c^6*d^2*e^6*g^3*i^2/(d*x +
c) - 126*(b*e*x + a*e)*B*a^2*b^7*c^5*d^3*e^6*g^3*i^2/(d*x + c) + 210*(b*e*
x + a*e)*B*a^3*b^6*c^4*d^4*e^6*g^3*i^2/(d*x + c) - 210*(b*e*x + a*e)*B*a^4
*b^5*c^3*d^5*e^6*g^3*i^2/(d*x + c) + 126*(b*e*x + a*e)*B*a^5*b^4*c^2*d^6*e
^6*g^3*i^2/(d*x + c) - 42*(b*e*x + a*e)*B*a^6*b^3*c*d^7*e^6*g^3*i^2/(d*x +
 c) + 6*(b*e*x + a*e)*B*a^7*b^2*d^8*e^6*g^3*i^2/(d*x + c) + 15*(b*e*x + a*
e)^2*B*b^8*c^7*d^2*e^5*g^3*i^2/(d*x + c)^2 - 105*(b*e*x + a*e)^2*B*a*b^7*c
^6*d^3*e^5*g^3*i^2/(d*x + c)^2 + 315*(b*e*x + a*e)^2*B*a^2*b^6*c^5*d^4*e^5
*g^3*i^2/(d*x + c)^2 - 525*(b*e*x + a*e)^2*B*a^3*b^5*c^4*d^5*e^5*g^3*i^2/(
d*x + c)^2 + 525*(b*e*x + a*e)^2*B*a^4*b^4*c^3*d^6*e^5*g^3*i^2/(d*x + c)^2
 - 315*(b*e*x + a*e)^2*B*a^5*b^3*c^2*d^7*e^5*g^3*i^2/(d*x + c)^2 + 105*(b*
e*x + a*e)^2*B*a^6*b^2*c*d^8*e^5*g^3*i^2/(d*x + c)^2 - 15*(b*e*x + a*e)^2*
B*a^7*b*d^9*e^5*g^3*i^2/(d*x + c)^2 - 20*(b*e*x + a*e)^3*B*b^7*c^7*d^3*e^4
*g^3*i^2/(d*x + c)^3 + 140*(b*e*x + a*e)^3*B*a*b^6*c^6*d^4*e^4*g^3*i^2/(d*
x + c)^3 - 420*(b*e*x + a*e)^3*B*a^2*b^5*c^5*d^5*e^4*g^3*i^2/(d*x + c)^3 +
 700*(b*e*x + a*e)^3*B*a^3*b^4*c^4*d^6*e^4*g^3*i^2/(d*x + c)^3 - 700*(b...
```

Mupad [B] (verification not implemented)

Time = 27.93 (sec) , antiderivative size = 2473, normalized size of antiderivative = 5.85

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input

```
int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x
)
```

output

```
x^3*((g^3*i^2*(16*A*a^3*d^3 + 4*A*b^3*c^3 + 3*B*a^3*d^3 - B*b^3*c^3 + 48*A
*a*b^2*c^2*d + 72*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2))/(12*
d) + ((60*a*d + 60*b*c)*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d - B
*b*c))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*
b*d) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2 - 2*B*b^2*c^2
 + 60*A*a*b*c*d - B*a*b*c*d))/5 + A*a*b^2*c*d*g^3*i^2))/(180*b*d) - (a*c*(
(b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d - B*b*c))/6 - (A*b^2*d*g^3*i^2
*(60*a*d + 60*b*c))/60))/(3*b*d)) - x^4*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A
*b*c + B*a*d - B*b*c))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d
 + 60*b*c))/(240*b*d) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*
d^2 - 2*B*b^2*c^2 + 60*A*a*b*c*d - B*a*b*c*d))/20 + (A*a*b^2*c*d*g^3*i^2)/
4) + x^2*((a*c*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d - B*b*c))/6
- (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (b
*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2 - 2*B*b^2*c^2 + 60*A*a
*b*c*d - B*a*b*c*d))/5 + A*a*b^2*c*d*g^3*i^2))/(2*b*d) - ((60*a*d + 60*b*c
)*((g^3*i^2*(16*A*a^3*d^3 + 4*A*b^3*c^3 + 3*B*a^3*d^3 - B*b^3*c^3 + 48*A*a
*b^2*c^2*d + 72*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2))/(4*d)
+ ((60*a*d + 60*b*c)*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d - B*b*
c))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d
) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2 - 2*B*b^2*c^2...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1036, normalized size of antiderivative = 2.45

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input

```
int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output

```
(g**3*( - 6*log(c + d*x)*a**6*d**6 + 36*log(c + d*x)*a**5*b*c*d**5 - 90*lo
g(c + d*x)*a**4*b**2*c**2*d**4 + 120*log(c + d*x)*a**3*b**3*c**3*d**3 - 90
*log(c + d*x)*a**2*b**4*c**4*d**2 + 36*log(c + d*x)*a*b**5*c**5*d - 6*log(
c + d*x)*b**6*c**6 - 6*log((a*e + b*e*x)/(c + d*x))*a**6*d**6 + 36*log((a*
e + b*e*x)/(c + d*x))*a**5*b*c*d**5 - 90*log((a*e + b*e*x)/(c + d*x))*a**4
*b**2*c**2*d**4 - 360*log((a*e + b*e*x)/(c + d*x))*a**3*b**3*c**2*d**4*x -
 360*log((a*e + b*e*x)/(c + d*x))*a**3*b**3*c*d**5*x**2 - 120*log((a*e + b
*e*x)/(c + d*x))*a**3*b**3*d**6*x**3 - 540*log((a*e + b*e*x)/(c + d*x))*a*
*2*b**4*c**2*d**4*x**2 - 720*log((a*e + b*e*x)/(c + d*x))*a**2*b**4*c*d**5
*x**3 - 270*log((a*e + b*e*x)/(c + d*x))*a**2*b**4*d**6*x**4 - 360*log((a*
e + b*e*x)/(c + d*x))*a*b**5*c**2*d**4*x**3 - 540*log((a*e + b*e*x)/(c + d
*x))*a*b**5*c*d**5*x**4 - 216*log((a*e + b*e*x)/(c + d*x))*a*b**5*d**6*x**
5 - 90*log((a*e + b*e*x)/(c + d*x))*b**6*c**2*d**4*x**4 - 144*log((a*e + b
*e*x)/(c + d*x))*b**6*c*d**5*x**5 - 60*log((a*e + b*e*x)/(c + d*x))*b**6*d
**6*x**6 + 6*a**5*b*d**6*x - 360*a**4*b**2*c**2*d**4*x - 360*a**4*b**2*c*d
**5*x**2 - 36*a**4*b**2*c*d**5*x - 120*a**4*b**2*d**6*x**3 - 3*a**4*b**2*d
**6*x**2 - 540*a**3*b**3*c**2*d**4*x**2 - 30*a**3*b**3*c**2*d**4*x - 720*a
**3*b**3*c*d**5*x**3 - 102*a**3*b**3*c*d**5*x**2 - 270*a**3*b**3*d**6*x**4
 - 38*a**3*b**3*d**6*x**3 + 90*a**2*b**4*c**3*d**3*x - 360*a**2*b**4*c**2*
d**4*x**3 + 90*a**2*b**4*c**2*d**4*x**2 - 540*a**2*b**4*c*d**5*x**4 - 4...
```

3.11 $\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 337

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx \\
&= -\frac{B(bc-ad)^4g^2i^2x}{30b^2d^2} - \frac{B(bc-ad)^3g^2i^2(c+dx)^2}{60bd^3} + \frac{B(bc-ad)^2g^2i^2(c+dx)^3}{10d^3} \\
&\quad - \frac{bB(bc-ad)g^2i^2(c+dx)^4}{20d^3} - \frac{B(bc-ad)^5g^2i^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3} \\
&\quad + \frac{(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^3} \\
&\quad - \frac{b(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^3} \\
&\quad + \frac{b^2g^2i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^3} - \frac{B(bc-ad)^5g^2i^2\log(c+dx)}{30b^3d^3}
\end{aligned}$$

output
```
-1/30*B*(-a*d+b*c)^4*g^2*i^2*x/b^2/d^2-1/60*B*(-a*d+b*c)^3*g^2*i^2*(d*x+c)
^2/b/d^3+1/10*B*(-a*d+b*c)^2*g^2*i^2*(d*x+c)^3/d^3-1/20*b*B*(-a*d+b*c)*g^2
*i^2*(d*x+c)^4/d^3-1/30*B*(-a*d+b*c)^5*g^2*i^2*ln((b*x+a)/(d*x+c))/b^3/d^3
+1/3*(-a*d+b*c)^2*g^2*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3-1/2*b*
(-a*d+b*c)*g^2*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3+1/5*b^2*g^2*i
^2*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3-1/30*B*(-a*d+b*c)^5*g^2*i^2*l
n(d*x+c)/b^3/d^3
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.07

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{g^2i^2\Big(20d^3(bc-ad)^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+30d^4(bc-ad)(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+$$

input
```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)]),x]
```

output
```
(g^2*i^2*(20*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d
*x)]) + 30*d^4*(b*c - a*d)*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]
) + 12*d^5*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 10*B*(b*c -
a*d)^3*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*
x]) - 5*B*(b*c - a*d)^2*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a +
 b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + B*(b*c - a*d
)*(12*b*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c -
 a*d)*(a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x])))/(
60*b^3*d^3)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2782, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\downarrow 2962$$

$$g^2i^2(bc-ad)^5\int\frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\downarrow 2782$$

$$g^2i^2(bc-ad)^5\left(-B\int\frac{(c+dx)\left(b^2-\frac{5d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{30d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}+\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right.}$$

$$\downarrow 27$$

$$g^2i^2(bc-ad)^5\left(-\frac{B\int\frac{(c+dx)\left(b^2-\frac{5d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{30d^3}+\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{B\,l}{3d}\right.$$

$$\downarrow 1195$$

$$g^2i^2(bc-ad)^5\left(-\frac{B\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{9d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{6bd}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a+bx}{c+dx}}{30d^3}+\right.$$

$$\downarrow 2009$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(\frac{b^2 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)}{5d^3 \left(b - \frac{d(a+bx)}{c+dx}\right)^5} - \frac{b\left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)}{2d^3 \left(b - \frac{d(a+bx)}{c+dx}\right)^4} + \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{3d^3 \left(b - \frac{d(a+bx)}{c+dx}\right)^3} - \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3} - \frac{\log\left(b - \frac{d(a+}{c+d}\right.\right.}{b^3}$$

input
```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x
]
```

output
```
(b*c - a*d)^5*g^2*i^2*((b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*d^3*(
b - (d*(a + b*x))/(c + d*x))^5) - (b*(A + B*Log[(e*(a + b*x))/(c + d*x)]))
/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^4) + (A + B*Log[(e*(a + b*x))/(c + d
*x)])/(3*d^3*(b - (d*(a + b*x))/(c + d*x))^3) - (B*((3*b)/(2*(b - (d*(a +
b*x))/(c + d*x))^4) - 3/(b - (d*(a + b*x))/(c + d*x))^3 + 1/(2*b*(b - (d*(
a + b*x))/(c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a +
 b*x)/(c + d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(30*d^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.89

method	result
risch	$\frac{i^2g^2dB\,a^3cx}{6b} - \frac{i^2g^2bBa\,c^3x}{6d} + \frac{i^2g^2bB\ln(dx+c)a\,c^4}{6d^2} - \frac{i^2g^2dB\ln(-bx-a)a^4c}{6b^2} + i^2g^2A\,a^2c^2x + \frac{4i^2g^2bdAa$
parallelrisch	Expression too large to display
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input

```
int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETU
RNVERBOSE)
```

output

```
1/6*i^2*g^2/b*d*B*a^3*c*x-1/6*i^2*g^2*b/d*B*a*c^3*x+1/6*i^2*g^2*b/d^2*B*ln
(d*x+c)*a*c^4-1/6*i^2*g^2/b^2*d*B*ln(-b*x-a)*a^4*c+i^2*g^2*A*a^2*c^2*x+4/3
*i^2*g^2*b*d*A*a*c*x^3+i^2*g^2*d*A*a^2*c*x^2+i^2*g^2*b*A*a*c^2*x^2+1/4*i^2
*g^2*d*B*a^2*c*x^2-1/4*i^2*g^2*b*B*a*c^2*x^2-1/30*i^2*g^2/b^2*d^2*B*a^4*x+
1/30*i^2*g^2*b^2/d^2*B*c^4*x-1/3*i^2*g^2/d*B*ln(d*x+c)*a^2*c^3+1/3*i^2*g^2
/b*B*ln(-b*x-a)*a^3*c^2+1/30*i^2*g^2/b^3*d^2*B*ln(-b*x-a)*a^5-1/30*i^2*g^2
*b^2/d^3*B*ln(d*x+c)*c^5+1/2*i^2*g^2*b*d^2*A*a*x^4+1/2*i^2*g^2*b^2*d*A*c*x
^4+1/20*i^2*g^2*b*d^2*B*a*x^4-1/20*i^2*g^2*b^2*d*B*c*x^4+1/3*i^2*g^2*d^2*A
*a^2*x^3+1/3*i^2*g^2*b^2*A*c^2*x^3+1/10*i^2*g^2*d^2*B*a^2*x^3-1/10*i^2*g^2
*b^2*B*c^2*x^3+1/60*i^2*g^2/b*d^2*B*a^3*x^2-1/60*i^2*g^2*b^2/d*B*c^3*x^2+1
/5*i^2*g^2*b^2*d^2*A*x^5+1/30*i^2*g^2*B*x*(6*b^2*d^2*x^4+15*a*b*d^2*x^3+15
*b^2*c*d*x^3+10*a^2*d^2*x^2+40*a*b*c*d*x^2+10*b^2*c^2*x^2+30*a^2*c*d*x+30*
a*b*c^2*x+30*a^2*c^2)*ln(e*(b*x+a)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.58

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{12\,Ab^5d^5g^2i^2x^5+3\left((10\,A-B)b^5cd^4+(10\,A+B)ab^4d^5\right)g^2i^2x^4+2\left((10\,A-3\,B)b^5c^2d^3+40\,Aab^4cd^4+\right.}{}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="fricas")
```

output
```
1/60*(12*A*b^5*d^5*g^2*i^2*x^5 + 3*((10*A - B)*b^5*c*d^4 + (10*A + B)*a*b^
4*d^5)*g^2*i^2*x^4 + 2*((10*A - 3*B)*b^5*c^2*d^3 + 40*A*a*b^4*c*d^4 + (10*
A + 3*B)*a^2*b^3*d^5)*g^2*i^2*x^3 - (B*b^5*c^3*d^2 - 15*(4*A - B)*a*b^4*c^
2*d^3 - 15*(4*A + B)*a^2*b^3*c*d^4 - B*a^3*b^2*d^5)*g^2*i^2*x^2 + 2*(B*b^5
*c^4*d - 5*B*a*b^4*c^3*d^2 + 30*A*a^2*b^3*c^2*d^3 + 5*B*a^3*b^2*c*d^4 - B*
a^4*b*d^5)*g^2*i^2*x + 2*(10*B*a^3*b^2*c^2*d^3 - 5*B*a^4*b*c*d^4 + B*a^5*d
^5)*g^2*i^2*log(b*x + a) - 2*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c
^3*d^2)*g^2*i^2*log(d*x + c) + 2*(6*B*b^5*d^5*g^2*i^2*x^5 + 30*B*a^2*b^3*c
^2*d^3*g^2*i^2*x + 15*(B*b^5*c*d^4 + B*a*b^4*d^5)*g^2*i^2*x^4 + 10*(B*b^5*
c^2*d^3 + 4*B*a*b^4*c*d^4 + B*a^2*b^3*d^5)*g^2*i^2*x^3 + 30*(B*a*b^4*c^2*d
^3 + B*a^2*b^3*c*d^4)*g^2*i^2*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^3*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. 2(311) = 622.

Time = 3.48 (sec) , antiderivative size = 1266, normalized size of antiderivative = 3.76

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output

```
A*b**2*d**2*g**2*i**2*x**5/5 + B*a**3*g**2*i**2*(a**2*d**2 - 5*a*b*c*d + 1
0*b**2*c**2)*log(x + (B*a**5*c*d**4*g**2*i**2 - 5*B*a**4*b*c**2*d**3*g**2*
i**2 + B*a**4*d**3*g**2*i**2*(a**2*d**2 - 5*a*b*c*d + 10*b**2*c**2)/b + 20
*B*a**3*b**2*c**3*d**2*g**2*i**2 - B*a**3*c*d**2*g**2*i**2*(a**2*d**2 - 5*
a*b*c*d + 10*b**2*c**2) - 5*B*a**2*b**3*c**4*d*g**2*i**2 + B*a*b**4*c**5*g
**2*i**2)/(B*a**5*d**5*g**2*i**2 - 5*B*a**4*b*c*d**4*g**2*i**2 + 10*B*a**3
*b**2*c**2*d**3*g**2*i**2 + 10*B*a**2*b**3*c**3*d**2*g**2*i**2 - 5*B*a*b**
4*c**4*d*g**2*i**2 + B*b**5*c**5*g**2*i**2))/(30*b**3) - B*c**3*g**2*i**2*
(10*a**2*d**2 - 5*a*b*c*d + b**2*c**2)*log(x + (B*a**5*c*d**4*g**2*i**2 -
5*B*a**4*b*c**2*d**3*g**2*i**2 + 20*B*a**3*b**2*c**3*d**2*g**2*i**2 - 5*B*
a**2*b**3*c**4*d*g**2*i**2 + B*a*b**4*c**5*g**2*i**2 - B*a*b**2*c**3*g**2*
i**2*(10*a**2*d**2 - 5*a*b*c*d + b**2*c**2) + B*b**3*c**4*g**2*i**2*(10*a*
*2*d**2 - 5*a*b*c*d + b**2*c**2)/d)/(B*a**5*d**5*g**2*i**2 - 5*B*a**4*b*c*
d**4*g**2*i**2 + 10*B*a**3*b**2*c**2*d**3*g**2*i**2 + 10*B*a**2*b**3*c**3*
d**2*g**2*i**2 - 5*B*a*b**4*c**4*d*g**2*i**2 + B*b**5*c**5*g**2*i**2))/(30
*d**3) + x**4*(A*a*b*d**2*g**2*i**2/2 + A*b**2*c*d*g**2*i**2/2 + B*a*b*d**
2*g**2*i**2/20 - B*b**2*c*d*g**2*i**2/20) + x**3*(A*a**2*d**2*g**2*i**2/3
+ 4*A*a*b*c*d*g**2*i**2/3 + A*b**2*c**2*g**2*i**2/3 + B*a**2*d**2*g**2*i**
2/10 - B*b**2*c**2*g**2*i**2/10) + x**2*(A*a**2*c*d*g**2*i**2 + A*a*b*c**2
*g**2*i**2 + B*a**3*d**2*g**2*i**2/(60*b) + B*a**2*c*d*g**2*i**2/4 - B*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1200 vs. $2(319) = 638$.

Time $= 0.07$ (sec) , antiderivative size $= 1200$, normalized size of antiderivative $= 3.56$

$$\int (ag + bgx)^2 (ci + dix)^2 \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="maxima")
```

output

```
1/5*A*b^2*d^2*g^2*i^2*x^5 + 1/2*A*b^2*c*d*g^2*i^2*x^4 + 1/2*A*a*b*d^2*g^2*
i^2*x^4 + 1/3*A*b^2*c^2*g^2*i^2*x^3 + 4/3*A*a*b*c*d*g^2*i^2*x^3 + 1/3*A*a^
2*d^2*g^2*i^2*x^3 + A*a*b*c^2*g^2*i^2*x^2 + A*a^2*c*d*g^2*i^2*x^2 + (x*log
(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B
*a^2*c^2*g^2*i^2 + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x
 + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a*b*c^2*g^2*i^2
+ 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3
 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^
2)*x)/(b^2*d^2))*B*b^2*c^2*g^2*i^2 + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x +
 c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*
B*a^2*c*d*g^2*i^2 + 2/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^
3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2
*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b*c*d*g^2*i^2 + 1/12*(6*x^4*log(b*e
*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c
)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6
*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^2*c*d*g^2*i^2 + 1/6*(2*x^3*log(b*e*
x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^
2*d^2*g^2*i^2 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*l
og(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3144 vs. $2(319) = 638$.

Time $= 0.32$ (sec) , antiderivative size $= 3144$, normalized size of antiderivative $= 9.33$

$$\int (ag + bgx)^2 (ci + dix)^2 \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="giac")
```

output

```
1/60*(2*(B*b^8*c^6*e^6*g^2*i^2 - 6*B*a*b^7*c^5*d*e^6*g^2*i^2 + 15*B*a^2*b^
6*c^4*d^2*e^6*g^2*i^2 - 20*B*a^3*b^5*c^3*d^3*e^6*g^2*i^2 + 15*B*a^4*b^4*c^
2*d^4*e^6*g^2*i^2 - 6*B*a^5*b^3*c*d^5*e^6*g^2*i^2 + B*a^6*b^2*d^6*e^6*g^2*
i^2 - 5*(b*e*x + a*e)*B*b^7*c^6*d*e^5*g^2*i^2/(d*x + c) + 30*(b*e*x + a*e)
*B*a*b^6*c^5*d^2*e^5*g^2*i^2/(d*x + c) - 75*(b*e*x + a*e)*B*a^2*b^5*c^4*d^
3*e^5*g^2*i^2/(d*x + c) + 100*(b*e*x + a*e)*B*a^3*b^4*c^3*d^4*e^5*g^2*i^2/
(d*x + c) - 75*(b*e*x + a*e)*B*a^4*b^3*c^2*d^5*e^5*g^2*i^2/(d*x + c) + 30*
(b*e*x + a*e)*B*a^5*b^2*c*d^6*e^5*g^2*i^2/(d*x + c) - 5*(b*e*x + a*e)*B*a^
6*b*d^7*e^5*g^2*i^2/(d*x + c) + 10*(b*e*x + a*e)^2*B*b^6*c^6*d^2*e^4*g^2*i
^2/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a*b^5*c^5*d^3*e^4*g^2*i^2/(d*x + c)^
2 + 150*(b*e*x + a*e)^2*B*a^2*b^4*c^4*d^4*e^4*g^2*i^2/(d*x + c)^2 - 200*(b
*e*x + a*e)^2*B*a^3*b^3*c^3*d^5*e^4*g^2*i^2/(d*x + c)^2 + 150*(b*e*x + a*e
)^2*B*a^4*b^2*c^2*d^6*e^4*g^2*i^2/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a^5*b
*c*d^7*e^4*g^2*i^2/(d*x + c)^2 + 10*(b*e*x + a*e)^2*B*a^6*d^8*e^4*g^2*i^2/
(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/(b^5*d^3*e^5 - 5*(b*e*x + a*e)*b
^4*d^4*e^4/(d*x + c) + 10*(b*e*x + a*e)^2*b^3*d^5*e^3/(d*x + c)^2 - 10*(b*
e*x + a*e)^3*b^2*d^6*e^2/(d*x + c)^3 + 5*(b*e*x + a*e)^4*b*d^7*e/(d*x + c)
^4 - (b*e*x + a*e)^5*d^8/(d*x + c)^5) + (2*A*b^10*c^6*e^6*g^2*i^2 - 12*A*a
*b^9*c^5*d*e^6*g^2*i^2 + 30*A*a^2*b^8*c^4*d^2*e^6*g^2*i^2 - 40*A*a^3*b^7*c
^3*d^3*e^6*g^2*i^2 + 30*A*a^4*b^6*c^2*d^4*e^6*g^2*i^2 - 12*A*a^5*b^5*c*...
```

Mupad [B] (verification not implemented)

Time = 27.25 (sec) , antiderivative size = 1287, normalized size of antiderivative = 3.82

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input

```
int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x
)
```

output

```
log((e*(a + b*x))/(c + d*x))*((B*g^2*i^2*x^3*(a^2*d^2 + b^2*c^2 + 4*a*b*c*
d))/3 + B*a^2*c^2*g^2*i^2*x + (B*b^2*d^2*g^2*i^2*x^5)/5 + B*a*c*g^2*i^2*x^
2*(a*d + b*c) + (B*b*d*g^2*i^2*x^4*(a*d + b*c))/2) - x^3*(((30*a*d + 30*b*
c)*((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d - B*b*c))/5 - (A*b*d*g^2*i^2
*(30*a*d + 30*b*c))/30))/(90*b*d) - (g^2*i^2*(6*A*a^2*d^2 + 6*A*b^2*c^2 +
B*a^2*d^2 - B*b^2*c^2 + 18*A*a*b*c*d))/6 + (A*a*b*c*d*g^2*i^2)/3) + x*((a*
c*(((30*a*d + 30*b*c)*((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d - B*b*c))
/5 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))/30))/(30*b*d) - (g^2*i^2*(6*A*a^2*d
^2 + 6*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 18*A*a*b*c*d))/2 + A*a*b*c*d*g^
2*i^2))/(b*d) - ((30*a*d + 30*b*c)*(((30*a*d + 30*b*c)*(((30*a*d + 30*b*c)
*((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d - B*b*c))/5 - (A*b*d*g^2*i^2*(
30*a*d + 30*b*c))/30))/(30*b*d) - (g^2*i^2*(6*A*a^2*d^2 + 6*A*b^2*c^2 + B*
a^2*d^2 - B*b^2*c^2 + 18*A*a*b*c*d))/2 + A*a*b*c*d*g^2*i^2))/(30*b*d) + (g
^2*i^2*(3*A*a^3*d^3 + 3*A*b^3*c^3 + B*a^3*d^3 - B*b^3*c^3 + 27*A*a*b^2*c^2
*d + 27*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2))/(3*b*d) - (a*c
*((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d - B*b*c))/5 - (A*b*d*g^2*i^2*(
30*a*d + 30*b*c))/30))/(b*d)))/(30*b*d) + (a*c*g^2*i^2*(3*A*a^2*d^2 + 3*A*
b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 9*A*a*b*c*d))/(b*d)) + x^2*(((30*a*d + 3
0*b*c)*(((30*a*d + 30*b*c)*((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d - B*
b*c))/5 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))/30))/(30*b*d) - (g^2*i^2*(6...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 754, normalized size of antiderivative = 2.24

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{g^2\left(-12\log\left(\frac{bex+ae}{dx+c}\right)b^5d^5x^5+2a^4b\,d^5x-a^3b^2d^5x^2-6a^2b^3d^5x^3-3a\,b^4d^5x^4-2b^5c^4dx+b^5c^3d^2x^2+6b^5c^2\right.}{}$$

input

```
int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output

```
(g**2*( - 2*log(c + d*x)*a**5*d**5 + 10*log(c + d*x)*a**4*b*c*d**4 - 20*lo
g(c + d*x)*a**3*b**2*c**2*d**3 + 20*log(c + d*x)*a**2*b**3*c**3*d**2 - 10*
log(c + d*x)*a*b**4*c**4*d + 2*log(c + d*x)*b**5*c**5 - 2*log((a*e + b*e*x
)/(c + d*x))*a**5*d**5 + 10*log((a*e + b*e*x)/(c + d*x))*a**4*b*c*d**4 - 2
0*log((a*e + b*e*x)/(c + d*x))*a**3*b**2*c**2*d**3 - 60*log((a*e + b*e*x)/
(c + d*x))*a**2*b**3*c**2*d**3*x - 60*log((a*e + b*e*x)/(c + d*x))*a**2*b*
*3*c*d**4*x**2 - 20*log((a*e + b*e*x)/(c + d*x))*a**2*b**3*d**5*x**3 - 60*
log((a*e + b*e*x)/(c + d*x))*a*b**4*c**2*d**3*x**2 - 80*log((a*e + b*e*x)/
(c + d*x))*a*b**4*c*d**4*x**3 - 30*log((a*e + b*e*x)/(c + d*x))*a*b**4*d**
5*x**4 - 20*log((a*e + b*e*x)/(c + d*x))*b**5*c**2*d**3*x**3 - 30*log((a*e
 + b*e*x)/(c + d*x))*b**5*c*d**4*x**4 - 12*log((a*e + b*e*x)/(c + d*x))*b*
*5*d**5*x**5 + 2*a**4*b*d**5*x - 60*a**3*b**2*c**2*d**3*x - 60*a**3*b**2*c
*d**4*x**2 - 10*a**3*b**2*c*d**4*x - 20*a**3*b**2*d**5*x**3 - a**3*b**2*d*
*5*x**2 - 60*a**2*b**3*c**2*d**3*x**2 - 80*a**2*b**3*c*d**4*x**3 - 15*a**2
*b**3*c*d**4*x**2 - 30*a**2*b**3*d**5*x**4 - 6*a**2*b**3*d**5*x**3 + 10*a*
b**4*c**3*d**2*x - 20*a*b**4*c**2*d**3*x**3 + 15*a*b**4*c**2*d**3*x**2 - 3
0*a*b**4*c*d**4*x**4 - 12*a*b**4*d**5*x**5 - 3*a*b**4*d**5*x**4 - 2*b**5*c
**4*d*x + b**5*c**3*d**2*x**2 + 6*b**5*c**2*d**3*x**3 + 3*b**5*c*d**4*x**4
))/(60*b**2*d**3)
```

3.12 $\int (ag+bgx)(ci+dix)^2 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

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Optimal result

Integrand size = 38, antiderivative size = 239

$$\begin{aligned}
&\int (ag+bgx)(ci+dix)^2 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \\
&= \frac{B(bc-ad)^3 gi^2 x}{12b^2 d} + \frac{B(bc-ad)^2 gi^2 (c+dx)^2}{24bd^2} - \frac{B(bc-ad) gi^2 (c+dx)^3}{12d^2} \\
&\quad + \frac{B(bc-ad)^4 gi^2 \log\left(\frac{a+bx}{c+dx}\right)}{12b^3 d^2} - \frac{(bc-ad) gi^2 (c+dx)^3 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^2} \\
&\quad + \frac{bgi^2 (c+dx)^4 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2} + \frac{B(bc-ad)^4 gi^2 \log(c+dx)}{12b^3 d^2}
\end{aligned}$$

output

```
1/12*B*(-a*d+b*c)^3*g*i^2*x/b^2/d+1/24*B*(-a*d+b*c)^2*g*i^2*(d*x+c)^2/b/d^
2-1/12*B*(-a*d+b*c)*g*i^2*(d*x+c)^3/d^2+1/12*B*(-a*d+b*c)^4*g*i^2*ln((b*x+
a)/(d*x+c))/b^3/d^2-1/3*(-a*d+b*c)*g*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+
c)))/d^2+1/4*b*g*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2+1/12*B*(-a*
d+b*c)^4*g*i^2*ln(d*x+c)/b^3/d^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.90

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{gi^2\left(\frac{4B(bc-ad)^2(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx))}{b^3}-\frac{B(bc-ad)(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-d}{b^3}\right.}{24d^2}$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)
]),x]
```

output
```
(g*i^2*((4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c
 - a*d)^2*Log[a + b*x]))/b^3 - (B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*b
^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b
*x]))/b^3 - 8*(b*c - a*d)*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])
 + 6*b*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/(24*d^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2962, 2782, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$
$$\downarrow 2962$$
$$gi^2(bc-ad)^4\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$
$$\downarrow 2782$$

$$gi^2(bc-ad)^4\left(-B\int -\frac{(c+dx)\left(b-\frac{4d(a+bx)}{c+dx}\right)}{12d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)$$

$$\downarrow 27$$

$$gi^2(bc-ad)^4\left(\frac{B\int\frac{(c+dx)\left(b-\frac{4d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{12d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)$$

$$\downarrow 86$$

$$gi^2(bc-ad)^4\left(\frac{B\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a+bx}{c+dx}}{12d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.$$

$$\downarrow 2009$$

$$gi^2(bc-ad)^4\left(-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^3}+\frac{1}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{1}{2b\left(b-\right.}\right.}{12d^2}$$

input

```
Int[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output

```
(b*c - a*d)^4*g*i^2*((b*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*d^2*(b -
(d*(a + b*x))/(c + d*x))^4) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(3*d^2*
(b - (d*(a + b*x))/(c + d*x))^3) + (B*(-(b - (d*(a + b*x))/(c + d*x))^(-3)
 + 1/(2*b*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c
+ d*x))) + Log[(a + b*x)/(c + d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]
/b^3))/(12*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.66

method	result
risch	$\frac{g i^2 B x \left(3 b d^2 x^3+4 x^2 a d^2+8 b c d x^2+12 a c d x+6 b c^2 x+12 a c^2\right) \ln\left(\frac{e(bx+a)}{dx+c}\right)}{12}+\frac{i^2 g b d^2 A x^4}{4}+\frac{i^2 g d^2 A a x^3}{3}+\frac{2 i^2 g b d}{3}$
parallelrisch	$\frac{2B a^4 d^4 g i^2+2B b^4 c^4 g i^2-7B a^3 b c d^3 g i^2-8B a^2 b^2 c^2 d^2 g i^2+11 B a b^3 c^3 d g i^2+8B x^3 \ln\left(\frac{e(bx+a)}{dx+c}\right) a b^3 d^4 g i^2+6B x^4 \ln$
parts	$Ag i^2\left(\frac{b d^2 x^4}{4}+\frac{(a d^2+2bcd) x^3}{3}+\frac{(2acd+b c^2) x^2}{2}+a c^2 x\right)-\frac{Bg i^2 (da-bc)^3 e^3\left(\left(\frac{1}{3b^2 e^2 d\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right.\right.\right.$
derivativedivides	$-\frac{e(da-bc)\left(-A d^2 e^2 g i^2\left(a^3 d^3-3a^2 bc d^2+3a b^2 c^2 d-b^3 c^3\right)\left(-\frac{1}{3d^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right) d\right)^3}+\frac{be}{4d^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right.}\right.\right.$
default	$-\frac{e(da-bc)\left(-A d^2 e^2 g i^2\left(a^3 d^3-3a^2 bc d^2+3a b^2 c^2 d-b^3 c^3\right)\left(-\frac{1}{3d^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right) d\right)^3}+\frac{be}{4d^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right.}\right.\right.$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURN
VERBOSE)
```

output
```
1/12*g*i^2*B*x*(3*b*d^2*x^3+4*a*d^2*x^2+8*b*c*d*x^2+12*a*c*d*x+6*b*c^2*x+1
2*a*c^2)*ln(e*(b*x+a)/(d*x+c))+1/4*i^2*g*b*d^2*A*x^4+1/3*i^2*g*d^2*A*a*x^3
+2/3*i^2*g*b*d*A*c*x^3+1/12*i^2*g*d^2*B*a*x^3-1/12*i^2*g*b*d*B*c*x^3+i^2*g
*d*A*a*c*x^2+1/2*i^2*g*b*A*c^2*x^2+1/24*i^2*g/b*d^2*B*a^2*x^2+1/6*i^2*g*d*
B*a*c*x^2-5/24*i^2*g*b*B*c^2*x^2+i^2*g*A*a*c^2*x+1/12*i^2*g/b^3*d^2*B*ln(-
b*x-a)*a^4-1/3*i^2*g/b^2*d*B*ln(-b*x-a)*a^3*c+1/2*i^2*g/b*B*ln(-b*x-a)*a^2
*c^2-1/3*i^2*g/d*B*ln(d*x+c)*a*c^3+1/12*i^2*g*b/d^2*B*ln(d*x+c)*c^4-1/12*i
^2*g/b^2*d^2*B*a^3*x+1/3*i^2*g/b*d*B*a^2*c*x-1/6*i^2*g*B*a*c^2*x-1/12*i^2*
g*b/d*B*c^3*x
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.54

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{6\,Ab^4d^4gi^2x^4+2\left((8\,A-B)b^4cd^3+(4\,A+B)ab^3d^4\right)gi^2x^3+\left((12\,A-5\,B)b^4c^2d^2+4\,(6\,A+B)ab^3cd^3+\right.}{}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="fricas")
```

output
```
1/24*(6*A*b^4*d^4*g*i^2*x^4 + 2*((8*A - B)*b^4*c*d^3 + (4*A + B)*a*b^3*d^4
)*g*i^2*x^3 + ((12*A - 5*B)*b^4*c^2*d^2 + 4*(6*A + B)*a*b^3*c*d^3 + B*a^2*
b^2*d^4)*g*i^2*x^2 - 2*(B*b^4*c^3*d - 2*(6*A - B)*a*b^3*c^2*d^2 - 4*B*a^2*
b^2*c*d^3 + B*a^3*b*d^4)*g*i^2*x + 2*(6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^
3 + B*a^4*d^4)*g*i^2*log(b*x + a) + 2*(B*b^4*c^4 - 4*B*a*b^3*c^3*d)*g*i^2*
log(d*x + c) + 2*(3*B*b^4*d^4*g*i^2*x^4 + 12*B*a*b^3*c^2*d^2*g*i^2*x + 4*(
2*B*b^4*c*d^3 + B*a*b^3*d^4)*g*i^2*x^3 + 6*(B*b^4*c^2*d^2 + 2*B*a*b^3*c*d^
3)*g*i^2*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^3*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(221) = 442$.

Time = 2.30 (sec) , antiderivative size = 850, normalized size of antiderivative = 3.56

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{Abd^2gi^2x^4}{4}$$
$$+\frac{Ba^2gi^2(a^2d^2-4abcd+6b^2c^2)\log\left(x+\frac{Ba^4cd^3gi^2-4Ba^3bc^2d^2gi^2+\frac{Ba^3d^2gi^2\left(a^2d^2-4abcd+6b^2c^2\right)}{b}+10Ba^2b^2c^3dgi^2-Ba^2cd}{Ba^4d^4gi^2-4Ba^3bcd^3gi^2+6Ba^2b^2c^2d^2gi^2+4Bab^3c^3dgi^2-}\right)}{12b^3}$$
$$-\frac{Bc^3gi^2\cdot(4ad-bc)\log\left(x+\frac{Ba^4cd^3gi^2-4Ba^3bc^2d^2gi^2+10Ba^2b^2c^3dgi^2-Bab^3c^4gi^2-Bab^2c^3gi^2\cdot(4ad-bc)+\frac{Bb^3c^4gi^2\cdot(4ad-bc)}{d}}{Ba^4d^4gi^2-4Ba^3bcd^3gi^2+6Ba^2b^2c^2d^2gi^2+4Bab^3c^3dgi^2-Bb^4c^4gi^2}\right)}{12d^2}$$
$$+x^3\left(\frac{Aad^2gi^2}{3}+\frac{2Abcdgi^2}{3}+\frac{Bad^2gi^2}{12}-\frac{Bbcdgi^2}{12}\right)$$
$$+x^2\left(Aacdgi^2+\frac{Abc^2gi^2}{2}+\frac{Ba^2d^2gi^2}{24b}+\frac{Bacdgi^2}{6}-\frac{5Bbc^2gi^2}{24}\right)$$
$$+x\left(Aac^2gi^2-\frac{Ba^3d^2gi^2}{12b^2}+\frac{Ba^2cdgi^2}{3b}-\frac{Bac^2gi^2}{6}-\frac{Bbc^3gi^2}{12d}\right)+\left(Bac^2gi^2x\right.$$
$$\left.+Bacdgi^2x^2+\frac{Bad^2gi^2x^3}{3}+\frac{Bbc^2gi^2x^2}{2}+\frac{2Bbcdgi^2x^3}{3}+\frac{Bbd^2gi^2x^4}{4}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output

```
A*b*d**2*g*i**2*x**4/4 + B*a**2*g*i**2*(a**2*d**2 - 4*a*b*c*d + 6*b**2*c**
2)*log(x + (B*a**4*c*d**3*g*i**2 - 4*B*a**3*b*c**2*d**2*g*i**2 + B*a**3*d*
*2*g*i**2*(a**2*d**2 - 4*a*b*c*d + 6*b**2*c**2)/b + 10*B*a**2*b**2*c**3*d*
g*i**2 - B*a**2*c*d*g*i**2*(a**2*d**2 - 4*a*b*c*d + 6*b**2*c**2) - B*a*b**
3*c**4*g*i**2)/(B*a**4*d**4*g*i**2 - 4*B*a**3*b*c*d**3*g*i**2 + 6*B*a**2*b
**2*c**2*d**2*g*i**2 + 4*B*a*b**3*c**3*d*g*i**2 - B*b**4*c**4*g*i**2))/(12
*b**3) - B*c**3*g*i**2*(4*a*d - b*c)*log(x + (B*a**4*c*d**3*g*i**2 - 4*B*a
**3*b*c**2*d**2*g*i**2 + 10*B*a**2*b**2*c**3*d*g*i**2 - B*a*b**3*c**4*g*i*
*2 - B*a*b**2*c**3*g*i**2*(4*a*d - b*c) + B*b**3*c**4*g*i**2*(4*a*d - b*c)
/d)/(B*a**4*d**4*g*i**2 - 4*B*a**3*b*c*d**3*g*i**2 + 6*B*a**2*b**2*c**2*d*
*2*g*i**2 + 4*B*a*b**3*c**3*d*g*i**2 - B*b**4*c**4*g*i**2))/(12*d**2) + x*
*3*(A*a*d**2*g*i**2/3 + 2*A*b*c*d*g*i**2/3 + B*a*d**2*g*i**2/12 - B*b*c*d*
g*i**2/12) + x**2*(A*a*c*d*g*i**2 + A*b*c**2*g*i**2/2 + B*a**2*d**2*g*i**2
/(24*b) + B*a*c*d*g*i**2/6 - 5*B*b*c**2*g*i**2/24) + x*(A*a*c**2*g*i**2 -
B*a**3*d**2*g*i**2/(12*b**2) + B*a**2*c*d*g*i**2/(3*b) - B*a*c**2*g*i**2/6
 - B*b*c**3*g*i**2/(12*d)) + (B*a*c**2*g*i**2*x + B*a*c*d*g*i**2*x**2 + B*
a*d**2*g*i**2*x**3/3 + B*b*c**2*g*i**2*x**2/2 + 2*B*b*c*d*g*i**2*x**3/3 +
B*b*d**2*g*i**2*x**4/4)*log(e*(a + b*x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(225) = 450$.

Time = 0.05 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.81

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$= \frac{1}{4}Abd^2gi^2x^4+\frac{2}{3}Abcdgi^2x^3+\frac{1}{3}Aad^2gi^2x^3+\frac{1}{2}Abc^2gi^2x^2+Aacdgi^2x^2$$
$$+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bac^2gi^2$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bbc^2gi^2$$
$$+\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bacdgi^2$$
$$+\frac{1}{3}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-}{b^2d^2}\right.$$
$$+\frac{1}{6}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-}{b^2d^2}\right.$$
$$+\frac{1}{24}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log(dx+c)}{d^4}-\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3}{}\right.$$
$$+Aac^2gi^2x$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="maxima")
```

output
```
1/4*A*b*d^2*g*i^2*x^4 + 2/3*A*b*c*d*g*i^2*x^3 + 1/3*A*a*d^2*g*i^2*x^3 + 1/
2*A*b*c^2*g*i^2*x^2 + A*a*c*d*g*i^2*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a*c^2*g*i^2 + 1/2*(x^2*lo
g(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x +
c)/d^2 - (b*c - a*d)*x/(b*d))*B*b*c^2*g*i^2 + (x^2*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*
x/(b*d))*B*a*c*d*g*i^2 + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^
2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b*c*d*g*i^2 + 1/6*(2*x^3*log(b*e
*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c
)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a
*d^2*g*i^2 + 1/24*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(
b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b*d^2
*g*i^2 + A*a*c^2*g*i^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1975 vs. $2(225) = 450$.

Time = 0.28 (sec) , antiderivative size = 1975, normalized size of antiderivative = 8.26

$$\int (ag + bgx)(ci + dix)^2 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="giac")
```

output

```
-1/24*(2*(B*b^6*c^5*e^5*g*i^2 - 5*B*a*b^5*c^4*d*e^5*g*i^2 + 10*B*a^2*b^4*c
^3*d^2*e^5*g*i^2 - 10*B*a^3*b^3*c^2*d^3*e^5*g*i^2 + 5*B*a^4*b^2*c*d^4*e^5*
g*i^2 - B*a^5*b*d^5*e^5*g*i^2 - 4*(b*e*x + a*e)*B*b^5*c^5*d*e^4*g*i^2/(d*x
 + c) + 20*(b*e*x + a*e)*B*a*b^4*c^4*d^2*e^4*g*i^2/(d*x + c) - 40*(b*e*x +
 a*e)*B*a^2*b^3*c^3*d^3*e^4*g*i^2/(d*x + c) + 40*(b*e*x + a*e)*B*a^3*b^2*c
^2*d^4*e^4*g*i^2/(d*x + c) - 20*(b*e*x + a*e)*B*a^4*b*c*d^5*e^4*g*i^2/(d*x
 + c) + 4*(b*e*x + a*e)*B*a^5*d^6*e^4*g*i^2/(d*x + c))*log((b*e*x + a*e)/(
d*x + c))/(b^4*d^2*e^4 - 4*(b*e*x + a*e)*b^3*d^3*e^3/(d*x + c) + 6*(b*e*x
+ a*e)^2*b^2*d^4*e^2/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b*d^5*e/(d*x + c)^3 +
 (b*e*x + a*e)^4*d^6/(d*x + c)^4) + (2*A*b^8*c^5*e^5*g*i^2 - B*b^8*c^5*e^5
*g*i^2 - 10*A*a*b^7*c^4*d*e^5*g*i^2 + 5*B*a*b^7*c^4*d*e^5*g*i^2 + 20*A*a^2
*b^6*c^3*d^2*e^5*g*i^2 - 10*B*a^2*b^6*c^3*d^2*e^5*g*i^2 - 20*A*a^3*b^5*c^2
*d^3*e^5*g*i^2 + 10*B*a^3*b^5*c^2*d^3*e^5*g*i^2 + 10*A*a^4*b^4*c*d^4*e^5*g
*i^2 - 5*B*a^4*b^4*c*d^4*e^5*g*i^2 - 2*A*a^5*b^3*d^5*e^5*g*i^2 + B*a^5*b^3
*d^5*e^5*g*i^2 - 8*(b*e*x + a*e)*A*b^7*c^5*d*e^4*g*i^2/(d*x + c) + 6*(b*e*
x + a*e)*B*b^7*c^5*d*e^4*g*i^2/(d*x + c) + 40*(b*e*x + a*e)*A*a*b^6*c^4*d^
2*e^4*g*i^2/(d*x + c) - 30*(b*e*x + a*e)*B*a*b^6*c^4*d^2*e^4*g*i^2/(d*x +
c) - 80*(b*e*x + a*e)*A*a^2*b^5*c^3*d^3*e^4*g*i^2/(d*x + c) + 60*(b*e*x +
a*e)*B*a^2*b^5*c^3*d^3*e^4*g*i^2/(d*x + c) + 80*(b*e*x + a*e)*A*a^3*b^4*c^
2*d^4*e^4*g*i^2/(d*x + c) - 60*(b*e*x + a*e)*B*a^3*b^4*c^2*d^4*e^4*g*i^...
```

Mupad [B] (verification not implemented)

Time = 26.63 (sec) , antiderivative size = 636, normalized size of antiderivative = 2.66

$$
\begin{aligned}
&\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx \\
&= x^3\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d-B\,b\,c)}{12}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{36}\right) \\
&\quad -x^2\left(\frac{\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{12}\right)(12\,a\,d+12\,b\,c)}{24\,b\,d}\right. \\
&\qquad \left.-\frac{g\,i^2\,(3\,A\,a^2\,d^2+9\,A\,b^2\,c^2+B\,a^2\,d^2-2\,B\,b^2\,c^2+18\,A\,a\,b\,c\,d+B\,a\,b\,c\,d)}{6\,b}\right. \\
&\qquad \left.+\frac{A\,a\,c\,d\,g\,i^2}{2}\right)+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(B\,a\,c^2\,g\,i^2\,x+\frac{B\,c\,g\,i^2\,x^2\,(2\,a\,d+b\,c)}{2}\right. \\
&\qquad \left.+\frac{B\,d\,g\,i^2\,x^3\,(a\,d+2\,b\,c)}{3}+\frac{B\,b\,d^2\,g\,i^2\,x^4}{4}\right) \\
&\quad +x\left(\frac{(12\,a\,d+12\,b\,c)\left(\frac{\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{12}\right)(12\,a\,d+12\,b\,c)}{12\,b\,d}-\frac{g\,i^2\,(3\,A\,a^2\,d^2+9\,A\,b^2\,c^2+B\,a^2}{12\,b\,d}\right. \\
&\qquad \left.-\frac{a\,c\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{12}\right)}{b\,d}\right. \\
&\qquad \left.+\frac{c\,g\,i^2\,(6\,A\,a^2\,d^2+2\,A\,b^2\,c^2+2\,B\,a^2\,d^2-B\,b^2\,c^2+12\,A\,a\,b\,c\,d-B\,a\,b\,c\,d)}{2\,b\,d}\right) \\
&\quad +\frac{\ln(a+b\,x)\,(B\,g\,a^4\,d^2\,i^2-4\,B\,g\,a^3\,b\,c\,d\,i^2+6\,B\,g\,a^2\,b^2\,c^2\,i^2)}{12\,b^3} \\
&\quad +\frac{\ln(c+d\,x)\,(B\,b\,c^4\,g\,i^2-4\,B\,a\,c^3\,d\,g\,i^2)}{12\,d^2}+\frac{A\,b\,d^2\,g\,i^2\,x^4}{4}
\end{aligned}
$$

input

```
int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output

```
x^3*((d*g*i^2*(8*A*a*d + 12*A*b*c + B*a*d - B*b*c))/12 - (A*d*g*i^2*(12*a*
d + 12*b*c))/36) - x^2*((((d*g*i^2*(8*A*a*d + 12*A*b*c + B*a*d - B*b*c))/4
 - (A*d*g*i^2*(12*a*d + 12*b*c))/12)*(12*a*d + 12*b*c))/(24*b*d) - (g*i^2*
(3*A*a^2*d^2 + 9*A*b^2*c^2 + B*a^2*d^2 - 2*B*b^2*c^2 + 18*A*a*b*c*d + B*a*
b*c*d))/(6*b) + (A*a*c*d*g*i^2)/2) + log((e*(a + b*x))/(c + d*x))*(B*a*c^2
*g*i^2*x + (B*c*g*i^2*x^2*(2*a*d + b*c))/2 + (B*d*g*i^2*x^3*(a*d + 2*b*c))
/3 + (B*b*d^2*g*i^2*x^4)/4) + x*(((12*a*d + 12*b*c)*((((d*g*i^2*(8*A*a*d +
 12*A*b*c + B*a*d - B*b*c))/4 - (A*d*g*i^2*(12*a*d + 12*b*c))/12)*(12*a*d
+ 12*b*c))/(12*b*d) - (g*i^2*(3*A*a^2*d^2 + 9*A*b^2*c^2 + B*a^2*d^2 - 2*B*
b^2*c^2 + 18*A*a*b*c*d + B*a*b*c*d))/(3*b) + A*a*c*d*g*i^2))/(12*b*d) - (a
*c*((d*g*i^2*(8*A*a*d + 12*A*b*c + B*a*d - B*b*c))/4 - (A*d*g*i^2*(12*a*d
+ 12*b*c))/12))/(b*d) + (c*g*i^2*(6*A*a^2*d^2 + 2*A*b^2*c^2 + 2*B*a^2*d^2
- B*b^2*c^2 + 12*A*a*b*c*d - B*a*b*c*d))/(2*b*d)) + (log(a + b*x)*(B*a^4*d
^2*g*i^2 + 6*B*a^2*b^2*c^2*g*i^2 - 4*B*a^3*b*c*d*g*i^2))/(12*b^3) + (log(c
 + d*x)*(B*b*c^4*g*i^2 - 4*B*a*c^3*d*g*i^2))/(12*d^2) + (A*b*d^2*g*i^2*x^4
)/4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 545, normalized size of antiderivative = 2.28

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{g\left(2a^3b\,d^4x-a^2b^2d^4x^2-2a\,b^3d^4x^3+2b^4c^3dx+5b^4c^2d^2x^2+2b^4c\,d^3x^3-24\log\left(\frac{bex+ae}{dx+c}\right)a\,b^3c^2d^2x+8\log\right.}{}$$

input

```
int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output

```
(g*( - 2*log(c + d*x)*a**4*d**4 + 8*log(c + d*x)*a**3*b*c*d**3 - 12*log(c
+ d*x)*a**2*b**2*c**2*d**2 + 8*log(c + d*x)*a*b**3*c**3*d - 2*log(c + d*x)
*b**4*c**4 - 2*log((a*e + b*e*x)/(c + d*x))*a**4*d**4 + 8*log((a*e + b*e*x
)/(c + d*x))*a**3*b*c*d**3 - 12*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*c**
2*d**2 - 24*log((a*e + b*e*x)/(c + d*x))*a*b**3*c**2*d**2*x - 24*log((a*e
+ b*e*x)/(c + d*x))*a*b**3*c*d**3*x**2 - 8*log((a*e + b*e*x)/(c + d*x))*a*
b**3*d**4*x**3 - 12*log((a*e + b*e*x)/(c + d*x))*b**4*c**2*d**2*x**2 - 16*
log((a*e + b*e*x)/(c + d*x))*b**4*c*d**3*x**3 - 6*log((a*e + b*e*x)/(c + d
*x))*b**4*d**4*x**4 + 2*a**3*b*d**4*x - 24*a**2*b**2*c**2*d**2*x - 24*a**2
*b**2*c*d**3*x**2 - 8*a**2*b**2*c*d**3*x - 8*a**2*b**2*d**4*x**3 - a**2*b*
*2*d**4*x**2 - 12*a*b**3*c**2*d**2*x**2 + 4*a*b**3*c**2*d**2*x - 16*a*b**3
*c*d**3*x**3 - 4*a*b**3*c*d**3*x**2 - 6*a*b**3*d**4*x**4 - 2*a*b**3*d**4*x
**3 + 2*b**4*c**3*d*x + 5*b**4*c**2*d**2*x**2 + 2*b**4*c*d**3*x**3))/(24*b
**2*d**2)
```

3.13 $\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

Optimal result 252
Mathematica [A] (verified) 252
Rubi [A] (verified) 253
Maple [A] (verified) 254
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Maxima [B] (verification not implemented) 257
Giac [B] (verification not implemented) 257
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Reduce [B] (verification not implemented) 260

Optimal result

Integrand size = 30, antiderivative size = 118

$$\begin{aligned}&\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx\\&=-\frac{B(bc-ad)^2i^2x}{3b^2}-\frac{B(bc-ad)i^2(c+dx)^2}{6bd}\\&\quad-\frac{B(bc-ad)^3i^2\log(a+bx)}{3b^3d}+\frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d}\end{aligned}$$

output

```
-1/3*B*(-a*d+b*c)^2*i^2*x/b^2-1/6*B*(-a*d+b*c)*i^2*(d*x+c)^2/b/d-1/3*B*(-a
*d+b*c)^3*i^2*ln(b*x+a)/b^3/d+1/3*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c))
)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

$$\begin{aligned}&\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx\\&=\frac{i^2\left(-\frac{B(bc-ad)\left(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx)\right)}{2b^3}+(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\right)}{3d}\end{aligned}$$

input
```
Integrate[(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(i^2*(-1/2*(B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c
- a*d)^2*Log[a + b*x]))/b^3 + (c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)])))/(3*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)^2 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right) dx$$

$$\downarrow 2948$$

$$\frac{i^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d} - \frac{B(bc-ad)\int \frac{i^3(c+dx)^2}{a+bx}dx}{3di}$$

$$\downarrow 27$$

$$\frac{i^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d} - \frac{Bi^2(bc-ad)\int \frac{(c+dx)^2}{a+bx}dx}{3d}$$

$$\downarrow 49$$

$$\frac{i^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d} - \frac{Bi^2(bc-ad)\int \left(\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(bc-ad)}{b^2}+\frac{d(c+dx)}{b}\right)dx}{3d}$$

$$\downarrow 2009$$

$$\frac{i^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d} - \frac{Bi^2(bc-ad)\left(\frac{(bc-ad)^2\log(a+bx)}{b^3}+\frac{dx(bc-ad)}{b^2}+\frac{(c+dx)^2}{2b}\right)}{3d}$$

input
```
Int[(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
-1/3*(B*(b*c - a*d)*i^2*((d*(b*c - a*d)*x)/b^2 + (c + d*x)^2/(2*b) + ((b*c
 - a*d)^2*Log[a + b*x])/b^3))/d + (i^2*(c + d*x)^3*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.75

method	result
risch	$\frac{i^2(dx+c)^3 B \ln\left(\frac{e(bx+a)}{dx+c}\right)}{3d} + \frac{i^2 d^2 A x^3}{3} + i^2 dAc x^2 + \frac{i^2 d^2 Ba x^2}{6b} - \frac{i^2 dBc x^2}{6} + i^2 A c^2 x + \frac{i^2 d^2 B \ln(bx+a}{3b^3}$
parts	$\frac{A i^2(dx+c)^3}{3d} - B i^2(da-bc)^3 e^3 \left(\frac{1}{3b^2e^2d\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)} - \frac{1}{6bed\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)^2} + \frac{\ln\left(\right.}{}$
parallelrisch	$\frac{-6A b^3c^3i^2-6B\ln(bx+a)a^2bcd^2i^2+6B\ln(bx+a)ab^2c^2di^2+6Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3cd^2i^2+6Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3c^2di^2+}{}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{Ade^2i^2\left(a^2d^2-2acdb+c^2b^2\right)}{3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^3}+Bd^2e^2i^2\left(a^2d^2-2acdb+c^2b^2\right)\left(-\frac{1}{6bed\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^2}-\frac{}{3b^2e^2d\left(be-\right.}\right.}{}$
default	$-\frac{e(da-bc)\left(\frac{Ade^2i^2\left(a^2d^2-2acdb+c^2b^2\right)}{3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^3}+Bd^2e^2i^2\left(a^2d^2-2acdb+c^2b^2\right)\left(-\frac{1}{6bed\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^2}-\frac{}{3b^2e^2d\left(be-\right.}\right.}{}$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/3*i^2*(d*x+c)^3*B/d*ln(e*(b*x+a)/(d*x+c))+1/3*i^2*d^2*A*x^3+i^2*d*A*c*x^
2+1/6*i^2/b*d^2*B*a*x^2-1/6*i^2*d*B*c*x^2+i^2*A*c^2*x+1/3*i^2/b^3*d^2*B*ln
(b*x+a)*a^3-i^2/b^2*d*B*ln(b*x+a)*a^2*c+i^2/b*B*ln(b*x+a)*a*c^2-1/3*i^2/d*
B*ln(b*x+a)*c^3-1/3*i^2/b^2*d^2*B*a^2*x+i^2/b*d*B*a*c*x-2/3*i^2*B*c^2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(110) = 220$.

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.89

$$\int (ci+dix)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$$
$$= \frac{2Ab^3d^3i^2x^3 - 2Bb^3c^3i^2\log(dx+c) + ((6A-B)b^3cd^2 + Bab^2d^3)i^2x^2 + 2((3A-2B)b^3c^2d + 3Bab^2c}{}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas"
)
```

output
```
1/6*(2*A*b^3*d^3*i^2*x^3 - 2*B*b^3*c^3*i^2*log(d*x + c) + ((6*A - B)*b^3*c
*d^2 + B*a*b^2*d^3)*i^2*x^2 + 2*((3*A - 2*B)*b^3*c^2*d + 3*B*a*b^2*c*d^2 -
 B*a^2*b*d^3)*i^2*x + 2*(3*B*a*b^2*c^2*d - 3*B*a^2*b*c*d^2 + B*a^3*d^3)*i^
2*log(b*x + a) + 2*(B*b^3*d^3*i^2*x^3 + 3*B*b^3*c*d^2*i^2*x^2 + 3*B*b^3*c^
2*d*i^2*x)*log((b*e*x + a*e)/(d*x + c)))/(b^3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(100) = 200$.

Time = 1.34 (sec) , antiderivative size = 491, normalized size of antiderivative = 4.16

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{Ad^2i^2x^3}{3} + \frac{Bai^2(a^2d^2-3abcd+3b^2c^2)\log\left(x+\frac{\frac{Ba^3cd^2i^2-3Ba^2bc^2di^2+\frac{Ba^2di^2\left(a^2d^2-3abcd+3b^2c^2\right)}{b}+4Bab^2c^3i^2-Baci^2\left(a^2d^2-3abcd-}{Ba^3d^3i^2-3Ba^2bcd^2i^2+3Bab^2c^2di^2+Bb^3c^3i^2}}{3b^3} - \frac{Bc^3i^2\log\left(x+\frac{\frac{Ba^3cd^2i^2-3Ba^2bc^2di^2+3Bab^2c^3i^2+\frac{Bb^3c^4i^2}{d}}{Ba^3d^3i^2-3Ba^2bcd^2i^2+3Bab^2c^2di^2+Bb^3c^3i^2}}\right)}{3d} + x^2\left(Acdi^2+\frac{Bad^2i^2}{6b}-\frac{Bcdi^2}{6}\right) + x\left(Ac^2i^2-\frac{Ba^2d^2i^2}{3b^2}+\frac{Bacdi^2}{b}-\frac{2Bc^2i^2}{3}\right) + \left(Bc^2i^2x+Bcdi^2x^2+\frac{Bd^2i^2x^3}{3}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*d**2*i**2*x**3/3 + B*a*i**2*(a**2*d**2 - 3*a*b*c*d + 3*b**2*c**2)*log(x
+ (B*a**3*c*d**2*i**2 - 3*B*a**2*b*c**2*d*i**2 + B*a**2*d*i**2*(a**2*d**2
- 3*a*b*c*d + 3*b**2*c**2)/b + 4*B*a*b**2*c**3*i**2 - B*a*c*i**2*(a**2*d**
2 - 3*a*b*c*d + 3*b**2*c**2))/(B*a**3*d**3*i**2 - 3*B*a**2*b*c*d**2*i**2 +
 3*B*a*b**2*c**2*d*i**2 + B*b**3*c**3*i**2))/(3*b**3) - B*c**3*i**2*log(x
+ (B*a**3*c*d**2*i**2 - 3*B*a**2*b*c**2*d*i**2 + 3*B*a*b**2*c**3*i**2 + B*
b**3*c**4*i**2/d)/(B*a**3*d**3*i**2 - 3*B*a**2*b*c*d**2*i**2 + 3*B*a*b**2*
c**2*d*i**2 + B*b**3*c**3*i**2))/(3*d) + x**2*(A*c*d*i**2 + B*a*d**2*i**2/
(6*b) - B*c*d*i**2/6) + x*(A*c**2*i**2 - B*a**2*d**2*i**2/(3*b**2) + B*a*c
*d*i**2/b - 2*B*c**2*i**2/3) + (B*c**2*i**2*x + B*c*d*i**2*x**2 + B*d**2*i
**2*x**3/3)*log(e*(a + b*x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(110) = 220$.

Time = 0.04 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.37

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{1}{3}Ad^2i^2x^3 + Acdi^2x^2$$
$$+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bc^2i^2$$
$$+\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bcdi^2$$
$$+\frac{1}{6}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-}{b^2d^2}\right.$$
$$+Ac^2i^2x$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima"
)
```

output
```
1/3*A*d^2*i^2*x^3 + A*c*d*i^2*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)
) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*c^2*i^2 + (x^2*log(b*e*x/(d*x +
 c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c
- a*d)*x/(b*d))*B*c*d*i^2 + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)
) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)
*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*d^2*i^2 + A*c^2*i^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1056 vs. $2(110) = 220$.

Time = 0.24 (sec) , antiderivative size = 1056, normalized size of antiderivative = 8.95

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output

```
1/6*(2*(B*b^4*c^4*e^4*i^2 - 4*B*a*b^3*c^3*d*e^4*i^2 + 6*B*a^2*b^2*c^2*d^2*
e^4*i^2 - 4*B*a^3*b*c*d^3*e^4*i^2 + B*a^4*d^4*e^4*i^2)*log((b*e*x + a*e)/(
d*x + c))/(b^3*d*e^3 - 3*(b*e*x + a*e)*b^2*d^2*e^2/(d*x + c) + 3*(b*e*x +
a*e)^2*b*d^3*e/(d*x + c)^2 - (b*e*x + a*e)^3*d^4/(d*x + c)^3) + (2*A*b^6*c
^4*e^4*i^2 - 3*B*b^6*c^4*e^4*i^2 - 8*A*a*b^5*c^3*d*e^4*i^2 + 12*B*a*b^5*c^
3*d*e^4*i^2 + 12*A*a^2*b^4*c^2*d^2*e^4*i^2 - 18*B*a^2*b^4*c^2*d^2*e^4*i^2
- 8*A*a^3*b^3*c*d^3*e^4*i^2 + 12*B*a^3*b^3*c*d^3*e^4*i^2 + 2*A*a^4*b^2*d^4
*e^4*i^2 - 3*B*a^4*b^2*d^4*e^4*i^2 + 5*(b*e*x + a*e)*B*b^5*c^4*d*e^3*i^2/(
d*x + c) - 20*(b*e*x + a*e)*B*a*b^4*c^3*d^2*e^3*i^2/(d*x + c) + 30*(b*e*x
+ a*e)*B*a^2*b^3*c^2*d^3*e^3*i^2/(d*x + c) - 20*(b*e*x + a*e)*B*a^3*b^2*c*
d^4*e^3*i^2/(d*x + c) + 5*(b*e*x + a*e)*B*a^4*b*d^5*e^3*i^2/(d*x + c) - 2*
(b*e*x + a*e)^2*B*b^4*c^4*d^2*e^2*i^2/(d*x + c)^2 + 8*(b*e*x + a*e)^2*B*a*
b^3*c^3*d^3*e^2*i^2/(d*x + c)^2 - 12*(b*e*x + a*e)^2*B*a^2*b^2*c^2*d^4*e^2
*i^2/(d*x + c)^2 + 8*(b*e*x + a*e)^2*B*a^3*b*c*d^5*e^2*i^2/(d*x + c)^2 - 2
*(b*e*x + a*e)^2*B*a^4*d^6*e^2*i^2/(d*x + c)^2)/(b^5*d*e^3 - 3*(b*e*x + a*
e)*b^4*d^2*e^2/(d*x + c) + 3*(b*e*x + a*e)^2*b^3*d^3*e/(d*x + c)^2 - (b*e*
x + a*e)^3*b^2*d^4/(d*x + c)^3) + 2*(B*b^4*c^4*e*i^2 - 4*B*a*b^3*c^3*d*e*i
^2 + 6*B*a^2*b^2*c^2*d^2*e*i^2 - 4*B*a^3*b*c*d^3*e*i^2 + B*a^4*d^4*e*i^2)*
log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b^3*d) - 2*(B*b^4*c^4*e*i^2 - 4*B*a
*b^3*c^3*d*e*i^2 + 6*B*a^2*b^2*c^2*d^2*e*i^2 - 4*B*a^3*b*c*d^3*e*i^2 + ...
```

Mupad [B] (verification not implemented)

Time = 26.44 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.46

$$\begin{aligned}
&\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx \\
&= x^2\left(\frac{di^2(3Aad+9Abc+Bad-Bbc)}{6b}-\frac{Adi^2(3ad+3bc)}{6b}\right) \\
&\quad - x\left(\frac{(3ad+3bc)\left(\frac{di^2(3Aad+9Abc+Bad-Bbc)}{3b}-\frac{Adi^2(3ad+3bc)}{3b}\right)}{3bd}\right. \\
&\qquad\qquad \left. -\frac{ci^2(3Aad+3Abc+Bad-Bbc)}{b}+\frac{Aacdi^2}{b}\right) \\
&\quad + \ln\left(\frac{e(a+bx)}{c+dx}\right)\left(Bc^2i^2x+Bcdi^2x^2+\frac{Bd^2i^2x^3}{3}\right) \\
&\quad + \frac{\ln(a+bx)\left(Ba^3d^2i^2-3Ba^2bcdi^2+3Bab^2c^2i^2\right)}{3b^3} \\
&\quad + \frac{Ad^2i^2x^3}{3}-\frac{Bc^3i^2\ln(c+dx)}{3d}
\end{aligned}$$

input
```
int((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x^2*((d*i^2*(3*A*a*d + 9*A*b*c + B*a*d - B*b*c))/(6*b) - (A*d*i^2*(3*a*d +
 3*b*c))/(6*b)) - x*(((3*a*d + 3*b*c)*((d*i^2*(3*A*a*d + 9*A*b*c + B*a*d -
 B*b*c))/(3*b) - (A*d*i^2*(3*a*d + 3*b*c))/(3*b)))/(3*b*d) - (c*i^2*(3*A*a
*d + 3*A*b*c + B*a*d - B*b*c))/b + (A*a*c*d*i^2)/b) + log((e*(a + b*x))/(c
 + d*x))*((B*d^2*i^2*x^3)/3 + B*c^2*i^2*x + B*c*d*i^2*x^2) + (log(a + b*x)
*(B*a^3*d^2*i^2 + 3*B*a*b^2*c^2*i^2 - 3*B*a^2*b*c*d*i^2))/(3*b^3) + (A*d^2
*i^2*x^3)/3 - (B*c^3*i^2*log(c + d*x))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.27

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{-2\log(bx+a)\,a^3d^3+6\log(bx+a)\,a^2bc\,d^2-6\log(bx+a)\,ab^2c^2d+2\log(bx+a)\,b^3c^3-2\log\left(\frac{bex+ae}{dx+c}\right)b^3$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output
```
( - 2*log(a + b*x)*a**3*d**3 + 6*log(a + b*x)*a**2*b*c*d**2 - 6*log(a + b*
x)*a*b**2*c**2*d + 2*log(a + b*x)*b**3*c**3 - 2*log((a*e + b*e*x)/(c + d*x
))*b**3*c**3 - 6*log((a*e + b*e*x)/(c + d*x))*b**3*c**2*d*x - 6*log((a*e +
 b*e*x)/(c + d*x))*b**3*c*d**2*x**2 - 2*log((a*e + b*e*x)/(c + d*x))*b**3*
d**3*x**3 + 2*a**2*b*d**3*x - 6*a*b**2*c**2*d*x - 6*a*b**2*c*d**2*x**2 - 6
*a*b**2*c*d**2*x - 2*a*b**2*d**3*x**3 - a*b**2*d**3*x**2 + 4*b**3*c**2*d*x
 + b**3*c*d**2*x**2)/(6*b**2*d)
```

3.14 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 276

$$\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx \\
&= -\frac{Bd(bc-ad)i^2x}{2b^2g} - \frac{B(bc-ad)^2i^2\log\left(\frac{a+bx}{c+dx}\right)}{2b^3g} \\
&\quad + \frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g} + \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg} \\
&\quad - \frac{3B(bc-ad)^2i^2\log(c+dx)}{2b^3g} - \frac{(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad + \frac{B(bc-ad)^2i^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}
\end{aligned}$$

output

```
-1/2*B*d*(-a*d+b*c)*i^2*x/b^2/g-1/2*B*(-a*d+b*c)^2*i^2*ln((b*x+a)/(d*x+c))
/b^3/g+d*(-a*d+b*c)*i^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/g+1/2*i^2*
(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g-3/2*B*(-a*d+b*c)^2*i^2*ln(d*x+c)
/b^3/g-(-a*d+b*c)^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+
a))/b^3/g+B*(-a*d+b*c)^2*i^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.91

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i^2\Big(2Abd(bc-ad)x - B(bc-ad)(bdx+(bc-ad)\log(a+bx)) + 2Bd(bc-ad)(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right) +$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x),x]
```

output
```
(i^2*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*(b*d*x + (b*c - a*d)*Log[a + b
*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + b^2*(c +
 d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*(b*c - a*d)^2*Log[g*(a +
b*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b*c - a*d)^2*Log[c + d*x
] + B*(b*c - a*d)^2*(-(Log[g*(a + b*x)]*(Log[g*(a + b*x)] - 2*Log[(b*(c +
d*x))/(b*c - a*d)])) + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(2*b^
3*g)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2962, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{ag+bgx}\,dx$$
$$\downarrow \text{2962}$$
$$\frac{i^2(bc-ad)^2\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{g}$$

$$\downarrow 2789$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\downarrow 2756$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\downarrow 54$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\downarrow 2009$$

$$\frac{i^2(bc-ad)^2\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}\right)}{g}$$

$\downarrow$ 2789

$$\frac{i^2(bc-ad)^2\left(\frac{\frac{d\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+\cdots}{c+d\cdots}\right.}{b^2}\right.}{2d}\cdots\right.}{b}\cdots\right.}{g}$$

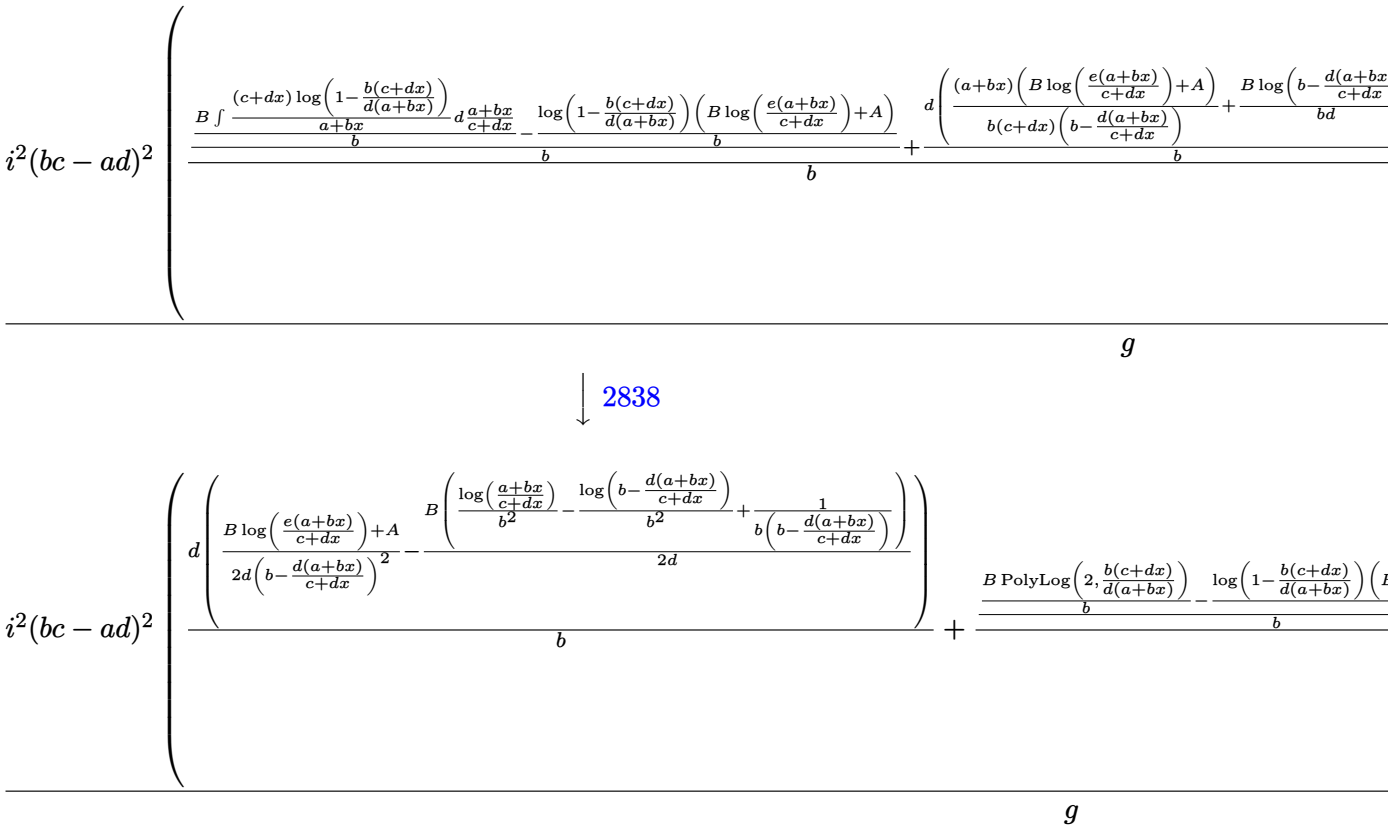
$\downarrow$ 2751

$$\frac{i^2(bc-ad)^2\left(\frac{\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{B\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\cdots\right.}{\cdots}\right.}{g}$$

$\downarrow$ 16

$$\frac{i^2(bc-ad)^2\left(\frac{\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{g}$$

$\downarrow$ 2779



input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x),x
]
```

output
```
((b*c - a*d)^2*i^2*((d*((A + B*Log[(e*(a + b*x))/(c + d*x)])/(2*d*(b - (d*
(a + b*x))/(c + d*x))^2) - (B*(1/(b*(b - (d*(a + b*x))/(c + d*x))) + Log[(
a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(2*d)))/b
 + ((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b*(c + d*x)*(b -
 (d*(a + b*x))/(c + d*x))) + (B*Log[b - (d*(a + b*x))/(c + d*x)])/(b*d)))/
b + (-(((A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a +
 b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b)/b))/g
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(268) = 536$.

Time = 3.18 (sec) , antiderivative size = 698, normalized size of antiderivative = 2.53

method	result
parts	$\frac{i^2 A\left(\frac{d\left(\frac{1}{2}bd\,x^2-xda+2bcx\right)}{b^2}+\frac{\left(a^2d^2-2acdb+c^2b^2\right)\ln(bx+a)}{b^3}\right)}{g}-\frac{i^2B(da-bc)^3e^3\left(\frac{\left(-\frac{1}{2bed\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}-\frac{\ln\left(\right.}{}\right.}{}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{A\,d^2e^2i^2(da-bc)\left(\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{b^3e^3}+\frac{1}{b^2e^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{b^3e^3}+\frac{}{2be\left(be-\right.}\right.}{g}$
default	$-\frac{e(da-bc)\left(-\frac{A\,d^2e^2i^2(da-bc)\left(\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{b^3e^3}+\frac{1}{b^2e^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{b^3e^3}+\frac{}{2be\left(be-\right.}\right.}{g}$
risch	Expression too large to display

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x,method=_RETURN
VERBOSE)
```

output
```
i^2*A/g*(d/b^2*(1/2*b*d*x^2-x*d*a+2*b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3
*ln(b*x+a))-i^2*B/g/d^4*(a*d-b*c)^3*e^3*((-1/2/b/e/d/((b*e/d+(a*d-b*c)*e/d
/(d*x+c))*d-b*e)-1/2/b^2/e^2*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/d+1/2
*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*((b*e/d+(a*
d-b*c)*e/d/(d*x+c))*d-2*b*e)/b^2/e^2/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)
^2)/(a*d-b*c)*d^5/b/e-(1/b/e*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/d-ln(
b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*
d-b*c)*e/d/(d*x+c))*d-b*e))/(a*d-b*c)*d^5/b^2/e^2-1/2*ln(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^2/(a*d-b*c)*d^4/b^3/e^3+(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))
*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(
d*x+c))*d-b*e)/b/e)/d)/(a*d-b*c)*d^5/b^3/e^3)
```

Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algori
thm="fricas")
```

output
```
integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B
*c*d*i^2*x + B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)))/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i^2\left(\int \frac{Ac^2}{a+bx}\,dx + \int \frac{Ad^2x^2}{a+bx}\,dx + \int \frac{Bc^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx + \int \frac{2Acdx}{a+bx}\,dx + \int \frac{Bd^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx + \int \frac{2Bcdx\log}{}\right.}{g}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
```

output
```
i**2*(Integral(A*c**2/(a + b*x), x) + Integral(A*d**2*x**2/(a + b*x), x) +
 Integral(B*c**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Inte
gral(2*A*c*d*x/(a + b*x), x) + Integral(B*d**2*x**2*log(a*e/(c + d*x) + b*
e*x/(c + d*x))/(a + b*x), x) + Integral(2*B*c*d*x*log(a*e/(c + d*x) + b*e*
x/(c + d*x))/(a + b*x), x))/g
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.88

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= 2\,Acdi^2\left(\frac{x}{bg}-\frac{a\log(bx+a)}{b^2g}\right)+\frac{1}{2}\,Ad^2i^2\left(\frac{2\,a^2\log(bx+a)}{b^3g}+\frac{bx^2-2\,ax}{b^2g}\right)$$
$$+\frac{Ac^2i^2\log(bgx+ag)}{bg}-\frac{(3\,bc^2i^2-2\,acdi^2)B\log(dx+c)}{2\,b^2g}$$
$$+\frac{(b^2c^2i^2-2\,abcdi^2+a^2d^2i^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^3g}$$
$$+\frac{Bb^2d^2i^2x^2\log(e)+(b^2c^2i^2-2\,abcdi^2+a^2d^2i^2)B\log(bx+a)^2+((4\,i^2\log(e)-i^2)b^2cd-(2\,i^2\log(e)$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algori
thm="maxima")
```

output

```
2*A*c*d*i^2*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + 1/2*A*d^2*i^2*(2*a^2*log(
b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A*c^2*i^2*log(b*g*x + a*g)/(
b*g) - 1/2*(3*b*c^2*i^2 - 2*a*c*d*i^2)*B*log(d*x + c)/(b^2*g) + (b^2*c^2*i
^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a
*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^3*g) + 1/2*(B*b^2*d^2*i
^2*x^2*log(e) + (b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*B*log(b*x + a)
^2 + ((4*i^2*log(e) - i^2)*b^2*c*d - (2*i^2*log(e) - i^2)*a*b*d^2)*B*x + (
B*b^2*d^2*i^2*x^2 + 2*(2*b^2*c*d*i^2 - a*b*d^2*i^2)*B*x + (2*b^2*c^2*i^2*l
og(e) - 4*(i^2*log(e) - i^2)*a*b*c*d + (2*i^2*log(e) - 3*i^2)*a^2*d^2)*B)*
log(b*x + a) - (B*b^2*d^2*i^2*x^2 + 2*(2*b^2*c*d*i^2 - a*b*d^2*i^2)*B*x +
2*(b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*B*log(b*x + a))*log(d*x + c)
)/(b^3*g)
```

Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{bgx+ag}\,dx$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algori
thm="giac")
```

output

```
integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{a\,g+b\,g\,x}\,dx$$

input

```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x),x
)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x),
x)
```

Reduce [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= \frac{-2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{bx+a}dx\right)b^4c^2 - 2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x^2}{bx+a}dx\right)b^4d^2 - 4\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{bx+a}dx\right)b^4cd - 2\log(bx+a)\,a^3d^2}{2b^3g}$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
```

output
```
( - 2*int(log((a*e + b*e*x)/(c + d*x))/(a + b*x),x)*b**4*c**2 - 2*int((log
((a*e + b*e*x)/(c + d*x))*x**2)/(a + b*x),x)*b**4*d**2 - 4*int((log((a*e +
 b*e*x)/(c + d*x))*x)/(a + b*x),x)*b**4*c*d - 2*log(a + b*x)*a**3*d**2 + 4
*log(a + b*x)*a**2*b*c*d - 2*log(a + b*x)*a*b**2*c**2 + 2*a**2*b*d**2*x -
4*a*b**2*c*d*x - a*b**2*d**2*x**2)/(2*b**3*g)
```

3.15 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 247

$$\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx \\
&= -\frac{B(bc-ad)i^2(c+dx)}{b^2g^2(a+bx)} + \frac{d^2i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2} \\
&\quad - \frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^2(a+bx)} - \frac{Bd(bc-ad)i^2\log(c+dx)}{b^3g^2} \\
&\quad - \frac{2d(bc-ad)i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&\quad + \frac{2Bd(bc-ad)i^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}
\end{aligned}$$

output

```
-B*(-a*d+b*c)*i^2*(d*x+c)/b^2/g^2/(b*x+a)+d^2*i^2*(b*x+a)*(A+B*ln(e*(b*x+a
)/(d*x+c)))/b^3/g^2-(-a*d+b*c)*i^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2
/g^2/(b*x+a)-B*d*(-a*d+b*c)*i^2*ln(d*x+c)/b^3/g^2-2*d*(-a*d+b*c)*i^2*(A+B*
ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^2+2*B*d*(-a*d+b*c)*
i^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.89

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$$
$$= \frac{i^2\left(Abd^2x - \frac{B(bc-ad)^2}{a+bx} + Bd(-bc+ad)\log(a+bx) + Bd^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right) - \frac{(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{a+bx}\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^2,x]
```

output
```
(i^2*(A*b*d^2*x - (B*(b*c - a*d)^2)/(a + b*x) + B*d*(-(b*c) + a*d)*Log[a +
 b*x] + B*d^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - ((b*c - a*d)^2*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) + 2*d*(b*c - a*d)*Log[a + b*x]
*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + B*d*(-(b*c) + a*d)*(Log[a + b*x]*(
Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*
x))/(-(b*c) + a*d)])))/(b^3*g^2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2962, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^2}\,dx$$
$$\downarrow \text{2962}$$
$$\frac{i^2(bc-ad)\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g^2}$$

$$\downarrow 2793$$

$$\frac{i^2(bc-ad)\int\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d^2}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d}{b^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{g^2}$$

$$\downarrow 2009$$

$$\frac{i^2(bc-ad)\left(\frac{d^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2d\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3}-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^2(a+bx)}+\frac{2Bd\,\mathrm{Poly}}{g^2}$$

input

```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^2
,x]
```

output

```
((b*c - a*d)*i^2*(-((B*(c + d*x))/(b^2*(a + b*x))) - ((c + d*x)*(A + B*Log
[(e*(a + b*x))/(c + d*x)]))/(b^2*(a + b*x)) + (d^2*(a + b*x)*(A + B*Log[(e
*(a + b*x))/(c + d*x)]))/(b^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (
B*d*Log[b - (d*(a + b*x))/(c + d*x)])/b^3 - (2*d*(A + B*Log[(e*(a + b*x))/
(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b^3 + (2*B*d*PolyLog[2,
(b*(c + d*x))/(d*(a + b*x))])/b^3))/g^2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(247) = 494$.

Time = 3.06 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.30

method	result
parts	$\frac{i^2A\left(\frac{x\,d^2}{b^2}-\frac{2d(da-bc)\ln(bx+a)}{b^3}-\frac{a^2d^2-2acdb+c^2b^2}{b^3(bx+a)}\right)}{g^2}-\frac{i^2B(da-bc)^3e^3\left(\frac{\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)d^4}{(da-bc)^2b^2e^2}+$
derivativedivides	$-\frac{e(da-bc)\left(\frac{i^2d^2e^2A\left(-\frac{1}{b^2e^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{2d\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{b^3e^3}+\frac{d}{b^2e^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}-\frac{2d\ln\left(be-\left(\frac{be}{d}+\frac{(d}{b^3e^3}\right.\right.\right.\right.}{g^2}$
default	$-\frac{e(da-bc)\left(\frac{i^2d^2e^2A\left(-\frac{1}{b^2e^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{2d\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{b^3e^3}+\frac{d}{b^2e^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}-\frac{2d\ln\left(be-\left(\frac{be}{d}+\frac{(d}{b^3e^3}\right.\right.\right.\right.}{g^2}$
risch	Expression too large to display

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x,method=_RETU
RNVERBOSE)
```

output
```
i^2*A/g^2*(x*d^2/b^2-2/b^3*d*(a*d-b*c)*ln(b*x+a)-1/b^3*(a^2*d^2-2*a*b*c*d+
b^2*c^2)/(b*x+a))-i^2*B/g^2/d^4*(a*d-b*c)^3*e^3*((-1/(b*e/d+(a*d-b*c)*e/d/
(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))/
(a*d-b*c)^2*d^4/b^2/e^2+(1/b/e*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/d-l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(
a*d-b*c)*e/d/(d*x+c))*d-b*e))/(a*d-b*c)^2*d^6/b^2/e^2+ln(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^2/(a*d-b*c)^2*d^5/b^3/e^3-2*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x
+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e
/d/(d*x+c))*d-b*e)/b/e)/d)/(a*d-b*c)^2*d^6/b^3/e^3)
```

Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algo
rithm="fricas")
```

output
```
integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B
*c*d*i^2*x + B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)))/(b^2*g^2*x^2 + 2*a*b
*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(246) = 492$.

Time = 0.11 (sec) , antiderivative size = 992, normalized size of antiderivative = 4.02

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algo
rithm="maxima")
```

output
```
-A*(a^2/(b^4*g^2*x + a*b^3*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^2)
)*d^2*i^2 + 2*A*c*d*i^2*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2
)) - B*c^2*i^2*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2)
 + 1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(
d*x + c)/((b^2*c - a*b*d)*g^2)) - A*c^2*i^2/(b^2*g^2*x + a*b*g^2) - (b^2*c
^2*d*i^2 + a*b*c*d^2*i^2 - a^2*d^3*i^2)*B*log(d*x + c)/(b^4*c*g^2 - a*b^3*
d*g^2) + ((b^3*c*d^2*i^2*log(e) - a*b^2*d^3*i^2*log(e))*B*x^2 + (a*b^2*c*d
^2*i^2*log(e) - a^2*b*d^3*i^2*log(e))*B*x + ((b^3*c^2*d*i^2 - 2*a*b^2*c*d^
2*i^2 + a^2*b*d^3*i^2)*B*x + (a*b^2*c^2*d*i^2 - 2*a^2*b*c*d^2*i^2 + a^3*d^
3*i^2)*B)*log(b*x + a)^2 + (2*(i^2*log(e) + i^2)*a*b^2*c^2*d - 3*(i^2*log(
e) + i^2)*a^2*b*c*d^2 + (i^2*log(e) + i^2)*a^3*d^3)*B + ((b^3*c*d^2*i^2 -
a*b^2*d^3*i^2)*B*x^2 + (2*b^3*c^2*d*i^2*log(e) - 4*(i^2*log(e) - i^2)*a*b^
2*c*d^2 + (2*i^2*log(e) - 3*i^2)*a^2*b*d^3)*B*x - (4*a^2*b*c*d^2*i^2*log(e
) - 2*(i^2*log(e) + i^2)*a*b^2*c^2*d - (2*i^2*log(e) - i^2)*a^3*d^3)*B)*lo
g(b*x + a) - ((b^3*c*d^2*i^2 - a*b^2*d^3*i^2)*B*x^2 + (a*b^2*c*d^2*i^2 - a
^2*b*d^3*i^2)*B*x + (2*a*b^2*c^2*d*i^2 - 3*a^2*b*c*d^2*i^2 + a^3*d^3*i^2)*
B + 2*((b^3*c^2*d*i^2 - 2*a*b^2*c*d^2*i^2 + a^2*b*d^3*i^2)*B*x + (a*b^2*c^
2*d*i^2 - 2*a^2*b*c*d^2*i^2 + a^3*d^3*i^2)*B)*log(b*x + a))*log(d*x + c))/
(a*b^4*c*g^2 - a^2*b^3*d*g^2 + (b^5*c*g^2 - a*b^4*d*g^2)*x) + 2*(b*c*d*i^2
 - a*d^2*i^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(...
```

Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algo
rithm="giac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)
^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(a\,g+b\,g\,x)^2}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2
,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2
, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x)
```

output

```
( - int((log((a*e + b*e*x)/(c + d*x))*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x
)*a**3*b**4*d**3 + int((log((a*e + b*e*x)/(c + d*x))*x**2)/(a**2 + 2*a*b*x
 + b**2*x**2),x)*a**2*b**5*c*d**2 - int((log((a*e + b*e*x)/(c + d*x))*x**2
)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**5*d**3*x + int((log((a*e + b*e*x
)/(c + d*x))*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a*b**6*c*d**2*x - 2*int
((log((a*e + b*e*x)/(c + d*x))*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**3*b**
4*c*d**2 + 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a**2 + 2*a*b*x + b**2*x
**2),x)*a**2*b**5*c**2*d - 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a**2 +
2*a*b*x + b**2*x**2),x)*a**2*b**5*c*d**2*x + 2*int((log((a*e + b*e*x)/(c +
 d*x))*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a*b**6*c**2*d*x + 2*log(a + b*x)
*a**5*d**3 - 4*log(a + b*x)*a**4*b*c*d**2 + 2*log(a + b*x)*a**4*b*d**3*x +
 2*log(a + b*x)*a**3*b**2*c**2*d - 4*log(a + b*x)*a**3*b**2*c*d**2*x + 2*l
og(a + b*x)*a**2*b**3*c**2*d*x - log(a + b*x)*a*b**4*c**3 - log(a + b*x)*b
**5*c**3*x + log(c + d*x)*a*b**4*c**3 + log(c + d*x)*b**5*c**3*x - log((a*
e + b*e*x)/(c + d*x))*a*b**4*c**2*d*x + log((a*e + b*e*x)/(c + d*x))*b**5*
c**3*x - 2*a**4*b*d**3*x + 4*a**3*b**2*c*d**2*x - a**3*b**2*d**3*x**2 - 3*
a**2*b**3*c**2*d*x + a**2*b**3*c*d**2*x**2 + a*b**4*c**3*x - a*b**4*c**2*d
*x + b**5*c**3*x)/(a*b**3*g**2*(a**2*d - a*b*c + a*b*d*x - b**2*c*x))
```

3.16 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 230

$$\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx \\
&= -\frac{Bdi^2(c+dx)}{b^2g^3(a+bx)} - \frac{Bi^2(c+dx)^2}{4bg^3(a+bx)^2} - \frac{di^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^3(a+bx)} \\
&\quad - \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2} \\
&\quad - \frac{d^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} + \frac{Bd^2i^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}
\end{aligned}$$

output

```
-B*d*i^2*(d*x+c)/b^2/g^3/(b*x+a)-1/4*B*i^2*(d*x+c)^2/b/g^3/(b*x+a)^2-d*i^2
*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g^3/(b*x+a)-1/2*i^2*(d*x+c)^2*(A+
B*ln(e*(b*x+a)/(d*x+c)))/b/g^3/(b*x+a)^2-d^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)
))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^3+B*d^2*i^2*polylog(2,b*(d*x+c)/d/(b*x+
a))/b^3/g^3
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.06

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$$
$$= \frac{i^2\left(-\frac{B(bc-ad)^2}{(a+bx)^2}+\frac{6Bd(-bc+ad)}{a+bx}-6Bd^2\log(a+bx)-\frac{2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}+\frac{8d(-bc+ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx}\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^3,x]
```

output
```
(i^2*(-((B*(b*c - a*d)^2)/(a + b*x)^2) + (6*B*d*(-(b*c) + a*d))/(a + b*x)
- 6*B*d^2*Log[a + b*x] - (2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/(a + b*x)^2 + (8*d*(-(b*c) + a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)
]))/(a + b*x) + 4*d^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) +
6*B*d^2*Log[c + d*x] - 2*B*d^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
 d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(4*b^
3*g^3)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {2962, 2780, 2741, 2780, 2741, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^3}\,dx$$
$$\downarrow \text{2962}$$
$$\frac{i^2\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g^3}$$

$$\downarrow 2780$$

$$\frac{i^2\left(\frac{\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^3}$$

$$\downarrow 2741$$

$$\frac{i^2\left(\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}}{b}\right)}{g^3}$$

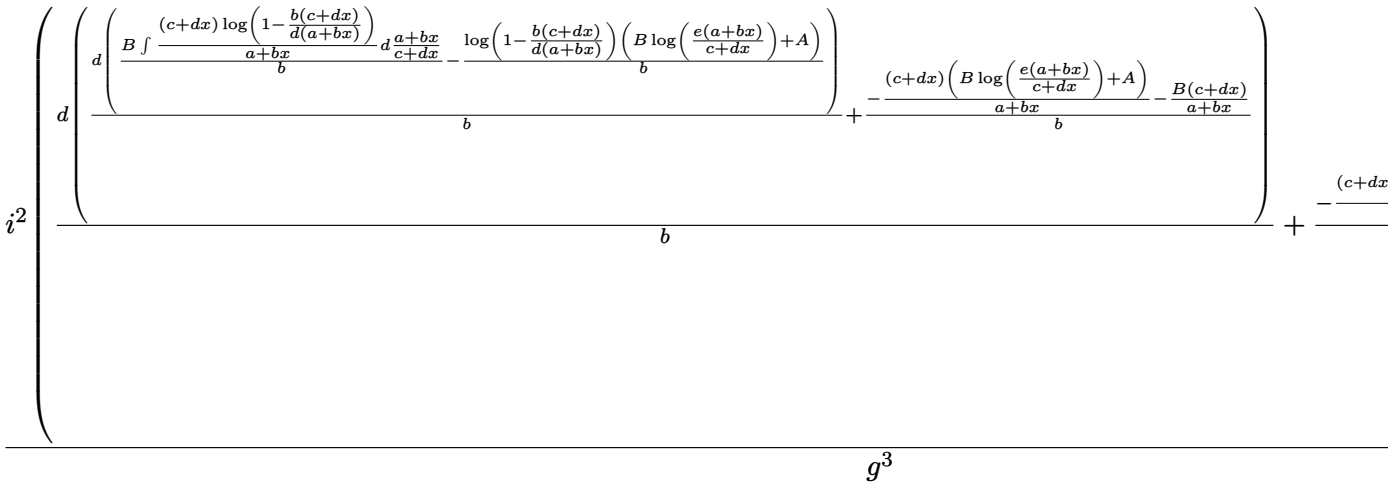
$$\downarrow 2780$$

$$\frac{i^2\left(\frac{d\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}}{b}\right)}{g^3}$$

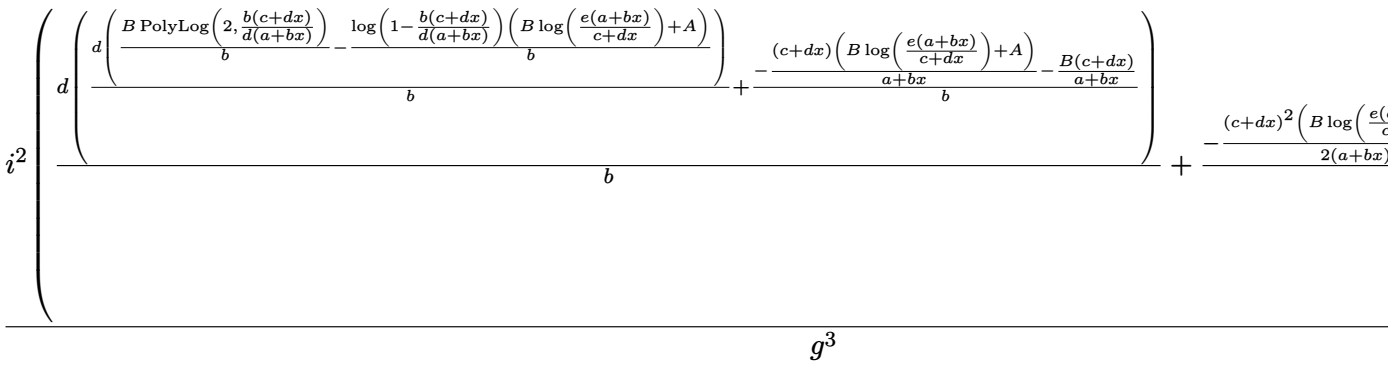
$$\downarrow 2741$$

$$\frac{i^2\left(\frac{d\left(\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}}{b}\right)}{g^3}$$

$$\downarrow 2779$$



↓ 2838



input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^3
,x]
```

output
```
(i^2*((-1/4*(B*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[(e*(a +
b*x))/(c + d*x)]))/(2*(a + b*x)^2))/b + (d*((-((B*(c + d*x))/(a + b*x)) -
((c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x))/b + (d*(-(((A
+ B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b)
 + (B*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b))/b))/b))/g^3
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(226) = 452$.

Time = 3.11 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.27

method	result
parts	$\frac{i^2 A\left(-\frac{a^2d^2-2acdb+c^2b^2}{2b^3(bx+a)^2}+\frac{d^2\ln(bx+a)}{b^3}+\frac{2d(da-bc)}{b^3(bx+a)}\right)}{g^3}-\frac{i^2B(da-bc)^3e^3\left(-\frac{\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)d^5}{(da-bc)^3b^2e^2}\right.$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{i^2d^2e^2A\left(-\frac{d^2\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{b^3e^3}-\frac{1}{2be\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{d^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{b^3e^3}-\frac{d}{b^2e^2\left(\frac{be}{d}+\frac{(da-b}{d(dx+}\right.\right.\right.\right.}{(da-bc)g^3}$
default	$-\frac{e(da-bc)\left(-\frac{i^2d^2e^2A\left(-\frac{d^2\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{b^3e^3}-\frac{1}{2be\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{d^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{b^3e^3}-\frac{d}{b^2e^2\left(\frac{be}{d}+\frac{(da-b}{d(dx+}\right.\right.\right.\right.}{(da-bc)g^3}$
risch	Expression too large to display

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x,method=_RETU
RNVERBOSE)
```

output
```
i^2*A/g^3*(-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^2+d^2/b^3*ln(b*x+a
)+2/b^3*d*(a*d-b*c)/(b*x+a))-i^2*B/g^3/d^4*(a*d-b*c)^3*e^3*(-(-1/(b*e/d+(a
*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/
d/(d*x+c)))/(a*d-b*c)^3*d^5/b^2/e^2-1/2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/
(a*d-b*c)^3*d^6/b^3/e^3-(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)/(a*d-b*c)^3*d^4/b
/e+(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)
*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)/(a*d-b*c)^
3*d^7/b^3/e^3)
```

Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algo
rithm="fricas")
```

output
```
integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B
*c*d*i^2*x + B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)))/(b^3*g^3*x^3 + 3*a*b
^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$$

$$= \frac{i^2\left(\int \frac{Ac^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{Ad^2x^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{Bc^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{2Acdx}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\right.}{g^3}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3,x)
```

output
```
i**2*(Integral(A*c**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x)
+ Integral(A*d**2*x**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x)
 + Integral(B*c**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x
 + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*c*d*x/(a**3 + 3*a**2*b*x
+ 3*a*b**2*x**2 + b**3*x**3), x) + Integral(B*d**2*x**2*log(a*e/(c + d*x)
+ b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + I
ntegral(2*B*c*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x
+ 3*a*b**2*x**2 + b**3*x**3), x))/g**3
```

Maxima [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algo
rithm="maxima")
```

output

```
-1/2*B*d^2*i^2*((4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a
))*log(d*x + c)/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) - 2*integrate(
1/2*(2*b^3*d*x^3*log(e) + 7*a^2*b*d*x + 3*a^3*d + 2*(b^3*c*log(e) + 2*a*b^
2*d)*x^2 + 2*(2*b^3*d*x^3 + 3*a^2*b*d*x + a^3*d + (b^3*c + 3*a*b^2*d)*x^2)
*log(b*x + a))/(b^6*d*g^3*x^4 + a^3*b^3*c*g^3 + (b^6*c*g^3 + 3*a*b^5*d*g^3
)*x^3 + 3*(a*b^5*c*g^3 + a^2*b^4*d*g^3)*x^2 + (3*a^2*b^4*c*g^3 + a^3*b^3*d
*g^3)*x), x)) - 1/2*B*c*d*i^2*(2*(2*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*
x + c))/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + (3*a*b*c - a^2*d + 2
*(2*b^2*c - a*b*d)*x)/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)
*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/(
(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x +
 c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3)) + 1/2*A*d^2*i^2*((4*a*b*x
 + 3*a^2)/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^
3*g^3)) + 1/4*B*c^2*i^2*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^
2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e
*x/(d*x + c) + a*e/(d*x + c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) +
2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d
*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - (2*b*x + a)*A*c*d*i^2
/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - 1/2*A*c^2*i^2/(b^3*g^3*x^2
+ 2*a*b^2*g^3*x + a^2*b*g^3)
```

Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^3}\,dx$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algo
rithm="giac")
```

output

```
integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)
^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(a\,g+b\,g\,x)^3}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^3
,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^3
, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x)
```

output

```
( - 4*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**
2*x**2 + b**3*x**3),x)*a**6*b**4*d**4 + 8*int((log((a*e + b*e*x)/(c + d*x)
)*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**5*b**5*c*d**
3 - 8*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**
2*x**2 + b**3*x**3),x)*a**5*b**5*d**4*x - 4*int((log((a*e + b*e*x)/(c + d*
x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**4*b**6*c**
2*d**2 + 16*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(a**3 + 3*a**2*b*x + 3
*a*b**2*x**2 + b**3*x**3),x)*a**4*b**6*c*d**3*x - 4*int((log((a*e + b*e*x)
/(c + d*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**4*
b**6*d**4*x**2 - 8*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(a**3 + 3*a**2*
b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**3*b**7*c**2*d**2*x + 8*int((log((a*
e + b*e*x)/(c + d*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3
),x)*a**3*b**7*c*d**3*x**2 - 4*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(a*
*3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**2*b**8*c**2*d**2*x**2 -
 4*log(a + b*x)*a**7*d**4 + 8*log(a + b*x)*a**6*b*c*d**3 - 8*log(a + b*x)*
a**6*b*d**4*x - 4*log(a + b*x)*a**5*b**2*c**2*d**2 + 16*log(a + b*x)*a**5*
b**2*c*d**3*x - 4*log(a + b*x)*a**5*b**2*d**4*x**2 - 8*log(a + b*x)*a**4*b
**3*c**2*d**2*x + 8*log(a + b*x)*a**4*b**3*c*d**3*x**2 - 4*log(a + b*x)*a*
*3*b**4*c**2*d**2*x**2 + 2*log(a + b*x)*a**2*b**5*c**4 + 4*log(a + b*x)*a*
b**6*c**4*x + 2*log(a + b*x)*b**7*c**4*x**2 - 2*log(c + d*x)*a**2*b**5*...
```

3.17 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 89

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = -\frac{Bi^2(c+dx)^3}{9(bc-ad)g^4(a+bx)^3} - \frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)g^4(a+bx)^3}$$

output
```
-1/9*B*i^2*(d*x+c)^3/(-a*d+b*c)/g^4/(b*x+a)^3-1/3*i^2*(d*x+c)^3*(A+B*ln(e*
(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^4/(b*x+a)^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 315 vs. 2(89) = 178.

Time = 0.37 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.54

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = -\frac{i^2\left(3Ab^3c^3+b^3Bc^3-3a^3Ad^3-a^3Bd^3+9Ab^3c^2dx+3b^3Bc^2dx-9a^2Abd^3x-3a^2bBd^3x+9Ab^3cd^2x\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^4,x]
```

output
```
-1/9*(i^2*(3*A*b^3*c^3 + b^3*B*c^3 - 3*a^3*A*d^3 - a^3*B*d^3 + 9*A*b^3*c^2
*d*x + 3*b^3*B*c^2*d*x - 9*a^2*A*b*d^3*x - 3*a^2*b*B*d^3*x + 9*A*b^3*c*d^2
*x^2 + 3*b^3*B*c*d^2*x^2 - 9*a*A*b^2*d^3*x^2 - 3*a*b^2*B*d^3*x^2 + 3*B*d^3
*(a + b*x)^3*Log[a + b*x] + 3*B*(b*c - a*d)*(a^2*d^2 + a*b*d*(c + 3*d*x) +
 b^2*(c^2 + 3*c*d*x + 3*d^2*x^2))*Log[(e*(a + b*x))/(c + d*x)] - 3*a^3*B*d
^3*Log[c + d*x] - 9*a^2*b*B*d^3*x*Log[c + d*x] - 9*a*b^2*B*d^3*x^2*Log[c +
 d*x] - 3*b^3*B*d^3*x^3*Log[c + d*x]))/(b^3*(b*c - a*d)*g^4*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2962, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^4}\,dx$$

$$\downarrow 2962$$

$$\frac{i^2\int \frac{(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{i^2\left(-\frac{(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}-\frac{B(c+dx)^3}{9(a+bx)^3}\right)}{g^4(bc-ad)}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^4
,x]
```

output
```
(i^2*(-1/9*(B*(c + d*x)^3)/(a + b*x)^3 - ((c + d*x)^3*(A + B*Log[(e*(a + b
*x))/(c + d*x)]))/(3*(a + b*x)^3)))/((b*c - a*d)*g^4)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(85) = 170$.

Time = 1.64 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.08

method	result
derivativedivides	$-\frac{e(da-bc)\left(-\frac{i^2d^2e^2A}{3(da-bc)^2g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}+\frac{i^2d^2e^2B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}\right)}{(da-bc)^2g^4}\right)}{d^2}$
default	$-\frac{e(da-bc)\left(-\frac{i^2d^2e^2A}{3(da-bc)^2g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}+\frac{i^2d^2e^2B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}\right)}{(da-bc)^2g^4}\right)}{d^2}$
parts	$\frac{i^2A\left(\frac{d(da-bc)}{b^3(bx+a)^2}-\frac{d^2}{b^3(bx+a)}-\frac{a^2d^2-2acdb+c^2b^2}{3b^3(bx+a)^3}\right)}{g^4}-\frac{i^2Be^3\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}\right)}{g^4(da-bc)}$
norman	$\frac{Bcd^2i^2x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(da-bc)}+\frac{Bc^2di^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(da-bc)}-\frac{3Aacdi^2+3Abc^2i^2+Bacdi^2+Bbc^2i^2}{9gb^2}+\frac{\left(3i^2Ad^2+Bd^2i^2\right)x^3}{9ag}-\frac{\left(3Acdi^2+\right.}{3g}$
parallelrisch	$-\frac{-3Bx^3\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5d^4i^2+9Ax^2ab^4d^4i^2-9Ax^2b^5cd^3i^2+3Bx^2ab^4d^4i^2-3Bx^2b^5cd^3i^2+9Axa^2b^3d^4i^2-9Axb^5$
risch	$-\frac{Bi^2\left(3d^2b^2x^2+3abd^2x+3b^2cdx+a^2d^2+acdb+c^2b^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{3(bx+a)^3g^4b^3}-\frac{i^2\left(3B\ln(dx+c)b^3d^3x^3-3B\ln(-bx-a)b^3d^3x\right.}{}$
orering	$-\frac{(bx+a)\left(2ab^2d^3x^4+2b^3cd^2x^4-5a^2bd^3x^3+8ab^2cd^2x^3+4b^3c^2dx^3-15a^2bcd^2x^2+12ab^2c^2dx^2-11a^2bc^2dx+2a^3c^2\right.}{9a^2b(da-bc)(dx+c)^2(bgx+ag)^4}$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x,method=_RETU
RNVERBOSE)
```

output
```
-1/d^2*e*(a*d-b*c)*(-1/3*i^2*d^2*e^2/(a*d-b*c)^2/g^4*A/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^3+i^2*d^2*e^2/(a*d-b*c)^2/g^4*B*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(85) = 170$.

Time = 0.09 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.04

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{3\left((3A+B)b^3cd^2-(3A+B)ab^2d^3\right)i^2x^2+3\left((3A+B)b^3c^2d-(3A+B)a^2bd^3\right)i^2x+\left((3A+B)b^3c^{3}\right.}{9\left((b^7c-ab^6d)g^4x^3+3\left(ab^6c-a^2b^5d\right)g^4x^2+3\left(a\right.\right.}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algo
rithm="fricas")
```

output
```
-1/9*(3*((3*A + B)*b^3*c*d^2 - (3*A + B)*a*b^2*d^3)*i^2*x^2 + 3*((3*A + B)
*b^3*c^2*d - (3*A + B)*a^2*b*d^3)*i^2*x + ((3*A + B)*b^3*c^3 - (3*A + B)*a
^3*d^3)*i^2 + 3*(B*b^3*d^3*i^2*x^3 + 3*B*b^3*c*d^2*i^2*x^2 + 3*B*b^3*c^2*d
*i^2*x + B*b^3*c^3*i^2)*log((b*e*x + a*e)/(d*x + c)))/((b^7*c - a*b^6*d)*g
^4*x^3 + 3*(a*b^6*c - a^2*b^5*d)*g^4*x^2 + 3*(a^2*b^5*c - a^3*b^4*d)*g^4*x
 + (a^3*b^4*c - a^4*b^3*d)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(76) = 152$.

Time = 9.79 (sec) , antiderivative size = 614, normalized size of antiderivative = 6.90

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$$

$$= -\frac{Bd^3i^2\log\left(x+\frac{-\frac{Ba^2d^5i^2}{ad-bc}+\frac{2Babcd^4i^2}{ad-bc}+Bad^4i^2-\frac{Bb^2c^2d^3i^2}{ad-bc}+Bbcd^3i^2}{2Bbd^4i^2}\right)}{3b^3g^4(ad-bc)}$$

$$+\frac{Bd^3i^2\log\left(x+\frac{\frac{Ba^2d^5i^2}{ad-bc}-\frac{2Babcd^4i^2}{ad-bc}+Bad^4i^2+\frac{Bb^2c^2d^3i^2}{ad-bc}+Bbcd^3i^2}{2Bbd^4i^2}\right)}{3b^3g^4(ad-bc)}$$

$$+\frac{-3Aa^2d^2i^2-3Aabcdi^2-3Ab^2c^2i^2-Ba^2d^2i^2-Babcdi^2-Bb^2c^2i^2+x^2(-9Ab^2d^2i^2-3Bb^2d^2i^2)+x}{9a^3b^3g^4+27a^2b^4g^4x+27ab^5g^4x^2+9b^6g^4x^3}$$

$$+\frac{(-Ba^2d^2i^2-Babcdi^2-3Babd^2i^2x-Bb^2c^2i^2-3Bb^2cdi^2x-3Bb^2d^2i^2x^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{3a^3b^3g^4+9a^2b^4g^4x+9ab^5g^4x^2+3b^6g^4x^3}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4,x)
```

output
```
-B*d**3*i**2*log(x + (-B*a**2*d**5*i**2/(a*d - b*c) + 2*B*a*b*c*d**4*i**2/
(a*d - b*c) + B*a*d**4*i**2 - B*b**2*c**2*d**3*i**2/(a*d - b*c) + B*b*c*d*
*3*i**2)/(2*B*b*d**4*i**2))/(3*b**3*g**4*(a*d - b*c)) + B*d**3*i**2*log(x
+ (B*a**2*d**5*i**2/(a*d - b*c) - 2*B*a*b*c*d**4*i**2/(a*d - b*c) + B*a*d*
*4*i**2 + B*b**2*c**2*d**3*i**2/(a*d - b*c) + B*b*c*d**3*i**2)/(2*B*b*d**4
*i**2))/(3*b**3*g**4*(a*d - b*c)) + (-3*A*a**2*d**2*i**2 - 3*A*a*b*c*d*i**
2 - 3*A*b**2*c**2*i**2 - B*a**2*d**2*i**2 - B*a*b*c*d*i**2 - B*b**2*c**2*i
**2 + x**2*(-9*A*b**2*d**2*i**2 - 3*B*b**2*d**2*i**2) + x*(-9*A*a*b*d**2*i
**2 - 9*A*b**2*c*d*i**2 - 3*B*a*b*d**2*i**2 - 3*B*b**2*c*d*i**2))/(9*a**3*
b**3*g**4 + 27*a**2*b**4*g**4*x + 27*a*b**5*g**4*x**2 + 9*b**6*g**4*x**3)
+ (-B*a**2*d**2*i**2 - B*a*b*c*d*i**2 - 3*B*a*b*d**2*i**2*x - B*b**2*c**2*
i**2 - 3*B*b**2*c*d*i**2*x - 3*B*b**2*d**2*i**2*x**2)*log(e*(a + b*x)/(c +
 d*x))/(3*a**3*b**3*g**4 + 9*a**2*b**4*g**4*x + 9*a*b**5*g**4*x**2 + 3*b**
6*g**4*x**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1515 vs. $2(85) = 170$.

Time = 0.10 (sec) , antiderivative size = 1515, normalized size of antiderivative = 17.02

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algo
rithm="maxima")
```

output
```
-1/18*B*d^2*i^2*(6*(3*b^2*x^2 + 3*a*b*x + a^2)*log(b*e*x/(d*x + c) + a*e/(
d*x + c))/(b^6*g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3*g^4)
+ (11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*c^2 - 3*a*b^3*c*d +
 a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a^3*b*d^2)*x)/((b^8
*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c^2 - 2*a^2*b^6*c*d +
 a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d + a^4*b^4*d^2)*g^4*
x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6*(3*b^2*c^2*d - 3*
a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*
d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(d*x
+ c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4)) - 1/
18*B*c*d*i^2*(6*(3*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^5*g^4*
x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) + (5*a*b^2*c^2 - 22
*a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16
*a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^
3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 -
 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b
^2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*
d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x +
c)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4)) - 1/18
*B*c^2*i^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^...
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.57

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{1}{9}\left(\frac{3\,(dx+c)^3Be^4i^2\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^3g^4}+\frac{(3\,Ae^4i^2+Be^4i^2)(dx+c)^3}{(bex+ae)^3g^4}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{c}{(bce-ad}\right.$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algo
rithm="giac")
```

output
```
-1/9*(3*(d*x + c)^3*B*e^4*i^2*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^
3*g^4) + (3*A*e^4*i^2 + B*e^4*i^2)*(d*x + c)^3/((b*e*x + a*e)^3*g^4))*(b*c
/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 28.26 (sec) , antiderivative size = 423, normalized size of antiderivative = 4.75

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{x^2\,(3\,A\,b^2\,d^2\,i^2+B\,b^2\,d^2\,i^2)+x\,(3\,A\,a\,b\,d^2\,i^2+B\,a\,b\,d^2\,i^2+3\,A\,b^2\,c\,d\,i^2+B\,b^2\,c\,d\,i^2)+A\,a^2\,d^2\,i^2+A\,b}{3\,a^3\,b^3\,g^4+9\,a^2\,b^4\,g^4\,x+9\,a\,b^5\,g^4\,x^2+3\,b^6\,g^4\,x^3}$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(a\left(\frac{B\,a\,d^2\,i^2}{3\,b^4\,g^4}+\frac{B\,c\,d\,i^2}{3\,b^3\,g^4}\right)+x\left(b\left(\frac{B\,a\,d^2\,i^2}{3\,b^4\,g^4}+\frac{B\,c\,d\,i^2}{3\,b^3\,g^4}\right)+\frac{2\,B\,a\,d^2\,i^2}{3\,b^3\,g^4}+\frac{2\,B\,c\,d\,i^2}{3\,b^2\,g^4}\right)+\frac{B\,c^2\,i^2}{3\,b^2\,g^4}+\frac{B\,d^2\,i^2\,x^2}{b^2\,g^4}\right.}{3\,a^2\,x+\frac{a^3}{b}+b^2\,x^3+3\,a\,b\,x^2}$$
$$-\frac{B\,d^3\,i^2\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{3\,b^3\,g^4\,\left(a\,d-b\,c\right)}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^4
,x)
```

output

```
- (x^2*(3*A*b^2*d^2*i^2 + B*b^2*d^2*i^2) + x*(3*A*a*b*d^2*i^2 + B*a*b*d^2*
i^2 + 3*A*b^2*c*d*i^2 + B*b^2*c*d*i^2) + A*a^2*d^2*i^2 + A*b^2*c^2*i^2 + (
B*a^2*d^2*i^2)/3 + (B*b^2*c^2*i^2)/3 + A*a*b*c*d*i^2 + (B*a*b*c*d*i^2)/3)/
(3*a^3*b^3*g^4 + 3*b^6*g^4*x^3 + 9*a^2*b^4*g^4*x + 9*a*b^5*g^4*x^2) - (log
((e*(a + b*x))/(c + d*x))*(a*((B*a*d^2*i^2)/(3*b^4*g^4) + (B*c*d*i^2)/(3*b
^3*g^4)) + x*(b*((B*a*d^2*i^2)/(3*b^4*g^4) + (B*c*d*i^2)/(3*b^3*g^4)) + (2
*B*a*d^2*i^2)/(3*b^3*g^4) + (2*B*c*d*i^2)/(3*b^2*g^4)) + (B*c^2*i^2)/(3*b^
2*g^4) + (B*d^2*i^2*x^2)/(b^2*g^4)))/(3*a^2*x + a^3/b + b^2*x^3 + 3*a*b*x^
2) - (B*d^3*i^2*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(3*b^3*g^4*
(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 534, normalized size of antiderivative = 6.00

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$$
$$= \frac{3a^4cd^2 - 3a^2b^2c^3 + a^3bcd^2 - ab^3c^3 - ab^3d^3x^3 + b^4cd^2x^3 - 3\log\left(\frac{bex+ae}{dx+c}\right)ab^3c^3 - 3a^2b^2d^3x^3 - 9a^2b^2c^2d$$

input

```
int((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x)
```

output

```
( - 3*log(a + b*x)*a**3*b*c*d**2 - 9*log(a + b*x)*a**2*b**2*c*d**2*x - 9*l
og(a + b*x)*a*b**3*c*d**2*x**2 - 3*log(a + b*x)*b**4*c*d**2*x**3 + 3*log(c
 + d*x)*a**3*b*c*d**2 + 9*log(c + d*x)*a**2*b**2*c*d**2*x + 9*log(c + d*x)
*a*b**3*c*d**2*x**2 + 3*log(c + d*x)*b**4*c*d**2*x**3 + 3*log((a*e + b*e*x
)/(c + d*x))*a**3*b*c*d**2 + 9*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*c*d*
*2*x - 3*log((a*e + b*e*x)/(c + d*x))*a*b**3*c**3 - 9*log((a*e + b*e*x)/(c
 + d*x))*a*b**3*c**2*d*x - 3*log((a*e + b*e*x)/(c + d*x))*a*b**3*d**3*x**3
 + 3*log((a*e + b*e*x)/(c + d*x))*b**4*c*d**2*x**3 + 3*a**4*c*d**2 + 9*a**
3*b*c*d**2*x + a**3*b*c*d**2 - 3*a**2*b**2*c**3 - 9*a**2*b**2*c**2*d*x + 3
*a**2*b**2*c*d**2*x - 3*a**2*b**2*d**3*x**3 - a*b**3*c**3 - 3*a*b**3*c**2*
d*x + 3*a*b**3*c*d**2*x**3 - a*b**3*d**3*x**3 + b**4*c*d**2*x**3)/(9*a*b**
2*g**4*(a**4*d - a**3*b*c + 3*a**3*b*d*x - 3*a**2*b**2*c*x + 3*a**2*b**2*d
*x**2 - 3*a*b**3*c*x**2 + a*b**3*d*x**3 - b**4*c*x**3))
```

3.18 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 181

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx &= \frac{Bdi^2(c+dx)^3}{9(bc-ad)^2g^5(a+bx)^3}\\ &\quad-\frac{bBi^2(c+dx)^4}{16(bc-ad)^2g^5(a+bx)^4}\\ &\quad+\frac{di^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^2g^5(a+bx)^3}\\ &\quad-\frac{bi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^2g^5(a+bx)^4}\end{aligned}$$

output

```
1/9*B*d*i^2*(d*x+c)^3/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/16*b*B*i^2*(d*x+c)^4/(-
a*d+b*c)^2/g^5/(b*x+a)^4+1/3*d*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(
-a*d+b*c)^2/g^5/(b*x+a)^3-1/4*b*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/
(-a*d+b*c)^2/g^5/(b*x+a)^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 454 vs. $2(181) = 362$.

Time $= 0.42$ (sec) , antiderivative size $= 454$, normalized size of antiderivative $= 2.51$

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{i^2\Big(36Ab^4c^4+9b^4Bc^4-48aAb^3c^3d-16ab^3Bc^3d+12a^4Ad^4+7a^4Bd^4+96Ab^4c^3dx+20b^4Bc^3dx-1$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^5,x]
```

output
```
-1/144*(i^2*(36*A*b^4*c^4 + 9*b^4*B*c^4 - 48*a*A*b^3*c^3*d - 16*a*b^3*B*c^
3*d + 12*a^4*A*d^4 + 7*a^4*B*d^4 + 96*A*b^4*c^3*d*x + 20*b^4*B*c^3*d*x - 1
44*a*A*b^3*c^2*d^2*x - 48*a*b^3*B*c^2*d^2*x + 48*a^3*A*b*d^4*x + 28*a^3*b*
B*d^4*x + 72*A*b^4*c^2*d^2*x^2 + 6*b^4*B*c^2*d^2*x^2 - 144*a*A*b^3*c*d^3*x
^2 - 48*a*b^3*B*c*d^3*x^2 + 72*a^2*A*b^2*d^4*x^2 + 42*a^2*b^2*B*d^4*x^2 -
12*b^4*B*c*d^3*x^3 + 12*a*b^3*B*d^4*x^3 - 12*B*d^4*(a + b*x)^4*Log[a + b*x
] + 12*B*(b*c - a*d)^2*(a^2*d^2 + 2*a*b*d*(c + 2*d*x) + b^2*(3*c^2 + 8*c*d
*x + 6*d^2*x^2))*Log[(e*(a + b*x))/(c + d*x)] + 12*a^4*B*d^4*Log[c + d*x]
+ 48*a^3*b*B*d^4*x*Log[c + d*x] + 72*a^2*b^2*B*d^4*x^2*Log[c + d*x] + 48*a
*b^3*B*d^4*x^3*Log[c + d*x] + 12*b^4*B*d^4*x^4*Log[c + d*x]))/(b^3*(b*c -
a*d)^2*g^5*(a + b*x)^4)
```

Rubi [A] (verified)

Time $= 0.37$ (sec) , antiderivative size $= 138$, normalized size of antiderivative $= 0.76$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used $= \{2962, 2772, 27, 53, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^5}\,dx$$

$$\downarrow 2962$$

$$\frac{i^2\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^2}$$

$$\downarrow 2772$$

$$\frac{i^2\left(-B\int -\frac{(c+dx)^5\left(3b-\frac{4d(a+bx)}{c+dx}\right)}{12(a+bx)^5}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}\right)}{g^5(bc-ad)^2}$$

$$\downarrow 27$$

$$\frac{i^2\left(\frac{1}{12}B\int \frac{(c+dx)^5\left(3b-\frac{4d(a+bx)}{c+dx}\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}\right)}{g^5(bc-ad)^2}$$

$$\downarrow 53$$

$$\frac{i^2\left(\frac{1}{12}B\int \left(\frac{3b(c+dx)^5}{(a+bx)^5}-\frac{4d(c+dx)^4}{(a+bx)^4}\right)d\frac{a+bx}{c+dx}-\frac{b(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}\right)}{g^5(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{i^2\left(-\frac{b(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{1}{12}B\left(\frac{4d(c+dx)^3}{3(a+bx)^3}-\frac{3b(c+dx)^4}{4(a+bx)^4}\right)\right)}{g^5(bc-ad)^2}$$

input

```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^5
,x]
```

output

```
(i^2*((B*((4*d*(c + d*x)^3)/(3*(a + b*x)^3) - (3*b*(c + d*x)^4)/(4*(a + b*
x)^4)))/12 + (d*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(a +
b*x)^3) - (b*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*(a + b*x
)^4)))/((b*c - a*d)^2*g^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.76

method	result
parts	$\frac{i^2A\left(-\frac{d^2}{2b^3(bx+a)^2}+\frac{2d(da-bc)}{3b^3(bx+a)^3}-\frac{a^2d^2-2acdb+c^2b^2}{4b^3(bx+a)^4}\right)}{g^5}-\frac{i^2B(da-bc)^3e^3\left(\frac{d^5\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}\right.\right.}{(da-bc)^5}}{g^5d}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{i^2d^2e^3Ab}{4(da-bc)^3g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{i^2d^3e^2A}{3(da-bc)^3g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{i^2d^2e^3Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{}{16\left(\frac{be}{d}+\right.\right.}\right.}{(da-bc)^3g^5}}{d^2}$
default	$-\frac{e(da-bc)\left(\frac{i^2d^2e^3Ab}{4(da-bc)^3g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{i^2d^3e^2A}{3(da-bc)^3g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{i^2d^2e^3Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{}{16\left(\frac{be}{d}+\right.\right.}\right.}{(da-bc)^3g^5}}{d^2}$
orering	$-\frac{(bx+a)\left(-48b^4d^4x^4-150ab^3d^4x^3-42b^4cd^3x^3-450ab^3cd^3x^2+162b^4c^2d^2x^2+35a^3bd^4x-105a^2b^2cd^3x-345ab^3c^2\right.}{144b^3(a^2d^2-2acdb+c^2b^2)(d}$
risch	$-\frac{i^2B\left(6d^2b^2x^2+4abd^2x+8b^2cdx+a^2d^2+2acdb+3c^2b^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{12(bx+a)^4g^5b^3}-\frac{(6Bb^4c^2d^2x^2+12Bab^3d^4x^3-12Bb^4cd^3x^3}{}$
norman	$\frac{\left(12Aac^2di^2-12Abc^3i^2+4Bac^2di^2-3Bbc^3i^2\right)x}{12ga(da-bc)}+\frac{\left(24Aa^2cd^2i^2+12Aabc^2di^2-36Ab^2c^3i^2+8Ba^2cd^2i^2+7Babc^2di^2-9Bb^2c^3\right.}{24ga^2(da-bc)}$
parallelrisch	$\frac{12Bx^4\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6bcd^4i^2-72Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6b^3cd^2i^2-96Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6b^2c^4di^2+36Ax^4a^2b^5c^5i^2+9Bx^4}{}$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x,method=_RETU
RNVERBOSE)
```

output
```
i^2*A/g^5*(-1/2*d^2/b^3/(b*x+a)^2+2/3*d*(a*d-b*c)/b^3/(b*x+a)^3-1/4*(a^2*d
^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^4)-i^2*B/g^5/d^4*(a*d-b*c)^3*e^3*(d^5/(a
*d-b*c)^5*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-d^4/(a*d-b*c)^5*b*e*(-1/4/(b*e/
d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*
d-b*c)*e/d/(d*x+c))^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(173) = 346$.

Time $= 0.09$ (sec) , antiderivative size $= 510$, normalized size of antiderivative $= 2.82$

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{12\left(Bb^4cd^3-Bab^3d^4\right)i^2x^3-6\left(\left(12\,A+B\right)b^4c^2d^2-8\left(3\,A+B\right)ab^3cd^3+\left(12\,A+7\,B\right)a^2b^2d^4\right)i^2x^2-4\left(\left(2\right.\right.}{144\left(\left(b^9c\right.\right.}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algo
rithm="fricas")
```

output
```
1/144*(12*(B*b^4*c*d^3 - B*a*b^3*d^4)*i^2*x^3 - 6*((12*A + B)*b^4*c^2*d^2
- 8*(3*A + B)*a*b^3*c*d^3 + (12*A + 7*B)*a^2*b^2*d^4)*i^2*x^2 - 4*((24*A +
 5*B)*b^4*c^3*d - 12*(3*A + B)*a*b^3*c^2*d^2 + (12*A + 7*B)*a^3*b*d^4)*i^2
*x - (9*(4*A + B)*b^4*c^4 - 16*(3*A + B)*a*b^3*c^3*d + (12*A + 7*B)*a^4*d^
4)*i^2 + 12*(B*b^4*d^4*i^2*x^4 + 4*B*a*b^3*d^4*i^2*x^3 - 6*(B*b^4*c^2*d^2
- 2*B*a*b^3*c*d^3)*i^2*x^2 - 4*(2*B*b^4*c^3*d - 3*B*a*b^3*c^2*d^2)*i^2*x -
 (3*B*b^4*c^4 - 4*B*a*b^3*c^3*d)*i^2)*log((b*e*x + a*e)/(d*x + c)))/((b^9*
c^2 - 2*a*b^8*c*d + a^2*b^7*d^2)*g^5*x^4 + 4*(a*b^8*c^2 - 2*a^2*b^7*c*d +
a^3*b^6*d^2)*g^5*x^3 + 6*(a^2*b^7*c^2 - 2*a^3*b^6*c*d + a^4*b^5*d^2)*g^5*x
^2 + 4*(a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*g^5*x + (a^4*b^5*c^2 -
2*a^5*b^4*c*d + a^6*b^3*d^2)*g^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(165) = 330$.

Time = 18.93 (sec) , antiderivative size = 928, normalized size of antiderivative = 5.13

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**5,x)
```

output

```
-B*d**4*i**2*log(x + (-B*a**3*d**7*i**2/(a*d - b*c)**2 + 3*B*a**2*b*c*d**6
*i**2/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**5*i**2/(a*d - b*c)**2 + B*a*d**5
*i**2 + B*b**3*c**3*d**4*i**2/(a*d - b*c)**2 + B*b*c*d**4*i**2)/(2*B*b*d**
5*i**2))/(12*b**3*g**5*(a*d - b*c)**2) + B*d**4*i**2*log(x + (B*a**3*d**7*
i**2/(a*d - b*c)**2 - 3*B*a**2*b*c*d**6*i**2/(a*d - b*c)**2 + 3*B*a*b**2*c
**2*d**5*i**2/(a*d - b*c)**2 + B*a*d**5*i**2 - B*b**3*c**3*d**4*i**2/(a*d
- b*c)**2 + B*b*c*d**4*i**2)/(2*B*b*d**5*i**2))/(12*b**3*g**5*(a*d - b*c)*
*2) + (-12*A*a**3*d**3*i**2 - 12*A*a**2*b*c*d**2*i**2 - 12*A*a*b**2*c**2*d
*i**2 + 36*A*b**3*c**3*i**2 - 7*B*a**3*d**3*i**2 - 7*B*a**2*b*c*d**2*i**2
- 7*B*a*b**2*c**2*d*i**2 + 9*B*b**3*c**3*i**2 - 12*B*b**3*d**3*i**2*x**3 +
 x**2*(-72*A*a*b**2*d**3*i**2 + 72*A*b**3*c*d**2*i**2 - 42*B*a*b**2*d**3*i
**2 + 6*B*b**3*c*d**2*i**2) + x*(-48*A*a**2*b*d**3*i**2 - 48*A*a*b**2*c*d*
*2*i**2 + 96*A*b**3*c**2*d*i**2 - 28*B*a**2*b*d**3*i**2 - 28*B*a*b**2*c*d*
*2*i**2 + 20*B*b**3*c**2*d*i**2))/(144*a**5*b**3*d*g**5 - 144*a**4*b**4*c*
g**5 + x**4*(144*a*b**7*d*g**5 - 144*b**8*c*g**5) + x**3*(576*a**2*b**6*d*
g**5 - 576*a*b**7*c*g**5) + x**2*(864*a**3*b**5*d*g**5 - 864*a**2*b**6*c*g
**5) + x*(576*a**4*b**4*d*g**5 - 576*a**3*b**5*c*g**5)) + (-B*a**2*d**2*i*
*2 - 2*B*a*b*c*d*i**2 - 4*B*a*b*d**2*i**2*x - 3*B*b**2*c**2*i**2 - 8*B*b**
2*c*d*i**2*x - 6*B*b**2*d**2*i**2*x**2)*log(e*(a + b*x)/(c + d*x))/(12*a**
4*b**3*g**5 + 48*a**3*b**4*g**5*x + 72*a**2*b**5*g**5*x**2 + 48*a*b**6*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2218 vs. $2(173) = 346$.

Time $= 0.14$ (sec) , antiderivative size $= 2218$, normalized size of antiderivative $= 12.25$

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algo
rithm="maxima")
```

output
```
-1/144*B*d^2*i^2*(12*(6*b^2*x^2 + 4*a*b*x + a^2)*log(b*e*x/(d*x + c) + a*e
/(d*x + c))/(b^7*g^5*x^4 + 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*a^3*b^4
*g^5*x + a^4*b^3*g^5) + (13*a^2*b^3*c^3 - 75*a^3*b^2*c^2*d + 33*a^4*b*c*d^
2 - 7*a^5*d^3 - 12*(6*b^5*c^2*d - 4*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3 + 6*(6*
b^5*c^3 - 46*a*b^4*c^2*d + 29*a^2*b^3*c*d^2 - 7*a^3*b^2*d^3)*x^2 + 4*(10*a
*b^4*c^3 - 63*a^2*b^3*c^2*d + 33*a^3*b^2*c*d^2 - 7*a^4*b*d^3)*x)/((b^10*c^
3 - 3*a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*g^5*x^4 + 4*(a*b^9*c^3
- 3*a^2*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*g^5*x^3 + 6*(a^2*b^8*c^
3 - 3*a^3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*g^5*x^2 + 4*(a^3*b^7*
c^3 - 3*a^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*g^5*x + (a^4*b^6*c^
3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3)*g^5) - 12*(6*b^2*c^2*
d^2 - 4*a*b*c*d^3 + a^2*d^4)*log(b*x + a)/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^
2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^5) + 12*(6*b^2*c^2*d^2 -
4*a*b*c*d^3 + a^2*d^4)*log(d*x + c)/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*
c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^5)) - 1/72*B*c*d*i^2*(12*(4*b*x
 + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^6*g^5*x^4 + 4*a*b^5*g^5*x^3
+ 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5) + (7*a*b^3*c^3 - 33*a
^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 13*a^4*d^3 + 12*(4*b^4*c*d^2 - a*b^3*d^3)*
x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*x^2 + 4*(4*b^4*c^3
- 21*a*b^3*c^2*d + 57*a^2*b^2*c*d^2 - 13*a^3*b*d^3)*x)/((b^9*c^3 - 3*a*...
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.56

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{1}{144}\left(\frac{12\left(3\,Bbe^5i^2-\frac{4\,(bex+ae)Bde^4i^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^4bcg^5}{(dx+c)^4}-\frac{(bex+ae)^4adg^5}{(dx+c)^4}}+\frac{36\,Abe^5i^2+9\,Bbe^5i^2-\frac{48\,(bex+ae)Ade^4i^2}{dx+c}-\frac{16\,(bex+a}{dx}}{\frac{(bex+ae)^4bcg^5}{(dx+c)^4}-\frac{(bex+ae)^4adg^5}{(dx+c)^4}}\right.$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algo
rithm="giac")
```

output
```
-1/144*(12*(3*B*b*e^5*i^2 - 4*(b*e*x + a*e)*B*d*e^4*i^2/(d*x + c))*log((b*
e*x + a*e)/(d*x + c))/((b*e*x + a*e)^4*b*c*g^5/(d*x + c)^4 - (b*e*x + a*e)
^4*a*d*g^5/(d*x + c)^4) + (36*A*b*e^5*i^2 + 9*B*b*e^5*i^2 - 48*(b*e*x + a*
e)*A*d*e^4*i^2/(d*x + c) - 16*(b*e*x + a*e)*B*d*e^4*i^2/(d*x + c))/((b*e*x
 + a*e)^4*b*c*g^5/(d*x + c)^4 - (b*e*x + a*e)^4*a*d*g^5/(d*x + c)^4))*(b*c
/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 28.89 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.57

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{\frac{12\,A\,a^3\,d^3\,i^2-36\,A\,b^3\,c^3\,i^2+7\,B\,a^3\,d^3\,i^2-9\,B\,b^3\,c^3\,i^2+12\,A\,a\,b^2\,c^2\,d\,i^2+12\,A\,a^2\,b\,c\,d^2\,i^2+7\,B\,a\,b^2\,c^2\,d\,i^2+7\,B\,a^2\,b\,c\,d^2\,i^2}{12\,(a\,d-b\,c)}+\frac{x^2\,(12\,A\,a\,b^2\,d^3}{}}{12\,a^4\,b^3\,g^5+48\,a^3\,b^4\,g^5\,x}$$

$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(a\left(\frac{B\,a\,d^2\,i^2}{12\,b^4\,g^5}+\frac{B\,c\,d\,i^2}{6\,b^3\,g^5}\right)+x\left(b\left(\frac{B\,a\,d^2\,i^2}{12\,b^4\,g^5}+\frac{B\,c\,d\,i^2}{6\,b^3\,g^5}\right)+\frac{B\,a\,d^2\,i^2}{4\,b^3\,g^5}+\frac{B\,c\,d\,i^2}{2\,b^2\,g^5}\right)+\frac{B\,c^2\,i^2}{4\,b^2\,g^5}+\frac{B\,d^2\,i^2\,x^2}{2\,b^2\,g^5}\right)}{4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3}$$

$$-\frac{B\,d^4\,i^2\,\mathrm{atanh}\left(\frac{12\,b^5\,c^2\,g^5-12\,a^2\,b^3\,d^2\,g^5}{12\,b^3\,g^5\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{6\,b^3\,g^5\,(a\,d-b\,c)^2}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^5
,x)
```

output

```
- ((12*A*a^3*d^3*i^2 - 36*A*b^3*c^3*i^2 + 7*B*a^3*d^3*i^2 - 9*B*b^3*c^3*i^
2 + 12*A*a*b^2*c^2*d*i^2 + 12*A*a^2*b*c*d^2*i^2 + 7*B*a*b^2*c^2*d*i^2 + 7*
B*a^2*b*c*d^2*i^2)/(12*(a*d - b*c)) + (x^2*(12*A*a*b^2*d^3*i^2 + 7*B*a*b^2
*d^3*i^2 - 12*A*b^3*c*d^2*i^2 - B*b^3*c*d^2*i^2))/(2*(a*d - b*c)) + (x*(12
*A*a^2*b*d^3*i^2 + 7*B*a^2*b*d^3*i^2 - 24*A*b^3*c^2*d*i^2 - 5*B*b^3*c^2*d*
i^2 + 12*A*a*b^2*c*d^2*i^2 + 7*B*a*b^2*c*d^2*i^2))/(3*(a*d - b*c)) + (B*b^
3*d^3*i^2*x^3)/(a*d - b*c))/(12*a^4*b^3*g^5 + 12*b^7*g^5*x^4 + 48*a^3*b^4*
g^5*x + 48*a*b^6*g^5*x^3 + 72*a^2*b^5*g^5*x^2) - (log((e*(a + b*x))/(c + d
*x))*(a*((B*a*d^2*i^2)/(12*b^4*g^5) + (B*c*d*i^2)/(6*b^3*g^5)) + x*(b*((B*
a*d^2*i^2)/(12*b^4*g^5) + (B*c*d*i^2)/(6*b^3*g^5)) + (B*a*d^2*i^2)/(4*b^3*
g^5) + (B*c*d*i^2)/(2*b^2*g^5)) + (B*c^2*i^2)/(4*b^2*g^5) + (B*d^2*i^2*x^2
)/(2*b^2*g^5)))/(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3) -
(B*d^4*i^2*atanh((12*b^5*c^2*g^5 - 12*a^2*b^3*d^2*g^5)/(12*b^3*g^5*(a*d -
b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(6*b^3*g^5*(a*d - b*c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1274, normalized size of antiderivative = 7.04

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x)
```

output

```
( - 24*log(a + b*x)*a**5*b**2*c*d**3 + 12*log(a + b*x)*a**4*b**3*c**2*d**2
 - 96*log(a + b*x)*a**4*b**3*c*d**3*x + 48*log(a + b*x)*a**3*b**4*c**2*d**
2*x - 144*log(a + b*x)*a**3*b**4*c*d**3*x**2 + 72*log(a + b*x)*a**2*b**5*c
**2*d**2*x**2 - 96*log(a + b*x)*a**2*b**5*c*d**3*x**3 + 48*log(a + b*x)*a*
b**6*c**2*d**2*x**3 - 24*log(a + b*x)*a*b**6*c*d**3*x**4 + 12*log(a + b*x)
*b**7*c**2*d**2*x**4 + 24*log(c + d*x)*a**5*b**2*c*d**3 - 12*log(c + d*x)*
a**4*b**3*c**2*d**2 + 96*log(c + d*x)*a**4*b**3*c*d**3*x - 48*log(c + d*x)
*a**3*b**4*c**2*d**2*x + 144*log(c + d*x)*a**3*b**4*c*d**3*x**2 - 72*log(c
 + d*x)*a**2*b**5*c**2*d**2*x**2 + 96*log(c + d*x)*a**2*b**5*c*d**3*x**3 -
 48*log(c + d*x)*a*b**6*c**2*d**2*x**3 + 24*log(c + d*x)*a*b**6*c*d**3*x**
4 - 12*log(c + d*x)*b**7*c**2*d**2*x**4 + 24*log((a*e + b*e*x)/(c + d*x))*
a**5*b**2*c*d**3 - 12*log((a*e + b*e*x)/(c + d*x))*a**4*b**3*c**2*d**2 + 9
6*log((a*e + b*e*x)/(c + d*x))*a**4*b**3*c*d**3*x - 48*log((a*e + b*e*x)/(
c + d*x))*a**3*b**4*c**3*d - 192*log((a*e + b*e*x)/(c + d*x))*a**3*b**4*c*
*2*d**2*x - 48*log((a*e + b*e*x)/(c + d*x))*a**3*b**4*d**4*x**3 + 36*log((
a*e + b*e*x)/(c + d*x))*a**2*b**5*c**4 + 96*log((a*e + b*e*x)/(c + d*x))*a
**2*b**5*c**3*d*x + 96*log((a*e + b*e*x)/(c + d*x))*a**2*b**5*c*d**3*x**3
- 12*log((a*e + b*e*x)/(c + d*x))*a**2*b**5*d**4*x**4 - 48*log((a*e + b*e*
x)/(c + d*x))*a*b**6*c**2*d**2*x**3 + 24*log((a*e + b*e*x)/(c + d*x))*a*b*
*6*c*d**3*x**4 - 12*log((a*e + b*e*x)/(c + d*x))*b**7*c**2*d**2*x**4 + ...
```

3.19 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 281

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = &-\frac{Bd^2i^2(c+dx)^3}{9(bc-ad)^3g^6(a+bx)^3}\\ &+\frac{bBdi^2(c+dx)^4}{8(bc-ad)^3g^6(a+bx)^4}\\ &-\frac{b^2Bi^2(c+dx)^5}{25(bc-ad)^3g^6(a+bx)^5}\\ &-\frac{d^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^3g^6(a+bx)^3}\\ &+\frac{bdi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^6(a+bx)^4}\\ &-\frac{b^2i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^3g^6(a+bx)^5}\end{aligned}$$

output

```
-1/9*B*d^2*i^2*(d*x+c)^3/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/8*b*B*d*i^2*(d*x+c)^
4/(-a*d+b*c)^3/g^6/(b*x+a)^4-1/25*b^2*B*i^2*(d*x+c)^5/(-a*d+b*c)^3/g^6/(b*
x+a)^5-1/3*d^2*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^6/
(b*x+a)^3+1/2*b*d*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g
^6/(b*x+a)^4-1/5*b^2*i^2*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^
3/g^6/(b*x+a)^5
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.22

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$$

$$= \frac{i^2\left(-\frac{360Ab^2c^2}{(a+bx)^5}-\frac{72b^2Bc^2}{(a+bx)^5}+\frac{720aAbcd}{(a+bx)^5}+\frac{144abBcd}{(a+bx)^5}-\frac{360a^2Ad^2}{(a+bx)^5}-\frac{72a^2Bd^2}{(a+bx)^5}-\frac{900Abcd}{(a+bx)^4}-\frac{135bBcd}{(a+bx)^4}+\frac{900aAd^2}{(a+bx)^4}+\frac{135aBd^2}{(a+bx)^4}\right.}{}$$

input

```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^6,x]
```

output

```
(i^2*((-360*A*b^2*c^2)/(a + b*x)^5 - (72*b^2*B*c^2)/(a + b*x)^5 + (720*a*A
*b*c*d)/(a + b*x)^5 + (144*a*b*B*c*d)/(a + b*x)^5 - (360*a^2*A*d^2)/(a + b
*x)^5 - (72*a^2*B*d^2)/(a + b*x)^5 - (900*A*b*c*d)/(a + b*x)^4 - (135*b*B*
c*d)/(a + b*x)^4 + (900*a*A*d^2)/(a + b*x)^4 + (135*a*B*d^2)/(a + b*x)^4 -
 (600*A*d^2)/(a + b*x)^3 - (20*B*d^2)/(a + b*x)^3 + (30*B*d^3)/((b*c - a*d
)*(a + b*x)^2) - (60*B*d^4)/((b*c - a*d)^2*(a + b*x)) - (60*B*d^5*Log[a +
b*x])/(b*c - a*d)^3 - (60*B*(a^2*d^2 + a*b*d*(3*c + 5*d*x) + b^2*(6*c^2 +
15*c*d*x + 10*d^2*x^2))*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^5 + (60*B*
d^5*Log[c + d*x])/(b*c - a*d)^3))/(1800*b^3*g^6)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2772, 27, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^6}\,dx$$

$$\downarrow 2962$$

$$\frac{i^2\int \frac{(c+dx)^6\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^6}d\frac{a+bx}{c+dx}}{g^6(bc-ad)^3}$$

$$\downarrow 2772$$

$$\frac{i^2\left(-B\int -\frac{(c+dx)^6\left(6b^2-\frac{15d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{30(a+bx)^6}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\right.}{g^6(bc-ad)^3}$$

$$\downarrow 27$$

$$\frac{i^2\left(\frac{1}{30}B\int \frac{(c+dx)^6\left(6b^2-\frac{15d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^6}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\right.}{g^6(bc-ad)^3}$$

$$\downarrow 1140$$

$$\frac{i^2\left(\frac{1}{30}B\int \left(\frac{6b^2(c+dx)^6}{(a+bx)^6}-\frac{15bd(c+dx)^5}{(a+bx)^5}+\frac{10d^2(c+dx)^4}{(a+bx)^4}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{g^6(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{i^2\left(-\frac{b^2(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{bd(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^4}+\frac{1}{30}B\left(-\frac{6b^2(c+dx)^5}{5(a+bx)^5}-\right.}{g^6(bc-ad)^3}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^6
,x]
```

output
```
(i^2*((B*((-10*d^2*(c + d*x)^3)/(3*(a + b*x)^3) + (15*b*d*(c + d*x)^4)/(4*
(a + b*x)^4) - (6*b^2*(c + d*x)^5)/(5*(a + b*x)^5)))/30 - (d^2*(c + d*x)^3
*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(a + b*x)^3) + (b*d*(c + d*x)^4*
(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^4) - (b^2*(c + d*x)^5*(
A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*(a + b*x)^5)))/((b*c - a*d)^3*g^6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.53

method	result
parts	$\frac{i^2A\left(-\frac{a^2d^2-2acdb+c^2b^2}{5b^3(bx+a)^5}+\frac{d(da-bc)}{2b^3(bx+a)^4}-\frac{d^2}{3b^3(bx+a)^3}\right)}{g^6}-\frac{i^2B(da-bc)^3e^3\left(\frac{d^6\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}\right.\right.}{(da-bc)^6}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{i^2d^2e^4Ab^2}{5(da-bc)^4g^6\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^5}+\frac{i^2d^3e^3Ab}{2(da-bc)^4g^6\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{i^2d^4e^2A}{3(da-bc)^4g^6\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}+\frac{i^2d^2e^4B}{}\right.}{}$
default	$-\frac{e(da-bc)\left(-\frac{i^2d^2e^4Ab^2}{5(da-bc)^4g^6\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^5}+\frac{i^2d^3e^3Ab}{2(da-bc)^4g^6\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{i^2d^4e^2A}{3(da-bc)^4g^6\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}+\frac{i^2d^2e^4B}{}\right.}{}$
risch	$-\frac{i^2B\left(10d^2b^2x^2+5abd^2x+15b^2cdx+a^2d^2+3acdb+6c^2b^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{30(bx+a)^5g^6b^3}-\frac{(-300Ba\,b^4cd^4x^3-600B\,a^2b^3cd^4x^2+15}{}$
parallelrisch	$-\frac{60A\,a^5b^4d^6i^2-360A\,b^9c^5di^2+47B\,a^5b^4d^6i^2-72B\,b^9c^5di^2-200B\,a^2b^7c^3d^3i^2+225Ba\,b^8c^4d^2i^2-300B\,x^4\ln\left(\frac{e(bx+}{dx+}\right.}{}$
norman	Expression too large to display
orering	Expression too large to display

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x,method=_RETU
RNVERBOSE)
```

output
```
i^2*A/g^6*(-1/5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^5+1/2*d*(a*d-b*c)/
b^3/(b*x+a)^4-1/3*d^2/b^3/(b*x+a)^3)-i^2*B/g^6/d^4*(a*d-b*c)^3*e^3*(d^6/(a
*d-b*c)^6*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-2*d^5/(a*d-b*c)^6*b*e*(-1/4/(b*
e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(
a*d-b*c)*e/d/(d*x+c))^4)+d^4/(a*d-b*c)^6*b^2*e^2*(-1/5/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/25/(b*e/d+(a*d-b*c)*e/d/(d*
x+c))^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(269) = 538$.

Time = 0.10 (sec) , antiderivative size = 807, normalized size of antiderivative = 2.87

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algo
rithm="fricas")
```

output
```
-1/1800*(60*(B*b^5*c*d^4 - B*a*b^4*d^5)*i^2*x^4 - 30*(B*b^5*c^2*d^3 - 10*B
*a*b^4*c*d^4 + 9*B*a^2*b^3*d^5)*i^2*x^3 + 10*(2*(30*A + B)*b^5*c^3*d^2 - 1
5*(12*A + B)*a*b^4*c^2*d^3 + 60*(3*A + B)*a^2*b^3*c*d^4 - (60*A + 47*B)*a^
3*b^2*d^5)*i^2*x^2 + 5*(9*(20*A + 3*B)*b^5*c^4*d - 20*(24*A + 5*B)*a*b^4*c
^3*d^2 + 120*(3*A + B)*a^2*b^3*c^2*d^3 - (60*A + 47*B)*a^4*b*d^5)*i^2*x +
(72*(5*A + B)*b^5*c^5 - 225*(4*A + B)*a*b^4*c^4*d + 200*(3*A + B)*a^2*b^3*
c^3*d^2 - (60*A + 47*B)*a^5*d^5)*i^2 + 60*(B*b^5*d^5*i^2*x^5 + 5*B*a*b^4*d
^5*i^2*x^4 + 10*B*a^2*b^3*d^5*i^2*x^3 + 10*(B*b^5*c^3*d^2 - 3*B*a*b^4*c^2*
d^3 + 3*B*a^2*b^3*c*d^4)*i^2*x^2 + 5*(3*B*b^5*c^4*d - 8*B*a*b^4*c^3*d^2 +
6*B*a^2*b^3*c^2*d^3)*i^2*x + (6*B*b^5*c^5 - 15*B*a*b^4*c^4*d + 10*B*a^2*b^
3*c^3*d^2)*i^2)*log((b*e*x + a*e)/(d*x + c)))/((b^11*c^3 - 3*a*b^10*c^2*d
+ 3*a^2*b^9*c*d^2 - a^3*b^8*d^3)*g^6*x^5 + 5*(a*b^10*c^3 - 3*a^2*b^9*c^2*d
 + 3*a^3*b^8*c*d^2 - a^4*b^7*d^3)*g^6*x^4 + 10*(a^2*b^9*c^3 - 3*a^3*b^8*c^
2*d + 3*a^4*b^7*c*d^2 - a^5*b^6*d^3)*g^6*x^3 + 10*(a^3*b^8*c^3 - 3*a^4*b^7
*c^2*d + 3*a^5*b^6*c*d^2 - a^6*b^5*d^3)*g^6*x^2 + 5*(a^4*b^7*c^3 - 3*a^5*b
^6*c^2*d + 3*a^6*b^5*c*d^2 - a^7*b^4*d^3)*g^6*x + (a^5*b^6*c^3 - 3*a^6*b^5
*c^2*d + 3*a^7*b^4*c*d^2 - a^8*b^3*d^3)*g^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1300 vs. $2(258) = 516$.

Time = 41.37 (sec) , antiderivative size = 1300, normalized size of antiderivative = 4.63

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**6,x)
```

output
```
-B*d**5*i**2*log(x + (-B*a**4*d**9*i**2/(a*d - b*c)**3 + 4*B*a**3*b*c*d**8
*i**2/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**7*i**2/(a*d - b*c)**3 + 4*B*a
*b**3*c**3*d**6*i**2/(a*d - b*c)**3 + B*a*d**6*i**2 - B*b**4*c**4*d**5*i**
2/(a*d - b*c)**3 + B*b*c*d**5*i**2)/(2*B*b*d**6*i**2))/(30*b**3*g**6*(a*d
- b*c)**3) + B*d**5*i**2*log(x + (B*a**4*d**9*i**2/(a*d - b*c)**3 - 4*B*a*
*3*b*c*d**8*i**2/(a*d - b*c)**3 + 6*B*a**2*b**2*c**2*d**7*i**2/(a*d - b*c)
**3 - 4*B*a*b**3*c**3*d**6*i**2/(a*d - b*c)**3 + B*a*d**6*i**2 + B*b**4*c*
*4*d**5*i**2/(a*d - b*c)**3 + B*b*c*d**5*i**2)/(2*B*b*d**6*i**2))/(30*b**3
*g**6*(a*d - b*c)**3) + (-60*A*a**4*d**4*i**2 - 60*A*a**3*b*c*d**3*i**2 -
60*A*a**2*b**2*c**2*d**2*i**2 + 540*A*a*b**3*c**3*d*i**2 - 360*A*b**4*c**4
*i**2 - 47*B*a**4*d**4*i**2 - 47*B*a**3*b*c*d**3*i**2 - 47*B*a**2*b**2*c**
2*d**2*i**2 + 153*B*a*b**3*c**3*d*i**2 - 72*B*b**4*c**4*i**2 - 60*B*b**4*d
**4*i**2*x**4 + x**3*(-270*B*a*b**3*d**4*i**2 + 30*B*b**4*c*d**3*i**2) + x
**2*(-600*A*a**2*b**2*d**4*i**2 + 1200*A*a*b**3*c*d**3*i**2 - 600*A*b**4*c
**2*d**2*i**2 - 470*B*a**2*b**2*d**4*i**2 + 130*B*a*b**3*c*d**3*i**2 - 20*
B*b**4*c**2*d**2*i**2) + x*(-300*A*a**3*b*d**4*i**2 - 300*A*a**2*b**2*c*d*
*3*i**2 + 1500*A*a*b**3*c**2*d**2*i**2 - 900*A*b**4*c**3*d*i**2 - 235*B*a*
*3*b*d**4*i**2 - 235*B*a**2*b**2*c*d**3*i**2 + 365*B*a*b**3*c**2*d**2*i**2
 - 135*B*b**4*c**3*d*i**2))/(1800*a**7*b**3*d**2*g**6 - 3600*a**6*b**4*c*d
*g**6 + 1800*a**5*b**5*c**2*g**6 + x**5*(1800*a**2*b**8*d**2*g**6 - 360...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3029 vs. $2(269) = 538$.

Time $= 0.18$ (sec) , antiderivative size $= 3029$, normalized size of antiderivative $= 10.78$

$$\int \frac{(ci + dix)^2 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag + bgx)^6}\, dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algo
rithm="maxima")
```

output

```
-1/1800*B*d^2*i^2*(60*(10*b^2*x^2 + 5*a*b*x + a^2)*log(b*e*x/(d*x + c) + a
*e/(d*x + c))/(b^8*g^6*x^5 + 5*a*b^7*g^6*x^4 + 10*a^2*b^6*g^6*x^3 + 10*a^3
*b^5*g^6*x^2 + 5*a^4*b^4*g^6*x + a^5*b^3*g^6) + (47*a^2*b^4*c^4 - 278*a^3*
b^3*c^3*d + 822*a^4*b^2*c^2*d^2 - 278*a^5*b*c*d^3 + 47*a^6*d^4 + 60*(10*b^
6*c^2*d^2 - 5*a*b^5*c*d^3 + a^2*b^4*d^4)*x^4 - 30*(10*b^6*c^3*d - 95*a*b^5
*c^2*d^2 + 46*a^2*b^4*c*d^3 - 9*a^3*b^3*d^4)*x^3 + 10*(20*b^6*c^4 - 140*a*
b^5*c^3*d + 537*a^2*b^4*c^2*d^2 - 248*a^3*b^3*c*d^3 + 47*a^4*b^2*d^4)*x^2
+ 5*(35*a*b^5*c^4 - 218*a^2*b^4*c^3*d + 702*a^3*b^3*c^2*d^2 - 278*a^4*b^2*
c*d^3 + 47*a^5*b*d^4)*x)/((b^12*c^4 - 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*d^2
- 4*a^3*b^9*c*d^3 + a^4*b^8*d^4)*g^6*x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c^3*
d + 6*a^3*b^9*c^2*d^2 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*g^6*x^4 + 10*(a^2*b
^10*c^4 - 4*a^3*b^9*c^3*d + 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6*
d^4)*g^6*x^3 + 10*(a^3*b^9*c^4 - 4*a^4*b^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a
^6*b^6*c*d^3 + a^7*b^5*d^4)*g^6*x^2 + 5*(a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6
*a^6*b^6*c^2*d^2 - 4*a^7*b^5*c*d^3 + a^8*b^4*d^4)*g^6*x + (a^5*b^7*c^4 - 4
*a^6*b^6*c^3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a^9*b^3*d^4)*g^6) +
 60*(10*b^2*c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5)*log(b*x + a)/((b^8*c^5 - 5*a*
b^7*c^4*d + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 + 5*a^4*b^4*c*d^4 - a^
5*b^3*d^5)*g^6) - 60*(10*b^2*c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5)*log(d*x + c)
/((b^8*c^5 - 5*a*b^7*c^4*d + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 + ...
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.58

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx =$$

$$-\frac{1}{1800}\left(\frac{60\left(6\,Bb^2e^6i^2-\frac{15\,(bex+ae)Bbde^5i^2}{dx+c}+\frac{10\,(bex+ae)^2Bd^2e^4i^2}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^5b^2c^2g^6}{(dx+c)^5}-\frac{2\,(bex+ae)^5abcdg^6}{(dx+c)^5}+\frac{(bex+ae)^5a^2d^2g^6}{(dx+c)^5}}+\frac{360\,Ab^2e^6i^2+72\,Bb^2e^6i^2}{}\right.$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algo
rithm="giac")
```

output

```
-1/1800*(60*(6*B*b^2*e^6*i^2 - 15*(b*e*x + a*e)*B*b*d*e^5*i^2/(d*x + c) +
10*(b*e*x + a*e)^2*B*d^2*e^4*i^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))
/((b*e*x + a*e)^5*b^2*c^2*g^6/(d*x + c)^5 - 2*(b*e*x + a*e)^5*a*b*c*d*g^6/
(d*x + c)^5 + (b*e*x + a*e)^5*a^2*d^2*g^6/(d*x + c)^5) + (360*A*b^2*e^6*i^
2 + 72*B*b^2*e^6*i^2 - 900*(b*e*x + a*e)*A*b*d*e^5*i^2/(d*x + c) - 225*(b*
e*x + a*e)*B*b*d*e^5*i^2/(d*x + c) + 600*(b*e*x + a*e)^2*A*d^2*e^4*i^2/(d*
x + c)^2 + 200*(b*e*x + a*e)^2*B*d^2*e^4*i^2/(d*x + c)^2)/((b*e*x + a*e)^5
*b^2*c^2*g^6/(d*x + c)^5 - 2*(b*e*x + a*e)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*
e*x + a*e)^5*a^2*d^2*g^6/(d*x + c)^5))*(b*c/((b*c*e - a*d*e)*(b*c - a*d))
- a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 29.82 (sec) , antiderivative size = 941, normalized size of antiderivative = 3.35

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$$

$$= \frac{B\,d^5\,i^2\,\operatorname{atanh}\left(\frac{30\,a^3\,b^3\,d^3\,g^6-30\,a^2\,b^4\,c\,d^2\,g^6-30\,a\,b^5\,c^2\,d\,g^6+30\,b^6\,c^3\,g^6}{30\,b^3\,g^6\,(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{15\,b^3\,g^6\,(a\,d-b\,c)^3}$$

$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(a\left(\frac{B\,a\,d^2\,i^2}{30\,b^4\,g^6}+\frac{B\,c\,d\,i^2}{10\,b^3\,g^6}\right)+x\left(b\left(\frac{B\,a\,d^2\,i^2}{30\,b^4\,g^6}+\frac{B\,c\,d\,i^2}{10\,b^3\,g^6}\right)+\frac{2\,B\,a\,d^2\,i^2}{15\,b^3\,g^6}+\frac{2\,B\,c\,d\,i^2}{5\,b^2\,g^6}\right)+\frac{B\,c^2\,i^2}{5\,b^2\,g^6}+\frac{B\,d^2\,i^2\,x^2}{3\,b^2\,g^6}\right.}{5\,a^4\,x+\frac{a^5}{b}+b^4\,x^5+10\,a^3\,b\,x^2+5\,a\,b^3\,x^4+10\,a^2\,b^2\,x^3}$$

$$-\frac{60\,A\,a^4\,d^4\,i^2+360\,A\,b^4\,c^4\,i^2+47\,B\,a^4\,d^4\,i^2+72\,B\,b^4\,c^4\,i^2+60\,A\,a^2\,b^2\,c^2\,d^2\,i^2+47\,B\,a^2\,b^2\,c^2\,d^2\,i^2-540\,A\,a\,b^3\,c^3\,d\,i^2+60\,A\,a^3\,b\,c\,d^3\,i^2-153\,B\,a}{60\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}$$

input

```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^6
,x)
```

output

```
(B*d^5*i^2*atanh((30*b^6*c^3*g^6 + 30*a^3*b^3*d^3*g^6 - 30*a*b^5*c^2*d*g^6
 - 30*a^2*b^4*c*d^2*g^6)/(30*b^3*g^6*(a*d - b*c)^3) + (2*b*d*x*(a^2*d^2 +
b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(15*b^3*g^6*(a*d - b*c)^3) - (log((e
*(a + b*x))/(c + d*x))*(a*((B*a*d^2*i^2)/(30*b^4*g^6) + (B*c*d*i^2)/(10*b^
3*g^6)) + x*(b*((B*a*d^2*i^2)/(30*b^4*g^6) + (B*c*d*i^2)/(10*b^3*g^6)) + (
2*B*a*d^2*i^2)/(15*b^3*g^6) + (2*B*c*d*i^2)/(5*b^2*g^6)) + (B*c^2*i^2)/(5*
b^2*g^6) + (B*d^2*i^2*x^2)/(3*b^2*g^6)))/(5*a^4*x + a^5/b + b^4*x^5 + 10*a
^3*b*x^2 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3) - ((60*A*a^4*d^4*i^2 + 360*A*b^4*
c^4*i^2 + 47*B*a^4*d^4*i^2 + 72*B*b^4*c^4*i^2 + 60*A*a^2*b^2*c^2*d^2*i^2 +
 47*B*a^2*b^2*c^2*d^2*i^2 - 540*A*a*b^3*c^3*d*i^2 + 60*A*a^3*b*c*d^3*i^2 -
 153*B*a*b^3*c^3*d*i^2 + 47*B*a^3*b*c*d^3*i^2)/(60*(a^2*d^2 + b^2*c^2 - 2*
a*b*c*d)) + (x^2*(60*A*a^2*b^2*d^4*i^2 + 47*B*a^2*b^2*d^4*i^2 + 60*A*b^4*c
^2*d^2*i^2 + 2*B*b^4*c^2*d^2*i^2 - 120*A*a*b^3*c*d^3*i^2 - 13*B*a*b^3*c*d^
3*i^2))/(6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(60*A*a^3*b*d^4*i^2 + 47*
B*a^3*b*d^4*i^2 + 180*A*b^4*c^3*d*i^2 + 27*B*b^4*c^3*d*i^2 - 300*A*a*b^3*c
^2*d^2*i^2 + 60*A*a^2*b^2*c*d^3*i^2 - 73*B*a*b^3*c^2*d^2*i^2 + 47*B*a^2*b^
2*c*d^3*i^2))/(12*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d*x^3*(9*B*a*b^3*d^3
*i^2 - B*b^4*c*d^2*i^2))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B*b^4*d^4*
i^2*x^4)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(30*a^5*b^3*g^6 + 30*b^8*g^6*x^5
 + 150*a^4*b^4*g^6*x + 150*a*b^7*g^6*x^4 + 300*a^3*b^5*g^6*x^2 + 300*a^...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1323, normalized size of antiderivative = 4.71

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x)
```

output

```
( - 60*log(a + b*x)*a**6*b*d**5 - 300*log(a + b*x)*a**5*b**2*d**5*x - 600*
log(a + b*x)*a**4*b**3*d**5*x**2 - 600*log(a + b*x)*a**3*b**4*d**5*x**3 -
300*log(a + b*x)*a**2*b**5*d**5*x**4 - 60*log(a + b*x)*a*b**6*d**5*x**5 +
60*log(c + d*x)*a**6*b*d**5 + 300*log(c + d*x)*a**5*b**2*d**5*x + 600*log(
c + d*x)*a**4*b**3*d**5*x**2 + 600*log(c + d*x)*a**3*b**4*d**5*x**3 + 300*
log(c + d*x)*a**2*b**5*d**5*x**4 + 60*log(c + d*x)*a*b**6*d**5*x**5 + 60*l
og((a*e + b*e*x)/(c + d*x))*a**6*b*d**5 + 300*log((a*e + b*e*x)/(c + d*x))
*a**5*b**2*d**5*x + 600*log((a*e + b*e*x)/(c + d*x))*a**4*b**3*d**5*x**2 -
 600*log((a*e + b*e*x)/(c + d*x))*a**3*b**4*c**3*d**2 - 1800*log((a*e + b*
e*x)/(c + d*x))*a**3*b**4*c**2*d**3*x - 1800*log((a*e + b*e*x)/(c + d*x))*
a**3*b**4*c*d**4*x**2 + 900*log((a*e + b*e*x)/(c + d*x))*a**2*b**5*c**4*d
+ 2400*log((a*e + b*e*x)/(c + d*x))*a**2*b**5*c**3*d**2*x + 1800*log((a*e
+ b*e*x)/(c + d*x))*a**2*b**5*c**2*d**3*x**2 - 360*log((a*e + b*e*x)/(c +
d*x))*a*b**6*c**5 - 900*log((a*e + b*e*x)/(c + d*x))*a*b**6*c**4*d*x - 600
*log((a*e + b*e*x)/(c + d*x))*a*b**6*c**3*d**2*x**2 + 60*a**7*d**5 + 300*a
**6*b*d**5*x + 35*a**6*b*d**5 + 12*a**5*b**2*c*d**4 + 600*a**5*b**2*d**5*x
**2 + 175*a**5*b**2*d**5*x - 600*a**4*b**3*c**3*d**2 - 1800*a**4*b**3*c**2
*d**3*x - 1800*a**4*b**3*c*d**4*x**2 + 60*a**4*b**3*c*d**4*x + 350*a**4*b*
*3*d**5*x**2 + 900*a**3*b**4*c**4*d + 2400*a**3*b**4*c**3*d**2*x - 200*a**
3*b**4*c**3*d**2 + 1800*a**3*b**4*c**2*d**3*x**2 - 600*a**3*b**4*c**2*d...
```

3.20 $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 457

$$
\begin{aligned}
&\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx \\
&= \frac{B(bc-ad)^6g^3i^3x}{140b^3d^3}+\frac{B(bc-ad)^5g^3i^3(c+dx)^2}{280b^2d^4}+\frac{B(bc-ad)^4g^3i^3(c+dx)^3}{420bd^4} \\
&\quad -\frac{17B(bc-ad)^3g^3i^3(c+dx)^4}{280d^4}+\frac{bB(bc-ad)^2g^3i^3(c+dx)^5}{14d^4} \\
&\quad -\frac{b^2B(bc-ad)g^3i^3(c+dx)^6}{42d^4}+\frac{B(bc-ad)^7g^3i^3\log\left(\frac{a+bx}{c+dx}\right)}{140b^4d^4} \\
&\quad -\frac{(bc-ad)^3g^3i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^4} \\
&\quad +\frac{3b(bc-ad)^2g^3i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^4} \\
&\quad -\frac{b^2(bc-ad)g^3i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^4} \\
&\quad +\frac{b^3g^3i^3(c+dx)^7\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{7d^4}+\frac{B(bc-ad)^7g^3i^3\log(c+dx)}{140b^4d^4}
\end{aligned}
$$

output

```
1/140*B*(-a*d+b*c)^6*g^3*i^3*x/b^3/d^3+1/280*B*(-a*d+b*c)^5*g^3*i^3*(d*x+c
)^2/b^2/d^4+1/420*B*(-a*d+b*c)^4*g^3*i^3*(d*x+c)^3/b/d^4-17/280*B*(-a*d+b*
c)^3*g^3*i^3*(d*x+c)^4/d^4+1/14*b*B*(-a*d+b*c)^2*g^3*i^3*(d*x+c)^5/d^4-1/4
2*b^2*B*(-a*d+b*c)*g^3*i^3*(d*x+c)^6/d^4+1/140*B*(-a*d+b*c)^7*g^3*i^3*ln((
b*x+a)/(d*x+c))/b^4/d^4-1/4*(-a*d+b*c)^3*g^3*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+
a)/(d*x+c)))/d^4+3/5*b*(-a*d+b*c)^2*g^3*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d
*x+c)))/d^4-1/2*b^2*(-a*d+b*c)*g^3*i^3*(d*x+c)^6*(A+B*ln(e*(b*x+a)/(d*x+c)
))/d^4+1/7*b^3*g^3*i^3*(d*x+c)^7*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4+1/140*B*(
-a*d+b*c)^7*g^3*i^3*ln(d*x+c)/b^4/d^4
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.28

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{g^3i^3\left(\frac{120b^2Bc(bc-ad)^5x}{d^3}-\frac{126bB(bc-ad)^6x}{d^3}+\frac{120abB(-bc+ad)^5x}{d^2}-\frac{60bBc(bc-ad)^4(a+bx)^2}{d^2}+\frac{60aB(bc-ad)^4(a+bx)^2}{d}+\frac{63B(bc-}{}\right.}{}$$

input

```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)]),x]
```

output

```
(g^3*i^3*((120*b^2*B*c*(b*c - a*d)^5*x)/d^3 - (126*b*B*(b*c - a*d)^6*x)/d^
3 + (120*a*b*B*(-(b*c) + a*d)^5*x)/d^2 - (60*b*B*c*(b*c - a*d)^4*(a + b*x)
^2)/d^2 + (60*a*B*(b*c - a*d)^4*(a + b*x)^2)/d + (63*B*(b*c - a*d)^5*(a +
b*x)^2)/d^2 + (40*b*B*c*(b*c - a*d)^3*(a + b*x)^3)/d - (42*B*(b*c - a*d)^4
*(a + b*x)^3)/d + 40*a*B*(-(b*c) + a*d)^3*(a + b*x)^3 - 30*b*B*c*(b*c - a*
d)^2*(a + b*x)^4 + 30*a*B*d*(b*c - a*d)^2*(a + b*x)^4 + 21*B*(-(b*c) + a*d
)^3*(a + b*x)^4 + 24*b*B*c*d*(b*c - a*d)*(a + b*x)^5 - 84*B*d*(b*c - a*d)^
2*(a + b*x)^5 + 24*a*B*d^2*(-(b*c) + a*d)*(a + b*x)^5 - 20*b*B*c*d^2*(a +
b*x)^6 + 20*a*B*d^3*(a + b*x)^6 + 210*(b*c - a*d)^3*(a + b*x)^4*(A + B*Log
[(e*(a + b*x))/(c + d*x)]) + 504*d*(b*c - a*d)^2*(a + b*x)^5*(A + B*Log[(e
*(a + b*x))/(c + d*x)]) + 420*d^2*(b*c - a*d)*(a + b*x)^6*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]) + 120*d^3*(a + b*x)^7*(A + B*Log[(e*(a + b*x))/(c + d
*x)]) - (120*b*B*c*(b*c - a*d)^6*Log[c + d*x])/d^4 + (120*a*B*(b*c - a*d)^
6*Log[c + d*x])/d^3 + (126*B*(b*c - a*d)^7*Log[c + d*x])/d^4))/(840*b^4)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2782, 27, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\downarrow 2962$$

$$g^3i^3(bc-ad)^7\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^8}d\frac{a+bx}{c+dx}$$

$$\downarrow 2782$$

$$g^3i^3(bc-ad)^7\left(-B\int-\frac{(c+dx)\left(b^3-\frac{7d(a+bx)b^2}{c+dx}+\frac{21d^2(a+bx)^2b}{(c+dx)^2}-\frac{35d^3(a+bx)^3}{(c+dx)^3}\right)}{140d^4(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}+\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{7d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{b^2}{}\right.$$

$$\downarrow 27$$

$$g^3i^3(bc-ad)^7\left(\frac{B\int\frac{(c+dx)\left(b^3-\frac{7d(a+bx)b^2}{c+dx}+\frac{21d^2(a+bx)^2b}{(c+dx)^2}-\frac{35d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}}{140d^4}+\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{7d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.$$

$$\downarrow 2123$$

$$g^3i^3(bc-ad)^7\left(\frac{B\int\left(-\frac{20db^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^7}+\frac{50db}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{34d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4b}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3b^2}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2b^3}+\frac{c+}{(a+}\right.}{140d^4}\right.$$

$$\downarrow 2009$$

$$g^3 i^3 (bc -$$

$$ad)^7 \left(\frac{b^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)}{7d^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^7} - \frac{b^2 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)}{2d^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^6} + \frac{3b \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)}{5d^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^5} - \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{4d^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^4}$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x
]
```

output
```
(b*c - a*d)^7*g^3*i^3*((b^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(7*d^4*(
b - (d*(a + b*x))/(c + d*x))^7) - (b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^6) + (3*b*(A + B*Log[(e*(a + b*x))
/(c + d*x)]))/(5*d^4*(b - (d*(a + b*x))/(c + d*x))^5) - (A + B*Log[(e*(a +
 b*x))/(c + d*x)])/(4*d^4*(b - (d*(a + b*x))/(c + d*x))^4) + (B*((-10*b^2)
/(3*(b - (d*(a + b*x))/(c + d*x))^6) + (10*b)/(b - (d*(a + b*x))/(c + d*x)
)^5 - 17/(2*(b - (d*(a + b*x))/(c + d*x))^4) + 1/(3*b*(b - (d*(a + b*x))/(
c + d*x))^3) + 1/(2*b^2*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^3*(b - (d*
(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^4 - Log[b - (d*(a + b*
x))/(c + d*x)]/b^4))/(140*d^4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2782

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1193 vs. $2(433) = 866$.

Time = 2.13 (sec) , antiderivative size = 1194, normalized size of antiderivative = 2.61

method	result	size
risch	Expression too large to display	1194
parallelrisch	Expression too large to display	1958
parts	Expression too large to display	2513
derivativedivides	Expression too large to display	2536
default	Expression too large to display	2536

input

```
int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETU
RNVERBOSE)
```

output

```
9/5*i^3*g^3*d^2*b^2*A*a*c*x^5+i^3*g^3*b^2*A*a*c^3*x^3+7/30*i^3*g^3*d^2*B*a
^3*c*x^3-7/30*i^3*g^3*b^2*B*a*c^3*x^3+3/2*i^3*g^3*d*A*a^3*c^2*x^2+3/2*i^3*
g^3*b*A*a^2*c^3*x^2+1/40*i^3*g^3*d^2/b*B*a^4*c*x^2+3/10*i^3*g^3*d*B*a^3*c^
2*x^2-3/10*i^3*g^3*b*B*a^2*c^3*x^2-1/40*i^3*g^3/d*b^2*B*a*c^4*x^2+i^3*g^3*
A*a^3*c^3*x+9/4*i^3*g^3*d*b^2*A*a*c^2*x^4+7/40*i^3*g^3*d^2*b*B*a^2*c*x^4-7
/40*i^3*g^3*d*b^2*B*a*c^2*x^4+1/4*i^3*g^3/b*B*ln(b*x+a)*a^4*c^3+1/140*i^3*
g^3/d^4*b^3*B*ln(-d*x-c)*c^7-1/140*i^3*g^3*d^3/b^4*B*ln(b*x+a)*a^7+17/280*
i^3*g^3*d^3*B*a^3*x^4+i^3*g^3*d^2*A*a^3*c*x^3+3*i^3*g^3*d*b*A*a^2*c^2*x^3+
9/4*i^3*g^3*d^2*b*A*a^2*c*x^4-1/20*i^3*g^3*d^2/b^2*B*a^5*c*x+3/20*i^3*g^3*
d/b*B*a^4*c^2*x-3/20*i^3*g^3/d*b*B*a^2*c^4*x+1/20*i^3*g^3/d^2*b^2*B*a*c^5*
x+3/20*i^3*g^3/d^2*b*B*ln(-d*x-c)*a^2*c^5-1/20*i^3*g^3/d^3*b^2*B*ln(-d*x-c
)*a*c^6+1/20*i^3*g^3*d^2/b^3*B*ln(b*x+a)*a^6*c-3/20*i^3*g^3*d/b^2*B*ln(b*x
+a)*a^5*c^2+3/5*i^3*g^3*d^3*b*A*a^2*x^5+1/2*i^3*g^3*d^3*b^2*A*a*x^6+1/2*i^
3*g^3*d^2*b^3*A*c*x^6+3/5*i^3*g^3*d*b^3*A*c^2*x^5+1/14*i^3*g^3*d^3*b*B*a^2
*x^5-1/14*i^3*g^3*d*b^3*B*c^2*x^5+1/4*i^3*g^3*d^3*A*a^3*x^4+1/4*i^3*g^3*b^
3*A*c^3*x^4-17/280*i^3*g^3*b^3*B*c^3*x^4+1/420*i^3*g^3*d^3/b*B*a^4*x^3-1/4
20*i^3*g^3/d*b^3*B*c^4*x^3-1/280*i^3*g^3*d^3/b^2*B*a^5*x^2+1/280*i^3*g^3/d
^2*b^3*B*c^5*x^2+1/140*i^3*g^3*d^3/b^3*B*a^6*x-1/140*i^3*g^3/d^3*b^3*B*c^6
*x-1/4*i^3*g^3/d*B*ln(-d*x-c)*a^3*c^4+1/42*i^3*g^3*d^3*b^2*B*a*x^6-1/42*i^
3*g^3*d^2*b^3*B*c*x^6+1/140*i^3*g^3*B*x*(20*b^3*d^3*x^6+70*a*b^2*d^3*x^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(433) = 866$.

Time $= 0.31$ (sec) , antiderivative size $= 912$, normalized size of antiderivative $= 2.00$

$$\int (ag + bgx)^3 (ci + dix)^3 \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="fricas")
```

output

```
1/840*(120*A*b^7*d^7*g^3*i^3*x^7 + 20*((21*A - B)*b^7*c*d^6 + (21*A + B)*a
*b^6*d^7)*g^3*i^3*x^6 + 12*((42*A - 5*B)*b^7*c^2*d^5 + 126*A*a*b^6*c*d^6 +
 (42*A + 5*B)*a^2*b^5*d^7)*g^3*i^3*x^5 + 3*((70*A - 17*B)*b^7*c^3*d^4 + 7*
(90*A - 7*B)*a*b^6*c^2*d^5 + 7*(90*A + 7*B)*a^2*b^5*c*d^6 + (70*A + 17*B)*
a^3*b^4*d^7)*g^3*i^3*x^4 - 2*(B*b^7*c^4*d^3 - 14*(30*A - 7*B)*a*b^6*c^3*d^
4 - 1260*A*a^2*b^5*c^2*d^5 - 14*(30*A + 7*B)*a^3*b^4*c*d^6 - B*a^4*b^3*d^7
)*g^3*i^3*x^3 + 3*(B*b^7*c^5*d^2 - 7*B*a*b^6*c^4*d^3 + 84*(5*A - B)*a^2*b^
5*c^3*d^4 + 84*(5*A + B)*a^3*b^4*c^2*d^5 + 7*B*a^4*b^3*c*d^6 - B*a^5*b^2*d
^7)*g^3*i^3*x^2 - 6*(B*b^7*c^6*d - 7*B*a*b^6*c^5*d^2 + 21*B*a^2*b^5*c^4*d^
3 - 140*A*a^3*b^4*c^3*d^4 - 21*B*a^4*b^3*c^2*d^5 + 7*B*a^5*b^2*c*d^6 - B*a
^6*b*d^7)*g^3*i^3*x + 6*(35*B*a^4*b^3*c^3*d^4 - 21*B*a^5*b^2*c^2*d^5 + 7*B
*a^6*b*c*d^6 - B*a^7*d^7)*g^3*i^3*log(b*x + a) + 6*(B*b^7*c^7 - 7*B*a*b^6*
c^6*d + 21*B*a^2*b^5*c^5*d^2 - 35*B*a^3*b^4*c^4*d^3)*g^3*i^3*log(d*x + c)
+ 6*(20*B*b^7*d^7*g^3*i^3*x^7 + 140*B*a^3*b^4*c^3*d^4*g^3*i^3*x + 70*(B*b^
7*c*d^6 + B*a*b^6*d^7)*g^3*i^3*x^6 + 84*(B*b^7*c^2*d^5 + 3*B*a*b^6*c*d^6 +
 B*a^2*b^5*d^7)*g^3*i^3*x^5 + 35*(B*b^7*c^3*d^4 + 9*B*a*b^6*c^2*d^5 + 9*B*
a^2*b^5*c*d^6 + B*a^3*b^4*d^7)*g^3*i^3*x^4 + 140*(B*a*b^6*c^3*d^4 + 3*B*a^
2*b^5*c^2*d^5 + B*a^3*b^4*c*d^6)*g^3*i^3*x^3 + 210*(B*a^2*b^5*c^3*d^4 + B*
a^3*b^4*c^2*d^5)*g^3*i^3*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^4*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2161 vs. $2(427) = 854$.

Time $= 9.20$ (sec) , antiderivative size $= 2161$, normalized size of antiderivative $= 4.73$

$$\int (ag + bgx)^3 (ci + dix)^3 \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output

```
A*b**3*d**3*g**3*i**3*x**7/7 - B*a**4*g**3*i**3*(a**3*d**3 - 7*a**2*b*c*d*
*2 + 21*a*b**2*c**2*d - 35*b**3*c**3)*log(x + (B*a**7*c*d**6*g**3*i**3 - 7
*B*a**6*b*c**2*d**5*g**3*i**3 + 21*B*a**5*b**2*c**3*d**4*g**3*i**3 + B*a**
5*d**4*g**3*i**3*(a**3*d**3 - 7*a**2*b*c*d**2 + 21*a*b**2*c**2*d - 35*b**3
*c**3)/b - 70*B*a**4*b**3*c**4*d**3*g**3*i**3 - B*a**4*c*d**3*g**3*i**3*(a
**3*d**3 - 7*a**2*b*c*d**2 + 21*a*b**2*c**2*d - 35*b**3*c**3) + 21*B*a**3*
b**4*c**5*d**2*g**3*i**3 - 7*B*a**2*b**5*c**6*d*g**3*i**3 + B*a*b**6*c**7*
g**3*i**3)/(B*a**7*d**7*g**3*i**3 - 7*B*a**6*b*c*d**6*g**3*i**3 + 21*B*a**
5*b**2*c**2*d**5*g**3*i**3 - 35*B*a**4*b**3*c**3*d**4*g**3*i**3 - 35*B*a**
3*b**4*c**4*d**3*g**3*i**3 + 21*B*a**2*b**5*c**5*d**2*g**3*i**3 - 7*B*a*b*
*6*c**6*d*g**3*i**3 + B*b**7*c**7*g**3*i**3))/(140*b**4) - B*c**4*g**3*i**
3*(35*a**3*d**3 - 21*a**2*b*c*d**2 + 7*a*b**2*c**2*d - b**3*c**3)*log(x +
(B*a**7*c*d**6*g**3*i**3 - 7*B*a**6*b*c**2*d**5*g**3*i**3 + 21*B*a**5*b**2
*c**3*d**4*g**3*i**3 - 70*B*a**4*b**3*c**4*d**3*g**3*i**3 + 21*B*a**3*b**4
*c**5*d**2*g**3*i**3 - 7*B*a**2*b**5*c**6*d*g**3*i**3 + B*a*b**6*c**7*g**3
*i**3 + B*a*b**3*c**4*g**3*i**3*(35*a**3*d**3 - 21*a**2*b*c*d**2 + 7*a*b**
2*c**2*d - b**3*c**3) - B*b**4*c**5*g**3*i**3*(35*a**3*d**3 - 21*a**2*b*c*
d**2 + 7*a*b**2*c**2*d - b**3*c**3)/d)/(B*a**7*d**7*g**3*i**3 - 7*B*a**6*b
*c*d**6*g**3*i**3 + 21*B*a**5*b**2*c**2*d**5*g**3*i**3 - 35*B*a**4*b**3*c*
*3*d**4*g**3*i**3 - 35*B*a**3*b**4*c**4*d**3*g**3*i**3 + 21*B*a**2*b**5...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2637 vs. $2(433) = 866$.

Time $= 0.10$ (sec) , antiderivative size $= 2637$, normalized size of antiderivative $= 5.77$

$$\int (ag + bgx)^3 (ci + dix)^3 \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="maxima")
```

output

```
1/7*A*b^3*d^3*g^3*i^3*x^7 + 1/2*A*b^3*c*d^2*g^3*i^3*x^6 + 1/2*A*a*b^2*d^3*
g^3*i^3*x^6 + 3/5*A*b^3*c^2*d*g^3*i^3*x^5 + 9/5*A*a*b^2*c*d^2*g^3*i^3*x^5
+ 3/5*A*a^2*b*d^3*g^3*i^3*x^5 + 1/4*A*b^3*c^3*g^3*i^3*x^4 + 9/4*A*a*b^2*c^
2*d*g^3*i^3*x^4 + 9/4*A*a^2*b*c*d^2*g^3*i^3*x^4 + 1/4*A*a^3*d^3*g^3*i^3*x^
4 + A*a*b^2*c^3*g^3*i^3*x^3 + 3*A*a^2*b*c^2*d*g^3*i^3*x^3 + A*a^3*c*d^2*g^
3*i^3*x^3 + 3/2*A*a^2*b*c^3*g^3*i^3*x^2 + 3/2*A*a^3*c^2*d*g^3*i^3*x^2 + (x
*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/
d)*B*a^3*c^3*g^3*i^3 + 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2
*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a^2*b*c^
3*g^3*i^3 + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*
x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^
2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b^2*c^3*g^3*i^3 + 1/24*(6*x^4*log(b*e*x/(d*
x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4
- (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*
c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*c^3*g^3*i^3 + 3/2*(x^2*log(b*e*x/(d*x +
 c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c
- a*d)*x/(b*d))*B*a^3*c^2*d*g^3*i^3 + 3/2*(2*x^3*log(b*e*x/(d*x + c) + a*e
/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d
- a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^2*b*c^2*d*g^3*i^3
 + 3/8*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5520 vs. $2(433) = 866$.

Time $= 0.43$ (sec) , antiderivative size $= 5520$, normalized size of antiderivative $= 12.08$

$$\int (ag + bgx)^3 (ci + dix)^3 \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="giac")
```

output

```
-1/840*(6*(B*b^11*c^8*e^8*g^3*i^3 - 8*B*a*b^10*c^7*d*e^8*g^3*i^3 + 28*B*a^
2*b^9*c^6*d^2*e^8*g^3*i^3 - 56*B*a^3*b^8*c^5*d^3*e^8*g^3*i^3 + 70*B*a^4*b^
7*c^4*d^4*e^8*g^3*i^3 - 56*B*a^5*b^6*c^3*d^5*e^8*g^3*i^3 + 28*B*a^6*b^5*c^
2*d^6*e^8*g^3*i^3 - 8*B*a^7*b^4*c*d^7*e^8*g^3*i^3 + B*a^8*b^3*d^8*e^8*g^3*
i^3 - 7*(b*e*x + a*e)*B*b^10*c^8*d*e^7*g^3*i^3/(d*x + c) + 56*(b*e*x + a*e
)*B*a*b^9*c^7*d^2*e^7*g^3*i^3/(d*x + c) - 196*(b*e*x + a*e)*B*a^2*b^8*c^6*
d^3*e^7*g^3*i^3/(d*x + c) + 392*(b*e*x + a*e)*B*a^3*b^7*c^5*d^4*e^7*g^3*i^
3/(d*x + c) - 490*(b*e*x + a*e)*B*a^4*b^6*c^4*d^5*e^7*g^3*i^3/(d*x + c) +
392*(b*e*x + a*e)*B*a^5*b^5*c^3*d^6*e^7*g^3*i^3/(d*x + c) - 196*(b*e*x + a
*e)*B*a^6*b^4*c^2*d^7*e^7*g^3*i^3/(d*x + c) + 56*(b*e*x + a*e)*B*a^7*b^3*c
*d^8*e^7*g^3*i^3/(d*x + c) - 7*(b*e*x + a*e)*B*a^8*b^2*d^9*e^7*g^3*i^3/(d*
x + c) + 21*(b*e*x + a*e)^2*B*b^9*c^8*d^2*e^6*g^3*i^3/(d*x + c)^2 - 168*(b
*e*x + a*e)^2*B*a*b^8*c^7*d^3*e^6*g^3*i^3/(d*x + c)^2 + 588*(b*e*x + a*e)^
2*B*a^2*b^7*c^6*d^4*e^6*g^3*i^3/(d*x + c)^2 - 1176*(b*e*x + a*e)^2*B*a^3*b
^6*c^5*d^5*e^6*g^3*i^3/(d*x + c)^2 + 1470*(b*e*x + a*e)^2*B*a^4*b^5*c^4*d^
6*e^6*g^3*i^3/(d*x + c)^2 - 1176*(b*e*x + a*e)^2*B*a^5*b^4*c^3*d^7*e^6*g^3
*i^3/(d*x + c)^2 + 588*(b*e*x + a*e)^2*B*a^6*b^3*c^2*d^8*e^6*g^3*i^3/(d*x
+ c)^2 - 168*(b*e*x + a*e)^2*B*a^7*b^2*c*d^9*e^6*g^3*i^3/(d*x + c)^2 + 21*
(b*e*x + a*e)^2*B*a^8*b*d^10*e^6*g^3*i^3/(d*x + c)^2 - 35*(b*e*x + a*e)^3*
B*b^8*c^8*d^3*e^5*g^3*i^3/(d*x + c)^3 + 280*(b*e*x + a*e)^3*B*a*b^7*c^7...
```

Mupad [B] (verification not implemented)

Time = 29.20 (sec) , antiderivative size = 4347, normalized size of antiderivative = 9.51

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input

```
int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x
)
```

output

```
x*(((140*a*d + 140*b*c)*(((140*a*d + 140*b*c)*((a*c*((((b^2*d^2*g^3*i^3*(2
8*A*a*d + 28*A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140
*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 +
12*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^
3*i^3))/(b*d) - ((140*a*d + 140*b*c)*((g^3*i^3*(20*A*a^3*d^3 + 20*A*b^3*c^
3 + 3*B*a^3*d^3 - 3*B*b^3*c^3 + 120*A*a*b^2*c^2*d + 120*A*a^2*b*c*d^2 - 6*
B*a*b^2*c^2*d + 6*B*a^2*b*c*d^2))/5 + ((140*a*d + 140*b*c)*((((b^2*d^2*g^3
*i^3*(28*A*a*d + 28*A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*
d + 140*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(12*A*a^2
*d^2 + 12*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d))/2 + A*a*b^2*c
*d^2*g^3*i^3))/(140*b*d) - (a*c*((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B
*a*d - B*b*c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140))/(b*d)))/(
140*b*d) + (g^3*i^3*(4*A*a^4*d^4 + 4*A*b^4*c^4 + B*a^4*d^4 - B*b^4*c^4 + 1
44*A*a^2*b^2*c^2*d^2 + 64*A*a*b^3*c^3*d + 64*A*a^3*b*c*d^3 - 8*B*a*b^3*c^3
*d + 8*B*a^3*b*c*d^3))/(4*b*d)))/(140*b*d) + (a*c*((g^3*i^3*(20*A*a^3*d^3
+ 20*A*b^3*c^3 + 3*B*a^3*d^3 - 3*B*b^3*c^3 + 120*A*a*b^2*c^2*d + 120*A*a^2
*b*c*d^2 - 6*B*a*b^2*c^2*d + 6*B*a^2*b*c*d^2))/5 + ((140*a*d + 140*b*c)*((
((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^
3*i^3*(140*a*d + 140*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*
i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1347, normalized size of antiderivative = 2.95

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \text{Too large to display}$$

input

```
int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output

```
(g**3*i*(6*log(a + b*x)*a**7*d**7 - 42*log(a + b*x)*a**6*b*c*d**6 + 126*lo
g(a + b*x)*a**5*b**2*c**2*d**5 - 210*log(a + b*x)*a**4*b**3*c**3*d**4 + 21
0*log(a + b*x)*a**3*b**4*c**4*d**3 - 126*log(a + b*x)*a**2*b**5*c**5*d**2
+ 42*log(a + b*x)*a*b**6*c**6*d - 6*log(a + b*x)*b**7*c**7 - 210*log((a*e
+ b*e*x)/(c + d*x))*a**3*b**4*c**4*d**3 - 840*log((a*e + b*e*x)/(c + d*x))
*a**3*b**4*c**3*d**4*x - 1260*log((a*e + b*e*x)/(c + d*x))*a**3*b**4*c**2*
d**5*x**2 - 840*log((a*e + b*e*x)/(c + d*x))*a**3*b**4*c*d**6*x**3 - 210*l
og((a*e + b*e*x)/(c + d*x))*a**3*b**4*d**7*x**4 + 126*log((a*e + b*e*x)/(c
 + d*x))*a**2*b**5*c**5*d**2 - 1260*log((a*e + b*e*x)/(c + d*x))*a**2*b**5
*c**3*d**4*x**2 - 2520*log((a*e + b*e*x)/(c + d*x))*a**2*b**5*c**2*d**5*x*
*3 - 1890*log((a*e + b*e*x)/(c + d*x))*a**2*b**5*c*d**6*x**4 - 504*log((a*
e + b*e*x)/(c + d*x))*a**2*b**5*d**7*x**5 - 42*log((a*e + b*e*x)/(c + d*x)
)*a*b**6*c**6*d - 840*log((a*e + b*e*x)/(c + d*x))*a*b**6*c**3*d**4*x**3 -
 1890*log((a*e + b*e*x)/(c + d*x))*a*b**6*c**2*d**5*x**4 - 1512*log((a*e +
 b*e*x)/(c + d*x))*a*b**6*c*d**6*x**5 - 420*log((a*e + b*e*x)/(c + d*x))*a
*b**6*d**7*x**6 + 6*log((a*e + b*e*x)/(c + d*x))*b**7*c**7 - 210*log((a*e
+ b*e*x)/(c + d*x))*b**7*c**3*d**4*x**4 - 504*log((a*e + b*e*x)/(c + d*x))
*b**7*c**2*d**5*x**5 - 420*log((a*e + b*e*x)/(c + d*x))*b**7*c*d**6*x**6 -
 120*log((a*e + b*e*x)/(c + d*x))*b**7*d**7*x**7 - 6*a**6*b*d**7*x + 42*a*
*5*b**2*c*d**6*x + 3*a**5*b**2*d**7*x**2 - 840*a**4*b**3*c**3*d**4*x - ...
```

3.21 $\int (ag+bgx)^2(ci+dix)^3 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

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Optimal result

Integrand size = 40, antiderivative size = 371

$$
\begin{aligned}
&\int (ag+bgx)^2(ci+dix)^3 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \\
&= -\frac{B(bc-ad)^5 g^2 i^3 x}{60b^3d^2} - \frac{B(bc-ad)^4 g^2 i^3 (c+dx)^2}{120b^2d^3} - \frac{B(bc-ad)^3 g^2 i^3 (c+dx)^3}{180bd^3} \\
&\quad + \frac{7B(bc-ad)^2 g^2 i^3 (c+dx)^4}{120d^3} - \frac{bB(bc-ad) g^2 i^3 (c+dx)^5}{30d^3} \\
&\quad - \frac{B(bc-ad)^6 g^2 i^3 \log\left(\frac{a+bx}{c+dx}\right)}{60b^4d^3} + \frac{(bc-ad)^2 g^2 i^3 (c+dx)^4 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^3} \\
&\quad - \frac{2b(bc-ad) g^2 i^3 (c+dx)^5 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^3} \\
&\quad + \frac{b^2 g^2 i^3 (c+dx)^6 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^3} - \frac{B(bc-ad)^6 g^2 i^3 \log(c+dx)}{60b^4d^3}
\end{aligned}
$$

output

```
-1/60*B*(-a*d+b*c)^5*g^2*i^3*x/b^3/d^2-1/120*B*(-a*d+b*c)^4*g^2*i^3*(d*x+c
)^2/b^2/d^3-1/180*B*(-a*d+b*c)^3*g^2*i^3*(d*x+c)^3/b/d^3+7/120*B*(-a*d+b*c
)^2*g^2*i^3*(d*x+c)^4/d^3-1/30*b*B*(-a*d+b*c)*g^2*i^3*(d*x+c)^5/d^3-1/60*B
*(-a*d+b*c)^6*g^2*i^3*ln((b*x+a)/(d*x+c))/b^4/d^3+1/4*(-a*d+b*c)^2*g^2*i^3
*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3-2/5*b*(-a*d+b*c)*g^2*i^3*(d*x+c
)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3+1/6*b^2*g^2*i^3*(d*x+c)^6*(A+B*ln(e*(b
*x+a)/(d*x+c)))/d^3-1/60*B*(-a*d+b*c)^6*g^2*i^3*ln(d*x+c)/b^4/d^3
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.16

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{g^2i^3\Big(-15B(bc-ad)^3\left(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx)\right.$$

input

```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)]),x]
```

output

```
(g^2*i^3*(-15*B*(b*c - a*d)^3*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(
c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]) + 12*B*(b*c
 - a*d)^2*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2*(c + d*x)^2 + 4*b^
3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c - a*d)^4*Log[a + b
*x]) - B*(b*c - a*d)*(60*b*d*(b*c - a*d)^4*x + 30*b^2*(b*c - a*d)^3*(c + d
*x)^2 + 20*b^3*(b*c - a*d)^2*(c + d*x)^3 + 15*b^4*(b*c - a*d)*(c + d*x)^4
+ 12*b^5*(c + d*x)^5 + 60*(b*c - a*d)^5*Log[a + b*x]) + 90*b^4*(b*c - a*d)
^2*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 144*b^5*(b*c - a*d)*
(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 60*b^6*(c + d*x)^6*(A +
 B*Log[(e*(a + b*x))/(c + d*x)])))/(360*b^4*d^3)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2782, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\downarrow 2962$$

$$g^2i^3(bc-ad)^6\int\frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}$$

$$\downarrow 2782$$

$$g^2i^3(bc-ad)^6\left(-B\int\frac{(c+dx)\left(b^2-\frac{6d(a+bx)b}{c+dx}+\frac{15d^2(a+bx)^2}{(c+dx)^2}\right)}{60d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}+\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{2b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right.}$$

$$\downarrow 27$$

$$g^2i^3(bc-ad)^6\left(-\frac{B\int\frac{(c+dx)\left(b^2-\frac{6d(a+bx)b}{c+dx}+\frac{15d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{60d^3}+\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{2b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{B}{4}\right.$$

$$\downarrow 1195$$

$$g^2i^3(bc-ad)^6\left(-\frac{B\int\left(\frac{d}{b^4\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{14d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{10bd}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6}+\frac{c}{b^4(}\right.\right.}{60d^3}$$

$$\downarrow 2009$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(\frac{b^2 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)}{6d^3 \left(b - \frac{d(a+bx)}{c+dx} \right)^6} - \frac{2b \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)}{5d^3 \left(b - \frac{d(a+bx)}{c+dx} \right)^5} + \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{4d^3 \left(b - \frac{d(a+bx)}{c+dx} \right)^4} - \frac{B \left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^4} - \frac{\log\left(b - \frac{d(a}{c+}\right.\right.}{b^4}$$

input
```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x
]
```

output
```
(b*c - a*d)^6*g^2*i^3*((b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(6*d^3*(
b - (d*(a + b*x))/(c + d*x))^6) - (2*b*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(5*d^3*(b - (d*(a + b*x))/(c + d*x))^5) + (A + B*Log[(e*(a + b*x))/(c +
 d*x)])/(4*d^3*(b - (d*(a + b*x))/(c + d*x))^4) - (B*((2*b)/(b - (d*(a + b
*x))/(c + d*x))^5 - 7/(2*(b - (d*(a + b*x))/(c + d*x))^4) + 1/(3*b*(b - (d
*(a + b*x))/(c + d*x))^3) + 1/(2*b^2*(b - (d*(a + b*x))/(c + d*x))^2) + 1/
(b^3*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^4 - Log[b
 - (d*(a + b*x))/(c + d*x)]/b^4))/(60*d^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2782

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(351) = 702$.

Time = 1.68 (sec) , antiderivative size = 925, normalized size of antiderivative = 2.49

method	result
risch	$\frac{3i^3g^2b^2d^2Acx^5}{5}+\frac{i^3g^2bd^3Bax^5}{30}-\frac{i^3g^2b^2d^2Bcx^5}{30}+\frac{i^3g^2d^3Aa^2x^4}{4}+\frac{3i^3g^2b^2dAc^2x^4}{4}+\frac{i^3g^2bB\ln(-dx-c)a}{10d^2}$
parallelrisch	Expression too large to display
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input

```
int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETU
RNVERBOSE)
```

output

```
3/5*i^3*g^2*b^2*d^2*A*c*x^5+1/30*i^3*g^2*b*d^3*B*a*x^5-1/30*i^3*g^2*b^2*d^
2*B*c*x^5+1/4*i^3*g^2*d^3*A*a^2*x^4+3/4*i^3*g^2*b^2*d*A*c^2*x^4+1/10*i^3*g
^2*b/d^2*B*ln(-d*x-c)*a*c^5+3/2*i^3*g^2*b*d^2*A*a*c*x^4+1/20*i^3*g^2*b*d^2
*B*a*c*x^4+i^3*g^2*d^2*A*a^2*c*x^3+2*i^3*g^2*b*d*A*a*c^2*x^3+13/60*i^3*g^2
*d^2*B*a^2*c*x^3-7/60*i^3*g^2*b*d*B*a*c^2*x^3+3/2*i^3*g^2*d*A*a^2*c^2*x^2+
i^3*g^2*b*A*a*c^3*x^2+2/5*i^3*g^2*b*d^3*A*a*x^5+1/4*i^3*g^2/b*d*B*a^3*c^2*
x-1/12*i^3*g^2*B*a^2*c^3*x-1/10*i^3*g^2*b/d*B*a*c^4*x+1/10*i^3*g^2/b^3*d^2
*B*ln(b*x+a)*a^5*c-1/4*i^3*g^2/b^2*d*B*ln(b*x+a)*a^4*c^2+1/20*i^3*g^2/b*d^
2*B*a^3*c*x^2+1/4*i^3*g^2*d*B*a^2*c^2*x^2-17/60*i^3*g^2*b*B*a*c^3*x^2+i^3*
g^2*A*a^2*c^3*x-1/10*i^3*g^2/b^2*d^2*B*a^4*c*x+7/120*i^3*g^2*d^3*B*a^2*x^4
-13/120*i^3*g^2*b^2*d*B*c^2*x^4+1/3*i^3*g^2*b^2*A*c^3*x^3+1/180*i^3*g^2/b*
d^3*B*a^3*x^3-19/180*i^3*g^2*b^2*B*c^3*x^3-1/120*i^3*g^2/b^2*d^3*B*a^4*x^2
-1/120*i^3*g^2*b^2/d*B*c^4*x^2+1/60*i^3*g^2/b^3*d^3*B*a^5*x+1/60*i^3*g^2*b
^2/d^2*B*c^5*x+1/3*i^3*g^2/b*B*ln(b*x+a)*a^3*c^3-1/4*i^3*g^2/d*B*ln(-d*x-c
)*a^2*c^4-1/60*i^3*g^2/b^4*d^3*B*ln(b*x+a)*a^6-1/60*i^3*g^2*b^2/d^3*B*ln(-
d*x-c)*c^6+1/60*i^3*g^2*B*x*(10*b^2*d^3*x^5+24*a*b*d^3*x^4+36*b^2*c*d^2*x^
4+15*a^2*d^3*x^3+90*a*b*c*d^2*x^3+45*b^2*c^2*d*x^3+60*a^2*c*d^2*x^2+120*a*
b*c^2*d*x^2+20*b^2*c^3*x^2+90*a^2*c^2*d*x+60*a*b*c^3*x+60*a^2*c^3)*ln(e*(b
*x+a)/(d*x+c))+1/6*i^3*g^2*b^2*d^3*A*x^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(351) = 702$.

Time $= 0.19$ (sec) , antiderivative size $= 722$, normalized size of antiderivative $= 1.95$

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{60\,Ab^6d^6g^2i^3x^6+12\left((18\,A-B)b^6cd^5+(12\,A+B)ab^5d^6\right)g^2i^3x^5+3\left((90\,A-13\,B)b^6c^2d^4+6\,(30\,A+$$

input

```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="fricas")
```

output

```
1/360*(60*A*b^6*d^6*g^2*i^3*x^6 + 12*((18*A - B)*b^6*c*d^5 + (12*A + B)*a*
b^5*d^6)*g^2*i^3*x^5 + 3*((90*A - 13*B)*b^6*c^2*d^4 + 6*(30*A + B)*a*b^5*c
*d^5 + (30*A + 7*B)*a^2*b^4*d^6)*g^2*i^3*x^4 + 2*((60*A - 19*B)*b^6*c^3*d^
3 + 3*(120*A - 7*B)*a*b^5*c^2*d^4 + 3*(60*A + 13*B)*a^2*b^4*c*d^5 + B*a^3*
b^3*d^6)*g^2*i^3*x^3 - 3*(B*b^6*c^4*d^2 - 2*(60*A - 17*B)*a*b^5*c^3*d^3 -
30*(6*A + B)*a^2*b^4*c^2*d^4 - 6*B*a^3*b^3*c*d^5 + B*a^4*b^2*d^6)*g^2*i^3*
x^2 + 6*(B*b^6*c^5*d - 6*B*a*b^5*c^4*d^2 + 5*(12*A - B)*a^2*b^4*c^3*d^3 +
15*B*a^3*b^3*c^2*d^4 - 6*B*a^4*b^2*c*d^5 + B*a^5*b*d^6)*g^2*i^3*x + 6*(20*
B*a^3*b^3*c^3*d^3 - 15*B*a^4*b^2*c^2*d^4 + 6*B*a^5*b*c*d^5 - B*a^6*d^6)*g^
2*i^3*log(b*x + a) - 6*(B*b^6*c^6 - 6*B*a*b^5*c^5*d + 15*B*a^2*b^4*c^4*d^2
)*g^2*i^3*log(d*x + c) + 6*(10*B*b^6*d^6*g^2*i^3*x^6 + 60*B*a^2*b^4*c^3*d^
3*g^2*i^3*x + 12*(3*B*b^6*c*d^5 + 2*B*a*b^5*d^6)*g^2*i^3*x^5 + 15*(3*B*b^6
*c^2*d^4 + 6*B*a*b^5*c*d^5 + B*a^2*b^4*d^6)*g^2*i^3*x^4 + 20*(B*b^6*c^3*d^
3 + 6*B*a*b^5*c^2*d^4 + 3*B*a^2*b^4*c*d^5)*g^2*i^3*x^3 + 30*(2*B*a*b^5*c^3
*d^3 + 3*B*a^2*b^4*c^2*d^4)*g^2*i^3*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^
4*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1727 vs. $2(347) = 694$.

Time = 5.84 (sec) , antiderivative size = 1727, normalized size of antiderivative = 4.65

$$\int (ag + bgx)^2 (ci + dix)^3 \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output

```
A*b**2*d**3*g**2*i**3*x**6/6 - B*a**3*g**2*i**3*(a**3*d**3 - 6*a**2*b*c*d*
*2 + 15*a*b**2*c**2*d - 20*b**3*c**3)*log(x + (B*a**6*c*d**5*g**2*i**3 - 6
*B*a**5*b*c**2*d**4*g**2*i**3 + 15*B*a**4*b**2*c**3*d**3*g**2*i**3 + B*a**
4*d**3*g**2*i**3*(a**3*d**3 - 6*a**2*b*c*d**2 + 15*a*b**2*c**2*d - 20*b**3
*c**3)/b - 35*B*a**3*b**3*c**4*d**2*g**2*i**3 - B*a**3*c*d**2*g**2*i**3*(a
**3*d**3 - 6*a**2*b*c*d**2 + 15*a*b**2*c**2*d - 20*b**3*c**3) + 6*B*a**2*b
**4*c**5*d*g**2*i**3 - B*a*b**5*c**6*g**2*i**3)/(B*a**6*d**6*g**2*i**3 - 6
*B*a**5*b*c*d**5*g**2*i**3 + 15*B*a**4*b**2*c**2*d**4*g**2*i**3 - 20*B*a**
3*b**3*c**3*d**3*g**2*i**3 - 15*B*a**2*b**4*c**4*d**2*g**2*i**3 + 6*B*a*b*
*5*c**5*d*g**2*i**3 - B*b**6*c**6*g**2*i**3))/(60*b**4) - B*c**4*g**2*i**3
*(15*a**2*d**2 - 6*a*b*c*d + b**2*c**2)*log(x + (B*a**6*c*d**5*g**2*i**3 -
 6*B*a**5*b*c**2*d**4*g**2*i**3 + 15*B*a**4*b**2*c**3*d**3*g**2*i**3 - 35*
B*a**3*b**3*c**4*d**2*g**2*i**3 + 6*B*a**2*b**4*c**5*d*g**2*i**3 - B*a*b**
5*c**6*g**2*i**3 + B*a*b**3*c**4*g**2*i**3*(15*a**2*d**2 - 6*a*b*c*d + b**
2*c**2) - B*b**4*c**5*g**2*i**3*(15*a**2*d**2 - 6*a*b*c*d + b**2*c**2)/d)/
(B*a**6*d**6*g**2*i**3 - 6*B*a**5*b*c*d**5*g**2*i**3 + 15*B*a**4*b**2*c**2
*d**4*g**2*i**3 - 20*B*a**3*b**3*c**3*d**3*g**2*i**3 - 15*B*a**2*b**4*c**4
*d**2*g**2*i**3 + 6*B*a*b**5*c**5*d*g**2*i**3 - B*b**6*c**6*g**2*i**3))/(6
0*d**3) + x**5*(2*A*a*b*d**3*g**2*i**3/5 + 3*A*b**2*c*d**2*g**2*i**3/5 + B
*a*b*d**3*g**2*i**3/30 - B*b**2*c*d**2*g**2*i**3/30) + x**4*(A*a**2*d**...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1789 vs. $2(351) = 702$.

Time $= 0.08$ (sec) , antiderivative size $= 1789$, normalized size of antiderivative $= 4.82$

$$\int (ag + bgx)^2 (ci + dix)^3 \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="maxima")
```

output

```
1/6*A*b^2*d^3*g^2*i^3*x^6 + 3/5*A*b^2*c*d^2*g^2*i^3*x^5 + 2/5*A*a*b*d^3*g^
2*i^3*x^5 + 3/4*A*b^2*c^2*d*g^2*i^3*x^4 + 3/2*A*a*b*c*d^2*g^2*i^3*x^4 + 1/
4*A*a^2*d^3*g^2*i^3*x^4 + 1/3*A*b^2*c^3*g^2*i^3*x^3 + 2*A*a*b*c^2*d*g^2*i^
3*x^3 + A*a^2*c*d^2*g^2*i^3*x^3 + A*a*b*c^3*g^2*i^3*x^2 + 3/2*A*a^2*c^2*d*
g^2*i^3*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b -
 c*log(d*x + c)/d)*B*a^2*c^3*g^2*i^3 + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x
 + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d)
)*B*a*b*c^3*g^2*i^3 + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*
a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 -
 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*c^3*g^2*i^3 + 3/2*(x^2*log(b*e*
x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2
 - (b*c - a*d)*x/(b*d))*B*a^2*c^2*d*g^2*i^3 + (2*x^3*log(b*e*x/(d*x + c) +
 a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*
c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b*c^2*d*g^2*i
^3 + 1/8*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/
b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2
*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^2*c^2*d*g^2*
i^3 + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)
/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^
2*d^2)*x)/(b^2*d^2))*B*a^2*c*d^2*g^2*i^3 + 1/4*(6*x^4*log(b*e*x/(d*x + ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4110 vs. $2(351) = 702$.

Time $= 0.36$ (sec) , antiderivative size $= 4110$, normalized size of antiderivative $= 11.08$

$$\int (ag + bgx)^2 (ci + dix)^3 \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="giac")
```

output

```
1/360*(6*(B*b^9*c^7*e^7*g^2*i^3 - 7*B*a*b^8*c^6*d*e^7*g^2*i^3 + 21*B*a^2*b
^7*c^5*d^2*e^7*g^2*i^3 - 35*B*a^3*b^6*c^4*d^3*e^7*g^2*i^3 + 35*B*a^4*b^5*c
^3*d^4*e^7*g^2*i^3 - 21*B*a^5*b^4*c^2*d^5*e^7*g^2*i^3 + 7*B*a^6*b^3*c*d^6*
e^7*g^2*i^3 - B*a^7*b^2*d^7*e^7*g^2*i^3 - 6*(b*e*x + a*e)*B*b^8*c^7*d*e^6*
g^2*i^3/(d*x + c) + 42*(b*e*x + a*e)*B*a*b^7*c^6*d^2*e^6*g^2*i^3/(d*x + c)
 - 126*(b*e*x + a*e)*B*a^2*b^6*c^5*d^3*e^6*g^2*i^3/(d*x + c) + 210*(b*e*x
+ a*e)*B*a^3*b^5*c^4*d^4*e^6*g^2*i^3/(d*x + c) - 210*(b*e*x + a*e)*B*a^4*b
^4*c^3*d^5*e^6*g^2*i^3/(d*x + c) + 126*(b*e*x + a*e)*B*a^5*b^3*c^2*d^6*e^6
*g^2*i^3/(d*x + c) - 42*(b*e*x + a*e)*B*a^6*b^2*c*d^7*e^6*g^2*i^3/(d*x + c
) + 6*(b*e*x + a*e)*B*a^7*b*d^8*e^6*g^2*i^3/(d*x + c) + 15*(b*e*x + a*e)^2
*B*b^7*c^7*d^2*e^5*g^2*i^3/(d*x + c)^2 - 105*(b*e*x + a*e)^2*B*a*b^6*c^6*d
^3*e^5*g^2*i^3/(d*x + c)^2 + 315*(b*e*x + a*e)^2*B*a^2*b^5*c^5*d^4*e^5*g^2
*i^3/(d*x + c)^2 - 525*(b*e*x + a*e)^2*B*a^3*b^4*c^4*d^5*e^5*g^2*i^3/(d*x
+ c)^2 + 525*(b*e*x + a*e)^2*B*a^4*b^3*c^3*d^6*e^5*g^2*i^3/(d*x + c)^2 - 3
15*(b*e*x + a*e)^2*B*a^5*b^2*c^2*d^7*e^5*g^2*i^3/(d*x + c)^2 + 105*(b*e*x
+ a*e)^2*B*a^6*b*c*d^8*e^5*g^2*i^3/(d*x + c)^2 - 15*(b*e*x + a*e)^2*B*a^7*
d^9*e^5*g^2*i^3/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/(b^6*d^3*e^6 - 6
*(b*e*x + a*e)*b^5*d^4*e^5/(d*x + c) + 15*(b*e*x + a*e)^2*b^4*d^5*e^4/(d*x
 + c)^2 - 20*(b*e*x + a*e)^3*b^3*d^6*e^3/(d*x + c)^3 + 15*(b*e*x + a*e)^4*
b^2*d^7*e^2/(d*x + c)^4 - 6*(b*e*x + a*e)^5*b*d^8*e/(d*x + c)^5 + (b*e*...
```

Mupad [B] (verification not implemented)

Time = 28.41 (sec) , antiderivative size = 2465, normalized size of antiderivative = 6.64

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input

```
int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x
)
```

output

```
x^3*((g^2*i^3*(4*A*a^3*d^3 + 16*A*b^3*c^3 + B*a^3*d^3 - 3*B*b^3*c^3 + 72*A
*a*b^2*c^2*d + 48*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d + 5*B*a^2*b*c*d^2))/(12*
b) + ((60*a*d + 60*b*c)*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d - B
*b*c))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*
b*d) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2 - 3*B*b^2*c^2
 + 60*A*a*b*c*d + B*a*b*c*d))/5 + A*a*b*c*d^2*g^2*i^3))/(180*b*d) - (a*c*(
(b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d - B*b*c))/6 - (A*b*d^2*g^2*i^3
*(60*a*d + 60*b*c))/60))/(3*b*d)) - x^4*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A
*b*c + B*a*d - B*b*c))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d
 + 60*b*c))/(240*b*d) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*
d^2 - 3*B*b^2*c^2 + 60*A*a*b*c*d + B*a*b*c*d))/20 + (A*a*b*c*d^2*g^2*i^3)/
4) + x^2*((a*c*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d - B*b*c))/6
 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (d
*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2 - 3*B*b^2*c^2 + 60*A*a
*b*c*d + B*a*b*c*d))/5 + A*a*b*c*d^2*g^2*i^3))/(2*b*d) - ((60*a*d + 60*b*c
)*((g^2*i^3*(4*A*a^3*d^3 + 16*A*b^3*c^3 + B*a^3*d^3 - 3*B*b^3*c^3 + 72*A*a
*b^2*c^2*d + 48*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d + 5*B*a^2*b*c*d^2))/(4*b)
+ ((60*a*d + 60*b*c)*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d - B*b*
c))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d
) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2 - 3*B*b^2*c^2...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1040, normalized size of antiderivative = 2.80

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input

```
int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output

```
(g**2*i*(6*log(a + b*x)*a**6*d**6 - 36*log(a + b*x)*a**5*b*c*d**5 + 90*log
(a + b*x)*a**4*b**2*c**2*d**4 - 120*log(a + b*x)*a**3*b**3*c**3*d**3 + 90*
log(a + b*x)*a**2*b**4*c**4*d**2 - 36*log(a + b*x)*a*b**5*c**5*d + 6*log(a
 + b*x)*b**6*c**6 - 90*log((a*e + b*e*x)/(c + d*x))*a**2*b**4*c**4*d**2 -
360*log((a*e + b*e*x)/(c + d*x))*a**2*b**4*c**3*d**3*x - 540*log((a*e + b*
e*x)/(c + d*x))*a**2*b**4*c**2*d**4*x**2 - 360*log((a*e + b*e*x)/(c + d*x)
)*a**2*b**4*c*d**5*x**3 - 90*log((a*e + b*e*x)/(c + d*x))*a**2*b**4*d**6*x
**4 + 36*log((a*e + b*e*x)/(c + d*x))*a*b**5*c**5*d - 360*log((a*e + b*e*x
)/(c + d*x))*a*b**5*c**3*d**3*x**2 - 720*log((a*e + b*e*x)/(c + d*x))*a*b*
*5*c**2*d**4*x**3 - 540*log((a*e + b*e*x)/(c + d*x))*a*b**5*c*d**5*x**4 -
144*log((a*e + b*e*x)/(c + d*x))*a*b**5*d**6*x**5 - 6*log((a*e + b*e*x)/(c
 + d*x))*b**6*c**6 - 120*log((a*e + b*e*x)/(c + d*x))*b**6*c**3*d**3*x**3
- 270*log((a*e + b*e*x)/(c + d*x))*b**6*c**2*d**4*x**4 - 216*log((a*e + b*
e*x)/(c + d*x))*b**6*c*d**5*x**5 - 60*log((a*e + b*e*x)/(c + d*x))*b**6*d*
*6*x**6 - 6*a**5*b*d**6*x + 36*a**4*b**2*c*d**5*x + 3*a**4*b**2*d**6*x**2
- 360*a**3*b**3*c**3*d**3*x - 540*a**3*b**3*c**2*d**4*x**2 - 90*a**3*b**3*
c**2*d**4*x - 360*a**3*b**3*c*d**5*x**3 - 18*a**3*b**3*c*d**5*x**2 - 90*a*
*3*b**3*d**6*x**4 - 2*a**3*b**3*d**6*x**3 - 360*a**2*b**4*c**3*d**3*x**2 +
 30*a**2*b**4*c**3*d**3*x - 720*a**2*b**4*c**2*d**4*x**3 - 90*a**2*b**4*c*
*2*d**4*x**2 - 540*a**2*b**4*c*d**5*x**4 - 78*a**2*b**4*c*d**5*x**3 - 1...
```

3.22 $\int (ag+bgx)(ci+dix)^3 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

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Optimal result

Integrand size = 38, antiderivative size = 271

$$\begin{aligned}
&\int (ag+bgx)(ci+dix)^3 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \\
&= \frac{B(bc-ad)^4 gi^3 x}{20b^3 d} + \frac{B(bc-ad)^3 gi^3 (c+dx)^2}{40b^2 d^2} + \frac{B(bc-ad)^2 gi^3 (c+dx)^3}{60bd^2} \\
&\quad - \frac{B(bc-ad) gi^3 (c+dx)^4}{20d^2} + \frac{B(bc-ad)^5 gi^3 \log\left(\frac{a+bx}{c+dx}\right)}{20b^4 d^2} \\
&\quad - \frac{(bc-ad) gi^3 (c+dx)^4 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2} \\
&\quad + \frac{bgi^3 (c+dx)^5 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^2} + \frac{B(bc-ad)^5 gi^3 \log(c+dx)}{20b^4 d^2}
\end{aligned}$$

output

```
1/20*B*(-a*d+b*c)^4*g*i^3*x/b^3/d+1/40*B*(-a*d+b*c)^3*g*i^3*(d*x+c)^2/b^2/
d^2+1/60*B*(-a*d+b*c)^2*g*i^3*(d*x+c)^3/b/d^2-1/20*B*(-a*d+b*c)*g*i^3*(d*x
+c)^4/d^2+1/20*B*(-a*d+b*c)^5*g*i^3*ln((b*x+a)/(d*x+c))/b^4/d^2-1/4*(-a*d+
b*c)*g*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2+1/5*b*g*i^3*(d*x+c)^5
*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2+1/20*B*(-a*d+b*c)^5*g*i^3*ln(d*x+c)/b^4/d
^2
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.96

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{gi^3\left(\frac{5B(bc-ad)^2(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx))}{b^4}-\frac{2B(bc-ad)(12bd(bc-ad)^3x+6b^2(bc-ad)^2(c+\right.}{}$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)
]),x]
```

output
```
(g*i^3*((5*B*(b*c - a*d)^2*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c +
 d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]))/b^4 - (2*B*(b
*c - a*d)*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2*(c + d*x)^2 + 4*b^
3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c - a*d)^4*Log[a + b
*x]))/b^4 - 30*(b*c - a*d)*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]
) + 24*b*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/(120*d^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2962, 2782, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$
$$\downarrow 2962$$
$$gi^3(bc-ad)^5\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$
$$\downarrow 2782$$

$$gi^3(bc-ad)^5\left(-B\int -\frac{(c+dx)\left(b-\frac{5d(a+bx)}{c+dx}\right)}{20d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}\right)$$

$$\downarrow 27$$

$$gi^3(bc-ad)^5\left(\frac{B\int\frac{(c+dx)\left(b-\frac{5d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{20d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}\right)$$

$$\downarrow 86$$

$$gi^3(bc-ad)^5\left(\frac{B\int\left(\frac{d}{b^4\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{4d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{c+dx}{b^4(a+bx)}\right)d\frac{a+bx}{c+dx}}{20d^2}-\right.$$

$$\downarrow 2009$$

$$gi^3(bc-ad)^5\left(-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^4}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^4}+\frac{1}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{}{2b^2\left(\right.}\right.}{20d^2}$$

input

```
Int[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output

```
(b*c - a*d)^5*g*i^3*((b*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*d^2*(b -
(d*(a + b*x))/(c + d*x))^5) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(4*d^2*
(b - (d*(a + b*x))/(c + d*x))^4) + (B*(-(b - (d*(a + b*x))/(c + d*x))^(-4)
 + 1/(3*b*(b - (d*(a + b*x))/(c + d*x))^3) + 1/(2*b^2*(b - (d*(a + b*x))/(
c + d*x))^2) + 1/(b^3*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c +
d*x)]/b^4 - Log[b - (d*(a + b*x))/(c + d*x)]/b^4))/(20*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. 2(255) = 510.

Time = 1.44 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.10

method	result
risch	$-\frac{i^3gBa\,c^3x}{4}+\frac{i^3g\,d^2B\ln(bx+a)a^4c}{4b^3}-\frac{i^3gdB\ln(bx+a)a^3c^2}{2b^2}+i^3gAa\,c^3x+\frac{i^3g\,d^2Bacx^3}{6}+\frac{3i^3gdAa\,c^2x^2}{2}$
parallelrisch	$\frac{27Ba^4bcd^4gi^3-45Ba^3b^2c^2d^3gi^3-45Ba^2b^3c^3d^2gi^3+63Bab^4c^4dgi^3+120Axab^4c^3d^2gi^3-300Aa^2b^3c^3d^2gi^3-180A}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURN
VERBOSE)
```

output
```
-1/4*i^3*g*B*a*c^3*x+1/4*i^3*g/b^3*d^2*B*ln(b*x+a)*a^4*c-1/2*i^3*g/b^2*d*B
*ln(b*x+a)*a^3*c^2+i^3*g*A*a*c^3*x+1/6*i^3*g*d^2*B*a*c*x^3+3/2*i^3*g*d*A*a
*c^2*x^2+1/8*i^3*g/b*d^2*B*a^2*c*x^2+1/8*i^3*g*d*B*a*c^2*x^2+1/4*i^3*g*d^3
*A*a*x^4+i^3*g*d^2*A*a*c*x^3-1/4*i^3*g/b^2*d^2*B*a^3*c*x+1/2*i^3*g/b*d*B*a
^2*c^2*x-1/20*i^3*g*b/d*B*c^4*x-1/4*i^3*g/d*B*ln(-d*x-c)*a*c^4+1/2*i^3*g/b
*B*ln(b*x+a)*a^2*c^3+1/20*i^3*g*b/d^2*B*ln(-d*x-c)*c^5+3/4*i^3*g*b*d^2*A*c
*x^4+1/20*i^3*g*d^3*B*a*x^4-1/20*i^3*g*b*d^2*B*c*x^4+i^3*g*b*d*A*c^2*x^3+1
/60*i^3*g/b*d^3*B*a^2*x^3-11/60*i^3*g*b*d*B*c^2*x^3+1/2*i^3*g*b*A*c^3*x^2-
1/40*i^3*g/b^2*d^3*B*a^3*x^2-9/40*i^3*g*b*B*c^3*x^2+1/20*i^3*g/b^3*d^3*B*a
^4*x-1/20*i^3*g/b^4*d^3*B*ln(b*x+a)*a^5+1/20*g*i^3*B*x*(4*b*d^3*x^4+5*a*d^
3*x^3+15*b*c*d^2*x^3+20*a*c*d^2*x^2+20*b*c^2*d*x^2+30*a*c^2*d*x+10*b*c^3*x
+20*a*c^3)*ln(e*(b*x+a)/(d*x+c))+1/5*i^3*g*b*d^3*A*x^5
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.85

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{24\,Ab^5d^5gi^3x^5+6\left((15\,A-B)b^5cd^4+(5\,A+B)ab^4d^5\right)gi^3x^4+2\left((60\,A-11\,B)b^5c^2d^3+10\,(6\,A+B)ab\right.}{}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="fricas")
```

output
```
1/120*(24*A*b^5*d^5*g*i^3*x^5 + 6*((15*A - B)*b^5*c*d^4 + (5*A + B)*a*b^4*
d^5)*g*i^3*x^4 + 2*((60*A - 11*B)*b^5*c^2*d^3 + 10*(6*A + B)*a*b^4*c*d^4 +
 B*a^2*b^3*d^5)*g*i^3*x^3 + 3*((20*A - 9*B)*b^5*c^3*d^2 + 5*(12*A + B)*a*b
^4*c^2*d^3 + 5*B*a^2*b^3*c*d^4 - B*a^3*b^2*d^5)*g*i^3*x^2 - 6*(B*b^5*c^4*d
 - 5*(4*A - B)*a*b^4*c^3*d^2 - 10*B*a^2*b^3*c^2*d^3 + 5*B*a^3*b^2*c*d^4 -
B*a^4*b*d^5)*g*i^3*x + 6*(10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*
B*a^4*b*c*d^4 - B*a^5*d^5)*g*i^3*log(b*x + a) + 6*(B*b^5*c^5 - 5*B*a*b^4*c
^4*d)*g*i^3*log(d*x + c) + 6*(4*B*b^5*d^5*g*i^3*x^5 + 20*B*a*b^4*c^3*d^2*g
*i^3*x + 5*(3*B*b^5*c*d^4 + B*a*b^4*d^5)*g*i^3*x^4 + 20*(B*b^5*c^2*d^3 + B
*a*b^4*c*d^4)*g*i^3*x^3 + 10*(B*b^5*c^3*d^2 + 3*B*a*b^4*c^2*d^3)*g*i^3*x^2
)*log((b*e*x + a*e)/(d*x + c)))/(b^4*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1158 vs. $2(252) = 504$.

Time $= 3.53$ (sec) , antiderivative size $= 1158$, normalized size of antiderivative $= 4.27$

$$\int (ag + bgx)(ci + dix)^3 \left(A + B \log\left(\frac{e(a + bx)}{c + dx}\right)\right) dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output

```
A*b*d**3*g*i**3*x**5/5 - B*a**2*g*i**3*(a**3*d**3 - 5*a**2*b*c*d**2 + 10*a
*b**2*c**2*d - 10*b**3*c**3)*log(x + (B*a**5*c*d**4*g*i**3 - 5*B*a**4*b*c*
*2*d**3*g*i**3 + 10*B*a**3*b**2*c**3*d**2*g*i**3 + B*a**3*d**2*g*i**3*(a**
3*d**3 - 5*a**2*b*c*d**2 + 10*a*b**2*c**2*d - 10*b**3*c**3)/b - 15*B*a**2*
b**3*c**4*d*g*i**3 - B*a**2*c*d*g*i**3*(a**3*d**3 - 5*a**2*b*c*d**2 + 10*a
*b**2*c**2*d - 10*b**3*c**3) + B*a*b**4*c**5*g*i**3)/(B*a**5*d**5*g*i**3 -
 5*B*a**4*b*c*d**4*g*i**3 + 10*B*a**3*b**2*c**2*d**3*g*i**3 - 10*B*a**2*b*
*3*c**3*d**2*g*i**3 - 5*B*a*b**4*c**4*d*g*i**3 + B*b**5*c**5*g*i**3))/(20*
b**4) - B*c**4*g*i**3*(5*a*d - b*c)*log(x + (B*a**5*c*d**4*g*i**3 - 5*B*a*
*4*b*c**2*d**3*g*i**3 + 10*B*a**3*b**2*c**3*d**2*g*i**3 - 15*B*a**2*b**3*c
**4*d*g*i**3 + B*a*b**4*c**5*g*i**3 + B*a*b**3*c**4*g*i**3*(5*a*d - b*c) -
 B*b**4*c**5*g*i**3*(5*a*d - b*c)/d)/(B*a**5*d**5*g*i**3 - 5*B*a**4*b*c*d*
*4*g*i**3 + 10*B*a**3*b**2*c**2*d**3*g*i**3 - 10*B*a**2*b**3*c**3*d**2*g*i
**3 - 5*B*a*b**4*c**4*d*g*i**3 + B*b**5*c**5*g*i**3))/(20*d**2) + x**4*(A*
a*d**3*g*i**3/4 + 3*A*b*c*d**2*g*i**3/4 + B*a*d**3*g*i**3/20 - B*b*c*d**2*
g*i**3/20) + x**3*(A*a*c*d**2*g*i**3 + A*b*c**2*d*g*i**3 + B*a**2*d**3*g*i
**3/(60*b) + B*a*c*d**2*g*i**3/6 - 11*B*b*c**2*d*g*i**3/60) + x**2*(3*A*a*
c**2*d*g*i**3/2 + A*b*c**3*g*i**3/2 - B*a**3*d**3*g*i**3/(40*b**2) + B*a**
2*c*d**2*g*i**3/(8*b) + B*a*c**2*d*g*i**3/8 - 9*B*b*c**3*g*i**3/40) + x*(A
*a*c**3*g*i**3 + B*a**4*d**3*g*i**3/(20*b**3) - B*a**3*c*d**2*g*i**3/(4...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1022 vs. $2(255) = 510$.

Time $= 0.06$ (sec) , antiderivative size $= 1022$, normalized size of antiderivative $= 3.77$

$$\int (ag + bgx)(ci + dix)^3 \left(A + B \log\left(\frac{e(a + bx)}{c + dx}\right)\right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="maxima")
```

output

```
1/5*A*b*d^3*g*i^3*x^5 + 3/4*A*b*c*d^2*g*i^3*x^4 + 1/4*A*a*d^3*g*i^3*x^4 +
A*b*c^2*d*g*i^3*x^3 + A*a*c*d^2*g*i^3*x^3 + 1/2*A*b*c^3*g*i^3*x^2 + 3/2*A*
a*c^2*d*g*i^3*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x +
a)/b - c*log(d*x + c)/d)*B*a*c^3*g*i^3 + 1/2*(x^2*log(b*e*x/(d*x + c) + a*
e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x
/(b*d))*B*b*c^3*g*i^3 + 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^
2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a*c^2*d
*g*i^3 + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x +
 a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 -
 a^2*d^2)*x)/(b^2*d^2))*B*b*c^2*d*g*i^3 + 1/2*(2*x^3*log(b*e*x/(d*x + c) +
 a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*
c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*c*d^2*g*i^3 +
 1/8*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4
+ 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d -
 a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b*c*d^2*g*i^3 + 1/
24*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 +
6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a
^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*a*d^3*g*i^3 + 1/60*(
12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12
*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2589 vs. $2(255) = 510$.

Time $= 0.30$ (sec) , antiderivative size $= 2589$, normalized size of antiderivative $= 9.55$

$$\int (ag + bgx)(ci + dix)^3 \left(A + B \log\left(\frac{e(a + bx)}{c + dx}\right)\right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="giac")
```

output

```
-1/120*(6*(B*b^7*c^6*e^6*g*i^3 - 6*B*a*b^6*c^5*d*e^6*g*i^3 + 15*B*a^2*b^5*
c^4*d^2*e^6*g*i^3 - 20*B*a^3*b^4*c^3*d^3*e^6*g*i^3 + 15*B*a^4*b^3*c^2*d^4*
e^6*g*i^3 - 6*B*a^5*b^2*c*d^5*e^6*g*i^3 + B*a^6*b*d^6*e^6*g*i^3 - 5*(b*e*x
 + a*e)*B*b^6*c^6*d*e^5*g*i^3/(d*x + c) + 30*(b*e*x + a*e)*B*a*b^5*c^5*d^2
*e^5*g*i^3/(d*x + c) - 75*(b*e*x + a*e)*B*a^2*b^4*c^4*d^3*e^5*g*i^3/(d*x +
 c) + 100*(b*e*x + a*e)*B*a^3*b^3*c^3*d^4*e^5*g*i^3/(d*x + c) - 75*(b*e*x
 + a*e)*B*a^4*b^2*c^2*d^5*e^5*g*i^3/(d*x + c) + 30*(b*e*x + a*e)*B*a^5*b*c*
d^6*e^5*g*i^3/(d*x + c) - 5*(b*e*x + a*e)*B*a^6*d^7*e^5*g*i^3/(d*x + c))*l
og((b*e*x + a*e)/(d*x + c))/(b^5*d^2*e^5 - 5*(b*e*x + a*e)*b^4*d^3*e^4/(d*
x + c) + 10*(b*e*x + a*e)^2*b^3*d^4*e^3/(d*x + c)^2 - 10*(b*e*x + a*e)^3*b
^2*d^5*e^2/(d*x + c)^3 + 5*(b*e*x + a*e)^4*b*d^6*e/(d*x + c)^4 - (b*e*x +
a*e)^5*d^7/(d*x + c)^5) + (6*A*b^10*c^6*e^6*g*i^3 - 5*B*b^10*c^6*e^6*g*i^3
 - 36*A*a*b^9*c^5*d*e^6*g*i^3 + 30*B*a*b^9*c^5*d*e^6*g*i^3 + 90*A*a^2*b^8*
c^4*d^2*e^6*g*i^3 - 75*B*a^2*b^8*c^4*d^2*e^6*g*i^3 - 120*A*a^3*b^7*c^3*d^3
*e^6*g*i^3 + 100*B*a^3*b^7*c^3*d^3*e^6*g*i^3 + 90*A*a^4*b^6*c^2*d^4*e^6*g*
i^3 - 75*B*a^4*b^6*c^2*d^4*e^6*g*i^3 - 36*A*a^5*b^5*c*d^5*e^6*g*i^3 + 30*B
*a^5*b^5*c*d^5*e^6*g*i^3 + 6*A*a^6*b^4*d^6*e^6*g*i^3 - 5*B*a^6*b^4*d^6*e^6
*g*i^3 - 30*(b*e*x + a*e)*A*b^9*c^6*d*e^5*g*i^3/(d*x + c) + 31*(b*e*x + a*
e)*B*b^9*c^6*d*e^5*g*i^3/(d*x + c) + 180*(b*e*x + a*e)*A*a*b^8*c^5*d^2*e^5
*g*i^3/(d*x + c) - 186*(b*e*x + a*e)*B*a*b^8*c^5*d^2*e^5*g*i^3/(d*x + c...
```

Mupad [B] (verification not implemented)

Time = 27.98 (sec) , antiderivative size = 1192, normalized size of antiderivative = 4.40

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input

```
int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output

```
x^4*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d - B*b*c))/20 - (A*d^2*g*i^3*(
20*a*d + 20*b*c))/80) + x*((a*c*(((20*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d
+ 20*A*b*c + B*a*d - B*b*c))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(20*
b*d) - (d*g*i^3*(4*A*a^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2 - 3*B*b^2*c^2 + 32
*A*a*b*c*d + 2*B*a*b*c*d))/(4*b) + A*a*c*d^2*g*i^3))/(b*d) - ((20*a*d + 20
*b*c)*(((20*a*d + 20*b*c)*(((20*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d + 20*A
*b*c + B*a*d - B*b*c))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(20*b*d) -
 (d*g*i^3*(4*A*a^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2 - 3*B*b^2*c^2 + 32*A*a*b
*c*d + 2*B*a*b*c*d))/(4*b) + A*a*c*d^2*g*i^3))/(20*b*d) - (a*c*((d^2*g*i^3
*(10*A*a*d + 20*A*b*c + B*a*d - B*b*c))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c)
)/20))/(b*d) + (c*g*i^3*(4*A*a^2*d^2 + 4*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2
 + 12*A*a*b*c*d))/b))/(20*b*d) + (c^2*g*i^3*(12*A*a^2*d^2 + 2*A*b^2*c^2 +
3*B*a^2*d^2 - B*b^2*c^2 + 16*A*a*b*c*d - 2*B*a*b*c*d))/(2*b*d)) - x^3*(((2
0*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d - B*b*c))/5 - (A*
d^2*g*i^3*(20*a*d + 20*b*c))/20))/(60*b*d) - (d*g*i^3*(4*A*a^2*d^2 + 24*A*
b^2*c^2 + B*a^2*d^2 - 3*B*b^2*c^2 + 32*A*a*b*c*d + 2*B*a*b*c*d))/(12*b) +
(A*a*c*d^2*g*i^3)/3) + x^2*(((20*a*d + 20*b*c)*(((20*a*d + 20*b*c)*((d^2*g
*i^3*(10*A*a*d + 20*A*b*c + B*a*d - B*b*c))/5 - (A*d^2*g*i^3*(20*a*d + 20*
b*c))/20))/(20*b*d) - (d*g*i^3*(4*A*a^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2 - 3
*B*b^2*c^2 + 32*A*a*b*c*d + 2*B*a*b*c*d))/(4*b) + A*a*c*d^2*g*i^3))/(40...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 703, normalized size of antiderivative = 2.59

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{gi\left(-24\log\left(\frac{bex+ae}{dx+c}\right)b^5d^5x^5-6a^4bd^5x+3a^3b^2d^5x^2-2a^2b^3d^5x^3-6ab^4d^5x^4+6b^5c^4dx+27b^5c^3d^2x^2+22\right.}{}$$

input

```
int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output

```
(g*i*(6*log(a + b*x)*a**5*d**5 - 30*log(a + b*x)*a**4*b*c*d**4 + 60*log(a
+ b*x)*a**3*b**2*c**2*d**3 - 60*log(a + b*x)*a**2*b**3*c**3*d**2 + 30*log(
a + b*x)*a*b**4*c**4*d - 6*log(a + b*x)*b**5*c**5 - 30*log((a*e + b*e*x)/(
c + d*x))*a*b**4*c**4*d - 120*log((a*e + b*e*x)/(c + d*x))*a*b**4*c**3*d**
2*x - 180*log((a*e + b*e*x)/(c + d*x))*a*b**4*c**2*d**3*x**2 - 120*log((a*
e + b*e*x)/(c + d*x))*a*b**4*c*d**4*x**3 - 30*log((a*e + b*e*x)/(c + d*x))
*a*b**4*d**5*x**4 + 6*log((a*e + b*e*x)/(c + d*x))*b**5*c**5 - 60*log((a*e
 + b*e*x)/(c + d*x))*b**5*c**3*d**2*x**2 - 120*log((a*e + b*e*x)/(c + d*x)
)*b**5*c**2*d**3*x**3 - 90*log((a*e + b*e*x)/(c + d*x))*b**5*c*d**4*x**4 -
 24*log((a*e + b*e*x)/(c + d*x))*b**5*d**5*x**5 - 6*a**4*b*d**5*x + 30*a**
3*b**2*c*d**4*x + 3*a**3*b**2*d**5*x**2 - 120*a**2*b**3*c**3*d**2*x - 180*
a**2*b**3*c**2*d**3*x**2 - 60*a**2*b**3*c**2*d**3*x - 120*a**2*b**3*c*d**4
*x**3 - 15*a**2*b**3*c*d**4*x**2 - 30*a**2*b**3*d**5*x**4 - 2*a**2*b**3*d*
*5*x**3 - 60*a*b**4*c**3*d**2*x**2 + 30*a*b**4*c**3*d**2*x - 120*a*b**4*c*
*2*d**3*x**3 - 15*a*b**4*c**2*d**3*x**2 - 90*a*b**4*c*d**4*x**4 - 20*a*b**
4*c*d**4*x**3 - 24*a*b**4*d**5*x**5 - 6*a*b**4*d**5*x**4 + 6*b**5*c**4*d*x
 + 27*b**5*c**3*d**2*x**2 + 22*b**5*c**2*d**3*x**3 + 6*b**5*c*d**4*x**4))/
(120*b**3*d**2)
```

3.23 $\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

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Optimal result

Integrand size = 30, antiderivative size = 149

$$\begin{aligned}&\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \\ &= -\frac{B(bc-ad)^3 i^3 x}{4b^3} - \frac{B(bc-ad)^2 i^3 (c+dx)^2}{8b^2 d} - \frac{B(bc-ad) i^3 (c+dx)^3}{12bd} \\ &\quad - \frac{B(bc-ad)^4 i^3 \log(a+bx)}{4b^4 d} + \frac{i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d}\end{aligned}$$

output

```
-1/4*B*(-a*d+b*c)^3*i^3*x/b^3-1/8*B*(-a*d+b*c)^2*i^3*(d*x+c)^2/b^2/d-1/12*
B*(-a*d+b*c)*i^3*(d*x+c)^3/b/d-1/4*B*(-a*d+b*c)^4*i^3*ln(b*x+a)/b^4/d+1/4*
i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\begin{aligned}&\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \\ &= \frac{i^3\left(-\frac{B(bc-ad)(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx))}{6b^4} + (c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\right)}{4d}\end{aligned}$$

input
```
Integrate[(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(i^3*(-1/6*(B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c +
d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]))/b^4 + (c + d*x
)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/(4*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right) dx$$

$$\downarrow 2948$$

$$\frac{i^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d} - \frac{B(bc-ad)\int \frac{i^4(c+dx)^3}{a+bx}dx}{4di}$$

$$\downarrow 27$$

$$\frac{i^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d} - \frac{Bi^3(bc-ad)\int \frac{(c+dx)^3}{a+bx}dx}{4d}$$

$$\downarrow 49$$

$$\frac{i^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d} - \frac{Bi^3(bc-ad)\int \left(\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)^2}{b^3}+\frac{d(c+dx)(bc-ad)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx}{4d}$$

$$\downarrow 2009$$

$$\frac{i^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d} - \frac{Bi^3(bc-ad)\left(\frac{(bc-ad)^3\log(a+bx)}{b^4}+\frac{dx(bc-ad)^2}{b^3}+\frac{(c+dx)^2(bc-ad)}{2b^2}+\frac{(c+dx)^3}{3b}\right)}{4d}$$

input
```
Int[(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
-1/4*(B*(b*c - a*d)*i^3*((d*(b*c - a*d)^2*x)/b^3 + ((b*c - a*d)*(c + d*x)^
2)/(2*b^2) + (c + d*x)^3/(3*b) + ((b*c - a*d)^3*Log[a + b*x])/b^4))/d + (i
^3*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(139) = 278$.

Time = 1.24 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.11

method	result
risch	$\frac{i^3(dx+c)^4 B \ln\left(\frac{e(bx+a)}{dx+c}\right)}{4d} + \frac{i^3 d^3 A x^4}{4} + i^3 d^2 Ac x^3 + \frac{i^3 d^3 Ba x^3}{12b} - \frac{i^3 d^2 Bc x^3}{12} + \frac{3i^3 dA c^2 x^2}{2} - \frac{i^3 d^3 B a^2 x}$
parts	$\frac{A i^3(dx+c)^4}{4d} - B i^3(da-bc)^4 e^4 \left(- \frac{1}{4b^3 e^3 d\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)} + \frac{1}{8b^2 e^2 d\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)^2} -$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{Ad\,e^3 i^3\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{4\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^4} - B\,d^2 e^3 i^3\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)\left(-\frac{}{12bed\left(be-\left(\frac{be}{d}\right.\right.\right.\right.}$
default	$-\frac{e(da-bc)\left(-\frac{Ad\,e^3 i^3\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{4\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^4} - B\,d^2 e^3 i^3\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)\left(-\frac{}{12bed\left(be-\left(\frac{be}{d}\right.\right.\right.\right.}$
parallelrisch	$\frac{-6Ba^4d^4i^3+18Bb^4c^4i^3+21Ba^3bcd^3i^3-24Ba^2b^2c^2d^2i^3-9Bab^3c^3di^3+24B\ln(bx+a)a^3bcd^3i^3-36B\ln(bx+a)a^2b^2}{}$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/4*i^3*(d*x+c)^4*B/d*ln(e*(b*x+a)/(d*x+c))+1/4*i^3*d^3*A*x^4+i^3*d^2*A*c*
x^3+1/12*i^3/b*d^3*B*a*x^3-1/12*i^3*d^2*B*c*x^3+3/2*i^3*d*A*c^2*x^2-1/8*i^
3/b^2*d^3*B*a^2*x^2+1/2*i^3/b*d^2*B*a*c*x^2-3/8*i^3*d*B*c^2*x^2+i^3*A*c^3*
x-1/4*i^3/b^4*d^3*B*ln(b*x+a)*a^4+i^3/b^3*d^2*B*ln(b*x+a)*a^3*c-3/2*i^3/b^
2*d*B*ln(b*x+a)*a^2*c^2+i^3/b*B*ln(b*x+a)*a*c^3-1/4*i^3/d*B*ln(b*x+a)*c^4+
1/4*i^3/b^3*d^3*B*a^3*x-i^3/b^2*d^2*B*a^2*c*x+3/2*i^3/b*d*B*a*c^2*x-3/4*i^
3*B*c^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(139) = 278$.

Time = 0.10 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.16

$$\int (ci+dix)^3 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$$
$$= \frac{6\,Ab^4d^4i^3x^4 - 6\,Bb^4c^4i^3\log(dx+c) + 2\left((12\,A-B)b^4cd^3 + Bab^3d^4\right)i^3x^3 + 3\left(3\left(4\,A-B\right)b^4c^2d^2 + 4\,Ba\right.}{}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas"
)
```

output
```
1/24*(6*A*b^4*d^4*i^3*x^4 - 6*B*b^4*c^4*i^3*log(d*x + c) + 2*((12*A - B)*b
^4*c*d^3 + B*a*b^3*d^4)*i^3*x^3 + 3*(3*(4*A - B)*b^4*c^2*d^2 + 4*B*a*b^3*c
*d^3 - B*a^2*b^2*d^4)*i^3*x^2 + 6*((4*A - 3*B)*b^4*c^3*d + 6*B*a*b^3*c^2*d
^2 - 4*B*a^2*b^2*c*d^3 + B*a^3*b*d^4)*i^3*x + 6*(4*B*a*b^3*c^3*d - 6*B*a^2
*b^2*c^2*d^2 + 4*B*a^3*b*c*d^3 - B*a^4*d^4)*i^3*log(b*x + a) + 6*(B*b^4*d^
4*i^3*x^4 + 4*B*b^4*c*d^3*i^3*x^3 + 6*B*b^4*c^2*d^2*i^3*x^2 + 4*B*b^4*c^3*
d*i^3*x)*log((b*e*x + a*e)/(d*x + c)))/(b^4*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. 2(128) = 256.

Time = 1.98 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.74

$$\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{Ad^3i^3x^4}{4}$$
$$-\frac{Bai^3(ad-2bc)\left(a^2d^2-2abcd+2b^2c^2\right)\log\left(x+\frac{Ba^4cd^3i^3-4Ba^3bc^2d^2i^3+6Ba^2b^2c^3di^3+\frac{Ba^2di^3(ad-2bc)\left(a^2d^2-2abcd+2b}{b}}{Ba^4d^4i^3-4Ba^3bcd^3i^3+6Ba^2b^2c^2d^2i^3-}\right)}{4b^4}$$
$$-\frac{Bc^4i^3\log\left(x+\frac{Ba^4cd^3i^3-4Ba^3bc^2d^2i^3+6Ba^2b^2c^3di^3-4Bab^3c^4i^3-\frac{Bb^4c^5i^3}{d}}{Ba^4d^4i^3-4Ba^3bcd^3i^3+6Ba^2b^2c^2d^2i^3-4Bab^3c^3di^3-Bb^4c^4i^3}\right)}{4d}$$
$$+x^3\left(Acd^2i^3+\frac{Bad^3i^3}{12b}-\frac{Bcd^2i^3}{12}\right)+x^2\cdot\left(\frac{3Ac^2di^3}{2}-\frac{Ba^2d^3i^3}{8b^2}+\frac{Bacd^2i^3}{2b}-\frac{3Bc^2di^3}{8}\right)$$
$$+x\left(Ac^3i^3+\frac{Ba^3d^3i^3}{4b^3}-\frac{Ba^2cd^2i^3}{b^2}+\frac{3Bac^2di^3}{2b}-\frac{3Bc^3i^3}{4}\right)$$
$$+\left(Bc^3i^3x+\frac{3Bc^2di^3x^2}{2}+Bcd^2i^3x^3+\frac{Bd^3i^3x^4}{4}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output

```
A*d**3*i**3*x**4/4 - B*a*i**3*(a*d - 2*b*c)*(a**2*d**2 - 2*a*b*c*d + 2*b**
2*c**2)*log(x + (B*a**4*c*d**3*i**3 - 4*B*a**3*b*c**2*d**2*i**3 + 6*B*a**2
*b**2*c**3*d*i**3 + B*a**2*d*i**3*(a*d - 2*b*c)*(a**2*d**2 - 2*a*b*c*d + 2
*b**2*c**2)/b - 5*B*a*b**3*c**4*i**3 - B*a*c*i**3*(a*d - 2*b*c)*(a**2*d**2
 - 2*a*b*c*d + 2*b**2*c**2))/(B*a**4*d**4*i**3 - 4*B*a**3*b*c*d**3*i**3 +
6*B*a**2*b**2*c**2*d**2*i**3 - 4*B*a*b**3*c**3*d*i**3 - B*b**4*c**4*i**3))
/(4*b**4) - B*c**4*i**3*log(x + (B*a**4*c*d**3*i**3 - 4*B*a**3*b*c**2*d**2
*i**3 + 6*B*a**2*b**2*c**3*d*i**3 - 4*B*a*b**3*c**4*i**3 - B*b**4*c**5*i**
3/d)/(B*a**4*d**4*i**3 - 4*B*a**3*b*c*d**3*i**3 + 6*B*a**2*b**2*c**2*d**2*
i**3 - 4*B*a*b**3*c**3*d*i**3 - B*b**4*c**4*i**3))/(4*d) + x**3*(A*c*d**2*
i**3 + B*a*d**3*i**3/(12*b) - B*c*d**2*i**3/12) + x**2*(3*A*c**2*d*i**3/2
- B*a**2*d**3*i**3/(8*b**2) + B*a*c*d**2*i**3/(2*b) - 3*B*c**2*d*i**3/8) +
 x*(A*c**3*i**3 + B*a**3*d**3*i**3/(4*b**3) - B*a**2*c*d**2*i**3/b**2 + 3*
B*a*c**2*d*i**3/(2*b) - 3*B*c**3*i**3/4) + (B*c**3*i**3*x + 3*B*c**2*d*i**
3*x**2/2 + B*c*d**2*i**3*x**3 + B*d**3*i**3*x**4/4)*log(e*(a + b*x)/(c + d
*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(139) = 278$.

Time $= 0.04$ (sec) , antiderivative size $= 439$, normalized size of antiderivative $= 2.95$

$$\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{1}{4}Ad^3i^3x^4 + Acd^2i^3x^3 + \frac{3}{2}Ac^2di^3x^2$$
$$+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bc^3i^3$$
$$+\frac{3}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bc^2di^3$$
$$+\frac{1}{2}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-}{b^2d^2}\right.$$
$$+\frac{1}{24}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log(dx+c)}{d^4}-\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3}{}\right.$$
$$+Ac^3i^3x$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima"
)
```

output

```
1/4*A*d^3*i^3*x^4 + A*c*d^2*i^3*x^3 + 3/2*A*c^2*d*i^3*x^2 + (x*log(b*e*x/(
d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*c^3*i^3
 + 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 +
c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*c^2*d*i^3 + 1/2*(2*x^3*log(b
*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x +
 c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B
*c*d^2*i^3 + 1/24*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(
b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*d^3*i
^3 + A*c^3*i^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(139) = 278$.

Time $= 0.27$ (sec) , antiderivative size $= 1506$, normalized size of antiderivative $= 10.11$

$$\int (ci + dix)^3 \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right) dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output

```
1/24*(6*(B*b^5*c^5*e^5*i^3 - 5*B*a*b^4*c^4*d*e^5*i^3 + 10*B*a^2*b^3*c^3*d^
2*e^5*i^3 - 10*B*a^3*b^2*c^2*d^3*e^5*i^3 + 5*B*a^4*b*c*d^4*e^5*i^3 - B*a^5
*d^5*e^5*i^3)*log((b*e*x + a*e)/(d*x + c))/(b^4*d*e^4 - 4*(b*e*x + a*e)*b^
3*d^2*e^3/(d*x + c) + 6*(b*e*x + a*e)^2*b^2*d^3*e^2/(d*x + c)^2 - 4*(b*e*x
 + a*e)^3*b*d^4*e/(d*x + c)^3 + (b*e*x + a*e)^4*d^5/(d*x + c)^4) + (6*A*b^
8*c^5*e^5*i^3 - 11*B*b^8*c^5*e^5*i^3 - 30*A*a*b^7*c^4*d*e^5*i^3 + 55*B*a*b
^7*c^4*d*e^5*i^3 + 60*A*a^2*b^6*c^3*d^2*e^5*i^3 - 110*B*a^2*b^6*c^3*d^2*e^
5*i^3 - 60*A*a^3*b^5*c^2*d^3*e^5*i^3 + 110*B*a^3*b^5*c^2*d^3*e^5*i^3 + 30*
A*a^4*b^4*c*d^4*e^5*i^3 - 55*B*a^4*b^4*c*d^4*e^5*i^3 - 6*A*a^5*b^3*d^5*e^5
*i^3 + 11*B*a^5*b^3*d^5*e^5*i^3 + 26*(b*e*x + a*e)*B*b^7*c^5*d*e^4*i^3/(d*
x + c) - 130*(b*e*x + a*e)*B*a*b^6*c^4*d^2*e^4*i^3/(d*x + c) + 260*(b*e*x
+ a*e)*B*a^2*b^5*c^3*d^3*e^4*i^3/(d*x + c) - 260*(b*e*x + a*e)*B*a^3*b^4*c
^2*d^4*e^4*i^3/(d*x + c) + 130*(b*e*x + a*e)*B*a^4*b^3*c*d^5*e^4*i^3/(d*x
+ c) - 26*(b*e*x + a*e)*B*a^5*b^2*d^6*e^4*i^3/(d*x + c) - 21*(b*e*x + a*e)
^2*B*b^6*c^5*d^2*e^3*i^3/(d*x + c)^2 + 105*(b*e*x + a*e)^2*B*a*b^5*c^4*d^3
*e^3*i^3/(d*x + c)^2 - 210*(b*e*x + a*e)^2*B*a^2*b^4*c^3*d^4*e^3*i^3/(d*x
+ c)^2 + 210*(b*e*x + a*e)^2*B*a^3*b^3*c^2*d^5*e^3*i^3/(d*x + c)^2 - 105*(
b*e*x + a*e)^2*B*a^4*b^2*c*d^6*e^3*i^3/(d*x + c)^2 + 21*(b*e*x + a*e)^2*B*
a^5*b*d^7*e^3*i^3/(d*x + c)^2 + 6*(b*e*x + a*e)^3*B*b^5*c^5*d^3*e^2*i^3/(d
*x + c)^3 - 30*(b*e*x + a*e)^3*B*a*b^4*c^4*d^4*e^2*i^3/(d*x + c)^3 + 60...
```

Mupad [B] (verification not implemented)

Time = 27.08 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.80

$$\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$

$$=x\left(\frac{(4ad+4bc)\left(\frac{\left(\frac{d^2i^3(4Aad+16Abc+Bad-Bbc)}{4b}-\frac{Ad^2i^3(4ad+4bc)}{4b}\right)(4ad+4bc)}{4bd}-\frac{cdi^3(4Aad+6Abc+Bad-Bbc)}{b}+\frac{A}{4bd}\right.}{}\right.$$

$$\left.+\frac{c^2i^3(12Aad+8Abc+3Bad-3Bbc)}{2b}-\frac{ac\left(\frac{d^2i^3(4Aad+16Abc+Bad-Bbc)}{4b}-\frac{Ad^2i^3(4ad+4bc)}{4b}\right)}{bd}\right)$$

$$-x^2\left(\frac{\left(\frac{d^2i^3(4Aad+16Abc+Bad-Bbc)}{4b}-\frac{Ad^2i^3(4ad+4bc)}{4b}\right)(4ad+4bc)}{8bd}\right.$$

$$\left.-\frac{cdi^3(4Aad+6Abc+Bad-Bbc)}{2b}+\frac{Aacd^2i^3}{2b}\right)$$

$$+\ln\left(\frac{e(a+bx)}{c+dx}\right)\left(Bc^3i^3x+\frac{3Bc^2di^3x^2}{2}+Bcd^2i^3x^3+\frac{Bd^3i^3x^4}{4}\right)$$

$$+x^3\left(\frac{d^2i^3(4Aad+16Abc+Bad-Bbc)}{12b}-\frac{Ad^2i^3(4ad+4bc)}{12b}\right)$$

$$-\frac{\ln(a+bx)\left(Ba^4d^3i^3-4Ba^3bcd^2i^3+6Ba^2b^2c^2di^3-4Bab^3c^3i^3\right)}{4b^4}$$

$$+\frac{Ad^3i^3x^4}{4}-\frac{Bc^4i^3\ln(c+dx)}{4d}$$

input

```
int((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output

```
x*(((4*a*d + 4*b*c)*((((d^2*i^3*(4*A*a*d + 16*A*b*c + B*a*d - B*b*c))/(4*b
) - (A*d^2*i^3*(4*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(4*b*d) - (c*d*i^3
*(4*A*a*d + 6*A*b*c + B*a*d - B*b*c))/b + (A*a*c*d^2*i^3)/b))/(4*b*d) + (c
^2*i^3*(12*A*a*d + 8*A*b*c + 3*B*a*d - 3*B*b*c))/(2*b) - (a*c*((d^2*i^3*(4
*A*a*d + 16*A*b*c + B*a*d - B*b*c))/(4*b) - (A*d^2*i^3*(4*a*d + 4*b*c))/(4
*b)))/(b*d)) - x^2*((((d^2*i^3*(4*A*a*d + 16*A*b*c + B*a*d - B*b*c))/(4*b)
 - (A*d^2*i^3*(4*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(8*b*d) - (c*d*i^3*
(4*A*a*d + 6*A*b*c + B*a*d - B*b*c))/(2*b) + (A*a*c*d^2*i^3)/(2*b)) + log(
(e*(a + b*x))/(c + d*x))*((B*d^3*i^3*x^4)/4 + B*c^3*i^3*x + (3*B*c^2*d*i^3
*x^2)/2 + B*c*d^2*i^3*x^3) + x^3*((d^2*i^3*(4*A*a*d + 16*A*b*c + B*a*d - B
*b*c))/(12*b) - (A*d^2*i^3*(4*a*d + 4*b*c))/(12*b)) - (log(a + b*x)*(B*a^4
*d^3*i^3 - 4*B*a*b^3*c^3*i^3 + 6*B*a^2*b^2*c^2*d*i^3 - 4*B*a^3*b*c*d^2*i^3
))/(4*b^4) + (A*d^3*i^3*x^4)/4 - (B*c^4*i^3*log(c + d*x))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.20

$$\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{i\left(-6a^3b\,d^4x+3a^2b^2d^4x^2-2a\,b^3d^4x^3+18b^4c^3dx+9b^4c^2d^2x^2+2b^4c\,d^3x^3-24\log(dx+c)\,a^3bc\,d^3+36\right.}{}$$

input

```
int((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x)
```

output

```
(i*(6*log(c + d*x)*a**4*d**4 - 24*log(c + d*x)*a**3*b*c*d**3 + 36*log(c +
d*x)*a**2*b**2*c**2*d**2 - 24*log(c + d*x)*a*b**3*c**3*d + 6*log(c + d*x)*
b**4*c**4 + 6*log((a*e + b*e*x)/(c + d*x))*a**4*d**4 - 24*log((a*e + b*e*x
)/(c + d*x))*a**3*b*c*d**3 + 36*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*c**
2*d**2 - 24*log((a*e + b*e*x)/(c + d*x))*a*b**3*c**3*d - 24*log((a*e + b*e
*x)/(c + d*x))*b**4*c**3*d*x - 36*log((a*e + b*e*x)/(c + d*x))*b**4*c**2*d
**2*x**2 - 24*log((a*e + b*e*x)/(c + d*x))*b**4*c*d**3*x**3 - 6*log((a*e +
 b*e*x)/(c + d*x))*b**4*d**4*x**4 - 6*a**3*b*d**4*x + 24*a**2*b**2*c*d**3*
x + 3*a**2*b**2*d**4*x**2 - 24*a*b**3*c**3*d*x - 36*a*b**3*c**2*d**2*x**2
- 36*a*b**3*c**2*d**2*x - 24*a*b**3*c*d**3*x**3 - 12*a*b**3*c*d**3*x**2 -
6*a*b**3*d**4*x**4 - 2*a*b**3*d**4*x**3 + 18*b**4*c**3*d*x + 9*b**4*c**2*d
**2*x**2 + 2*b**4*c*d**3*x**3))/(24*b**3*d)
```

3.24 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

Optimal result . . . 369
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Optimal result

Integrand size = 40, antiderivative size = 356

$$\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx \\
&= -\frac{5Bd(bc-ad)^2i^3x}{6b^3g} - \frac{B(bc-ad)i^3(c+dx)^2}{6b^2g} - \frac{5B(bc-ad)^3i^3\log\left(\frac{a+bx}{c+dx}\right)}{6b^4g} \\
&\quad + \frac{d(bc-ad)^2i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g} \\
&\quad + \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g} \\
&\quad + \frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bg} - \frac{11B(bc-ad)^3i^3\log(c+dx)}{6b^4g} \\
&\quad - \frac{(bc-ad)^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} \\
&\quad + \frac{B(bc-ad)^3i^3\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}
\end{aligned}$$

output

```
-5/6*B*d*(-a*d+b*c)^2*i^3*x/b^3/g-1/6*B*(-a*d+b*c)*i^3*(d*x+c)^2/b^2/g-5/6
*B*(-a*d+b*c)^3*i^3*ln((b*x+a)/(d*x+c))/b^4/g+d*(-a*d+b*c)^2*i^3*(b*x+a)*(
A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/g+1/2*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(b
*x+a)/(d*x+c)))/b^2/g+1/3*i^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g-11
/6*B*(-a*d+b*c)^3*i^3*ln(d*x+c)/b^4/g-(-a*d+b*c)^3*i^3*(A+B*ln(e*(b*x+a)/(
d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g+B*(-a*d+b*c)^3*i^3*polylog(2,b*(d
*x+c)/d/(b*x+a))/b^4/g
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.99

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i^3\Big(6Abd(bc-ad)^2x-3B(bc-ad)^2(bdx+(bc-ad)\log(a+bx))-B(bc-ad)\,(2bd(bc-ad)x+b^2(c+$$

input

```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x),x]
```

output

```
(i^3*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*(b*d*x + (b*c - a*d)*Log
[a + b*x]) - B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c
 - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/
(c + d*x)] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d
*x)]) + 2*b^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c -
a*d)^3*Log[g*(a + b*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c -
a*d)^3*Log[c + d*x] - 3*B*(b*c - a*d)^3*(Log[g*(a + b*x)]*(Log[g*(a + b*x)
] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
 + a*d)])))/(6*b^4*g)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2962, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{ag+bgx}\,dx$$

$$\downarrow 2962$$

$$\frac{i^3(bc-ad)^3\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{g}$$

$$\downarrow 2789$$

$$i^3(bc-ad)^3\left(\frac{d\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)$$

$$\downarrow 2756$$

$$i^3(bc-ad)^3\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\int \frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)$$

$$\downarrow 54$$

$$i^3(bc-ad)^3\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a+bx}{c+dx}}{3d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\right.}{(a+bx)\left(b-\frac{d}{}\right.}}{g}$$

$$\downarrow 2009$$

$$i^3(bc-ad)^3\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^3}+\frac{1}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{}{2b\left(b-\frac{d}{}\right.}\right.}{3d}\right)}{b}\right)}{g}$$

$$\downarrow 2789$$

$$i^3(bc-ad)^3\left(\frac{d\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\frac{d(a+}{c+d}\right.}{b^3}\right.}{b}\right)}{g}$$

$$\downarrow 2756$$

$$\frac{i^3(bc-ad)^3\left(\frac{\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\right.}{}\right.}{g}$$

$\downarrow$ 54

$$\frac{i^3(bc-ad)^3\left(\frac{\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{b}+\right.}{g}$$

$\downarrow$ 2009

$$\frac{i^3(bc-ad)^3\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}}{b}+\frac{d\left(\right.}{}\right.}{g}$$

$\downarrow$ 2789

$$i^3(bc-ad)^3\left(\frac{\frac{d\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}\right.}{2d}\right.}{b}}{b}\right.$$

$$g$$

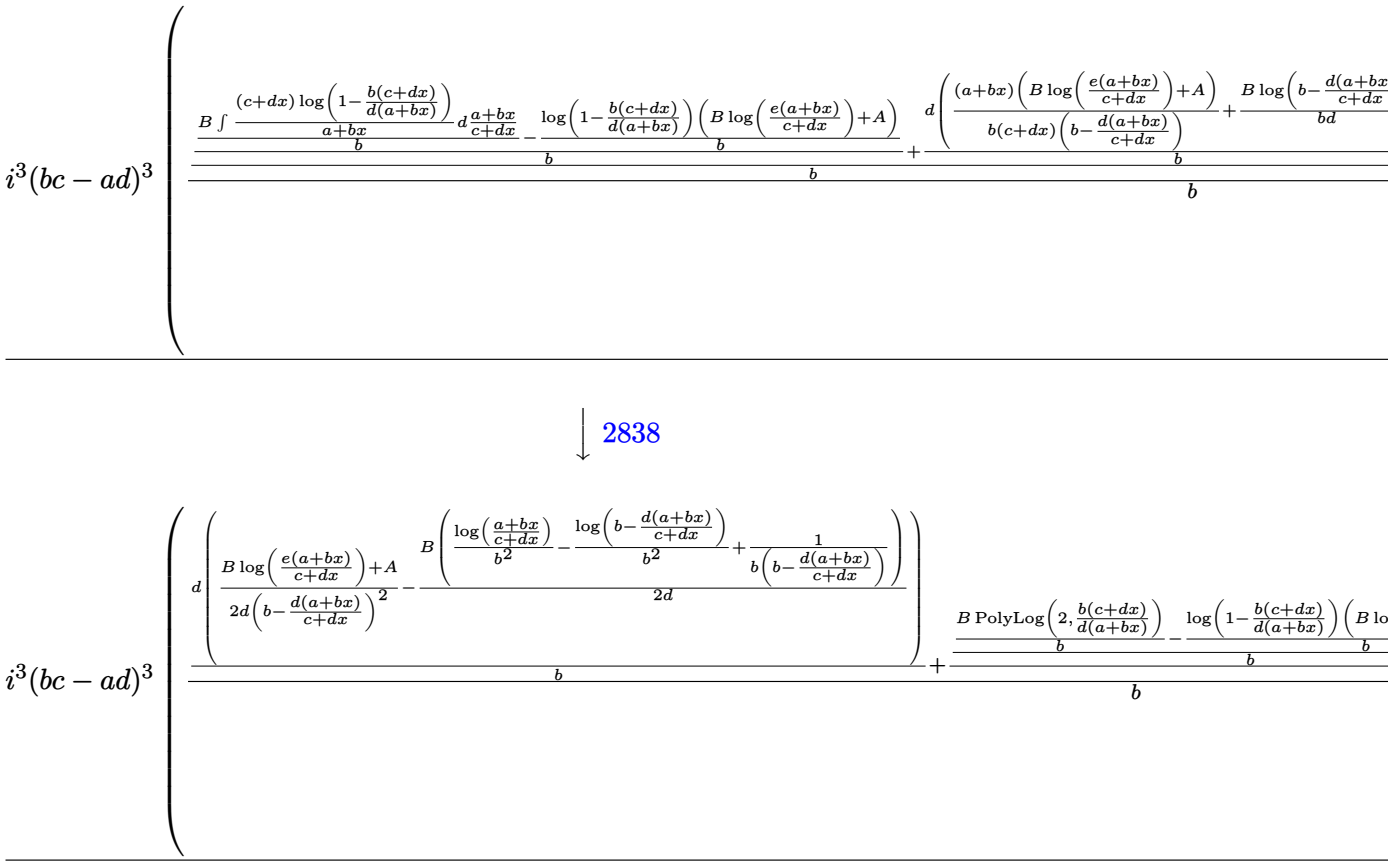
$$\downarrow 2751$$

$$i^3(bc-ad)^3\left(\frac{\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{B\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B}{}\right.}{b}\right.$$

$$\downarrow 16$$

$$i^3(bc-ad)^3\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\log\right.\right.}{}\right.}{b}\right.$$

$$\downarrow 2779$$



↓ 2838

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x),x
]
```

output
```
((b*c - a*d)^3*i^3*((d*((A + B*Log[(e*(a + b*x))/(c + d*x)])/(3*d*(b - (d*
(a + b*x))/(c + d*x))^3) - (B*(1/(2*b*(b - (d*(a + b*x))/(c + d*x))^2) + 1
/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^3 - Log[
b - (d*(a + b*x))/(c + d*x)]/b^3))/(3*d)))/b + ((d*((A + B*Log[(e*(a + b*x
))/(c + d*x)])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(1/(b*(b - (d*(a
 + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x)
)/(c + d*x)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c
 + d*x)]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*Log[b - (d*(a
+ b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[(e*(a + b*x))/(c + d*x)])*L
og[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(d*(
a + b*x))])/b)/b)/b)/b))/g
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(344) = 688$.

Time = 3.31 (sec) , antiderivative size = 1086, normalized size of antiderivative = 3.05

method	result	size
parts	Expression too large to display	1086
derivativedivides	Expression too large to display	1178
default	Expression too large to display	1178
risch	Expression too large to display	5361

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x,method=_RETURN
VERBOSE)
```

output

```
i^3*A/g*(d/b^3*(1/3*d^2*b^2*x^3-1/2*x^2*a*b*d^2+3/2*x^2*b^2*c*d+x*a^2*d^2-
3*x*a*c*d*b+3*x*c^2*b^2)+(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^
4*ln(b*x+a))-i^3*B/g/d^5*(a*d-b*c)^4*e^4*((1/3/b^2/e^2/d/((b*e/d+(a*d-b*c)
*e/d/(d*x+c))*d-b*e)-1/6/b/e/d/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)^2+1/3
/b^3/e^3*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/d-1/3*ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(3*b^2*e^2-3*(b*e/d+(a*d-b*c)*e
/d/(d*x+c))*d*b*e+d^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)/b^3/e^3/((b*e/d+(a*
d-b*c)*e/d/(d*x+c))*d-b*e)^3)/(a*d-b*c)*d^6/b/e-(-1/2/b/e/d/((b*e/d+(a*d-b
*c)*e/d/(d*x+c))*d-b*e)-1/2/b^2/e^2*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e
)/d+1/2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*((b*
e/d+(a*d-b*c)*e/d/(d*x+c))*d-2*b*e)/b^2/e^2/((b*e/d+(a*d-b*c)*e/d/(d*x+c))
*d-b*e)^2)/(a*d-b*c)*d^6/b^2/e^2+(1/b/e*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d
-b*e)/d-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/
((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e))/(a*d-b*c)*d^6/b^3/e^3+1/2*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^2/(a*d-b*c)*d^5/b^4/e^4-(dilog(-((b*e/d+(a*d-b*c)*e
/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d
-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)/(a*d-b*c)*d^6/b^4/e^4)
```

Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{bgx+ag}\,dx$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algori
thm="fricas")
```

output

```
integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3
+ (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log((b
*e*x + a*e)/(d*x + c)))/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i^3\left(\int \frac{Ac^3}{a+bx}\,dx+\int \frac{Ad^3x^3}{a+bx}\,dx+\int \frac{Bc^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx+\int \frac{3Acd^2x^2}{a+bx}\,dx+\int \frac{3Ac^2dx}{a+bx}\,dx+\int \frac{Bd^3x^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\right.}{g}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
```

output
```
i**3*(Integral(A*c**3/(a + b*x), x) + Integral(A*d**3*x**3/(a + b*x), x) +
 Integral(B*c**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Inte
gral(3*A*c*d**2*x**2/(a + b*x), x) + Integral(3*A*c**2*d*x/(a + b*x), x) +
 Integral(B*d**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) +
 Integral(3*B*c*d**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x),
x) + Integral(3*B*c**2*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x),
 x))/g
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(343) = 686$.

Time $= 0.11$ (sec) , antiderivative size $= 850$, normalized size of antiderivative $= 2.39$

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algori
thm="maxima")
```

output

```
3*A*c^2*d*i^3*(x/(b*g) - a*log(b*x + a)/(b^2*g)) - 1/6*A*d^3*i^3*(6*a^3*lo
g(b*x + a)/(b^4*g) - (2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/(b^3*g)) + 3/2*A*c*
d^2*i^3*(2*a^2*log(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A*c^3*i^3
*log(b*g*x + a*g)/(b*g) - 1/6*(11*b^2*c^3*i^3 - 15*a*b*c^2*d*i^3 + 6*a^2*c
*d^2*i^3)*B*log(d*x + c)/(b^3*g) + (b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^
2*b*c*d^2*i^3 - a^3*d^3*i^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) +
 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^4*g) + 1/6*(2*B*b^3*d^3*i^3*
x^3*log(e) + ((9*i^3*log(e) - i^3)*b^3*c*d^2 - (3*i^3*log(e) - i^3)*a*b^2*
d^3)*B*x^2 + 3*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*d^2*i^3 - a^3*
d^3*i^3)*B*log(b*x + a)^2 + ((18*i^3*log(e) - 7*i^3)*b^3*c^2*d - 6*(3*i^3*
log(e) - 2*i^3)*a*b^2*c*d^2 + (6*i^3*log(e) - 5*i^3)*a^2*b*d^3)*B*x + (2*B
*b^3*d^3*i^3*x^3 + 3*(3*b^3*c*d^2*i^3 - a*b^2*d^3*i^3)*B*x^2 + 6*(3*b^3*c^
2*d*i^3 - 3*a*b^2*c*d^2*i^3 + a^2*b*d^3*i^3)*B*x + (6*b^3*c^3*i^3*log(e) -
 18*(i^3*log(e) - i^3)*a*b^2*c^2*d + 9*(2*i^3*log(e) - 3*i^3)*a^2*b*c*d^2
 - (6*i^3*log(e) - 11*i^3)*a^3*d^3)*B)*log(b*x + a) - (2*B*b^3*d^3*i^3*x^3
 + 3*(3*b^3*c*d^2*i^3 - a*b^2*d^3*i^3)*B*x^2 + 6*(3*b^3*c^2*d*i^3 - 3*a*b^2
*c*d^2*i^3 + a^2*b*d^3*i^3)*B*x + 6*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a
^2*b*c*d^2*i^3 - a^3*d^3*i^3)*B*log(b*x + a))*log(d*x + c))/(b^4*g)
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{bgx+ag}\,dx$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algori
thm="giac")
```

output

```
integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x),x
)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x),
x)
```

Reduce [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i\left(-6\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{bx+a}dx\right)b^5c^3 - 6\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x^3}{bx+a}dx\right)b^5d^3 - 18\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x^2}{bx+a}dx\right)b^5c\,d^2 - 18\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{bx+a}dx\right)b^5c^2d + 6\log(a+bx)a^4d^3 - 18\log(a+bx)a^3bcd^2 + 18\log(a+bx)a^2b^2c^2d - 6\log(a+bx)ab^3c^3 - 6a^3bd^3x + 18a^2b^2cd^2x + 3a^2b^2d^3x^2 - 18ab^3c^2dx - 9ab^3cd^2x^2 - 2ab^3d^3x^3\right)}{6b^4g}$$

input
```
int((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
```

output
```
(i*( - 6*int(log((a*e + b*e*x)/(c + d*x))/(a + b*x),x)*b**5*c**3 - 6*int((
log((a*e + b*e*x)/(c + d*x))*x**3)/(a + b*x),x)*b**5*d**3 - 18*int((log((a
*e + b*e*x)/(c + d*x))*x**2)/(a + b*x),x)*b**5*c*d**2 - 18*int((log((a*e +
 b*e*x)/(c + d*x))*x)/(a + b*x),x)*b**5*c**2*d + 6*log(a + b*x)*a**4*d**3
 - 18*log(a + b*x)*a**3*b*c*d**2 + 18*log(a + b*x)*a**2*b**2*c**2*d - 6*log
(a + b*x)*a*b**3*c**3 - 6*a**3*b*d**3*x + 18*a**2*b**2*c*d**2*x + 3*a**2*b
**2*d**3*x**2 - 18*a*b**3*c**2*d*x - 9*a*b**3*c*d**2*x**2 - 2*a*b**3*d**3*
x**3))/(6*b**4*g)
```

3.25 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 373

$$\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx \\
&= -\frac{Bd^2(bc-ad)i^3x}{2b^3g^2} - \frac{B(bc-ad)^2i^3(c+dx)}{b^3g^2(a+bx)} - \frac{Bd(bc-ad)^2i^3\log\left(\frac{a+bx}{c+dx}\right)}{2b^4g^2} \\
&\quad + \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^2} \\
&\quad - \frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2(a+bx)} \\
&\quad + \frac{di^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^2} - \frac{5Bd(bc-ad)^2i^3\log(c+dx)}{2b^4g^2} \\
&\quad - \frac{3d(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} \\
&\quad + \frac{3Bd(bc-ad)^2i^3\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}
\end{aligned}$$

output
```
-1/2*B*d^2*(-a*d+b*c)*i^3*x/b^3/g^2-B*(-a*d+b*c)^2*i^3*(d*x+c)/b^3/g^2/(b*
x+a)-1/2*B*d*(-a*d+b*c)^2*i^3*ln((b*x+a)/(d*x+c))/b^4/g^2+2*d^2*(-a*d+b*c)
*i^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/g^2-(-a*d+b*c)^2*i^3*(d*x+c)*
(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/g^2/(b*x+a)+1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*
(b*x+a)/(d*x+c)))/b^2/g^2-5/2*B*d*(-a*d+b*c)^2*i^3*ln(d*x+c)/b^4/g^2-3*d*(
-a*d+b*c)^2*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/
g^2+3*B*d*(-a*d+b*c)^2*i^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^2
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$$

$$= \frac{i^3\left(2Abd^2(3bc-2ad)x - bBd^2(bc-ad)x - \frac{2B(bc-ad)^3}{a+bx} - a^2Bd^3\log(a+bx) - 2Bd(bc-ad)^2\log(a+bx)\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^2,x]
```

output
```
(i^3*(2*A*b*d^2*(3*b*c - 2*a*d)*x - b*B*d^2*(b*c - a*d)*x - (2*B*(b*c - a*
d)^3)/(a + b*x) - a^2*B*d^3*Log[a + b*x] - 2*B*d*(b*c - a*d)^2*Log[a + b*x
] + 2*B*d^2*(3*b*c - 2*a*d)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + b^2*d
^3*x^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (2*(b*c - a*d)^3*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/(a + b*x) + 6*d*(b*c - a*d)^2*Log[a + b*x]*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]) + b^2*B*c^2*d*Log[c + d*x] + 2*B*d*(b*c -
 a*d)^2*Log[c + d*x] - 2*B*d*(-(b*c) + a*d)*(-3*b*c + 2*a*d)*Log[c + d*x]
- 3*B*d*(b*c - a*d)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b
*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(2*b^4*g^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2962, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^2}\,dx$$

$$\downarrow 2962$$

$$\frac{i^3(bc-ad)^2\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{g^2}$$

$$\downarrow 2793$$

$$\frac{i^3(bc-ad)^2\int\left(\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d^2}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d^2}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d}{b^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{b^3(a+bx)^2}\right.}{g^2}$$

$$\downarrow 2009$$

$$\frac{i^3(bc-ad)^2\left(\frac{2d^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{3d\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4}-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3(a+bx)}+\frac{d\left(B\log\right.}{2b^2(}\right.}{g^2}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^2
,x]
```

output
```
((b*c - a*d)^2*i^3*(-((B*(c + d*x))/(b^3*(a + b*x))) - (B*d)/(2*b^3*(b - (
d*(a + b*x))/(c + d*x))) - (B*d*Log[(a + b*x)/(c + d*x)])/(2*b^4) - ((c +
d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^3*(a + b*x)) + (d*(A + B*Log
[(e*(a + b*x))/(c + d*x)]))/(2*b^2*(b - (d*(a + b*x))/(c + d*x))^2) + (2*d
^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^4*(c + d*x)*(b - (d*
(a + b*x))/(c + d*x))) + (5*B*d*Log[b - (d*(a + b*x))/(c + d*x)])/(2*b^4)
- (3*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a +
b*x))])/b^4 + (3*B*d*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b^4))/g^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 862 vs. $2(365) = 730$.

Time = 3.30 (sec) , antiderivative size = 863, normalized size of antiderivative = 2.31

method	result
parts	$\frac{i^3 A\left(\frac{d^2\left(\frac{1}{2}bd\,x^2-2xda+3bcx\right)}{b^3}+\frac{3d\left(a^2d^2-2acdb+c^2b^2\right)\ln(bx+a)}{b^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{b^4(bx+a)}\right)}{g^2}-\frac{i^3B(da-bc)^4e^4}{}$
derivativedivides	$e(da-bc)\left(-\frac{A\,d^2e^3i^3(da-bc)\left(-\frac{3d\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{b^4e^4}+\frac{2d}{b^3e^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}+\frac{d}{2b^2e^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}\right.\right.$
default	$e(da-bc)\left(-\frac{A\,d^2e^3i^3(da-bc)\left(-\frac{3d\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{b^4e^4}+\frac{2d}{b^3e^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}+\frac{d}{2b^2e^2\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}\right.\right.$
risch	Expression too large to display

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x,method=_RETU
RNVERBOSE)
```

output
```
i^3*A/g^2*(d^2/b^3*(1/2*b*d*x^2-2*x*d*a+3*b*c*x)+3/b^4*d*(a^2*d^2-2*a*b*c*
d+b^2*c^2)*ln(b*x+a)-1/b^4*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/
(b*x+a))-i^3*B/g^2/d^5*(a*d-b*c)^4*e^4*((-1/2/b/e/d/((b*e/d+(a*d-b*c)*e/d/
(d*x+c))*d-b*e)-1/2/b^2/e^2*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/d+1/2*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*((b*e/d+(a*d
-b*c)*e/d/(d*x+c))*d-2*b*e)/b^2/e^2/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)^
2)/(a*d-b*c)^2*d^7/b^2/e^2-(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d
-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))/(a*d-b*c)^2*d^5/b^3/e^
3-2*(1/b/e*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/d-ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c)
)*d-b*e))/(a*d-b*c)^2*d^7/b^3/e^3-3/2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/(a
*d-b*c)^2*d^6/b^4/e^4+3*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)
/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*
e)/b/e)/d)/(a*d-b*c)^2*d^7/b^4/e^4)
```

Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algo
rithm="fricas")
```

output
```
integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3
+ (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log((b
*e*x + a*e)/(d*x + c)))/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1501 vs. $2(364) = 728$.

Time = 0.12 (sec) , antiderivative size = 1501, normalized size of antiderivative = 4.02

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algo
rithm="maxima")
```

output

```
-3*A*(a^2/(b^4*g^2*x + a*b^3*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^
2))*c*d^2*i^3 + 1/2*(2*a^3/(b^5*g^2*x + a*b^4*g^2) + 6*a^2*log(b*x + a)/(b
^4*g^2) + (b*x^2 - 4*a*x)/(b^3*g^2))*A*d^3*i^3 + 3*A*c^2*d*i^3*(a/(b^3*g^2
*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - B*c^3*i^3*(log(b*e*x/(d*x + c)
 + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1/(b^2*g^2*x + a*b*g^2) + d*log(
b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - A
*c^3*i^3/(b^2*g^2*x + a*b*g^2) - 1/2*(5*b^3*c^3*d*i^3 - 3*a*b^2*c^2*d^2*i^
3 - 2*a^2*b*c*d^3*i^3 + 2*a^3*d^4*i^3)*B*log(d*x + c)/(b^5*c*g^2 - a*b^4*d
*g^2) + 1/2*((b^4*c*d^3*i^3*log(e) - a*b^3*d^4*i^3*log(e))*B*x^3 + ((6*i^3
*log(e) - i^3)*b^4*c^2*d^2 - (9*i^3*log(e) - 2*i^3)*a*b^3*c*d^3 + (3*i^3*l
og(e) - i^3)*a^2*b^2*d^4)*B*x^2 + ((6*i^3*log(e) - i^3)*a*b^3*c^2*d^2 - 2*
(5*i^3*log(e) - i^3)*a^2*b^2*c*d^3 + (4*i^3*log(e) - i^3)*a^3*b*d^4)*B*x +
 3*((b^4*c^3*d*i^3 - 3*a*b^3*c^2*d^2*i^3 + 3*a^2*b^2*c*d^3*i^3 - a^3*b*d^4
*i^3)*B*x + (a*b^3*c^3*d*i^3 - 3*a^2*b^2*c^2*d^2*i^3 + 3*a^3*b*c*d^3*i^3 -
 a^4*d^4*i^3)*B)*log(b*x + a)^2 + 2*(3*(i^3*log(e) + i^3)*a*b^3*c^3*d - 6*
(i^3*log(e) + i^3)*a^2*b^2*c^2*d^2 + 4*(i^3*log(e) + i^3)*a^3*b*c*d^3 - (i
^3*log(e) + i^3)*a^4*d^4)*B + ((b^4*c*d^3*i^3 - a*b^3*d^4*i^3)*B*x^3 + 3*(
2*b^4*c^2*d^2*i^3 - 3*a*b^3*c*d^3*i^3 + a^2*b^2*d^4*i^3)*B*x^2 + (6*b^4*c^
3*d*i^3*log(e) - 18*(i^3*log(e) - i^3)*a*b^3*c^2*d^2 + 9*(2*i^3*log(e) - 3
*i^3)*a^2*b^2*c*d^3 - (6*i^3*log(e) - 11*i^3)*a^3*b*d^4)*B*x - (18*a^2*...
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algo
rithm="giac")
```

output

```
integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)
^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(a\,g+b\,g\,x)^2}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2
,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2
, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

input
```
int((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x)
```

output

```
(i*( - 2*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(a**2 + 2*a*b*x + b**2*x*
*2),x)*a**3*b**5*d**4 + 2*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(a**2 +
2*a*b*x + b**2*x**2),x)*a**2*b**6*c*d**3 - 2*int((log((a*e + b*e*x)/(c + d
*x))*x**3)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**6*d**4*x + 2*int((log((
a*e + b*e*x)/(c + d*x))*x**3)/(a**2 + 2*a*b*x + b**2*x**2),x)*a*b**7*c*d**
3*x - 6*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(a**2 + 2*a*b*x + b**2*x**
2),x)*a**3*b**5*c*d**3 + 6*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(a**2 +
 2*a*b*x + b**2*x**2),x)*a**2*b**6*c**2*d**2 - 6*int((log((a*e + b*e*x)/(c
 + d*x))*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**6*c*d**3*x + 6*int(
(log((a*e + b*e*x)/(c + d*x))*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a*b**7
*c**2*d**2*x - 6*int((log((a*e + b*e*x)/(c + d*x))*x)/(a**2 + 2*a*b*x + b*
*2*x**2),x)*a**3*b**5*c**2*d**2 + 6*int((log((a*e + b*e*x)/(c + d*x))*x)/(
a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**6*c**3*d - 6*int((log((a*e + b*e*x)
/(c + d*x))*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**6*c**2*d**2*x + 6*i
nt((log((a*e + b*e*x)/(c + d*x))*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a*b**7
*c**3*d*x - 6*log(a + b*x)*a**6*d**4 + 18*log(a + b*x)*a**5*b*c*d**3 - 6*l
og(a + b*x)*a**5*b*d**4*x - 18*log(a + b*x)*a**4*b**2*c**2*d**2 + 18*log(a
 + b*x)*a**4*b**2*c*d**3*x + 6*log(a + b*x)*a**3*b**3*c**3*d - 18*log(a +
b*x)*a**3*b**3*c**2*d**2*x + 6*log(a + b*x)*a**2*b**4*c**3*d*x - 2*log(a +
 b*x)*a*b**5*c**4 - 2*log(a + b*x)*b**6*c**4*x + 2*log(c + d*x)*a*b**5*...
```

3.26 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 345

$$\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx \\
&= -\frac{2Bd(bc-ad)i^3(c+dx)}{b^3g^3(a+bx)} - \frac{B(bc-ad)i^3(c+dx)^2}{4b^2g^3(a+bx)^2} \\
&\quad + \frac{d^3i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^3} \\
&\quad - \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^3(a+bx)} \\
&\quad - \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^3(a+bx)^2} - \frac{Bd^2(bc-ad)i^3\log(c+dx)}{b^4g^3} \\
&\quad - \frac{3d^2(bc-ad)i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} \\
&\quad + \frac{3Bd^2(bc-ad)i^3\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}
\end{aligned}$$

output
```
-2*B*d*(-a*d+b*c)*i^3*(d*x+c)/b^3/g^3/(b*x+a)-1/4*B*(-a*d+b*c)*i^3*(d*x+c)
^2/b^2/g^3/(b*x+a)^2+d^3*i^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/g^3-2
*d*(-a*d+b*c)*i^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/g^3/(b*x+a)-1/2*
(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g^3/(b*x+a)^2-B*d
^2*(-a*d+b*c)*i^3*ln(d*x+c)/b^4/g^3-3*d^2*(-a*d+b*c)*i^3*(A+B*ln(e*(b*x+a)
/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^3+3*B*d^2*(-a*d+b*c)*i^3*polylo
g(2,b*(d*x+c)/d/(b*x+a))/b^4/g^3
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.91

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$$
$$= \frac{i^3\left(4Abd^3x - \frac{B(bc-ad)^3}{(a+bx)^2} - \frac{10Bd(bc-ad)^2}{a+bx} + 10Bd^2(-bc+ad)\log(a+bx) + 4Bd^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right) - \frac{2(}{}\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^3,x]
```

output
```
(i^3*(4*A*b*d^3*x - (B*(b*c - a*d)^3)/(a + b*x)^2 - (10*B*d*(b*c - a*d)^2)
/(a + b*x) + 10*B*d^2*(-(b*c) + a*d)*Log[a + b*x] + 4*B*d^3*(a + b*x)*Log[
(e*(a + b*x))/(c + d*x)] - (2*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c +
d*x)]))/(a + b*x)^2 - (12*d*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/(a + b*x) + 12*d^2*(b*c - a*d)*Log[a + b*x]*(A + B*Log[(e*(a + b*x))
/(c + d*x)]) + 6*B*d^2*(b*c - a*d)*Log[c + d*x] + 6*B*d^2*(-(b*c) + a*d)*(
Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog
[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(4*b^4*g^3)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2962, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{i^3(bc-ad)\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g^3}$$

$$\downarrow 2793$$

$$\frac{i^3(bc-ad)\int \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d^3}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d^2}{b^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d}{b^3(a+bx)^2}+\frac{(c+dx)^3\left(A+B\log\left(\right.\right.\right.}{b^2(a+bx)}}{g^3}$$

$$\downarrow 2009$$

$$\frac{i^3(bc-ad)\left(\frac{d^3(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{3d^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4}-\frac{2d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3(a+bx)}-\frac{(c+dx}{}\right.}{g^3}$$

input

```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^3
,x]
```

output
```
((b*c - a*d)*i^3*((-2*B*d*(c + d*x))/(b^3*(a + b*x)) - (B*(c + d*x)^2)/(4*
b^2*(a + b*x)^2) - (2*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b
^3*(a + b*x)) - ((c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b^2*
(a + b*x)^2) + (d^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^4*(
c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*d^2*Log[b - (d*(a + b*x))/(c
+ d*x)])/b^4 - (3*d^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c +
 d*x))/(d*(a + b*x))])/b^4 + (3*B*d^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x
))])/b^4))/g^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(341) = 682$.

Time = 3.22 (sec) , antiderivative size = 732, normalized size of antiderivative = 2.12

method	result
parts	$\frac{i^3 A\left(\frac{x\,d^3}{b^3}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{2b^4(bx+a)^2}-\frac{3d^2(da-bc)\ln(bx+a)}{b^4}-\frac{3d\left(a^2d^2-2acdb+c^2b^2\right)}{b^4(bx+a)}\right)}{g^3}-\frac{i^3B(da-bc)^4e^4\left(\frac{2\left(-\right.\right.}{}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{i^3d^2e^3A\left(\frac{d^2}{b^3e^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}-\frac{3d^2\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{b^4e^4}-\frac{1}{2b^2e^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{3d^2\ln\left(\frac{b}{d}\right.\right.}{g^3}$
default	$-\frac{e(da-bc)\left(\frac{i^3d^2e^3A\left(\frac{d^2}{b^3e^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}-\frac{3d^2\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{b^4e^4}-\frac{1}{2b^2e^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{3d^2\ln\left(\frac{b}{d}\right.\right.}{g^3}$
risch	Expression too large to display

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x,method=_RETU
RNVERBOSE)
```

output
```
i^3*A/g^3*(x*d^3/b^3-1/2/b^4*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3
)/(b*x+a)^2-3/b^4*d^2*(a*d-b*c)*ln(b*x+a)-3/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c
^2)/(b*x+a))-i^3*B/g^3/d^5*(a*d-b*c)^4*e^4*(2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*
x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))/(a*
d-b*c)^3*d^6/b^3/e^3+(1/b/e*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/d-ln(b
*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d
-b*c)*e/d/(d*x+c))*d-b*e))/(a*d-b*c)^3*d^8/b^3/e^3+3/2*ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))^2/(a*d-b*c)^3*d^7/b^4/e^4+(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))
^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)/(a
*d-b*c)^3*d^5/b^2/e^2-3*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)
/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*
e)/b/e)/d)/(a*d-b*c)^3*d^8/b^4/e^4)
```

Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algo
rithm="fricas")
```

output
```
integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3
+ (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log((b
*e*x + a*e)/(d*x + c)))/(b^3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a
^3*g^3), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$$

$$= \frac{i^3\left(\int \frac{Ac^3}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{Ad^3x^3}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{Bc^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{3Acd^2x^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\right.}{}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3,x)
```

output
```
i**3*(Integral(A*c**3/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x)
+ Integral(A*d**3*x**3/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x)
 + Integral(B*c**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x
 + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(3*A*c*d**2*x**2/(a**3 + 3*a**
2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(3*A*c**2*d*x/(a**3 + 3*a
**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(B*d**3*x**3*log(a*e/(c
 + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)
, x) + Integral(3*B*c*d**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3
 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(3*B*c**2*d*x*log
(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**
3*x**3), x))/g**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2302 vs. $2(340) = 680$.

Time $= 0.17$ (sec) , antiderivative size $= 2302$, normalized size of antiderivative $= 6.67$

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algo
rithm="maxima")
```

output

```
-3/4*B*c^2*d*i^3*(2*(2*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*
g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + (3*a*b*c - a^2*d + 2*(2*b^2*c - a
*b*d)*x)/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2
*b^3*c - a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*
a*b^3*c*d + a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2
 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3)) - 1/2*A*d^3*i^3*((6*a^2*b*x + 5*a^3)/(
b^6*g^3*x^2 + 2*a*b^5*g^3*x + a^2*b^4*g^3) - 2*x/(b^3*g^3) + 6*a*log(b*x +
 a)/(b^4*g^3)) + 3/2*A*c*d^2*i^3*((4*a*b*x + 3*a^2)/(b^5*g^3*x^2 + 2*a*b^4
*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3)) + 1/4*B*c^3*i^3*((2*b*d*
x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*
x + (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b
^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d
+ a^2*b*d^2)*g^3)) - 3/2*(2*b*x + a)*A*c^2*d*i^3/(b^4*g^3*x^2 + 2*a*b^3*g^
3*x + a^2*b^2*g^3) - 1/2*A*c^3*i^3/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^
3) - 1/2*(2*b^3*c^3*d^2*i^3 + 8*a*b^2*c^2*d^3*i^3 - 13*a^2*b*c*d^4*i^3 + 5
*a^3*d^5*i^3)*B*log(d*x + c)/(b^6*c^2*g^3 - 2*a*b^5*c*d*g^3 + a^2*b^4*d^2*
g^3) + 1/4*(4*(b^5*c^2*d^3*i^3*log(e) - 2*a*b^4*c*d^4*i^3*log(e) + a^2*b^3
*d^5*i^3*log(e))*B*x^3 + 8*(a*b^4*c^2*d^3*i^3*log(e) - 2*a^2*b^3*c*d^4*i^3
*log(e) + a^3*b^2*d^5*i^3*log(e))*B*x^2 + 2*(12*(i^3*log(e) + i^3)*a*b^...
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^3}\,dx$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algo
rithm="giac")
```

output

```
integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)
^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(a\,g+b\,g\,x)^3}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^3
,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^3
, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \text{too large to display}$$

input
```
int((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x)
```

output

```
(i*( - 4*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(a**3 + 3*a**2*b*x + 3*a*
b**2*x**2 + b**3*x**3),x)*a**6*b**5*d**5 + 8*int((log((a*e + b*e*x)/(c + d
*x))*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**5*b**6*c*
d**4 - 8*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(a**3 + 3*a**2*b*x + 3*a*
b**2*x**2 + b**3*x**3),x)*a**5*b**6*d**5*x - 4*int((log((a*e + b*e*x)/(c +
 d*x))*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**4*b**7*
c**2*d**3 + 16*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(a**3 + 3*a**2*b*x
+ 3*a*b**2*x**2 + b**3*x**3),x)*a**4*b**7*c*d**4*x - 4*int((log((a*e + b*e
*x)/(c + d*x))*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a*
*4*b**7*d**5*x**2 - 8*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(a**3 + 3*a*
*2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**3*b**8*c**2*d**3*x + 8*int((log(
(a*e + b*e*x)/(c + d*x))*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x
**3),x)*a**3*b**8*c*d**4*x**2 - 4*int((log((a*e + b*e*x)/(c + d*x))*x**3)/
(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**2*b**9*c**2*d**3*x**
2 - 12*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b*
*2*x**2 + b**3*x**3),x)*a**6*b**5*c*d**4 + 24*int((log((a*e + b*e*x)/(c +
d*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**5*b**6*c
**2*d**3 - 24*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(a**3 + 3*a**2*b*x +
 3*a*b**2*x**2 + b**3*x**3),x)*a**5*b**6*c*d**4*x - 12*int((log((a*e + b*e
*x)/(c + d*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)...
```

3.27 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 310

$$\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx \\
&= -\frac{Bd^2i^3(c+dx)}{b^3g^4(a+bx)} - \frac{Bdi^3(c+dx)^2}{4b^2g^4(a+bx)^2} - \frac{Bi^3(c+dx)^3}{9bg^4(a+bx)^3} \\
&\quad - \frac{d^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^4(a+bx)} \\
&\quad - \frac{di^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^4(a+bx)^2} - \frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bg^4(a+bx)^3} \\
&\quad - \frac{d^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} + \frac{Bd^3i^3\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4}
\end{aligned}$$

output

```
-B*d^2*i^3*(d*x+c)/b^3/g^4/(b*x+a)-1/4*B*d*i^3*(d*x+c)^2/b^2/g^4/(b*x+a)^2
-1/9*B*i^3*(d*x+c)^3/b/g^4/(b*x+a)^3-d^2*i^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*
x+c)))/b^3/g^4/(b*x+a)-1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2
/g^4/(b*x+a)^2-1/3*i^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g^4/(b*x+a)
^3-d^3*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^4+B
*d^3*i^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^4
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.99

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$$

$$= \frac{i^3\left(-\frac{4B(bc-ad)^3}{(a+bx)^3}-\frac{21Bd(bc-ad)^2}{(a+bx)^2}+\frac{66Bd^2(-bc+ad)}{a+bx}-66Bd^3\log(a+bx)-\frac{12(bc-ad)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3}-\frac{54d(bc-a}{}\right.}{}$$

input

```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^4,x]
```

output

```
(i^3*((-4*B*(b*c - a*d)^3)/(a + b*x)^3 - (21*B*d*(b*c - a*d)^2)/(a + b*x)^
2 + (66*B*d^2*(-(b*c) + a*d))/(a + b*x) - 66*B*d^3*Log[a + b*x] - (12*(b*c
 - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)^3 - (54*d*(b*c -
 a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)^2 + (108*d^2*(-(b*
c) + a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) + 36*d^3*Log[a +
 b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 66*B*d^3*Log[c + d*x] - 18*B*
d^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Po
lyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(36*b^4*g^4)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.87, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {2962, 2780, 2741, 2780, 2741, 2780, 2741, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^4}\,dx$$

$$\downarrow 2962$$

$$\frac{i^3 \int \frac{(c+dx)^4 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)} d\frac{a+bx}{c+dx}}{g^4}$$

$$\downarrow 2780$$

$$\frac{i^3 \left(\frac{\int \frac{(c+dx)^4 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^4} d\frac{a+bx}{c+dx}}{b} + \frac{d \int \frac{(c+dx)^3 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)} d\frac{a+bx}{c+dx}}{b} \right)}{g^4}$$

$$\downarrow 2741$$

$$\frac{i^3 \left(\frac{d \int \frac{(c+dx)^3 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)} d\frac{a+bx}{c+dx}}{b} + \frac{-\frac{(c+dx)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3} - \frac{B(c+dx)^3}{9(a+bx)^3}}{b} \right)}{g^4}$$

$$\downarrow 2780$$

$$\frac{i^3 \left(\frac{d \left(\frac{\int \frac{(c+dx)^3 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3} d\frac{a+bx}{c+dx}}{b} + \frac{d \int \frac{(c+dx)^2 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)} d\frac{a+bx}{c+dx}}{b} \right)}{b} + \frac{-\frac{(c+dx)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3} - \frac{B(c+dx)^3}{9(a+bx)^3}}{b} \right)}{g^4}$$

$$\downarrow 2741$$

$$\frac{i^3 \left(\frac{d \left(\frac{d \int \frac{(c+dx)^2 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)} d\frac{a+bx}{c+dx}}{b} + \frac{-\frac{(c+dx)^2 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2} - \frac{B(c+dx)^2}{4(a+bx)^2}}{b} \right)}{b} + \frac{-\frac{(c+dx)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3} - \frac{B(c+dx)^3}{9(a+bx)^3}}{b} \right)}{g^4}$$

$$\downarrow 2780$$

$$\frac{i^3\left(d\left(\frac{d\left(\frac{\int\frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}}{b}\right)}{b}+$$

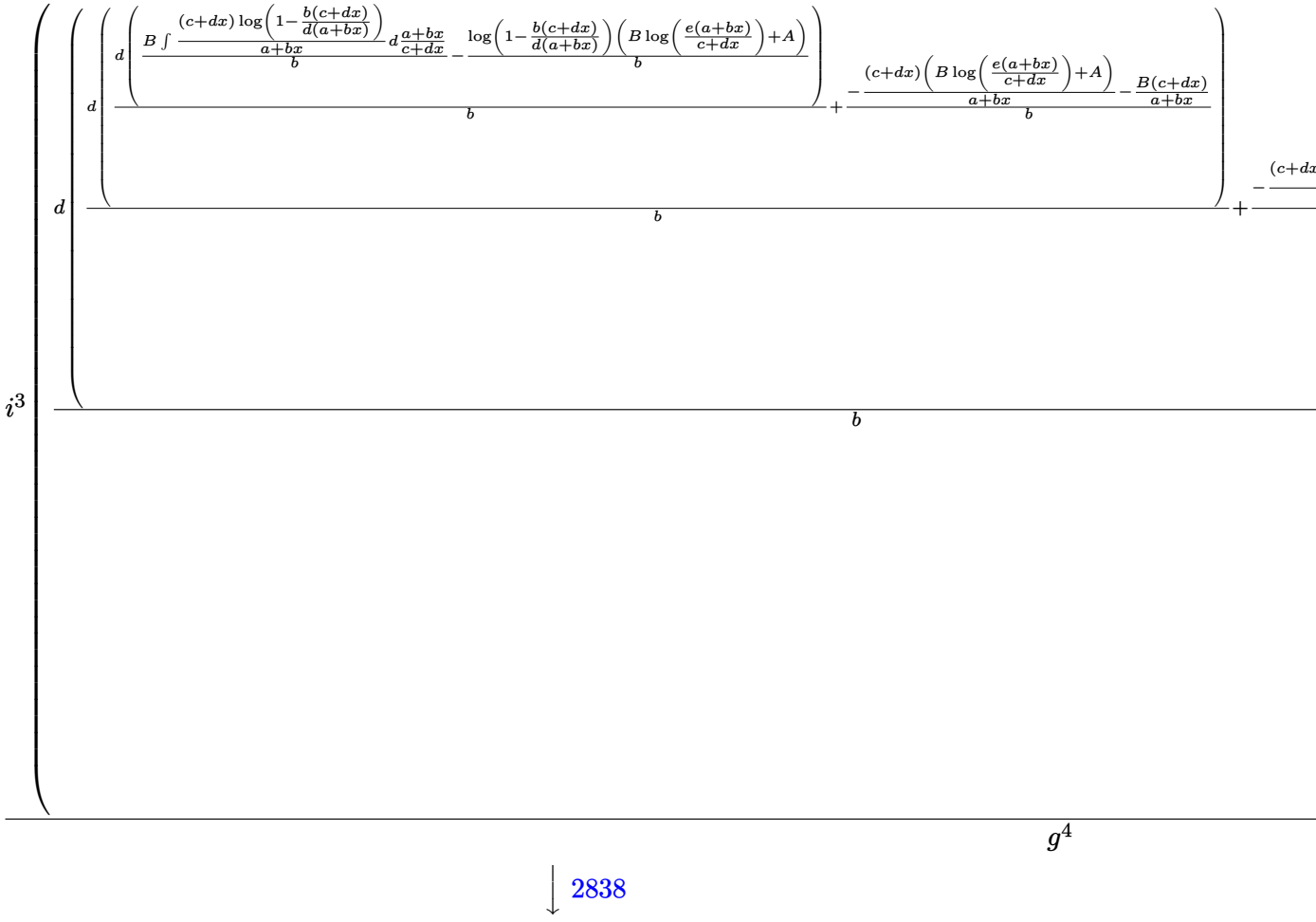
$$g^4$$

$$\downarrow 2741$$

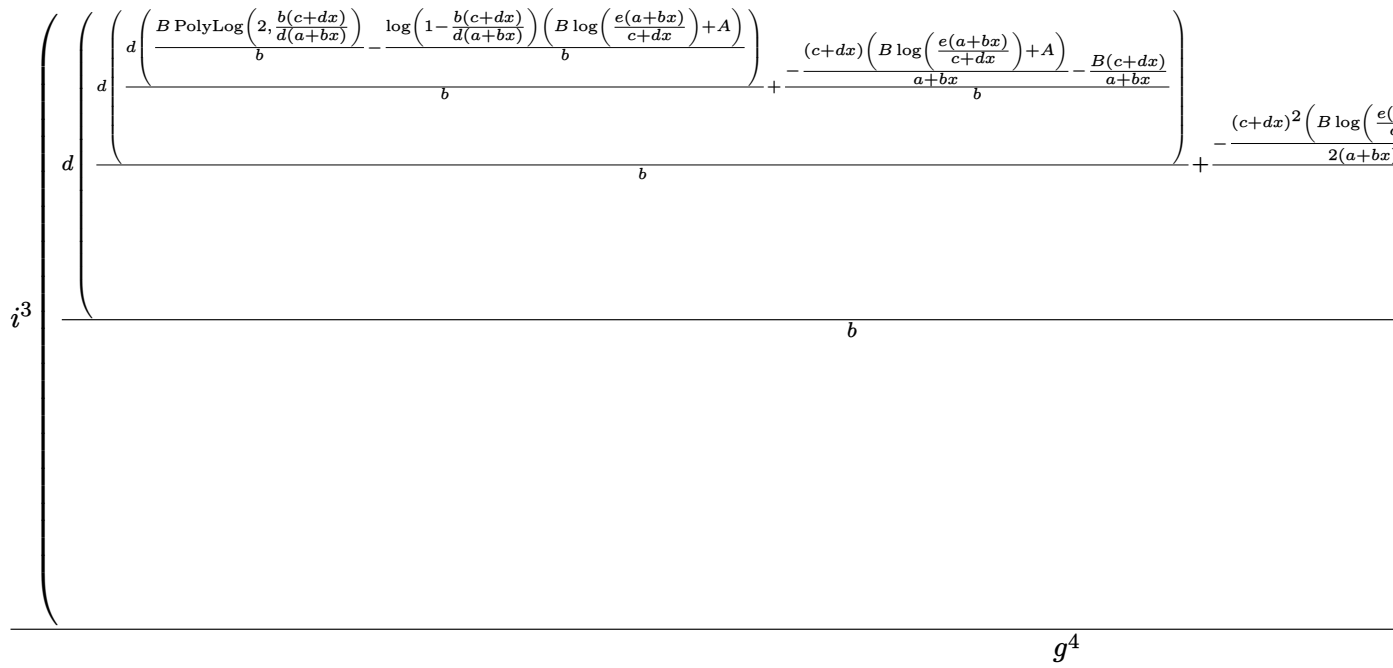
$$\frac{i^3\left(d\left(\frac{d\left(\frac{d\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}}{b}\right)}{b}+$$

$$g^4$$

$$\downarrow 2779$$



$$\downarrow 2838$$



input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^4
,x]
```

output
```
(i^3*((-1/9*(B*(c + d*x)^3)/(a + b*x)^3 - ((c + d*x)^3*(A + B*Log[(e*(a +
b*x))/(c + d*x)]))/(3*(a + b*x)^3))/b + (d*((-1/4*(B*(c + d*x)^2)/(a + b*x
)^2 - ((c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^2))/
b + (d*((-((B*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[(e*(a + b*x))/
(c + d*x)]))/(a + b*x))/b + (d*(-(((A + B*Log[(e*(a + b*x))/(c + d*x)])*Lo
g[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(d*(a
 + b*x))])/b))/b))/b))/b))/g^4
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2779

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(302) = 604$.

Time = 3.26 (sec) , antiderivative size = 685, normalized size of antiderivative = 2.21

method	result
parts	$\frac{i^3 A\left(-\frac{3d\left(a^2d^2-2acdb+c^2b^2\right)}{2b^4(bx+a)^2}+\frac{d^3\ln(bx+a)}{b^4}+\frac{3d^2(da-bc)}{b^4(bx+a)}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{3b^4(bx+a)^3}\right)}{g^4}-\frac{i^3B(da-bc)^4e^4\left(-\frac{\left(\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{i^3d^2e^3A\left(-\frac{d^3\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{b^4e^4}-\frac{1}{3be\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}+\frac{d^3\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{b^4e^4}-\frac{d^2}{b^3e^3\left(\frac{be}{d}+\frac{(da-b\cdots}{d(dx+\cdots}\cdots\right.}\cdots\right.}{(da-bc)g^4}\cdots\right.}{\cdots}$
default	$-\frac{e(da-bc)\left(-\frac{i^3d^2e^3A\left(-\frac{d^3\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{b^4e^4}-\frac{1}{3be\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}+\frac{d^3\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{b^4e^4}-\frac{d^2}{b^3e^3\left(\frac{be}{d}+\frac{(da-b\cdots}{d(dx+\cdots}\cdots\right.}\cdots\right.}{(da-bc)g^4}\cdots\right.}{\cdots}$
risch	Expression too large to display

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x,method=_RETU
RNVERBOSE)
```

output
```
i^3*A/g^4*(-3/2*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^2+d^3/b^4*ln(b*x
+a)+3/b^4*d^2*(a*d-b*c)/(b*x+a)-1/3*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+
b^3*c^3)/b^4/(b*x+a)^3)-i^3*B/g^4/d^5*(a*d-b*c)^4*e^4*(-(-1/(b*e/d+(a*d-b*
c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*
x+c)))/(a*d-b*c)^4*d^7/b^3/e^3-1/2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/(a*d-
b*c)^4*d^8/b^4/e^4-(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c
)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)/(a*d-b*c)^4*d^6/b^2/e^
2-(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/
9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)/(a*d-b*c)^4*d^5/b/e+(dilog(-((b*e/d+(a*
d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*
e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)/(a*d-b*c)^4*d^9/b^4/e^4)
```

Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^4}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algo
rithm="fricas")
```

output
```
integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3
+ (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log((b
*e*x + a*e)/(d*x + c)))/(b^4*g^4*x^4 + 4*a*b^3*g^4*x^3 + 6*a^2*b^2*g^4*x^2
 + 4*a^3*b*g^4*x + a^4*g^4), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$$
$$= \frac{i^3\left(\int \frac{Ac^3}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx+\int \frac{Ad^3x^3}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx+\int \frac{Bc^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx\right.}{}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4,x)
```

output
```
i**3*(Integral(A*c**3/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**
3 + b**4*x**4), x) + Integral(A*d**3*x**3/(a**4 + 4*a**3*b*x + 6*a**2*b**2
*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(B*c**3*log(a*e/(c + d*x)
 + b*e*x/(c + d*x))/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3
+ b**4*x**4), x) + Integral(3*A*c*d**2*x**2/(a**4 + 4*a**3*b*x + 6*a**2*b*
*2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(3*A*c**2*d*x/(a**4 + 4
*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(B
*d**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**4 + 4*a**3*b*x + 6*a**
2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(3*B*c*d**2*x**2*lo
g(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 +
 4*a*b**3*x**3 + b**4*x**4), x) + Integral(3*B*c**2*d*x*log(a*e/(c + d*x)
+ b*e*x/(c + d*x))/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 +
 b**4*x**4), x))/g**4
```

Maxima [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^4}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algo
rithm="maxima")
```

output

```
-1/6*B*d^3*i^3*((18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3 + 6*(b^3*x^3 + 3*a*b^2
*x^2 + 3*a^2*b*x + a^3)*log(b*x + a))*log(d*x + c)/(b^7*g^4*x^3 + 3*a*b^6*
g^4*x^2 + 3*a^2*b^5*g^4*x + a^3*b^4*g^4) - 6*integrate(1/6*(6*b^4*d*x^4*lo
g(e) + 45*a^2*b^2*d*x^2 + 38*a^3*b*d*x + 11*a^4*d + 6*(b^4*c*log(e) + 3*a*
b^3*d)*x^3 + 6*(2*b^4*d*x^4 + 6*a^2*b^2*d*x^2 + 4*a^3*b*d*x + a^4*d + (b^4
*c + 4*a*b^3*d)*x^3)*log(b*x + a))/(b^8*d*g^4*x^5 + a^4*b^4*c*g^4 + (b^8*c
*g^4 + 4*a*b^7*d*g^4)*x^4 + 2*(2*a*b^7*c*g^4 + 3*a^2*b^6*d*g^4)*x^3 + 2*(3
*a^2*b^6*c*g^4 + 2*a^3*b^5*d*g^4)*x^2 + (4*a^3*b^5*c*g^4 + a^4*b^4*d*g^4)*
x), x)) - 1/6*B*c*d^2*i^3*(6*(3*b^2*x^2 + 3*a*b*x + a^2)*log(b*e*x/(d*x +
c) + a*e/(d*x + c))/(b^6*g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3
*b^3*g^4) + (11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*c^2 - 3*a
*b^3*c*d + a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a^3*b*d^2
)*x)/((b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c^2 - 2*a^2
*b^6*c*d + a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d + a^4*b^4
*d^2)*g^4*x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6*(3*b^2*
c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*
a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3
)*log(d*x + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*
g^4)) - 1/12*B*c^2*d*i^3*(6*(3*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c
))/(b^5*g^4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) + (5...
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^4}\,dx$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algo
rithm="giac")
```

output

```
integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)
^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = \int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(a\,g+b\,g\,x)^4}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^4
,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^4
, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = \text{too large to display}$$

input
```
int((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x)
```

output

```
(i*( - 36*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(a**4 + 4*a**3*b*x + 6*a
**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4),x)*a**8*b**5*d**6 + 108*int((lo
g((a*e + b*e*x)/(c + d*x))*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4
*a*b**3*x**3 + b**4*x**4),x)*a**7*b**6*c*d**5 - 108*int((log((a*e + b*e*x)
/(c + d*x))*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 +
b**4*x**4),x)*a**7*b**6*d**6*x - 108*int((log((a*e + b*e*x)/(c + d*x))*x**
3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4),x)*a
**6*b**7*c**2*d**4 + 324*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(a**4 + 4
*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4),x)*a**6*b**7*c*d
**5*x - 108*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(a**4 + 4*a**3*b*x + 6
*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4),x)*a**6*b**7*d**6*x**2 + 36*i
nt((log((a*e + b*e*x)/(c + d*x))*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x*
*2 + 4*a*b**3*x**3 + b**4*x**4),x)*a**5*b**8*c**3*d**3 - 324*int((log((a*e
 + b*e*x)/(c + d*x))*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**
3*x**3 + b**4*x**4),x)*a**5*b**8*c**2*d**4*x + 324*int((log((a*e + b*e*x)/
(c + d*x))*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b
**4*x**4),x)*a**5*b**8*c*d**5*x**2 - 36*int((log((a*e + b*e*x)/(c + d*x))*
x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4),x
)*a**5*b**8*d**6*x**3 + 108*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(a**4
+ 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4),x)*a**4*b*...
```

3.28 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 89

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = -\frac{Bi^3(c+dx)^4}{16(bc-ad)g^5(a+bx)^4} - \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)g^5(a+bx)^4}$$

output

```
-1/16*B*i^3*(d*x+c)^4/(-a*d+b*c)/g^5/(b*x+a)^4-1/4*i^3*(d*x+c)^4*(A+B*ln(e
*(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^5/(b*x+a)^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 427 vs. 2(89) = 178.

Time = 0.54 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.80

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = -\frac{i^3\left(4Ab^4c^4+b^4Bc^4-4a^4Ad^4-a^4Bd^4+16Ab^4c^3dx+4b^4Bc^3dx-16a^3Abd^4x-4a^3bBd^4x+24Ab^4c^{}\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^5,x]
```

output
```
-1/16*(i^3*(4*A*b^4*c^4 + b^4*B*c^4 - 4*a^4*A*d^4 - a^4*B*d^4 + 16*A*b^4*c
^3*d*x + 4*b^4*B*c^3*d*x - 16*a^3*A*b*d^4*x - 4*a^3*b*B*d^4*x + 24*A*b^4*c
^2*d^2*x^2 + 6*b^4*B*c^2*d^2*x^2 - 24*a^2*A*b^2*d^4*x^2 - 6*a^2*b^2*B*d^4*
x^2 + 16*A*b^4*c*d^3*x^3 + 4*b^4*B*c*d^3*x^3 - 16*a*A*b^3*d^4*x^3 - 4*a*b^
3*B*d^4*x^3 + 4*B*d^4*(a + b*x)^4*Log[a + b*x] + 4*B*(-(a^4*d^4) - 4*a^3*b
*d^4*x - 6*a^2*b^2*d^4*x^2 - 4*a*b^3*d^4*x^3 + b^4*c*(c^3 + 4*c^2*d*x + 6*
c*d^2*x^2 + 4*d^3*x^3))*Log[(e*(a + b*x))/(c + d*x)] - 4*a^4*B*d^4*Log[c +
 d*x] - 16*a^3*b*B*d^4*x*Log[c + d*x] - 24*a^2*b^2*B*d^4*x^2*Log[c + d*x]
- 16*a*b^3*B*d^4*x^3*Log[c + d*x] - 4*b^4*B*d^4*x^4*Log[c + d*x]))/(b^4*(b
*c - a*d)*g^5*(a + b*x)^4)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2962, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^5}\,dx$$

$$\downarrow 2962$$

$$\frac{i^3\int \frac{(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{i^3\left(-\frac{(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}-\frac{B(c+dx)^4}{16(a+bx)^4}\right)}{g^5(bc-ad)}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^5
,x]
```

output
```
(i^3*(-1/16*(B*(c + d*x)^4)/(a + b*x)^4 - ((c + d*x)^4*(A + B*Log[(e*(a +
b*x))/(c + d*x)]))/(4*(a + b*x)^4)))/((b*c - a*d)*g^5)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(85) = 170$.

Time = 2.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.08

method	result
derivativedivides	$-\frac{e(da-bc)\left(-\frac{i^3d^2e^3A}{4(da-bc)^2g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}+\frac{i^3d^2e^3B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{1}{16\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}\right)}{(da-bc)^2g^5}\right)}{d^2}$
default	$-\frac{e(da-bc)\left(-\frac{i^3d^2e^3A}{4(da-bc)^2g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}+\frac{i^3d^2e^3B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{1}{16\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}\right)}{(da-bc)^2g^5}\right)}{d^2}$
parts	$\frac{i^3A\left(-\frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{4b^4(bx+a)^4}+\frac{3d^2(da-bc)}{2b^4(bx+a)^2}-\frac{d^3}{b^4(bx+a)}-\frac{d\left(a^2d^2-2acdb+c^2b^2\right)}{b^4(bx+a)^3}\right)}{g^5}-\frac{i^3Be^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)}{d(dx+c)}\right.\right.}{4\left(\frac{be}{d}+\frac{(da-bc)}{d(dx+c)}\right.}\right.$
parallelrisch	$\frac{4Bx^3a^6c^2d^3i^3-4Bx^3a^3b^3c^5i^3+24Ax^2a^6c^3d^2i^3-24Ax^2a^4b^2c^5i^3+6Bx^2a^6c^3d^2i^3-6Bx^2a^4b^2c^5i^3+16Axa^6c^4di^3-}{}$
norman	$\frac{Bcd^3i^3x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(da-bc)g}+\frac{Bc^3di^3x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(da-bc)}+\frac{\left(4Ac^3i^3+i^3c^3B\right)x}{4ga}+\frac{3\left(4Aac^2di^3+4Abc^3i^3+Bac^2di^3+Bbc^3i^3\right)x^2}{8ga^2}+\frac{(4}{}$
risch	$-\frac{i^3B\left(4b^3d^3x^3+6ab^2d^3x^2+6b^3cd^2x^2+4a^2bd^3x+4ab^2cd^2x+4b^3c^2dx+a^3d^3+a^2bcd^2+ab^2c^2d+b^3c^3\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{4(bx+a)^4g^5b^4}$
orering	$\frac{(bx+a)\left(-2a^3bd^4x^5-2a^2b^2cd^3x^5-2ab^3c^2d^2x^5-2b^4c^3dx^5+7a^4d^4x^4-10a^3bcd^3x^4-10a^2b^2c^2d^2x^4-10ab^3c^3dx^4-5b}{}$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x,method=_RETU
RNVERBOSE)
```

output
```
-1/d^2*e*(a*d-b*c)*(-1/4*i^3*d^2*e^3/(a*d-b*c)^2/g^5*A/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^4+i^3*d^2*e^3/(a*d-b*c)^2/g^5*B*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(85) = 170.

Time = 0.09 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.99

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$
$$-\frac{4\left((4A+B)b^4cd^3-(4A+B)ab^3d^4\right)i^3x^3+6\left((4A+B)b^4c^2d^2-(4A+B)a^2b^2d^4\right)i^3x^2+4\left((4A+B\right.}{16\left((b^9c-ab^8d\right)g^5x^4+4\left(ab^8c\right.}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorithm="fricas")
```

output
```
-1/16*(4*((4*A + B)*b^4*c*d^3 - (4*A + B)*a*b^3*d^4)*i^3*x^3 + 6*((4*A + B)*b^4*c^2*d^2 - (4*A + B)*a^2*b^2*d^4)*i^3*x^2 + 4*((4*A + B)*b^4*c^3*d - (4*A + B)*a^3*b*d^4)*i^3*x + ((4*A + B)*b^4*c^4 - (4*A + B)*a^4*d^4)*i^3 + 4*(B*b^4*d^4*i^3*x^4 + 4*B*b^4*c*d^3*i^3*x^3 + 6*B*b^4*c^2*d^2*i^3*x^2 + 4*B*b^4*c^3*d*i^3*x + B*b^4*c^4*i^3)*log((b*e*x + a*e)/(d*x + c)))/((b^9*c - a*b^8*d)*g^5*x^4 + 4*(a*b^8*c - a^2*b^7*d)*g^5*x^3 + 6*(a^2*b^7*c - a^3*b^6*d)*g^5*x^2 + 4*(a^3*b^6*c - a^4*b^5*d)*g^5*x + (a^4*b^5*c - a^5*b^4*d)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3107 vs. $2(85) = 170$.

Time = 0.18 (sec) , antiderivative size = 3107, normalized size of antiderivative = 34.91

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algo
rithm="maxima")
```

output
```
-1/48*B*d^3*i^3*(12*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)*log(b*e*x/
(d*x + c) + a*e/(d*x + c))/(b^8*g^5*x^4 + 4*a*b^7*g^5*x^3 + 6*a^2*b^6*g^5*
x^2 + 4*a^3*b^5*g^5*x + a^4*b^4*g^5) + (25*a^3*b^3*c^3 - 23*a^4*b^2*c^2*d
+ 13*a^5*b*c*d^2 - 3*a^6*d^3 + 12*(4*b^6*c^3 - 6*a*b^5*c^2*d + 4*a^2*b^4*c
*d^2 - a^3*b^3*d^3)*x^3 + 6*(18*a*b^5*c^3 - 22*a^2*b^4*c^2*d + 13*a^3*b^3*
c*d^2 - 3*a^4*b^2*d^3)*x^2 + 4*(22*a^2*b^4*c^3 - 23*a^3*b^3*c^2*d + 13*a^4
*b^2*c*d^2 - 3*a^5*b*d^3)*x)/((b^11*c^3 - 3*a*b^10*c^2*d + 3*a^2*b^9*c*d^2
 - a^3*b^8*d^3)*g^5*x^4 + 4*(a*b^10*c^3 - 3*a^2*b^9*c^2*d + 3*a^3*b^8*c*d^
2 - a^4*b^7*d^3)*g^5*x^3 + 6*(a^2*b^9*c^3 - 3*a^3*b^8*c^2*d + 3*a^4*b^7*c*
d^2 - a^5*b^6*d^3)*g^5*x^2 + 4*(a^3*b^8*c^3 - 3*a^4*b^7*c^2*d + 3*a^5*b^6*
c*d^2 - a^6*b^5*d^3)*g^5*x + (a^4*b^7*c^3 - 3*a^5*b^6*c^2*d + 3*a^6*b^5*c*
d^2 - a^7*b^4*d^3)*g^5) + 12*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*d^
3 - a^3*d^4)*log(b*x + a)/((b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 -
4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*g^5) - 12*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 +
4*a^2*b*c*d^3 - a^3*d^4)*log(d*x + c)/((b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^
6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*g^5)) - 1/48*B*c*d^2*i^3*(12*(6
*b^2*x^2 + 4*a*b*x + a^2)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^7*g^5*x^
4 + 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*a^3*b^4*g^5*x + a^4*b^3*g^5) +
 (13*a^2*b^3*c^3 - 75*a^3*b^2*c^2*d + 33*a^4*b*c*d^2 - 7*a^5*d^3 - 12*(6*b
^5*c^2*d - 4*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3 + 6*(6*b^5*c^3 - 46*a*b^4*c...
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.57

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$
$$-\frac{1}{16}\left(\frac{4\,(dx+c)^4Be^5i^3\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^4g^5}+\frac{(4\,Ae^5i^3+Be^5i^3)(dx+c)^4}{(bex+ae)^4g^5}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{}{(bce-ad}\right.$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algo
rithm="giac")
```

output
```
-1/16*(4*(d*x + c)^4*B*e^5*i^3*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)
^4*g^5) + (4*A*e^5*i^3 + B*e^5*i^3)*(d*x + c)^4/((b*e*x + a*e)^4*g^5))*(b*
c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 29.16 (sec) , antiderivative size = 780, normalized size of antiderivative = 8.76

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$
$$-\frac{x^3\,(4\,A\,b^3\,d^3\,i^3+B\,b^3\,d^3\,i^3)+x^2\left(6\,A\,a\,b^2\,d^3\,i^3+\frac{3\,B\,a\,b^2\,d^3\,i^3}{2}+6\,A\,b^3\,c\,d^2\,i^3+\frac{3\,B\,b^3\,c\,d^2\,i^3}{2}\right)+x\,(4\,A\,a^2\,b\,c}{}$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(x^2\left(b\left(b\left(\frac{B\,a\,d^3\,i^3}{4\,b^5\,g^5}+\frac{B\,c\,d^2\,i^3}{4\,b^4\,g^5}\right)+\frac{B\,a\,d^3\,i^3}{2\,b^4\,g^5}+\frac{B\,c\,d^2\,i^3}{2\,b^3\,g^5}\right)+\frac{3\,B\,a\,d^3\,i^3}{4\,b^3\,g^5}+\frac{3\,B\,c\,d^2\,i^3}{4\,b^2\,g^5}\right)+x\left(b\left(a\left(\frac{B\,a\,c}{4\,b^5}\right.\right.\right.\right.}{}$$
$$-\frac{B\,d^4\,i^3\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,1\mathrm{i}}{2\,b^4\,g^5\,(a\,d-b\,c)}$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^5
,x)
```

output

```
- (x^3*(4*A*b^3*d^3*i^3 + B*b^3*d^3*i^3) + x^2*(6*A*a*b^2*d^3*i^3 + (3*B*a
*b^2*d^3*i^3)/2 + 6*A*b^3*c*d^2*i^3 + (3*B*b^3*c*d^2*i^3)/2) + x*(4*A*a^2*
b*d^3*i^3 + B*a^2*b*d^3*i^3 + 4*A*b^3*c^2*d*i^3 + B*b^3*c^2*d*i^3 + 4*A*a*
b^2*c*d^2*i^3 + B*a*b^2*c*d^2*i^3) + A*a^3*d^3*i^3 + A*b^3*c^3*i^3 + (B*a^
3*d^3*i^3)/4 + (B*b^3*c^3*i^3)/4 + A*a*b^2*c^2*d*i^3 + A*a^2*b*c*d^2*i^3 +
 (B*a*b^2*c^2*d*i^3)/4 + (B*a^2*b*c*d^2*i^3)/4)/(4*a^4*b^4*g^5 + 4*b^8*g^5
*x^4 + 16*a^3*b^5*g^5*x + 16*a*b^7*g^5*x^3 + 24*a^2*b^6*g^5*x^2) - (log((e
*(a + b*x))/(c + d*x))*(x^2*(b*(b*((B*a*d^3*i^3)/(4*b^5*g^5) + (B*c*d^2*i^
3)/(4*b^4*g^5)) + (B*a*d^3*i^3)/(2*b^4*g^5) + (B*c*d^2*i^3)/(2*b^3*g^5)) +
 (3*B*a*d^3*i^3)/(4*b^3*g^5) + (3*B*c*d^2*i^3)/(4*b^2*g^5)) + x*(b*(a*((B*
a*d^3*i^3)/(4*b^5*g^5) + (B*c*d^2*i^3)/(4*b^4*g^5)) + (B*c^2*d*i^3)/(4*b^3
*g^5)) + a*(b*((B*a*d^3*i^3)/(4*b^5*g^5) + (B*c*d^2*i^3)/(4*b^4*g^5)) + (B
*a*d^3*i^3)/(2*b^4*g^5) + (B*c*d^2*i^3)/(2*b^3*g^5)) + (3*B*c^2*d*i^3)/(4*
b^2*g^5)) + a*(a*((B*a*d^3*i^3)/(4*b^5*g^5) + (B*c*d^2*i^3)/(4*b^4*g^5)) +
 (B*c^2*d*i^3)/(4*b^3*g^5)) + (B*c^3*i^3)/(4*b^2*g^5) + (B*d^3*i^3*x^3)/(b
^2*g^5)))/(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3) - (B*d^4
*i^3*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*1i)/(2*b^4*g^5*(a*d - b*c)
)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 727, normalized size of antiderivative = 8.17

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{i\left(4a^5cd^3-4a^2b^3c^4-4a^2b^3d^4x^4-4\log(bx+a)\,a^4bcd^3-4\log(bx+a)\,b^5cd^3x^4+4\log(dx+c)\,b^5cd^3x^4\right.}{}$$

input

```
int((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x)
```

output

```
(i*( - 4*log(a + b*x)*a**4*b*c*d**3 - 16*log(a + b*x)*a**3*b**2*c*d**3*x -
 24*log(a + b*x)*a**2*b**3*c*d**3*x**2 - 16*log(a + b*x)*a*b**4*c*d**3*x**
3 - 4*log(a + b*x)*b**5*c*d**3*x**4 + 4*log(c + d*x)*a**4*b*c*d**3 + 16*lo
g(c + d*x)*a**3*b**2*c*d**3*x + 24*log(c + d*x)*a**2*b**3*c*d**3*x**2 + 16
*log(c + d*x)*a*b**4*c*d**3*x**3 + 4*log(c + d*x)*b**5*c*d**3*x**4 + 4*log
((a*e + b*e*x)/(c + d*x))*a**4*b*c*d**3 + 16*log((a*e + b*e*x)/(c + d*x))*
a**3*b**2*c*d**3*x + 24*log((a*e + b*e*x)/(c + d*x))*a**2*b**3*c*d**3*x**2
 - 4*log((a*e + b*e*x)/(c + d*x))*a*b**4*c**4 - 16*log((a*e + b*e*x)/(c +
d*x))*a*b**4*c**3*d*x - 24*log((a*e + b*e*x)/(c + d*x))*a*b**4*c**2*d**2*x
**2 - 4*log((a*e + b*e*x)/(c + d*x))*a*b**4*d**4*x**4 + 4*log((a*e + b*e*x
)/(c + d*x))*b**5*c*d**3*x**4 + 4*a**5*c*d**3 + 16*a**4*b*c*d**3*x + a**4*
b*c*d**3 + 24*a**3*b**2*c*d**3*x**2 + 4*a**3*b**2*c*d**3*x - 4*a**2*b**3*c
**4 - 16*a**2*b**3*c**3*d*x - 24*a**2*b**3*c**2*d**2*x**2 + 6*a**2*b**3*c*
d**3*x**2 - 4*a**2*b**3*d**4*x**4 - a*b**4*c**4 - 4*a*b**4*c**3*d*x - 6*a*
b**4*c**2*d**2*x**2 + 4*a*b**4*c*d**3*x**4 - a*b**4*d**4*x**4 + b**5*c*d**
3*x**4))/(16*a*b**3*g**5*(a**5*d - a**4*b*c + 4*a**4*b*d*x - 4*a**3*b**2*c
*x + 6*a**3*b**2*d*x**2 - 6*a**2*b**3*c*x**2 + 4*a**2*b**3*d*x**3 - 4*a*b*
*4*c*x**3 + a*b**4*d*x**4 - b**5*c*x**4))
```

3.29 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 181

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = \frac{Bdi^3(c+dx)^4}{16(bc-ad)^2g^6(a+bx)^4} - \frac{bBi^3(c+dx)^5}{25(bc-ad)^2g^6(a+bx)^5} + \frac{di^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^2g^6(a+bx)^4} - \frac{bi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^2g^6(a+bx)^5}$$

output

```
1/16*B*d*i^3*(d*x+c)^4/(-a*d+b*c)^2/g^6/(b*x+a)^4-1/25*b*B*i^3*(d*x+c)^5/(
-a*d+b*c)^2/g^6/(b*x+a)^5+1/4*d*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/
(-a*d+b*c)^2/g^6/(b*x+a)^4-1/5*b*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))
/(-a*d+b*c)^2/g^6/(b*x+a)^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 608 vs. $2(181) = 362$.

Time $= 0.62$ (sec) , antiderivative size $= 608$, normalized size of antiderivative $= 3.36$

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx =$$

$$-\frac{i^3\Big(80Ab^5c^5+16b^5Bc^5-100aAb^4c^4d-25ab^4Bc^4d+20a^5Ad^5+9a^5Bd^5+300Ab^5c^4dx+55b^5Bc^4dx$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^6,x]
```

output
```
-1/400*(i^3*(80*A*b^5*c^5 + 16*b^5*B*c^5 - 100*a*A*b^4*c^4*d - 25*a*b^4*B*
c^4*d + 20*a^5*A*d^5 + 9*a^5*B*d^5 + 300*A*b^5*c^4*d*x + 55*b^5*B*c^4*d*x
- 400*a*A*b^4*c^3*d^2*x - 100*a*b^4*B*c^3*d^2*x + 100*a^4*A*b*d^5*x + 45*a
^4*b*B*d^5*x + 400*A*b^5*c^3*d^2*x^2 + 60*b^5*B*c^3*d^2*x^2 - 600*a*A*b^4*
c^2*d^3*x^2 - 150*a*b^4*B*c^2*d^3*x^2 + 200*a^3*A*b^2*d^5*x^2 + 90*a^3*b^2
*B*d^5*x^2 + 200*A*b^5*c^2*d^3*x^3 + 10*b^5*B*c^2*d^3*x^3 - 400*a*A*b^4*c*
d^4*x^3 - 100*a*b^4*B*c*d^4*x^3 + 200*a^2*A*b^3*d^5*x^3 + 90*a^2*b^3*B*d^5
*x^3 - 20*b^5*B*c*d^4*x^4 + 20*a*b^4*B*d^5*x^4 - 20*B*d^5*(a + b*x)^5*Log[
a + b*x] + 20*B*(b*c - a*d)^2*(a^3*d^3 + a^2*b*d^2*(2*c + 5*d*x) + a*b^2*d
*(3*c^2 + 10*c*d*x + 10*d^2*x^2) + b^3*(4*c^3 + 15*c^2*d*x + 20*c*d^2*x^2
+ 10*d^3*x^3))*Log[(e*(a + b*x))/(c + d*x)] + 20*a^5*B*d^5*Log[c + d*x] +
100*a^4*b*B*d^5*x*Log[c + d*x] + 200*a^3*b^2*B*d^5*x^2*Log[c + d*x] + 200*
a^2*b^3*B*d^5*x^3*Log[c + d*x] + 100*a*b^4*B*d^5*x^4*Log[c + d*x] + 20*b^5
*B*d^5*x^5*Log[c + d*x]))/(b^4*(b*c - a*d)^2*g^6*(a + b*x)^5)
```

Rubi [A] (verified)

Time $= 0.37$ (sec) , antiderivative size $= 138$, normalized size of antiderivative $= 0.76$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used $= \{2962, 2772, 27, 53, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^6}\,dx$$

$$\downarrow 2962$$

$$\frac{i^3\int \frac{(c+dx)^6\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^6}d\frac{a+bx}{c+dx}}{g^6(bc-ad)^2}$$

$$\downarrow 2772$$

$$\frac{i^3\left(-B\int -\frac{(c+dx)^6\left(4b-\frac{5d(a+bx)}{c+dx}\right)}{20(a+bx)^6}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}+\frac{d(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}\right)}{g^6(bc-ad)^2}$$

$$\downarrow 27$$

$$\frac{i^3\left(\frac{1}{20}B\int \frac{(c+dx)^6\left(4b-\frac{5d(a+bx)}{c+dx}\right)}{(a+bx)^6}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}+\frac{d(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}\right)}{g^6(bc-ad)^2}$$

$$\downarrow 53$$

$$\frac{i^3\left(\frac{1}{20}B\int \left(\frac{4b(c+dx)^6}{(a+bx)^6}-\frac{5d(c+dx)^5}{(a+bx)^5}\right)d\frac{a+bx}{c+dx}-\frac{b(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}+\frac{d(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}\right)}{g^6(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{i^3\left(-\frac{b(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}+\frac{d(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}+\frac{1}{20}B\left(\frac{5d(c+dx)^4}{4(a+bx)^4}-\frac{4b(c+dx)^5}{5(a+bx)^5}\right)\right)}{g^6(bc-ad)^2}$$

input

```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^6
,x]
```

output

```
(i^3*((B*((5*d*(c + d*x)^4)/(4*(a + b*x)^4) - (4*b*(c + d*x)^5)/(5*(a + b*
x)^5)))/20 + (d*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*(a +
b*x)^4) - (b*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*(a + b*x
)^5)))/((b*c - a*d)^2*g^6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(173) = 346.

Time = 2.26 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.97

method	result
derivativedivides	$-\frac{e(da-bc)\left(\frac{i^3d^2e^4Ab}{5(da-bc)^3g^6\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^5}-\frac{i^3d^3e^3A}{4(da-bc)^3g^6\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{i^3d^2e^4Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^5}-\frac{25\left(\frac{be}{d}+\right.}{(da-bc)^3g^6}\right.}{d^2}$
default	$-\frac{e(da-bc)\left(\frac{i^3d^2e^4Ab}{5(da-bc)^3g^6\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^5}-\frac{i^3d^3e^3A}{4(da-bc)^3g^6\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{i^3d^2e^4Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^5}-\frac{25\left(\frac{be}{d}+\right.}{(da-bc)^3g^6}\right.}{d^2}$
parts	$\frac{i^3A\left(-\frac{3d\left(a^2d^2-2acdb+c^2b^2\right)}{4b^4(bx+a)^4}-\frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{5b^4(bx+a)^5}-\frac{d^3}{2b^4(bx+a)^2}+\frac{d^2(da-bc)}{b^4(bx+a)^3}\right)}{g^6}-\frac{i^3B(da-bc)^4e^4\left(\frac{d^6\left(-\right.}{}\right.}{}$
risch	$-\frac{i^3B\left(10b^3d^3x^3+10ab^2d^3x^2+20b^3cd^2x^2+5a^2bd^3x+10ab^2cd^2x+15b^3c^2dx+a^3d^3+2a^2bcd^2+3ab^2c^2d+4b^3c^3\right)\ln\left(9\right.}{20(bx+a)^5g^6b^4}$
parallelrisch	$-\frac{-400Ax^3ab^7cd^5i^3-100Bx^3ab^7cd^5i^3+400Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^8c^3d^3i^3+20Aa^5b^3d^6i^3+80Ab^8c^5di^3+9Ba^5b^3d^6i^3}{}$
orering	$\frac{(bx+a)\left(-27a^4b^2d^5x^6-27a^3b^3cd^4x^6-27a^2b^4c^2d^3x^6-27ab^5c^3d^2x^6+48b^6c^4dx^6-28a^5bd^5x^5-162a^4b^2cd^4x^5-162a^3\right.}{}$
norman	$\frac{\left(20Aac^3di^3-20Abc^4i^3+5Bac^3di^3-4Bbc^4i^3\right)x}{20ga(da-bc)}+\frac{\left(60Aa^2c^2d^2i^3+20Aabc^3di^3-80Ab^2c^4i^3+15Ba^2c^2d^2i^3+9Babc^3di^3-16B\right.}{40ga^2(da-bc)}$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x,method=_RETU
RNVERBOSE)
```

output

```
-1/d^2*e*(a*d-b*c)*(1/5*i^3*d^2*e^4/(a*d-b*c)^3/g^6*A*b/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^5-1/4*i^3*d^3*e^3/(a*d-b*c)^3/g^6*A/(b*e/d+(a*d-b*c)*e/d/(d*x+
c))^4-i^3*d^2*e^4/(a*d-b*c)^3/g^6*B*b*(-1/5/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/25/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5)+i^
3*d^3*e^3/(a*d-b*c)^3/g^6*B*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d
+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(173) = 346$.

Time $= 0.14$ (sec) , antiderivative size $= 644$, normalized size of antiderivative $= 3.56$

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$$
$$= \frac{20\left(Bb^5cd^4-Bab^4d^5\right)i^3x^4-10\left((20\,A+B)b^5c^2d^3-10\left(4\,A+B\right)ab^4cd^4+(20\,A+9\,B)a^2b^3d^5\right)i^3x^3-10}{}$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algo
rithm="fricas")
```

output

```
1/400*(20*(B*b^5*c*d^4 - B*a*b^4*d^5)*i^3*x^4 - 10*((20*A + B)*b^5*c^2*d^3
 - 10*(4*A + B)*a*b^4*c*d^4 + (20*A + 9*B)*a^2*b^3*d^5)*i^3*x^3 - 10*(2*(2
0*A + 3*B)*b^5*c^3*d^2 - 15*(4*A + B)*a*b^4*c^2*d^3 + (20*A + 9*B)*a^3*b^2
*d^5)*i^3*x^2 - 5*((60*A + 11*B)*b^5*c^4*d - 20*(4*A + B)*a*b^4*c^3*d^2 +
(20*A + 9*B)*a^4*b*d^5)*i^3*x - (16*(5*A + B)*b^5*c^5 - 25*(4*A + B)*a*b^4
*c^4*d + (20*A + 9*B)*a^5*d^5)*i^3 + 20*(B*b^5*d^5*i^3*x^5 + 5*B*a*b^4*d^5
*i^3*x^4 - 10*(B*b^5*c^2*d^3 - 2*B*a*b^4*c*d^4)*i^3*x^3 - 10*(2*B*b^5*c^3*
d^2 - 3*B*a*b^4*c^2*d^3)*i^3*x^2 - 5*(3*B*b^5*c^4*d - 4*B*a*b^4*c^3*d^2)*i
^3*x - (4*B*b^5*c^5 - 5*B*a*b^4*c^4*d)*i^3)*log((b*e*x + a*e)/(d*x + c)))/
((b^11*c^2 - 2*a*b^10*c*d + a^2*b^9*d^2)*g^6*x^5 + 5*(a*b^10*c^2 - 2*a^2*b
^9*c*d + a^3*b^8*d^2)*g^6*x^4 + 10*(a^2*b^9*c^2 - 2*a^3*b^8*c*d + a^4*b^7*
d^2)*g^6*x^3 + 10*(a^3*b^8*c^2 - 2*a^4*b^7*c*d + a^5*b^6*d^2)*g^6*x^2 + 5*
(a^4*b^7*c^2 - 2*a^5*b^6*c*d + a^6*b^5*d^2)*g^6*x + (a^5*b^6*c^2 - 2*a^6*b
^5*c*d + a^7*b^4*d^2)*g^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**6,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4218 vs. $2(173) = 346$.

Time = 0.25 (sec) , antiderivative size = 4218, normalized size of antiderivative = 23.30

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algo
rithm="maxima")
```

output

```
-1/1200*B*d^3*i^3*(60*(10*b^3*x^3 + 10*a*b^2*x^2 + 5*a^2*b*x + a^3)*log(b*
e*x/(d*x + c) + a*e/(d*x + c))/(b^9*g^6*x^5 + 5*a*b^8*g^6*x^4 + 10*a^2*b^7
*g^6*x^3 + 10*a^3*b^6*g^6*x^2 + 5*a^4*b^5*g^6*x + a^5*b^4*g^6) + (77*a^3*b
^4*c^4 - 548*a^4*b^3*c^3*d + 352*a^5*b^2*c^2*d^2 - 148*a^6*b*c*d^3 + 27*a^
7*d^4 - 60*(10*b^7*c^3*d - 10*a*b^6*c^2*d^2 + 5*a^2*b^5*c*d^3 - a^3*b^4*d^
4)*x^4 + 30*(10*b^7*c^4 - 100*a*b^6*c^3*d + 95*a^2*b^5*c^2*d^2 - 46*a^3*b^
4*c*d^3 + 9*a^4*b^3*d^4)*x^3 + 10*(50*a*b^6*c^4 - 410*a^2*b^5*c^3*d + 337*
a^3*b^4*c^2*d^2 - 148*a^4*b^3*c*d^3 + 27*a^5*b^2*d^4)*x^2 + 5*(65*a^2*b^5*
c^4 - 488*a^3*b^4*c^3*d + 352*a^4*b^3*c^2*d^2 - 148*a^5*b^2*c*d^3 + 27*a^6
*b*d^4)*x)/((b^13*c^4 - 4*a*b^12*c^3*d + 6*a^2*b^11*c^2*d^2 - 4*a^3*b^10*c
*d^3 + a^4*b^9*d^4)*g^6*x^5 + 5*(a*b^12*c^4 - 4*a^2*b^11*c^3*d + 6*a^3*b^1
0*c^2*d^2 - 4*a^4*b^9*c*d^3 + a^5*b^8*d^4)*g^6*x^4 + 10*(a^2*b^11*c^4 - 4*
a^3*b^10*c^3*d + 6*a^4*b^9*c^2*d^2 - 4*a^5*b^8*c*d^3 + a^6*b^7*d^4)*g^6*x^
3 + 10*(a^3*b^10*c^4 - 4*a^4*b^9*c^3*d + 6*a^5*b^8*c^2*d^2 - 4*a^6*b^7*c*d
^3 + a^7*b^6*d^4)*g^6*x^2 + 5*(a^4*b^9*c^4 - 4*a^5*b^8*c^3*d + 6*a^6*b^7*c
^2*d^2 - 4*a^7*b^6*c*d^3 + a^8*b^5*d^4)*g^6*x + (a^5*b^8*c^4 - 4*a^6*b^7*c
^3*d + 6*a^7*b^6*c^2*d^2 - 4*a^8*b^5*c*d^3 + a^9*b^4*d^4)*g^6) - 60*(10*b^
3*c^3*d^2 - 10*a*b^2*c^2*d^3 + 5*a^2*b*c*d^4 - a^3*d^5)*log(b*x + a)/((b^9
*c^5 - 5*a*b^8*c^4*d + 10*a^2*b^7*c^3*d^2 - 10*a^3*b^6*c^2*d^3 + 5*a^4*b^5
*c*d^4 - a^5*b^4*d^5)*g^6) + 60*(10*b^3*c^3*d^2 - 10*a*b^2*c^2*d^3 + 5*...
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.56

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx =$$

$$-\frac{1}{400}\left(\frac{20\left(4\,Bbe^6i^3-\frac{5\,(bex+ae)Bde^5i^3}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^5bcg^6}{(dx+c)^5}-\frac{(bex+ae)^5adg^6}{(dx+c)^5}}+\frac{80\,Abe^6i^3+16\,Bbe^6i^3-\frac{100\,(bex+ae)Ade^5i^3}{dx+c}-\frac{25\,(bex}{\frac{(bex+ae)^5bcg^6}{(dx+c)^5}-\frac{(bex+ae)^5adg^6}{(dx+c)^5}}\right.$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algo
rithm="giac")
```

output

```
-1/400*(20*(4*B*b*e^6*i^3 - 5*(b*e*x + a*e)*B*d*e^5*i^3/(d*x + c))*log((b*
e*x + a*e)/(d*x + c))/((b*e*x + a*e)^5*b*c*g^6/(d*x + c)^5 - (b*e*x + a*e)
^5*a*d*g^6/(d*x + c)^5) + (80*A*b*e^6*i^3 + 16*B*b*e^6*i^3 - 100*(b*e*x +
a*e)*A*d*e^5*i^3/(d*x + c) - 25*(b*e*x + a*e)*B*d*e^5*i^3/(d*x + c))/((b*e
*x + a*e)^5*b*c*g^6/(d*x + c)^5 - (b*e*x + a*e)^5*a*d*g^6/(d*x + c)^5))*(b
*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 30.18 (sec) , antiderivative size = 1053, normalized size of antiderivative = 5.82

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input

```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^6
,x)
```

output

```
- ((20*A*a^4*d^4*i^3 - 80*A*b^4*c^4*i^3 + 9*B*a^4*d^4*i^3 - 16*B*b^4*c^4*i
^3 + 20*A*a^2*b^2*c^2*d^2*i^3 + 9*B*a^2*b^2*c^2*d^2*i^3 + 20*A*a*b^3*c^3*d
*i^3 + 20*A*a^3*b*c*d^3*i^3 + 9*B*a*b^3*c^3*d*i^3 + 9*B*a^3*b*c*d^3*i^3)/(
20*(a*d - b*c)) + (x^2*(20*A*a^2*b^2*d^4*i^3 + 9*B*a^2*b^2*d^4*i^3 - 40*A*
b^4*c^2*d^2*i^3 - 6*B*b^4*c^2*d^2*i^3 + 20*A*a*b^3*c*d^3*i^3 + 9*B*a*b^3*c
*d^3*i^3))/(2*(a*d - b*c)) + (x*(20*A*a^3*b*d^4*i^3 + 9*B*a^3*b*d^4*i^3 -
60*A*b^4*c^3*d*i^3 - 11*B*b^4*c^3*d*i^3 + 20*A*a*b^3*c^2*d^2*i^3 + 20*A*a^
2*b^2*c*d^3*i^3 + 9*B*a*b^3*c^2*d^2*i^3 + 9*B*a^2*b^2*c*d^3*i^3))/(4*(a*d
- b*c)) + (x^3*(20*A*a*b^3*d^4*i^3 + 9*B*a*b^3*d^4*i^3 - 20*A*b^4*c*d^3*i^
3 - B*b^4*c*d^3*i^3))/(2*(a*d - b*c)) + (B*b^4*d^4*i^3*x^4)/(a*d - b*c))/(
20*a^5*b^4*g^6 + 20*b^9*g^6*x^5 + 100*a^4*b^5*g^6*x + 100*a*b^8*g^6*x^4 +
200*a^3*b^6*g^6*x^2 + 200*a^2*b^7*g^6*x^3) - (log((e*(a + b*x))/(c + d*x))
*(x^2*(b*(b*((B*a*d^3*i^3)/(20*b^5*g^6) + (B*c*d^2*i^3)/(10*b^4*g^6)) + (3
*B*a*d^3*i^3)/(20*b^4*g^6) + (3*B*c*d^2*i^3)/(10*b^3*g^6)) + (3*B*a*d^3*i^
3)/(10*b^3*g^6) + (3*B*c*d^2*i^3)/(5*b^2*g^6)) + x*(b*(a*((B*a*d^3*i^3)/(2
0*b^5*g^6) + (B*c*d^2*i^3)/(10*b^4*g^6)) + (3*B*c^2*d*i^3)/(20*b^3*g^6)) +
 a*(b*((B*a*d^3*i^3)/(20*b^5*g^6) + (B*c*d^2*i^3)/(10*b^4*g^6)) + (3*B*a*d
^3*i^3)/(20*b^4*g^6) + (3*B*c*d^2*i^3)/(10*b^3*g^6)) + (3*B*c^2*d*i^3)/(5*
b^2*g^6)) + a*(a*((B*a*d^3*i^3)/(20*b^5*g^6) + (B*c*d^2*i^3)/(10*b^4*g^6))
 + (3*B*c^2*d*i^3)/(20*b^3*g^6)) + (B*c^3*i^3)/(5*b^2*g^6) + (B*d^3*i^3...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1647, normalized size of antiderivative = 9.10

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x)
```

output

```
(i*( - 40*log(a + b*x)*a**6*b**2*c*d**4 + 20*log(a + b*x)*a**5*b**3*c**2*d
**3 - 200*log(a + b*x)*a**5*b**3*c*d**4*x + 100*log(a + b*x)*a**4*b**4*c**
2*d**3*x - 400*log(a + b*x)*a**4*b**4*c*d**4*x**2 + 200*log(a + b*x)*a**3*
b**5*c**2*d**3*x**2 - 400*log(a + b*x)*a**3*b**5*c*d**4*x**3 + 200*log(a +
 b*x)*a**2*b**6*c**2*d**3*x**3 - 200*log(a + b*x)*a**2*b**6*c*d**4*x**4 +
100*log(a + b*x)*a*b**7*c**2*d**3*x**4 - 40*log(a + b*x)*a*b**7*c*d**4*x**
5 + 20*log(a + b*x)*b**8*c**2*d**3*x**5 + 40*log(c + d*x)*a**6*b**2*c*d**4
 - 20*log(c + d*x)*a**5*b**3*c**2*d**3 + 200*log(c + d*x)*a**5*b**3*c*d**4
*x - 100*log(c + d*x)*a**4*b**4*c**2*d**3*x + 400*log(c + d*x)*a**4*b**4*c
*d**4*x**2 - 200*log(c + d*x)*a**3*b**5*c**2*d**3*x**2 + 400*log(c + d*x)*
a**3*b**5*c*d**4*x**3 - 200*log(c + d*x)*a**2*b**6*c**2*d**3*x**3 + 200*lo
g(c + d*x)*a**2*b**6*c*d**4*x**4 - 100*log(c + d*x)*a*b**7*c**2*d**3*x**4
+ 40*log(c + d*x)*a*b**7*c*d**4*x**5 - 20*log(c + d*x)*b**8*c**2*d**3*x**5
 + 40*log((a*e + b*e*x)/(c + d*x))*a**6*b**2*c*d**4 - 20*log((a*e + b*e*x)
/(c + d*x))*a**5*b**3*c**2*d**3 + 200*log((a*e + b*e*x)/(c + d*x))*a**5*b*
*3*c*d**4*x - 100*log((a*e + b*e*x)/(c + d*x))*a**4*b**4*c**2*d**3*x + 400
*log((a*e + b*e*x)/(c + d*x))*a**4*b**4*c*d**4*x**2 - 100*log((a*e + b*e*x
)/(c + d*x))*a**3*b**5*c**4*d - 400*log((a*e + b*e*x)/(c + d*x))*a**3*b**5
*c**3*d**2*x - 800*log((a*e + b*e*x)/(c + d*x))*a**3*b**5*c**2*d**3*x**2 -
 100*log((a*e + b*e*x)/(c + d*x))*a**3*b**5*d**5*x**4 + 80*log((a*e + b...
```

3.30 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 281

$$\begin{aligned}\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx = &-\frac{Bd^2i^3(c+dx)^4}{16(bc-ad)^3g^7(a+bx)^4}\\ &+\frac{2bBdi^3(c+dx)^5}{25(bc-ad)^3g^7(a+bx)^5}\\ &-\frac{b^2Bi^3(c+dx)^6}{36(bc-ad)^3g^7(a+bx)^6}\\ &-\frac{d^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3g^7(a+bx)^4}\\ &+\frac{2bdi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^3g^7(a+bx)^5}\\ &-\frac{b^2i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bc-ad)^3g^7(a+bx)^6}\end{aligned}$$

output

```
-1/16*B*d^2*i^3*(d*x+c)^4/(-a*d+b*c)^3/g^7/(b*x+a)^4+2/25*b*B*d*i^3*(d*x+c
)^5/(-a*d+b*c)^3/g^7/(b*x+a)^5-1/36*b^2*B*i^3*(d*x+c)^6/(-a*d+b*c)^3/g^7/(
b*x+a)^6-1/4*d^2*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^
7/(b*x+a)^4+2/5*b*d*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3
/g^7/(b*x+a)^5-1/6*b^2*i^3*(d*x+c)^6*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c
)^3/g^7/(b*x+a)^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 642 vs. $2(281) = 562$.

Time = 1.09 (sec) , antiderivative size = 642, normalized size of antiderivative = 2.28

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx$$
$$= \frac{i^3\Big(-100B(bc-ad)^6+432aBd(-bc+ad)^5-432bBd(bc-ad)^5x+540aBd^2(bc-ad)^4(a+bx)+120B$$

input

```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^7,x]
```

output

```
(i^3*(-100*B*(b*c - a*d)^6 + 432*a*B*d*(-(b*c) + a*d)^5 - 432*b*B*d*(b*c -
 a*d)^5*x + 540*a*B*d^2*(b*c - a*d)^4*(a + b*x) + 120*B*d*(b*c - a*d)^5*(a
 + b*x) + 540*b*B*d^2*(b*c - a*d)^4*x*(a + b*x) - 825*B*d^2*(b*c - a*d)^4*
(a + b*x)^2 + 720*a*B*d^3*(-(b*c) + a*d)^3*(a + b*x)^2 - 720*b*B*d^3*(b*c
 - a*d)^3*x*(a + b*x)^2 + 1080*a*B*d^4*(b*c - a*d)^2*(a + b*x)^3 + 700*B*d^
3*(b*c - a*d)^3*(a + b*x)^3 + 1080*b*B*d^4*(b*c - a*d)^2*x*(a + b*x)^3 - 1
050*B*d^4*(b*c - a*d)^2*(a + b*x)^4 + 2160*a*B*d^5*(-(b*c) + a*d)*(a + b*x
)^4 - 2160*b*B*d^5*(b*c - a*d)*x*(a + b*x)^4 + 2100*B*d^5*(b*c - a*d)*(a +
 b*x)^5 - 2160*a*B*d^6*(a + b*x)^5*Log[a + b*x] - 2160*b*B*d^6*x*(a + b*x)
^5*Log[a + b*x] + 2100*B*d^6*(a + b*x)^6*Log[a + b*x] - 600*(b*c - a*d)^6*
(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2160*d*(-(b*c) + a*d)^5*(a + b*x)*(
A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2700*d^2*(b*c - a*d)^4*(a + b*x)^2*(
A + B*Log[(e*(a + b*x))/(c + d*x)]) + 1200*d^3*(-(b*c) + a*d)^3*(a + b*x)^
3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2160*a*B*d^6*(a + b*x)^5*Log[c +
d*x] + 2160*b*B*d^6*x*(a + b*x)^5*Log[c + d*x] - 2100*B*d^6*(a + b*x)^6*Lo
g[c + d*x]))/(3600*b^4*(b*c - a*d)^3*g^7*(a + b*x)^6)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2772, 27, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^7}\,dx$$

$$\downarrow 2962$$

$$\frac{i^3\int \frac{(c+dx)^7\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^7}d\frac{a+bx}{c+dx}}{g^7(bc-ad)^3}$$

$$\downarrow 2772$$

$$\frac{i^3\left(-B\int -\frac{(c+dx)^7\left(10b^2-\frac{24d(a+bx)b}{c+dx}+\frac{15d^2(a+bx)^2}{(c+dx)^2}\right)}{60(a+bx)^7}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6(a+bx)^6}-\frac{d^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}+\right.}{g^7(bc-ad)^3}$$

$$\downarrow 27$$

$$\frac{i^3\left(\frac{1}{60}B\int \frac{(c+dx)^7\left(10b^2-\frac{24d(a+bx)b}{c+dx}+\frac{15d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^7}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6(a+bx)^6}-\frac{d^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}+\right.}{g^7(bc-ad)^3}$$

$$\downarrow 1140$$

$$\frac{i^3\left(\frac{1}{60}B\int \left(\frac{10b^2(c+dx)^7}{(a+bx)^7}-\frac{24bd(c+dx)^6}{(a+bx)^6}+\frac{15d^2(c+dx)^5}{(a+bx)^5}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6(a+bx)^6}-\frac{d^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{g^7(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{i^3\left(-\frac{b^2(c+dx)^6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6(a+bx)^6}-\frac{d^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}+\frac{2bd(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}+\frac{1}{60}B\left(-\frac{5b^2(c+dx)^6}{3(a+bx)^6}\right.\right.}{g^7(bc-ad)^3}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^7
,x]
```

output
```
(i^3*((B*((-15*d^2*(c + d*x)^4)/(4*(a + b*x)^4) + (24*b*d*(c + d*x)^5)/(5*
(a + b*x)^5) - (5*b^2*(c + d*x)^6)/(3*(a + b*x)^6)))/60 - (d^2*(c + d*x)^4
*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*(a + b*x)^4) + (2*b*d*(c + d*x)^
5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*(a + b*x)^5) - (b^2*(c + d*x)^6
*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(6*(a + b*x)^6)))/((b*c - a*d)^3*g^
7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.71

method	result
parts	$\frac{i^3 A\left(\frac{3d^2(da-bc)}{4b^4(bx+a)^4}-\frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{6b^4(bx+a)^6}-\frac{3d\left(a^2d^2-2acdb+c^2b^2\right)}{5b^4(bx+a)^5}-\frac{d^3}{3b^4(bx+a)^3}\right)}{g^7}-\frac{i^3B(da-bc)^4e^4\left(\frac{d^7\left(-\right.}{}\right.}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{i^3d^2e^5Ab^2}{6(da-bc)^4g^7\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^6}+\frac{2i^3d^3e^4Ab}{5(da-bc)^4g^7\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^5}-\frac{i^3d^4e^3A}{4(da-bc)^4g^7\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}+\frac{i^3d^2e^5B}{}\right.}$
default	$-\frac{e(da-bc)\left(-\frac{i^3d^2e^5Ab^2}{6(da-bc)^4g^7\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^6}+\frac{2i^3d^3e^4Ab}{5(da-bc)^4g^7\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^5}-\frac{i^3d^4e^3A}{4(da-bc)^4g^7\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}+\frac{i^3d^2e^5B}{}\right.}$
orering	$-\frac{(bx+a)(-300b^6d^6x^6-1500ab^5d^6x^5-300b^6cd^5x^5-2870a^2b^4d^6x^4-1760ab^5cd^5x^4+130b^6c^2d^4x^4-11480a^2b^4cd^5x}$
risch	$-\frac{i^3B(20b^3d^3x^3+15ab^2d^3x^2+45b^3cd^2x^2+6a^2bd^3x+18ab^2cd^2x+36b^3c^2dx+a^3d^3+3a^2bcd^2+6ab^2c^2d+10b^3c^3)\ln\left(}{60(bx+a)^6g^7b^4}$
parallelrisch	Expression too large to display
norman	Expression too large to display

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^7,x,method=_RETU
RNVERBOSE)
```

output
```
i^3*A/g^7*(3/4*d^2*(a*d-b*c)/b^4/(b*x+a)^4-1/6*(-a^3*d^3+3*a^2*b*c*d^2-3*a
*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^6-3/5*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b
*x+a)^5-1/3*d^3/b^4/(b*x+a)^3)-i^3*B/g^7/d^5*(a*d-b*c)^4*e^4*(d^7/(a*d-b*c
)^7*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-
1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4)-2*d^6/(a*d-b*c)^7*b*e*(-1/5/(b*e/d+(
a*d-b*c)*e/d/(d*x+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/25/(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^5)+d^5/(a*d-b*c)^7*b^2*e^2*(-1/6/(b*e/d+(a*d-b*c)*e/d/(d*
x+c))^6*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/36/(b*e/d+(a*d-b*c)*e/d/(d*x+c))
^6))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 991 vs. $2(269) = 538$.

Time $= 0.10$ (sec) , antiderivative size $= 991$, normalized size of antiderivative $= 3.53$

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^7,x, algo
rithm="fricas")
```

output

```
-1/3600*(60*(B*b^6*c*d^5 - B*a*b^5*d^6)*i^3*x^5 - 30*(B*b^6*c^2*d^4 - 12*B
*a*b^5*c*d^5 + 11*B*a^2*b^4*d^6)*i^3*x^4 + 20*((60*A + B)*b^6*c^3*d^3 - 9*
(20*A + B)*a*b^5*c^2*d^4 + 45*(4*A + B)*a^2*b^4*c*d^5 - (60*A + 37*B)*a^3*
b^3*d^6)*i^3*x^3 + 15*((180*A + 19*B)*b^6*c^4*d^2 - 24*(20*A + 3*B)*a*b^5*
c^3*d^3 + 90*(4*A + B)*a^2*b^4*c^2*d^4 - (60*A + 37*B)*a^4*b^2*d^6)*i^3*x^
2 + 6*(4*(90*A + 13*B)*b^6*c^5*d - 15*(60*A + 11*B)*a*b^5*c^4*d^2 + 150*(4
*A + B)*a^2*b^4*c^3*d^3 - (60*A + 37*B)*a^5*b*d^6)*i^3*x + (100*(6*A + B)*
b^6*c^6 - 288*(5*A + B)*a*b^5*c^5*d + 225*(4*A + B)*a^2*b^4*c^4*d^2 - (60*
A + 37*B)*a^6*d^6)*i^3 + 60*(B*b^6*d^6*i^3*x^6 + 6*B*a*b^5*d^6*i^3*x^5 + 1
5*B*a^2*b^4*d^6*i^3*x^4 + 20*(B*b^6*c^3*d^3 - 3*B*a*b^5*c^2*d^4 + 3*B*a^2*
b^4*c*d^5)*i^3*x^3 + 15*(3*B*b^6*c^4*d^2 - 8*B*a*b^5*c^3*d^3 + 6*B*a^2*b^4
*c^2*d^4)*i^3*x^2 + 6*(6*B*b^6*c^5*d - 15*B*a*b^5*c^4*d^2 + 10*B*a^2*b^4*c
^3*d^3)*i^3*x + (10*B*b^6*c^6 - 24*B*a*b^5*c^5*d + 15*B*a^2*b^4*c^4*d^2)*i
^3)*log((b*e*x + a*e)/(d*x + c)))/((b^13*c^3 - 3*a*b^12*c^2*d + 3*a^2*b^11
*c*d^2 - a^3*b^10*d^3)*g^7*x^6 + 6*(a*b^12*c^3 - 3*a^2*b^11*c^2*d + 3*a^3*
b^10*c*d^2 - a^4*b^9*d^3)*g^7*x^5 + 15*(a^2*b^11*c^3 - 3*a^3*b^10*c^2*d +
3*a^4*b^9*c*d^2 - a^5*b^8*d^3)*g^7*x^4 + 20*(a^3*b^10*c^3 - 3*a^4*b^9*c^2*
d + 3*a^5*b^8*c*d^2 - a^6*b^7*d^3)*g^7*x^3 + 15*(a^4*b^9*c^3 - 3*a^5*b^8*c
^2*d + 3*a^6*b^7*c*d^2 - a^7*b^6*d^3)*g^7*x^2 + 6*(a^5*b^8*c^3 - 3*a^6*b^7
*c^2*d + 3*a^7*b^6*c*d^2 - a^8*b^5*d^3)*g^7*x + (a^6*b^7*c^3 - 3*a^7*b^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx = \text{Timed out}$$

input

```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**7,x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5524 vs. $2(269) = 538$.

Time $= 0.35$ (sec) , antiderivative size $= 5524$, normalized size of antiderivative $= 19.66$

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^7,x, algo
rithm="maxima")
```

output
```
-1/3600*B*d^3*i^3*(60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)*log(b*
e*x/(d*x + c) + a*e/(d*x + c))/(b^10*g^7*x^6 + 6*a*b^9*g^7*x^5 + 15*a^2*b^
8*g^7*x^4 + 20*a^3*b^7*g^7*x^3 + 15*a^4*b^6*g^7*x^2 + 6*a^5*b^5*g^7*x + a^
6*b^4*g^7) + (57*a^3*b^5*c^5 - 405*a^4*b^4*c^4*d + 1470*a^5*b^3*c^3*d^2 -
730*a^6*b^2*c^2*d^3 + 245*a^7*b*c*d^4 - 37*a^8*d^5 + 60*(20*b^8*c^3*d^2 -
15*a*b^7*c^2*d^3 + 6*a^2*b^6*c*d^4 - a^3*b^5*d^5)*x^5 - 30*(20*b^8*c^4*d -
 235*a*b^7*c^3*d^2 + 171*a^2*b^6*c^2*d^3 - 67*a^3*b^5*c*d^4 + 11*a^4*b^4*d
^5)*x^4 + 20*(20*b^8*c^5 - 175*a*b^7*c^4*d + 866*a^2*b^6*c^3*d^2 - 604*a^3
*b^5*c^2*d^3 + 230*a^4*b^4*c*d^4 - 37*a^5*b^3*d^5)*x^3 + 15*(35*a*b^7*c^5
- 271*a^2*b^6*c^4*d + 1128*a^3*b^5*c^3*d^2 - 700*a^4*b^4*c^2*d^3 + 245*a^5
*b^3*c*d^4 - 37*a^6*b^2*d^5)*x^2 + 6*(47*a^2*b^6*c^5 - 345*a^3*b^5*c^4*d +
 1320*a^4*b^4*c^3*d^2 - 730*a^5*b^3*c^2*d^3 + 245*a^6*b^2*c*d^4 - 37*a^7*b
*d^5)*x)/((b^15*c^5 - 5*a*b^14*c^4*d + 10*a^2*b^13*c^3*d^2 - 10*a^3*b^12*c
^2*d^3 + 5*a^4*b^11*c*d^4 - a^5*b^10*d^5)*g^7*x^6 + 6*(a*b^14*c^5 - 5*a^2*
b^13*c^4*d + 10*a^3*b^12*c^3*d^2 - 10*a^4*b^11*c^2*d^3 + 5*a^5*b^10*c*d^4
- a^6*b^9*d^5)*g^7*x^5 + 15*(a^2*b^13*c^5 - 5*a^3*b^12*c^4*d + 10*a^4*b^11
*c^3*d^2 - 10*a^5*b^10*c^2*d^3 + 5*a^6*b^9*c*d^4 - a^7*b^8*d^5)*g^7*x^4 +
20*(a^3*b^12*c^5 - 5*a^4*b^11*c^4*d + 10*a^5*b^10*c^3*d^2 - 10*a^6*b^9*c^2
*d^3 + 5*a^7*b^8*c*d^4 - a^8*b^7*d^5)*g^7*x^3 + 15*(a^4*b^11*c^5 - 5*a^5*b
^10*c^4*d + 10*a^6*b^9*c^3*d^2 - 10*a^7*b^8*c^2*d^3 + 5*a^8*b^7*c*d^4 -...
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.58

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx =$$
$$-\frac{1}{3600}\left(\frac{60\left(10\,Bb^2e^7i^3-\frac{24\,(bex+ae)Bbde^6i^3}{dx+c}+\frac{15\,(bex+ae)^2Bd^2e^5i^3}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^6b^2c^2g^7}{(dx+c)^6}-\frac{2\,(bex+ae)^6abcdg^7}{(dx+c)^6}+\frac{(bex+ae)^6a^2d^2g^7}{(dx+c)^6}}+\frac{600\,Ab^2e^7i^3+100\,Bb^2e^7}{}\right.$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^7,x, algo
rithm="giac")
```

output
```
-1/3600*(60*(10*B*b^2*e^7*i^3 - 24*(b*e*x + a*e)*B*b*d*e^6*i^3/(d*x + c) +
 15*(b*e*x + a*e)^2*B*d^2*e^5*i^3/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c)
)/((b*e*x + a*e)^6*b^2*c^2*g^7/(d*x + c)^6 - 2*(b*e*x + a*e)^6*a*b*c*d*g^7
/(d*x + c)^6 + (b*e*x + a*e)^6*a^2*d^2*g^7/(d*x + c)^6) + (600*A*b^2*e^7*i
^3 + 100*B*b^2*e^7*i^3 - 1440*(b*e*x + a*e)*A*b*d*e^6*i^3/(d*x + c) - 288*
(b*e*x + a*e)*B*b*d*e^6*i^3/(d*x + c) + 900*(b*e*x + a*e)^2*A*d^2*e^5*i^3/
(d*x + c)^2 + 225*(b*e*x + a*e)^2*B*d^2*e^5*i^3/(d*x + c)^2)/((b*e*x + a*e
)^6*b^2*c^2*g^7/(d*x + c)^6 - 2*(b*e*x + a*e)^6*a*b*c*d*g^7/(d*x + c)^6 +
(b*e*x + a*e)^6*a^2*d^2*g^7/(d*x + c)^6))*(b*c/((b*c*e - a*d*e)*(b*c - a*d
)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 31.99 (sec) , antiderivative size = 1396, normalized size of antiderivative = 4.97

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx = \text{Too large to display}$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^7
,x)
```

output

```
(B*d^6*i^3*atanh((60*b^7*c^3*g^7 + 60*a^3*b^4*d^3*g^7 - 60*a*b^6*c^2*d*g^7
 - 60*a^2*b^5*c*d^2*g^7)/(60*b^4*g^7*(a*d - b*c)^3) + (2*b*d*x*(a^2*d^2 +
b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(30*b^4*g^7*(a*d - b*c)^3) - (log((e
*(a + b*x))/(c + d*x))*(x^2*(b*(b*((B*a*d^3*i^3)/(60*b^5*g^7) + (B*c*d^2*i
^3)/(20*b^4*g^7)) + (B*a*d^3*i^3)/(15*b^4*g^7) + (B*c*d^2*i^3)/(5*b^3*g^7)
) + (B*a*d^3*i^3)/(6*b^3*g^7) + (B*c*d^2*i^3)/(2*b^2*g^7)) + x*(b*(a*((B*a
*d^3*i^3)/(60*b^5*g^7) + (B*c*d^2*i^3)/(20*b^4*g^7)) + (B*c^2*d*i^3)/(10*b
^3*g^7)) + a*(b*((B*a*d^3*i^3)/(60*b^5*g^7) + (B*c*d^2*i^3)/(20*b^4*g^7))
+ (B*a*d^3*i^3)/(15*b^4*g^7) + (B*c*d^2*i^3)/(5*b^3*g^7)) + (B*c^2*d*i^3)/
(2*b^2*g^7)) + a*(a*((B*a*d^3*i^3)/(60*b^5*g^7) + (B*c*d^2*i^3)/(20*b^4*g^
7)) + (B*c^2*d*i^3)/(10*b^3*g^7)) + (B*c^3*i^3)/(6*b^2*g^7) + (B*d^3*i^3*x
^3)/(3*b^2*g^7)))/(6*a^5*x + a^6/b + b^5*x^6 + 15*a^4*b*x^2 + 6*a*b^4*x^5
+ 20*a^3*b^2*x^3 + 15*a^2*b^3*x^4) - ((60*A*a^5*d^5*i^3 + 600*A*b^5*c^5*i^
3 + 37*B*a^5*d^5*i^3 + 100*B*b^5*c^5*i^3 + 60*A*a^2*b^3*c^3*d^2*i^3 + 60*A
*a^3*b^2*c^2*d^3*i^3 + 37*B*a^2*b^3*c^3*d^2*i^3 + 37*B*a^3*b^2*c^2*d^3*i^3
 - 840*A*a*b^4*c^4*d*i^3 + 60*A*a^4*b*c*d^4*i^3 - 188*B*a*b^4*c^4*d*i^3 +
37*B*a^4*b*c*d^4*i^3)/(60*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x^2*(60*A*a^
3*b^2*d^5*i^3 + 37*B*a^3*b^2*d^5*i^3 + 180*A*b^5*c^3*d^2*i^3 + 19*B*b^5*c^
3*d^2*i^3 - 300*A*a*b^4*c^2*d^3*i^3 + 60*A*a^2*b^3*c*d^4*i^3 - 53*B*a*b^4*
c^2*d^3*i^3 + 37*B*a^2*b^3*c*d^4*i^3))/(4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1696, normalized size of antiderivative = 6.04

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^7,x)
```

output

```
(i*( - 60*log(a + b*x)*a**7*b*d**6 - 360*log(a + b*x)*a**6*b**2*d**6*x - 9
00*log(a + b*x)*a**5*b**3*d**6*x**2 - 1200*log(a + b*x)*a**4*b**4*d**6*x**
3 - 900*log(a + b*x)*a**3*b**5*d**6*x**4 - 360*log(a + b*x)*a**2*b**6*d**6
*x**5 - 60*log(a + b*x)*a*b**7*d**6*x**6 + 60*log(c + d*x)*a**7*b*d**6 + 3
60*log(c + d*x)*a**6*b**2*d**6*x + 900*log(c + d*x)*a**5*b**3*d**6*x**2 +
1200*log(c + d*x)*a**4*b**4*d**6*x**3 + 900*log(c + d*x)*a**3*b**5*d**6*x*
*4 + 360*log(c + d*x)*a**2*b**6*d**6*x**5 + 60*log(c + d*x)*a*b**7*d**6*x*
*6 + 60*log((a*e + b*e*x)/(c + d*x))*a**7*b*d**6 + 360*log((a*e + b*e*x)/(
c + d*x))*a**6*b**2*d**6*x + 900*log((a*e + b*e*x)/(c + d*x))*a**5*b**3*d*
*6*x**2 + 1200*log((a*e + b*e*x)/(c + d*x))*a**4*b**4*d**6*x**3 - 900*log(
(a*e + b*e*x)/(c + d*x))*a**3*b**5*c**4*d**2 - 3600*log((a*e + b*e*x)/(c +
 d*x))*a**3*b**5*c**3*d**3*x - 5400*log((a*e + b*e*x)/(c + d*x))*a**3*b**5
*c**2*d**4*x**2 - 3600*log((a*e + b*e*x)/(c + d*x))*a**3*b**5*c*d**5*x**3
+ 1440*log((a*e + b*e*x)/(c + d*x))*a**2*b**6*c**5*d + 5400*log((a*e + b*e
*x)/(c + d*x))*a**2*b**6*c**4*d**2*x + 7200*log((a*e + b*e*x)/(c + d*x))*a
**2*b**6*c**3*d**3*x**2 + 3600*log((a*e + b*e*x)/(c + d*x))*a**2*b**6*c**2
*d**4*x**3 - 600*log((a*e + b*e*x)/(c + d*x))*a*b**7*c**6 - 2160*log((a*e
+ b*e*x)/(c + d*x))*a*b**7*c**5*d*x - 2700*log((a*e + b*e*x)/(c + d*x))*a*
b**7*c**4*d**2*x**2 - 1200*log((a*e + b*e*x)/(c + d*x))*a*b**7*c**3*d**3*x
**3 + 60*a**8*d**6 + 360*a**7*b*d**6*x + 27*a**7*b*d**6 + 10*a**6*b**2*...
```

3.31 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$

Optimal result . . . 445
Mathematica [A] (verified) . . . 446
Rubi [A] (verified) . . . 446
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Mupad [F(-1)] . . . 453
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Optimal result

Integrand size = 40, antiderivative size = 252

$$
\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx \\
&= \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3di} \\
&\quad - \frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^2i} \\
&\quad + \frac{(bc-ad)^2g^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^3i} \\
&\quad + \frac{(bc-ad)^3g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^4i} \\
&\quad + \frac{B(bc-ad)^3g^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}
\end{aligned}
$$

output

```
1/3*g^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i-1/6*(-a*d+b*c)*g^3*(b*x+
a)^2*(3*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/d^2/i+1/6*(-a*d+b*c)^2*g^3*(b*x+a)*
(6*A+5*B+6*B*ln(e*(b*x+a)/(d*x+c)))/d^3/i+1/6*(-a*d+b*c)^3*g^3*ln((-a*d+b*
c)/b/(d*x+c))*(6*A+11*B+6*B*ln(e*(b*x+a)/(d*x+c)))/d^4/i+B*(-a*d+b*c)^3*g^
3*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.40

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$$
$$= \frac{g^3\left(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+3d^2(-bc+ad)(a+bx)^2\left(A+B\log\left(\frac{e(a+}{c+}\right.\right.\right.$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*
i*x),x]
```

output
```
(g^3*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a +
b*x))/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) -
6*B*(b*c - a*d)^3*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*
(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (
-(b*c) + a*d)*Log[c + d*x]) - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c
+ d*x)])*Log[i*(c + d*x)] + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c
) + a*d)] - Log[i*(c + d*x)])*Log[i*(c + d*x)] + 2*PolyLog[2, (b*(c + d*x)
)/(b*c - a*d)])))/(6*d^4*i)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2962, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{ci+dix}\,dx$$
$$\downarrow 2962$$

$$\frac{g^3(bc-ad)^3\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{i}$$

$$\downarrow 2784$$

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int\frac{(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{i}$$

$$\downarrow 2784$$

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}}{3d}\right)}{i}$$

$$\downarrow 2784$$

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+5B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{6A+11B+6B\log}{b-\frac{d(a-}{c+}}}{2d}}{3d}\right)}{i}$$

$$\downarrow 2754$$

$$g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+5B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{6B\int\frac{(c+dx)\log\left($$

$$\Big\downarrow 2838$$

$$g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+5B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{}$$

input

```
Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x),x
]
```

output

```
((b*c - a*d)^3*g^3*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*
d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a + b*x)^2*(3*A + B +
3*B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c
+ d*x))^2) - (((a + b*x)*(6*A + 5*B + 6*B*Log[(e*(a + b*x))/(c + d*x)]))/(
d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((6*A + 11*B + 6*B*Log[(e*
(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (6*B*Pol
yLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d))/(3*d)))/i
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1104 vs. $2(244) = 488$.

Time = 3.35 (sec) , antiderivative size = 1105, normalized size of antiderivative = 4.38

method	result	size
derivativedivides	Expression too large to display	1105
default	Expression too large to display	1105
parts	Expression too large to display	1116
risch	Expression too large to display	3731

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x,method=_RETURN
VERBOSE)
```

output
```
-1/d^2*e*(a*d-b*c)*(A*d^2*g^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/e/i*(-3/2*b^2/d^
4*e^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2+3*b*e/d^4/(b*e-(b*e/d+(a*d-b
*c)*e/d/(d*x+c))*d)+1/d^4*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)+1/3*b^3*
e^3/d^4/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^3)+B*d^2*g^3*(a^2*d^2-2*a*b*
c*d+b^2*c^2)/e/i*(3*(1/2/b/e/d/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)-1/2/b
^2/e^2/d*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)-1/2*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(2*b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+
c))*d)/b^2/e^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2)*b^2/d^3*e^2+3*(1/b
/e/d*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c
))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)
)*b*e/d^3+(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a
*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)/d^3
+(-1/6/b/e/d/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2-1/3/b^2/e^2/d/(b*e-(b
*e/d+(a*d-b*c)*e/d/(d*x+c))*d)+1/3/b^3/e^3/d*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(
d*x+c))*d)+1/3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c
))*(3*b^2*e^2-3*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d*b*e+d^2*(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^2)/b^3/e^3/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^3)*b^3*e^3/d^
3))
```

Fricas [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algori
thm="fricas")
```

output
```
integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
+ (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log((b
*e*x + a*e)/(d*x + c)))/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$$
$$= \frac{g^3\left(\int \frac{Aa^3}{c+dx}\,dx+\int \frac{Ab^3x^3}{c+dx}\,dx+\int \frac{Ba^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx+\int \frac{3Aab^2x^2}{c+dx}\,dx+\int \frac{3Aa^2bx}{c+dx}\,dx+\int \frac{Bb^3x^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right.}{c+dx}\right.}{i}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x)
```

output
```
g**3*(Integral(A*a**3/(c + d*x), x) + Integral(A*b**3*x**3/(c + d*x), x) +
 Integral(B*a**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Inte
gral(3*A*a*b**2*x**2/(c + d*x), x) + Integral(3*A*a**2*b*x/(c + d*x), x) +
 Integral(B*b**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) +
 Integral(3*B*a*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x),
x) + Integral(3*B*a**2*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x),
 x))/i
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. $2(243) = 486$.

Time $= 0.12$ (sec) , antiderivative size $= 790$, normalized size of antiderivative $= 3.13$

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algori
thm="maxima")
```

output

```
3*A*a^2*b*g^3*(x/(d*i) - c*log(d*x + c)/(d^2*i)) - 1/6*A*b^3*g^3*(6*c^3*lo
g(d*x + c)/(d^4*i) - (2*d^2*x^3 - 3*c*d*x^2 + 6*c^2*x)/(d^3*i)) + 3/2*A*a*
b^2*g^3*(2*c^2*log(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A*a^3*g^3
*log(d*i*x + c*i)/(d*i) - (b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2
*g^3 - a^3*d^3*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dil
og(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^4*i) + 1/6*(6*a^3*d^3*g^3*log(e) - (6
*g^3*log(e) + 11*g^3)*b^3*c^3 + 9*(2*g^3*log(e) + 3*g^3)*a*b^2*c^2*d - 18*
(g^3*log(e) + g^3)*a^2*b*c*d^2)*B*log(d*x + c)/(d^4*i) + 1/6*(2*B*b^3*d^3*
g^3*x^3*log(e) - ((3*g^3*log(e) + g^3)*b^3*c*d^2 - (9*g^3*log(e) + g^3)*a*
b^2*d^3)*B*x^2 + 3*(b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 -
a^3*d^3*g^3)*B*log(d*x + c)^2 + ((6*g^3*log(e) + 5*g^3)*b^3*c^2*d - 6*(3*g
^3*log(e) + 2*g^3)*a*b^2*c*d^2 + (18*g^3*log(e) + 7*g^3)*a^2*b*d^3)*B*x +
(2*B*b^3*d^3*g^3*x^3 - 3*(b^3*c*d^2*g^3 - 3*a*b^2*d^3*g^3)*B*x^2 + 6*(b^3*
c^2*d*g^3 - 3*a*b^2*c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B*x + (6*a*b^2*c^2*d*g^3
- 15*a^2*b*c*d^2*g^3 + 11*a^3*d^3*g^3)*B)*log(b*x + a) - (2*B*b^3*d^3*g^3*
x^3 - 3*(b^3*c*d^2*g^3 - 3*a*b^2*d^3*g^3)*B*x^2 + 6*(b^3*c^2*d*g^3 - 3*a*b
^2*c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B*x)*log(d*x + c))/(d^4*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3552 vs. $2(243) = 486$.

Time $= 66.59$ (sec) , antiderivative size $= 3552$, normalized size of antiderivative $= 14.10$

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algori
thm="giac")
```

output

```
-1/120*(6*(B*b^9*c^6*e^6*g^3 - 6*B*a*b^8*c^5*d*e^6*g^3 + 15*B*a^2*b^7*c^4*
d^2*e^6*g^3 - 20*B*a^3*b^6*c^3*d^3*e^6*g^3 + 15*B*a^4*b^5*c^2*d^4*e^6*g^3
- 6*B*a^5*b^4*c*d^5*e^6*g^3 + B*a^6*b^3*d^6*e^6*g^3 - 5*(b*e*x + a*e)*B*b^
8*c^6*d*e^5*g^3/(d*x + c) + 30*(b*e*x + a*e)*B*a*b^7*c^5*d^2*e^5*g^3/(d*x
+ c) - 75*(b*e*x + a*e)*B*a^2*b^6*c^4*d^3*e^5*g^3/(d*x + c) + 100*(b*e*x +
 a*e)*B*a^3*b^5*c^3*d^4*e^5*g^3/(d*x + c) - 75*(b*e*x + a*e)*B*a^4*b^4*c^2
*d^5*e^5*g^3/(d*x + c) + 30*(b*e*x + a*e)*B*a^5*b^3*c*d^6*e^5*g^3/(d*x + c
) - 5*(b*e*x + a*e)*B*a^6*b^2*d^7*e^5*g^3/(d*x + c) + 10*(b*e*x + a*e)^2*B
*b^7*c^6*d^2*e^4*g^3/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a*b^6*c^5*d^3*e^4*
g^3/(d*x + c)^2 + 150*(b*e*x + a*e)^2*B*a^2*b^5*c^4*d^4*e^4*g^3/(d*x + c)^
2 - 200*(b*e*x + a*e)^2*B*a^3*b^4*c^3*d^5*e^4*g^3/(d*x + c)^2 + 150*(b*e*x
 + a*e)^2*B*a^4*b^3*c^2*d^6*e^4*g^3/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a^5
*b^2*c*d^7*e^4*g^3/(d*x + c)^2 + 10*(b*e*x + a*e)^2*B*a^6*b*d^8*e^4*g^3/(d
*x + c)^2 - 10*(b*e*x + a*e)^3*B*b^6*c^6*d^3*e^3*g^3/(d*x + c)^3 + 60*(b*e
*x + a*e)^3*B*a*b^5*c^5*d^4*e^3*g^3/(d*x + c)^3 - 150*(b*e*x + a*e)^3*B*a^
2*b^4*c^4*d^5*e^3*g^3/(d*x + c)^3 + 200*(b*e*x + a*e)^3*B*a^3*b^3*c^3*d^6*
e^3*g^3/(d*x + c)^3 - 150*(b*e*x + a*e)^3*B*a^4*b^2*c^2*d^7*e^3*g^3/(d*x +
 c)^3 + 60*(b*e*x + a*e)^3*B*a^5*b*c*d^8*e^3*g^3/(d*x + c)^3 - 10*(b*e*x +
 a*e)^3*B*a^6*d^9*e^3*g^3/(d*x + c)^3)*log((b*e*x + a*e)/(d*x + c))/(b^5*d
^4*e^5*i - 5*(b*e*x + a*e)*b^4*d^5*e^4*i/(d*x + c) + 10*(b*e*x + a*e)^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \int \frac{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{c\,i+d\,i\,x}\,dx$$

input

```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x),x
)
```

output

```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x),
x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$$

$$= \frac{g^3 i\left(-6\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{dx+c}dx\right)a^3 b\,d^4 - 6\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x^3}{dx+c}dx\right)b^4 d^4 - 18\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x^2}{dx+c}dx\right)a\,b^3 d^4 - 18\left(\int \frac{\log}{}\right.\right.}{}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x)
```

output
```
(g**3*i*( - 6*int(log((a*e + b*e*x)/(c + d*x))/(c + d*x),x)*a**3*b*d**4 -
6*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(c + d*x),x)*b**4*d**4 - 18*int(
(log((a*e + b*e*x)/(c + d*x))*x**2)/(c + d*x),x)*a*b**3*d**4 - 18*int((log
((a*e + b*e*x)/(c + d*x))*x)/(c + d*x),x)*a**2*b**2*d**4 - 6*log(c + d*x)*
a**4*d**3 + 18*log(c + d*x)*a**3*b*c*d**2 - 18*log(c + d*x)*a**2*b**2*c**2
*d + 6*log(c + d*x)*a*b**3*c**3 - 18*a**3*b*d**3*x + 18*a**2*b**2*c*d**2*x
 - 9*a**2*b**2*d**3*x**2 - 6*a*b**3*c**2*d*x + 3*a*b**3*c*d**2*x**2 - 2*a*
b**3*d**3*x**3))/(6*d**4)
```

3.32 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 198

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx \\
&= \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di} \\
&\quad - \frac{(bc-ad)g^2(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i} \\
&\quad - \frac{(bc-ad)^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^3i} \\
&\quad - \frac{B(bc-ad)^2g^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}
\end{aligned}$$

output

```
1/2*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i-1/2*(-a*d+b*c)*g^2*(b*x+
a)*(2*A+B+2*B*ln(e*(b*x+a)/(d*x+c)))/d^2/i-1/2*(-a*d+b*c)^2*g^2*ln((-a*d+b
*c)/b/(d*x+c))*(2*A+3*B+2*B*ln(e*(b*x+a)/(d*x+c)))/d^3/i-B*(-a*d+b*c)^2*g^
2*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.28

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$$
$$=\frac{g^2\left(-2Abd(bc-ad)x+2Bd(-bc+ad)(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2I\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*
i*x),x]
```

output
```
(g^2*(-2*A*b*d*(b*c - a*d)*x + 2*B*d*(-(b*c) + a*d)*(a + b*x)*Log[(e*(a +
b*x))/(c + d*x)] + d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) +
2*B*(b*c - a*d)^2*Log[c + d*x] - B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log
[c + d*x]) + 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[i*(c
 + d*x)] - B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[i*(
c + d*x)])*Log[i*(c + d*x)] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(
2*d^3*i)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{ci+dix}\,dx$$
$$\downarrow \text{2962}$$
$$\frac{g^2(bc-ad)^2\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{i}$$

$$\downarrow 2784$$

$$\frac{g^2(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{i}$$

$$\downarrow 2784$$

$$\frac{g^2(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}}{2d}\right)}{i}$$

$$\downarrow 2754$$

$$\frac{g^2(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2B\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{d}}{2d}\right)}{i}$$

$$\downarrow 2838$$

$$\frac{g^2(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+3B\right)}{d}}{d}-\frac{2B\,\mathrm{Pol}}{}}{2d}\right)}{i}$$

input
```
Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x),x
]
```

output
```
((b*c - a*d)^2*g^2*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*
d*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(2*A + B + 2*
B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)
)) - (-(((2*A + 3*B + 2*B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*
x))/(b*(c + d*x))])/d) - (2*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/
d)/(2*d)))/i
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. 2(192) = 384.

Time = 3.20 (sec) , antiderivative size = 657, normalized size of antiderivative = 3.32

method	result
parts	$\frac{g^2 A\left(\frac{b\left(\frac{1}{2}bd\,x^2+2xda-bcx\right)}{d^2}+\frac{\left(a^2d^2-2acdb+c^2b^2\right)\ln(dx+c)}{d^3}\right)}{i}-g^2B\left(\frac{\left(-\frac{1}{2bed\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}-\frac{\ln\left(\left(\frac{be}{d}+\frac{(da-b}{d(dx-}\right.\right.}{2b^2e^2}\right.$
derivativedivides	$-e(da-bc)\left(-\frac{A\,d^2g^2(da-bc)\left(\frac{b^2e^2}{2d^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^2}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{d^3}-\frac{2be}{d^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}\right.}{ei}\right.$
default	$-e(da-bc)\left(-\frac{A\,d^2g^2(da-bc)\left(\frac{b^2e^2}{2d^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)^2}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{d^3}-\frac{2be}{d^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}\right.}{ei}\right.$
risch	Expression too large to display

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x,method=_RETURNVERBOSE)
```

output
```
g^2*A/i*(b/d^2*(1/2*b*d*x^2+2*x*d*a-b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3
*ln(d*x+c))-g^2*B/i/d*((-1/2/b/e/d/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-1
/2/b^2/e^2*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/d+1/2*ln(b*e/d+(a*d-b*c
)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*((b*e/d+(a*d-b*c)*e/d/(d*x+c)
)*d-2*b*e)/b^2/e^2/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)^2)*b^2*e^2*(a^2*d
^2-2*a*b*c*d+b^2*c^2)/d+2*(1/b/e*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/d
-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d
+(a*d-b*c)*e/d/(d*x+c))*d-b*e))*b*e*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d+(dilog(-
((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c
))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)*(a^2*d^2-2*a*b*c*d+b^
2*c^2)/d)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algori
thm="fricas")
```

output
```
integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B
*a*b*g^2*x + B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)))/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$$
$$= \frac{g^2\left(\int \frac{Aa^2}{c+dx}\,dx + \int \frac{Ab^2x^2}{c+dx}\,dx + \int \frac{Ba^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx + \int \frac{2Aabx}{c+dx}\,dx + \int \frac{Bb^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx + \int \frac{2Babx\,\text{lo}}{i}\right.}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x)
```

output

```
g**2*(Integral(A*a**2/(c + d*x), x) + Integral(A*b**2*x**2/(c + d*x), x) +
 Integral(B*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Inte
gral(2*A*a*b*x/(c + d*x), x) + Integral(B*b**2*x**2*log(a*e/(c + d*x) + b*
e*x/(c + d*x))/(c + d*x), x) + Integral(2*B*a*b*x*log(a*e/(c + d*x) + b*e*
x/(c + d*x))/(c + d*x), x))/i
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(191) = 382$.

Time $= 0.10$ (sec) , antiderivative size $= 477$, normalized size of antiderivative $= 2.41$

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = 2\,Aabg^2\left(\frac{x}{di}-\frac{c\log(dx+c)}{d^2i}\right)$$
$$+\frac{1}{2}\,Ab^2g^2\left(\frac{2\,c^2\log(dx+c)}{d^3i}+\frac{dx^2-2\,cx}{d^2i}\right)+\frac{Aa^2g^2\log(dix+ci)}{di}$$
$$+\frac{(b^2c^2g^2-2\,abcdg^2+a^2d^2g^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^3i}$$
$$+\frac{(2\,a^2d^2g^2\log(e)+(2\,g^2\log(e)+3\,g^2)b^2c^2-4\,(g^2\log(e)+g^2)abcd)B\log(dx+c)}{2\,d^3i}$$
$$+\frac{Bb^2d^2g^2x^2\log(e)-(b^2c^2g^2-2\,abcdg^2+a^2d^2g^2)B\log(dx+c)^2-((2\,g^2\log(e)+g^2)b^2cd-(4\,g^2\log($$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algori
thm="maxima")
```

output

```
2*A*a*b*g^2*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + 1/2*A*b^2*g^2*(2*c^2*log(
d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A*a^2*g^2*log(d*i*x + c*i)/(
d*i) + (b^2*c^2*g^2 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)*(log(b*x + a)*log((b*d*
x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^3*i) +
 1/2*(2*a^2*d^2*g^2*log(e) + (2*g^2*log(e) + 3*g^2)*b^2*c^2 - 4*(g^2*log(e
) + g^2)*a*b*c*d)*B*log(d*x + c)/(d^3*i) + 1/2*(B*b^2*d^2*g^2*x^2*log(e) -
 (b^2*c^2*g^2 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)*B*log(d*x + c)^2 - ((2*g^2*lo
g(e) + g^2)*b^2*c*d - (4*g^2*log(e) + g^2)*a*b*d^2)*B*x + (B*b^2*d^2*g^2*x
^2 - 2*(b^2*c*d*g^2 - 2*a*b*d^2*g^2)*B*x - (2*a*b*c*d*g^2 - 3*a^2*d^2*g^2)
*B)*log(b*x + a) - (B*b^2*d^2*g^2*x^2 - 2*(b^2*c*d*g^2 - 2*a*b*d^2*g^2)*B*
x)*log(d*x + c))/(d^3*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2364 vs. $2(191) = 382$.

Time = 55.51 (sec) , antiderivative size = 2364, normalized size of antiderivative = 11.94

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algori
thm="giac")
```

output

```
1/24*(2*(B*b^7*c^5*e^5*g^2 - 5*B*a*b^6*c^4*d*e^5*g^2 + 10*B*a^2*b^5*c^3*d^
2*e^5*g^2 - 10*B*a^3*b^4*c^2*d^3*e^5*g^2 + 5*B*a^4*b^3*c*d^4*e^5*g^2 - B*a
^5*b^2*d^5*e^5*g^2 - 4*(b*e*x + a*e)*B*b^6*c^5*d*e^4*g^2/(d*x + c) + 20*(b
*e*x + a*e)*B*a*b^5*c^4*d^2*e^4*g^2/(d*x + c) - 40*(b*e*x + a*e)*B*a^2*b^4
*c^3*d^3*e^4*g^2/(d*x + c) + 40*(b*e*x + a*e)*B*a^3*b^3*c^2*d^4*e^4*g^2/(d
*x + c) - 20*(b*e*x + a*e)*B*a^4*b^2*c*d^5*e^4*g^2/(d*x + c) + 4*(b*e*x +
a*e)*B*a^5*b*d^6*e^4*g^2/(d*x + c) + 6*(b*e*x + a*e)^2*B*b^5*c^5*d^2*e^3*g
^2/(d*x + c)^2 - 30*(b*e*x + a*e)^2*B*a*b^4*c^4*d^3*e^3*g^2/(d*x + c)^2 +
60*(b*e*x + a*e)^2*B*a^2*b^3*c^3*d^4*e^3*g^2/(d*x + c)^2 - 60*(b*e*x + a*e
)^2*B*a^3*b^2*c^2*d^5*e^3*g^2/(d*x + c)^2 + 30*(b*e*x + a*e)^2*B*a^4*b*c*d
^6*e^3*g^2/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^5*d^7*e^3*g^2/(d*x + c)^2)*
log((b*e*x + a*e)/(d*x + c))/(b^4*d^3*e^4*i - 4*(b*e*x + a*e)*b^3*d^4*e^3*
i/(d*x + c) + 6*(b*e*x + a*e)^2*b^2*d^5*e^2*i/(d*x + c)^2 - 4*(b*e*x + a*e
)^3*b*d^6*e*i/(d*x + c)^3 + (b*e*x + a*e)^4*d^7*i/(d*x + c)^4) + (2*A*b^8*
c^5*e^5*g^2 + B*b^8*c^5*e^5*g^2 - 10*A*a*b^7*c^4*d*e^5*g^2 - 5*B*a*b^7*c^4
*d*e^5*g^2 + 20*A*a^2*b^6*c^3*d^2*e^5*g^2 + 10*B*a^2*b^6*c^3*d^2*e^5*g^2 -
 20*A*a^3*b^5*c^2*d^3*e^5*g^2 - 10*B*a^3*b^5*c^2*d^3*e^5*g^2 + 10*A*a^4*b^
4*c*d^4*e^5*g^2 + 5*B*a^4*b^4*c*d^4*e^5*g^2 - 2*A*a^5*b^3*d^5*e^5*g^2 - B*
a^5*b^3*d^5*e^5*g^2 - 8*(b*e*x + a*e)*A*b^7*c^5*d*e^4*g^2/(d*x + c) - 2*(b
*e*x + a*e)*B*b^7*c^5*d*e^4*g^2/(d*x + c) + 40*(b*e*x + a*e)*A*a*b^6*c^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x),x
)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x),
x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$$
$$= \frac{g^2 i\left(-2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{dx+c}dx\right)a^2 b\,d^3 - 2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x^2}{dx+c}dx\right)b^3 d^3 - 4\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{dx+c}dx\right)a\,b^2 d^3 - 2\log(dx+\right.}{2d^3}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x)
```

output
```
(g**2*i*( - 2*int(log((a*e + b*e*x)/(c + d*x))/(c + d*x),x)*a**2*b*d**3 -
2*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(c + d*x),x)*b**3*d**3 - 4*int((
log((a*e + b*e*x)/(c + d*x))*x)/(c + d*x),x)*a*b**2*d**3 - 2*log(c + d*x)*
a**3*d**2 + 4*log(c + d*x)*a**2*b*c*d - 2*log(c + d*x)*a*b**2*c**2 - 4*a**
2*b*d**2*x + 2*a*b**2*c*d*x - a*b**2*d**2*x**2))/(2*d**3)
```

3.33 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$

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Optimal result

Integrand size = 38, antiderivative size = 125

$$\begin{aligned}
&\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx \\
&= \frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di} \\
&\quad + \frac{(bc-ad)g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2 i} \\
&\quad + \frac{B(bc-ad)g\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2 i}
\end{aligned}$$

output

```
g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i+(-a*d+b*c)*g*ln((-a*d+b*c)/b/(d*
x+c))*(A+B+B*ln(e*(b*x+a)/(d*x+c)))/d^2/i+B*(-a*d+b*c)*g*polylog(2,d*(b*x+
a)/b/(d*x+c))/d^2/i
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.30

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$$
$$=\frac{g\left(2Abdx+2Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-2B(bc-ad)\log(c+dx)-2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{lo}\right.}{2d^2i}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*
x),x]
```

output
```
(g*(2*A*b*d*x + 2*B*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 2*B*(b*c -
a*d)*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log
[c + d*x] + B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c +
d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(2*d^2*i)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2962, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{ci+dix}\,dx$$
$$\downarrow 2962$$
$$\frac{g(bc-ad)\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i}$$
$$\downarrow 2784$$

$$\frac{g(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{i}$$

$$\downarrow 2754$$

$$\frac{g(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{B\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A+B\right)}{d}}{d}\right)}{i}$$

$$\downarrow 2838$$

$$\frac{g(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A+B\right)}{d}-\frac{B\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d}}{d}\right)}{i}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x),x]
```

output
```
((b*c - a*d)*g*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c + d
*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((A + B + B*Log[(e*(a + b*x))/(c +
 d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (B*PolyLog[2, (d*(a + b
*x))/(b*(c + d*x))])/d)/d))/i
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2784

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(125) = 250$.

Time = 2.96 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.66

method	result
parts	$\frac{gA\left(\frac{bx}{d}+\frac{(da-bc)\ln(dx+c)}{d^2}\right)}{i} - \frac{gB\left(\left(\frac{\ln\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}{bed}-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)}\right)be(da-bc)+\left($
derivativedivides	$-\frac{e(da-bc)\left(\frac{gA\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{ei}+\frac{gAb}{i\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}+\frac{gB\,\mathrm{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be}{be}\right)}{ei}+\frac{gB\ln\left(\frac{be}{d}+\right.}{d^2}$
default	$-\frac{e(da-bc)\left(\frac{gA\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{ei}+\frac{gAb}{i\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}+\frac{gB\,\mathrm{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be}{be}\right)}{ei}+\frac{gB\ln\left(\frac{be}{d}+\right.}{d^2}$
risch	$\frac{gAbx}{id}+\frac{gA\ln(dx+c)a}{id}-\frac{gA\ln(dx+c)bc}{i\,d^2}-\frac{gB\ln\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)a}{id}+\frac{gBb\ln\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be\right)c}{i\,d^2}+$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x,method=_RETURNVE
RBOSE)
```

output
```
g*A/i*(b*x/d+(a*d-b*c)/d^2*ln(d*x+c))-g*B/i/d*((1/b/e*ln((b*e/d+(a*d-b*c)*
e/d/(d*x+c))*d-b*e)/d-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d
/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e))*b*e*(a*d-b*c)+(dilog(
-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)*(a*d-b*c))
```

Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorith
m="fricas")
```

output
```
integral((A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b*e*x + a*e)/(d*x + c))
)/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \frac{g\left(\int \frac{Aa}{c+dx}\,dx + \int \frac{Abx}{c+dx}\,dx + \int \frac{Ba\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx + \int \frac{Bbx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx\right)}{i}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x)
```

output
```
g*(Integral(A*a/(c + d*x), x) + Integral(A*b*x/(c + d*x), x) + Integral(B*
a*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Integral(B*b*x*log(
a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x))/i
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.77

$$\begin{aligned}\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx &= Abg\left(\frac{x}{di}-\frac{c\log(dx+c)}{d^2i}\right)\\ &+\frac{Aag\log(dix+ci)}{di}-\frac{(bcg-adg)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^2i}\\ &+\frac{(adg\log(e)-(g\log(e)+g)bc)B\log(dx+c)}{d^2i}\\ &-\frac{2\,Bbdgx\log(dx+c)-2\,Bbdgx\log(e)-(bcg-adg)B\log(dx+c)^2-2\,(Bbdgx+Badg)\log(bx+a}{2\,d^2i}\end{aligned}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorith
m="maxima")
```

output

```
A*b*g*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + A*a*g*log(d*i*x + c*i)/(d*i) -
(b*c*g - a*d*g)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-
(b*d*x + a*d)/(b*c - a*d)))*B/(d^2*i) + (a*d*g*log(e) - (g*log(e) + g)*b*c
)*B*log(d*x + c)/(d^2*i) - 1/2*(2*B*b*d*g*x*log(d*x + c) - 2*B*b*d*g*x*log
(e) - (b*c*g - a*d*g)*B*log(d*x + c)^2 - 2*(B*b*d*g*x + B*a*d*g)*log(b*x +
 a))/(d^2*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1230 vs. $2(124) = 248$.

Time $= 43.19$ (sec) , antiderivative size $= 1230$, normalized size of antiderivative $= 9.84$

$$\int \frac{(ag + bgx)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci + dix}\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorith
m="giac")
```

output

```
-1/6*((B*b^5*c^4*e^4*g - 4*B*a*b^4*c^3*d*e^4*g + 6*B*a^2*b^3*c^2*d^2*e^4*g
 - 4*B*a^3*b^2*c*d^3*e^4*g + B*a^4*b*d^4*e^4*g - 3*(b*e*x + a*e)*B*b^4*c^4
*d*e^3*g/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^3*c^3*d^2*e^3*g/(d*x + c) - 18
*(b*e*x + a*e)*B*a^2*b^2*c^2*d^3*e^3*g/(d*x + c) + 12*(b*e*x + a*e)*B*a^3*
b*c*d^4*e^3*g/(d*x + c) - 3*(b*e*x + a*e)*B*a^4*d^5*e^3*g/(d*x + c))*log((
b*e*x + a*e)/(d*x + c))/(b^3*d^2*e^3*i - 3*(b*e*x + a*e)*b^2*d^3*e^2*i/(d*
x + c) + 3*(b*e*x + a*e)^2*b*d^4*e*i/(d*x + c)^2 - (b*e*x + a*e)^3*d^5*i/(
d*x + c)^3) + (A*b^6*c^4*e^4*g - 4*A*a*b^5*c^3*d*e^4*g + 6*A*a^2*b^4*c^2*d
^2*e^4*g - 4*A*a^3*b^3*c*d^3*e^4*g + A*a^4*b^2*d^4*e^4*g - 3*(b*e*x + a*e)
*A*b^5*c^4*d*e^3*g/(d*x + c) + (b*e*x + a*e)*B*b^5*c^4*d*e^3*g/(d*x + c) +
 12*(b*e*x + a*e)*A*a*b^4*c^3*d^2*e^3*g/(d*x + c) - 4*(b*e*x + a*e)*B*a*b^
4*c^3*d^2*e^3*g/(d*x + c) - 18*(b*e*x + a*e)*A*a^2*b^3*c^2*d^3*e^3*g/(d*x
 + c) + 6*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3*g/(d*x + c) + 12*(b*e*x + a*e
)*A*a^3*b^2*c*d^4*e^3*g/(d*x + c) - 4*(b*e*x + a*e)*B*a^3*b^2*c*d^4*e^3*g/
(d*x + c) - 3*(b*e*x + a*e)*A*a^4*b*d^5*e^3*g/(d*x + c) + (b*e*x + a*e)*B*
a^4*b*d^5*e^3*g/(d*x + c) - (b*e*x + a*e)^2*B*b^4*c^4*d^2*e^2*g/(d*x + c)^
2 + 4*(b*e*x + a*e)^2*B*a*b^3*c^3*d^3*e^2*g/(d*x + c)^2 - 6*(b*e*x + a*e)^
2*B*a^2*b^2*c^2*d^4*e^2*g/(d*x + c)^2 + 4*(b*e*x + a*e)^2*B*a^3*b*c*d^5*e^
2*g/(d*x + c)^2 - (b*e*x + a*e)^2*B*a^4*d^6*e^2*g/(d*x + c)^2)/(b^4*d^2*e^
3*i - 3*(b*e*x + a*e)*b^3*d^3*e^2*i/(d*x + c) + 3*(b*e*x + a*e)^2*b^2*d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{c\,i+d\,i\,x}\,dx$$

input

```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x),x)
```

output

```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x), x)
```

Reduce [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$$
$$= \frac{gi\left(-\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{dx+c}dx\right)ab\,d^2-\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{dx+c}dx\right)b^2d^2-\log(dx+c)\,a^2d+\log(dx+c)\,abc-abdx\right)}{d^2}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x)
```

output
```
(g*i*( - int(log((a*e + b*e*x)/(c + d*x))/(c + d*x),x)*a*b*d**2 - int((log
((a*e + b*e*x)/(c + d*x))*x)/(c + d*x),x)*b**2*d**2 - log(c + d*x)*a**2*d
+ log(c + d*x)*a*b*c - a*b*d*x))/d**2
```

3.34 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx = -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di} - \frac{B\,\mathrm{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{di}$$

output

```
-ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i-B*polylog(2,1-(-
a*d+b*c)/b/(d*x+c))/d/i
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx = \frac{\log(i(c+dx))\left(2A-2B\log\left(\frac{d(a+bx)}{-bc+ad}\right)+2B\log\left(\frac{e(a+bx)}{c+dx}\right)+B\log(i(c+dx))\right)-2B\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right.}{2di}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x),x]
```

output
```
(Log[i*(c + d*x)]*(2*A - 2*B*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*B*Log[(
e*(a + b*x))/(c + d*x)] + B*Log[i*(c + d*x)]) - 2*B*PolyLog[2, (b*(c + d*x
))/(b*c - a*d)])/(2*d*i)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2944, 2858, 27, 25, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{ci + dix}\, dx$$

$$\downarrow 2944$$

$$\frac{B(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx}{di} - \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di}$$

$$\downarrow 2858$$

$$\frac{B(bc-ad)\int \frac{d\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)\left(\left(a-\frac{bc}{d}\right)d+b(c+dx)\right)}d(c+dx)}{d^2 i} - \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di}$$

$$\downarrow 27$$

$$\frac{B(bc-ad)\int -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{di} - \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di}$$

$$\downarrow 25$$

$$-\frac{B(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{di} - \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di}$$

$$\downarrow 2778$$

$$\frac{B(bc-ad)\int \frac{(c+dx)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bc-ad-b(c+dx)}d\frac{1}{c+dx}}{di} - \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di}$$

$$\downarrow 2005$$

$$\frac{B(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{\frac{bc-ad}{c+dx}-b}d\frac{1}{c+dx}}{di} - \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di}$$

$$\downarrow 2752$$

$$-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di} - \frac{B\,\mathrm{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{di}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x),x]
```

output
```
-((Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d
*i)) - (B*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(d*i)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2944

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c +
 d*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
- a*d)/g)   Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x
] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && EqQ[d*f - c*g, 0]
```

Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.80

method	result
parts	$\frac{A\ln(dx+c)}{id}+\frac{B\left(-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be}{be}\right)}{d}-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be}{be}\right)}{d}\right)}{i}$
risch	$\frac{A\ln(dx+c)}{id}-\frac{B\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be}{be}\right)}{id}-\frac{B\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be}{be}\right)}{id}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{dA\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{ie(da-bc)}-\frac{d^2B\left(-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be}{be}\right)}{d}-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(-\frac{\left(\frac{be}{d}+\frac{(da}{d(d}\right.\right.\right.}{ie(da-bc)}\right.}{d^2}$
default	$-\frac{e(da-bc)\left(\frac{dA\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{ie(da-bc)}-\frac{d^2B\left(-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d-be}{be}\right)}{d}-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(-\frac{\left(\frac{be}{d}+\frac{(da}{d(d}\right.\right.\right.}{ie(da-bc)}\right.}{d^2}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x,method=_RETURNVERBOSE)
```

output
```
A/i*ln(d*x+c)/d+B/i*(-dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d-
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/
b/e)/d)
```

Fricas [F]

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{ci + dix}\, dx = \int \frac{B \log\left(\frac{(bx+a)e}{dx+c}\right) + A}{dix + ci}\, dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorithm="fricas")
```

output
```
integral((B*log((b*e*x + a*e)/(d*x + c)) + A)/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{ci + dix}\, dx = \frac{\int \frac{A}{c+dx}\, dx + \int \frac{B \log\left(\frac{ae}{c+dx} + \frac{bex}{c+dx}\right)}{c+dx}\, dx}{i}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x)
```

output
```
(Integral(A/(c + d*x), x) + Integral(B*log(a*e/(c + d*x) + b*e*x/(c + d*x)
)/(c + d*x), x))/i
```

Maxima [F]

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{ci + dix}\, dx = \int \frac{B \log\left(\frac{(bx+a)e}{dx+c}\right) + A}{dix + ci}\, dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorithm="maxima")
```

output
```
-1/2*B*(log(d*x + c)^2/(d*i) - 2*integrate((log(b*x + a) + log(e))/(d*i*x
+ c*i), x)) + A*log(d*i*x + c*i)/(d*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(80) = 160.

Time = 36.84 (sec) , antiderivative size = 617, normalized size of antiderivative = 7.62

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx$$

$$=\frac{1}{2}\left(\frac{\left(Bb^3c^3e^3-3\,Bab^2c^2de^3+3\,Ba^2bcd^2e^3-Ba^3d^3e^3\right)\log\left(\frac{bex+ae}{dx+c}\right)}{b^2de^2i-\frac{2\,(bex+ae)bd^2ei}{dx+c}+\frac{(bex+ae)^2d^3i}{(dx+c)^2}}+\frac{Ab^4c^3e^3-Bb^4c^3e^3-3\,Aab^3c^2de^3}{}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorithm="giac")
```

output
```
1/2*((B*b^3*c^3*e^3 - 3*B*a*b^2*c^2*d*e^3 + 3*B*a^2*b*c*d^2*e^3 - B*a^3*d^
3*e^3)*log((b*e*x + a*e)/(d*x + c))/(b^2*d*e^2*i - 2*(b*e*x + a*e)*b*d^2*e
*i/(d*x + c) + (b*e*x + a*e)^2*d^3*i/(d*x + c)^2) + (A*b^4*c^3*e^3 - B*b^4
*c^3*e^3 - 3*A*a*b^3*c^2*d*e^3 + 3*B*a*b^3*c^2*d*e^3 + 3*A*a^2*b^2*c*d^2*e
^3 - 3*B*a^2*b^2*c*d^2*e^3 - A*a^3*b*d^3*e^3 + B*a^3*b*d^3*e^3 + (b*e*x +
a*e)*B*b^3*c^3*d*e^2/(d*x + c) - 3*(b*e*x + a*e)*B*a*b^2*c^2*d^2*e^2/(d*x
+ c) + 3*(b*e*x + a*e)*B*a^2*b*c*d^3*e^2/(d*x + c) - (b*e*x + a*e)*B*a^3*d
^4*e^2/(d*x + c))/(b^3*d*e^2*i - 2*(b*e*x + a*e)*b^2*d^2*e*i/(d*x + c) + (
b*e*x + a*e)^2*b*d^3*i/(d*x + c)^2) + (B*b^3*c^3*e - 3*B*a*b^2*c^2*d*e + 3
*B*a^2*b*c*d^2*e - B*a^3*d^3*e)*log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b^2
*d*i) - (B*b^3*c^3*e - 3*B*a*b^2*c^2*d*e + 3*B*a^2*b*c*d^2*e - B*a^3*d^3*e
)*log((b*e*x + a*e)/(d*x + c))/(b^2*d*i))*(b*c/((b*c*e - a*d*e)*(b*c - a*d
)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx=\int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{c\,i+d\,i\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(c*i + d*i*x),x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(c*i + d*i*x), x)
```

Reduce [F]

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci + dix}\,dx = -\frac{i\left(\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{dx+c}dx\right)bd + \log(dx+c)\,a\right)}{d}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x)
```

output
```
( - i*(int(log((a*e + b*e*x)/(c + d*x))/(c + d*x),x)*b*d + log(c + d*x)*a)
)/d
```

3.35 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 44

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2B(bc-ad)gi}$$

output
```
1/2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/B/(-a*d+b*c)/g/i
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 4.70

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx$$
$$= \frac{2A\log(a+bx) - B\log^2(a+bx) + 2B\log(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right) - 2A\log(c+dx) + 2B\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x
)),x]
```

output
```
(2*A*Log[a + b*x] - B*Log[a + b*x]^2 + 2*B*Log[a + b*x]*Log[(e*(a + b*x))/
(c + d*x)] - 2*A*Log[c + d*x] + 2*B*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[
c + d*x] - 2*B*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] - B*Log[c + d*x]^
2 + 2*B*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 2*B*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)] + 2*B*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(2*(
b*c - a*d)*g*i)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2962, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{(ag+bgx)(ci+dix)}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx} d\frac{a+bx}{c+dx}}{gi(bc-ad)}$$

$$\downarrow 2738$$

$$\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2Bgi(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x)),x]
```

output
```
(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(2*B*(b*c - a*d)*g*i)
```

Defintions of rubi rules used

rule 2738

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.64

method	result	size
norman	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2gi(da-bc)}-\frac{A\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi(da-bc)}$	72
parallelrisch	$-\frac{a^2c^2B\ln\left(\frac{e(bx+a)}{dx+c}\right)^2+2A\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2c^2}{2a^2c^2gi(da-bc)}$	75
parts	$\frac{A\left(\frac{\ln(dx+c)}{da-bc}-\frac{\ln(bx+a)}{da-bc}\right)}{gi}-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2gi(da-bc)}$	82
risch	$\frac{A\ln(dx+c)}{gi(da-bc)}-\frac{A\ln(bx+a)}{gi(da-bc)}-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2gi(da-bc)}$	87
derivativedivides	$-\frac{e(da-bc)\left(\frac{d^2A\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei(da-bc)^2g}+\frac{d^2B\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2ei(da-bc)^2g}\right)}{d^2}$	123
default	$-\frac{e(da-bc)\left(\frac{d^2A\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei(da-bc)^2g}+\frac{d^2B\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2ei(da-bc)^2g}\right)}{d^2}$	123

input

```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i),x,method=_RETURNVERBOSE)
```

output
```
-1/2*B/g/i/(a*d-b*c)*ln(e*(b*x+a)/(d*x+c))^2-A/g/i/(a*d-b*c)*ln(e*(b*x+a)/
(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx = \frac{B \log\left(\frac{bex+ae}{dx+c}\right)^2 + 2\,A \log\left(\frac{bex+ae}{dx+c}\right)}{2\,(bc-ad)gi}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i),x, algorith
m="fricas")
```

output
```
1/2*(B*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*log((b*e*x + a*e)/(d*x + c)))/
((b*c - a*d)*g*i)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(31) = 62$.

Time = 0.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.86

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx = A\left(\frac{\log\left(x + \frac{-\frac{a^2d^2}{ad-bc} + \frac{2abcd}{ad-bc} + ad - \frac{b^2c^2}{ad-bc} + bc}{2bd}\right)}{gi\,(ad-bc)} - \frac{\log\left(x + \frac{\frac{a^2d^2}{ad-bc} - \frac{2abcd}{ad-bc} + ad + \frac{b^2c^2}{ad-bc} + bc}{2bd}\right)}{gi\,(ad-bc)}\right) - \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2adgi - 2bcgi}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i),x)
```

output
```
A*(log(x + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c*
*2/(a*d - b*c) + b*c)/(2*b*d))/(g*i*(a*d - b*c)) - log(x + (a**2*d**2/(a*d
 - b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*
d))/(g*i*(a*d - b*c))) - B*log(e*(a + b*x)/(c + d*x))**2/(2*a*d*g*i - 2*b*
c*g*i)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(42) = 84$.

Time $= 0.04$ (sec) , antiderivative size $= 172$, normalized size of antiderivative $= 3.91$

$$\begin{aligned}
&\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx \\
&= B\left(\frac{\log(bx+a)}{(bc-ad)gi} - \frac{\log(dx+c)}{(bc-ad)gi}\right)\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) \\
&\quad + A\left(\frac{\log(bx+a)}{(bc-ad)gi} - \frac{\log(dx+c)}{(bc-ad)gi}\right) \\
&\quad - \frac{\left(\log(bx+a)^2 - 2\log(bx+a)\log(dx+c) + \log(dx+c)^2\right)B}{2(bcgi-adgi)}
\end{aligned}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i),x, algorith
m="maxima")
```

output
```
B*(log(b*x + a)/((b*c - a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))*log(b*
e*x/(d*x + c) + a*e/(d*x + c)) + A*(log(b*x + a)/((b*c - a*d)*g*i) - log(d
*x + c)/((b*c - a*d)*g*i)) - 1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(d*x
 + c) + log(d*x + c)^2)*B/(b*c*g*i - a*d*g*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(42) = 84$.

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.36

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx = \frac{\left(Be\log\left(\frac{bex+ae}{dx+c}\right)^2 + 2\,Ae\log\left(\frac{bex+ae}{dx+c}\right)\right)\left(\frac{bc}{(bce-ade)(bc-ad)} - \frac{ad}{(bce-ade)(bc-ad)}\right)}{2\,gi}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i),x, algorithm="giac")
```

output
```
1/2*(B*e*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*e*log((b*e*x + a*e)/(d*x + c)))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(g*i)
```

Mupad [B] (verification not implemented)

Time = 27.50 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.57

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx = -\frac{B\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2 - A\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c} + 1\mathrm{i}\right)\,4\mathrm{i}}{2\,g\,i\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)*(c*i + d*i*x)),x)
```

output
```
-(B*log((e*(a + b*x))/(c + d*x))^2 - A*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*4i)/(2*g*i*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx = \frac{i\left(2\log(bx+a)\,a-2\log(dx+c)\,a+\log\left(\frac{bex+ae}{dx+c}\right)^2 b\right)}{2g\,(ad-bc)}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i),x)
```

output
```
(i*(2*log(a + b*x)*a - 2*log(c + d*x)*a + log((a*e + b*e*x)/(c + d*x))**2*
b))/(2*g*(a*d - b*c))
```

3.36 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 173

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{bB(c+dx)}{(bc-ad)^2g^2i(a+bx)} + \frac{Bd\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^2g^2i} - \frac{b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{d\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i}$$

output

```
-b*B*(d*x+c)/(-a*d+b*c)^2/g^2/i/(b*x+a)+1/2*B*d*ln((b*x+a)/(d*x+c))^2/(-a*
d+b*c)^2/g^2/i-b*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^2/g^2/i/(b
*x+a)-d*ln((b*x+a)/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^2/g^2/i
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.69

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx =$$
$$-\frac{2(bc-ad)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) + 2d(a+bx)\log(a+bx)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) - 2d(a+bx)\left(A +$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i
*x)),x]
```

output
```
-1/2*(2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d*(a + b*x)*L
og[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*d*(a + b*x)*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*B*(b*c - a*d + d*(a + b*x)*Lo
g[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[
a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)]) + B*d*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - L
og[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*
c - a*d)^2*g^2*i*(a + b*x))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2772, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right) + A}{(ag+bgx)^2(ci+dix)}\,dx$$
$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i(bc-ad)^2}$$

$$\downarrow 2772$$

$$\frac{-B\int -\frac{(c+dx)^2\left(b+\frac{d(a+bx)\log\left(\frac{a+bx}{c+dx}\right)}{c+dx}\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{b(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2i(bc-ad)^2}$$

$$\downarrow 25$$

$$\frac{B\int \frac{(c+dx)^2\left(b+\frac{d(a+bx)\log\left(\frac{a+bx}{c+dx}\right)}{c+dx}\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{b(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2i(bc-ad)^2}$$

$$\downarrow 2010$$

$$\frac{B\int \left(\frac{b(c+dx)^2}{(a+bx)^2}+\frac{d\log\left(\frac{a+bx}{c+dx}\right)(c+dx)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2i(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{-\frac{b(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+B\left(\frac{1}{2}d\log^2\left(\frac{a+bx}{c+dx}\right)-\frac{b(c+dx)}{a+bx}\right)}{g^2i(bc-ad)^2}$$

input

```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i*x)),x
]
```

output

```
(B*(-((b*(c + d*x))/(a + b*x)) + (d*Log[(a + b*x)/(c + d*x)]^2)/2) - (b*(c
 + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) - d*Log[(a + b*x)/
(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^2*g^2*i)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.33

method	result
parts	$\frac{A\left(\frac{d\ln(dx+c)}{(da-bc)^2}+\frac{1}{(bx+a)(da-bc)}-\frac{d\ln(bx+a)}{(da-bc)^2}\right)}{g^2 i}-\frac{B\left(\frac{d^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2(da-bc)^2}-\frac{dbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)}{(da-bc)^2}\right.}{g^2 id}$
norman	$\frac{-\frac{(Ada+Bbc)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2acdb+c^2b^2\right)}-\frac{Bad\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2gi\left(a^2d^2-2acdb+c^2b^2\right)}-\frac{b(Ad+Bd)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2acdb+c^2b^2\right)}-\frac{(A+B)bx}{gia(da-bc)}-\frac{bBdx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2gi\left(a^2d^2-2acdb+c^2b^2\right)}}{g(bx+a)}$
parallelrisch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)^2 a^4c^2d-2Ax\,a^2b^2c^3+2A\ln\left(\frac{e(bx+a)}{dx+c}\right)a^4c^2d-2Bx\,a^2b^2c^3+2B\ln\left(\frac{e(bx+a)}{dx+c}\right)a^3bc^3+2Ax\,a^3bc^2d+2B$ $2i\,g^2(bx+a)(a^2d^2-2acdb+c^2b^2$
risch	$\frac{Ad\ln(dx+c)}{g^2i(da-bc)^2}+\frac{A}{g^2i(bx+a)(da-bc)}-\frac{Ad\ln(bx+a)}{g^2i(da-bc)^2}-\frac{Bd\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2g^2i(da-bc)^2}-\frac{Bbe\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{g^2i(da-bc)^2\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{ebc}{d(dx+$
derivativedivides	$-\frac{e(da-bc)\left(\frac{d^2Ab}{i(da-bc)^3g^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{d^3A\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei(da-bc)^3g^2}-\frac{d^2Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)}{i(da-bc)^3g^2}+\frac{d^3B}{}\right.}{d^2}$
default	$-\frac{e(da-bc)\left(\frac{d^2Ab}{i(da-bc)^3g^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{d^3A\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei(da-bc)^3g^2}-\frac{d^2Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)}{i(da-bc)^3g^2}+\frac{d^3B}{}\right.}{d^2}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i),x,method=_RETURNVERBOSE)
```

output
```
1/g^2*A/i*(d/(a*d-b*c)^2*ln(d*x+c)+1/(b*x+a)/(a*d-b*c)-d/(a*d-b*c)^2*ln(b*x+a))-B/g^2/i/d*(1/2*d^2/(a*d-b*c)^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-d/(a*d-b*c)^2*b*e*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.83

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx =$$
$$-\frac{2\,(A+B)bc - 2\,(A+B)ad + (Bbdx + Bad)\log\left(\frac{bex+ae}{dx+c}\right)^2 + 2\,((A+B)bdx + Bbc + Aad)\log\left(\frac{bex+ae}{dx+c}\right.}{2\,((b^3c^2 - 2\,ab^2cd + a^2bd^2)g^2ix + (ab^2c^2 - 2\,a^2bcd + a^3d^2)g^2i)}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i),x, algori
thm="fricas")
```

output
```
-1/2*(2*(A + B)*b*c - 2*(A + B)*a*d + (B*b*d*x + B*a*d)*log((b*e*x + a*e)/
(d*x + c))^2 + 2*((A + B)*b*d*x + B*b*c + A*a*d)*log((b*e*x + a*e)/(d*x +
c)))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^2*i*x + (a*b^2*c^2 - 2*a^2*b*c
*d + a^3*d^2)*g^2*i)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(144) = 288$.

Time = 0.61 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.23

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx$$
$$= -\frac{Bd\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^2d^2g^2i-4abcdg^2i+2b^2c^2g^2i}+\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a^2dg^2i-abcg^2i+abdg^2ix-b^2cg^2ix}$$
$$+(A+B)\left(\frac{d\log\left(x+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{g^2i\,(ad-bc)^2}\right.$$
$$-\frac{d\log\left(x+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{g^2i\,(ad-bc)^2}$$
$$\left.+\frac{1}{a^2dg^2i-abcg^2i+x\,(abdg^2i-b^2cg^2i)}\right)$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2/(d*i*x+c*i),x)
```

output
```
-B*d*log(e*(a + b*x)/(c + d*x))**2/(2*a**2*d**2*g**2*i - 4*a*b*c*d*g**2*i
+ 2*b**2*c**2*g**2*i) + B*log(e*(a + b*x)/(c + d*x))/(a**2*d*g**2*i - a*b*
c*g**2*i + a*b*d*g**2*i*x - b**2*c*g**2*i*x) + (A + B)*(d*log(x + (-a**3*d
**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 - 3*a*b**2*c**2*d**2/(
a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(
g**2*i*(a*d - b*c)**2) - d*log(x + (a**3*d**4/(a*d - b*c)**2 - 3*a**2*b*c*
d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 - b**3*c*
*3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(g**2*i*(a*d - b*c)**2) + 1/(a**2
*d*g**2*i - a*b*c*g**2*i + x*(a*b*d*g**2*i - b**2*c*g**2*i)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(171) = 342$.

Time $= 0.06$ (sec) , antiderivative size $= 424$, normalized size of antiderivative $= 2.45$

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx =$$

$$-B\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)\log\left(\right.$$

$$\left.+\frac{ae}{dx+c}\right)$$

$$-A\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)$$

$$+\frac{\left((bdx+ad)\log(bx+a)^2+(bdx+ad)\log(dx+c)^2-2\,bc+2\,ad-2\,(bdx+ad)\log(bx+a)+2\,(bdx\right.}{2\left(ab^2c^2g^2i-2\,a^2bcdg^2i+a^3d^2g^2i+(b^3c^2g^2i-2\,ab^2cdg^2i+a^2\right.}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i),x, algori
thm="maxima")
```

output

```
-B*(1/((b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/(
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*g^2*i))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - A*(1/((b^2*c
 - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*g^2*i)) + 1/2*((b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)
^2 - 2*b*c + 2*a*d - 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*
x + a*d)*log(b*x + a))*log(d*x + c))*B/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*
i + a^3*d^2*g^2*i + (b^3*c^2*g^2*i - 2*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*
x)
```

Giac [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx = \int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{(bgx+ag)^2(dix+ci)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i),x, algori
thm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)/((b*g*x + a*g)^2*(d*i*x + c*i
)), x)
```

Mupad [B] (verification not implemented)

Time = 27.53 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.39

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx = {} & \frac{A+B}{(a\,d-b\,c)\,(a\,g^2\,i+b\,g^2\,i\,x)} \\ & - \frac{B\,d\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2}{2\,g^2\,i\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)} \\ & + \frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,(a\,d-b\,c)}{b\,d\,g^2\,i\,\left(\frac{x}{d}+\frac{a}{b\,d}\right)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)} \\ & + \frac{d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{a^2\,d^2\,g^2\,i-b^2\,c^2\,g^2\,i}{g^2\,i\,(a\,d-b\,c)}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A+B)\,2\mathrm{i}}{g^2\,i\,(a\,d-b\,c)^2}\end{aligned}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^2*(c*i + d*i*x)),x
)
```

output
```
(A + B)/((a*d - b*c)*(a*g^2*i + b*g^2*i*x)) + (d*atan(((2*b*d*x + (a^2*d^2
*g^2*i - b^2*c^2*g^2*i)/(g^2*i*(a*d - b*c)))*1i)/(a*d - b*c))*(A + B)*2i)/
(g^2*i*(a*d - b*c)^2) - (B*d*log((e*(a + b*x))/(c + d*x))^2)/(2*g^2*i*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B*log((e*(a + b*x))/(c + d*x))*(a*d - b*c)
)/(b*d*g^2*i*(x/d + a/(b*d))*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.72

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx$$

$$= \frac{i\Big(2\log(bx+a)\,a^3d + 2\log(bx+a)\,a^2bdx + 2\log(bx+a)\,ab^2c + 2\log(bx+a)\,b^3cx - 2\log(dx+c)\,a^3d -$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i),x)
```

output

```
(i*(2*log(a + b*x)*a**3*d + 2*log(a + b*x)*a**2*b*d*x + 2*log(a + b*x)*a*b
**2*c + 2*log(a + b*x)*b**3*c*x - 2*log(c + d*x)*a**3*d - 2*log(c + d*x)*a
**2*b*d*x - 2*log(c + d*x)*a*b**2*c - 2*log(c + d*x)*b**3*c*x + log((a*e +
 b*e*x)/(c + d*x))**2*a**2*b*d + log((a*e + b*e*x)/(c + d*x))**2*a*b**2*d*
x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*d*x - 2*log((a*e + b*e*x)/(c + d
*x))*b**3*c*x + 2*a**2*b*d*x - 2*a*b**2*c*x + 2*a*b**2*d*x - 2*b**3*c*x))/
(2*a*g**2*(a**3*d**2 - 2*a**2*b*c*d + a**2*b*d**2*x + a*b**2*c**2 - 2*a*b*
*2*c*d*x + b**3*c**2*x))
```

3.37 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 255

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx &= -\frac{B(c+dx)^2\left(b-\frac{4d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3g^3i(a+bx)^2}-\frac{Bd^2\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^3g^3i}\\ &\quad+\frac{2bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i(a+bx)}\\ &\quad-\frac{b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2}\\ &\quad+\frac{d^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i}\end{aligned}$$

output

```
-1/4*B*(d*x+c)^2*(b-4*d*(b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^3/i/(b*x+a)^2-1/
2*B*d^2*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^3/i+2*b*d*(d*x+c)*(A+B*ln(e*(
b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^3/i/(b*x+a)-1/2*b^2*(d*x+c)^2*(A+B*ln(e*(b
*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^3/i/(b*x+a)^2+d^2*ln((b*x+a)/(d*x+c))*(A+B*
ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^3/i
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.43 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.64

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx$$
$$= \frac{-2(bc-ad)^2\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) + 4d(bc-ad)(a+bx)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) + 4d^2(a+bx)^2\log(a$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i
*x)),x]
```

output
```
(-2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d*(b*c - a*d)*(
a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d^2*(a + b*x)^2*Log[a +
b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 4*d^2*(a + b*x)^2*(A + B*Log[(
e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 4*B*d*(a + b*x)*(b*c - a*d + d*(a
+ b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*((b*c - a*d)^2 + 2*d*(
-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)
^2*Log[c + d*x]) - 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log
[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])
 + 2*B*d^2*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x
])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4*(b*c - a*d)
^3*g^3*i*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2772, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^3(ci+dix)}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3i(bc-ad)^3}$$

$$\downarrow 2772$$

$$\frac{-B\int -\frac{(c+dx)^3\left(b^2-\frac{4d(a+bx)b}{c+dx}-\frac{2d^2(a+bx)^2\log\left(\frac{a+bx}{c+dx}\right)}{(c+dx)^2}\right)}{2(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+}{g^3i(bc-ad)^3}$$

$$\downarrow 27$$

$$\frac{\frac{1}{2}B\int \frac{(c+dx)^3\left(b^2-\frac{4d(a+bx)b}{c+dx}-\frac{2d^2(a+bx)^2\log\left(\frac{a+bx}{c+dx}\right)}{(c+dx)^2}\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right.\right.}{g^3i(bc-ad)^3}$$

$$\downarrow 2010$$

$$\frac{\frac{1}{2}B\int\left(\frac{b(c+dx)^3\left(b-\frac{4d(a+bx)}{c+dx}\right)}{(a+bx)^3}-\frac{2d^2(c+dx)\log\left(\frac{a+bx}{c+dx}\right)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(}{g^3i(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+\frac{2bd(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}+\frac{1}{2}B\left(-d^2\log\right.}{g^3i(bc-ad)^3}$$

input

```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i*x)),x
]
```

output
```
((B*(-1/2*((c + d*x)^2*(b - (4*d*(a + b*x))/(c + d*x))^2)/(a + b*x)^2 - d^
2*Log[(a + b*x)/(c + d*x)]^2))/2 + (2*b*d*(c + d*x)*(A + B*Log[(e*(a + b*x
))/(c + d*x)]))/(a + b*x) - (b^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c +
 d*x)]))/(2*(a + b*x)^2) + d^2*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]))/((b*c - a*d)^3*g^3*i)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.43

method	result
parts	$\frac{A\left(\frac{d^2\ln(dx+c)}{(da-bc)^3}+\frac{1}{2(da-bc)(bx+a)^2}+\frac{d}{(da-bc)^2(bx+a)}-\frac{d^2\ln(bx+a)}{(da-bc)^3}\right)}{g^3 i}-B\left(\frac{d^3\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2(da-bc)^3}-\frac{2d^2be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da}{d(}\right.\right.}{\frac{be}{d}+\frac{(da-}{(}$
risch	$\frac{A\,d^2\ln(dx+c)}{g^3 i(da-bc)^3}+\frac{A}{2g^3 i(da-bc)(bx+a)^2}+\frac{Ad}{g^3 i(da-bc)^2(bx+a)}-\frac{A\,d^2\ln(bx+a)}{g^3 i(da-bc)^3}-\frac{B\,d^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2g^3 i(da-bc)^3}-$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{d^2eA\,b^2}{2i(da-bc)^4g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{2d^3Ab}{i(da-bc)^4g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{d^4A\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei(da-bc)^4g^3}+\frac{d^2eB\,b^2\left(-\frac{\ln\left(\frac{b}{c}\right.\right.}{2\left(\frac{be}{d}\right.}\right.}{d^2}$
default	$-\frac{e(da-bc)\left(-\frac{d^2eA\,b^2}{2i(da-bc)^4g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{2d^3Ab}{i(da-bc)^4g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{d^4A\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei(da-bc)^4g^3}+\frac{d^2eB\,b^2\left(-\frac{\ln\left(\frac{b}{c}\right.\right.}{2\left(\frac{be}{d}\right.}\right.}{d^2}$
parallelrisch	$-\frac{-2B\ln\left(\frac{e(bx+a)}{dx+c}\right)a^4b^2c^4+2A\,x^2a^2b^4c^4+B\,x^2a^2b^4c^4+2B\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a^6c^2d^2+4Ax\,a^3b^3c^4+4A\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6c^2$
norman	$\frac{6Aa\,b^2d-2A\,b^3c+7Ba\,b^2d-B\,b^3c}{4gi(da-bc)^2b^2}-\frac{\left(2A\,a^2d^2+4Babcd-B\,b^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2ig\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{\left(2A\,b^2d+3B\,b^2d\right)x}{2ig\left(a^2d^2-2acdb+c^2b^2\right)b}-\frac{B\,a^2d^2\ln\left(\frac{e}{}\right.}{2ig\left(a^3d^3-3a^2bc\,d^2\right.}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i),x,method=_RETURNVERBOSE)
```

output
```
A/g^3/i*(d^2/(a*d-b*c)^3*ln(d*x+c)+1/2/(a*d-b*c)/(b*x+a)^2+d/(a*d-b*c)^2/(b*x+a)-d^2/(a*d-b*c)^3*ln(b*x+a))-B/g^3/i/d*(1/2*d^3/(a*d-b*c)^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*d^2/(a*d-b*c)^3*b*e*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+d/(a*d-b*c)^3*b^2*e^2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.37

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx =$$
$$-\frac{(2A+B)b^2c^2 - 8(A+B)abcd + (6A+7B)a^2d^2 - 2(Bb^2d^2x^2 + 2Babd^2x + Ba^2d^2)\log\left(\frac{bex+ae}{dx+c}\right)^2 -}{4((b^5c^3 - 3ab^4c^2d + 3a^2b^3cd^2 - a^3b^2d^3)g^3ix^2 + 2($$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i),x, algori
thm="fricas")
```

output
```
-1/4*((2*A + B)*b^2*c^2 - 8*(A + B)*a*b*c*d + (6*A + 7*B)*a^2*d^2 - 2*(B*b
^2*d^2*x^2 + 2*B*a*b*d^2*x + B*a^2*d^2)*log((b*e*x + a*e)/(d*x + c))^2 - 2
*((2*A + 3*B)*b^2*c*d - (2*A + 3*B)*a*b*d^2)*x - 2*((2*A + 3*B)*b^2*d^2*x^
2 - B*b^2*c^2 + 4*B*a*b*c*d + 2*A*a^2*d^2 + 2*(B*b^2*c*d + 2*(A + B)*a*b*d
^2)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3
*c*d^2 - a^3*b^2*d^3)*g^3*i*x^2 + 2*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b
^2*c*d^2 - a^4*b*d^3)*g^3*i*x + (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c
*d^2 - a^5*d^3)*g^3*i)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. 2(221) = 442.

Time = 2.62 (sec) , antiderivative size = 889, normalized size of antiderivative = 3.49

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3/(d*i*x+c*i),x)
```

output

```
-B*d**2*log(e*(a + b*x)/(c + d*x))**2/(2*a**3*d**3*g**3*i - 6*a**2*b*c*d**
2*g**3*i + 6*a*b**2*c**2*d*g**3*i - 2*b**3*c**3*g**3*i) + d**2*(2*A + 3*B)
*log(x + (2*A*a*d**3 + 2*A*b*c*d**2 + 3*B*a*d**3 + 3*B*b*c*d**2 - a**4*d**
6*(2*A + 3*B)/(a*d - b*c)**3 + 4*a**3*b*c*d**5*(2*A + 3*B)/(a*d - b*c)**3
- 6*a**2*b**2*c**2*d**4*(2*A + 3*B)/(a*d - b*c)**3 + 4*a*b**3*c**3*d**3*(2
*A + 3*B)/(a*d - b*c)**3 - b**4*c**4*d**2*(2*A + 3*B)/(a*d - b*c)**3)/(4*A
*b*d**3 + 6*B*b*d**3))/(2*g**3*i*(a*d - b*c)**3) - d**2*(2*A + 3*B)*log(x
+ (2*A*a*d**3 + 2*A*b*c*d**2 + 3*B*a*d**3 + 3*B*b*c*d**2 + a**4*d**6*(2*A
+ 3*B)/(a*d - b*c)**3 - 4*a**3*b*c*d**5*(2*A + 3*B)/(a*d - b*c)**3 + 6*a**
2*b**2*c**2*d**4*(2*A + 3*B)/(a*d - b*c)**3 - 4*a*b**3*c**3*d**3*(2*A + 3*
B)/(a*d - b*c)**3 + b**4*c**4*d**2*(2*A + 3*B)/(a*d - b*c)**3)/(4*A*b*d**3
 + 6*B*b*d**3))/(2*g**3*i*(a*d - b*c)**3) + (3*B*a*d - B*b*c + 2*B*b*d*x)*
log(e*(a + b*x)/(c + d*x))/(2*a**4*d**2*g**3*i - 4*a**3*b*c*d*g**3*i + 4*a
**3*b*d**2*g**3*i*x + 2*a**2*b**2*c**2*g**3*i - 8*a**2*b**2*c*d*g**3*i*x +
 2*a**2*b**2*d**2*g**3*i*x**2 + 4*a*b**3*c**2*g**3*i*x - 4*a*b**3*c*d*g**3
*i*x**2 + 2*b**4*c**2*g**3*i*x**2) + (6*A*a*d - 2*A*b*c + 7*B*a*d - B*b*c
+ x*(4*A*b*d + 6*B*b*d))/(4*a**4*d**2*g**3*i - 8*a**3*b*c*d*g**3*i + 4*a**
2*b**2*c**2*g**3*i + x**2*(4*a**2*b**2*d**2*g**3*i - 8*a*b**3*c*d*g**3*i +
 4*b**4*c**2*g**3*i) + x*(8*a**3*b*d**2*g**3*i - 16*a**2*b**2*c*d*g**3*i +
 8*a*b**3*c**2*g**3*i))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 885 vs. $2(249) = 498$.

Time = 0.09 (sec) , antiderivative size = 885, normalized size of antiderivative = 3.47

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^3(ci + dix)}\, dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i),x, algori
thm="maxima")
```

output

```
1/2*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3*
i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a^2*b^2*c^2 -
 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(b*e*x/(d*x + c) + a*e/(d
*x + c)) + 1/2*A*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^
2*d^2)*g^3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a^
2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i)) - 1/4*(b^2*c^2 -
8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x +
a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d
 - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(
3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a
^2*d^2)*log(b*x + a))*log(d*x + c))*B/(a^2*b^3*c^3*g^3*i - 3*a^3*b^2*c^2*d
*g^3*i + 3*a^4*b*c*d^2*g^3*i - a^5*d^3*g^3*i + (b^5*c^3*g^3*i - 3*a*b^4*c^
2*d*g^3*i + 3*a^2*b^3*c*d^2*g^3*i - a^3*b^2*d^3*g^3*i)*x^2 + 2*(a*b^4*c^3*
g^3*i - 3*a^2*b^3*c^2*d*g^3*i + 3*a^3*b^2*c*d^2*g^3*i - a^4*b*d^3*g^3*i)*x
)
```

Giac [A] (verification not implemented)

Time = 41.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.55

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx =$$
$$-\frac{1}{4}\left(\frac{2\,(dx+c)^2 Be^3\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^2 g^3 i} + \frac{(2\,Ae^3+Be^3)(dx+c)^2}{(bex+ae)^2 g^3 i}\right)\left(\frac{bc}{(bce-ade)(bc-ad)} - \frac{ad}{(bce-ade)(bc}\right.$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i),x, algori
thm="giac")
```

output

```
-1/4*(2*(d*x + c)^2*B*e^3*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^2*g^
3*i) + (2*A*e^3 + B*e^3)*(d*x + c)^2/((b*e*x + a*e)^2*g^3*i))*(b*c/((b*c*e
 - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [B] (verification not implemented)

Time = 28.84 (sec) , antiderivative size = 545, normalized size of antiderivative = 2.14

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx = \frac{3Aad}{2g^3i(ad-bc)^2(a+bx)^2} - \frac{Bd^2\ln\left(\frac{e(a+bx)}{c+dx}\right)^2}{2g^3i(ad-bc)^3} - \frac{Abc}{2g^3i(ad-bc)^2(a+bx)^2} + \frac{7Bad}{4g^3i(ad-bc)^2(a+bx)^2} - \frac{Bbc}{4g^3i(ad-bc)^2(a+bx)^2} + \frac{3Ba^2d^2\ln\left(\frac{e(a+bx)}{c+dx}\right)}{2g^3i(ad-bc)^3(a+bx)^2} + \frac{Bb^2c^2\ln\left(\frac{e(a+bx)}{c+dx}\right)}{2g^3i(ad-bc)^3(a+bx)^2} + \frac{Abdx}{g^3i(ad-bc)^2(a+bx)^2} + \frac{3Bbdx}{2g^3i(ad-bc)^2(a+bx)^2} + \frac{Babd^2x\ln\left(\frac{e(a+bx)}{c+dx}\right)}{g^3i(ad-bc)^3(a+bx)^2} - \frac{Bb^2cdx\ln\left(\frac{e(a+bx)}{c+dx}\right)}{g^3i(ad-bc)^3(a+bx)^2} - \frac{2Babcd\ln\left(\frac{e(a+bx)}{c+dx}\right)}{g^3i(ad-bc)^3(a+bx)^2} + \frac{Ad^2\operatorname{atan}\left(\frac{ad\,1i+bc\,1i+bdx\,2i}{ad-bc}\right)\,2i}{g^3i(ad-bc)^3} + \frac{Bd^2\operatorname{atan}\left(\frac{ad\,1i+bc\,1i+bdx\,2i}{ad-bc}\right)\,3i}{g^3i(ad-bc)^3}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^3*(c*i + d*i*x)),x
)
```

output

```
(A*d^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g^3*i*(a*d - b*
c)^3) + (B*d^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*3i)/(g^3*i*(
a*d - b*c)^3) - (B*d^2*log((e*(a + b*x))/(c + d*x))^2)/(2*g^3*i*(a*d - b*c
)^3) + (3*A*a*d)/(2*g^3*i*(a*d - b*c)^2*(a + b*x)^2) - (A*b*c)/(2*g^3*i*(a
*d - b*c)^2*(a + b*x)^2) + (7*B*a*d)/(4*g^3*i*(a*d - b*c)^2*(a + b*x)^2) -
 (B*b*c)/(4*g^3*i*(a*d - b*c)^2*(a + b*x)^2) + (3*B*a^2*d^2*log((e*(a + b*
x))/(c + d*x)))/(2*g^3*i*(a*d - b*c)^3*(a + b*x)^2) + (B*b^2*c^2*log((e*(a
 + b*x))/(c + d*x)))/(2*g^3*i*(a*d - b*c)^3*(a + b*x)^2) + (A*b*d*x)/(g^3*
i*(a*d - b*c)^2*(a + b*x)^2) + (3*B*b*d*x)/(2*g^3*i*(a*d - b*c)^2*(a + b*x
)^2) + (B*a*b*d^2*x*log((e*(a + b*x))/(c + d*x)))/(g^3*i*(a*d - b*c)^3*(a
+ b*x)^2) - (B*b^2*c*d*x*log((e*(a + b*x))/(c + d*x)))/(g^3*i*(a*d - b*c)^
3*(a + b*x)^2) - (2*B*a*b*c*d*log((e*(a + b*x))/(c + d*x)))/(g^3*i*(a*d -
b*c)^3*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 679, normalized size of antiderivative = 2.66

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx$$
$$= \frac{i\Big(4\log(bx+a)\,a^2b^2d^2x^2 + 12\log(bx+a)\,a^2b^2d^2x + 6\log(bx+a)\,ab^3d^2x^2 - 8\log(dx+c)\,a^3bd^2x - 4\lo$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i),x)
```

output

```
(i*(4*log(a + b*x)*a**4*d**2 + 8*log(a + b*x)*a**3*b*d**2*x + 6*log(a + b*
x)*a**3*b*d**2 + 4*log(a + b*x)*a**2*b**2*d**2*x**2 + 12*log(a + b*x)*a**2
*b**2*d**2*x + 6*log(a + b*x)*a*b**3*d**2*x**2 - 4*log(c + d*x)*a**4*d**2
- 8*log(c + d*x)*a**3*b*d**2*x - 6*log(c + d*x)*a**3*b*d**2 - 4*log(c + d*
x)*a**2*b**2*d**2*x**2 - 12*log(c + d*x)*a**2*b**2*d**2*x - 6*log(c + d*x)
*a*b**3*d**2*x**2 + 2*log((a*e + b*e*x)/(c + d*x))**2*a**3*b*d**2 + 4*log(
(a*e + b*e*x)/(c + d*x))**2*a**2*b**2*d**2*x + 2*log((a*e + b*e*x)/(c + d*
x))**2*a*b**3*d**2*x**2 - 6*log((a*e + b*e*x)/(c + d*x))*a**3*b*d**2 + 8*l
og((a*e + b*e*x)/(c + d*x))*a**2*b**2*c*d - 4*log((a*e + b*e*x)/(c + d*x))
*a**2*b**2*d**2*x - 2*log((a*e + b*e*x)/(c + d*x))*a*b**3*c**2 + 4*log((a*
e + b*e*x)/(c + d*x))*a*b**3*c*d*x - 4*a**4*d**2 + 6*a**3*b*c*d - 4*a**3*b
*d**2 - 2*a**2*b**2*c**2 + 5*a**2*b**2*c*d + 2*a**2*b**2*d**2*x**2 - a*b**
3*c**2 - 2*a*b**3*c*d*x**2 + 3*a*b**3*d**2*x**2 - 3*b**4*c*d*x**2))/(4*a*g
**3*(a**5*d**3 - 3*a**4*b*c*d**2 + 2*a**4*b*d**3*x + 3*a**3*b**2*c**2*d -
6*a**3*b**2*c*d**2*x + a**3*b**2*d**3*x**2 - a**2*b**3*c**3 + 6*a**2*b**3*
c**2*d*x - 3*a**2*b**3*c*d**2*x**2 - 2*a*b**4*c**3*x + 3*a*b**4*c**2*d*x**
2 - b**5*c**3*x**2))
```

3.38 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 373

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx = &-\frac{3bBd^2(c+dx)}{(bc-ad)^4g^4i(a+bx)}+\frac{3b^2Bd(c+dx)^2}{4(bc-ad)^4g^4i(a+bx)^2}\\ &-\frac{b^3B(c+dx)^3}{9(bc-ad)^4g^4i(a+bx)^3}+\frac{Bd^3\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^4i}\\ &-\frac{3bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i(a+bx)}\\ &+\frac{3b^2d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2}\\ &-\frac{b^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3}\\ &-\frac{d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i}\end{aligned}$$

output

```
-3*b*B*d^2*(d*x+c)/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/4*b^2*B*d*(d*x+c)^2/(-a*d+
b*c)^4/g^4/i/(b*x+a)^2-1/9*b^3*B*(d*x+c)^3/(-a*d+b*c)^4/g^4/i/(b*x+a)^3+1/
2*B*d^3*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^4/i-3*b*d^2*(d*x+c)*(A+B*ln(e
*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/2*b^2*d*(d*x+c)^2*(A+B*ln(
e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i/(b*x+a)^2-1/3*b^3*(d*x+c)^3*(A+B*ln
(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i/(b*x+a)^3-d^3*ln((b*x+a)/(d*x+c))*
(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.73 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.32

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx$$

$$= \frac{-\frac{12A(bc-ad)^3}{(a+bx)^3} - \frac{4B(bc-ad)^3}{(a+bx)^3} + \frac{18Ad(bc-ad)^2}{(a+bx)^2} + \frac{15Bd(bc-ad)^2}{(a+bx)^2} + \frac{36Ad^2(-bc+ad)}{a+bx} + \frac{66Bd^2(-bc+ad)}{a+bx} - 36Ad^3\log(a+bx}$$

input

```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i
*x)),x]
```

output

```
((-12*A*(b*c - a*d)^3)/(a + b*x)^3 - (4*B*(b*c - a*d)^3)/(a + b*x)^3 + (18
*A*d*(b*c - a*d)^2)/(a + b*x)^2 + (15*B*d*(b*c - a*d)^2)/(a + b*x)^2 + (36
*A*d^2*(-(b*c) + a*d))/(a + b*x) + (66*B*d^2*(-(b*c) + a*d))/(a + b*x) - 3
6*A*d^3*Log[a + b*x] - 66*B*d^3*Log[a + b*x] + 18*B*d^3*Log[a + b*x]^2 - (
12*B*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^3 + (18*B*d*(b*
c - a*d)^2*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^2 + (36*B*d^2*(-(b*c) +
 a*d)*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x) - 36*B*d^3*Log[a + b*x]*Log[
(e*(a + b*x))/(c + d*x)] + 36*A*d^3*Log[c + d*x] + 66*B*d^3*Log[c + d*x] -
 36*B*d^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 36*B*d^3*Log[(e
*(a + b*x))/(c + d*x)]*Log[c + d*x] + 18*B*d^3*Log[c + d*x]^2 - 36*B*d^3*L
og[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 36*B*d^3*PolyLog[2, (d*(a + b
*x))/(-(b*c) + a*d)] - 36*B*d^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(36
*(b*c - a*d)^4*g^4*i)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2772, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^4(ci+dix)}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4i(bc-ad)^4}$$

$$\downarrow 2772$$

$$\frac{-B\int -\frac{(c+dx)^4\left(2b^3-\frac{9d(a+bx)b^2}{c+dx}+\frac{18d^2(a+bx)^2b}{(c+dx)^2}+\frac{6d^3(a+bx)^3\log\left(\frac{a+bx}{c+dx}\right)}{(c+dx)^3}\right)}{6(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B}{2(a}}{g^4i(bc-ad)^4}$$

$$\downarrow 27$$

$$\frac{\frac{1}{6}B\int \frac{(c+dx)^4\left(2b^3-\frac{9d(a+bx)b^2}{c+dx}+\frac{18d^2(a+bx)^2b}{(c+dx)^2}+\frac{6d^3(a+bx)^3\log\left(\frac{a+bx}{c+dx}\right)}{(c+dx)^3}\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B\log\right.}{2(a+b}}{g^4i(bc-ad)^4}$$

$$\downarrow 2010$$

$$\frac{\frac{1}{6}B\int\left(\frac{b\left(2b^2-\frac{9d(a+bx)b}{c+dx}+\frac{18d^2(a+bx)^2}{(c+dx)^2}\right)(c+dx)^4}{(a+bx)^4}+\frac{6d^3\log\left(\frac{a+bx}{c+dx}\right)(c+dx)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2}{}}{g^4i(bc-ad)^4}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+d^3\left(-\log\left(\frac{a+bx}{c+dx}\right)\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-\frac{3bd^2(c}{}}{g^4i(bc-ad)^4}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i*x)),x
]
```

output
```
((B*((-18*b*d^2*(c + d*x))/(a + b*x) + (9*b^2*d*(c + d*x)^2)/(2*(a + b*x)^
2) - (2*b^3*(c + d*x)^3)/(3*(a + b*x)^3) + 3*d^3*Log[(a + b*x)/(c + d*x)]^
2))/6 - (3*b*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)
 + (3*b^2*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)
^2) - (b^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(a + b*x)^
3) - d^3*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((
b*c - a*d)^4*g^4*i)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.34

method	result
parts	$\frac{A\left(\frac{d^3\ln(dx+c)}{(da-bc)^4}+\frac{1}{3(da-bc)(bx+a)^3}+\frac{d}{2(da-bc)^2(bx+a)^2}+\frac{d^2}{(da-bc)^3(bx+a)}-\frac{d^3\ln(bx+a)}{(da-bc)^4}\right)}{g^4 i}-\frac{B\left(\frac{d^4\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2(da-bc)^4}-\right.$
risch	$\frac{A\,d^3\ln(dx+c)}{g^4 i(da-bc)^4}+\frac{A}{3g^4 i(da-bc)(bx+a)^3}+\frac{Ad}{2g^4 i(da-bc)^2(bx+a)^2}+\frac{A\,d^2}{g^4 i(da-bc)^3(bx+a)}-\frac{A\,d^3\ln(bx+a)}{g^4 i(da-bc)^4}-\frac{B}{}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{d^2e^2Ab^3}{3i(da-bc)^5g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{3d^3eAb^2}{2i(da-bc)^5g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{3d^4Ab}{i(da-bc)^5g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{d^5A\ln\left(\frac{be}{d}\right.}{ei(da}\right.}{}$
default	$-\frac{e(da-bc)\left(\frac{d^2e^2Ab^3}{3i(da-bc)^5g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{3d^3eAb^2}{2i(da-bc)^5g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{3d^4Ab}{i(da-bc)^5g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{d^5A\ln\left(\frac{be}{d}\right.}{ei(da}\right.}{}$
parallelrisch	$-\frac{36Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a^5b^3c^3d^2+54Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a^7bc^2d^3+108Ax\ln\left(\frac{e(bx+a)}{dx+c}\right)a^7bc^2d^3+108Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^7b}{}$
norman	Expression too large to display

input

```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i),x,method=_RETURN
VERBOSE)
```

output

```
A/g^4/i*(d^3/(a*d-b*c)^4*ln(d*x+c)+1/3/(a*d-b*c)/(b*x+a)^3+1/2*d/(a*d-b*c)
^2/(b*x+a)^2+d^2/(a*d-b*c)^3/(b*x+a)-d^3/(a*d-b*c)^4*ln(b*x+a))-B/g^4/i/d*
(1/2*d^4/(a*d-b*c)^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-3*d^3/(a*d-b*c)^4*b
*e*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*
e/d+(a*d-b*c)*e/d/(d*x+c)))+3*d^2/(a*d-b*c)^4*b^2*e^2*(-1/2/(b*e/d+(a*d-b*
c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d
/(d*x+c))^2)-d/(a*d-b*c)^4*b^3*e^3*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.64

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx =$$

$$-\frac{4\,(3\,A+B)b^3c^3 - 27\,(2\,A+B)ab^2c^2d + 108\,(A+B)a^2bcd^2 - (66\,A+85\,B)a^3d^3 + 6\,((6\,A+11\,B)b^3c}{36\,((}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i),x, algori
thm="fricas")
```

output

```
-1/36*(4*(3*A + B)*b^3*c^3 - 27*(2*A + B)*a*b^2*c^2*d + 108*(A + B)*a^2*b*
c*d^2 - (66*A + 85*B)*a^3*d^3 + 6*((6*A + 11*B)*b^3*c*d^2 - (6*A + 11*B)*a
*b^2*d^3)*x^2 + 18*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x^2 + 3*B*a^2*b*d^3*x +
B*a^3*d^3)*log((b*e*x + a*e)/(d*x + c))^2 - 3*((6*A + 5*B)*b^3*c^2*d - 18*
(2*A + 3*B)*a*b^2*c*d^2 + (30*A + 49*B)*a^2*b*d^3)*x + 6*((6*A + 11*B)*b^3
*d^3*x^3 + 2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2 + 6*A*a^3*d^3
+ 3*(2*B*b^3*c*d^2 + 3*(2*A + 3*B)*a*b^2*d^3)*x^2 - 3*(B*b^3*c^2*d - 6*B*a
*b^2*c*d^2 - 6*(A + B)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^7*c
^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^
4*i*x^3 + 3*(a*b^6*c^4 - 4*a^2*b^5*c^3*d + 6*a^3*b^4*c^2*d^2 - 4*a^4*b^3*c
*d^3 + a^5*b^2*d^4)*g^4*i*x^2 + 3*(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b
^3*c^2*d^2 - 4*a^5*b^2*c*d^3 + a^6*b*d^4)*g^4*i*x + (a^3*b^4*c^4 - 4*a^4*b
^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*g^4*i)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1392 vs. $2(332) = 664$.

Time = 7.73 (sec) , antiderivative size = 1392, normalized size of antiderivative = 3.73

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4/(d*i*x+c*i),x)
```

output

```
-B*d**3*log(e*(a + b*x)/(c + d*x))**2/(2*a**4*d**4*g**4*i - 8*a**3*b*c*d**
3*g**4*i + 12*a**2*b**2*c**2*d**2*g**4*i - 8*a*b**3*c**3*d*g**4*i + 2*b**4
*c**4*g**4*i) + d**3*(6*A + 11*B)*log(x + (6*A*a*d**4 + 6*A*b*c*d**3 + 11*
B*a*d**4 + 11*B*b*c*d**3 - a**5*d**8*(6*A + 11*B)/(a*d - b*c)**4 + 5*a**4*
b*c*d**7*(6*A + 11*B)/(a*d - b*c)**4 - 10*a**3*b**2*c**2*d**6*(6*A + 11*B)
/(a*d - b*c)**4 + 10*a**2*b**3*c**3*d**5*(6*A + 11*B)/(a*d - b*c)**4 - 5*a
*b**4*c**4*d**4*(6*A + 11*B)/(a*d - b*c)**4 + b**5*c**5*d**3*(6*A + 11*B)/
(a*d - b*c)**4)/(12*A*b*d**4 + 22*B*b*d**4))/(6*g**4*i*(a*d - b*c)**4) - d
**3*(6*A + 11*B)*log(x + (6*A*a*d**4 + 6*A*b*c*d**3 + 11*B*a*d**4 + 11*B*b
*c*d**3 + a**5*d**8*(6*A + 11*B)/(a*d - b*c)**4 - 5*a**4*b*c*d**7*(6*A + 1
1*B)/(a*d - b*c)**4 + 10*a**3*b**2*c**2*d**6*(6*A + 11*B)/(a*d - b*c)**4 -
 10*a**2*b**3*c**3*d**5*(6*A + 11*B)/(a*d - b*c)**4 + 5*a*b**4*c**4*d**4*(
6*A + 11*B)/(a*d - b*c)**4 - b**5*c**5*d**3*(6*A + 11*B)/(a*d - b*c)**4)/(
12*A*b*d**4 + 22*B*b*d**4))/(6*g**4*i*(a*d - b*c)**4) + (11*B*a**2*d**2 -
7*B*a*b*c*d + 15*B*a*b*d**2*x + 2*B*b**2*c**2 - 3*B*b**2*c*d*x + 6*B*b**2*
d**2*x**2)*log(e*(a + b*x)/(c + d*x))/(6*a**6*d**3*g**4*i - 18*a**5*b*c*d*
*2*g**4*i + 18*a**5*b*d**3*g**4*i*x + 18*a**4*b**2*c**2*d*g**4*i - 54*a**4
*b**2*c*d**2*g**4*i*x + 18*a**4*b**2*d**3*g**4*i*x**2 - 6*a**3*b**3*c**3*g
**4*i + 54*a**3*b**3*c**2*d*g**4*i*x - 54*a**3*b**3*c*d**2*g**4*i*x**2 + 6
*a**3*b**3*d**3*g**4*i*x**3 - 18*a**2*b**4*c**3*g**4*i*x + 54*a**2*b**4...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1469 vs. $2(363) = 726$.

Time $= 0.15$ (sec) , antiderivative size $= 1469$, normalized size of antiderivative $= 3.94$

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i),x, algori
thm="maxima")
```

output
```
-1/6*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*
g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3
)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d
^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*g^
4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i))*log(b*e*x/(d*x
 + c) + a*e/(d*x + c)) - 1/6*A*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 1
1*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b
^4*c*d^2 - a^3*b^3*d^3)*g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3
*b^3*c*d^2 - a^4*b^2*d^3)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3
*a^4*b^2*c*d^2 - a^5*b*d^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a
^5*b*c*d^2 - a^6*d^3)*g^4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*
d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x +
c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4
)*g^4*i)) - 1/36*(4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^
3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3
*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2
 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*...
```

Giac [A] (verification not implemented)

Time = 53.52 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.72

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx =$$
$$-\frac{1}{36}\left(\frac{6\left(2\,Bbe^4-\frac{3\,(bex+ae)Bde^3}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^3bcg^4i}{(dx+c)^3}-\frac{(bex+ae)^3adg^4i}{(dx+c)^3}}+\frac{12\,Abe^4+4\,Bbe^4-\frac{18\,(bex+ae)Ade^3}{dx+c}-\frac{9\,(bex+ae)Bde^3}{dx+c}}{\frac{(bex+ae)^3bcg^4i}{(dx+c)^3}-\frac{(bex+ae)^3adg^4i}{(dx+c)^3}}\right)\left(\frac{}{(}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i),x, algori
thm="giac")
```

output
```
-1/36*(6*(2*B*b*e^4 - 3*(b*e*x + a*e)*B*d*e^3/(d*x + c))*log((b*e*x + a*e)
/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4*i/(d*x + c)^3 - (b*e*x + a*e)^3*a*d*g
^4*i/(d*x + c)^3) + (12*A*b*e^4 + 4*B*b*e^4 - 18*(b*e*x + a*e)*A*d*e^3/(d*
x + c) - 9*(b*e*x + a*e)*B*d*e^3/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4*i/(d*
x + c)^3 - (b*e*x + a*e)^3*a*d*g^4*i/(d*x + c)^3))*(b*c/((b*c*e - a*d*e)*(
b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [B] (verification not implemented)

Time = 31.78 (sec) , antiderivative size = 970, normalized size of antiderivative = 2.60

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^4*(c*i + d*i*x)),x
)
```

output

```
(A*d^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g^4*i*(a*d - b*
c)^4) + (B*d^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*11i)/(3*g^4*
i*(a*d - b*c)^4) - (B*d^3*log((e*(a + b*x))/(c + d*x))^2)/(2*g^4*i*(a*d -
b*c)^4) + (11*A*a^2*d^2)/(6*g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (A*b^2*c^2)
/(3*g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (85*B*a^2*d^2)/(36*g^4*i*(a*d - b*c
)^3*(a + b*x)^3) + (B*b^2*c^2)/(9*g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (11*B
*a^3*d^3*log((e*(a + b*x))/(c + d*x)))/(6*g^4*i*(a*d - b*c)^4*(a + b*x)^3)
 - (B*b^3*c^3*log((e*(a + b*x))/(c + d*x)))/(3*g^4*i*(a*d - b*c)^4*(a + b*
x)^3) + (A*b^2*d^2*x^2)/(g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (11*B*b^2*d^2*
x^2)/(6*g^4*i*(a*d - b*c)^3*(a + b*x)^3) - (7*A*a*b*c*d)/(6*g^4*i*(a*d - b
*c)^3*(a + b*x)^3) - (23*B*a*b*c*d)/(36*g^4*i*(a*d - b*c)^3*(a + b*x)^3) +
 (5*A*a*b*d^2*x)/(2*g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (49*B*a*b*d^2*x)/(1
2*g^4*i*(a*d - b*c)^3*(a + b*x)^3) - (A*b^2*c*d*x)/(2*g^4*i*(a*d - b*c)^3*
(a + b*x)^3) - (5*B*b^2*c*d*x)/(12*g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (3*B
*a*b^2*c^2*d*log((e*(a + b*x))/(c + d*x)))/(2*g^4*i*(a*d - b*c)^4*(a + b*x
)^3) - (3*B*a^2*b*c*d^2*log((e*(a + b*x))/(c + d*x)))/(g^4*i*(a*d - b*c)^4
*(a + b*x)^3) + (5*B*a^2*b*d^3*x*log((e*(a + b*x))/(c + d*x)))/(2*g^4*i*(a
*d - b*c)^4*(a + b*x)^3) + (B*b^3*c^2*d*x*log((e*(a + b*x))/(c + d*x)))/(2
*g^4*i*(a*d - b*c)^4*(a + b*x)^3) + (B*a*b^2*d^3*x^2*log((e*(a + b*x))/(c
+ d*x)))/(g^4*i*(a*d - b*c)^4*(a + b*x)^3) - (B*b^3*c*d^2*x^2*log((e*(a...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1284, normalized size of antiderivative = 3.44

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i),x)
```

output

```
(i*(36*log(a + b*x)*a**5*d**3 + 108*log(a + b*x)*a**4*b*d**3*x + 54*log(a
+ b*x)*a**4*b*d**3 + 12*log(a + b*x)*a**3*b**2*c*d**2 + 108*log(a + b*x)*a
**3*b**2*d**3*x**2 + 162*log(a + b*x)*a**3*b**2*d**3*x + 36*log(a + b*x)*a
**2*b**3*c*d**2*x + 36*log(a + b*x)*a**2*b**3*d**3*x**3 + 162*log(a + b*x)
*a**2*b**3*d**3*x**2 + 36*log(a + b*x)*a*b**4*c*d**2*x**2 + 54*log(a + b*x
)*a*b**4*d**3*x**3 + 12*log(a + b*x)*b**5*c*d**2*x**3 - 36*log(c + d*x)*a*
*5*d**3 - 108*log(c + d*x)*a**4*b*d**3*x - 54*log(c + d*x)*a**4*b*d**3 - 1
2*log(c + d*x)*a**3*b**2*c*d**2 - 108*log(c + d*x)*a**3*b**2*d**3*x**2 - 1
62*log(c + d*x)*a**3*b**2*d**3*x - 36*log(c + d*x)*a**2*b**3*c*d**2*x - 36
*log(c + d*x)*a**2*b**3*d**3*x**3 - 162*log(c + d*x)*a**2*b**3*d**3*x**2 -
 36*log(c + d*x)*a*b**4*c*d**2*x**2 - 54*log(c + d*x)*a*b**4*d**3*x**3 - 1
2*log(c + d*x)*b**5*c*d**2*x**3 + 18*log((a*e + b*e*x)/(c + d*x))**2*a**4*
b*d**3 + 54*log((a*e + b*e*x)/(c + d*x))**2*a**3*b**2*d**3*x + 54*log((a*e
 + b*e*x)/(c + d*x))**2*a**2*b**3*d**3*x**2 + 18*log((a*e + b*e*x)/(c + d*
x))**2*a*b**4*d**3*x**3 - 54*log((a*e + b*e*x)/(c + d*x))*a**4*b*d**3 + 96
*log((a*e + b*e*x)/(c + d*x))*a**3*b**2*c*d**2 - 54*log((a*e + b*e*x)/(c +
 d*x))*a**3*b**2*d**3*x - 54*log((a*e + b*e*x)/(c + d*x))*a**2*b**3*c**2*d
 + 72*log((a*e + b*e*x)/(c + d*x))*a**2*b**3*c*d**2*x + 12*log((a*e + b*e*
x)/(c + d*x))*a*b**4*c**3 - 18*log((a*e + b*e*x)/(c + d*x))*a*b**4*c**2*d*
x + 12*log((a*e + b*e*x)/(c + d*x))*a*b**4*d**3*x**3 - 12*log((a*e + b*...
```

3.39 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$

Optimal result . . . 520
Mathematica [A] (verified) . . . 521
Rubi [A] (verified) . . . 522
Maple [B] (verified) . . . 524
Fricas [F] . . . 526
Sympy [F(-1)] . . . 527
Maxima [B] (verification not implemented) . . . 527
Giac [B] (verification not implemented) . . . 528
Mupad [F(-1)] . . . 529
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Optimal result

Integrand size = 40, antiderivative size = 341

$$
\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx \\
&= \frac{3B(bc-ad)^2g^3(a+bx)}{d^3i^2(c+dx)} - \frac{(6A+5B)(bc-ad)^2g^3(a+bx)}{2d^3i^2(c+dx)} \\
&\quad - \frac{3B(bc-ad)^2g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^2(c+dx)} + \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^2(c+dx)} \\
&\quad - \frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^2(c+dx)} \\
&\quad - \frac{b(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^4i^2} \\
&\quad - \frac{3bB(bc-ad)^2g^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}
\end{aligned}
$$

output
```
3*B*(-a*d+b*c)^2*g^3*(b*x+a)/d^3/i^2/(d*x+c)-1/2*(6*A+5*B)*(-a*d+b*c)^2*g^
3*(b*x+a)/d^3/i^2/(d*x+c)-3*B*(-a*d+b*c)^2*g^3*(b*x+a)*ln(e*(b*x+a)/(d*x+c
))/d^3/i^2/(d*x+c)+1/2*g^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i^2/(d*
x+c)-1/2*(-a*d+b*c)*g^3*(b*x+a)^2*(3*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/d^2/i^
2/(d*x+c)-1/2*b*(-a*d+b*c)^2*g^3*ln((-a*d+b*c)/b/(d*x+c))*(6*A+5*B+6*B*ln(
e*(b*x+a)/(d*x+c)))/d^4/i^2-3*b*B*(-a*d+b*c)^2*g^3*polylog(2,d*(b*x+a)/b/(
d*x+c))/d^4/i^2
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.05

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$$
$$= \frac{g^3\left(-2Ab^2d(2bc-3ad)x-2bBd(2bc-3ad)(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+b^3d^2x^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{2(b$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*
i*x)^2,x]
```

output
```
(g^3*(-2*A*b^2*d*(2*b*c - 3*a*d)*x - 2*b*B*d*(2*b*c - 3*a*d)*(a + b*x)*Log
[(e*(a + b*x))/(c + d*x)] + b^3*d^2*x^2*(A + B*Log[(e*(a + b*x))/(c + d*x)
]) + (2*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) + 2*
b*B*(2*b*c - 3*a*d)*(b*c - a*d)*Log[c + d*x] + 6*b*(b*c - a*d)^2*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 2*B*(b*c - a*d)^2*((b*c - a*d)/
(c + d*x) + b*Log[a + b*x] - b*Log[c + d*x]) + b*B*(-(a^2*d^2*Log[a + b*x]
) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 3*b*B*(b*c - a*d)^2*((2
*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLo
g[2, (b*(c + d*x))/(b*c - a*d)])))/(2*d^4*i^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2784, 2784, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ci+dix)^2}\,dx$$

$$\downarrow 2962$$

$$\frac{g^3(bc-ad)^2\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{i^2}$$

$$\downarrow 2784$$

$$g^3(bc-ad)^2\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int \frac{(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)$$

$$\downarrow 2784$$

$$g^3(bc-ad)^2\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int \frac{(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{d}}{2d}\right)$$

$$\downarrow 2793$$

$$\frac{g^3(bc-ad)^2\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\left(-\frac{6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)}{d}-\frac{b\left(6A+5B+6B\log\left(\frac{e(a}{c}\right.\right.\right.}{d\left(\frac{d(a+bx)}{c+dx}-b\right.}}{d}}{2d}\right)}{i^2}$$

↓ 2009

$$\frac{g^3(bc-ad)^2\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+5B\right)}{d^2}-\frac{(6A}{}}{2d}\right)}{i^2}$$

input

```
Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^2
,x]
```

output

```
((b*c - a*d)^2*g^3*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*
d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)^2*(3*A + B +
3*B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c + d*x)^2*(b - (d*(a + b*x))/(c +
d*x))) - ((6*B*(a + b*x))/(d*(c + d*x)) - ((6*A + 5*B)*(a + b*x))/(d*(c +
d*x)) - (6*B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d*(c + d*x)) - (b*(6
*A + 5*B + 6*B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c +
 d*x))])/d^2 - (6*b*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^2)/d)/(2*
d)))/i^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 845 vs. $2(333) = 666$.

Time = 3.29 (sec) , antiderivative size = 846, normalized size of antiderivative = 2.48

method	result
derivativedivides	$-\frac{e(da-bc)\left(-\frac{A d^2 g^3 (da-bc)\left(-\frac{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}{d^3}-\frac{3be\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{d^4}-\frac{3b^2e^2}{d^4\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}+\frac{\cdots}{2d^4\left(be-\left(\cdots\right)\right)}\cdots\right)}{e^2 i^2}\cdots\right)}{\cdots}$
default	$-\frac{e(da-bc)\left(-\frac{A d^2 g^3 (da-bc)\left(-\frac{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}{d^3}-\frac{3be\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{d^4}-\frac{3b^2e^2}{d^4\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}+\frac{\cdots}{2d^4\left(be-\left(\cdots\right)\right)}\cdots\right)}{e^2 i^2}\cdots\right)}{\cdots}$
parts	$\frac{g^3 A\left(\frac{b^2\left(\frac{1}{2}bdx^2+3xda-2bcx\right)}{d^3}+\frac{3b\left(a^2d^2-2acdb+c^2b^2\right)\ln(dx+c)}{d^4}-\frac{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{d^4(dx+c)}\right)}{i^2}-\frac{g^3 B\left(\frac{\left(\left(\frac{be}{d}+\frac{(da-\cdots}{d(dx\cdots}\cdots\right)\cdots\right)\cdots}{\cdots}\cdots\right)}{\cdots}$
risch	Expression too large to display

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x,method=_RETU
RNVERBOSE)
```

output
```
-1/d^2*e*(a*d-b*c)*(-A*d^2*g^3*(a*d-b*c)/e^2/i^2*(-(b*e/d+(a*d-b*c)*e/d/(d
*x+c))/d^3-3/d^4*b*e*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)-3/d^4*b^2*e^2
/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)+1/2*b^3*e^3/d^4/(b*e-(b*e/d+(a*d-b*
c)*e/d/(d*x+c))*d)^2)-B*d^2*g^3*(a*d-b*c)/e^2/i^2*(-((b*e/d+(a*d-b*c)*e/d/
(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)/d^3-
3*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)/d^3*b*e-3*(
1/b/e/d*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)+ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))
*d))/d^3*b^2*e^2-(1/2/b/e/d/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)-1/2/b^2/
e^2/d*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)-1/2*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(2*b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))
*d)/b^2/e^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2)*b^3*e^3/d^3))
```

Fricas [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algo
rithm="fricas")
```

output
```
integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
+ (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log((b
*e*x + a*e)/(d*x + c)))/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1341 vs. $2(332) = 664$.

Time = 0.11 (sec) , antiderivative size = 1341, normalized size of antiderivative = 3.93

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algo
rithm="maxima")
```

output

```
1/2*(2*c^3/(d^5*i^2*x + c*d^4*i^2) + 6*c^2*log(d*x + c)/(d^4*i^2) + (d*x^2
 - 4*c*x)/(d^3*i^2))*A*b^3*g^3 - 3*A*a*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2) -
x/(d^2*i^2) + 2*c*log(d*x + c)/(d^3*i^2))*g^3 + 3*A*a^2*b*g^3*(c/(d^3*i^2*
x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2)) - B*a^3*g^3*(log(b*e*x/(d*x + c)
+ a*e/(d*x + c))/(d^2*i^2*x + c*d*i^2) - 1/(d^2*i^2*x + c*d*i^2) - b*log(b
*x + a)/((b*c*d - a*d^2)*i^2) + b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) - A*
a^3*g^3/(d^2*i^2*x + c*d*i^2) - 1/2*(6*a^3*b*d^3*g^3*log(e) - (6*g^3*log(e
) + 7*g^3)*b^4*c^3 + (18*g^3*log(e) + 17*g^3)*a*b^3*c^2*d - 6*(3*g^3*log(e
) + 2*g^3)*a^2*b^2*c*d^2)*B*log(d*x + c)/(b*c*d^4*i^2 - a*d^5*i^2) + 1/2*(
(b^4*c*d^3*g^3*log(e) - a*b^3*d^4*g^3*log(e))*B*x^3 - ((3*g^3*log(e) + g^3
)*b^4*c^2*d^2 - (9*g^3*log(e) + 2*g^3)*a*b^3*c*d^3 + (6*g^3*log(e) + g^3)*
a^2*b^2*d^4)*B*x^2 - ((4*g^3*log(e) + g^3)*b^4*c^3*d - 2*(5*g^3*log(e) + g
^3)*a*b^3*c^2*d^2 + (6*g^3*log(e) + g^3)*a^2*b^2*c*d^3)*B*x - 3*((b^4*c^3*
d*g^3 - 3*a*b^3*c^2*d^2*g^3 + 3*a^2*b^2*c*d^3*g^3 - a^3*b*d^4*g^3)*B*x + (
b^4*c^4*g^3 - 3*a*b^3*c^3*d*g^3 + 3*a^2*b^2*c^2*d^2*g^3 - a^3*b*c*d^3*g^3)
*B)*log(d*x + c)^2 + 2*((g^3*log(e) - g^3)*b^4*c^4 - 4*(g^3*log(e) - g^3)*
a*b^3*c^3*d + 6*(g^3*log(e) - g^3)*a^2*b^2*c^2*d^2 - 3*(g^3*log(e) - g^3)*
a^3*b*c*d^3)*B + ((b^4*c*d^3*g^3 - a*b^3*d^4*g^3)*B*x^3 - 3*(b^4*c^2*d^2*g
^3 - 3*a*b^3*c*d^3*g^3 + 2*a^2*b^2*d^4*g^3)*B*x^2 - (6*b^4*c^3*d*g^3 - 12*
a*b^3*c^2*d^2*g^3 + 3*a^2*b^2*c*d^3*g^3 + 5*a^3*b*d^4*g^3)*B*x - (6*a*b...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2814 vs. $2(332) = 664$.

Time $= 69.45$ (sec) , antiderivative size $= 2814$, normalized size of antiderivative $= 8.25$

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algo
rithm="giac")
```

output

```
-1/24*(6*(B*b^8*c^5*e^5*g^3 - 5*B*a*b^7*c^4*d*e^5*g^3 + 10*B*a^2*b^6*c^3*d
^2*e^5*g^3 - 10*B*a^3*b^5*c^2*d^3*e^5*g^3 + 5*B*a^4*b^4*c*d^4*e^5*g^3 - B*
a^5*b^3*d^5*e^5*g^3 - 4*(b*e*x + a*e)*B*b^7*c^5*d*e^4*g^3/(d*x + c) + 20*(
b*e*x + a*e)*B*a*b^6*c^4*d^2*e^4*g^3/(d*x + c) - 40*(b*e*x + a*e)*B*a^2*b^
5*c^3*d^3*e^4*g^3/(d*x + c) + 40*(b*e*x + a*e)*B*a^3*b^4*c^2*d^4*e^4*g^3/(
d*x + c) - 20*(b*e*x + a*e)*B*a^4*b^3*c*d^5*e^4*g^3/(d*x + c) + 4*(b*e*x +
 a*e)*B*a^5*b^2*d^6*e^4*g^3/(d*x + c) + 6*(b*e*x + a*e)^2*B*b^6*c^5*d^2*e^
3*g^3/(d*x + c)^2 - 30*(b*e*x + a*e)^2*B*a*b^5*c^4*d^3*e^3*g^3/(d*x + c)^2
 + 60*(b*e*x + a*e)^2*B*a^2*b^4*c^3*d^4*e^3*g^3/(d*x + c)^2 - 60*(b*e*x +
a*e)^2*B*a^3*b^3*c^2*d^5*e^3*g^3/(d*x + c)^2 + 30*(b*e*x + a*e)^2*B*a^4*b^
2*c*d^6*e^3*g^3/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^5*b*d^7*e^3*g^3/(d*x +
 c)^2 - 4*(b*e*x + a*e)^3*B*b^5*c^5*d^3*e^2*g^3/(d*x + c)^3 + 20*(b*e*x +
a*e)^3*B*a*b^4*c^4*d^4*e^2*g^3/(d*x + c)^3 - 40*(b*e*x + a*e)^3*B*a^2*b^3*
c^3*d^5*e^2*g^3/(d*x + c)^3 + 40*(b*e*x + a*e)^3*B*a^3*b^2*c^2*d^6*e^2*g^3
/(d*x + c)^3 - 20*(b*e*x + a*e)^3*B*a^4*b*c*d^7*e^2*g^3/(d*x + c)^3 + 4*(b
*e*x + a*e)^3*B*a^5*d^8*e^2*g^3/(d*x + c)^3)*log((b*e*x + a*e)/(d*x + c))/
(b^4*d^4*e^4*i^2 - 4*(b*e*x + a*e)*b^3*d^5*e^3*i^2/(d*x + c) + 6*(b*e*x +
a*e)^2*b^2*d^6*e^2*i^2/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b*d^7*e*i^2/(d*x +
c)^3 + (b*e*x + a*e)^4*d^8*i^2/(d*x + c)^4) + (6*A*b^8*c^5*e^5*g^3 + 11*B*
b^8*c^5*e^5*g^3 - 30*A*a*b^7*c^4*d*e^5*g^3 - 55*B*a*b^7*c^4*d*e^5*g^3 +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(c\,i+d\,i\,x)^2}\,dx$$

input

```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2
,x)
```

output

```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2
, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
int((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x)
```

output

```
(g**3*( - 2*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(c**2 + 2*c*d*x + d**2
*x**2),x)*a*b**4*c**2*d**5 - 2*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(c*
*2 + 2*c*d*x + d**2*x**2),x)*a*b**4*c*d**6*x + 2*int((log((a*e + b*e*x)/(c
 + d*x))*x**3)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**5*c**3*d**4 + 2*int((log
((a*e + b*e*x)/(c + d*x))*x**3)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**5*c**2*
d**5*x - 6*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(c**2 + 2*c*d*x + d**2*
x**2),x)*a**2*b**3*c**2*d**5 - 6*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(
c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b**3*c*d**6*x + 6*int((log((a*e + b*e*
x)/(c + d*x))*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**4*c**3*d**4 + 6*i
nt((log((a*e + b*e*x)/(c + d*x))*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b
**4*c**2*d**5*x - 6*int((log((a*e + b*e*x)/(c + d*x))*x)/(c**2 + 2*c*d*x +
 d**2*x**2),x)*a**3*b**2*c**2*d**5 - 6*int((log((a*e + b*e*x)/(c + d*x))*x
)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**3*b**2*c*d**6*x + 6*int((log((a*e + b
*e*x)/(c + d*x))*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b**3*c**3*d**4 +
6*int((log((a*e + b*e*x)/(c + d*x))*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**
2*b**3*c**2*d**5*x + 2*log(a + b*x)*a**4*b*c*d**4 + 2*log(a + b*x)*a**4*b*
d**5*x - 6*log(c + d*x)*a**4*b*c**2*d**3 - 6*log(c + d*x)*a**4*b*c*d**4*x
- 2*log(c + d*x)*a**4*b*c*d**4 - 2*log(c + d*x)*a**4*b*d**5*x + 18*log(c +
 d*x)*a**3*b**2*c**3*d**2 + 18*log(c + d*x)*a**3*b**2*c**2*d**3*x - 18*log
(c + d*x)*a**2*b**3*c**4*d - 18*log(c + d*x)*a**2*b**3*c**3*d**2*x + 6*...
```

3.40 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$

Optimal result . . . 531
Mathematica [A] (verified) . . . 532
Rubi [A] (verified) . . . 532
Maple [B] (verified) . . . 534
Fricas [F] . . . 536
Sympy [F(-1)] . . . 536
Maxima [B] (verification not implemented) . . . 537
Giac [B] (verification not implemented) . . . 538
Mupad [F(-1)] . . . 539
Reduce [F] . . . 539

Optimal result

Integrand size = 40, antiderivative size = 260

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx \\
&= -\frac{2B(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)} + \frac{(2A+B)(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)} \\
&\quad + \frac{2B(bc-ad)g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^2(c+dx)} + \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di^2(c+dx)} \\
&\quad + \frac{b(bc-ad)g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^2} \\
&\quad + \frac{2bB(bc-ad)g^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}
\end{aligned}$$

output

```
-2*B*(-a*d+b*c)*g^2*(b*x+a)/d^2/i^2/(d*x+c)+(2*A+B)*(-a*d+b*c)*g^2*(b*x+a)
/d^2/i^2/(d*x+c)+2*B*(-a*d+b*c)*g^2*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d^2/i^2/
(d*x+c)+g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i^2/(d*x+c)+b*(-a*d+b*
c)*g^2*ln((-a*d+b*c)/b/(d*x+c))*(2*A+B+2*B*ln(e*(b*x+a)/(d*x+c)))/d^3/i^2+
2*b*B*(-a*d+b*c)*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^2
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.92

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$$
$$= \frac{g^2\left(Ab^2dx+\frac{B(bc-ad)^2}{c+dx}+bB(bc-ad)\log(a+bx)+bBd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-\frac{(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{c+dx}\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*
i*x)^2,x]
```

output
```
(g^2*(A*b^2*d*x + (B*(b*c - a*d)^2)/(c + d*x) + b*B*(b*c - a*d)*Log[a + b*
x] + b*B*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - ((b*c - a*d)^2*(A + B*
Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) - 2*b*B*(b*c - a*d)*Log[c + d*x]
- 2*b*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + b*B*
(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(d^3*i^2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2962, 2784, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ci+dix)^2}\,dx$$
$$\downarrow 2962$$
$$\frac{g^2(bc-ad)\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i^2}$$

$$\downarrow 2784$$

$$\frac{g^2(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{d}\right)}{i^2}$$

$$\downarrow 2793$$

$$\frac{g^2(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\left(-\frac{2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)}{d}-\frac{b\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{d}\right)}{i^2}$$

$$\downarrow 2009$$

$$\frac{g^2(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d^2}-\frac{(2A+B)(a+bx)}{d(c+dx)}-\frac{2bB\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2}-}{d}\right)}{i^2}$$

input

```
Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^2
,x]
```

output

```
((b*c - a*d)*g^2*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c
 + d*x)^2*(b - (d*(a + b*x))/(c + d*x))) - ((2*B*(a + b*x))/(d*(c + d*x))
- ((2*A + B)*(a + b*x))/(d*(c + d*x)) - (2*B*(a + b*x)*Log[(e*(a + b*x))/(
c + d*x)])/(d*(c + d*x)) - (b*(2*A + B + 2*B*Log[(e*(a + b*x))/(c + d*x)])
*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 - (2*b*B*PolyLog[2, (d*(a + b*x
))/(b*(c + d*x))])/d^2)/d))/i^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(260) = 520$.

Time = 3.06 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.05

method	result
parts	$\frac{g^2 A\left(\frac{x\,b^2}{d^2}+\frac{2b(da-bc)\ln(dx+c)}{d^3}-\frac{a^2d^2-2acdb+c^2b^2}{d^3(dx+c)}\right)}{i^2} - \frac{g^2 B\left(\frac{\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-\frac{(da-bc)e}{d(dx+c)}-\frac{be}{d}\right)(a^2$
derivativedivides	$-\frac{e(da-bc)\left(\frac{g^2d^2A\left(\frac{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}{d^2}+\frac{2be\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{d^3}+\frac{b^2e^2}{d^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}\right)}{e^2i^2}+\frac{g^2d^2B\left(\frac{\left(\frac{be}{d}+\frac{(da-b}{d(dx+}\right.\right.}$
default	$-\frac{e(da-bc)\left(\frac{g^2d^2A\left(\frac{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}{d^2}+\frac{2be\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{d^3}+\frac{b^2e^2}{d^3\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}\right)}{e^2i^2}+\frac{g^2d^2B\left(\frac{\left(\frac{be}{d}+\frac{(da-b}{d(dx+}\right.\right.}$
risch	Expression too large to display

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x,method=_RETU
RNVERBOSE)
```

output
```
g^2*A/i^2*(x*b^2/d^2+2*b/d^3*(a*d-b*c)*ln(d*x+c)-1/d^3*(a^2*d^2-2*a*b*c*d+
b^2*c^2)/(d*x+c))-g^2*B/i^2/(a*d-b*c)/e*(((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln
(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)/d^2*(a^2*d^2-2*
a*b*c*d+b^2*c^2)+(1/b/e*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/d-ln(b*e/d
+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c
)*e/d/(d*x+c))*d-b*e))*b^2*e^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2+2*(dilog(-(
(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)*b/d^2*e*(a^2*d^2-2*a*b
*c*d+b^2*c^2))
```

Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algo
rithm="fricas")
```

output
```
integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B
*a*b*g^2*x + B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)))/(d^2*i^2*x^2 + 2*c*d
*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 886 vs. 2(259) = 518.

Time = 0.10 (sec) , antiderivative size = 886, normalized size of antiderivative = 3.41

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algo
rithm="maxima")
```

output
```
-A*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2) - x/(d^2*i^2) + 2*c*log(d*x + c)/(d^3*
i^2))*g^2 + 2*A*a*b*g^2*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2
)) - B*a^2*g^2*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x + c*d*i^2)
 - 1/(d^2*i^2*x + c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^2) + b*log(
d*x + c)/((b*c*d - a*d^2)*i^2)) - A*a^2*g^2/(d^2*i^2*x + c*d*i^2) - (2*a^2
*b*d^2*g^2*log(e) + 2*(g^2*log(e) + g^2)*b^3*c^2 - (4*g^2*log(e) + 3*g^2)*
a*b^2*c*d)*B*log(d*x + c)/(b*c*d^3*i^2 - a*d^4*i^2) + ((b^3*c*d^2*g^2*log(
e) - a*b^2*d^3*g^2*log(e))*B*x^2 + (b^3*c^2*d*g^2*log(e) - a*b^2*c*d^2*g^2
*log(e))*B*x + ((b^3*c^2*d*g^2 - 2*a*b^2*c*d^2*g^2 + a^2*b*d^3*g^2)*B*x +
(b^3*c^3*g^2 - 2*a*b^2*c^2*d*g^2 + a^2*b*c*d^2*g^2)*B)*log(d*x + c)^2 - ((
g^2*log(e) - g^2)*b^3*c^3 - 3*(g^2*log(e) - g^2)*a*b^2*c^2*d + 2*(g^2*log(
e) - g^2)*a^2*b*c*d^2)*B + ((b^3*c*d^2*g^2 - a*b^2*d^3*g^2)*B*x^2 + (2*b^3
*c^2*d*g^2 - 2*a*b^2*c*d^2*g^2 - a^2*b*d^3*g^2)*B*x + (2*a*b^2*c^2*d*g^2 -
 3*a^2*b*c*d^2*g^2)*B)*log(b*x + a) - ((b^3*c*d^2*g^2 - a*b^2*d^3*g^2)*B*x
^2 + (b^3*c^2*d*g^2 - a*b^2*c*d^2*g^2)*B*x - (b^3*c^3*g^2 - 3*a*b^2*c^2*d*
g^2 + 2*a^2*b*c*d^2*g^2)*B)*log(d*x + c))/(b*c^2*d^3*i^2 - a*c*d^4*i^2 + (
b*c*d^4*i^2 - a*d^5*i^2)*x) - 2*(b^2*c*g^2 - a*b*d*g^2)*(log(b*x + a)*log(
(b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^3
*i^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1774 vs. $2(259) = 518$.

Time = 59.34 (sec) , antiderivative size = 1774, normalized size of antiderivative = 6.82

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algo
rithm="giac")
```

output

```
1/6*(2*(B*b^6*c^4*e^4*g^2 - 4*B*a*b^5*c^3*d*e^4*g^2 + 6*B*a^2*b^4*c^2*d^2*
e^4*g^2 - 4*B*a^3*b^3*c*d^3*e^4*g^2 + B*a^4*b^2*d^4*e^4*g^2 - 3*(b*e*x + a
*e)*B*b^5*c^4*d*e^3*g^2/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^4*c^3*d^2*e^3*g
^2/(d*x + c) - 18*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3*g^2/(d*x + c) + 12*(
b*e*x + a*e)*B*a^3*b^2*c*d^4*e^3*g^2/(d*x + c) - 3*(b*e*x + a*e)*B*a^4*b*d
^5*e^3*g^2/(d*x + c) + 3*(b*e*x + a*e)^2*B*b^4*c^4*d^2*e^2*g^2/(d*x + c)^2
 - 12*(b*e*x + a*e)^2*B*a*b^3*c^3*d^3*e^2*g^2/(d*x + c)^2 + 18*(b*e*x + a*
e)^2*B*a^2*b^2*c^2*d^4*e^2*g^2/(d*x + c)^2 - 12*(b*e*x + a*e)^2*B*a^3*b*c*
d^5*e^2*g^2/(d*x + c)^2 + 3*(b*e*x + a*e)^2*B*a^4*d^6*e^2*g^2/(d*x + c)^2)
*log((b*e*x + a*e)/(d*x + c))/(b^3*d^3*e^3*i^2 - 3*(b*e*x + a*e)*b^2*d^4*e
^2*i^2/(d*x + c) + 3*(b*e*x + a*e)^2*b*d^5*e*i^2/(d*x + c)^2 - (b*e*x + a*
e)^3*d^6*i^2/(d*x + c)^3) + (2*A*b^6*c^4*e^4*g^2 + 3*B*b^6*c^4*e^4*g^2 - 8
*A*a*b^5*c^3*d*e^4*g^2 - 12*B*a*b^5*c^3*d*e^4*g^2 + 12*A*a^2*b^4*c^2*d^2*e
^4*g^2 + 18*B*a^2*b^4*c^2*d^2*e^4*g^2 - 8*A*a^3*b^3*c*d^3*e^4*g^2 - 12*B*a
^3*b^3*c*d^3*e^4*g^2 + 2*A*a^4*b^2*d^4*e^4*g^2 + 3*B*a^4*b^2*d^4*e^4*g^2 -
 6*(b*e*x + a*e)*A*b^5*c^4*d*e^3*g^2/(d*x + c) - 7*(b*e*x + a*e)*B*b^5*c^4
*d*e^3*g^2/(d*x + c) + 24*(b*e*x + a*e)*A*a*b^4*c^3*d^2*e^3*g^2/(d*x + c)
+ 28*(b*e*x + a*e)*B*a*b^4*c^3*d^2*e^3*g^2/(d*x + c) - 36*(b*e*x + a*e)*A*
a^2*b^3*c^2*d^3*e^3*g^2/(d*x + c) - 42*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3
*g^2/(d*x + c) + 24*(b*e*x + a*e)*A*a^3*b^2*c*d^4*e^3*g^2/(d*x + c) + 2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(c\,i+d\,i\,x)^2}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2
,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2
, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x)
```

output

```
(g**2*( - int((log((a*e + b*e*x)/(c + d*x))*x**2)/(c**2 + 2*c*d*x + d**2*x
**2),x)*a*b**3*c**2*d**4 - int((log((a*e + b*e*x)/(c + d*x))*x**2)/(c**2 +
 2*c*d*x + d**2*x**2),x)*a*b**3*c*d**5*x + int((log((a*e + b*e*x)/(c + d*x
))*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**4*c**3*d**3 + int((log((a*e +
b*e*x)/(c + d*x))*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**4*c**2*d**4*x -
 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*
*2*b**2*c**2*d**4 - 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(c**2 + 2*c*d*x
 + d**2*x**2),x)*a**2*b**2*c*d**5*x + 2*int((log((a*e + b*e*x)/(c + d*x))*
x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**3*c**3*d**3 + 2*int((log((a*e + b*
e*x)/(c + d*x))*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**3*c**2*d**4*x + lo
g(a + b*x)*a**3*b*c*d**3 + log(a + b*x)*a**3*b*d**4*x - 2*log(c + d*x)*a**
3*b*c**2*d**2 - 2*log(c + d*x)*a**3*b*c*d**3*x - log(c + d*x)*a**3*b*c*d**
3 - log(c + d*x)*a**3*b*d**4*x + 4*log(c + d*x)*a**2*b**2*c**3*d + 4*log(c
 + d*x)*a**2*b**2*c**2*d**2*x - 2*log(c + d*x)*a*b**3*c**4 - 2*log(c + d*x
)*a*b**3*c**3*d*x - log((a*e + b*e*x)/(c + d*x))*a**3*b*d**4*x + log((a*e
+ b*e*x)/(c + d*x))*a**2*b**2*c*d**3*x - a**4*d**4*x + 3*a**3*b*c*d**3*x +
 a**3*b*d**4*x - 4*a**2*b**2*c**2*d**2*x - a**2*b**2*c*d**3*x**2 - a**2*b*
*2*c*d**3*x + 2*a*b**3*c**3*d*x + a*b**3*c**2*d**2*x**2))/(c*d**3*(a*c*d +
 a*d**2*x - b*c**2 - b*c*d*x))
```

3.41 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$

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Optimal result

Integrand size = 38, antiderivative size = 160

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = -\frac{Ag(a+bx)}{di^2(c+dx)} + \frac{Bg(a+bx)}{di^2(c+dx)} - \frac{Bg(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{di^2(c+dx)} - \frac{bg\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i^2} - \frac{bBg\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}$$

output

```
-A*g*(b*x+a)/d/i^2/(d*x+c)+B*g*(b*x+a)/d/i^2/(d*x+c)-B*g*(b*x+a)*ln(e*(b*x
+a)/(d*x+c))/d/i^2/(d*x+c)-b*g*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/
(d*x+c)))/d^2/i^2-b*B*g*polylog(2,d*(b*x+a)/b/(d*x+c))/d^2/i^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.09

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$$
$$= \frac{g\left(\frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{c+dx}+2b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)-2B\left(\frac{bc-ad}{c+dx}+b\log(a+bx)-b\log(c\right.\right.}{2d^2i^2}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*
x)^2,x]
```

output
```
(g*((2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) + 2*b*(
A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 2*B*((b*c - a*d)/(c + d
*x) + b*Log[a + b*x] - b*Log[c + d*x]) - b*B*((2*Log[(d*(a + b*x))/(-(b*c)
 + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)])))/(2*d^2*i^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2962, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ci+dix)^2}\,dx$$
$$\downarrow \text{2962}$$
$$\frac{g\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{i^2}$$
$$\downarrow \text{2793}$$

$$\frac{g\int\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{d}-\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{i^2}$$

$$\downarrow 2009$$

$$\frac{g\left(-\frac{b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^2}-\frac{A(a+bx)}{d(c+dx)}-\frac{bB\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2}-\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d(c+dx)}+\frac{B(a+bx)}{d(c+dx)}\right)}{i^2}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^2,x
]
```

output
```
(g*(-((A*(a + b*x))/(d*(c + d*x))) + (B*(a + b*x))/(d*(c + d*x)) - (B*(a +
 b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d*(c + d*x)) - (b*(A + B*Log[(e*(a +
b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 - (b*B*PolyLog
[2, (d*(a + b*x))/(b*(c + d*x))])/d^2))/i^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.84

method	result
parts	$\frac{gA\left(\frac{b\ln(dx+c)}{d^2}-\frac{da-bc}{d^2(dx+c)}\right)}{i^2}-\frac{gB\left(\frac{\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-\frac{(da-bc)e}{d(dx+c)}-\frac{be}{d}\right)(da-bc)}{d}+\frac{\left(\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(da\cdots}{d(d\cdots}\right.}{\cdots}\right.}{d}\right.}{\cdots}\right.}{i^2(da-bc)e\cdots}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{g\,d^2A\left(-\frac{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}{d}-\frac{be\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{d^2}\right)}{(da-bc)e^2i^2}-\frac{g\,d^2B\left(-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-\cdots}{d}\right.}{\cdots}\right.}{d^2}$
default	$-\frac{e(da-bc)\left(-\frac{g\,d^2A\left(-\frac{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}{d}-\frac{be\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{d^2}\right)}{(da-bc)e^2i^2}-\frac{g\,d^2B\left(-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-\cdots}{d}\right.}{\cdots}\right.}{d^2}$
risch	$\frac{gAb\ln(dx+c)}{i^2d^2}-\frac{gAa}{i^2d(dx+c)}+\frac{gAbc}{i^2d^2(dx+c)}-\frac{gB\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)ba}{i^2(da-bc)d}+\frac{gB\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)b^2c}{i^2(da-bc)d^2}-\frac{gB\ln\left(\frac{be}{d}\cdots\right.}{i^2(da\cdots}$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x,method=_RETURN
VERBOSE)
```

output
```
g*A/i^2*(b/d^2*ln(d*x+c)-(a*d-b*c)/d^2/(d*x+c))-g*B/i^2/(a*d-b*c)/e*(((b*e
/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d
*x+c)-b*e/d)*(a*d-b*c)/d+(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e
)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b
*e)/b/e)/d)*b*e*(a*d-b*c)/d)
```

Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algori
thm="fricas")
```

output
```
integral((A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b*e*x + a*e)/(d*x + c))
)/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algori
thm="maxima")
```

output
```
-1/2*B*b*g*(((d*x + c)*log(d*x + c)^2 + 2*c*log(d*x + c))/(d^3*i^2*x + c*d
^2*i^2) - 2*integrate((d*x*log(b*x + a) + d*x*log(e) + c)/(d^3*i^2*x^2 + 2
*c*d^2*i^2*x + c^2*d*i^2), x)) + A*b*g*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*
x + c)/(d^2*i^2)) - B*a*g*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x
 + c*d*i^2) - 1/(d^2*i^2*x + c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^
2) + b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) - A*a*g/(d^2*i^2*x + c*d*i^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(159) = 318$.

Time $= 46.76$ (sec) , antiderivative size $= 895$, normalized size of antiderivative $= 5.59$

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algori
thm="giac")
```

output

```
-1/2*((B*b^4*c^3*e^3*g - 3*B*a*b^3*c^2*d*e^3*g + 3*B*a^2*b^2*c*d^2*e^3*g -
 B*a^3*b*d^3*e^3*g - 2*(b*e*x + a*e)*B*b^3*c^3*d*e^2*g/(d*x + c) + 6*(b*e*
x + a*e)*B*a*b^2*c^2*d^2*e^2*g/(d*x + c) - 6*(b*e*x + a*e)*B*a^2*b*c*d^3*e
^2*g/(d*x + c) + 2*(b*e*x + a*e)*B*a^3*d^4*e^2*g/(d*x + c))*log((b*e*x + a
*e)/(d*x + c))/(b^2*d^2*e^2*i^2 - 2*(b*e*x + a*e)*b*d^3*e*i^2/(d*x + c) +
(b*e*x + a*e)^2*d^4*i^2/(d*x + c)^2) + (A*b^4*c^3*e^3*g + B*b^4*c^3*e^3*g
- 3*A*a*b^3*c^2*d*e^3*g - 3*B*a*b^3*c^2*d*e^3*g + 3*A*a^2*b^2*c*d^2*e^3*g
+ 3*B*a^2*b^2*c*d^2*e^3*g - A*a^3*b*d^3*e^3*g - B*a^3*b*d^3*e^3*g - 2*(b*e
*x + a*e)*A*b^3*c^3*d*e^2*g/(d*x + c) - (b*e*x + a*e)*B*b^3*c^3*d*e^2*g/(d
*x + c) + 6*(b*e*x + a*e)*A*a*b^2*c^2*d^2*e^2*g/(d*x + c) + 3*(b*e*x + a*e
)*B*a*b^2*c^2*d^2*e^2*g/(d*x + c) - 6*(b*e*x + a*e)*A*a^2*b*c*d^3*e^2*g/(d
*x + c) - 3*(b*e*x + a*e)*B*a^2*b*c*d^3*e^2*g/(d*x + c) + 2*(b*e*x + a*e)*
A*a^3*d^4*e^2*g/(d*x + c) + (b*e*x + a*e)*B*a^3*d^4*e^2*g/(d*x + c))/(b^2*
d^2*e^2*i^2 - 2*(b*e*x + a*e)*b*d^3*e*i^2/(d*x + c) + (b*e*x + a*e)^2*d^4*
i^2/(d*x + c)^2) + (B*b^3*c^3*e*g - 3*B*a*b^2*c^2*d*e*g + 3*B*a^2*b*c*d^2*
e*g - B*a^3*d^3*e*g)*log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b*d^2*i^2) - (
B*b^3*c^3*e*g - 3*B*a*b^2*c^2*d*e*g + 3*B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)
*log((b*e*x + a*e)/(d*x + c))/(b*d^2*i^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*
d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(c\,i+d\,i\,x)^2}\,dx$$

input

```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2,x
)
```

output

```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2,
x)
```

Reduce [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$$

$$= \frac{g\left(-\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{d^2x^2+2cdx+c^2}dx\right)ab^2c^2d^3-\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{d^2x^2+2cdx+c^2}dx\right)ab^2cd^4x+\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{d^2x^2+2cdx+c^2}dx\right)b^3c^3d^2+\left(\int \frac{\log(}{d^2x^2}\right.\right.}{}$$

input

```
int((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x)
```

output

```
(g*( - int((log((a*e + b*e*x)/(c + d*x))*x)/(c**2 + 2*c*d*x + d**2*x**2),x
)*a*b**2*c**2*d**3 - int((log((a*e + b*e*x)/(c + d*x))*x)/(c**2 + 2*c*d*x
+ d**2*x**2),x)*a*b**2*c*d**4*x + int((log((a*e + b*e*x)/(c + d*x))*x)/(c*
*2 + 2*c*d*x + d**2*x**2),x)*b**3*c**3*d**2 + int((log((a*e + b*e*x)/(c +
d*x))*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**3*c**2*d**3*x + log(a + b*x)*a
**2*b*c*d**2 + log(a + b*x)*a**2*b*d**3*x - log(c + d*x)*a**2*b*c**2*d - l
og(c + d*x)*a**2*b*c*d**2*x - log(c + d*x)*a**2*b*c*d**2 - log(c + d*x)*a*
*2*b*d**3*x + log(c + d*x)*a*b**2*c**3 + log(c + d*x)*a*b**2*c**2*d*x - lo
g((a*e + b*e*x)/(c + d*x))*a**2*b*d**3*x + log((a*e + b*e*x)/(c + d*x))*a*
b**2*c*d**2*x - a**3*d**3*x + 2*a**2*b*c*d**2*x + a**2*b*d**3*x - a*b**2*c
**2*d*x - a*b**2*c*d**2*x))/(c*d**2*(a*c*d + a*d**2*x - b*c**2 - b*c*d*x))
```

3.42 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx$

Optimal result 550
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Optimal result

Integrand size = 30, antiderivative size = 98

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx = \frac{A(a+bx)}{(bc-ad)i^2(c+dx)} - \frac{B(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)i^2(c+dx)}$$

output

```
A*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)-B*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)+B*(b*x+a
)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)/i^2/(d*x+c)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx = \frac{Abc-bBc-aAd+aBd-bB(c+dx)\log(a+bx)+B(bc-ad)\log\left(\frac{e(a+bx)}{c+dx}\right)+bBc\log(c+dx)+bBd}{d(-bc+ad)i^2(c+dx)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x)^2,x]
```

output
```
(A*b*c - b*B*c - a*A*d + a*B*d - b*B*(c + d*x)*Log[a + b*x] + B*(b*c - a*d
)*Log[(e*(a + b*x))/(c + d*x)] + b*B*c*Log[c + d*x] + b*B*d*x*Log[c + d*x]
)/(d*(-(b*c) + a*d)*i^2*(c + d*x))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2952, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{(ci+dix)^2}\,dx$$

$$\downarrow 2952$$

$$\frac{\int \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2009$$

$$\frac{\frac{A(a+bx)}{c+dx} + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx} - \frac{B(a+bx)}{c+dx}}{i^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x)^2,x]
```

output
```
((A*(a + b*x))/(c + d*x) - (B*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[(e*(
a + b*x))/(c + d*x)])/(c + d*x))/((b*c - a*d)*i^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2952

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84

method	result
parts	$-\frac{A}{i^2(dx+c)d}-\frac{B\left(\frac{\ln\left(\frac{e(bx+a)}{dx+c}\right)e(bx+a)}{dx+c}-\frac{e(bx+a)}{dx+c}\right)}{i^2e(da-bc)}$
norman	$\frac{\frac{(A-B)x}{ic}-\frac{aB\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(da-bc)i}-\frac{Bbx\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(da-bc)i}}{i(dx+c)}$
parallelrisch	$-\frac{-Babd^3+Bb^2cd^2+Aabd^3-Ab^2cd^2+Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2d^3+B\ln\left(\frac{e(bx+a)}{dx+c}\right)abd^3}{i^2(dx+c)bd^3(da-bc)}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{di^2(dx+c)}-\frac{B\ln(bx+a)bdx-B\ln(-dx-c)bdx+B\ln(bx+a)bc-B\ln(-dx-c)bc+Ada-Abc-Bad+Bbc}{i^2(dx+c)d(da-bc)}$
orering	$-\frac{3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)(bx+a)(dx+c)}{(dix+ci)^2(da-bc)}-\frac{(bx+a)(dx+c)^2\left(\frac{B\left(\frac{eb}{dx+c}-\frac{e(bx+a)d}{(dx+c)^2}\right)(dx+c)}{e(bx+a)(dix+ci)^2}-\frac{2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)di}{(dix+ci)^3}\right)}{(da-bc)d}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{d^2A\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{(da-bc)^2e^2i^2}+\frac{d^2B\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-\frac{(da-bc)e}{d(dx+c)}-\frac{be}{d}\right)}{(da-bc)^2e^2i^2}\right)}{d^2}$
default	$-\frac{e(da-bc)\left(\frac{d^2A\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{(da-bc)^2e^2i^2}+\frac{d^2B\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-\frac{(da-bc)e}{d(dx+c)}-\frac{be}{d}\right)}{(da-bc)^2e^2i^2}\right)}{d^2}$

input

```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x,method=_RETURNVERBOSE)
```

output
```
-A/i^2/(d*x+c)/d-B/i^2/e/(a*d-b*c)*(ln(e*(b*x+a)/(d*x+c))*e*(b*x+a)/(d*x+c
)-e*(b*x+a)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.90

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx = -\frac{(A-B)bc-(A-B)ad-(Bbdx+Bad)\log\left(\frac{bex+ae}{dx+c}\right)}{(bcd^2-ad^3)i^2x+(bc^2d-acd^2)i^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algorithm="fricas"
)
```

output
```
-((A - B)*b*c - (A - B)*a*d - (B*b*d*x + B*a*d)*log((b*e*x + a*e)/(d*x + c
)))/((b*c*d^2 - a*d^3)*i^2*x + (b*c^2*d - a*c*d^2)*i^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(78) = 156.

Time = 0.63 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.36

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx = {} & \frac{Bb\log\left(x+\frac{-\frac{Ba^2bd^2}{ad-bc}+\frac{2Bab^2cd}{ad-bc}+Babd-\frac{Bb^3c^2}{ad-bc}+Bb^2c}{2Bb^2d}\right)}{di^2\left(ad-bc\right)} \\ & - \frac{Bb\log\left(x+\frac{\frac{Ba^2bd^2}{ad-bc}-\frac{2Bab^2cd}{ad-bc}+Babd+\frac{Bb^3c^2}{ad-bc}+Bb^2c}{2Bb^2d}\right)}{di^2\left(ad-bc\right)} \\ & - \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{cdi^2+d^2i^2x}+\frac{-A+B}{cdi^2+d^2i^2x}\end{aligned}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**2,x)
```

output
```
B*b*log(x + (-B*a**2*b*d**2/(a*d - b*c) + 2*B*a*b**2*c*d/(a*d - b*c) + B*a
*b*d - B*b**3*c**2/(a*d - b*c) + B*b**2*c)/(2*B*b**2*d))/(d*i**2*(a*d - b*
c)) - B*b*log(x + (B*a**2*b*d**2/(a*d - b*c) - 2*B*a*b**2*c*d/(a*d - b*c)
+ B*a*b*d + B*b**3*c**2/(a*d - b*c) + B*b**2*c)/(2*B*b**2*d))/(d*i**2*(a*d
 - b*c)) - B*log(e*(a + b*x)/(c + d*x))/(c*d*i**2 + d**2*i**2*x) + (-A + B
)/(c*d*i**2 + d**2*i**2*x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx$$
$$= -B\left(\frac{\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right)}{d^2i^2x + cdi^2} - \frac{1}{d^2i^2x + cdi^2} - \frac{b\log(bx+a)}{(bcd - ad^2)i^2} + \frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right)$$
$$- \frac{A}{d^2i^2x + cdi^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algorithm="maxima"
)
```

output
```
-B*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x + c*d*i^2) - 1/(d^2*i^
2*x + c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^2) + b*log(d*x + c)/((b
*c*d - a*d^2)*i^2)) - A/(d^2*i^2*x + c*d*i^2)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx$$
$$= \left(\frac{bc}{(bce - ade)(bc - ad)} - \frac{ad}{(bce - ade)(bc - ad)}\right)\left(\frac{(bex + ae)B\log\left(\frac{bex+ae}{dx+c}\right)}{(dx+c)i^2} + \frac{(bex+ae)(A-B)}{(dx+c)i^2}\right)$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algorithm="giac")
```

output
```
(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))*((
b*e*x + a*e)*B*log((b*e*x + a*e)/(d*x + c))/((d*x + c)*i^2) + (b*e*x + a*e
)*(A - B)/((d*x + c)*i^2))
```

Mupad [B] (verification not implemented)

Time = 26.59 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx = -\frac{A-B}{x\,d^2\,i^2+c\,d\,i^2}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{d^2\,i^2\,\left(x+\frac{c}{d}\right)} + \frac{B\,b\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{d\,i^2\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(c*i + d*i*x)^2,x)
```

output
```
(B*b*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(d*i^2*(a*d - b*c)) -
(B*log((e*(a + b*x))/(c + d*x)))/(d^2*i^2*(x + c/d)) - (A - B)/(d^2*i^2*x
+ c*d*i^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.47

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx$$
$$= \frac{\log(bx+a)\,abc+\log(bx+a)\,abdx-\log(dx+c)\,abc-\log(dx+c)\,abdx-\log\left(\frac{bex+ae}{dx+c}\right)\,abdx+\log\left(\frac{bex+a}{dx+c}\right.}{c\,(a\,d^2x-bcdx+acd-b\,c^2)}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x)
```

output

```
(log(a + b*x)*a*b*c + log(a + b*x)*a*b*d*x - log(c + d*x)*a*b*c - log(c +
d*x)*a*b*d*x - log((a*e + b*e*x)/(c + d*x))*a*b*d*x + log((a*e + b*e*x)/(c
 + d*x))*b**2*c*x - a**2*d*x + a*b*c*x + a*b*d*x - b**2*c*x)/(c*(a*c*d + a
*d**2*x - b*c**2 - b*c*d*x))
```

3.43 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 156

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx = -\frac{Ad(a+bx)}{(bc-ad)^2gi^2(c+dx)} + \frac{Bd(a+bx)}{(bc-ad)^2gi^2(c+dx)} - \frac{Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^2gi^2(c+dx)} + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2B(bc-ad)^2gi^2}$$

output

```
-A*d*(b*x+a)/(-a*d+b*c)^2/g/i^2/(d*x+c)+B*d*(b*x+a)/(-a*d+b*c)^2/g/i^2/(d*
x+c)-B*d*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)^2/g/i^2/(d*x+c)+1/2*b*(A
+B*ln(e*(b*x+a)/(d*x+c)))^2/B/(-a*d+b*c)^2/g/i^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.87

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2b(c+dx)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2b(c+dx)\left(A+B\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x
)^2),x]
```

output
```
(2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*b*(c + d*x)*Log[a
+ b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b*(c + d*x)*(A + B*Log[(e*
(a + b*x))/(c + d*x)])*Log[c + d*x] - 2*B*(b*c - a*d + b*(c + d*x)*Log[a +
 b*x] - b*(c + d*x)*Log[c + d*x]) - b*B*(c + d*x)*(Log[a + b*x]*(Log[a + b
*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*
c) + a*d)]) + b*B*(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
+ d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(2*(b*c -
 a*d)^2*g*i^2*(c + d*x))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2962, 2788, 2009, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{(ag+bgx)(ci+dix)^2}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx} d\frac{a+bx}{c+dx}}{gi^2(bc-ad)^2}$$

$$\downarrow 2788$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx} d\frac{a+bx}{c+dx} - d\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) d\frac{a+bx}{c+dx}}{gi^2(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx} d\frac{a+bx}{c+dx} - d\left(\frac{A(a+bx)}{c+dx} + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx} - \frac{B(a+bx)}{c+dx}\right)}{gi^2(bc-ad)^2}$$

$$\downarrow 2738$$

$$\frac{\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2B}-d\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}-\frac{B(a+bx)}{c+dx}\right)}{gi^2(bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x)^2),x
]
```

output
```
((b*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*B) - d*((A*(a + b*x))/(c +
d*x) - (B*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)]
)/(c + d*x)))/((b*c - a*d)^2*g*i^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.48

method	result
parts	$\frac{A\left(-\frac{1}{(da-bc)(dx+c)}-\frac{b\ln(dx+c)}{(da-bc)^2}+\frac{b\ln(bx+a)}{(da-bc)^2}\right)}{g\,i^2}-\frac{B\left(\frac{d\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-\frac{(da-bc)e}{d(dx+c)}-\frac{be}{d}\right)}{da-bc}-\frac{be\ln\left(\frac{be}{d}+\right.}{2(d}\right.}{g\,i^2(da-bc)e}$
parallelrisch	$\frac{-2Aa\,b^2d^4+2A\,b^3cd^3+Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b^3d^4+2Ax\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3d^4-2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3d^4+B\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b^3cd^3}{2i^2g(dx+c)b^2d^3(a^2d^2-2acdb+c^2b^2)}$
norman	$\frac{\frac{(Abc-Bad)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{d(Ab-Bb)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{(A-B)dx}{gic(da-bc)}+\frac{Bbc\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2gi\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{bBdx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2gi\left(a^2d^2-2acdb+c^2b^2\right)}}{i(dx+c)}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{d^2Ab\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e\,i^2(da-bc)^3g}+\frac{d^3A\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e^2i^2(da-bc)^3g}-\frac{d^2Bb\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2e\,i^2(da-bc)^3g}+\frac{d^3B\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\right.\right.}{e^2i^2(da}\right.}{d^2}$
default	$-\frac{e(da-bc)\left(-\frac{d^2Ab\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e\,i^2(da-bc)^3g}+\frac{d^3A\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e^2i^2(da-bc)^3g}-\frac{d^2Bb\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2e\,i^2(da-bc)^3g}+\frac{d^3B\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\right.\right.}{e^2i^2(da}\right.}{d^2}$
risch	$-\frac{A}{g\,i^2(da-bc)(dx+c)}-\frac{Ab\ln(dx+c)}{g\,i^2(da-bc)^2}+\frac{Ab\ln(bx+a)}{g\,i^2(da-bc)^2}-\frac{B\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)b}{g\,i^2(da-bc)^2}-\frac{Bd\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)a}{g\,i^2(da-bc)^2(dx+c)}+\frac{B}{g}$

input

```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^2,x,method=_RETURN
VERBOSE)
```

output

```
A/g/i^2*(-1/(a*d-b*c)/(d*x+c)-b/(a*d-b*c)^2*ln(d*x+c)+b/(a*d-b*c)^2*ln(b*x
+a))-B/g/i^2/(a*d-b*c)/e*(d/(a*d-b*c)*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)-1/2/(a*d-b*c)*b*e*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.97

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{2\,(A-B)bc - 2\,(A-B)ad + (Bbdx + Bbc)\log\left(\frac{bex+ae}{dx+c}\right)^2 + 2\,((A-B)bdx + Abc - Bad)\log\left(\frac{bex+ae}{dx+c}\right)}{2\,((b^2c^2d - 2\,abcd^2 + a^2d^3)gi^2x + (b^2c^3 - 2\,abc^2d + a^2cd^2)gi^2)}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algori
thm="fricas")
```

output
```
1/2*(2*(A - B)*b*c - 2*(A - B)*a*d + (B*b*d*x + B*b*c)*log((b*e*x + a*e)/(
d*x + c))^2 + 2*((A - B)*b*d*x + A*b*c - B*a*d)*log((b*e*x + a*e)/(d*x + c
)))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g*i^2*x + (b^2*c^3 - 2*a*b*c^2*d
+ a^2*c*d^2)*g*i^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(131) = 262.

Time = 0.60 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.47

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{Bb\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^2d^2gi^2-4abcdgi^2+2b^2c^2gi^2} - \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{acdgi^2+ad^2gi^2x-bc^2gi^2-bcdgi^2x}$$
$$+(A-B)\left(-\frac{b\log\left(x+\frac{-\frac{a^3bd^3}{(ad-bc)^2}+\frac{3a^2b^2cd^2}{(ad-bc)^2}-\frac{3ab^3c^2d}{(ad-bc)^2}+abd+\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{gi^2(ad-bc)^2}\right.$$
$$+\frac{b\log\left(x+\frac{\frac{a^3bd^3}{(ad-bc)^2}-\frac{3a^2b^2cd^2}{(ad-bc)^2}+\frac{3ab^3c^2d}{(ad-bc)^2}+abd-\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{gi^2(ad-bc)^2}$$
$$\left.-\frac{1}{acdgi^2-bc^2gi^2+x\left(ad^2gi^2-bcdgi^2\right)}\right)$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)**2,x)
```

output
```
B*b*log(e*(a + b*x)/(c + d*x))**2/(2*a**2*d**2*g*i**2 - 4*a*b*c*d*g*i**2 +
 2*b**2*c**2*g*i**2) - B*log(e*(a + b*x)/(c + d*x))/(a*c*d*g*i**2 + a*d**2
*g*i**2*x - b*c**2*g*i**2 - b*c*d*g*i**2*x) + (A - B)*(-b*log(x + (-a**3*b
*d**3/(a*d - b*c)**2 + 3*a**2*b**2*c*d**2/(a*d - b*c)**2 - 3*a*b**3*c**2*d
/(a*d - b*c)**2 + a*b*d + b**4*c**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(
g*i**2*(a*d - b*c)**2) + b*log(x + (a**3*b*d**3/(a*d - b*c)**2 - 3*a**2*b*
*2*c*d**2/(a*d - b*c)**2 + 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d - b**4*c
**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(g*i**2*(a*d - b*c)**2) - 1/(a*c*
d*g*i**2 - b*c**2*g*i**2 + x*(a*d**2*g*i**2 - b*c*d*g*i**2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. 2(154) = 308.

Time = 0.06 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.70

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$$

$$= B\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)\log\left(\frac{b}{dx}\right.$$

$$\left.+\frac{ae}{dx+c}\right)$$

$$+A\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)$$

$$-\frac{\left((bdx+bc)\log(bx+a)^2+(bdx+bc)\log(dx+c)^2+2\,bc-2\,ad+2\,(bdx+bc)\log(bx+a)-2\,(bdx+\right.}{2\left(b^2c^3gi^2-2\,abc^2dgi^2+a^2cd^2gi^2+(b^2c^2dgi^2-2\,abcd^2gi^2+a^2\right.}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algori
thm="maxima")
```

output

```
B*(1/((b*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*g*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + A*(1/((b*c*d
 - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*g*i^2)) - 1/2*((b*d*x + b*c)*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^
2 + 2*b*c - 2*a*d + 2*(b*d*x + b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*d*x
 + b*c)*log(b*x + a))*log(d*x + c))*B/(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 +
 a^2*c*d^2*g*i^2 + (b^2*c^2*d*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x
)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.49

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$=\frac{1}{2}\left(\frac{Bbe\log\left(\frac{bex+ae}{dx+c}\right)^2}{bcgi^2-adgi^2}+\frac{2\,Abe\log\left(\frac{bex+ae}{dx+c}\right)}{bcgi^2-adgi^2}-\frac{2\,(bex+ae)Bd\log\left(\frac{bex+ae}{dx+c}\right)}{(bcgi^2-adgi^2)(dx+c)}-\frac{2\,(bex+ae)(Ad-Bd)}{(bcgi^2-adgi^2)(dx+c)}\right)\left($$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algori
thm="giac")
```

output
```
1/2*(B*b*e*log((b*e*x + a*e)/(d*x + c))^2/(b*c*g*i^2 - a*d*g*i^2) + 2*A*b*
e*log((b*e*x + a*e)/(d*x + c))/(b*c*g*i^2 - a*d*g*i^2) - 2*(b*e*x + a*e)*B
*d*log((b*e*x + a*e)/(d*x + c))/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)) - 2*(b
*e*x + a*e)*(A*d - B*d)/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)))*(b*c/((b*c*e
- a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 27.67 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.58

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx = \frac{B\,b\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2}{2\,g\,i^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}$$
$$-\frac{A-B}{(a\,d-b\,c)\,\left(c\,g\,i^2+d\,g\,i^2\,x\right)}$$
$$-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,(a\,d-b\,c)}{b\,d\,g\,i^2\,\left(\frac{x}{b}+\frac{c}{b\,d}\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}$$
$$-\frac{b\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{a^2\,d^2\,g\,i^2-b^2\,c^2\,g\,i^2}{g\,i^2\,(a\,d-b\,c)}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A-B)\,2\mathrm{i}}{g\,i^2\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)*(c*i + d*i*x)^2),x
)
```

output
```
(B*b*log((e*(a + b*x))/(c + d*x))^2)/(2*g*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c
*d)) - (b*atan(((2*b*d*x + (a^2*d^2*g*i^2 - b^2*c^2*g*i^2)/(g*i^2*(a*d - b
*c)))*1i)/(a*d - b*c))*(A - B)*2i)/(g*i^2*(a*d - b*c)^2) - (A - B)/((a*d -
 b*c)*(c*g*i^2 + d*g*i^2*x)) - (B*log((e*(a + b*x))/(c + d*x))*(a*d - b*c)
)/(b*d*g*i^2*(x/b + c/(b*d))*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.93

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{-2\log(bx+a)\,ab\,c^2 - 2\log(bx+a)\,abcdx + 2\log(bx+a)\,abcd + 2\log(bx+a)\,ab\,d^2x + 2\log(dx+c)\,ab$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^2,x)
```

output
```
( - 2*log(a + b*x)*a*b*c**2 - 2*log(a + b*x)*a*b*c*d*x + 2*log(a + b*x)*a*
b*c*d + 2*log(a + b*x)*a*b*d**2*x + 2*log(c + d*x)*a*b*c**2 + 2*log(c + d*
x)*a*b*c*d*x - 2*log(c + d*x)*a*b*c*d - 2*log(c + d*x)*a*b*d**2*x - log((a
*e + b*e*x)/(c + d*x))**2*b**2*c**2 - log((a*e + b*e*x)/(c + d*x))**2*b**2
*c*d*x - 2*log((a*e + b*e*x)/(c + d*x))*a*b*d**2*x + 2*log((a*e + b*e*x)/(
c + d*x))*b**2*c*d*x - 2*a**2*d**2*x + 2*a*b*c*d*x + 2*a*b*d**2*x - 2*b**2
*c*d*x)/(2*c*g*(a**2*c*d**2 + a**2*d**3*x - 2*a*b*c**2*d - 2*a*b*c*d**2*x
+ b**2*c**3 + b**2*c**2*d*x))
```

3.44 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 261

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx &= -\frac{Bd^2(a+bx)}{(bc-ad)^3g^2i^2(c+dx)} - \frac{b^2B(c+dx)}{(bc-ad)^3g^2i^2(a+bx)} \\ &+ \frac{bBd\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2} + \frac{d^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2(c+dx)} \\ &- \frac{b^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2(a+bx)} \\ &- \frac{2bd\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2}\end{aligned}$$

output

```
-B*d^2*(b*x+a)/(-a*d+b*c)^3/g^2/i^2/(d*x+c)-b^2*B*(d*x+c)/(-a*d+b*c)^3/g^2
/i^2/(b*x+a)+b*B*d*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^2/i^2+d^2*(b*x+a)*
(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^2/i^2/(d*x+c)-b^2*(d*x+c)*(A+B*
ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^2/i^2/(b*x+a)-2*b*d*ln((b*x+a)/(d*x+
c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^2/i^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.42 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.24

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$$

$$= \frac{-\frac{b^2Bc}{a+bx} + \frac{abBd}{a+bx} + \frac{bBcd}{c+dx} - \frac{aBd^2}{c+dx} - \frac{b(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx} + \frac{d(-bc+ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{c+dx} - 2bd\log(a+bx)\left(A\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i
*x)^2),x]
```

output
```
(-((b^2*B*c)/(a + b*x)) + (a*b*B*d)/(a + b*x) + (b*B*c*d)/(c + d*x) - (a*B
*d^2)/(c + d*x) - (b*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a
+ b*x) + (d*(-(b*c) + a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x)
 - 2*b*d*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*b*d*(A + B*
Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + b*B*d*(Log[a + b*x]*(Log[a +
b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b
*c) + a*d)]) - b*B*d*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])
*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^3*g
^2*i^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2962, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right) + A}{(ag+bgx)^2(ci+dix)^2}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i^2(bc-ad)^3}$$

$$\downarrow 2772$$

$$\frac{-B\int\left(d^2-\frac{2b(c+dx)\log\left(\frac{a+bx}{c+dx}\right)d}{a+bx}-\frac{b^2(c+dx)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}+\frac{d^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{c+dx}-2b}{g^2i^2(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}+\frac{d^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{c+dx}-2bd\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-B\left(\frac{b^2(c+dx)}{a+bx}+\right.}{g^2i^2(bc-ad)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i*x)^2)
,x]
```

output
```
(-(B*((d^2*(a + b*x))/(c + d*x) + (b^2*(c + d*x))/(a + b*x) - b*d*Log[(a +
 b*x)/(c + d*x)]^2)) + (d^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)/(c + d*x) - (b^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*
x) - 2*b*d*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/
((b*c - a*d)^3*g^2*i^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.40

method	result
parts	$\frac{A\left(-\frac{d}{(da-bc)^2(dx+c)}-\frac{2db\ln(dx+c)}{(da-bc)^3}-\frac{b}{(da-bc)^2(bx+a)}+\frac{2db\ln(bx+a)}{(da-bc)^3}\right)}{g^2i^2}-B\left(\frac{d^2\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-\frac{(d}{(da-bc)^2}\right.\right.$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{d^2Ab^2}{i^2(da-bc)^4g^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}-\frac{2d^3Ab\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei^2(da-bc)^4g^2}+\frac{d^4A\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e^2i^2(da-bc)^4g^2}+\frac{d^2Bb^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-}{d(dx}\right.\right.}{i^2(da-}$
default	$-\frac{e(da-bc)\left(-\frac{d^2Ab^2}{i^2(da-bc)^4g^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}-\frac{2d^3Ab\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei^2(da-bc)^4g^2}+\frac{d^4A\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e^2i^2(da-bc)^4g^2}+\frac{d^2Bb^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-}{d(dx}\right.\right.}{i^2(da-}$
risch	$-\frac{Ad}{g^2i^2(da-bc)^2(dx+c)}-\frac{2Adb\ln(dx+c)}{g^2i^2(da-bc)^3}-\frac{Ab}{g^2i^2(da-bc)^2(bx+a)}+\frac{2Adb\ln(bx+a)}{g^2i^2(da-bc)^3}-\frac{Bd\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)b}{g^2i^2(da-bc)^3}$
norman	$\frac{\left(2Aabcd-Ba^2d^2+Bb^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}+\frac{\left(2Aabd^2+2Ab^2cd-2Babd^2+2Bb^2cd\right)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}+\frac{b^2Bd^2x^2\ln\left(\frac{e(bx+a)}{dx+c}\right.}{ig\left(a^3d^3-3a^2bcd^2+3ab^2c^2\right.}$
parallelrisch	$\frac{2Ax\ln\left(\frac{e(bx+a)}{dx+c}\right)a^4bc^3d^2+2Ax\ln\left(\frac{e(bx+a)}{dx+c}\right)a^3b^2c^4d-2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^4bc^3d^2+2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^3b^2c^4d+Axa^5c$

input

```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,method=_RETU
RNVERBOSE)
```

output
```
1/g^2*A/i^2*(-d/(a*d-b*c)^2/(d*x+c)-2*d/(a*d-b*c)^3*b*ln(d*x+c)-b/(a*d-b*c
)^2/(b*x+a)+2*d/(a*d-b*c)^3*b*ln(b*x+a))-B/g^2/i^2/(a*d-b*c)/e*(d^2/(a*d-b
*c)^2*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-
b*c)*e/d/(d*x+c)-b*e/d)-1/(a*d-b*c)^2*b*d*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^2+1/(a*d-b*c)^2*b^2*e^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-
b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.28

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx =$$
$$-\frac{(A+B)b^2c^2 - 2\,Babcd - (A-B)a^2d^2 + (Bb^2d^2x^2 + Babcd + (Bb^2cd + Babd^2)x)\log\left(\frac{bex+ae}{dx+c}\right)^2 + 2\,(}{(b^4c^3d - 3\,ab^3c^2d^2 + 3\,a^2b^2cd^3 - a^3bd^4)g^2i^2x^2 + (b^4c^4 - 2\,ab^3c^3$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, algo
rithm="fricas")
```

output
```
-((A + B)*b^2*c^2 - 2*B*a*b*c*d - (A - B)*a^2*d^2 + (B*b^2*d^2*x^2 + B*a*b
*c*d + (B*b^2*c*d + B*a*b*d^2)*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*b^
2*c*d - A*a*b*d^2)*x + (2*A*b^2*d^2*x^2 + B*b^2*c^2 + 2*A*a*b*c*d - B*a^2*
d^2 + 2*((A + B)*b^2*c*d + (A - B)*a*b*d^2)*x)*log((b*e*x + a*e)/(d*x + c)
))/((b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*g^2*i^2*x^
2 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*g^2*i^2*x + (a*b^3
*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*g^2*i^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. 2(235) = 470.

Time = 1.88 (sec) , antiderivative size = 828, normalized size of antiderivative = 3.17

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$$

$$= -\frac{2Abd\log\left(x + \frac{-\frac{2Aa^4bd^5}{(ad-bc)^3} + \frac{8Aa^3b^2cd^4}{(ad-bc)^3} - \frac{12Aa^2b^3c^2d^3}{(ad-bc)^3} + \frac{8Aab^4c^3d^2}{(ad-bc)^3} + 2Aabd^2 - \frac{2Ab^5c^4d}{(ad-bc)^3} + 2Ab^2cd}{4Ab^2d^2}\right)}{g^2i^2\left(ad-bc\right)^3}$$

$$+ \frac{2Abd\log\left(x + \frac{\frac{2Aa^4bd^5}{(ad-bc)^3} - \frac{8Aa^3b^2cd^4}{(ad-bc)^3} + \frac{12Aa^2b^3c^2d^3}{(ad-bc)^3} - \frac{8Aab^4c^3d^2}{(ad-bc)^3} + 2Aabd^2 + \frac{2Ab^5c^4d}{(ad-bc)^3} + 2Ab^2cd}{4Ab^2d^2}\right)}{g^2i^2\left(ad-bc\right)^3}$$

$$+ \frac{Bbd\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{a^3d^3g^2i^2 - 3a^2bcd^2g^2i^2 + 3ab^2c^2dg^2i^2 - b^3c^3g^2i^2}$$

$$+ \frac{(-Bad - Bbc - 2Bbdx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{a^3cd^2g^2i^2 + a^3d^3g^2i^2x - 2a^2bc^2dg^2i^2 - a^2bcd^2g^2i^2x + a^2bd^3g^2i^2x^2 + ab^2c^3g^2i^2 - ab^2c^2dg^2i^2x - 2ab^2c}$$

$$- \frac{Aad + Abc + 2Abdx - Bad + Bbc}{a^3cd^2g^2i^2 - 2a^2bc^2dg^2i^2 + ab^2c^3g^2i^2 + x^2\left(a^2bd^3g^2i^2 - 2ab^2cd^2g^2i^2 + b^3c^2dg^2i^2\right) + x\left(a^3d^3g^2i^2 - a^2bcd\right.}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2/(d*i*x+c*i)**2,x)
```

output

```
-2*A*b*d*log(x + (-2*A*a**4*b*d**5/(a*d - b*c)**3 + 8*A*a**3*b**2*c*d**4/(
a*d - b*c)**3 - 12*A*a**2*b**3*c**2*d**3/(a*d - b*c)**3 + 8*A*a*b**4*c**3*
d**2/(a*d - b*c)**3 + 2*A*a*b*d**2 - 2*A*b**5*c**4*d/(a*d - b*c)**3 + 2*A*
b**2*c*d)/(4*A*b**2*d**2))/(g**2*i**2*(a*d - b*c)**3) + 2*A*b*d*log(x + (2
*A*a**4*b*d**5/(a*d - b*c)**3 - 8*A*a**3*b**2*c*d**4/(a*d - b*c)**3 + 12*A
*a**2*b**3*c**2*d**3/(a*d - b*c)**3 - 8*A*a*b**4*c**3*d**2/(a*d - b*c)**3
+ 2*A*a*b*d**2 + 2*A*b**5*c**4*d/(a*d - b*c)**3 + 2*A*b**2*c*d)/(4*A*b**2*
d**2))/(g**2*i**2*(a*d - b*c)**3) + B*b*d*log(e*(a + b*x)/(c + d*x))**2/(a
**3*d**3*g**2*i**2 - 3*a**2*b*c*d**2*g**2*i**2 + 3*a*b**2*c**2*d*g**2*i**2
 - b**3*c**3*g**2*i**2) + (-B*a*d - B*b*c - 2*B*b*d*x)*log(e*(a + b*x)/(c
+ d*x))/(a**3*c*d**2*g**2*i**2 + a**3*d**3*g**2*i**2*x - 2*a**2*b*c**2*d*g
**2*i**2 - a**2*b*c*d**2*g**2*i**2*x + a**2*b*d**3*g**2*i**2*x**2 + a*b**2
*c**3*g**2*i**2 - a*b**2*c**2*d*g**2*i**2*x - 2*a*b**2*c*d**2*g**2*i**2*x*
*2 + b**3*c**3*g**2*i**2*x + b**3*c**2*d*g**2*i**2*x**2) - (A*a*d + A*b*c
+ 2*A*b*d*x - B*a*d + B*b*c)/(a**3*c*d**2*g**2*i**2 - 2*a**2*b*c**2*d*g**2
*i**2 + a*b**2*c**3*g**2*i**2 + x**2*(a**2*b*d**3*g**2*i**2 - 2*a*b**2*c*d
**2*g**2*i**2 + b**3*c**2*d*g**2*i**2) + x*(a**3*d**3*g**2*i**2 - a**2*b*c
*d**2*g**2*i**2 - a*b**2*c**2*d*g**2*i**2 + b**3*c**3*g**2*i**2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(261) = 522$.

Time $= 0.08$ (sec) , antiderivative size $= 859$, normalized size of antiderivative $= 3.29$

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^2(ci + dix)^2}\, dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, algo
rithm="maxima")
```

output

```
-B*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i^2
*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*g^2*i^2*x + (a*b^2*
c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d*log(b*x + a)/((b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2) - 2*b*d*log(d*x + c)/((
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2))*log(b*e*x/(d*
x + c) + a*e/(d*x + c)) - A*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c
*d^2 + a^2*b*d^3)*g^2*i^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3
*d^3)*g^2*i^2*x + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d
*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2
) - 2*b*d*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*g^2*i^2)) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*d^2*x^2 + a*b*c*d + (b
^2*c*d + a*b*d^2)*x)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d
+ a*b*d^2)*x)*log(b*x + a)*log(d*x + c) - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*
d + a*b*d^2)*x)*log(d*x + c)^2)*B/(a*b^3*c^4*g^2*i^2 - 3*a^2*b^2*c^3*d*g^2
*i^2 + 3*a^3*b*c^2*d^2*g^2*i^2 - a^4*c*d^3*g^2*i^2 + (b^4*c^3*d*g^2*i^2 -
3*a*b^3*c^2*d^2*g^2*i^2 + 3*a^2*b^2*c*d^3*g^2*i^2 - a^3*b*d^4*g^2*i^2)*x^2
 + (b^4*c^4*g^2*i^2 - 2*a*b^3*c^3*d*g^2*i^2 + 2*a^3*b*c*d^3*g^2*i^2 - a^4*
d^4*g^2*i^2)*x)
```

Giac [A] (verification not implemented)

Time = 42.64 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.51

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx =$$
$$-\left(\frac{bc}{(bce-ade)(bc-ad)} - \frac{ad}{(bce-ade)(bc-ad)}\right)^2\left(\frac{(dx+c)Be^2\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)g^2i^2} + \frac{(Ae^2+Be^2)(dx+c)}{(bex+ae)g^2i^2}\right.$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, algo
rithm="giac")
```

output

```
-(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
*((d*x + c)*B*e^2*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)*g^2*i^2) + (
A*e^2 + B*e^2)*(d*x + c)/((b*e*x + a*e)*g^2*i^2))
```

Mupad [B] (verification not implemented)

Time = 27.99 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.59

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = \frac{B\,b\,d\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2}{g^2\,i^2\,(a\,d-b\,c)^3} - \frac{A\,a\,d}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{A\,b\,c}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} + \frac{B\,a\,d}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{B\,b\,c}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{2\,A\,b\,d\,x}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{B\,a\,d\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{B\,b\,c\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{2\,B\,b\,d\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{A\,b\,d\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,4\mathrm{i}}{g^2\,i^2\,(a\,d-b\,c)^3}$$

input

```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^2*(c*i + d*i*x)^2)
,x)
```

output

```
(B*b*d*log((e*(a + b*x))/(c + d*x))^2)/(g^2*i^2*(a*d - b*c)^3) - (A*b*d*at
an((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*4i)/(g^2*i^2*(a*d - b*c)^3) -
 (A*a*d)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (A*b*c)/(g^2*i^2*(a
*d - b*c)^2*(a + b*x)*(c + d*x)) + (B*a*d)/(g^2*i^2*(a*d - b*c)^2*(a + b*x
)*(c + d*x)) - (B*b*c)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (2*A*
b*d*x)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (B*a*d*log((e*(a + b*
x))/(c + d*x)))/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (B*b*c*log((
e*(a + b*x))/(c + d*x)))/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (2*
B*b*d*x*log((e*(a + b*x))/(c + d*x)))/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c
+ d*x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 894, normalized size of antiderivative = 3.43

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x)
```

output

```
( - 2*log(a + b*x)*a**3*b*c*d**2 - 2*log(a + b*x)*a**3*b*d**3*x - 2*log(a
+ b*x)*a**2*b**2*c**2*d - 4*log(a + b*x)*a**2*b**2*c*d**2*x - 2*log(a + b*
x)*a**2*b**2*d**3*x**2 - 2*log(a + b*x)*a*b**3*c**2*d*x - 2*log(a + b*x)*a
*b**3*c*d**2*x**2 + 2*log(c + d*x)*a**3*b*c*d**2 + 2*log(c + d*x)*a**3*b*d
**3*x + 2*log(c + d*x)*a**2*b**2*c**2*d + 4*log(c + d*x)*a**2*b**2*c*d**2*
x + 2*log(c + d*x)*a**2*b**2*d**3*x**2 + 2*log(c + d*x)*a*b**3*c**2*d*x +
2*log(c + d*x)*a*b**3*c*d**2*x**2 - log((a*e + b*e*x)/(c + d*x))**2*a**2*b
**2*c*d**2 - log((a*e + b*e*x)/(c + d*x))**2*a**2*b**2*d**3*x - log((a*e +
 b*e*x)/(c + d*x))**2*a*b**3*c**2*d - 2*log((a*e + b*e*x)/(c + d*x))**2*a*
b**3*c*d**2*x - log((a*e + b*e*x)/(c + d*x))**2*a*b**3*d**3*x**2 - log((a*
e + b*e*x)/(c + d*x))**2*b**4*c**2*d*x - log((a*e + b*e*x)/(c + d*x))**2*b
**4*c*d**2*x**2 + log((a*e + b*e*x)/(c + d*x))*a**3*b*d**3 + log((a*e + b*
e*x)/(c + d*x))*a**2*b**2*c*d**2 + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b**
2*d**3*x - log((a*e + b*e*x)/(c + d*x))*a*b**3*c**2*d - log((a*e + b*e*x)/
(c + d*x))*b**4*c**3 - 2*log((a*e + b*e*x)/(c + d*x))*b**4*c**2*d*x + a**4
*d**3 - a**3*b*c*d**2 - a**3*b*d**3 + a**2*b**2*c**2*d + a**2*b**2*c*d**2
- 2*a**2*b**2*d**3*x**2 - a*b**3*c**3 + a*b**3*c**2*d + 2*a*b**3*c*d**2*x*
*2 - b**4*c**3)/(g**2*(a**5*c*d**4 + a**5*d**5*x - 2*a**4*b*c**2*d**3 - a*
*4*b*c*d**4*x + a**4*b*d**5*x**2 - 2*a**3*b**2*c**2*d**3*x - 2*a**3*b**2*c
*d**4*x**2 + 2*a**2*b**3*c**4*d + 2*a**2*b**3*c**3*d**2*x - a*b**4*c**5...
```

3.45 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx$

Optimal result 577
Mathematica [C] (verified) 578
Rubi [A] (verified) 579
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Reduce [B] (verification not implemented) 586

Optimal result

Integrand size = 40, antiderivative size = 346

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = {}& \frac{Bd^3(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} - \frac{bB(c+dx)^2\left(b-\frac{6d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^4g^3i^2(a+bx)^2} \\ & - \frac{3bBd^2\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2} - \frac{d^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(c+dx)} \\ & + \frac{3b^2d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} \\ & - \frac{b^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} \\ & + \frac{3bd^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2}\end{aligned}$$

output

```
B*d^3*(b*x+a)/(-a*d+b*c)^4/g^3/i^2/(d*x+c)-1/4*b*B*(d*x+c)^2*(b-6*d*(b*x+a
)/(d*x+c))^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2-3/2*b*B*d^2*ln((b*x+a)/(d*x+c)
)^2/(-a*d+b*c)^4/g^3/i^2-d^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c
)^4/g^3/i^2/(d*x+c)+3*b^2*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)
^4/g^3/i^2/(b*x+a)-1/2*b^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c
)^4/g^3/i^2/(b*x+a)^2+3*b*d^2*ln((b*x+a)/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c
)))/(-a*d+b*c)^4/g^3/i^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.65 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.31

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx$$

$$= \frac{-\frac{bB(bc-ad)^2}{(a+bx)^2} + \frac{8b^2Bcd}{a+bx} - \frac{8abBd^2}{a+bx} + \frac{2bBd(bc-ad)}{a+bx} - \frac{4bBcd^2}{c+dx} + \frac{4aBd^3}{c+dx} + 6bBd^2\log(a+bx) - \frac{2b(bc-ad)^2\left(A+B\log\left(\frac{e(a+}{c+}\right.\right.}{(a+bx)^2}$$

input

```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i
*x)^2),x]
```

output

```
(-((b*B*(b*c - a*d)^2)/(a + b*x)^2) + (8*b^2*B*c*d)/(a + b*x) - (8*a*b*B*d
^2)/(a + b*x) + (2*b*B*d*(b*c - a*d))/(a + b*x) - (4*b*B*c*d^2)/(c + d*x)
+ (4*a*B*d^3)/(c + d*x) + 6*b*B*d^2*Log[a + b*x] - (2*b*(b*c - a*d)^2*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)^2 + (8*b*d*(b*c - a*d)*(A + B*
Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) + (4*d^2*(b*c - a*d)*(A + B*Log[(
e*(a + b*x))/(c + d*x)]))/(c + d*x) + 12*b*d^2*Log[a + b*x]*(A + B*Log[(e*
(a + b*x))/(c + d*x)]) - 6*b*B*d^2*Log[c + d*x] - 12*b*d^2*(A + B*Log[(e*(
a + b*x))/(c + d*x)])*Log[c + d*x] - 6*b*B*d^2*(Log[a + b*x]*(Log[a + b*x]
 - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
+ a*d)]) + 6*b*B*d^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])
*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4*(b*c - a*d)^4
*g^3*i^2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2772, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^3(ci+dix)^2}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3i^2(bc-ad)^4}$$

$$\downarrow 2772$$

$$\frac{-B\int -\frac{(c+dx)^3\left(b^3-\frac{6d(a+bx)b^2}{c+dx}-\frac{6d^2(a+bx)^2\log\left(\frac{a+bx}{c+dx}\right)b}{(c+dx)^2}+\frac{2d^3(a+bx)^3}{(c+dx)^3}\right)}{2(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx)\left(B\log\right.}{a+b}}{g^3i^2(bc-ad)^4}$$

$$\downarrow 27$$

$$\frac{\frac{1}{2}B\int \frac{(c+dx)^3\left(b^3-\frac{6d(a+bx)b^2}{c+dx}-\frac{6d^2(a+bx)^2\log\left(\frac{a+bx}{c+dx}\right)b}{(c+dx)^2}+\frac{2d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx)\left(B\log\left(\right.\right.}{a+bx}}{g^3i^2(bc-ad)^4}$$

$$\downarrow 2010$$

$$\frac{\frac{1}{2}B\int \left(\frac{(c+dx)^3\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{2d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)^3}-\frac{6bd^2(c+dx)\log\left(\frac{a+bx}{c+dx}\right)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx)\left(\right.}{g^3i^2(bc-ad)^4}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^3(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{d^3(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{c+dx}+3bd^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\right.}{g^3i^2(bc-ad)^4}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i*x)^2)
,x]
```

output
```
((B*((2*d^3*(a + b*x))/(c + d*x) + (6*b^2*d*(c + d*x))/(a + b*x) - (b^3*(c
 + d*x)^2)/(2*(a + b*x)^2) - 3*b*d^2*Log[(a + b*x)/(c + d*x)]^2))/2 - (d^3
*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) + (3*b^2*d*(c +
 d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) - (b^3*(c + d*x)^2*(
A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^2) + 3*b*d^2*Log[(a + b*
x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^4*g^3*i^2
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.46

method	result
parts	$\frac{A\left(-\frac{d^2}{(da-bc)^3(dx+c)}-\frac{3d^2b\ln(dx+c)}{(da-bc)^4}-\frac{b}{2(da-bc)^2(bx+a)^2}+\frac{3d^2b\ln(bx+a)}{(da-bc)^4}-\frac{2bd}{(da-bc)^3(bx+a)}\right)}{g^3i^2}-B\left(\frac{d^3\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right.\right.\right.}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{d^2eAb^3}{2i^2(da-bc)^5g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{3d^3Ab^2}{i^2(da-bc)^5g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}-\frac{3d^4Ab\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei^2(da-bc)^5g^3}+\frac{d^5A\left(\frac{be}{d}+\frac{(da}{e^2i^2(da-bc}\right.\right.\right.}$
default	$-\frac{e(da-bc)\left(\frac{d^2eAb^3}{2i^2(da-bc)^5g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{3d^3Ab^2}{i^2(da-bc)^5g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}-\frac{3d^4Ab\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei^2(da-bc)^5g^3}+\frac{d^5A\left(\frac{be}{d}+\frac{(da}{e^2i^2(da-bc}\right.\right.\right.}$
risch	$-\frac{Ad^2}{g^3i^2(da-bc)^3(dx+c)}-\frac{3Ad^2b\ln(dx+c)}{g^3i^2(da-bc)^4}-\frac{Ab}{2g^3i^2(da-bc)^2(bx+a)^2}+\frac{3Ad^2b\ln(bx+a)}{g^3i^2(da-bc)^4}-\frac{2Abd}{g^3i^2(da-bc)^3(bx+}$
parallelrisch	$\frac{6Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b^7cd^5+24Ax^2\ln\left(\frac{e(bx+a)}{dx+c}\right)ab^6d^6+12Ax^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^7cd^5+18Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^7cd^5+6Bx}$
norman	Expression too large to display

input

```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,method=_RETU
RNVERBOSE)
```

output

```
A/g^3/i^2*(-d^2/(a*d-b*c)^3/(d*x+c)-3*d^2/(a*d-b*c)^4*b*ln(d*x+c)-1/2*b/(a
*d-b*c)^2/(b*x+a)^2+3*d^2/(a*d-b*c)^4*b*ln(b*x+a)-2*b/(a*d-b*c)^3*d/(b*x+a
))-B/g^3/i^2/(a*d-b*c)/e*(d^3/(a*d-b*c)^3*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)-3/2/(a*d-b*c)^
3*b*d^2*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+3/(a*d-b*c)^3*b^2*d*e^2*(-1/(b
*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-
b*c)*e/d/(d*x+c)))-1/(a*d-b*c)^3*b^3*e^3*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c
))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.92

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx =$$

$$-\frac{(2\,A+B)b^3c^3 - 12\,(A+B)ab^2c^2d + 3\,(2\,A+5\,B)a^2bcd^2 + 4\,(A-B)a^3d^3 - 6\,((2\,A+B)b^3cd^2 - (2\,A}{4\,((b^6c^4d - 4\,a}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, algo
rithm="fricas")
```

output

```
-1/4*((2*A + B)*b^3*c^3 - 12*(A + B)*a*b^2*c^2*d + 3*(2*A + 5*B)*a^2*b*c*d
^2 + 4*(A - B)*a^3*d^3 - 6*((2*A + B)*b^3*c*d^2 - (2*A + B)*a*b^2*d^3)*x^2
 - 6*(B*b^3*d^3*x^3 + B*a^2*b*c*d^2 + (B*b^3*c*d^2 + 2*B*a*b^2*d^3)*x^2 +
(2*B*a*b^2*c*d^2 + B*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 3*((2*
A + 3*B)*b^3*c^2*d + 2*(2*A - B)*a*b^2*c*d^2 - (6*A + B)*a^2*b*d^3)*x - 2*
(3*(2*A + B)*b^3*d^3*x^3 - B*b^3*c^3 + 6*B*a*b^2*c^2*d + 6*A*a^2*b*c*d^2 -
 2*B*a^3*d^3 + 3*((2*A + 3*B)*b^3*c*d^2 + 4*A*a*b^2*d^3)*x^2 + 3*(B*b^3*c^
2*d + 4*(A + B)*a*b^2*c*d^2 + 2*(A - B)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d
*x + c)))/((b^6*c^4*d - 4*a*b^5*c^3*d^2 + 6*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*
d^4 + a^4*b^2*d^5)*g^3*i^2*x^3 + (b^6*c^5 - 2*a*b^5*c^4*d - 2*a^2*b^4*c^3*
d^2 + 8*a^3*b^3*c^2*d^3 - 7*a^4*b^2*c*d^4 + 2*a^5*b*d^5)*g^3*i^2*x^2 + (2*
a*b^5*c^5 - 7*a^2*b^4*c^4*d + 8*a^3*b^3*c^3*d^2 - 2*a^4*b^2*c^2*d^3 - 2*a^
5*b*c*d^4 + a^6*d^5)*g^3*i^2*x + (a^2*b^4*c^5 - 4*a^3*b^3*c^4*d + 6*a^4*b^
2*c^3*d^2 - 4*a^5*b*c^2*d^3 + a^6*c*d^4)*g^3*i^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1562 vs. $2(316) = 632$.

Time = 12.13 (sec) , antiderivative size = 1562, normalized size of antiderivative = 4.51

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3/(d*i*x+c*i)**2,x)
```

output

```
3*B*b*d**2*log(e*(a + b*x)/(c + d*x))**2/(2*a**4*d**4*g**3*i**2 - 8*a**3*b
*c*d**3*g**3*i**2 + 12*a**2*b**2*c**2*d**2*g**3*i**2 - 8*a*b**3*c**3*d*g**
3*i**2 + 2*b**4*c**4*g**3*i**2) - 3*b*d**2*(2*A + B)*log(x + (6*A*a*b*d**3
 + 6*A*b**2*c*d**2 + 3*B*a*b*d**3 + 3*B*b**2*c*d**2 - 3*a**5*b*d**7*(2*A +
 B)/(a*d - b*c)**4 + 15*a**4*b**2*c*d**6*(2*A + B)/(a*d - b*c)**4 - 30*a**
3*b**3*c**2*d**5*(2*A + B)/(a*d - b*c)**4 + 30*a**2*b**4*c**3*d**4*(2*A +
B)/(a*d - b*c)**4 - 15*a*b**5*c**4*d**3*(2*A + B)/(a*d - b*c)**4 + 3*b**6*
c**5*d**2*(2*A + B)/(a*d - b*c)**4)/(12*A*b**2*d**3 + 6*B*b**2*d**3))/(2*g
**3*i**2*(a*d - b*c)**4) + 3*b*d**2*(2*A + B)*log(x + (6*A*a*b*d**3 + 6*A*
b**2*c*d**2 + 3*B*a*b*d**3 + 3*B*b**2*c*d**2 + 3*a**5*b*d**7*(2*A + B)/(a*
d - b*c)**4 - 15*a**4*b**2*c*d**6*(2*A + B)/(a*d - b*c)**4 + 30*a**3*b**3*
c**2*d**5*(2*A + B)/(a*d - b*c)**4 - 30*a**2*b**4*c**3*d**4*(2*A + B)/(a*d
 - b*c)**4 + 15*a*b**5*c**4*d**3*(2*A + B)/(a*d - b*c)**4 - 3*b**6*c**5*d*
*2*(2*A + B)/(a*d - b*c)**4)/(12*A*b**2*d**3 + 6*B*b**2*d**3))/(2*g**3*i**
2*(a*d - b*c)**4) + (-2*B*a**2*d**2 - 5*B*a*b*c*d - 9*B*a*b*d**2*x + B*b**
2*c**2 - 3*B*b**2*c*d*x - 6*B*b**2*d**2*x**2)*log(e*(a + b*x)/(c + d*x))/(
2*a**5*c*d**3*g**3*i**2 + 2*a**5*d**4*g**3*i**2*x - 6*a**4*b*c**2*d**2*g**
3*i**2 - 2*a**4*b*c*d**3*g**3*i**2*x + 4*a**4*b*d**4*g**3*i**2*x**2 + 6*a*
*3*b**2*c**3*d*g**3*i**2 - 6*a**3*b**2*c**2*d**2*g**3*i**2*x - 10*a**3*b**
2*c*d**3*g**3*i**2*x**2 + 2*a**3*b**2*d**4*g**3*i**2*x**3 - 2*a**2*b**3...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1721 vs. $2(340) = 680$.

Time $= 0.16$ (sec) , antiderivative size $= 1721$, normalized size of antiderivative $= 4.97$

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, algo
rithm="maxima")
```

output

```
1/2*B*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a
*b*d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*
g^3*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3
 - 2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c
^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d
 + 3*a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4
- 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) -
6*b*d^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)*g^3*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 1/2*A
*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*b*d^
2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*g^3*i
^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 - 2*
a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^
2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*
a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4 - 4*a
*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) - 6*b*d
^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*
d^3 + a^4*d^4)*g^3*i^2)) - 1/4*(b^3*c^3 - 12*a*b^2*c^2*d + 15*a^2*b*c*d^2
- 4*a^3*d^3 - 6*(b^3*c*d^2 - a*b^2*d^3)*x^2 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2
 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b...
```

Giac [A] (verification not implemented)

Time = 53.40 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.80

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx =$$
$$-\frac{1}{4}\left(\frac{2\left(Bbe^3 - \frac{2\,(bex+ae)Bde^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^2bcg^3i^2}{(dx+c)^2} - \frac{(bex+ae)^2adg^3i^2}{(dx+c)^2}} + \frac{2\,Abe^3 + Bbe^3 - \frac{4\,(bex+ae)Ade^2}{dx+c} - \frac{4\,(bex+ae)Bde^2}{dx+c}}{\frac{(bex+ae)^2bcg^3i^2}{(dx+c)^2} - \frac{(bex+ae)^2adg^3i^2}{(dx+c)^2}}\right)\left(\frac{}{(bce - d\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, algo
rithm="giac")
```

output
```
-1/4*(2*(B*b*e^3 - 2*(b*e*x + a*e)*B*d*e^2/(d*x + c))*log((b*e*x + a*e)/(d
*x + c))/((b*e*x + a*e)^2*b*c*g^3*i^2/(d*x + c)^2 - (b*e*x + a*e)^2*a*d*g^
3*i^2/(d*x + c)^2) + (2*A*b*e^3 + B*b*e^3 - 4*(b*e*x + a*e)*A*d*e^2/(d*x +
 c) - 4*(b*e*x + a*e)*B*d*e^2/(d*x + c))/((b*e*x + a*e)^2*b*c*g^3*i^2/(d*x
 + c)^2 - (b*e*x + a*e)^2*a*d*g^3*i^2/(d*x + c)^2))*(b*c/((b*c*e - a*d*e)*
(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [B] (verification not implemented)

Time = 31.11 (sec) , antiderivative size = 984, normalized size of antiderivative = 2.84

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^3*(c*i + d*i*x)^2)
,x)
```

output

```
(3*B*b*d^2*log((e*(a + b*x))/(c + d*x))^2)/(2*g^3*i^2*(a*d - b*c)^4) - (A*
b*d^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*6i)/(g^3*i^2*(a*d - b
*c)^4) - (B*b*d^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*3i)/(g^3*
i^2*(a*d - b*c)^4) - (A*a^2*d^2)/(g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d
*x)) + (A*b^2*c^2)/(2*g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) + (B*a^
2*d^2)/(g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) + (B*b^2*c^2)/(4*g^3*
i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (B*a*d*log((e*(a + b*x))/(c + d
*x)))/(g^3*i^2*(a*d - b*c)^2*(a + b*x)^2*(c + d*x)) - (B*b*c*log((e*(a + b
*x))/(c + d*x)))/(2*g^3*i^2*(a*d - b*c)^2*(a + b*x)^2*(c + d*x)) - (3*A*b^
2*d^2*x^2)/(g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (3*B*b^2*d^2*x^
2)/(2*g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (5*A*a*b*c*d)/(2*g^3*
i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (11*B*a*b*c*d)/(4*g^3*i^2*(a*d
- b*c)^3*(a + b*x)^2*(c + d*x)) - (3*B*b*d*x*log((e*(a + b*x))/(c + d*x)))
/(2*g^3*i^2*(a*d - b*c)^2*(a + b*x)^2*(c + d*x)) - (3*B*b^2*d^2*x^2*log((e
*(a + b*x))/(c + d*x)))/(g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (9
*A*a*b*d^2*x)/(2*g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (3*B*a*b*d
^2*x)/(4*g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (3*A*b^2*c*d*x)/(2
*g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (9*B*b^2*c*d*x)/(4*g^3*i^2
*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (3*B*a*b*c*d*log((e*(a + b*x))/(c
+ d*x)))/(g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (3*B*a*b*d^2*x...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1775, normalized size of antiderivative = 5.13

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x)
```

output

```
( - 24*log(a + b*x)*a**4*b*c*d**3 - 24*log(a + b*x)*a**4*b*d**4*x - 12*log
(a + b*x)*a**3*b**2*c**2*d**2 - 60*log(a + b*x)*a**3*b**2*c*d**3*x - 48*lo
g(a + b*x)*a**3*b**2*d**4*x**2 - 24*log(a + b*x)*a**2*b**3*c**2*d**2*x - 1
8*log(a + b*x)*a**2*b**3*c**2*d**2 - 48*log(a + b*x)*a**2*b**3*c*d**3*x**2
 - 18*log(a + b*x)*a**2*b**3*c*d**3*x - 24*log(a + b*x)*a**2*b**3*d**4*x**
3 - 12*log(a + b*x)*a*b**4*c**2*d**2*x**2 - 36*log(a + b*x)*a*b**4*c**2*d*
*2*x - 12*log(a + b*x)*a*b**4*c*d**3*x**3 - 36*log(a + b*x)*a*b**4*c*d**3*
x**2 - 18*log(a + b*x)*b**5*c**2*d**2*x**2 - 18*log(a + b*x)*b**5*c*d**3*x
**3 + 24*log(c + d*x)*a**4*b*c*d**3 + 24*log(c + d*x)*a**4*b*d**4*x + 12*l
og(c + d*x)*a**3*b**2*c**2*d**2 + 60*log(c + d*x)*a**3*b**2*c*d**3*x + 48*
log(c + d*x)*a**3*b**2*d**4*x**2 + 24*log(c + d*x)*a**2*b**3*c**2*d**2*x +
 18*log(c + d*x)*a**2*b**3*c**2*d**2 + 48*log(c + d*x)*a**2*b**3*c*d**3*x*
*2 + 18*log(c + d*x)*a**2*b**3*c*d**3*x + 24*log(c + d*x)*a**2*b**3*d**4*x
**3 + 12*log(c + d*x)*a*b**4*c**2*d**2*x**2 + 36*log(c + d*x)*a*b**4*c**2*
d**2*x + 12*log(c + d*x)*a*b**4*c*d**3*x**3 + 36*log(c + d*x)*a*b**4*c*d**
3*x**2 + 18*log(c + d*x)*b**5*c**2*d**2*x**2 + 18*log(c + d*x)*b**5*c*d**3
*x**3 - 12*log((a*e + b*e*x)/(c + d*x))**2*a**3*b**2*c*d**3 - 12*log((a*e
+ b*e*x)/(c + d*x))**2*a**3*b**2*d**4*x - 6*log((a*e + b*e*x)/(c + d*x))**
2*a**2*b**3*c**2*d**2 - 30*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**3*c*d**
3*x - 24*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**3*d**4*x**2 - 12*log((...
```

3.46 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx$

Optimal result . . . 588
Mathematica [C] (verified) . . . 589
Rubi [A] (verified) . . . 590
Maple [A] (verified) . . . 592
Fricas [B] (verification not implemented) . . . 593
Sympy [F(-1)] . . . 594
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Giac [A] (verification not implemented) . . . 596
Mupad [B] (verification not implemented) . . . 596
Reduce [B] (verification not implemented) . . . 597

Optimal result

Integrand size = 40, antiderivative size = 457

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx &= -\frac{Bd^4(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2Bd^2(c+dx)}{(bc-ad)^5g^4i^2(a+bx)} \\ &+ \frac{b^3Bd(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4B(c+dx)^3}{9(bc-ad)^5g^4i^2(a+bx)^3} \\ &+ \frac{2bBd^3\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2} + \frac{d^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(c+dx)} \\ &- \frac{6b^2d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)} \\ &+ \frac{2b^3d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} \\ &- \frac{b^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^5g^4i^2(a+bx)^3} \\ &- \frac{4bd^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2}\end{aligned}$$

output

```
-B*d^4*(b*x+a)/(-a*d+b*c)^5/g^4/i^2/(d*x+c)-6*b^2*B*d^2*(d*x+c)/(-a*d+b*c)
^5/g^4/i^2/(b*x+a)+b^3*B*d*(d*x+c)^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-1/9*b^
4*B*(d*x+c)^3/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3+2*b*B*d^3*ln((b*x+a)/(d*x+c))
^2/(-a*d+b*c)^5/g^4/i^2+d^4*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)
^5/g^4/i^2/(d*x+c)-6*b^2*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c
)^5/g^4/i^2/(b*x+a)+2*b^3*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*
c)^5/g^4/i^2/(b*x+a)^2-1/3*b^4*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d
+b*c)^5/g^4/i^2/(b*x+a)^3-4*b*d^3*ln((b*x+a)/(d*x+c))*(A+B*ln(e*(b*x+a)/(d
*x+c)))/(-a*d+b*c)^5/g^4/i^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.23 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.14

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx =$$

$$-\frac{\frac{bB(bc-ad)^3}{(a+bx)^3} - \frac{6bBd(bc-ad)^2}{(a+bx)^2} + \frac{27b^2Bcd^2}{a+bx} - \frac{27abBd^3}{a+bx} + \frac{12bBd^2(bc-ad)}{a+bx} - \frac{9bBcd^3}{c+dx} + \frac{9aBd^4}{c+dx} + 30bBd^3\log(a+bx) +$$

input

```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i
*x)^2),x]
```

output

```
-1/9*((b*B*(b*c - a*d)^3)/(a + b*x)^3 - (6*b*B*d*(b*c - a*d)^2)/(a + b*x)^
2 + (27*b^2*B*c*d^2)/(a + b*x) - (27*a*b*B*d^3)/(a + b*x) + (12*b*B*d^2*(b
*c - a*d))/(a + b*x) - (9*b*B*c*d^3)/(c + d*x) + (9*a*B*d^4)/(c + d*x) + 3
0*b*B*d^3*Log[a + b*x] + (3*b*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c +
d*x)]))/(a + b*x)^3 - (9*b*d*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d
*x)]))/(a + b*x)^2 + (27*b*d^2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d
*x)]))/(a + b*x) - (9*d^3*(-(b*c) + a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(c + d*x) + 36*b*d^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]
) - 30*b*B*d^3*Log[c + d*x] - 36*b*d^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)*Log[c + d*x] - 18*b*B*d^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*
x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 18*b*B*d
^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*
PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^5*g^4*i^2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.68, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2962, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{(ag+bgx)^4(ci+dix)^2}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^4} d\frac{a+bx}{c+dx}}{g^4 i^2 (bc-ad)^5}$$

$$\downarrow 2772$$

$$\frac{-B\int \left(d^4 - \frac{4b(c+dx)\log\left(\frac{a+bx}{c+dx}\right)d^3}{a+bx} - \frac{6b^2(c+dx)^2d^2}{(a+bx)^2} + \frac{2b^3(c+dx)^3 d}{(a+bx)^3} - \frac{b^4(c+dx)^4}{3(a+bx)^4}\right) d\frac{a+bx}{c+dx} - \frac{b^4(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3} + \frac{2}{}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^4(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3} + \frac{2b^3d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(a+bx)^2} - \frac{6b^2d^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx} + \frac{d^4(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{c+dx}}{g^4}$$

input

```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i*x)^2)
,x]
```

output
```
(-(B*((d^4*(a + b*x))/(c + d*x) + (6*b^2*d^2*(c + d*x))/(a + b*x) - (b^3*d
*(c + d*x)^2)/(a + b*x)^2 + (b^4*(c + d*x)^3)/(9*(a + b*x)^3) - 2*b*d^3*Lo
g[(a + b*x)/(c + d*x)]^2)) + (d^4*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c +
d*x)]))/(c + d*x) - (6*b^2*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(a + b*x) + (2*b^3*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)/(a + b*x)^2 - (b^4*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*
(a + b*x)^3) - 4*b*d^3*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(
c + d*x)]))/((b*c - a*d)^5*g^4*i^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.40

method	result
parts	$\frac{A\left(-\frac{d^3}{(da-bc)^4(dx+c)}-\frac{4d^3b\ln(dx+c)}{(da-bc)^5}-\frac{b}{3(da-bc)^2(bx+a)^3}+\frac{4d^3b\ln(bx+a)}{(da-bc)^5}-\frac{3bd^2}{(da-bc)^4(bx+a)}-\frac{bd}{(da-bc)^3(bx+a)^2}\right)}{g^4i^2}-B\left($
derivativedivides	$-\frac{e(da-bc)\left(-\frac{d^2e^2Ab^4}{3i^2(da-bc)^6g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}+\frac{2d^3eAb^3}{i^2(da-bc)^6g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{6d^4Ab^2}{i^2(da-bc)^6g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}-\frac{4d^5A}{}\right.}{}$
default	$-\frac{e(da-bc)\left(-\frac{d^2e^2Ab^4}{3i^2(da-bc)^6g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}+\frac{2d^3eAb^3}{i^2(da-bc)^6g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{6d^4Ab^2}{i^2(da-bc)^6g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}-\frac{4d^5A}{}\right.}{}$
risch	$-\frac{Ad^3}{g^4i^2(da-bc)^4(dx+c)}-\frac{4Ad^3b\ln(dx+c)}{g^4i^2(da-bc)^5}-\frac{Ab}{3g^4i^2(da-bc)^2(bx+a)^3}+\frac{4Ad^3b\ln(bx+a)}{g^4i^2(da-bc)^5}-\frac{3Abd^2}{g^4i^2(da-bc)^4(bx+}$
norman	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,method=_RETU
RNVERBOSE)
```

output

```
A/g^4/i^2*(-d^3/(a*d-b*c)^4/(d*x+c)-4*d^3/(a*d-b*c)^5*b*ln(d*x+c)-1/3*b/(a
*d-b*c)^2/(b*x+a)^3+4*d^3/(a*d-b*c)^5*b*ln(b*x+a)-3*b/(a*d-b*c)^4*d^2/(b*x
+a)-b/(a*d-b*c)^3*d/(b*x+a)^2)-B/g^4/i^2/(a*d-b*c)/e*(d^4/(a*d-b*c)^4*((b*
e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(
d*x+c)-b*e/d)-2/(a*d-b*c)^4*b*d^3*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+6/(a
*d-b*c)^4*b^2*d^2*e^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)
*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-4/(a*d-b*c)^4*b^3*d*e^3*(-1
/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*
e/d+(a*d-b*c)*e/d/(d*x+c))^2)+1/(a*d-b*c)^4*b^4*e^4*(-1/3/(b*e/d+(a*d-b*c)
*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. $2(453) = 906$.

Time $= 0.10$ (sec) , antiderivative size $= 1019$, normalized size of antiderivative $= 2.23$

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^4 (ci + dix)^2}\, dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, algo
rithm="fricas")
```

output

```
-1/9*((3*A + B)*b^4*c^4 - 9*(2*A + B)*a*b^3*c^3*d + 54*(A + B)*a^2*b^2*c^2
*d^2 - 5*(6*A + 11*B)*a^3*b*c*d^3 - 9*(A - B)*a^4*d^4 + 6*((6*A + 5*B)*b^4
*c*d^3 - (6*A + 5*B)*a*b^3*d^4)*x^3 + 3*((6*A + 11*B)*b^4*c^2*d^2 + 8*(3*A
 + B)*a*b^3*c*d^3 - (30*A + 19*B)*a^2*b^2*d^4)*x^2 + 18*(B*b^4*d^4*x^4 + B
*a^3*b*c*d^3 + (B*b^4*c*d^3 + 3*B*a*b^3*d^4)*x^3 + 3*(B*a*b^3*c*d^3 + B*a^
2*b^2*d^4)*x^2 + (3*B*a^2*b^2*c*d^3 + B*a^3*b*d^4)*x)*log((b*e*x + a*e)/(d
*x + c))^2 - ((6*A + 5*B)*b^4*c^3*d - 27*(2*A + 3*B)*a*b^3*c^2*d^2 - 3*(6*
A - 19*B)*a^2*b^2*c*d^3 + (66*A + 19*B)*a^3*b*d^4)*x + 3*(2*(6*A + 5*B)*b^
4*d^4*x^4 + B*b^4*c^4 - 6*B*a*b^3*c^3*d + 18*B*a^2*b^2*c^2*d^2 + 12*A*a^3*
b*c*d^3 - 3*B*a^4*d^4 + 2*((6*A + 11*B)*b^4*c*d^3 + 9*(2*A + B)*a*b^3*d^4)
*x^3 + 6*(B*b^4*c^2*d^2 + 3*(2*A + 3*B)*a*b^3*c*d^3 + 6*A*a^2*b^2*d^4)*x^2
 - 2*(B*b^4*c^3*d - 9*B*a*b^3*c^2*d^2 - 18*(A + B)*a^2*b^2*c*d^3 - 6*(A -
B)*a^3*b*d^4)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^8*c^5*d - 5*a*b^7*c^4*d
^2 + 10*a^2*b^6*c^3*d^3 - 10*a^3*b^5*c^2*d^4 + 5*a^4*b^4*c*d^5 - a^5*b^3*d
^6)*g^4*i^2*x^4 + (b^8*c^6 - 2*a*b^7*c^5*d - 5*a^2*b^6*c^4*d^2 + 20*a^3*b^
5*c^3*d^3 - 25*a^4*b^4*c^2*d^4 + 14*a^5*b^3*c*d^5 - 3*a^6*b^2*d^6)*g^4*i^2
*x^3 + 3*(a*b^7*c^6 - 4*a^2*b^6*c^5*d + 5*a^3*b^5*c^4*d^2 - 5*a^5*b^3*c^2*
d^4 + 4*a^6*b^2*c*d^5 - a^7*b*d^6)*g^4*i^2*x^2 + (3*a^2*b^6*c^6 - 14*a^3*b
^5*c^5*d + 25*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 5*a^6*b^2*c^2*d^4 + 2
*a^7*b*c*d^5 - a^8*d^6)*g^4*i^2*x + (a^3*b^5*c^6 - 5*a^4*b^4*c^5*d + 10...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Timed out}$$

input

```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4/(d*i*x+c*i)**2,x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2560 vs. $2(453) = 906$.

Time $= 0.27$ (sec) , antiderivative size $= 2560$, normalized size of antiderivative $= 5.60$

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, algo
rithm="maxima")
```

output
```
-1/3*B*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 13*a^2*b*c*d^2 + 3*a^3
*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 8*a*b^2*c*d^2 - 11
*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b
^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c
^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*x^
3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d^3
 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^4*
c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*g^
4*i^2*x + (a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2
*d^3 + a^7*c*d^4)*g^4*i^2) + 12*b*d^3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4
*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^
4*i^2) - 12*b*d^3*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*
d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2))*log(b*e*x/(d
*x + c) + a*e/(d*x + c)) - 1/3*A*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*
d + 13*a^2*b*c*d^2 + 3*a^3*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*
c^2*d - 8*a*b^2*c*d^2 - 11*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6
*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 -
 a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 +
 3*a^5*b^2*d^5)*g^4*i^2*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c
^3*d^2 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 +...
```

Giac [A] (verification not implemented)

Time = 64.85 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.95

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx =$$
$$-\frac{1}{18}\left(\frac{6\left(Bb^2e^4-\frac{3\,(bex+ae)Bbde^3}{dx+c}+\frac{3\,(bex+ae)^2Bd^2e^2}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^3b^2c^2g^4i^2}{(dx+c)^3}-\frac{2\,(bex+ae)^3abcdg^4i^2}{(dx+c)^3}+\frac{(bex+ae)^3a^2d^2g^4i^2}{(dx+c)^3}}+\frac{6\,Ab^2e^4+2\,Bb^2e^4-\frac{18\,(bex+ae)Abde^3}{dx+c}-}{\frac{(bex+ae)^3b^2c^2g^4i^2}{(dx+c)^3}-\frac{2(}{}}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, algo
rithm="giac")
```

output
```
-1/18*(6*(B*b^2*e^4 - 3*(b*e*x + a*e)*B*b*d*e^3/(d*x + c) + 3*(b*e*x + a*e
)^2*B*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^3*b
^2*c^2*g^4*i^2/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4*i^2/(d*x + c)^3
 + (b*e*x + a*e)^3*a^2*d^2*g^4*i^2/(d*x + c)^3) + (6*A*b^2*e^4 + 2*B*b^2*e
^4 - 18*(b*e*x + a*e)*A*b*d*e^3/(d*x + c) - 9*(b*e*x + a*e)*B*b*d*e^3/(d*x
 + c) + 18*(b*e*x + a*e)^2*A*d^2*e^2/(d*x + c)^2 + 18*(b*e*x + a*e)^2*B*d^
2*e^2/(d*x + c)^2)/((b*e*x + a*e)^3*b^2*c^2*g^4*i^2/(d*x + c)^3 - 2*(b*e*x
 + a*e)^3*a*b*c*d*g^4*i^2/(d*x + c)^3 + (b*e*x + a*e)^3*a^2*d^2*g^4*i^2/(d
*x + c)^3))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c
 - a*d)))^2
```

Mupad [B] (verification not implemented)

Time = 34.90 (sec) , antiderivative size = 1679, normalized size of antiderivative = 3.67

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^4*(c*i + d*i*x)^2)
,x)
```

output

```
(2*B*b*d^3*log((e*(a + b*x))/(c + d*x))^2)/(g^4*i^2*(a*d - b*c)^2*(a^3*d^3
 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (log((e*(a + b*x))/(c + d*x
))*(x*((4*B)/(3*g^4*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (4*B*b*d^3*(((2
*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2))*(a*
d + b*c) + (a*c*(a*d - b*c))/d^2))/(g^4*i^2*(a*d - b*c)^2*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) + (B*(3*a*d + b*c))/(3*g^4*i^2*(a^2*
b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (4*B*b^2*d^2*x^3)/(g^4*i^2*(a*d - b*
c)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (4*B*b*d^3*x^2*(
b*d*((2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^
2)) + ((a*d + b*c)*(a*d - b*c))/d^2))/(g^4*i^2*(a*d - b*c)^2*(a^3*d^3 - b^
3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (4*B*a*b*c*d^3*((2*a^2*d^2 + b^2
*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(g^4*i^2*(a*d -
b*c)^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))))/(b^2*x^4 + (
a^3*c)/(b*d) + (x*(a^3*d + 3*a^2*b*c))/(b*d) + (x^3*(b^3*c + 3*a*b^2*d))/(
b*d) + (x^2*(3*a*b^2*c + 3*a^2*b*d))/(b*d)) - (b*d^3*atan((b*d^3*((a^5*d^5
*g^4*i^2 + b^5*c^5*g^4*i^2 - 3*a*b^4*c^4*d*g^4*i^2 - 3*a^4*b*c*d^4*g^4*i^2
 + 2*a^2*b^3*c^3*d^2*g^4*i^2 + 2*a^3*b^2*c^2*d^3*g^4*i^2)/(a^4*d^4*g^4*i^2
 + b^4*c^4*g^4*i^2 - 4*a*b^3*c^3*d*g^4*i^2 - 4*a^3*b*c*d^3*g^4*i^2 + 6*a^2
*b^2*c^2*d^2*g^4*i^2) + 2*b*d*x)*(6*A + 5*B)*(a^4*d^4*g^4*i^2 + b^4*c^4*g^
4*i^2 - 4*a*b^3*c^3*d*g^4*i^2 - 4*a^3*b*c*d^3*g^4*i^2 + 6*a^2*b^2*c^2*d...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2879, normalized size of antiderivative = 6.30

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x)
```

output

```
( - 108*log(a + b*x)*a**5*b*c*d**4 - 108*log(a + b*x)*a**5*b*d**5*x - 36*l
og(a + b*x)*a**4*b**2*c**2*d**3 - 360*log(a + b*x)*a**4*b**2*c*d**4*x - 54
*log(a + b*x)*a**4*b**2*c*d**4 - 324*log(a + b*x)*a**4*b**2*d**5*x**2 - 54
*log(a + b*x)*a**4*b**2*d**5*x - 108*log(a + b*x)*a**3*b**3*c**2*d**3*x -
66*log(a + b*x)*a**3*b**3*c**2*d**3 - 432*log(a + b*x)*a**3*b**3*c*d**4*x*
*2 - 228*log(a + b*x)*a**3*b**3*c*d**4*x - 324*log(a + b*x)*a**3*b**3*d**5
*x**3 - 162*log(a + b*x)*a**3*b**3*d**5*x**2 - 108*log(a + b*x)*a**2*b**4*
c**2*d**3*x**2 - 198*log(a + b*x)*a**2*b**4*c**2*d**3*x - 216*log(a + b*x)
*a**2*b**4*c*d**4*x**3 - 360*log(a + b*x)*a**2*b**4*c*d**4*x**2 - 108*log(
a + b*x)*a**2*b**4*d**5*x**4 - 162*log(a + b*x)*a**2*b**4*d**5*x**3 - 36*l
og(a + b*x)*a*b**5*c**2*d**3*x**3 - 198*log(a + b*x)*a*b**5*c**2*d**3*x**2
 - 36*log(a + b*x)*a*b**5*c*d**4*x**4 - 252*log(a + b*x)*a*b**5*c*d**4*x**
3 - 54*log(a + b*x)*a*b**5*d**5*x**4 - 66*log(a + b*x)*b**6*c**2*d**3*x**3
 - 66*log(a + b*x)*b**6*c*d**4*x**4 + 108*log(c + d*x)*a**5*b*c*d**4 + 108
*log(c + d*x)*a**5*b*d**5*x + 36*log(c + d*x)*a**4*b**2*c**2*d**3 + 360*lo
g(c + d*x)*a**4*b**2*c*d**4*x + 54*log(c + d*x)*a**4*b**2*c*d**4 + 324*log
(c + d*x)*a**4*b**2*d**5*x**2 + 54*log(c + d*x)*a**4*b**2*d**5*x + 108*log
(c + d*x)*a**3*b**3*c**2*d**3*x + 66*log(c + d*x)*a**3*b**3*c**2*d**3 + 43
2*log(c + d*x)*a**3*b**3*c*d**4*x**2 + 228*log(c + d*x)*a**3*b**3*c*d**4*x
 + 324*log(c + d*x)*a**3*b**3*d**5*x**3 + 162*log(c + d*x)*a**3*b**3*d*...
```

3.47 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$

Optimal result . . . 599
Mathematica [A] (verified) . . . 600
Rubi [A] (verified) . . . 601
Maple [A] (verified) . . . 603
Fricas [F] . . . 605
Sympy [F] . . . 606
Maxima [B] (verification not implemented) . . . 606
Giac [F] . . . 607
Mupad [F(-1)] . . . 608
Reduce [F] . . . 608

Optimal result

Integrand size = 40, antiderivative size = 361

$$
\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx \\
&= -\frac{3B(bc-ad)g^3(a+bx)^2}{4d^2i^3(c+dx)^2} - \frac{3bB(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)} \\
&\quad + \frac{b(3A+B)(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)} + \frac{3bB(bc-ad)g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^3(c+dx)} \\
&\quad + \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di^3(c+dx)^2} \\
&\quad + \frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^3(c+dx)^2} \\
&\quad + \frac{b^2(bc-ad)g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i^3} \\
&\quad + \frac{3b^2B(bc-ad)g^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}
\end{aligned}
$$

output

```
-3/4*B*(-a*d+b*c)*g^3*(b*x+a)^2/d^2/i^3/(d*x+c)^2-3*b*B*(-a*d+b*c)*g^3*(b*
x+a)/d^3/i^3/(d*x+c)+b*(3*A+B)*(-a*d+b*c)*g^3*(b*x+a)/d^3/i^3/(d*x+c)+3*b*
B*(-a*d+b*c)*g^3*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d^3/i^3/(d*x+c)+g^3*(b*x+a)
^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i^3/(d*x+c)^2+1/2*(-a*d+b*c)*g^3*(b*x+a)^
2*(3*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/d^2/i^3/(d*x+c)^2+b^2*(-a*d+b*c)*g^3*l
n((-a*d+b*c)/b/(d*x+c))*(3*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/d^4/i^3+3*b^2*B*
(-a*d+b*c)*g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^3
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.88

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{g^3\left(4Ab^3dx - \frac{B(bc-ad)^3}{(c+dx)^2} + \frac{10bB(bc-ad)^2}{c+dx} + 10b^2B(bc-ad)\log(a+bx) + 4b^2Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right) + \frac{2(b}{}\right.}{}$$

input

```
Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*
i*x)^3,x]
```

output

```
(g^3*(4*A*b^3*d*x - (B*(b*c - a*d)^3)/(c + d*x)^2 + (10*b*B*(b*c - a*d)^2)
/(c + d*x) + 10*b^2*B*(b*c - a*d)*Log[a + b*x] + 4*b^2*B*d*(a + b*x)*Log[(
e*(a + b*x))/(c + d*x)] + (2*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d
*x)]))/(c + d*x)^2 - (12*b*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(c + d*x) - 14*b^2*B*(b*c - a*d)*Log[c + d*x] - 12*b^2*(b*c - a*d)*(A
 + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 6*b^2*B*(b*c - a*d)*((2*
Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog
[2, (b*(c + d*x))/(b*c - a*d)])))/(4*d^4*i^3)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2962, 2784, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ci+dix)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{g^3(bc-ad)\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i^3}$$

$$\downarrow 2784$$

$$g^3(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int \frac{(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{d}\right)$$
$$i^3$$

$$\downarrow 2793$$

$$g^3(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\left(-\frac{\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)}-\frac{\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{d^2}-\frac{(a+bx)\left(3A+B+3B\log\left(\frac{e}{}\right.\right.}{d(c+dx)}\right.}{d}$$
$$i^3$$

$$\downarrow 2009$$

$$g^3(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{b^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{d^3}-\frac{b(3A+B)(a+bx)}{d^2(c+dx)}-\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right.\right.}{2d(c+dx)^2}}{d}\right)$$
$$i^3$$

input
```
Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^3
,x]
```

output
```
((b*c - a*d)*g^3*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c
 + d*x)^3*(b - (d*(a + b*x))/(c + d*x))) - ((3*B*(a + b*x)^2)/(4*d*(c + d*
x)^2) + (3*b*B*(a + b*x))/(d^2*(c + d*x)) - (b*(3*A + B)*(a + b*x))/(d^2*(
c + d*x)) - (3*b*B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d^2*(c + d*x))
 - ((a + b*x)^2*(3*A + B + 3*B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d*(c + d*
x)^2) - (b^2*(3*A + B + 3*B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a +
b*x))/(b*(c + d*x))])/d^3 - (3*b^2*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x)
)])/d^3)/d))/i^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 3.20 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.89

method	result
derivativedivides	$e(da-bc)\left(\frac{g^3d^2A\left(\frac{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)be+\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2d}{2}}{d^3}+\frac{b^3e^3}{d^4\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}+\frac{3b^2e^2\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right)}{d^4}\right.\right.$
default	$e(da-bc)\left(\frac{g^3d^2A\left(\frac{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)be+\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2d}{2}}{d^3}+\frac{b^3e^3}{d^4\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}+\frac{3b^2e^2\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right)}{d^4}\right.\right.$
parts	$\frac{g^3A\left(\frac{xb^3}{d^3}-\frac{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{2d^4(dx+c)^2}+\frac{3b^2(da-bc)\ln(dx+c)}{d^4}-\frac{3b\left(a^2d^2-2acdb+c^2b^2\right)}{d^4(dx+c)}\right)}{i^3}-g^3Bd\left(\frac{\left(\left(\left(\frac{be}{d}+\frac{(da-bc)}{d(dx+c)}\right.\right.\right.\right.$
risch	Expression too large to display

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x,method=_RETU
RNVERBOSE)
```

output
```
-1/d^2*e*(a*d-b*c)*(g^3*d^2/e^3/i^3*A*(1/d^3*(2*(b*e/d+(a*d-b*c)*e/d/(d*x+
c))*b*e+1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*d)+b^3*e^3/d^4/(b*e-(b*e/d+(a*
d-b*c)*e/d/(d*x+c))*d)+3/d^4*b^2*e^2*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*
d))+g^3*d^2/e^3/i^3*B*((1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-
b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)/d^2+2*((b*e/d+(a*d-
b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*
e/d)*b*e/d^3+(1/b/e/d*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)+ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/(b*e-(b*e/d+(a*d-b*c
)*e/d/(d*x+c))*d))*b^3*e^3/d^3+3*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-
b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x
+c))*d-b*e)/b/e)/d)/d^3*b^2*e^2))
```

Fricas [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algo
rithm="fricas")
```

output
```
integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
+ (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log((b
*e*x + a*e)/(d*x + c)))/(d^3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c
^3*i^3), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$

$$= \frac{g^3\left(\int \frac{Aa^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{Ab^3x^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{Ba^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{3Aab^2x^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right.}{}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**3,x)
```

output
```
g**3*(Integral(A*a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x)
+ Integral(A*b**3*x**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x)
 + Integral(B*a**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x
 + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(3*A*a*b**2*x**2/(c**3 + 3*c**
2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(3*A*a**2*b*x/(c**3 + 3*c
**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(B*b**3*x**3*log(a*e/(c
 + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)
, x) + Integral(3*B*a*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3
 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(3*B*a**2*b*x*log
(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3), x))/i**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2037 vs. $2(356) = 712$.

Time $= 0.16$ (sec) , antiderivative size $= 2037$, normalized size of antiderivative $= 5.64$

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algo
rithm="maxima")
```

output

```
-3/4*B*a^2*b*g^3*(2*(2*d*x + c)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^4*
i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a
*d^2)*x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c
^3*d^2 - a*c^2*d^3)*i^3) - 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2
- 2*a*b*c*d^3 + a^2*d^4)*i^3) + 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2
*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3)) - 1/2*A*b^3*g^3*((6*c^2*d*x + 5*c^3)/(
d^6*i^3*x^2 + 2*c*d^5*i^3*x + c^2*d^4*i^3) - 2*x/(d^3*i^3) + 6*c*log(d*x +
 c)/(d^4*i^3)) + 1/4*B*a^3*g^3*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)
*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) - 2*
log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*
i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^
2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) + 3/2*A*a*b^2*g^
3*((4*c*d*x + 3*c^2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) + 2*log(d
*x + c)/(d^3*i^3)) - 3/2*(2*d*x + c)*A*a^2*b*g^3/(d^4*i^3*x^2 + 2*c*d^3*i^
3*x + c^2*d^2*i^3) - 1/2*A*a^3*g^3/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^
3) + 1/2*(6*a^3*b^2*d^3*g^3*log(e) - (6*g^3*log(e) + 7*g^3)*b^5*c^3 + (18*
g^3*log(e) + 19*g^3)*a*b^4*c^2*d - 2*(9*g^3*log(e) + 7*g^3)*a^2*b^3*c*d^2)
*B*log(d*x + c)/(b^2*c^2*d^4*i^3 - 2*a*b*c*d^5*i^3 + a^2*d^6*i^3) + 1/4*(4
*(b^5*c^2*d^3*g^3*log(e) - 2*a*b^4*c*d^4*g^3*log(e) + a^2*b^3*d^5*g^3*log(
e))*B*x^3 + 8*(b^5*c^3*d^2*g^3*log(e) - 2*a*b^4*c^2*d^3*g^3*log(e) + a^...
```

Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^3}\,dx$$

input

```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algo
rithm="giac")
```

output

```
integrate((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)/(d*i*x + c*i)
^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(c\,i+d\,i\,x)^3}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^3
,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^3
, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x)
```

output

```
(g**3*i*(4*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(c**3 + 3*c**2*d*x + 3*
c*d**2*x**2 + d**3*x**3),x)*a**2*b**4*c**4*d**6 + 8*int((log((a*e + b*e*x)
/(c + d*x))*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*
b**4*c**3*d**7*x + 4*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(c**3 + 3*c**
2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*b**4*c**2*d**8*x**2 - 8*int((lo
g((a*e + b*e*x)/(c + d*x))*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3),x)*a*b**5*c**5*d**5 - 16*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(c
**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**5*c**4*d**6*x - 8*in
t((log((a*e + b*e*x)/(c + d*x))*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
 d**3*x**3),x)*a*b**5*c**3*d**7*x**2 + 4*int((log((a*e + b*e*x)/(c + d*x))
*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**6*c**6*d**4 +
 8*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x
**2 + d**3*x**3),x)*b**6*c**5*d**5*x + 4*int((log((a*e + b*e*x)/(c + d*x))
*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**6*c**4*d**6*x
**2 + 12*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3),x)*a**3*b**3*c**4*d**6 + 24*int((log((a*e + b*e*x)/
(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**3*b
**3*c**3*d**7*x + 12*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(c**3 + 3*c**
2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**3*b**3*c**2*d**8*x**2 - 24*int((l
og((a*e + b*e*x)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + ...
```

3.48 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 251

$$\begin{aligned}\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx &= \frac{Bg^2(a+bx)^2}{4di^3(c+dx)^2}-\frac{Abg^2(a+bx)}{d^2i^3(c+dx)}\\ &+\frac{bBg^2(a+bx)}{d^2i^3(c+dx)}\\ &-\frac{bBg^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^3(c+dx)}\\ &-\frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^3(c+dx)^2}\\ &-\frac{b^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^3}\\ &-\frac{b^2Bg^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}\end{aligned}$$

output
```
1/4*B*g^2*(b*x+a)^2/d/i^3/(d*x+c)^2-A*b*g^2*(b*x+a)/d^2/i^3/(d*x+c)+b*B*g^
2*(b*x+a)/d^2/i^3/(d*x+c)-b*B*g^2*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d^2/i^3/(d
*x+c)-1/2*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i^3/(d*x+c)^2-b^2*g^
2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3/i^3-b^2*B*g^2*p
olylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^3
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.98

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{g^2\left(\frac{B(bc-ad)^2}{(c+dx)^2}-\frac{6bB(bc-ad)}{c+dx}-6b^2B\log(a+bx)-\frac{2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2}+\frac{8b(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{c+dx}+6\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*
i*x)^3,x]
```

output
```
(g^2*((B*(b*c - a*d)^2)/(c + d*x)^2 - (6*b*B*(b*c - a*d))/(c + d*x) - 6*b^
2*B*Log[a + b*x] - (2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/
(c + d*x)^2 + (8*b*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c +
d*x) + 6*b^2*B*Log[c + d*x] + 4*b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*L
og[c + d*x] - 2*b^2*B*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x]
)*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4*d^3*i^3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2962, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ci+dix)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{g^2\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{i^3}$$

$$\downarrow 2793$$

$$\frac{g^2\int\left(-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)}-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{d^2}-\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d(c+dx)}\right)d\frac{a+bx}{c+dx}}{i^3}$$

$$\downarrow 2009$$

$$\frac{g^2\left(-\frac{b^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3}-\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2}-\frac{Ab(a+bx)}{d^2(c+dx)}-\frac{b^2B\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3}-\frac{bB(a+bx)}{d^2(}\right.}{i^3}$$

input

```
Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^3
,x]
```

output

```
(g^2*((B*(a + b*x)^2)/(4*d*(c + d*x)^2) - (A*b*(a + b*x))/(d^2*(c + d*x))
+ (b*B*(a + b*x))/(d^2*(c + d*x)) - (b*B*(a + b*x)*Log[(e*(a + b*x))/(c +
d*x)])/(d^2*(c + d*x)) - ((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)/(2*d*(c + d*x)^2) - (b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d
*(a + b*x))/(b*(c + d*x))])/d^3 - (b^2*B*PolyLog[2, (d*(a + b*x))/(b*(c +
d*x))])/d^3))/i^3
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.84

method	result
parts	$\frac{g^2 A\left(-\frac{a^2d^2-2acdb+c^2b^2}{2d^3(dx+c)^2}+\frac{b^2\ln(dx+c)}{d^3}-\frac{2b(da-bc)}{d^3(dx+c)}\right)}{i^3}-\frac{g^2Bd\left(\frac{\left(\left(\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{4}\right.\right.\right.}{}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{g^2d^2A\left(-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)be+\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2d}{2}}{d^2}-\frac{b^2e^2\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{d^3}\right)}{(da-bc)e^3i^3}-\frac{g^2d^2B\left(-\frac{\left(\frac{be}{d}+\frac{(}{d}\right.\right.}{}$
default	$-\frac{e(da-bc)\left(-\frac{g^2d^2A\left(-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)be+\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2d}{2}}{d^2}-\frac{b^2e^2\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{d^3}\right)}{(da-bc)e^3i^3}-\frac{g^2d^2B\left(-\frac{\left(\frac{be}{d}+\frac{(}{d}\right.\right.}{}$
risch	Expression too large to display

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x,method=_RETU
RNVERBOSE)
```

output
```
g^2*A/i^3*(-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3/(d*x+c)^2+b^2/d^3*ln(d*x+c
)-2*b/d^3*(a*d-b*c)/(d*x+c))-g^2*B/i^3*d/(a*d-b*c)^2/e^2*(((1/2*(b*e/d+(a*
d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)
*e/d/(d*x+c))^2)*d+((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)*b*e)*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3+
(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)*e^2*(a^2*d^2-
2*a*b*c*d+b^2*c^2)/d^3*b^2)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algo
rithm="fricas")
```

output
```
integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B
*a*b*g^2*x + B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)))/(d^3*i^3*x^3 + 3*c*d
^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{g^2\left(\int \frac{Aa^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{Ab^2x^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{Ba^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2Aabx}{c^3+3c^2dx+3cd^2x^2+d^3}\right.}{i^3}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**3,x)
```

output
```
g**2*(Integral(A*a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x)
+ Integral(A*b**2*x**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x)
 + Integral(B*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x
 + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*a*b*x/(c**3 + 3*c**2*d*x
+ 3*c*d**2*x**2 + d**3*x**3), x) + Integral(B*b**2*x**2*log(a*e/(c + d*x)
+ b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + I
ntegral(2*B*a*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x
+ 3*c*d**2*x**2 + d**3*x**3), x))/i**3
```

Maxima [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algo
rithm="maxima")
```

output

```
-1/2*B*a*b*g^2*(2*(2*d*x + c)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^4*i^
3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d
^2)*x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3
*d^2 - a*c^2*d^3)*i^3) - 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 -
2*a*b*c*d^3 + a^2*d^4)*i^3) + 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d
^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3)) + 1/4*B*a^2*g^2*((2*b*d*x + 3*b*c - a*d)
/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a
*c^2*d^2)*i^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^3*i^3*x^2 + 2*c
*d^2*i^3*x + c^2*d*i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3
)) + 1/2*A*b^2*g^2*((4*c*d*x + 3*c^2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d
^3*i^3) + 2*log(d*x + c)/(d^3*i^3)) - 1/2*B*b^2*g^2*(((d^2*x^2 + 2*c*d*x +
 c^2)*log(d*x + c)^2 + (4*c*d*x + 3*c^2)*log(d*x + c))/(d^5*i^3*x^2 + 2*c*
d^4*i^3*x + c^2*d^3*i^3) - 2*integrate(1/2*(2*d^2*x^2*log(b*x + a) + 2*d^2
*x^2*log(e) + 4*c*d*x + 3*c^2)/(d^5*i^3*x^3 + 3*c*d^4*i^3*x^2 + 3*c^2*d^3*
i^3*x + c^3*d^2*i^3), x)) - (2*d*x + c)*A*a*b*g^2/(d^4*i^3*x^2 + 2*c*d^3*i
^3*x + c^2*d^2*i^3) - 1/2*A*a^2*g^2/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i
^3)
```

Giac [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^3}\,dx$$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algo
rithm="giac")
```

output

```
integrate((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)/(d*i*x + c*i)
^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(c\,i+d\,i\,x)^3}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^3
,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^3
, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x)
```

output

```
(g**2*i*(4*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*
c*d**2*x**2 + d**3*x**3),x)*a**2*b**3*c**4*d**5 + 8*int((log((a*e + b*e*x)
/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*
b**3*c**3*d**6*x + 4*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(c**3 + 3*c**
2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*b**3*c**2*d**7*x**2 - 8*int((lo
g((a*e + b*e*x)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3),x)*a*b**4*c**5*d**4 - 16*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(c
**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**4*c**4*d**5*x - 8*in
t((log((a*e + b*e*x)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
 d**3*x**3),x)*a*b**4*c**3*d**6*x**2 + 4*int((log((a*e + b*e*x)/(c + d*x))
*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**5*c**6*d**3 +
 8*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x
**2 + d**3*x**3),x)*b**5*c**5*d**4*x + 4*int((log((a*e + b*e*x)/(c + d*x))
*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**5*c**4*d**5*x
**2 - 2*log(a + b*x)*a**4*b*c**2*d**4 - 4*log(a + b*x)*a**4*b*c*d**5*x - 2
*log(a + b*x)*a**4*b*d**6*x**2 + 2*log(c + d*x)*a**4*b*c**2*d**4 + 4*log(c
 + d*x)*a**4*b*c*d**5*x + 2*log(c + d*x)*a**4*b*d**6*x**2 + 4*log(c + d*x)
*a**3*b**2*c**4*d**2 + 8*log(c + d*x)*a**3*b**2*c**3*d**3*x + 4*log(c + d*
x)*a**3*b**2*c**2*d**4*x**2 - 8*log(c + d*x)*a**2*b**3*c**5*d - 16*log(c +
 d*x)*a**2*b**3*c**4*d**2*x - 8*log(c + d*x)*a**2*b**3*c**3*d**3*x**2 +...
```

3.49 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$

Optimal result . . . 620
Mathematica [B] (verified) . . . 620
Rubi [A] (verified) . . . 621
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Giac [A] (verification not implemented) . . . 626
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Reduce [B] (verification not implemented) . . . 627

Optimal result

Integrand size = 38, antiderivative size = 85

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = -\frac{Bg(a+bx)^2}{4(bc-ad)i^3(c+dx)^2} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)i^3(c+dx)^2}$$

output

```
-1/4*B*g*(b*x+a)^2/(-a*d+b*c)/i^3/(d*x+c)^2+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x
+a)/(d*x+c)))/(-a*d+b*c)/i^3/(d*x+c)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 207 vs. 2(85) = 170.

Time = 0.18 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.44

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \frac{g\left(\frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2(c+dx)^2} - \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2(c+dx)} + \frac{bB\left(\frac{1}{c+dx}+\frac{b\log(a+bx)}{bc-ad}-\frac{b\log(c+dx)}{bc-ad}\right)}{d^2} - \frac{B\left(\frac{bc-ad}{(c+dx)^2}+\frac{2b}{c+dx}+\frac{2b^2\log(a+bx)}{bc-ad}\right.}{4d^2}\right.}{i^3}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*
x)^3,x]
```

output
```
(g*(((b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d^2*(c + d*x)^2)
 - (b*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d^2*(c + d*x)) + (b*B*((c + d
*x)^(-1) + (b*Log[a + b*x])/(b*c - a*d) - (b*Log[c + d*x])/(b*c - a*d)))/d
^2 - (B*((b*c - a*d)/(c + d*x)^2 + (2*b)/(c + d*x) + (2*b^2*Log[a + b*x])/
(b*c - a*d) - (2*b^2*Log[c + d*x])/(b*c - a*d)))/(4*d^2)))/i^3
```

Rubi [A] (verified)

Time $= 0.26$ (sec) , antiderivative size $= 72$, normalized size of antiderivative $= 0.85$, number of steps used $= 3$, number of rules used $= 2$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used $= \{2962, 2741\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ci+dix)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{g\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{c+dx}d\frac{a+bx}{c+dx}}{i^3(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{g\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(c+dx)^2}-\frac{B(a+bx)^2}{4(c+dx)^2}\right)}{i^3(bc-ad)}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^3,x
]
```

output
```
(g*(-1/4*(B*(a + b*x)^2)/(c + d*x)^2 + ((a + b*x)^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)]))/(2*(c + d*x)^2)))/((b*c - a*d)*i^3)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(81) = 162$.

Time = 1.34 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.05

method	result
norman	$\frac{-\frac{2Aadg+2Abcg-Badg-Bbcg}{4i\,d^2}-\frac{(2Abg-Bbg)x}{2id}-\frac{B\,a^2 g\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2i(da-bc)}-\frac{B\,b^2 g\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2(da-bc)i}-\frac{Babgx\ln\left(\frac{e(bx+a)}{dx+c}\right)}{i(da-bc)}}{i^2(dx+c)^2}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{g\,d^2A\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2(da-bc)^2e^3i^3}+\frac{g\,d^2B\left(\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{4}\right)}{(da-bc)^2e^3i^3}\right)}{d^2}$
default	$-\frac{e(da-bc)\left(\frac{g\,d^2A\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2(da-bc)^2e^3i^3}+\frac{g\,d^2B\left(\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{4}\right)}{(da-bc)^2e^3i^3}\right)}{d^2}$
parallelrisch	$-\frac{2A\,a^2b\,d^4g-2A\,b^3c^2d^2g-B\,a^2b\,d^4g+B\,b^3c^2d^2g+2B\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2b\,d^4g+2B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3d^4g+4Axa\,b^2d^4g-$
risch	$-\frac{Bg(2bdx+da+bc)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2d^2i^3(dx+c)^2}-\frac{g(2B\ln(bx+a)b^2d^2x^2-2B\ln(-dx-c)b^2d^2x^2+4B\ln(bx+a)b^2cdx-4B\ln(-dx-$
parts	$\frac{gA\left(-\frac{da-bc}{2d^2(dx+c)^2}-\frac{b}{d^2(dx+c)}\right)}{i^3}-\frac{gBd\left(a\left(\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{4}\right)-\frac{bc\left(\frac{\left(\frac{be}{d}+\frac{(da-}{d(d}\right.\right.}{i^3(da-bc)^2e^2}$
orering	$-\frac{(dx+c)(-2d^2b^2x^3-3x^2ab\,d^2+3x^2b^2cd+6xacdb+5a^2cd-ab\,c^2)(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{4c(da-bc)d(bx+a)(dix+ci)^3}-\frac{(-bd\,x^2+ac)(d}{}$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x,method=_RETURN
VERBOSE)
```

output
```
(-1/4*(2*A*a*d*g+2*A*b*c*g-B*a*d*g-B*b*c*g)/i/d^2-1/2*(2*A*b*g-B*b*g)/i/d*
x-1/2*B*a^2*g/i/(a*d-b*c)*ln(e*(b*x+a)/(d*x+c))-1/2*B*b^2*g/(a*d-b*c)/i*x^
2*ln(e*(b*x+a)/(d*x+c))-B*a*b*g/i/(a*d-b*c)*x*ln(e*(b*x+a)/(d*x+c)))/i^2/(
d*x+c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(81) = 162.

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.18

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx =$$
$$-\frac{2\left((2A-B)b^2cd-(2A-B)abd^2\right)gx+\left((2A-B)b^2c^2-(2A-B)a^2d^2\right)g-2\left(Bb^2d^2gx^2+2Babd^2g\right.}{4\left((bcd^4-ad^5)i^3x^2+2\left(bc^2d^3-acd^4\right)i^3x+\left(bc^3d^2-ac^2d^3\right)i^3\right)}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algori
thm="fricas")
```

output
```
-1/4*(2*((2*A - B)*b^2*c*d - (2*A - B)*a*b*d^2)*g*x + ((2*A - B)*b^2*c^2 -
 (2*A - B)*a^2*d^2)*g - 2*(B*b^2*d^2*g*x^2 + 2*B*a*b*d^2*g*x + B*a^2*d^2*g
)*log((b*e*x + a*e)/(d*x + c)))/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3
- a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(71) = 142.

Time = 2.28 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.49

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{Bb^2g\log\left(x+\frac{-\frac{Ba^2b^2d^2g}{ad-bc}+\frac{2Bab^3cdg}{ad-bc}+Bab^2dg-\frac{Bb^4c^2g}{ad-bc}+Bb^3cg}{2Bb^3dg}\right)}{2d^2i^3\left(ad-bc\right)}$$
$$- \frac{Bb^2g\log\left(x+\frac{\frac{Ba^2b^2d^2g}{ad-bc}-\frac{2Bab^3cdg}{ad-bc}+Bab^2dg+\frac{Bb^4c^2g}{ad-bc}+Bb^3cg}{2Bb^3dg}\right)}{2d^2i^3\left(ad-bc\right)}$$
$$+ \frac{-2Aadg-2Abcg+Badg+Bbcg+x(-4Abdg+2Bbdg)}{4c^2d^2i^3+8cd^3i^3x+4d^4i^3x^2}$$
$$+ \frac{(-Badg-Bbcg-2Bbdgx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2c^2d^2i^3+4cd^3i^3x+2d^4i^3x^2}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**3,x)
```

output
```
B*b**2*g*log(x + (-B*a**2*b**2*d**2*g/(a*d - b*c) + 2*B*a*b**3*c*d*g/(a*d
 - b*c) + B*a*b**2*d*g - B*b**4*c**2*g/(a*d - b*c) + B*b**3*c*g)/(2*B*b**3*
d*g))/(2*d**2*i**3*(a*d - b*c)) - B*b**2*g*log(x + (B*a**2*b**2*d**2*g/(a*
d - b*c) - 2*B*a*b**3*c*d*g/(a*d - b*c) + B*a*b**2*d*g + B*b**4*c**2*g/(a*
d - b*c) + B*b**3*c*g)/(2*B*b**3*d*g))/(2*d**2*i**3*(a*d - b*c)) + (-2*A*a
*d*g - 2*A*b*c*g + B*a*d*g + B*b*c*g + x*(-4*A*b*d*g + 2*B*b*d*g))/(4*c**2
*d**2*i**3 + 8*c*d**3*i**3*x + 4*d**4*i**3*x**2) + (-B*a*d*g - B*b*c*g - 2
*B*b*d*g*x)*log(e*(a + b*x)/(c + d*x))/(2*c**2*d**2*i**3 + 4*c*d**3*i**3*x
 + 2*d**4*i**3*x**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(81) = 162$.

Time $= 0.06$ (sec) , antiderivative size $= 567$, normalized size of antiderivative $= 6.67$

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx =$$
$$-\frac{1}{4}\,Bbg\left(\frac{2\,(2\,dx+c)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{d^4i^3x^2+2\,cd^3i^3x+c^2d^2i^3}-\frac{bc^2-3\,acd+2\,(bcd-2\,ad^2)x}{(bcd^4-ad^5)i^3x^2+2\,(bc^2d^3-acd^4)i^3x+(bc^3d^2-ac^2d^3)i^3}-\right.$$
$$+\frac{1}{4}\,Bag\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)i^3x^2+2\,(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3}-\frac{2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{d^3i^3x^2+2\,cd^2i^3x+c^2di^3}+\right.$$
$$-\frac{(2\,dx+c)Abg}{2\,(d^4i^3x^2+2\,cd^3i^3x+c^2d^2i^3)}-\frac{Aag}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algori
thm="maxima")
```

output

```
-1/4*B*b*g*(2*(2*d*x + c)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^4*i^3*x^
2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*
x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2
 - a*c^2*d^3)*i^3) - 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*
b*c*d^3 + a^2*d^4)*i^3) + 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*i^3)) + 1/4*B*a*g*((2*b*d*x + 3*b*c - a*d)/((b*c*d
^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2
)*i^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^3*i^3*x^2 + 2*c*d^2*i^3
*x + c^2*d*i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*
i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2
*(2*d*x + c)*A*b*g/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - 1/2*A*a*g
/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.55

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$

$$=\frac{1}{4}\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)\left(\frac{2\,(bex+ae)^2Bg\log\left(\frac{bex+ae}{dx+c}\right)}{(dx+c)^2ei^3}+\frac{(bex+ae)^2(2\,Ag-\right.$$

input

```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algori
thm="giac")
```

output

```
1/4*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d))
)*(2*(b*e*x + a*e)^2*B*g*log((b*e*x + a*e)/(d*x + c))/((d*x + c)^2*e*i^3)
+ (b*e*x + a*e)^2*(2*A*g - B*g)/((d*x + c)^2*e*i^3))
```

Mupad [B] (verification not implemented)

Time = 28.29 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.33

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$
$$= -\frac{x\,(2\,A\,b\,d\,g - B\,b\,d\,g) + A\,a\,d\,g + A\,b\,c\,g - \frac{B\,a\,d\,g}{2} - \frac{B\,b\,c\,g}{2}}{2\,c^2\,d^2\,i^3 + 4\,c\,d^3\,i^3\,x + 2\,d^4\,i^3\,x^2}$$
$$- \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{B\,a\,g}{2\,d^2\,i^3} + \frac{B\,b\,c\,g}{2\,d^3\,i^3} + \frac{B\,b\,g\,x}{d^2\,i^3}\right)}{2\,c\,x + d\,x^2 + \frac{c^2}{d}} + \frac{B\,b^2\,g\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c} + 1\mathrm{i}\right)\,1\mathrm{i}}{d^2\,i^3\,(a\,d-b\,c)}$$

input
```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^3,x
)
```

output
```
(B*b^2*g*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*1i)/(d^2*i^3*(a*d - b*
c)) - (log((e*(a + b*x))/(c + d*x))*((B*a*g)/(2*d^2*i^3) + (B*b*c*g)/(2*d^
3*i^3) + (B*b*g*x)/(d^2*i^3)))/(2*c*x + d*x^2 + c^2/d) - (x*(2*A*b*d*g - B
*b*d*g) + A*a*d*g + A*b*c*g - (B*a*d*g)/2 - (B*b*c*g)/2)/(2*c^2*d^2*i^3 +
2*d^4*i^3*x^2 + 4*c*d^3*i^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.99

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{gi\left(-2\log(bx+a)\,a\,b^2c^2 - 4\log(bx+a)\,a\,b^2cdx - 2\log(bx+a)\,a\,b^2d^2x^2 + 2\log(dx+c)\,a\,b^2c^2 + 4\log(d$$

input
```
int((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x)
```

output

```
(g*i*( - 2*log(a + b*x)*a*b**2*c**2 - 4*log(a + b*x)*a*b**2*c*d*x - 2*log(
a + b*x)*a*b**2*d**2*x**2 + 2*log(c + d*x)*a*b**2*c**2 + 4*log(c + d*x)*a*
b**2*c*d*x + 2*log(c + d*x)*a*b**2*d**2*x**2 - 2*log((a*e + b*e*x)/(c + d*
x))*a**2*b*c*d + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*c**2 + 2*log((a*e +
 b*e*x)/(c + d*x))*a*b**2*d**2*x**2 - 2*log((a*e + b*e*x)/(c + d*x))*b**3*
c*d*x**2 - 2*a**3*c*d + 2*a**2*b*c**2 + a**2*b*c*d + 2*a**2*b*d**2*x**2 -
a*b**2*c**2 - 2*a*b**2*c*d*x**2 - a*b**2*d**2*x**2 + b**3*c*d*x**2))/(4*c*
d*(a*c**2*d + 2*a*c*d**2*x + a*d**3*x**2 - b*c**3 - 2*b*c**2*d*x - b*c*d**
2*x**2))
```

3.50 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 144

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx = \frac{B}{4di^3(c+dx)^2}+\frac{bB}{2d(bc-ad)i^3(c+dx)}+\frac{b^2B\log(a+bx)}{2d(bc-ad)^2i^3} - \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2di^3(c+dx)^2}-\frac{b^2B\log(c+dx)}{2d(bc-ad)^2i^3}$$

output

```
1/4*B/d/i^3/(d*x+c)^2+1/2*b*B/d/(-a*d+b*c)/i^3/(d*x+c)+1/2*b^2*B*ln(b*x+a)
/d/(-a*d+b*c)^2/i^3-1/2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i^3/(d*x+c)^2-1/2*b^
2*B*ln(d*x+c)/d/(-a*d+b*c)^2/i^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx = \frac{-2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B\left((bc-ad)(3bc-ad+2bdx)+2b^2(c+dx)^2\log(a+bx)-2b^2(c+dx)^2\log(c+dx)\right)}{(bc-ad)^2}}{4di^3(c+dx)^2}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x)^3,x]
```

output
```
(-2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*((b*c - a*d)*(3*b*c - a*d +
2*b*d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*x]
))/(b*c - a*d)^2)/(4*d*i^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{(ci+dix)^3}\,dx$$

$$\downarrow 2948$$

$$\frac{B(bc-ad)\int \frac{1}{i^2(a+bx)(c+dx)^3}\,dx}{2di} - \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{2di^3(c+dx)^2}$$

$$\downarrow 27$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)(c+dx)^3}\,dx}{2di^3} - \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{2di^3(c+dx)^2}$$

$$\downarrow 54$$

$$\frac{B(bc-ad)\int \left(\frac{b^3}{(bc-ad)^3(a+bx)} - \frac{db^2}{(bc-ad)^3(c+dx)} - \frac{db}{(bc-ad)^2(c+dx)^2} - \frac{d}{(bc-ad)(c+dx)^3}\right)dx}{2di^3} - \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{2di^3(c+dx)^2}$$

$$\downarrow 2009$$

$$\frac{B(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)^3} - \frac{b^2\log(c+dx)}{(bc-ad)^3} + \frac{b}{(c+dx)(bc-ad)^2} + \frac{1}{2(c+dx)^2(bc-ad)}\right)}{2di^3} - \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{2di^3(c+dx)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x)^3,x]
```

output
```
-1/2*(A + B*Log[(e*(a + b*x))/(c + d*x)])/(d*i^3*(c + d*x)^2) + (B*(b*c -
a*d)*(1/(2*(b*c - a*d)*(c + d*x)^2) + b/((b*c - a*d)^2*(c + d*x)) + (b^2*L
og[a + b*x])/(b*c - a*d)^3 - (b^2*Log[c + d*x])/(b*c - a*d)^3))/(2*d*i^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.53

method	result
parts	$-\frac{A}{2i^3(dx+c)^2d}-\frac{Bd\left(\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{4}-\frac{be\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right.}{d}\right.}{i^3(da-bc)^2e^2}$
norman	$\frac{\frac{Bcb^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{\left(a^2d^2-2acdb+c^2b^2\right)i}-\frac{2Aad^2-2Abcd-Bad^2+3Bbcd}{4id^2(da-bc)}-\frac{Bbx}{2i(da-bc)}-\frac{Ba(da-2bc)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2i\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{b^2Bdx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2\left(a^2d^2-2acdb+c^2b^2\right)i}}{i^2(dx+c)^2}$
parallelrisch	$-\frac{2Aa^2bd^5+2Ab^3c^2d^3-Ba^2bd^5-3Bb^3c^2d^3-4Aab^2cd^4+4Bab^2cd^4-2Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3d^5+2Bxab^2d^5-2Bxb^3}{4i^3(dx+c)^2(a^2d^2-2acdb+c^2b^2)bd^4}$
orering	$-\frac{(dx+c)(-8b^2dx^2-3xabd-13b^2cx+5a^2d-13abc)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{4(a^2d^2-2acdb+c^2b^2)(dix+ci)^3}-\frac{(-2bdx+da-3bc)(bx+a)(dx+c)^2\left(\frac{B\left(\right.\right.}{4d(a^2}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2di^3(dx+c)^2}-\frac{2B\ln(dx+c)b^2d^2x^2-2B\ln(-bx-a)b^2d^2x^2+4B\ln(dx+c)b^2cdx-4B\ln(-bx-a)b^2cdx+2B\ln(d}{4(a^2d^2-2}$
derivativedivides	$e(da-bc)\left(-\frac{d^2Ab\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{(da-bc)^3e^2i^3}+\frac{d^3A\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2(da-bc)^3e^3i^3}-\frac{d^2Bb\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-\frac{(da-bc)e}{d(dx+c)}-\frac{be}{d}\right.}{(da-bc)^3e^2i^3}\right.$ $-\frac{}{d^2}$
default	$e(da-bc)\left(-\frac{d^2Ab\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{(da-bc)^3e^2i^3}+\frac{d^3A\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2(da-bc)^3e^3i^3}-\frac{d^2Bb\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-\frac{(da-bc)e}{d(dx+c)}-\frac{be}{d}\right.}{(da-bc)^3e^2i^3}\right.$ $-\frac{}{d^2}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/i^3/(d*x+c)^2/d-B/i^3*d/(a*d-b*c)^2/e^2*(1/2*(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^2-b*e/d*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(
a*d-b*c)*e/d/(d*x+c)-b*e/d))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.53

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx = -\frac{(2\,A-3\,B)b^2c^2-4\,(A-B)abcd+(2\,A-B)a^2d^2-2\,(Bb^2cd-Babd^2)x-2\,(Bb^2d^2x^2+2\,Bb^2cdx+}{4\,((b^2c^2d^3-2\,abcd^4+a^2d^5)i^3x^2+2\,(b^2c^3d^2-2\,abc^2d^3+a^2cd^4)i^3x+(b^2c^4d-2\,abc^3$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algorithm="fricas"
)
```

output
```
-1/4*((2*A - 3*B)*b^2*c^2 - 4*(A - B)*a*b*c*d + (2*A - B)*a^2*d^2 - 2*(B*b
^2*c*d - B*a*b*d^2)*x - 2*(B*b^2*d^2*x^2 + 2*B*b^2*c*d*x + 2*B*a*b*c*d - B
*a^2*d^2)*log((b*e*x + a*e)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*
d^5)*i^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*i^3*x + (b^2*c^
4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*i^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(122) = 244.

Time = 1.10 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.93

$$\begin{aligned}
&\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx \\
&= -\frac{Bb^2\log\left(x+\frac{-\frac{Ba^3b^2d^3}{(ad-bc)^2}+\frac{3Ba^2b^3cd^2}{(ad-bc)^2}-\frac{3Bab^4c^2d}{(ad-bc)^2}+Bab^2d+\frac{Bb^5c^3}{(ad-bc)^2}+Bb^3c}{2Bb^3d}\right)}{2di^3\,(ad-bc)^2} \\
&\quad + \frac{Bb^2\log\left(x+\frac{\frac{Ba^3b^2d^3}{(ad-bc)^2}-\frac{3Ba^2b^3cd^2}{(ad-bc)^2}+\frac{3Bab^4c^2d}{(ad-bc)^2}+Bab^2d-\frac{Bb^5c^3}{(ad-bc)^2}+Bb^3c}{2Bb^3d}\right)}{2di^3\,(ad-bc)^2} \\
&\quad - \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2c^2di^3+4cd^2i^3x+2d^3i^3x^2} \\
&\quad + \frac{-2Aad+2Abc+Bad-3Bbc-2Bbdx}{4ac^2d^2i^3-4bc^3di^3+x^2\cdot(4ad^4i^3-4bcd^3i^3)+x\,(8acd^3i^3-8bc^2d^2i^3)}
\end{aligned}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**3,x)
```

output
```
-B*b**2*log(x + (-B*a**3*b**2*d**3/(a*d - b*c)**2 + 3*B*a**2*b**3*c*d**2/(
a*d - b*c)**2 - 3*B*a*b**4*c**2*d/(a*d - b*c)**2 + B*a*b**2*d + B*b**5*c**
3/(a*d - b*c)**2 + B*b**3*c)/(2*B*b**3*d))/(2*d*i**3*(a*d - b*c)**2) + B*b
**2*log(x + (B*a**3*b**2*d**3/(a*d - b*c)**2 - 3*B*a**2*b**3*c*d**2/(a*d -
 b*c)**2 + 3*B*a*b**4*c**2*d/(a*d - b*c)**2 + B*a*b**2*d - B*b**5*c**3/(a*
d - b*c)**2 + B*b**3*c)/(2*B*b**3*d))/(2*d*i**3*(a*d - b*c)**2) - B*log(e*
(a + b*x)/(c + d*x))/(2*c**2*d*i**3 + 4*c*d**2*i**3*x + 2*d**3*i**3*x**2)
+ (-2*A*a*d + 2*A*b*c + B*a*d - 3*B*b*c - 2*B*b*d*x)/(4*a*c**2*d**2*i**3 -
 4*b*c**3*d*i**3 + x**2*(4*a*d**4*i**3 - 4*b*c*d**3*i**3) + x*(8*a*c*d**3*
i**3 - 8*b*c**2*d**2*i**3))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.77

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx$$
$$= \frac{1}{4}B\left(\frac{2\,bdx + 3\,bc - ad}{(bcd^3 - ad^4)i^3x^2 + 2\,(bc^2d^2 - acd^3)i^3x + (bc^3d - ac^2d^2)i^3} - \frac{2\,\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right)}{d^3i^3x^2 + 2\,cd^2i^3x + c^2di^3} + \frac{2\,b^2}{(b^2c^2d -}\right.$$
$$\left. - \frac{A}{2\,(d^3i^3x^2 + 2\,cd^2i^3x + c^2di^3)}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algorithm="maxima"
)
```

output
```
1/4*B*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 -
 a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) - 2*log(b*e*x/(d*x + c) + a*e
/(d*x + c))/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) + 2*b^2*log(b*x + a)
/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*A/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^
2*d*i^3)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.64

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx$$
$$= \frac{1}{4}\left(2\left(\frac{2\,(bex+ae)Bb}{(bci^3-adi^3)(dx+c)} - \frac{(bex+ae)^2Bd}{(bcei^3-adei^3)(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right) - \frac{(bex+ae)^2(2\,Ad-Bd)}{(bcei^3-adei^3)(dx+c)^2}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algorithm="giac")
```

output
```
1/4*(2*(2*(b*e*x + a*e)*B*b/((b*c*i^3 - a*d*i^3)*(d*x + c)) - (b*e*x + a*e
)^2*B*d/((b*c*e*i^3 - a*d*e*i^3)*(d*x + c)^2))*log((b*e*x + a*e)/(d*x + c)
) - (b*e*x + a*e)^2*(2*A*d - B*d)/((b*c*e*i^3 - a*d*e*i^3)*(d*x + c)^2) +
4*(b*e*x + a*e)*(A*b - B*b)/((b*c*i^3 - a*d*i^3)*(d*x + c)))*(b*c/((b*c*e
- a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 27.25 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.44

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx = \frac{B\,b^2\,\mathrm{atanh}\left(\frac{2\,a^2\,d^3\,i^3-2\,b^2\,c^2\,d\,i^3}{2\,d\,i^3\,(a\,d-b\,c)^2}+\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{d\,i^3\,(a\,d-b\,c)^2}$$
$$-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,d^2\,i^3\,\left(2\,c\,x+d\,x^2+\frac{c^2}{d}\right)}$$
$$-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c-B\,a\,d+3\,B\,b\,c}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,x}{a\,d-b\,c}}{2\,c^2\,d\,i^3+4\,c\,d^2\,i^3\,x+2\,d^3\,i^3\,x^2}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(c*i + d*i*x)^3,x)
```

output
```
(B*b^2*atanh((2*a^2*d^3*i^3 - 2*b^2*c^2*d*i^3)/(2*d*i^3*(a*d - b*c)^2) + (
2*b*d*x)/(a*d - b*c)))/(d*i^3*(a*d - b*c)^2) - (B*log((e*(a + b*x))/(c + d
*x)))/(2*d^2*i^3*(2*c*x + d*x^2 + c^2/d)) - ((2*A*a*d - 2*A*b*c - B*a*d +
3*B*b*c)/(2*(a*d - b*c)) + (B*b*d*x)/(a*d - b*c))/(2*c^2*d*i^3 + 2*d^3*i^3
*x^2 + 4*c*d^2*i^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 577, normalized size of antiderivative = 4.01

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx$$
$$= \frac{i\left(-3a\,b^2c^3d + a\,b^2c\,d^3x^2 - 4\log(bx+a)\,a^2bc\,d^3x + 8\log(bx+a)\,a\,b^2c^2d^2x + 4\log(bx+a)\,a\,b^2c\,d^3x^2 + 4\right.}{}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x)
```

output
```
(i*( - 2*log(a + b*x)*a**2*b*c**2*d**2 - 4*log(a + b*x)*a**2*b*c*d**3*x -
2*log(a + b*x)*a**2*b*d**4*x**2 + 4*log(a + b*x)*a*b**2*c**3*d + 8*log(a +
 b*x)*a*b**2*c**2*d**2*x + 4*log(a + b*x)*a*b**2*c*d**3*x**2 + 2*log(c + d
*x)*a**2*b*c**2*d**2 + 4*log(c + d*x)*a**2*b*c*d**3*x + 2*log(c + d*x)*a**
2*b*d**4*x**2 - 4*log(c + d*x)*a*b**2*c**3*d - 8*log(c + d*x)*a*b**2*c**2*
d**2*x - 4*log(c + d*x)*a*b**2*c*d**3*x**2 + 4*log((a*e + b*e*x)/(c + d*x)
)*a**2*b*c*d**3*x + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d**4*x**2 - 8*lo
g((a*e + b*e*x)/(c + d*x))*a*b**2*c**2*d**2*x - 4*log((a*e + b*e*x)/(c + d
*x))*a*b**2*c*d**3*x**2 + 4*log((a*e + b*e*x)/(c + d*x))*b**3*c**3*d*x + 2
*log((a*e + b*e*x)/(c + d*x))*b**3*c**2*d**2*x**2 - 2*a**3*c**2*d**2 + 4*a
**2*b*c**3*d + a**2*b*c**2*d**2 - 2*a*b**2*c**4 - 3*a*b**2*c**3*d + a*b**2
*c*d**3*x**2 + 2*b**3*c**4 - b**3*c**2*d**2*x**2))/(4*c**2*d*(a**2*c**2*d*
*2 + 2*a**2*c*d**3*x + a**2*d**4*x**2 - 2*a*b*c**3*d - 4*a*b*c**2*d**2*x -
 2*a*b*c*d**3*x**2 + b**2*c**4 + 2*b**2*c**3*d*x + b**2*c**2*d**2*x**2))
```

3.51 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 243

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx = &-\frac{B\left(4b-\frac{d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3gi^3}-\frac{b^2B\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^3gi^3}\\ &+\frac{d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2}\\ &-\frac{2bd(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3gi^3(c+dx)}\\ &+\frac{b^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3gi^3}\end{aligned}$$

output

```
-1/4*B*(4*b-d*(b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g/i^3-1/2*b^2*B*ln((b*x+a)/(
d*x+c))^2/(-a*d+b*c)^3/g/i^3+1/2*d^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))
/(-a*d+b*c)^3/g/i^3/(d*x+c)^2-2*b*d*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-
a*d+b*c)^3/g/i^3/(d*x+c)+b^2*ln((b*x+a)/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)
))/(-a*d+b*c)^3/g/i^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.46 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.72

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \frac{2(bc-ad)^2\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) + 4b(bc-ad)(c+dx)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) + 4b^2(c+dx)^2\log(a+$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x
)^3),x]
```

output
```
(2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*b*(b*c - a*d)*(c
 + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*b^2*(c + d*x)^2*Log[a + b
*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 4*b^2*(c + d*x)^2*(A + B*Log[(e
*(a + b*x))/(c + d*x)])*Log[c + d*x] - 4*b*B*(c + d*x)*(b*c - a*d + b*(c +
 d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]) - B*((b*c - a*d)^2 + 2*b*(b
*c - a*d)*(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*L
og[c + d*x]) - 2*b^2*B*(c + d*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*
(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2
*b^2*B*(c + d*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*L
og[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4*(b*c - a*d)^3*g
*i^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.75, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2962, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)(ci+dix)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx}d\frac{a+bx}{c+dx}}{gi^3(bc-ad)^3}$$

$$\downarrow 2772$$

$$\frac{-B\int\left(\frac{b^2(c+dx)\log\left(\frac{a+bx}{c+dx}\right)}{a+bx}-\frac{1}{2}d\left(4b-\frac{d(a+bx)}{c+dx}\right)\right)d\frac{a+bx}{c+dx}+b^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+\frac{d^2(a+bx)^2\left(B\log\left(\frac{e}{}\right.\right.}{2(c+dx)}}{gi^3(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{b^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+\frac{d^2(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(c+dx)^2}-\frac{2bd(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{c+dx}-B\left(\frac{1}{2}b^2\log^2\left(\frac{}{}\right.\right.}{gi^3(bc-ad)^3}$$

input

```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x)^3),x
]
```

output

```
(-(B*((4*b - (d*(a + b*x))/(c + d*x))^2/4 + (b^2*Log[(a + b*x)/(c + d*x)]^
2)/2)) + (d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(c + d*
x)^2) - (2*b*d*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) +
 b^2*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c
- a*d)^3*g*i^3)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.51

method	result
parts	$\frac{A\left(-\frac{1}{2(da-bc)(dx+c)^2}+\frac{b^2\ln(dx+c)}{(da-bc)^3}+\frac{b}{(da-bc)^2(dx+c)}-\frac{b^2\ln(bx+a)}{(da-bc)^3}\right)}{g\,i^3}-\frac{Bd\left(\frac{d\left(\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2}-\right.}{da-bc}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{d^2Ab^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e\,i^3(da-bc)^4g}-\frac{2d^3Ab\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e^2i^3(da-bc)^4g}+\frac{d^4A\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2e^3i^3(da-bc)^4g}+\frac{d^2Bb^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2e\,i^3(da-bc)^4g}-\frac{2}{}\right.}{}$
default	$-\frac{e(da-bc)\left(\frac{d^2Ab^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e\,i^3(da-bc)^4g}-\frac{2d^3Ab\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e^2i^3(da-bc)^4g}+\frac{d^4A\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2e^3i^3(da-bc)^4g}+\frac{d^2Bb^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2e\,i^3(da-bc)^4g}-\frac{2}{}\right.}{}$
parallelrisch	$-\frac{2B\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a^2b^2c^6-4Ax\,a^4c^3d^3+4A\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2b^2c^6+2Bx\,a^4c^3d^3+2B\ln\left(\frac{e(bx+a)}{dx+c}\right)a^4c^4d^2-2A\,x^2a^4c^2d}{}$
norman	$-\frac{2Aa\,d^3-6Abc\,d^2-Ba\,d^3+7Bbc\,d^2}{4gi(da-bc)^2d^2}-\frac{\left(2A\,b^2c^2+B\,a^2d^2-4Babcd\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2gi\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{\left(2Ab\,d^2-3Bb\,d^2\right)x}{2ig\left(a^2d^2-2acdb+c^2b^2\right)d}-\frac{B\,b^2c^2\ln}{2gi\left(a^3d^3-3a^2bc}$
risch	$-\frac{A}{2g\,i^3(da-bc)(dx+c)^2}+\frac{A\,b^2\ln(dx+c)}{g\,i^3(da-bc)^3}+\frac{Ab}{g\,i^3(da-bc)^2(dx+c)}-\frac{A\,b^2\ln(bx+a)}{g\,i^3(da-bc)^3}+\frac{3B\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)b^2}{2g\,i^3(da-bc)^3}+$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^3,x,method=_RETURN
VERBOSE)
```

output
```
A/g/i^3*(-1/2/(a*d-b*c)/(d*x+c)^2+b^2/(a*d-b*c)^3*ln(d*x+c)+b/(a*d-b*c)^2/
(d*x+c)-b^2/(a*d-b*c)^3*ln(b*x+a))-B/g/i^3*d/(a*d-b*c)^2/e^2*(d/(a*d-b*c)*
(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(
b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2/(a*d-b*c)*b*e*((b*e/d+(a*d-b*c)*e/d/(d*x
+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)+1/2/d/(a
*d-b*c)*b^2*e^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.46

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \frac{(6A-7B)b^2c^2 - 8(A-B)abcd + (2A-B)a^2d^2 + 2\left(Bb^2d^2x^2 + 2Bb^2cdx + Bb^2c^2\right)\log\left(\frac{bex+ae}{dx+c}\right)^2 + 2($$
$$4\left((b^3c^3d^2 - 3ab^2c^2d^3 + 3a^2bcd^4 - a^3d^5)gi^3x^2 + 2(b^3$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algori
thm="fricas")
```

output
```
1/4*((6*A - 7*B)*b^2*c^2 - 8*(A - B)*a*b*c*d + (2*A - B)*a^2*d^2 + 2*(B*b^
2*d^2*x^2 + 2*B*b^2*c*d*x + B*b^2*c^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*
((2*A - 3*B)*b^2*c*d - (2*A - 3*B)*a*b*d^2)*x + 2*((2*A - 3*B)*b^2*d^2*x^2
 + 2*A*b^2*c^2 - 4*B*a*b*c*d + B*a^2*d^2 + 2*(2*(A - B)*b^2*c*d - B*a*b*d^
2)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^
2*b*c*d^4 - a^3*d^5)*g*i^3*x^2 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*
c^2*d^3 - a^3*c*d^4)*g*i^3*x + (b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2
 - a^3*c^2*d^3)*g*i^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. 2(207) = 414.

Time = 2.57 (sec) , antiderivative size = 889, normalized size of antiderivative = 3.66

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)**3,x)
```

output

```
-B*b**2*log(e*(a + b*x)/(c + d*x))**2/(2*a**3*d**3*g*i**3 - 6*a**2*b*c*d**
2*g*i**3 + 6*a*b**2*c**2*d*g*i**3 - 2*b**3*c**3*g*i**3) + b**2*(2*A - 3*B)
*log(x + (2*A*a*b**2*d + 2*A*b**3*c - 3*B*a*b**2*d - 3*B*b**3*c - a**4*b**
2*d**4*(2*A - 3*B)/(a*d - b*c)**3 + 4*a**3*b**3*c*d**3*(2*A - 3*B)/(a*d -
b*c)**3 - 6*a**2*b**4*c**2*d**2*(2*A - 3*B)/(a*d - b*c)**3 + 4*a*b**5*c**3
*d*(2*A - 3*B)/(a*d - b*c)**3 - b**6*c**4*(2*A - 3*B)/(a*d - b*c)**3)/(4*A
*b**3*d - 6*B*b**3*d))/(2*g*i**3*(a*d - b*c)**3) - b**2*(2*A - 3*B)*log(x
+ (2*A*a*b**2*d + 2*A*b**3*c - 3*B*a*b**2*d - 3*B*b**3*c + a**4*b**2*d**4*
(2*A - 3*B)/(a*d - b*c)**3 - 4*a**3*b**3*c*d**3*(2*A - 3*B)/(a*d - b*c)**3
 + 6*a**2*b**4*c**2*d**2*(2*A - 3*B)/(a*d - b*c)**3 - 4*a*b**5*c**3*d*(2*A
 - 3*B)/(a*d - b*c)**3 + b**6*c**4*(2*A - 3*B)/(a*d - b*c)**3)/(4*A*b**3*d
 - 6*B*b**3*d))/(2*g*i**3*(a*d - b*c)**3) + (-B*a*d + 3*B*b*c + 2*B*b*d*x)
*log(e*(a + b*x)/(c + d*x))/(2*a**2*c**2*d**2*g*i**3 + 4*a**2*c*d**3*g*i**
3*x + 2*a**2*d**4*g*i**3*x**2 - 4*a*b*c**3*d*g*i**3 - 8*a*b*c**2*d**2*g*i*
*3*x - 4*a*b*c*d**3*g*i**3*x**2 + 2*b**2*c**4*g*i**3 + 4*b**2*c**3*d*g*i**
3*x + 2*b**2*c**2*d**2*g*i**3*x**2) + (-2*A*a*d + 6*A*b*c + B*a*d - 7*B*b*
c + x*(4*A*b*d - 6*B*b*d))/(4*a**2*c**2*d**2*g*i**3 - 8*a*b*c**3*d*g*i**3
+ 4*b**2*c**4*g*i**3 + x**2*(4*a**2*d**4*g*i**3 - 8*a*b*c*d**3*g*i**3 + 4*
b**2*c**2*d**2*g*i**3) + x*(8*a**2*c*d**3*g*i**3 - 16*a*b*c**2*d**2*g*i**3
 + 8*b**2*c**3*d*g*i**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 885 vs. $2(237) = 474$.

Time $= 0.09$ (sec) , antiderivative size $= 885$, normalized size of antiderivative $= 3.64$

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)(ci + dix)^3}\, dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algori
thm="maxima")
```

output

```
1/2*B*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g*i^
3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b^2*c^4 - 2*a
*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(b*e*x/(d*x + c) + a*e/(d
*x + c)) + 1/2*A*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^
2*d^4)*g*i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b^
2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3)) - 1/4*(7*b^2*c^2
- 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x +
a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d
 - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(
3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*log(b*x + a))*log(d*x + c))*B/(b^3*c^5*g*i^3 - 3*a*b^2*c^4*d*g*i^3
 + 3*a^2*b*c^3*d^2*g*i^3 - a^3*c^2*d^3*g*i^3 + (b^3*c^3*d^2*g*i^3 - 3*a*b^
2*c^2*d^3*g*i^3 + 3*a^2*b*c*d^4*g*i^3 - a^3*d^5*g*i^3)*x^2 + 2*(b^3*c^4*d*
g*i^3 - 3*a*b^2*c^3*d^2*g*i^3 + 3*a^2*b*c^2*d^3*g*i^3 - a^3*c*d^4*g*i^3)*x
)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.82

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \frac{1}{4}\left(\frac{2\,Bb^2e\log\left(\frac{bex+ae}{dx+c}\right)^2}{b^2c^2gi^3 - 2\,abcdgi^3 + a^2d^2gi^3} + \frac{4\,Ab^2e\log\left(\frac{bex+ae}{dx+c}\right)}{b^2c^2gi^3 - 2\,abcdgi^3 + a^2d^2gi^3} - 2\left(\frac{4\,(bex+ae)Bbd}{(b^2c^2gi^3 - 2\,abcdgi^3 + a^2d^2gi^3)}\right.\right.$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algori
thm="giac")
```

output

```
1/4*(2*B*b^2*e*log((b*e*x + a*e)/(d*x + c))^2/(b^2*c^2*g*i^3 - 2*a*b*c*d*g
*i^3 + a^2*d^2*g*i^3) + 4*A*b^2*e*log((b*e*x + a*e)/(d*x + c))/(b^2*c^2*g*
i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3) - 2*(4*(b*e*x + a*e)*B*b*d/((b^2*c^
2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)) - (b*e*x + a*e)^2*B*
d^2/((b^2*c^2*e*g*i^3 - 2*a*b*c*d*e*g*i^3 + a^2*d^2*e*g*i^3)*(d*x + c)^2))
*log((b*e*x + a*e)/(d*x + c)) + (2*A*d^2 - B*d^2)*(b*e*x + a*e)^2/((b^2*c^
2*e*g*i^3 - 2*a*b*c*d*e*g*i^3 + a^2*d^2*e*g*i^3)*(d*x + c)^2) - 8*(A*b*d -
 B*b*d)*(b*e*x + a*e)/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(
d*x + c)))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c
- a*d)))
```

Mupad [B] (verification not implemented)

Time = 29.27 (sec) , antiderivative size = 545, normalized size of antiderivative = 2.24

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx = \frac{3\,A\,b\,c}{2\,g\,i^3\,(a\,d-b\,c)^2\,(c+d\,x)^2} - \frac{A\,a\,d}{2\,g\,i^3\,(a\,d-b\,c)^2\,(c+d\,x)^2} - \frac{B\,b^2\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2}{2\,g\,i^3\,(a\,d-b\,c)^3} + \frac{B\,a\,d}{4\,g\,i^3\,(a\,d-b\,c)^2\,(c+d\,x)^2} - \frac{7\,B\,b\,c}{4\,g\,i^3\,(a\,d-b\,c)^2\,(c+d\,x)^2} - \frac{B\,a^2\,d^2\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g\,i^3\,(a\,d-b\,c)^3\,(c+d\,x)^2} - \frac{3\,B\,b^2\,c^2\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g\,i^3\,(a\,d-b\,c)^3\,(c+d\,x)^2} + \frac{A\,b\,d\,x}{g\,i^3\,(a\,d-b\,c)^2\,(c+d\,x)^2} - \frac{3\,B\,b\,d\,x}{2\,g\,i^3\,(a\,d-b\,c)^2\,(c+d\,x)^2} + \frac{B\,a\,b\,d^2\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g\,i^3\,(a\,d-b\,c)^3\,(c+d\,x)^2} - \frac{B\,b^2\,c\,d\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g\,i^3\,(a\,d-b\,c)^3\,(c+d\,x)^2} + \frac{2\,B\,a\,b\,c\,d\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g\,i^3\,(a\,d-b\,c)^3\,(c+d\,x)^2} + \frac{A\,b^2\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g\,i^3\,(a\,d-b\,c)^3} - \frac{B\,b^2\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,3\mathrm{i}}{g\,i^3\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)*(c*i + d*i*x)^3),x
)
```

output
```
(A*b^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g*i^3*(a*d - b*
c)^3) - (B*b^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*3i)/(g*i^3*(
a*d - b*c)^3) - (B*b^2*log((e*(a + b*x))/(c + d*x))^2)/(2*g*i^3*(a*d - b*c
)^3) - (A*a*d)/(2*g*i^3*(a*d - b*c)^2*(c + d*x)^2) + (3*A*b*c)/(2*g*i^3*(a
*d - b*c)^2*(c + d*x)^2) + (B*a*d)/(4*g*i^3*(a*d - b*c)^2*(c + d*x)^2) - (
7*B*b*c)/(4*g*i^3*(a*d - b*c)^2*(c + d*x)^2) - (B*a^2*d^2*log((e*(a + b*x)
)/(c + d*x)))/(2*g*i^3*(a*d - b*c)^3*(c + d*x)^2) - (3*B*b^2*c^2*log((e*(a
 + b*x))/(c + d*x)))/(2*g*i^3*(a*d - b*c)^3*(c + d*x)^2) + (A*b*d*x)/(g*i^
3*(a*d - b*c)^2*(c + d*x)^2) - (3*B*b*d*x)/(2*g*i^3*(a*d - b*c)^2*(c + d*x
)^2) + (B*a*b*d^2*x*log((e*(a + b*x))/(c + d*x)))/(g*i^3*(a*d - b*c)^3*(c
+ d*x)^2) - (B*b^2*c*d*x*log((e*(a + b*x))/(c + d*x)))/(g*i^3*(a*d - b*c)^
3*(c + d*x)^2) + (2*B*a*b*c*d*log((e*(a + b*x))/(c + d*x)))/(g*i^3*(a*d -
b*c)^3*(c + d*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 674, normalized size of antiderivative = 2.77

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx$$

$$= \frac{i\Big(-6\log(dx+c)\,b^3c^3 + 4\log(dx+c)\,a\,b^2c^3 + 3a\,b^2d^3x^2 - 3b^3c\,d^2x^2 + 8\log\left(\frac{bex+ae}{dx+c}\right)a\,b^2c^2d + 12\log(bx -$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^3,x)
```

output

```
(i*( - 4*log(a + b*x)*a*b**2*c**3 - 8*log(a + b*x)*a*b**2*c**2*d*x - 4*log
(a + b*x)*a*b**2*c*d**2*x**2 + 6*log(a + b*x)*b**3*c**3 + 12*log(a + b*x)*
b**3*c**2*d*x + 6*log(a + b*x)*b**3*c*d**2*x**2 + 4*log(c + d*x)*a*b**2*c*
*3 + 8*log(c + d*x)*a*b**2*c**2*d*x + 4*log(c + d*x)*a*b**2*c*d**2*x**2 -
6*log(c + d*x)*b**3*c**3 - 12*log(c + d*x)*b**3*c**2*d*x - 6*log(c + d*x)*
b**3*c*d**2*x**2 - 2*log((a*e + b*e*x)/(c + d*x))**2*b**3*c**3 - 4*log((a*
e + b*e*x)/(c + d*x))**2*b**3*c**2*d*x - 2*log((a*e + b*e*x)/(c + d*x))**2
*b**3*c*d**2*x**2 - 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*c*d**2 + 8*log((
a*e + b*e*x)/(c + d*x))*a*b**2*c**2*d + 4*log((a*e + b*e*x)/(c + d*x))*a*b
**2*c*d**2*x - 6*log((a*e + b*e*x)/(c + d*x))*b**3*c**3 - 4*log((a*e + b*e
*x)/(c + d*x))*b**3*c**2*d*x - 2*a**3*c*d**2 + 6*a**2*b*c**2*d + a**2*b*c*
d**2 - 2*a**2*b*d**3*x**2 - 4*a*b**2*c**3 - 5*a*b**2*c**2*d + 2*a*b**2*c*d
**2*x**2 + 3*a*b**2*d**3*x**2 + 4*b**3*c**3 - 3*b**3*c*d**2*x**2))/(4*c*g*
(a**3*c**2*d**3 + 2*a**3*c*d**4*x + a**3*d**5*x**2 - 3*a**2*b*c**3*d**2 -
6*a**2*b*c**2*d**3*x - 3*a**2*b*c*d**4*x**2 + 3*a*b**2*c**4*d + 6*a*b**2*c
**3*d**2*x + 3*a*b**2*c**2*d**3*x**2 - b**3*c**5 - 2*b**3*c**4*d*x - b**3*
c**3*d**2*x**2))
```

3.52 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$

Optimal result . . . 648
Mathematica [C] (verified) . . . 649
Rubi [A] (verified) . . . 650
Maple [A] (verified) . . . 651
Fricas [A] (verification not implemented) . . . 653
Sympy [B] (verification not implemented) . . . 653
Maxima [B] (verification not implemented) . . . 654
Giac [F] . . . 655
Mupad [B] (verification not implemented) . . . 656
Reduce [B] (verification not implemented) . . . 657

Optimal result

Integrand size = 40, antiderivative size = 365

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = {}& \frac{Bd^3(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2} - \frac{3bBd^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)} \\ & - \frac{b^3B(c+dx)}{(bc-ad)^4g^2i^3(a+bx)} + \frac{3b^2Bd\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^2i^3} \\ & - \frac{d^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2} \\ & + \frac{3bd^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3(c+dx)} \\ & - \frac{b^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3(a+bx)} \\ & - \frac{3b^2d\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3}\end{aligned}$$

output
```
1/4*B*d^3*(b*x+a)^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2-3*b*B*d^2*(b*x+a)/(-a*d
+b*c)^4/g^2/i^3/(d*x+c)-b^3*B*(d*x+c)/(-a*d+b*c)^4/g^2/i^3/(b*x+a)+3/2*b^2
*B*d*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^2/i^3-1/2*d^3*(b*x+a)^2*(A+B*ln(
e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2+3*b*d^2*(b*x+a)*(A+B*ln
(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3/(d*x+c)-b^3*(d*x+c)*(A+B*ln(e*(b
*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-3*b^2*d*ln((b*x+a)/(d*x+c))*(
A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.24

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$$

$$= \frac{-\frac{4b^3Bc}{a+bx} + \frac{4ab^2Bd}{a+bx} + \frac{Bd(bc-ad)^2}{(c+dx)^2} + \frac{8b^2Bcd}{c+dx} - \frac{8abBd^2}{c+dx} + \frac{2bBd(bc-ad)}{c+dx} + 6b^2Bd\log(a+bx) - \frac{4b^2(bc-ad)\left(A+B\log\left(\frac{e(a+}{c+}\right.\right.}{a+bx}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i
*x)^3),x]
```

output
```
((-4*b^3*B*c)/(a + b*x) + (4*a*b^2*B*d)/(a + b*x) + (B*d*(b*c - a*d)^2)/(c
 + d*x)^2 + (8*b^2*B*c*d)/(c + d*x) - (8*a*b*B*d^2)/(c + d*x) + (2*b*B*d*(
b*c - a*d))/(c + d*x) + 6*b^2*B*d*Log[a + b*x] - (4*b^2*(b*c - a*d)*(A + B
*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) - (2*d*(b*c - a*d)^2*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/(c + d*x)^2 - (8*b*d*(b*c - a*d)*(A + B*Log[(e*
(a + b*x))/(c + d*x)]))/(c + d*x) - 12*b^2*d*Log[a + b*x]*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]) - 6*b^2*B*d*Log[c + d*x] + 12*b^2*d*(A + B*Log[(e*(a
+ b*x))/(c + d*x)])*Log[c + d*x] + 6*b^2*B*d*(Log[a + b*x]*(Log[a + b*x] -
 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) +
a*d)]) - 6*b^2*B*d*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*L
og[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4*(b*c - a*d)^4*g
^2*i^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2962, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{(ag+bgx)^2(ci+dix)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2} d\frac{a+bx}{c+dx}}{g^2i^3(bc-ad)^4}$$

$$\downarrow 2772$$

$$\frac{-B\int\left(-\frac{(c+dx)^2b^3}{(a+bx)^2} - \frac{3d(c+dx)\log\left(\frac{a+bx}{c+dx}\right)b^2}{a+bx} + 3d^2b - \frac{d^3(a+bx)}{2(c+dx)}\right) d\frac{a+bx}{c+dx} - \frac{b^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx} - 3b^2d\log\left(\frac{a+bx}{c+dx}\right.}{g^2i^3(bc-ad)^4}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx} - 3b^2d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right) - \frac{d^3(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(c+dx)^2} + \frac{3bd^2(a+bx)\left(\right.}{g^2i^3(bc-ad)^4}$$

input

```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i*x)^3)
,x]
```

output

```
(-(B*(-1/4*(d^3*(a + b*x)^2)/(c + d*x)^2 + (3*b*d^2*(a + b*x))/(c + d*x) +
 (b^3*(c + d*x))/(a + b*x) - (3*b^2*d*Log[(a + b*x)/(c + d*x)]^2)/2)) - (d
^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(c + d*x)^2) + (3*
b*d^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) - (b^3*(c
+ d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) - 3*b^2*d*Log[(a +
b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^4*g^2*i
^3)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.39

method	result
parts	$\frac{A\left(-\frac{d}{2(da-bc)^2(dx+c)^2}+\frac{3d\,b^2\ln(dx+c)}{(da-bc)^4}+\frac{2db}{(da-bc)^3(dx+c)}+\frac{b^2}{(da-bc)^3(bx+a)}-\frac{3d\,b^2\ln(bx+a)}{(da-bc)^4}\right)}{g^2i^3}-\frac{Bd\left(\frac{d^2\left(\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right.}{}\right.}{}\right.}{}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{d^2Ab^3}{i^3(da-bc)^5g^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{3d^3Ab^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei^3(da-bc)^5g^2}-\frac{3d^4Ab\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e^2i^3(da-bc)^5g^2}+\frac{d^5A\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2e^3i^3(da-bc)^5g^2}\right.}{}$
default	$-\frac{e(da-bc)\left(\frac{d^2Ab^3}{i^3(da-bc)^5g^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{3d^3Ab^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei^3(da-bc)^5g^2}-\frac{3d^4Ab\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e^2i^3(da-bc)^5g^2}+\frac{d^5A\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2e^3i^3(da-bc)^5g^2}\right.}{}$
risch	$-\frac{Ad}{2g^2i^3(da-bc)^2(dx+c)^2}+\frac{3Ad\,b^2\ln(dx+c)}{g^2i^3(da-bc)^4}+\frac{2Adb}{g^2i^3(da-bc)^3(dx+c)}+\frac{A\,b^2}{g^2i^3(da-bc)^3(bx+a)}-\frac{3Ad\,b^2\ln(bx+}{g^2i^3(da-bc)^}$
parallelrisch	$-\frac{12Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a\,b^5c\,d^6+24Ax\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^5c\,d^6-24Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^5c\,d^6+18Ax\,b^6c^2d^5+9Bx\,a^2b^4d^7-3}{}$
norman	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,method=_RETU
RNVERBOSE)
```

output
```
1/g^2*A/i^3*(-1/2*d/(a*d-b*c)^2/(d*x+c)^2+3*d/(a*d-b*c)^4*b^2*ln(d*x+c)+2*
d/(a*d-b*c)^3*b/(d*x+c)+b^2/(a*d-b*c)^3/(b*x+a)-3*d/(a*d-b*c)^4*b^2*ln(b*x
+a))-B/g^2/i^3*d/(a*d-b*c)^2/e^2*(d^2/(a*d-b*c)^2*(1/2*(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^2)-3*d/(a*d-b*c)^2*b*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b
*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)+3/2/(a*d-b*c)^2*b^2*e^2*ln(b
*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/d/(a*d-b*c)^2*b^3*e^3*(-1/(b*e/d+(a*d-b*c)
*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+
c))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.84

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx =$$

$$-\frac{4\,(A+B)b^3c^3 + 3\,(2\,A-5\,B)ab^2c^2d - 12\,(A-B)a^2bcd^2 + (2\,A-B)a^3d^3 + 6\,((2\,A-B)b^3cd^2 - (2\,A}{4\,((b^5c^4d^2 - 4}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, algo
rithm="fricas")
```

output
```
-1/4*(4*(A + B)*b^3*c^3 + 3*(2*A - 5*B)*a*b^2*c^2*d - 12*(A - B)*a^2*b*c*d
^2 + (2*A - B)*a^3*d^3 + 6*((2*A - B)*b^3*c*d^2 - (2*A - B)*a*b^2*d^3)*x^2
 + 6*(B*b^3*d^3*x^3 + B*a*b^2*c^2*d + (2*B*b^3*c*d^2 + B*a*b^2*d^3)*x^2 +
(B*b^3*c^2*d + 2*B*a*b^2*c*d^2)*x)*log((b*e*x + a*e)/(d*x + c))^2 + 3*((6*
A - B)*b^3*c^2*d - 2*(2*A + B)*a*b^2*c*d^2 - (2*A - 3*B)*a^2*b*d^3)*x + 2*
(3*(2*A - B)*b^3*d^3*x^3 + 2*B*b^3*c^3 + 6*A*a*b^2*c^2*d - 6*B*a^2*b*c*d^2
 + B*a^3*d^3 + 3*(4*A*b^3*c*d^2 + (2*A - 3*B)*a*b^2*d^3)*x^2 + 3*(2*(A + B
)*b^3*c^2*d + 4*(A - B)*a*b^2*c*d^2 - B*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d
*x + c)))/((b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*
c*d^5 + a^4*b*d^6)*g^2*i^3*x^3 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^
3*c^3*d^3 - 2*a^3*b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*g^2*i^3*x^2 + (b^
5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^
2*d^4 + 2*a^5*c*d^5)*g^2*i^3*x + (a*b^4*c^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*
c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4)*g^2*i^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1562 vs. $2(335) = 670$.

Time = 12.23 (sec) , antiderivative size = 1562, normalized size of antiderivative = 4.28

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2/(d*i*x+c*i)**3,x)
```

output

```
-3*B*b**2*d*log(e*(a + b*x)/(c + d*x))**2/(2*a**4*d**4*g**2*i**3 - 8*a**3*
b*c*d**3*g**2*i**3 + 12*a**2*b**2*c**2*d**2*g**2*i**3 - 8*a*b**3*c**3*d*g*
*2*i**3 + 2*b**4*c**4*g**2*i**3) + 3*b**2*d*(2*A - B)*log(x + (6*A*a*b**2*
d**2 + 6*A*b**3*c*d - 3*B*a*b**2*d**2 - 3*B*b**3*c*d - 3*a**5*b**2*d**6*(2
*A - B)/(a*d - b*c)**4 + 15*a**4*b**3*c*d**5*(2*A - B)/(a*d - b*c)**4 - 30
*a**3*b**4*c**2*d**4*(2*A - B)/(a*d - b*c)**4 + 30*a**2*b**5*c**3*d**3*(2*
A - B)/(a*d - b*c)**4 - 15*a*b**6*c**4*d**2*(2*A - B)/(a*d - b*c)**4 + 3*b
**7*c**5*d*(2*A - B)/(a*d - b*c)**4)/(12*A*b**3*d**2 - 6*B*b**3*d**2))/(2*
g**2*i**3*(a*d - b*c)**4) - 3*b**2*d*(2*A - B)*log(x + (6*A*a*b**2*d**2 +
6*A*b**3*c*d - 3*B*a*b**2*d**2 - 3*B*b**3*c*d + 3*a**5*b**2*d**6*(2*A - B)
/(a*d - b*c)**4 - 15*a**4*b**3*c*d**5*(2*A - B)/(a*d - b*c)**4 + 30*a**3*b
**4*c**2*d**4*(2*A - B)/(a*d - b*c)**4 - 30*a**2*b**5*c**3*d**3*(2*A - B)/
(a*d - b*c)**4 + 15*a*b**6*c**4*d**2*(2*A - B)/(a*d - b*c)**4 - 3*b**7*c**
5*d*(2*A - B)/(a*d - b*c)**4)/(12*A*b**3*d**2 - 6*B*b**3*d**2))/(2*g**2*i*
*3*(a*d - b*c)**4) + (-B*a**2*d**2 + 5*B*a*b*c*d + 3*B*a*b*d**2*x + 2*B*b*
*2*c**2 + 9*B*b**2*c*d*x + 6*B*b**2*d**2*x**2)*log(e*(a + b*x)/(c + d*x))/
(2*a**4*c**2*d**3*g**2*i**3 + 4*a**4*c*d**4*g**2*i**3*x + 2*a**4*d**5*g**2
*i**3*x**2 - 6*a**3*b*c**3*d**2*g**2*i**3 - 10*a**3*b*c**2*d**3*g**2*i**3*
x - 2*a**3*b*c*d**4*g**2*i**3*x**2 + 2*a**3*b*d**5*g**2*i**3*x**3 + 6*a**2
*b**2*c**4*d*g**2*i**3 + 6*a**2*b**2*c**3*d**2*g**2*i**3*x - 6*a**2*b**...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1721 vs. $2(359) = 718$.

Time $= 0.16$ (sec) , antiderivative size $= 1721$, normalized size of antiderivative $= 4.72$

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^2(ci + dix)^3}\, dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, algo
rithm="maxima")
```

output

```
-1/2*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d +
a*b*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)
*g^2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*
c*d^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2
+ 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*d
+ 3*a^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c^4
 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3) -
 6*b^2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^
3*b*c*d^3 + a^4*d^4)*g^2*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/2*
A*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d
^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*g^2*
i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4
 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a
^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a
^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c^4 - 4*
a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3) - 6*b^
2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c
*d^3 + a^4*d^4)*g^2*i^3)) - 1/4*(4*b^3*c^3 - 15*a*b^2*c^2*d + 12*a^2*b*c*d
^2 - a^3*d^3 - 6*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 6*(b^3*d^3*x^3 + a*b^2*c^2*
d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(...
```

Giac [F]

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right) + A}{(bgx+ag)^2(dix+ci)^3}\,dx$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, algo
rithm="giac")
```

output

```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)/((b*g*x + a*g)^2*(d*i*x + c*i
)^3), x)
```

Mupad [B] (verification not implemented)

Time = 31.33 (sec) , antiderivative size = 983, normalized size of antiderivative = 2.69

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^2*(c*i + d*i*x)^3)
,x)
```

output

```
(A*b^2*d*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*6i)/(g^2*i^3*(a*d
- b*c)^4) - (3*B*b^2*d*log((e*(a + b*x))/(c + d*x))^2)/(2*g^2*i^3*(a*d - b
*c)^4) - (B*b^2*d*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*3i)/(g^2*
i^3*(a*d - b*c)^4) - (A*a^2*d^2)/(2*g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d
*x)^2) + (A*b^2*c^2)/(g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (B*a^
2*d^2)/(4*g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (B*b^2*c^2)/(g^2*
i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) - (B*a*d*log((e*(a + b*x))/(c + d
*x)))/(2*g^2*i^3*(a*d - b*c)^2*(a + b*x)*(c + d*x)^2) - (B*b*c*log((e*(a +
 b*x))/(c + d*x)))/(g^2*i^3*(a*d - b*c)^2*(a + b*x)*(c + d*x)^2) + (3*A*b^
2*d^2*x^2)/(g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) - (3*B*b^2*d^2*x^
2)/(2*g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (5*A*a*b*c*d)/(2*g^2*
i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) - (11*B*a*b*c*d)/(4*g^2*i^3*(a*d
- b*c)^3*(a + b*x)*(c + d*x)^2) - (3*B*b*d*x*log((e*(a + b*x))/(c + d*x)))
/(2*g^2*i^3*(a*d - b*c)^2*(a + b*x)*(c + d*x)^2) + (3*B*b^2*d^2*x^2*log((e
*(a + b*x))/(c + d*x)))/(g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (3
*A*a*b*d^2*x)/(2*g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) - (9*B*a*b*d
^2*x)/(4*g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (9*A*b^2*c*d*x)/(2
*g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) - (3*B*b^2*c*d*x)/(4*g^2*i^3
*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (3*B*a*b*c*d*log((e*(a + b*x))/(c
+ d*x)))/(g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (3*B*a*b*d^2*x...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1782, normalized size of antiderivative = 4.88

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x)
```

output

```
(i*( - 12*log(a + b*x)*a**3*b**2*c**2*d**2 - 24*log(a + b*x)*a**3*b**2*c*d
**3*x - 12*log(a + b*x)*a**3*b**2*d**4*x**2 - 24*log(a + b*x)*a**2*b**3*c*
*3*d - 60*log(a + b*x)*a**2*b**3*c**2*d**2*x + 18*log(a + b*x)*a**2*b**3*c
**2*d**2 - 48*log(a + b*x)*a**2*b**3*c*d**3*x**2 + 36*log(a + b*x)*a**2*b*
*3*c*d**3*x - 12*log(a + b*x)*a**2*b**3*d**4*x**3 + 18*log(a + b*x)*a**2*b
**3*d**4*x**2 - 24*log(a + b*x)*a*b**4*c**3*d*x - 48*log(a + b*x)*a*b**4*c
**2*d**2*x**2 + 18*log(a + b*x)*a*b**4*c**2*d**2*x - 24*log(a + b*x)*a*b**
4*c*d**3*x**3 + 36*log(a + b*x)*a*b**4*c*d**3*x**2 + 18*log(a + b*x)*a*b**
4*d**4*x**3 + 12*log(c + d*x)*a**3*b**2*c**2*d**2 + 24*log(c + d*x)*a**3*b
**2*c*d**3*x + 12*log(c + d*x)*a**3*b**2*d**4*x**2 + 24*log(c + d*x)*a**2*
b**3*c**3*d + 60*log(c + d*x)*a**2*b**3*c**2*d**2*x - 18*log(c + d*x)*a**2
*b**3*c**2*d**2 + 48*log(c + d*x)*a**2*b**3*c*d**3*x**2 - 36*log(c + d*x)*
a**2*b**3*c*d**3*x + 12*log(c + d*x)*a**2*b**3*d**4*x**3 - 18*log(c + d*x)
*a**2*b**3*d**4*x**2 + 24*log(c + d*x)*a*b**4*c**3*d*x + 48*log(c + d*x)*a
*b**4*c**2*d**2*x**2 - 18*log(c + d*x)*a*b**4*c**2*d**2*x + 24*log(c + d*x
)*a*b**4*c*d**3*x**3 - 36*log(c + d*x)*a*b**4*c*d**3*x**2 - 18*log(c + d*x
)*a*b**4*d**4*x**3 - 6*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**3*c**2*d**2
 - 12*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**3*c*d**3*x - 6*log((a*e + b*
e*x)/(c + d*x))**2*a**2*b**3*d**4*x**2 - 12*log((a*e + b*e*x)/(c + d*x))**
2*a*b**4*c**3*d - 30*log((a*e + b*e*x)/(c + d*x))**2*a*b**4*c**2*d**2*x...
```

3.53 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx$

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Maple [A] (verified) . . . 662
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Mupad [B] (verification not implemented) . . . 666
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Optimal result

Integrand size = 40, antiderivative size = 463

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = &-\frac{Bd^4(a+bx)^2}{4(bc-ad)^5g^3i^3(c+dx)^2}\\ &+\frac{4bBd^3(a+bx)}{(bc-ad)^5g^3i^3(c+dx)}+\frac{4b^3Bd(c+dx)}{(bc-ad)^5g^3i^3(a+bx)}\\ &-\frac{b^4B(c+dx)^2}{4(bc-ad)^5g^3i^3(a+bx)^2}-\frac{3b^2Bd^2\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}\\ &+\frac{d^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2}\\ &-\frac{4bd^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3(c+dx)}\\ &+\frac{4b^3d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3(a+bx)}\\ &-\frac{b^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2}\\ &+\frac{6b^2d^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3}\end{aligned}$$

output

```
-1/4*B*d^4*(b*x+a)^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2+4*b*B*d^3*(b*x+a)/(-a*
d+b*c)^5/g^3/i^3/(d*x+c)+4*b^3*B*d*(d*x+c)/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/
4*b^4*B*(d*x+c)^2/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2-3*b^2*B*d^2*ln((b*x+a)/(d
*x+c))^2/(-a*d+b*c)^5/g^3/i^3+1/2*d^4*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c))
)/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2-4*b*d^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)
))/(-a*d+b*c)^5/g^3/i^3/(d*x+c)+4*b^3*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c))
)/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/2*b^4*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)
))/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+6*b^2*d^2*ln((b*x+a)/(d*x+c))*(A+B*ln(e*
(b*x+a)/(d*x+c)))/(-a*d+b*c)^5/g^3/i^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.07 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.15

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx =$$

$$-\frac{\frac{b^2B(bc-ad)^2}{(a+bx)^2} - \frac{12b^3Bcd}{a+bx} + \frac{12ab^2Bd^2}{a+bx} - \frac{2b^2Bd(bc-ad)}{a+bx} + \frac{Bd^2(bc-ad)^2}{(c+dx)^2} + \frac{12b^2Bcd^2}{c+dx} - \frac{12abBd^3}{c+dx} + \frac{2bBd^2(bc-ad)}{c+dx} + \frac{2b^2(bc-}{}$$

input

```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i
*x)^3),x]
```

output

```
-1/4*((b^2*B*(b*c - a*d)^2)/(a + b*x)^2 - (12*b^3*B*c*d)/(a + b*x) + (12*a
*b^2*B*d^2)/(a + b*x) - (2*b^2*B*d*(b*c - a*d))/(a + b*x) + (B*d^2*(b*c -
a*d)^2)/(c + d*x)^2 + (12*b^2*B*c*d^2)/(c + d*x) - (12*a*b*B*d^3)/(c + d*x
) + (2*b*B*d^2*(b*c - a*d))/(c + d*x) + (2*b^2*(b*c - a*d)^2*(A + B*Log[(e
*(a + b*x))/(c + d*x)]))/(a + b*x)^2 - (12*b^2*d*(b*c - a*d)*(A + B*Log[(e
*(a + b*x))/(c + d*x)]))/(a + b*x) - (2*d^2*(b*c - a*d)^2*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/(c + d*x)^2 - (12*b*d^2*(b*c - a*d)*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/(c + d*x) - 24*b^2*d^2*Log[a + b*x]*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]) + 24*b^2*d^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log
[c + d*x] + 12*b^2*B*d^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))
/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 12*b^2*B*d^
2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*P
olyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^5*g^3*i^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2962, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{(ag+bgx)^3(ci+dix)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3} d\frac{a+bx}{c+dx}}{g^3 i^3 (bc-ad)^5}$$

$$\downarrow 2772$$

$$-B\int\left(\frac{6b^2(c+dx)\log\left(\frac{a+bx}{c+dx}\right)d^2}{a+bx} + \frac{1}{2}\left(-\frac{(c+dx)^3 b^4}{(a+bx)^3} + \frac{8d(c+dx)^2 b^3}{(a+bx)^2} - 8d^3 b + \frac{d^4(a+bx)}{c+dx}\right)\right) d\frac{a+bx}{c+dx} - \frac{b^4(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right.}{2(a+bx)^2}$$

$$\downarrow 2009$$

$$-\frac{b^4(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2} + \frac{4b^3 d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx} + 6b^2 d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right) + \frac{d^4(a+bx)}{g^3}$$

input

```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i*x)^3)
,x]
```

output
```
(-(B*((d^4*(a + b*x)^2)/(4*(c + d*x)^2) - (4*b*d^3*(a + b*x))/(c + d*x) -
(4*b^3*d*(c + d*x))/(a + b*x) + (b^4*(c + d*x)^2)/(4*(a + b*x)^2) + 3*b^2*
d^2*Log[(a + b*x)/(c + d*x)]^2)) + (d^4*(a + b*x)^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)]))/(2*(c + d*x)^2) - (4*b*d^3*(a + b*x)*(A + B*Log[(e*(a + b*
x))/(c + d*x)]))/(c + d*x) + (4*b^3*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(
c + d*x)]))/(a + b*x) - (b^4*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(2*(a + b*x)^2) + 6*b^2*d^2*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/((b*c - a*d)^5*g^3*i^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 649, normalized size of antiderivative = 1.40

method	result
parts	$\frac{A\left(-\frac{d^2}{2(da-bc)^3(dx+c)^2}+\frac{6d^2b^2\ln(dx+c)}{(da-bc)^5}+\frac{3d^2b}{(da-bc)^4(dx+c)}+\frac{b^2}{2(da-bc)^3(bx+a)^2}-\frac{6d^2b^2\ln(bx+a)}{(da-bc)^5}+\frac{3b^2d}{(da-bc)^4(bx+a)}\right)}{g^3i^3}-$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{d^2eAb^4}{2i^3(da-bc)^6g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{4d^3Ab^3}{i^3(da-bc)^6g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{6d^4Ab^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei^3(da-bc)^6g^3}-\frac{4d^5Ab\left(\frac{be}{d}\right.}{e^2i^3(d}$
default	$-\frac{e(da-bc)\left(-\frac{d^2eAb^4}{2i^3(da-bc)^6g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{4d^3Ab^3}{i^3(da-bc)^6g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{6d^4Ab^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei^3(da-bc)^6g^3}-\frac{4d^5Ab\left(\frac{be}{d}\right.}{e^2i^3(d}$
risch	Expression too large to display
parallelrisch	Expression too large to display
norman	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,method=_RETU
RNVERBOSE)
```

output
```
A/g^3/i^3*(-1/2*d^2/(a*d-b*c)^3/(d*x+c)^2+6*d^2/(a*d-b*c)^5*b^2*ln(d*x+c)+
3*d^2/(a*d-b*c)^4*b/(d*x+c)+1/2*b^2/(a*d-b*c)^3/(b*x+a)^2-6*d^2/(a*d-b*c)^
5*b^2*ln(b*x+a)+3*b^2/(a*d-b*c)^4*d/(b*x+a))-B/g^3/i^3*d/(a*d-b*c)^2/e^2*(
d^3/(a*d-b*c)^3*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-4*d^2/(a*d-b*c)^3*b*e*((b*
e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(
d*x+c)-b*e/d)+3*d/(a*d-b*c)^3*b^2*e^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-4/
(a*d-b*c)^3*b^3*e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e
/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+1/d/(a*d-b*c)^3*b^4*e^4*(-1/2
/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/
d+(a*d-b*c)*e/d/(d*x+c))^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1011 vs. $2(455) = 910$.

Time = 0.11 (sec) , antiderivative size = 1011, normalized size of antiderivative = 2.18

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^3(ci + dix)^3}\, dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, algo
rithm="fricas")
```

output

```
-1/4*((2*A + B)*b^4*c^4 - 16*(A + B)*a*b^3*c^3*d + 30*B*a^2*b^2*c^2*d^2 +
16*(A - B)*a^3*b*c*d^3 - (2*A - B)*a^4*d^4 - 24*(A*b^4*c*d^3 - A*a*b^3*d^4
)*x^3 - 12*((3*A + B)*b^4*c^2*d^2 - 2*B*a*b^3*c*d^3 - (3*A - B)*a^2*b^2*d^
4)*x^2 - 12*(B*b^4*d^4*x^4 + B*a^2*b^2*c^2*d^2 + 2*(B*b^4*c*d^3 + B*a*b^3*
d^4)*x^3 + (B*b^4*c^2*d^2 + 4*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*x^2 + 2*(B*a*
b^3*c^2*d^2 + B*a^2*b^2*c*d^3)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 4*((2*A
 + 3*B)*b^4*c^3*d + 3*(4*A - B)*a*b^3*c^2*d^2 - 3*(4*A + B)*a^2*b^2*c*d^3
 - (2*A - 3*B)*a^3*b*d^4)*x - 2*(12*A*b^4*d^4*x^4 - B*b^4*c^4 + 8*B*a*b^3*c
^3*d + 12*A*a^2*b^2*c^2*d^2 - 8*B*a^3*b*c*d^3 + B*a^4*d^4 + 12*((2*A + B)*
b^4*c*d^3 + (2*A - B)*a*b^3*d^4)*x^3 + 6*((2*A + 3*B)*b^4*c^2*d^2 + 8*A*a*
b^3*c*d^3 + (2*A - 3*B)*a^2*b^2*d^4)*x^2 + 4*(B*b^4*c^3*d + 6*(A + B)*a*b^
3*c^2*d^2 + 6*(A - B)*a^2*b^2*c*d^3 - B*a^3*b*d^4)*x)*log((b*e*x + a*e)/(d
*x + c)))/((b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^
4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*g^3*i^3*x^4 + 2*(b^7*c^6*d - 4*
a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 - 5*a^4*b^3*c^2*d^5 + 4*a^5*b^2*c*d^6 -
a^6*b*d^7)*g^3*i^3*x^3 + (b^7*c^7 - a*b^6*c^6*d - 9*a^2*b^5*c^5*d^2 + 25*a
^3*b^4*c^4*d^3 - 25*a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 - a^
7*d^7)*g^3*i^3*x^2 + 2*(a*b^6*c^7 - 4*a^2*b^5*c^6*d + 5*a^3*b^4*c^5*d^2 -
5*a^5*b^2*c^3*d^4 + 4*a^6*b*c^2*d^5 - a^7*c*d^6)*g^3*i^3*x + (a^2*b^5*c^7
- 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2106 vs. $2(430) = 860$.

Time = 19.85 (sec) , antiderivative size = 2106, normalized size of antiderivative = 4.55

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3/(d*i*x+c*i)**3,x)
```

output

```
6*A*b**2*d**2*log(x + (-6*A*a**6*b**2*d**8/(a*d - b*c)**5 + 36*A*a**5*b**3
*c*d**7/(a*d - b*c)**5 - 90*A*a**4*b**4*c**2*d**6/(a*d - b*c)**5 + 120*A*a
**3*b**5*c**3*d**5/(a*d - b*c)**5 - 90*A*a**2*b**6*c**4*d**4/(a*d - b*c)**
5 + 36*A*a*b**7*c**5*d**3/(a*d - b*c)**5 + 6*A*a*b**2*d**3 - 6*A*b**8*c**6
*d**2/(a*d - b*c)**5 + 6*A*b**3*c*d**2)/(12*A*b**3*d**3))/(g**3*i**3*(a*d
- b*c)**5) - 6*A*b**2*d**2*log(x + (6*A*a**6*b**2*d**8/(a*d - b*c)**5 - 36
*A*a**5*b**3*c*d**7/(a*d - b*c)**5 + 90*A*a**4*b**4*c**2*d**6/(a*d - b*c)*
*5 - 120*A*a**3*b**5*c**3*d**5/(a*d - b*c)**5 + 90*A*a**2*b**6*c**4*d**4/(
a*d - b*c)**5 - 36*A*a*b**7*c**5*d**3/(a*d - b*c)**5 + 6*A*a*b**2*d**3 + 6
*A*b**8*c**6*d**2/(a*d - b*c)**5 + 6*A*b**3*c*d**2)/(12*A*b**3*d**3))/(g**
3*i**3*(a*d - b*c)**5) - 3*B*b**2*d**2*log(e*(a + b*x)/(c + d*x))**2/(a**5
*d**5*g**3*i**3 - 5*a**4*b*c*d**4*g**3*i**3 + 10*a**3*b**2*c**2*d**3*g**3*
i**3 - 10*a**2*b**3*c**3*d**2*g**3*i**3 + 5*a*b**4*c**4*d*g**3*i**3 - b**5
*c**5*g**3*i**3) + (-B*a**3*d**3 + 7*B*a**2*b*c*d**2 + 4*B*a**2*b*d**3*x +
 7*B*a*b**2*c**2*d + 28*B*a*b**2*c*d**2*x + 18*B*a*b**2*d**3*x**2 - B*b**3
*c**3 + 4*B*b**3*c**2*d*x + 18*B*b**3*c*d**2*x**2 + 12*B*b**3*d**3*x**3)*l
og(e*(a + b*x)/(c + d*x))/(2*a**6*c**2*d**4*g**3*i**3 + 4*a**6*c*d**5*g**3
*i**3*x + 2*a**6*d**6*g**3*i**3*x**2 - 8*a**5*b*c**3*d**3*g**3*i**3 - 12*a
**5*b*c**2*d**4*g**3*i**3*x + 4*a**5*b*d**6*g**3*i**3*x**3 + 12*a**4*b**2*
c**4*d**2*g**3*i**3 + 8*a**4*b**2*c**3*d**3*g**3*i**3*x - 18*a**4*b**2*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2380 vs. $2(455) = 910$.

Time $= 0.18$ (sec) , antiderivative size $= 2380$, normalized size of antiderivative $= 5.14$

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, algo
rithm="maxima")
```

output

```
1/2*B*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3
 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b*d
^3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d
^5 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4
*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3 +
(b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^
6*d^6)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 +
2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^6
- 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g^3
*i^3) + 12*b^2*d^2*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3
*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d^2
*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*
c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3))*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) + 1/2*A*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2
 - a^3*d^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2
 + a^2*b*d^3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a
^3*b^3*c*d^5 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 +
 2*a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*
i^3*x^3 + (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^
2*d^4 + a^6*d^6)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b...
```

Giac [F]

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = \int \frac{B \log\left(\frac{(bx+a)e}{dx+c}\right) + A}{(bgx+ag)^3(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, algo
rithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)/((b*g*x + a*g)^3*(d*i*x + c*i
)^3), x)
```

Mupad [B] (verification not implemented)

Time = 35.02 (sec) , antiderivative size = 1443, normalized size of antiderivative = 3.12

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^3*(c*i + d*i*x)^3)
,x)
```

output

```
(A*b^2*d^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*12i)/(g^3*i^3*(a
*d - b*c)^5) - (3*B*b^2*d^2*log((e*(a + b*x))/(c + d*x))^2)/(g^3*i^3*(a*d
- b*c)^5) - (A*a^3*d^3)/(2*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2)
- (A*b^3*c^3)/(2*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (B*a^3*d
^3)/(4*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) - (B*b^3*c^3)/(4*g^3
*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) - (B*a*d*log((e*(a + b*x))/(c
+ d*x)))/(2*g^3*i^3*(a*d - b*c)^2*(a + b*x)^2*(c + d*x)^2) - (B*b*c*log((e
*(a + b*x))/(c + d*x)))/(2*g^3*i^3*(a*d - b*c)^2*(a + b*x)^2*(c + d*x)^2)
+ (6*A*b^3*d^3*x^3)/(g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (7*A
*a*b^2*c^2*d)/(2*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (7*A*a^2
*b*c*d^2)/(2*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (15*B*a*b^2*
c^2*d)/(4*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) - (15*B*a^2*b*c*d
^2)/(4*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (2*A*a^2*b*d^3*x)/
(g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) - (3*B*a^2*b*d^3*x)/(g^3*i
^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (2*A*b^3*c^2*d*x)/(g^3*i^3*(a*
d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (3*B*b^3*c^2*d*x)/(g^3*i^3*(a*d - b*
c)^4*(a + b*x)^2*(c + d*x)^2) + (9*A*a*b^2*d^3*x^2)/(g^3*i^3*(a*d - b*c)^4
*(a + b*x)^2*(c + d*x)^2) - (3*B*a*b^2*d^3*x^2)/(g^3*i^3*(a*d - b*c)^4*(a
+ b*x)^2*(c + d*x)^2) + (9*A*b^3*c*d^2*x^2)/(g^3*i^3*(a*d - b*c)^4*(a + b*
x)^2*(c + d*x)^2) + (3*B*b^3*c*d^2*x^2)/(g^3*i^3*(a*d - b*c)^4*(a + b*x...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2732, normalized size of antiderivative = 5.90

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^3(ci + dix)^3}\, dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x)
```

output

```
(i*( - 24*log(a + b*x)*a**4*b**2*c**2*d**3 - 48*log(a + b*x)*a**4*b**2*c*d
**4*x - 24*log(a + b*x)*a**4*b**2*d**5*x**2 - 24*log(a + b*x)*a**3*b**3*c*
*3*d**2 - 96*log(a + b*x)*a**3*b**3*c**2*d**3*x + 12*log(a + b*x)*a**3*b**
3*c**2*d**3 - 120*log(a + b*x)*a**3*b**3*c*d**4*x**2 + 24*log(a + b*x)*a**
3*b**3*c*d**4*x - 48*log(a + b*x)*a**3*b**3*d**5*x**3 + 12*log(a + b*x)*a*
*3*b**3*d**5*x**2 - 48*log(a + b*x)*a**2*b**4*c**3*d**2*x - 12*log(a + b*x
)*a**2*b**4*c**3*d**2 - 120*log(a + b*x)*a**2*b**4*c**2*d**3*x**2 - 96*log
(a + b*x)*a**2*b**4*c*d**4*x**3 + 36*log(a + b*x)*a**2*b**4*c*d**4*x**2 -
24*log(a + b*x)*a**2*b**4*d**5*x**4 + 24*log(a + b*x)*a**2*b**4*d**5*x**3
- 24*log(a + b*x)*a*b**5*c**3*d**2*x**2 - 24*log(a + b*x)*a*b**5*c**3*d**2
*x - 48*log(a + b*x)*a*b**5*c**2*d**3*x**3 - 36*log(a + b*x)*a*b**5*c**2*d
**3*x**2 - 24*log(a + b*x)*a*b**5*c*d**4*x**4 + 12*log(a + b*x)*a*b**5*d**
5*x**4 - 12*log(a + b*x)*b**6*c**3*d**2*x**2 - 24*log(a + b*x)*b**6*c**2*d
**3*x**3 - 12*log(a + b*x)*b**6*c*d**4*x**4 + 24*log(c + d*x)*a**4*b**2*c*
*2*d**3 + 48*log(c + d*x)*a**4*b**2*c*d**4*x + 24*log(c + d*x)*a**4*b**2*d
**5*x**2 + 24*log(c + d*x)*a**3*b**3*c**3*d**2 + 96*log(c + d*x)*a**3*b**3
*c**2*d**3*x - 12*log(c + d*x)*a**3*b**3*c**2*d**3 + 120*log(c + d*x)*a**3
*b**3*c*d**4*x**2 - 24*log(c + d*x)*a**3*b**3*c*d**4*x + 48*log(c + d*x)*a
**3*b**3*d**5*x**3 - 12*log(c + d*x)*a**3*b**3*d**5*x**2 + 48*log(c + d*x)
*a**2*b**4*c**3*d**2*x + 12*log(c + d*x)*a**2*b**4*c**3*d**2 + 120*log(...
```

3.54 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx$

Optimal result 670
Mathematica [C] (verified) 671
Rubi [A] (verified) 672
Maple [A] (verified) 674
Fricas [B] (verification not implemented) 675
Sympy [F(-1)] 676
Maxima [B] (verification not implemented) 677
Giac [F] 678
Mupad [B] (verification not implemented) 678
Reduce [B] (verification not implemented) 679

Optimal result

Integrand size = 40, antiderivative size = 563

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = {} & \frac{Bd^5(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2}-\frac{5bBd^4(a+bx)}{(bc-ad)^6g^4i^3(c+dx)}\\ & -\frac{10b^3Bd^2(c+dx)}{(bc-ad)^6g^4i^3(a+bx)}+\frac{5b^4Bd(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2}\\ & -\frac{b^5B(c+dx)^3}{9(bc-ad)^6g^4i^3(a+bx)^3}+\frac{5b^2Bd^3\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}\\ & -\frac{d^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2}\\ & +\frac{5bd^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(c+dx)}\\ & -\frac{10b^3d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(a+bx)}\\ & +\frac{5b^4d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2}\\ & -\frac{b^5(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3}\\ & -\frac{10b^2d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3}\end{aligned}$$

output

```
1/4*B*d^5*(b*x+a)^2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2-5*b*B*d^4*(b*x+a)/(-a*d
+b*c)^6/g^4/i^3/(d*x+c)-10*b^3*B*d^2*(d*x+c)/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+
5/4*b^4*B*d*(d*x+c)^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-1/9*b^5*B*(d*x+c)^3/(
-a*d+b*c)^6/g^4/i^3/(b*x+a)^3+5*b^2*B*d^3*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)
^6/g^4/i^3-1/2*d^5*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^6/g^4/
i^3/(d*x+c)^2+5*b*d^4*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^6/g^4
/i^3/(d*x+c)-10*b^3*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^6/g
^4/i^3/(b*x+a)+5/2*b^4*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^
6/g^4/i^3/(b*x+a)^2-1/3*b^5*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*
c)^6/g^4/i^3/(b*x+a)^3-10*b^2*d^3*ln((b*x+a)/(d*x+c))*(A+B*ln(e*(b*x+a)/(d
*x+c)))/(-a*d+b*c)^6/g^4/i^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.51 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.13

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx =$$

$$-\frac{\frac{4b^2B(bc-ad)^3}{(a+bx)^3} - \frac{33b^2Bd(bc-ad)^2}{(a+bx)^2} + \frac{216b^3Bcd^2}{a+bx} - \frac{216ab^2Bd^3}{a+bx} + \frac{66b^2Bd^2(bc-ad)}{a+bx} - \frac{9Bd^3(bc-ad)^2}{(c+dx)^2} - \frac{144b^2Bcd^3}{c+dx} + \frac{144abBd^4}{c+dx}$$

input

```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i
*x)^3),x]
```

output

```
-1/36*((4*b^2*B*(b*c - a*d)^3)/(a + b*x)^3 - (33*b^2*B*d*(b*c - a*d)^2)/(a
 + b*x)^2 + (216*b^3*B*c*d^2)/(a + b*x) - (216*a*b^2*B*d^3)/(a + b*x) + (6
6*b^2*B*d^2*(b*c - a*d))/(a + b*x) - (9*B*d^3*(b*c - a*d)^2)/(c + d*x)^2 -
 (144*b^2*B*c*d^3)/(c + d*x) + (144*a*b*B*d^4)/(c + d*x) - (18*b*B*d^3*(b*
c - a*d))/(c + d*x) + 120*b^2*B*d^3*Log[a + b*x] + (12*b^2*(b*c - a*d)^3*(
A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)^3 - (54*b^2*d*(b*c - a*d)^2
*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)^2 + (216*b^2*d^2*(b*c - a
*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) + (18*d^3*(b*c - a*d)^
2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x)^2 + (144*b*d^3*(b*c - a*
d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) + 360*b^2*d^3*Log[a + b
*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 120*b^2*B*d^3*Log[c + d*x] - 36
0*b^2*d^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 180*b^2*B*d^
3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Poly
Log[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 180*b^2*B*d^3*((2*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x)
)/(b*c - a*d)]))/((b*c - a*d)^6*g^4*i^3)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.69, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2772, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^4(ci+dix)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4i^3(bc-ad)^6}$$

$$\downarrow 2772$$

$$-B\int -\frac{(c+dx)^4\left(2b^5-\frac{15d(a+bx)b^4}{c+dx}+\frac{60d^2(a+bx)^2b^3}{(c+dx)^2}+\frac{60d^3(a+bx)^3\log\left(\frac{a+bx}{c+dx}\right)b^2}{(c+dx)^3}-\frac{30d^4(a+bx)^4b}{(c+dx)^4}+\frac{3d^5(a+bx)^5}{(c+dx)^5}\right)}{6(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b^5(c+dx)^3\left(B\log\left(\frac{e(}{3(a+bx)^3}\right.\right.$$

$$\downarrow 27$$

$$\frac{1}{6}B\int \frac{(c+dx)^4\left(2b^5-\frac{15d(a+bx)b^4}{c+dx}+\frac{60d^2(a+bx)^2b^3}{(c+dx)^2}+\frac{60d^3(a+bx)^3\log\left(\frac{a+bx}{c+dx}\right)b^2}{(c+dx)^3}-\frac{30d^4(a+bx)^4b}{(c+dx)^4}+\frac{3d^5(a+bx)^5}{(c+dx)^5}\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b^5(c+dx)^3\left(B\log\left(\frac{e(a+}{c+}\right.\right.$$

$$\downarrow 2010$$

$$\frac{1}{6}B\int \left(\frac{\left(2b^5-\frac{15d(a+bx)b^4}{c+dx}+\frac{60d^2(a+bx)^2b^3}{(c+dx)^2}-\frac{30d^4(a+bx)^4b}{(c+dx)^4}+\frac{3d^5(a+bx)^5}{(c+dx)^5}\right)(c+dx)^4}{(a+bx)^4}+\frac{60b^2d^3\log\left(\frac{a+bx}{c+dx}\right)(c+dx)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b^5(c+dx)^3\left(B\right.}{3(a}$$

$$\downarrow 2009$$

$$-\frac{b^5(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{5b^4d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{10b^3d^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-10b^2d^3\log\left(\frac{a+bx}{c+dx}\right)$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i*x)^3)
,x]
```

output
```
((B*((3*d^5*(a + b*x)^2)/(2*(c + d*x)^2) - (30*b*d^4*(a + b*x))/(c + d*x)
- (60*b^3*d^2*(c + d*x))/(a + b*x) + (15*b^4*d*(c + d*x)^2)/(2*(a + b*x)^2
) - (2*b^5*(c + d*x)^3)/(3*(a + b*x)^3) + 30*b^2*d^3*Log[(a + b*x)/(c + d*
x)]^2))/6 - (d^5*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(c +
 d*x)^2) + (5*b*d^4*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d
*x) - (10*b^3*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x
) + (5*b^4*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x
)^2) - (b^5*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(a + b*x)
^3) - 10*b^2*d^3*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/((b*c - a*d)^6*g^4*i^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 8.76 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.40

method	result
parts	$\frac{A\left(-\frac{d^3}{2(da-bc)^4(dx+c)^2}+\frac{10d^3b^2\ln(dx+c)}{(da-bc)^6}+\frac{4d^3b}{(da-bc)^5(dx+c)}+\frac{b^2}{3(da-bc)^3(bx+a)^3}-\frac{10d^3b^2\ln(bx+a)}{(da-bc)^6}+\frac{6b^2d^2}{(da-bc)^5(bx+a)}+\frac{2(da}{g^4i^3}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{d^2e^2Ab^5}{3i^3(da-bc)^7g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{5d^3eAb^4}{2i^3(da-bc)^7g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{10d^4Ab^3}{i^3(da-bc)^7g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{10d^5A}{}\right.}{}$
default	$-\frac{e(da-bc)\left(\frac{d^2e^2Ab^5}{3i^3(da-bc)^7g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{5d^3eAb^4}{2i^3(da-bc)^7g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{10d^4Ab^3}{i^3(da-bc)^7g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{10d^5A}{}\right.}{}$
risch	Expression too large to display
parallelrisch	Expression too large to display
norman	Expression too large to display

input

```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,method=_RETU
RNVERBOSE)
```

output

```
A/g^4/i^3*(-1/2*d^3/(a*d-b*c)^4/(d*x+c)^2+10*d^3/(a*d-b*c)^6*b^2*ln(d*x+c)
+4*d^3/(a*d-b*c)^5*b/(d*x+c)+1/3*b^2/(a*d-b*c)^3/(b*x+a)^3-10*d^3/(a*d-b*c
)^6*b^2*ln(b*x+a)+6*b^2/(a*d-b*c)^5*d^2/(b*x+a)+3/2*b^2/(a*d-b*c)^4*d/(b*x
+a)^2)-B/g^4/i^3*d/(a*d-b*c)^2/e^2*(d^4/(a*d-b*c)^4*(1/2*(b*e/d+(a*d-b*c)*
e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d
*x+c))^2)-5*d^3/(a*d-b*c)^4*b*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a
*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)+5*d^2/(a*d-b*c)^4*b^2*e^
2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-10*d/(a*d-b*c)^4*b^3*e^3*(-1/(b*e/d+(a
*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/
d/(d*x+c)))+5/(a*d-b*c)^4*b^4*e^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln
(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-1/d/(a*
d-b*c)^4*b^5*e^5*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1509 vs. $2(551) = 1102$.

Time $= 0.13$ (sec) , antiderivative size $= 1509$, normalized size of antiderivative $= 2.68$

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, algo
rithm="fricas")
```

output

```
-1/36*(4*(3*A + B)*b^5*c^5 - 45*(2*A + B)*a*b^4*c^4*d + 360*(A + B)*a^2*b^
3*c^3*d^2 - 10*(12*A + 49*B)*a^3*b^2*c^2*d^3 - 180*(A - B)*a^4*b*c*d^4 + 9
*(2*A - B)*a^5*d^5 + 120*((3*A + B)*b^5*c*d^4 - (3*A + B)*a*b^4*d^5)*x^4 +
 60*(3*(3*A + 2*B)*b^5*c^2*d^3 + 2*(3*A - 2*B)*a*b^4*c*d^4 - (15*A + 2*B)*
a^2*b^3*d^5)*x^3 + 20*((6*A + 11*B)*b^5*c^3*d^2 + 21*(3*A + B)*a*b^4*c^2*d
^3 - 3*(12*A + 13*B)*a^2*b^3*c*d^4 - (33*A - 7*B)*a^3*b^2*d^5)*x^2 + 180*(
B*b^5*d^5*x^5 + B*a^3*b^2*c^2*d^3 + (2*B*b^5*c*d^4 + 3*B*a*b^4*d^5)*x^4 +
(B*b^5*c^2*d^3 + 6*B*a*b^4*c*d^4 + 3*B*a^2*b^3*d^5)*x^3 + (3*B*a*b^4*c^2*d
^3 + 6*B*a^2*b^3*c*d^4 + B*a^3*b^2*d^5)*x^2 + (3*B*a^2*b^3*c^2*d^3 + 2*B*a
^3*b^2*c*d^4)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 5*((6*A + 5*B)*b^5*c^4*d
 - 36*(2*A + 3*B)*a*b^4*c^3*d^2 - 6*(24*A - 13*B)*a^2*b^3*c^2*d^3 + 4*(48*
A + 13*B)*a^3*b^2*c*d^4 + 9*(2*A - 3*B)*a^4*b*d^5)*x + 6*(20*(3*A + B)*b^5
*d^5*x^5 + 2*B*b^5*c^5 - 15*B*a*b^4*c^4*d + 60*B*a^2*b^3*c^3*d^2 + 60*A*a^
3*b^2*c^2*d^3 - 30*B*a^4*b*c*d^4 + 3*B*a^5*d^5 + 20*((6*A + 5*B)*b^5*c*d^4
 + 9*A*a*b^4*d^5)*x^4 + 10*((6*A + 11*B)*b^5*c^2*d^3 + 18*(2*A + B)*a*b^4*
c*d^4 + 9*(2*A - B)*a^2*b^3*d^5)*x^3 + 10*(2*B*b^5*c^3*d^2 + 9*(2*A + 3*B)
*a*b^4*c^2*d^3 + 36*A*a^2*b^3*c*d^4 + 3*(2*A - 3*B)*a^3*b^2*d^5)*x^2 - 5*(
B*b^5*c^4*d - 12*B*a*b^4*c^3*d^2 - 36*(A + B)*a^2*b^3*c^2*d^3 - 24*(A - B)
*a^3*b^2*c*d^4 + 3*B*a^4*b*d^5)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^9*c^6
*d^2 - 6*a*b^8*c^5*d^3 + 15*a^2*b^7*c^4*d^4 - 20*a^3*b^6*c^3*d^5 + 15*a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^4 (ci + dix)^3}\, dx = \text{Timed out}$$

input

```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4/(d*i*x+c*i)**3,x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3816 vs. $2(551) = 1102$.

Time = 0.38 (sec) , antiderivative size = 3816, normalized size of antiderivative = 6.78

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, algo
rithm="maxima")
```

output

```
-1/6*B*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2*b^2*c^2*d^2
+ 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 10*(2*
b^4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3*d - 11*a*b
^3*c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/((b^8*c^5*d^2 - 5*a*b^7*c^
4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^
3*d^7)*g^4*i^3*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 +
10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7
)*g^4*i^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c
^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*
d^7)*g^4*i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a
^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a
^8*d^7)*g^4*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d
^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^
6)*g^4*i^3*x + (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^
6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i^3) + 60*b^2*d^3*log(b
*x + a)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^
3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3) - 60*b^2*d^3*lo
g(d*x + c)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3
*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3))*log(b*e*x/(
d*x + c) + a*e/(d*x + c)) - 1/6*A*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b...
```

Giac [F]

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \int \frac{B \log\left(\frac{(bx+a)e}{dx+c}\right) + A}{(bgx+ag)^4(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, algo
rithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)/((b*g*x + a*g)^4*(d*i*x + c*i
)^3), x)
```

Mupad [B] (verification not implemented)

Time = 38.95 (sec) , antiderivative size = 2291, normalized size of antiderivative = 4.07

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^4*(c*i + d*i*x)^3)
,x)
```

output

```
((12*A*b^4*c^4 - 18*A*a^4*d^4 + 9*B*a^4*d^4 + 4*B*b^4*c^4 + 282*A*a^2*b^2*
c^2*d^2 + 319*B*a^2*b^2*c^2*d^2 - 78*A*a*b^3*c^3*d + 162*A*a^3*b*c*d^3 - 4
1*B*a*b^3*c^3*d - 171*B*a^3*b*c*d^3)/(6*(a*d - b*c)) + (10*x^2*(33*A*a^2*b
^2*d^4 - 7*B*a^2*b^2*d^4 + 6*A*b^4*c^2*d^2 + 11*B*b^4*c^2*d^2 + 69*A*a*b^3
*c*d^3 + 32*B*a*b^3*c*d^3))/(3*(a*d - b*c)) + (5*x*(18*A*a^3*b*d^4 - 27*B*
a^3*b*d^4 - 6*A*b^4*c^3*d - 5*B*b^4*c^3*d + 66*A*a*b^3*c^2*d^2 + 210*A*a^2
*b^2*c*d^3 + 103*B*a*b^3*c^2*d^2 + 25*B*a^2*b^2*c*d^3))/(6*(a*d - b*c)) +
(10*x^3*(15*A*a*b^3*d^4 + 2*B*a*b^3*d^4 + 9*A*b^4*c*d^3 + 6*B*b^4*c*d^3))/
(a*d - b*c) + (20*x^4*(3*A*b^4*d^4 + B*b^4*d^4))/(a*d - b*c))/(x^5*(6*a^4*
b^3*d^6*g^4*i^3 + 6*b^7*c^4*d^2*g^4*i^3 - 24*a*b^6*c^3*d^3*g^4*i^3 - 24*a^
3*b^4*c*d^5*g^4*i^3 + 36*a^2*b^5*c^2*d^4*g^4*i^3) + x*(18*a^2*b^5*c^6*g^4*
i^3 + 12*a^7*c*d^5*g^4*i^3 - 60*a^3*b^4*c^5*d*g^4*i^3 - 30*a^6*b*c^2*d^4*g
^4*i^3 + 60*a^4*b^3*c^4*d^2*g^4*i^3) + x^2*(6*a^7*d^6*g^4*i^3 + 18*a*b^6*c
^6*g^4*i^3 + 12*a^6*b*c*d^5*g^4*i^3 - 36*a^2*b^5*c^5*d*g^4*i^3 - 30*a^3*b^
4*c^4*d^2*g^4*i^3 + 120*a^4*b^3*c^3*d^3*g^4*i^3 - 90*a^5*b^2*c^2*d^4*g^4*i
^3) + x^3*(6*b^7*c^6*g^4*i^3 + 18*a^6*b*d^6*g^4*i^3 + 12*a*b^6*c^5*d*g^4*i
^3 - 36*a^5*b^2*c*d^5*g^4*i^3 - 90*a^2*b^5*c^4*d^2*g^4*i^3 + 120*a^3*b^4*c
^3*d^3*g^4*i^3 - 30*a^4*b^3*c^2*d^4*g^4*i^3) + x^4*(18*a^5*b^2*d^6*g^4*i^3
 + 12*b^7*c^5*d*g^4*i^3 - 30*a*b^6*c^4*d^2*g^4*i^3 - 60*a^4*b^3*c*d^5*g^4*
i^3 + 60*a^3*b^4*c^2*d^4*g^4*i^3) + 6*a^3*b^4*c^6*g^4*i^3 + 6*a^7*c^2*d...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 3982, normalized size of antiderivative = 7.07

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x)
```

output

```
(i*( - 1080*log(a + b*x)*a**5*b**2*c**2*d**4 - 2160*log(a + b*x)*a**5*b**2
*c*d**5*x - 1080*log(a + b*x)*a**5*b**2*d**6*x**2 - 720*log(a + b*x)*a**4*
b**3*c**3*d**3 - 4680*log(a + b*x)*a**4*b**3*c**2*d**4*x - 7200*log(a + b*
x)*a**4*b**3*c*d**5*x**2 - 3240*log(a + b*x)*a**4*b**3*d**6*x**3 - 2160*lo
g(a + b*x)*a**3*b**4*c**3*d**3*x - 600*log(a + b*x)*a**3*b**4*c**3*d**3 -
7560*log(a + b*x)*a**3*b**4*c**2*d**4*x**2 - 1200*log(a + b*x)*a**3*b**4*c
**2*d**4*x - 8640*log(a + b*x)*a**3*b**4*c*d**5*x**3 - 600*log(a + b*x)*a*
*3*b**4*c*d**5*x**2 - 3240*log(a + b*x)*a**3*b**4*d**6*x**4 - 2160*log(a +
 b*x)*a**2*b**5*c**3*d**3*x**2 - 1800*log(a + b*x)*a**2*b**5*c**3*d**3*x -
 5400*log(a + b*x)*a**2*b**5*c**2*d**4*x**3 - 3600*log(a + b*x)*a**2*b**5*
c**2*d**4*x**2 - 4320*log(a + b*x)*a**2*b**5*c*d**5*x**4 - 1800*log(a + b*
x)*a**2*b**5*c*d**5*x**3 - 1080*log(a + b*x)*a**2*b**5*d**6*x**5 - 720*log
(a + b*x)*a*b**6*c**3*d**3*x**3 - 1800*log(a + b*x)*a*b**6*c**3*d**3*x**2
- 1440*log(a + b*x)*a*b**6*c**2*d**4*x**4 - 3600*log(a + b*x)*a*b**6*c**2*
d**4*x**3 - 720*log(a + b*x)*a*b**6*c*d**5*x**5 - 1800*log(a + b*x)*a*b**6
*c*d**5*x**4 - 600*log(a + b*x)*b**7*c**3*d**3*x**3 - 1200*log(a + b*x)*b*
*7*c**2*d**4*x**4 - 600*log(a + b*x)*b**7*c*d**5*x**5 + 1080*log(c + d*x)*
a**5*b**2*c**2*d**4 + 2160*log(c + d*x)*a**5*b**2*c*d**5*x + 1080*log(c +
d*x)*a**5*b**2*d**6*x**2 + 720*log(c + d*x)*a**4*b**3*c**3*d**3 + 4680*log
(c + d*x)*a**4*b**3*c**2*d**4*x + 7200*log(c + d*x)*a**4*b**3*c*d**5*x*...
```

3.55 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result 682
Mathematica [A] (verified) 683
Rubi [A] (verified) 684
Maple [F] 690
Fricas [F] 690
Sympy [F(-1)] 691
Maxima [B] (verification not implemented) 691
Giac [F] 692
Mupad [F(-1)] 693
Reduce [F] 693

Optimal result

Integrand size = 40, antiderivative size = 539

$$\begin{aligned}
&\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= \frac{3B^2(bc-ad)^4g^3ix}{10bd^3} - \frac{3B^2(bc-ad)^3g^3i(c+dx)^2}{20d^4} + \frac{bB^2(bc-ad)^2g^3i(c+dx)^3}{30d^4} \\
&\quad - \frac{B(bc-ad)^2g^3i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^2d} \\
&\quad - \frac{B(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^2} \\
&\quad + \frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2} \\
&\quad + \frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} \\
&\quad + \frac{B(bc-ad)^3g^3i(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^2} \\
&\quad - \frac{B(bc-ad)^4g^3i(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^3} \\
&\quad - \frac{B(bc-ad)^5g^3i\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^4} \\
&\quad - \frac{B^2(bc-ad)^5g^3i\log(c+dx)}{10b^2d^4} - \frac{B^2(bc-ad)^5g^3i\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{10b^2d^4}
\end{aligned}$$

output

```
3/10*B^2*(-a*d+b*c)^4*g^3*i*x/b/d^3-3/20*B^2*(-a*d+b*c)^3*g^3*i*(d*x+c)^2/
d^4+1/30*b*B^2*(-a*d+b*c)^2*g^3*i*(d*x+c)^3/d^4-1/30*B*(-a*d+b*c)^2*g^3*i*
(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d-1/10*B*(-a*d+b*c)*g^3*i*(b*x+a
)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2+1/20*(-a*d+b*c)*g^3*i*(b*x+a)^4*(A+B*l
n(e*(b*x+a)/(d*x+c)))^2/b^2+1/5*g^3*i*(b*x+a)^4*(d*x+c)*(A+B*ln(e*(b*x+a)/
(d*x+c)))^2/b+1/60*B*(-a*d+b*c)^3*g^3*i*(b*x+a)^2*(3*A+B+3*B*ln(e*(b*x+a)/
(d*x+c)))/b^2/d^2-1/60*B*(-a*d+b*c)^4*g^3*i*(b*x+a)*(6*A+5*B+6*B*ln(e*(b*x
+a)/(d*x+c)))/b^2/d^3-1/60*B*(-a*d+b*c)^5*g^3*i*ln((-a*d+b*c)/b/(d*x+c))*(
6*A+11*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b^2/d^4-1/10*B^2*(-a*d+b*c)^5*g^3*i*ln
(d*x+c)/b^2/d^4-1/10*B^2*(-a*d+b*c)^5*g^3*i*polylog(2,d*(b*x+a)/b/(d*x+c))
/b^2/d^4
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 905, normalized size of antiderivative = 1.68

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{g^3 i\left(5(bc-ad)(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+4d(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{5B(bc-ad)^2\left(6Ab\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)
])^2,x]
```

output
```
(g^3*i*(5*(b*c - a*d)*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 +
 4*d*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (5*B*(b*c - a*d)
^2*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*
x))/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))
/(c + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*
B*(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c
 + d*x)])*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x
)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*c) +
 a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*
d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
)))/(3*d^4) + (B*(b*c - a*d)*(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d
)^3*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x
)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(
A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]) - 24*B*(b*c - a*d)^4*Log[c + d*x] - 24*(b*c - a*d)^4*(A
 + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 4*B*(b*c - a*d)^2*(2*b*d
*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c
- a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(
a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*(b*d*x + (
-(b*c) + a*d)*Log[c + d*x]) + 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/...
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {2962, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\downarrow 2962$$

$$g^3i(bc-ad)^5\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$g^3i(bc-ad)^5\left(-\frac{2B\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.$$

$$\downarrow 2773$$

$$g^3i(bc-ad)^5\left(-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int \frac{(a+bx)^3}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}\right)}{5b}+\frac{\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{(a\right.$$

$$\downarrow 49$$

$$g^3 i(bc-ad)^5 \left(-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int\left(\frac{b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{1}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}}{4b}\right)}{5b}\right)$$

$$\downarrow 2009$$

$$g^3 i(bc-ad)^5 \left(\frac{\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{b^3}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3b^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\right.}{4b}\right.}{5b}\right)$$

$$\downarrow 2781$$

$$g^3 i(bc-ad)^5 \left(\frac{\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}}{5b}-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\right.}{3d^4\left(b-\right.}\right.}{5b}\right)$$

$$\downarrow 2784$$

$$g^3 i(bc-$$

$$ad)^5\left(\frac{\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int\frac{(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{2b}}{5b}-\frac{2B\left(\frac{(a+bx}{4b(}\right.}{}\right.$$

$\downarrow$ 2784

$$g^3 i(bc-$$

$$ad)^5\left(\frac{\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(6A+5B+6B\log\left(\frac{e}{}\right.\right.}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right.}}{2d}}{3d}\right.}{2b}}{5b}\right.$$

$\downarrow$ 2784

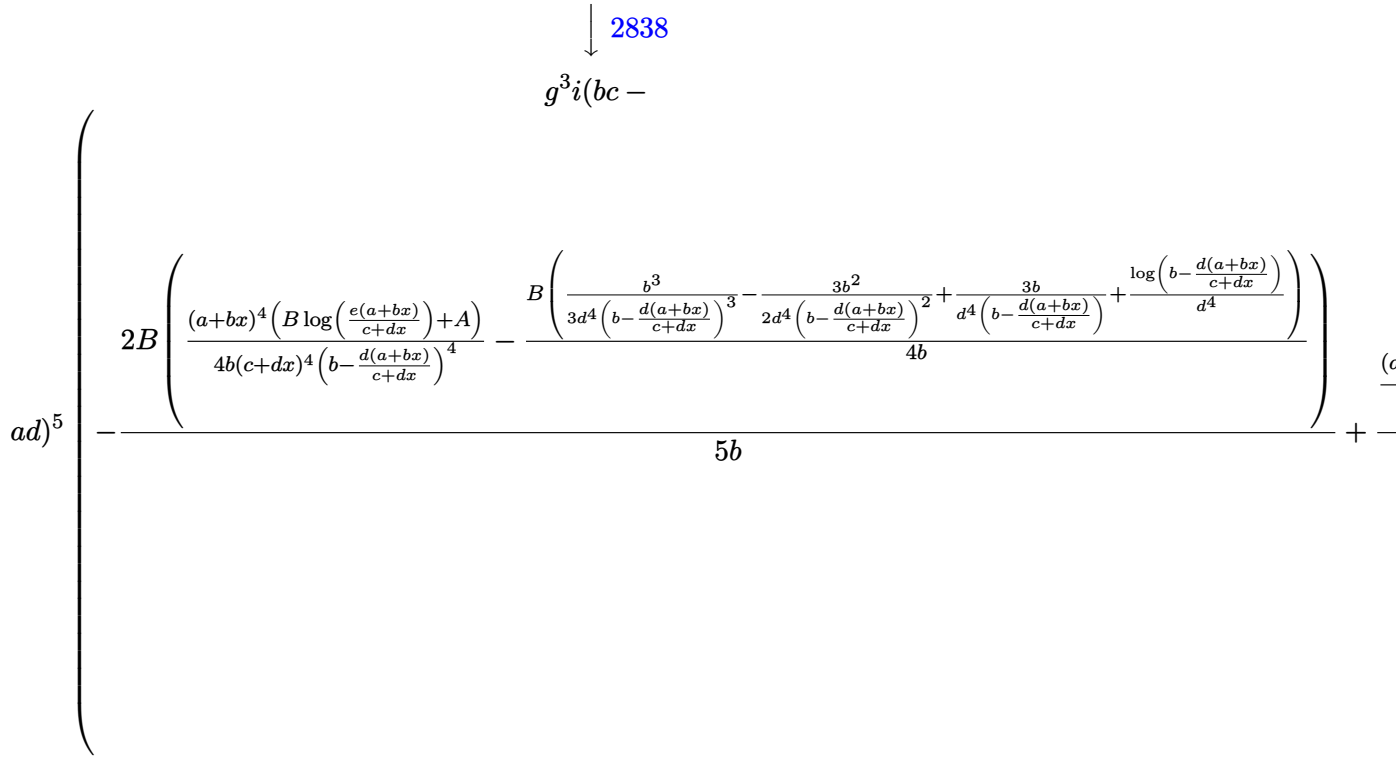
$$g^3 i(bc-$$

$$ad)^5\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6\right.}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right.}}{3d}\right.}{2b}\right.$$

$$\Big\downarrow 2754$$

$$g^3 i(bc-$$

$$ad)^5\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6\right.}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right.}}{3d}\right.}{2b}\right.$$



input

```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x
]
```

output

```
(b*c - a*d)^5*g^3*i*(((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/
(5*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*(((a + b*x)^4*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]))/(4*b*(c + d*x)^4*(b - (d*(a + b*x))/(c
+ d*x))^4) - (B*(b^3/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^3) - (3*b^2)/(2*
d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (3*b)/(d^4*(b - (d*(a + b*x))/(c +
d*x))) + Log[b - (d*(a + b*x))/(c + d*x)]/d^4))/(4*b)))/(5*b) + (((a + b*x
)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*b*(c + d*x)^4*(b - (d*(a +
b*x))/(c + d*x))^4) - (B*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a + b*x)^2*(3*A
+ B + 3*B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x
))/(c + d*x))^2) - (((a + b*x)*(6*A + 5*B + 6*B*Log[(e*(a + b*x))/(c + d*x
)]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((6*A + 11*B + 6*B*L
og[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (6
*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d))/(3*d)))/(2*b))/(5
*b))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)^3 (dix+ci) \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3 (ci+dix) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)^3 (dix+ci) \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="fricas")
```

output
```
integral(A^2*b^3*d*g^3*i*x^4 + A^2*a^3*c*g^3*i + (A^2*b^3*c + 3*A^2*a*b^2*
d)*g^3*i*x^3 + 3*(A^2*a*b^2*c + A^2*a^2*b*d)*g^3*i*x^2 + (3*A^2*a^2*b*c +
A^2*a^3*d)*g^3*i*x + (B^2*b^3*d*g^3*i*x^4 + B^2*a^3*c*g^3*i + (B^2*b^3*c +
 3*B^2*a*b^2*d)*g^3*i*x^3 + 3*(B^2*a*b^2*c + B^2*a^2*b*d)*g^3*i*x^2 + (3*B
^2*a^2*b*c + B^2*a^3*d)*g^3*i*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b
^3*d*g^3*i*x^4 + A*B*a^3*c*g^3*i + (A*B*b^3*c + 3*A*B*a*b^2*d)*g^3*i*x^3 +
 3*(A*B*a*b^2*c + A*B*a^2*b*d)*g^3*i*x^2 + (3*A*B*a^2*b*c + A*B*a^3*d)*g^3
*i*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3186 vs. $2(514) = 1028$.

Time $= 0.19$ (sec) , antiderivative size $= 3186$, normalized size of antiderivative $= 5.91$

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="maxima")
```

output

```
1/5*A^2*b^3*d*g^3*i*x^5 + 1/4*A^2*b^3*c*g^3*i*x^4 + 3/4*A^2*a*b^2*d*g^3*i*
x^4 + A^2*a*b^2*c*g^3*i*x^3 + A^2*a^2*b*d*g^3*i*x^3 + 3/2*A^2*a^2*b*c*g^3*
i*x^2 + 1/2*A^2*a^3*d*g^3*i*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)
) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^3*c*g^3*i + 3*(x^2*log(b*e*
x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2
 - (b*c - a*d)*x/(b*d))*A*B*a^2*b*c*g^3*i + (2*x^3*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*b^2*c*g^3*i +
 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4
 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d
- a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^3*c*g^3*i + (
x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(
d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^3*d*g^3*i + (2*x^3*log(b*e*x/(d*
x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3
- ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b
*d*g^3*i + 1/4*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x
 + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b
^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b^2*
d*g^3*i + 1/30*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b
*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 ...
```

Giac [F]

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)^3(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input

```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="giac")
```

output

```
integrate((b*g*x + a*g)^3*(d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)
^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^3\,(c\,i+d\,i\,x)\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x
)
```

output
```
int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,
x)
```

Reduce [F]

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
(g**3*i*( - 6*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x +
b*d*x**2),x)*a**5*b**2*d**6 + 30*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c
 + a*d*x + b*c*x + b*d*x**2),x)*a**4*b**3*c*d**5 - 60*int((log((a*e + b*e*
x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**4*c**2*d**4 +
 60*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),
x)*a**2*b**5*c**3*d**3 - 30*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*
d*x + b*c*x + b*d*x**2),x)*a*b**6*c**4*d**2 + 6*int((log((a*e + b*e*x)/(c
+ d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**7*c**5*d - 6*log(a + b*x
)*a**6*d**5 + 30*log(a + b*x)*a**5*b*c*d**4 - 5*log(a + b*x)*a**5*b*d**5 -
 60*log(a + b*x)*a**4*b**2*c**2*d**3 + 25*log(a + b*x)*a**4*b**2*c*d**4 +
60*log(a + b*x)*a**3*b**3*c**3*d**2 - 50*log(a + b*x)*a**3*b**3*c**2*d**3
- 30*log(a + b*x)*a**2*b**4*c**4*d + 50*log(a + b*x)*a**2*b**4*c**3*d**2 +
 6*log(a + b*x)*a*b**5*c**5 - 25*log(a + b*x)*a*b**5*c**4*d + 5*log(a + b*
x)*b**6*c**5 + 3*log((a*e + b*e*x)/(c + d*x))**2*a**4*b**2*c*d**4 + 18*log
((a*e + b*e*x)/(c + d*x))**2*a**3*b**3*c**2*d**3 + 60*log((a*e + b*e*x)/(c
 + d*x))**2*a**3*b**3*c*d**4*x + 30*log((a*e + b*e*x)/(c + d*x))**2*a**3*b
**3*d**5*x**2 - 12*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**4*c**3*d**2 + 9
0*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**4*c*d**4*x**2 + 60*log((a*e + b*
e*x)/(c + d*x))**2*a**2*b**4*d**5*x**3 + 3*log((a*e + b*e*x)/(c + d*x))**2
*a*b**5*c**4*d + 60*log((a*e + b*e*x)/(c + d*x))**2*a*b**5*c*d**4*x**3 ...
```

3.56 $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result . . . 695
Mathematica [A] (verified) . . . 696
Rubi [A] (verified) . . . 697
Maple [F] . . . 703
Fricas [F] . . . 703
Sympy [F(-1)] . . . 703
Maxima [B] (verification not implemented) . . . 704
Giac [F] . . . 705
Mupad [F(-1)] . . . 705
Reduce [F] . . . 706

Optimal result

Integrand size = 40, antiderivative size = 450

$$
\begin{aligned}
&\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= -\frac{B^2(bc-ad)^3g^2ix}{3bd^2} + \frac{B^2(bc-ad)^2g^2i(c+dx)^2}{12d^3} \\
&\quad - \frac{B(bc-ad)^2g^2i(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d} \\
&\quad - \frac{B(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^2} \\
&\quad + \frac{(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{12b^2} \\
&\quad + \frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} \\
&\quad + \frac{B(bc-ad)^3g^2i(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d^2} \\
&\quad + \frac{B(bc-ad)^4g^2i\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d^3} \\
&\quad + \frac{B^2(bc-ad)^4g^2i\log(c+dx)}{6b^2d^3} + \frac{B^2(bc-ad)^4g^2i\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{6b^2d^3}
\end{aligned}
$$

output

```
-1/3*B^2*(-a*d+b*c)^3*g^2*i*x/b/d^2+1/12*B^2*(-a*d+b*c)^2*g^2*i*(d*x+c)^2/
d^3-1/12*B*(-a*d+b*c)^2*g^2*i*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d-
1/6*B*(-a*d+b*c)*g^2*i*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2+1/12*(-a*
d+b*c)*g^2*i*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2+1/4*g^2*i*(b*x+a)
^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/12*B*(-a*d+b*c)^3*g^2*i*(b*x+
a)*(2*A+B+2*B*ln(e*(b*x+a)/(d*x+c)))/b^2/d^2+1/12*B*(-a*d+b*c)^4*g^2*i*ln(
(-a*d+b*c)/b/(d*x+c))*(2*A+3*B+2*B*ln(e*(b*x+a)/(d*x+c)))/b^2/d^3+1/6*B^2*
(-a*d+b*c)^4*g^2*i*ln(d*x+c)/b^2/d^3+1/6*B^2*(-a*d+b*c)^4*g^2*i*polylog(2,
d*(b*x+a)/b/(d*x+c))/b^2/d^3
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.51

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$

$$= \frac{g^2 i\left(4(bc-ad)(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+3d(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{4B(bc-ad)^2\left(2Ab\right.}{}\right.}{}$$

input

```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)
])^2,x]
```

output

```
(g^2*i*(4*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 +
 3*d*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (4*B*(b*c - a*d)
^2*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/
(c + d*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b
*c - a*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d
*x)])*Log[c + d*x] + B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) +
 B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log
[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3 - (B*(b*c - a*d
)*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x
))/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/
(c + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B
*(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c
 + d*x)])*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)
^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*c) +
a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
))/d^3))/(12*b^2)
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2962, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\downarrow 2962$$

$$g^2i(bc-ad)^4\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$g^2 i(bc-ad)^4\left(-\frac{B\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}+\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{4b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.$$

$\downarrow$ 2773

$$g^2 i(bc-ad)^4\left(-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\int \frac{(a+bx)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}\right)}{2b}+\frac{\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}+\frac{(a\ }{}\right.$$

$\downarrow$ 49

$$g^2 i(bc-ad)^4\left(-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\int \left(\frac{b^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{1}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}}{3b}\right)}{2b}+\frac{\int \frac{(a+bx)^2\left(A+\right.}{(c+dx)^2}}{}\right.$$

$\downarrow$ 2009

$$g^2 i(bc-ad)^4\left(\frac{\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\left(\frac{b^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\log\left(b\right.}{}\right.}{3b}\right.}{2b}\right.$$

$\downarrow$ 2781

$$g^2 i(bc-$$

$$ad)^4\left(\frac{\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\int\frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}}{4b}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\left(\frac{}{2d^3\left(b-\right.}\right.}{2}\right.$$

$$\downarrow 2784$$

$$g^2 i(bc-$$

$$ad)^4\left(\frac{\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{3b}}{4b}-\frac{B\left(\frac{(a+bx)^3}{3b(c-}\right.}{}$$

$$\downarrow 2784$$

$$g^2 i(bc-$$

$$ad)^4\left(\frac{\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)}{b-\frac{d(a+bx)}{c+dx}}}{d}}{2d}\right)}{3b}}{4b}$$

$$\downarrow 2754$$

$$g^2 i(bc-ad)^4\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2B\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}}{d}}{2d}\right)}{3b}\right)$$

$$\downarrow 2838$$

$$g^2 i(bc-ad)^4\left(-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\left(\frac{b^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^3}\right)}{3b}\right)}{2b}+\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{3b(c+dx)^3\left(b-\frac{d(a+b}{c+d}\right.}$$

input
```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x
]
```

output
```
(b*c - a*d)^4*g^2*i*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/
(4*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^4) - (B*(((a + b*x)^3*(A +
B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*(c + d*x)^3*(b - (d*(a + b*x))/(c +
d*x))^3) - (B*(b^2/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^2) - (2*b)/(d^3*(b
 - (d*(a + b*x))/(c + d*x))) - Log[b - (d*(a + b*x))/(c + d*x)]/d^3))/(3*b
)))/(2*b) + (((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*b*(c
+ d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*(((a + b*x)^2*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^
2) - (((a + b*x)*(2*A + B + 2*B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c + d*x
)*(b - (d*(a + b*x))/(c + d*x))) - (-(((2*A + 3*B + 2*B*Log[(e*(a + b*x))/
(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (2*B*PolyLog[2, (d*
(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d)))/(3*b))/(4*b))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2783

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)^2 (dix+ci) \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)^2(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="fricas")
```

output
```
integral(A^2*b^2*d*g^2*i*x^3 + A^2*a^2*c*g^2*i + (A^2*b^2*c + 2*A^2*a*b*d)
*g^2*i*x^2 + (2*A^2*a*b*c + A^2*a^2*d)*g^2*i*x + (B^2*b^2*d*g^2*i*x^3 + B^
2*a^2*c*g^2*i + (B^2*b^2*c + 2*B^2*a*b*d)*g^2*i*x^2 + (2*B^2*a*b*c + B^2*a
^2*d)*g^2*i*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^2*d*g^2*i*x^3 + A
*B*a^2*c*g^2*i + (A*B*b^2*c + 2*A*B*a*b*d)*g^2*i*x^2 + (2*A*B*a*b*c + A*B*
a^2*d)*g^2*i*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2243 vs. $2(429) = 858$.

Time $= 0.16$ (sec) , antiderivative size $= 2243$, normalized size of antiderivative $= 4.98$

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="maxima")
```

output
```
1/4*A^2*b^2*d*g^2*i*x^4 + 1/3*A^2*b^2*c*g^2*i*x^3 + 2/3*A^2*a*b*d*g^2*i*x^
3 + A^2*a*b*c*g^2*i*x^2 + 1/2*A^2*a^2*d*g^2*i*x^2 + 2*(x*log(b*e*x/(d*x +
c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^2*c*g^2*i
 + 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^
2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a*b*c*g^2*i + 1/3*(2*x^3*log
(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x
 + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))
*A*B*b^2*c*g^2*i + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x
 + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^2*d*g^2*i +
2/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 -
 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)
*x)/(b^2*d^2))*A*B*a*b*d*g^2*i + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2
- a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x
)/(b^3*d^3))*A*B*b^2*d*g^2*i + A^2*a^2*c*g^2*i*x - 1/12*(2*a^3*c*d^3*g^2*i
 + (2*g^2*i*log(e) + g^2*i)*b^3*c^4 - 2*(4*g^2*i*log(e) + g^2*i)*a*b^2*c^3
*d + (12*g^2*i*log(e) - g^2*i)*a^2*b*c^2*d^2)*B^2*log(d*x + c)/(b*d^3) - 1
/6*(b^4*c^4*g^2*i - 4*a*b^3*c^3*d*g^2*i + 6*a^2*b^2*c^2*d^2*g^2*i - 4*a^3*
b*c*d^3*g^2*i + a^4*d^4*g^2*i)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d)
 + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^3) + 1/12*(3*B^2*...
```

Giac [F]

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)^2(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="giac")
```

output
```
integrate((b*g*x + a*g)^2*(d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)
^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x
)
```

output
```
int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,
x)
```

Reduce [F]

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \text{Too large to display}$$

input

```
int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
(g**2*i*( - 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x +
b*d*x**2),x)*a**4*b**2*d**5 + 8*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c
+ a*d*x + b*c*x + b*d*x**2),x)*a**3*b**3*c*d**4 - 12*int((log((a*e + b*e*x
)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**4*c**2*d**3 +
8*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)
*a*b**5*c**3*d**2 - 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x +
b*c*x + b*d*x**2),x)*b**6*c**4*d - 2*log(a + b*x)*a**5*d**4 + 8*log(a + b*
x)*a**4*b*c*d**3 - log(a + b*x)*a**4*b*d**4 - 12*log(a + b*x)*a**3*b**2*c*
*2*d**2 + 4*log(a + b*x)*a**3*b**2*c*d**3 + 8*log(a + b*x)*a**2*b**3*c**3*
d - 6*log(a + b*x)*a**2*b**3*c**2*d**2 - 2*log(a + b*x)*a*b**4*c**4 + 4*lo
g(a + b*x)*a*b**4*c**3*d - log(a + b*x)*b**5*c**4 + log((a*e + b*e*x)/(c +
 d*x))**2*a**3*b**2*c*d**3 + 3*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**3*c
**2*d**2 + 12*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**3*c*d**3*x + 6*log((
a*e + b*e*x)/(c + d*x))**2*a**2*b**3*d**4*x**2 - log((a*e + b*e*x)/(c + d*
x))**2*a*b**4*c**3*d + 12*log((a*e + b*e*x)/(c + d*x))**2*a*b**4*c*d**3*x*
*2 + 8*log((a*e + b*e*x)/(c + d*x))**2*a*b**4*d**4*x**3 + 4*log((a*e + b*e
*x)/(c + d*x))**2*b**5*c*d**3*x**3 + 3*log((a*e + b*e*x)/(c + d*x))**2*b**
5*d**4*x**4 + 12*log((a*e + b*e*x)/(c + d*x))*a**3*b**2*c**2*d**2 + 24*log
((a*e + b*e*x)/(c + d*x))*a**3*b**2*c*d**3*x + 2*log((a*e + b*e*x)/(c + d*
x))*a**3*b**2*c*d**3 + 12*log((a*e + b*e*x)/(c + d*x))*a**3*b**2*d**4*x...
```

3.57 $\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result . . . 707
Mathematica [B] (verified) . . . 708
Rubi [A] (verified) . . . 709
Maple [F] . . . 714
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Sympy [F(-1)] . . . 715
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Giac [F] . . . 716
Mupad [F(-1)] . . . 717
Reduce [F] . . . 717

Optimal result

Integrand size = 38, antiderivative size = 343

$$\begin{aligned}
&\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^2gix}{3bd} - \frac{B(bc-ad)^2gi(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2d} \\
&\quad - \frac{B(bc-ad)gi(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2} \\
&\quad + \frac{(bc-ad)gi(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b^2} \\
&\quad + \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} \\
&\quad - \frac{B(bc-ad)^3gi\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2d^2} \\
&\quad - \frac{B^2(bc-ad)^3gi\log(c+dx)}{3b^2d^2} - \frac{B^2(bc-ad)^3gi\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^2d^2}
\end{aligned}$$

output

```
1/3*B^2*(-a*d+b*c)^2*g*i*x/b/d-1/3*B*(-a*d+b*c)^2*g*i*(b*x+a)*(A+B*ln(e*(b
*x+a)/(d*x+c)))/b^2/d-1/3*B*(-a*d+b*c)*g*i*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*
x+c)))/b^2+1/6*(-a*d+b*c)*g*i*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2+
1/3*g*i*(b*x+a)^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b-1/3*B*(-a*d+b*c)
^3*g*i*ln((-a*d+b*c)/b/(d*x+c))*(A+B+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d^2-1/3*
B^2*(-a*d+b*c)^3*g*i*ln(d*x+c)/b^2/d^2-1/3*B^2*(-a*d+b*c)^3*g*i*polylog(2,
d*(b*x+a)/b/(d*x+c))/b^2/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 869 vs. $2(343) = 686$.

Time $= 0.71$ (sec) , antiderivative size $= 869$, normalized size of antiderivative $= 2.53$

$$\int (ag + bgx)(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$

$$= \frac{gi\Big(-6Ab^2Bcd(bc-ad)x + 6aAbBd^2(-bc+ad)x + 4AbBd(bc-ad)(bc+ad)x - 6bB^2cd(bc-ad)(a+$$

input

```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])
^2,x]
```

output

```
(g*i*(-6*A*b^2*B*c*d*(b*c - a*d)*x + 6*a*A*b*B*d^2*(-(b*c) + a*d)*x + 4*A*
b*B*d*(b*c - a*d)*(b*c + a*d)*x - 6*b*B^2*c*d*(b*c - a*d)*(a + b*x)*Log[(e
*(a + b*x))/(c + d*x)] + 6*a*B^2*d^2*(-(b*c) + a*d)*(a + b*x)*Log[(e*(a +
b*x))/(c + d*x)] + 4*B^2*d*(b*c - a*d)*(b*c + a*d)*(a + b*x)*Log[(e*(a + b
*x))/(c + d*x)] - 2*b^2*B*d^2*(b*c - a*d)*x^2*(A + B*Log[(e*(a + b*x))/(c
+ d*x)]) + 6*a^2*b*B*c*d^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)
]) - 2*a^3*B*d^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*a*b
^2*c*d^2*x*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 3*b^2*d^2*(b*c + a*d)*
x^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 2*b^3*d^3*x^3*(A + B*Log[(e*(
a + b*x))/(c + d*x)])^2 + 6*b*B^2*c*(b*c - a*d)^2*Log[c + d*x] + 6*a*B^2*d
*(b*c - a*d)^2*Log[c + d*x] - 4*B^2*(b*c - a*d)^2*(b*c + a*d)*Log[c + d*x]
 + 2*b^3*B*c^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 6*a*b^2
*B*c^2*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*B^2*(b*c -
a*d)*(a^2*d^2*Log[a + b*x] - b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x]))
- 3*a^2*b*B^2*c*d^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c
 - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + a^3*B^2*d^3*(Log
[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2,
 (d*(a + b*x))/(-(b*c) + a*d)]) - b^3*B^2*c^3*((2*Log[(d*(a + b*x))/(-(b*c
) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c -
 a*d)]) + 3*a*b^2*B^2*c^2*d*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log...
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {2962, 2783, 2773, 49, 2009, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\downarrow 2962$$

$$gi(bc-ad)^3\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$gi(bc-ad)^3\left(-\frac{2B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{3b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)$$

$$\downarrow 2773$$

$$gi(bc-ad)^3\left(-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{a+bx}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2b}\right)}{3b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+}{}\right)$$

$$\downarrow 49$$

$$gi(bc-ad)^3\left(-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\left(\frac{b}{d\left(\frac{d(a+bx)}{c+dx}-b\right)^2}+\frac{1}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{2b}\right)}{3b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d}{3b}\right)$$

$$\downarrow 2009$$

$$gi(bc-ad)^3\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^2}\right)}{2b}\right)}{3b}+\right)$$

$$\downarrow 2781$$

$$gi(bc-ad)^3\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)\Bigg/3b-2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{b}{d^2\left(b-\frac{d}{}\right.\right.}\right.$$

$\downarrow$ 2784

$$gi(bc-ad)^3\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{b}\right)\Bigg/3b-2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a}{2b(c+dx)^2\left(b-\frac{d}{}\right.\right.}\right.$$

$\downarrow$ 2754

$$gi(bc-ad)^3\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{B\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+}{c+}\right.\right.\right.}{d}\right.}{b}\right)\Bigg/3b$$

$\downarrow$ 2838

$$gi(bc-ad)^3\left(-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^2}\right)}{2b}\right)}{3b}+\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{(a}{-}\right.$$

input
```
Int[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)^3*g*i*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3
*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*(((a + b*x)^2*(A +
B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c +
d*x))^2) - (B*(b/(d^2*(b - (d*(a + b*x))/(c + d*x))) + Log[b - (d*(a + b*x
))/(c + d*x)]/d^2))/(2*b)))/(3*b) + (((a + b*x)^2*(A + B*Log[(e*(a + b*x))
/(c + d*x)])^2)/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(((
a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c + d*x)*(b - (d*(a + b
*x))/(c + d*x))) - (-(((A + B + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d
*(a + b*x))/(b*(c + d*x))])/d) - (B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))
])/d)/d))/b)/(3*b))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2783

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)\,(dix+ci)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input

```
int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$=\int (bgx+ag)(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algori
thm="fricas")
```

output

```
integral(A^2*b*d*g*i*x^2 + A^2*a*c*g*i + (A^2*b*c + A^2*a*d)*g*i*x + (B^2*
b*d*g*i*x^2 + B^2*a*c*g*i + (B^2*b*c + B^2*a*d)*g*i*x)*log((b*e*x + a*e)/(
d*x + c))^2 + 2*(A*B*b*d*g*i*x^2 + A*B*a*c*g*i + (A*B*b*c + A*B*a*d)*g*i*x
)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1252 vs. $2(326) = 652$.

Time = 0.14 (sec) , antiderivative size = 1252, normalized size of antiderivative = 3.65

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algori
thm="maxima")
```

output

```
1/3*A^2*b*d*g*i*x^3 + 1/2*A^2*b*c*g*i*x^2 + 1/2*A^2*a*d*g*i*x^2 + 2*(x*log
(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A
*B*a*c*g*i + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/
b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*b*c*g*i + (x^2*log(b
*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/
d^2 - (b*c - a*d)*x/(b*d))*A*B*a*d*g*i + 1/3*(2*x^3*log(b*e*x/(d*x + c) +
a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c
*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b*d*g*i + A^2*
a*c*g*i*x + 1/3*(b^2*c^3*g*i*log(e) - a^2*c*d^2*g*i - (3*g*i*log(e) - g*i)
*a*b*c^2*d)*B^2*log(d*x + c)/(b*d^2) + 1/3*(b^3*c^3*g*i - 3*a*b^2*c^2*d*g*
i + 3*a^2*b*c*d^2*g*i - a^3*d^3*g*i)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
- a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/6*(2*B^
2*b^3*d^3*g*i*x^3*log(e)^2 + ((3*g*i*log(e)^2 - 2*g*i*log(e))*b^3*c*d^2 +
(3*g*i*log(e)^2 + 2*g*i*log(e))*a*b^2*d^3)*B^2*x^2 - 2*((g*i*log(e) - g*i)
*b^3*c^2*d - (3*g*i*log(e)^2 - 2*g*i)*a*b^2*c*d^2 - (g*i*log(e) + g*i)*a^2
*b*d^3)*B^2*x + (2*B^2*b^3*d^3*g*i*x^3 + 6*B^2*a*b^2*c*d^2*g*i*x + 3*(b^3*
c*d^2*g*i + a*b^2*d^3*g*i)*B^2*x^2 + (3*a^2*b*c*d^2*g*i - a^3*d^3*g*i)*B^2
)*log(b*x + a)^2 + (2*B^2*b^3*d^3*g*i*x^3 + 6*B^2*a*b^2*c*d^2*g*i*x + 3*(b
^3*c*d^2*g*i + a*b^2*d^3*g*i)*B^2*x^2 - (b^3*c^3*g*i - 3*a*b^2*c^2*d*g*i)*
B^2)*log(d*x + c)^2 + 2*(2*B^2*b^3*d^3*g*i*x^3*log(e) + ((3*g*i*log(e) ...
```

Giac [F]

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algori
thm="giac")
```

output

```
integrate((b*g*x + a*g)*(d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)^2
, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)\,(c\,i+d\,i\,x)\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

Reduce [F]

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
(g*i*( - 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d
*x**2),x)*a**3*b**2*d**4 + 6*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a
*d*x + b*c*x + b*d*x**2),x)*a**2*b**3*c*d**3 - 6*int((log((a*e + b*e*x)/(c
 + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**4*c**2*d**2 + 2*int((
log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**5*c
**3*d - 2*log(a + b*x)*a**4*d**3 + 6*log(a + b*x)*a**3*b*c*d**2 - 6*log(a
+ b*x)*a**2*b**2*c**2*d + 2*log(a + b*x)*a*b**3*c**3 + log((a*e + b*e*x)/(
c + d*x))**2*a**2*b**2*c*d**2 + log((a*e + b*e*x)/(c + d*x))**2*a*b**3*c**
2*d + 6*log((a*e + b*e*x)/(c + d*x))**2*a*b**3*c*d**2*x + 3*log((a*e + b*e
*x)/(c + d*x))**2*a*b**3*d**3*x**2 + 3*log((a*e + b*e*x)/(c + d*x))**2*b**
4*c*d**2*x**2 + 2*log((a*e + b*e*x)/(c + d*x))**2*b**4*d**3*x**3 + 6*log((
a*e + b*e*x)/(c + d*x))*a**2*b**2*c**2*d + 12*log((a*e + b*e*x)/(c + d*x))
*a**2*b**2*c*d**2*x + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*c*d**2 + 6*
log((a*e + b*e*x)/(c + d*x))*a**2*b**2*d**3*x**2 + 2*log((a*e + b*e*x)/(c
+ d*x))*a**2*b**2*d**3*x - 2*log((a*e + b*e*x)/(c + d*x))*a*b**3*c**3 - 2*
log((a*e + b*e*x)/(c + d*x))*a*b**3*c**2*d + 6*log((a*e + b*e*x)/(c + d*x)
)*a*b**3*c*d**2*x**2 + 4*log((a*e + b*e*x)/(c + d*x))*a*b**3*d**3*x**3 + 2
*log((a*e + b*e*x)/(c + d*x))*a*b**3*d**3*x**2 - 2*log((a*e + b*e*x)/(c +
d*x))*b**4*c**2*d*x - 2*log((a*e + b*e*x)/(c + d*x))*b**4*c*d**2*x**2 + 6*
a**3*b*c*d**2*x + 3*a**3*b*d**3*x**2 + 2*a**3*b*d**3*x + 3*a**2*b**2*c*...
```

3.58 $\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result . . . 719
Mathematica [A] (verified) . . . 720
Rubi [A] (verified) . . . 720
Maple [F] . . . 724
Fricas [F] . . . 724
Sympy [F(-1)] . . . 725
Maxima [B] (verification not implemented) . . . 725
Giac [F] . . . 726
Mupad [F(-1)] . . . 726
Reduce [F] . . . 727

Optimal result

Integrand size = 30, antiderivative size = 203

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= -\frac{B(bc-ad)i(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2} + \frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d}$$
$$+ \frac{B^2(bc-ad)^2 i\log(c+dx)}{b^2 d} + \frac{B(bc-ad)^2 i\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2 d}$$
$$- \frac{B^2(bc-ad)^2 i\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2 d}$$

output

```
-B*(-a*d+b*c)*i*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2+1/2*i*(d*x+c)^2*(A
+B*ln(e*(b*x+a)/(d*x+c)))^2/d+B^2*(-a*d+b*c)^2*i*ln(d*x+c)/b^2/d+B*(-a*d+b
*c)^2*i*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^2/d-B^2*(-
a*d+b*c)^2*i*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.01

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{i\left((c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B(bc-ad)\left((-bBc+aBd)\log^2(a+bx)+2\left(Abdx+Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+(-bBc+aBd)\right.}{2d}$$

input
```
Integrate[(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(i*((c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*((
-(b*B*c) + a*B*d)*Log[a + b*x]^2 + 2*(A*b*d*x + B*d*(a + b*x)*Log[(e*(a +
b*x))/(c + d*x)] + (-(b*B*c) + a*B*d)*Log[c + d*x]) + 2*(b*c - a*d)*Log[a
+ b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)] + B*Log[(b*(c + d*x))/(b*c - a*
d)]) + 2*B*(b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/b^2))/(2
*d)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2952, 2756, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$
$$\downarrow 2952$$
$$i(bc-ad)^2\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3} d\frac{a+bx}{c+dx}$$
$$\downarrow 2756$$

$$i(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{d}\right)$$

$$\downarrow 2789$$

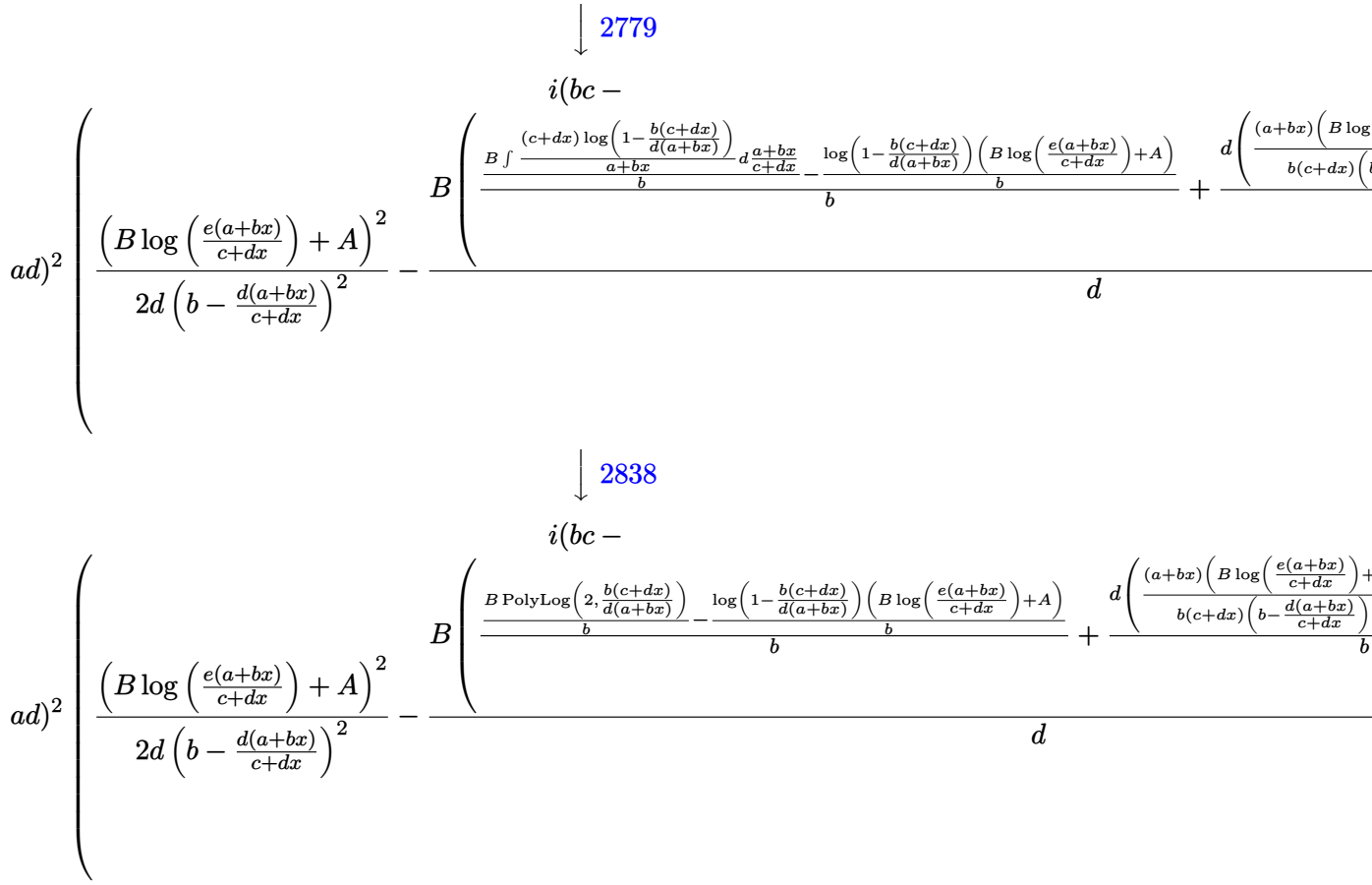
$$ad)^2\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{i(bc-\\ B\left(\frac{d\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{d}\right)$$

$$\downarrow 2751$$

$$ad)^2\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{i(bc-\\ B\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{B\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a}{c}}{b}\right)}{d}\right)$$

$$\downarrow 16$$

$$ad)^2\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{i(bc-\\ B\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}\right)}{d}\right)$$



input
```
Int[(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)^2*i*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(2*d*(b - (d*(a +
b*x))/(c + d*x))^2) - (B*((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*Log[b - (d*(a + b*x)
)/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 -
 (b*(c + d*x))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(d*(a + b*
x))])/b)/b))/d)
```

Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (dix + ci)\left(A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input

```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (dix + ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2 dx$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas"
)
```

output

```
integral(A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log((b*e*x + a*e)/(d*
x + c))^2 + 2*(A*B*d*i*x + A*B*c*i)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(200) = 400$.

Time = 0.12 (sec) , antiderivative size = 633, normalized size of antiderivative = 3.12

$$\begin{aligned}
&\int (ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= \frac{1}{2}A^2 dix^2 + 2\left(x\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) + \frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right)ABci \\
&\quad + \left(x^2\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) - \frac{a^2\log(bx+a)}{b^2} + \frac{c^2\log(dx+c)}{d^2} - \frac{(bc-ad)x}{bd}\right)ABdi \\
&\quad + A^2cix - \frac{((i\log(e)-i)bc^2 + acdi)B^2\log(dx+c)}{bd} \\
&\quad - \frac{(b^2c^2i - 2\,abcdi + a^2d^2i)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{b^2d} \\
&\quad + \frac{B^2b^2d^2ix^2\log(e)^2 + 2\left(abd^2i\log(e) + \left(i\log(e)^2 - i\log(e)\right)b^2cd\right)B^2x + (B^2b^2d^2ix^2 + 2\,B^2b^2cdix + (2}{}
\end{aligned}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima"
)
```

output
```
1/2*A^2*d*i*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x +
a)/b - c*log(d*x + c)/d)*A*B*c*i + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c
)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*
B*d*i + A^2*c*i*x - ((i*log(e) - i)*b*c^2 + a*c*d*i)*B^2*log(d*x + c)/(b*d
) - (b^2*c^2*i - 2*a*b*c*d*i + a^2*d^2*i)*(log(b*x + a)*log((b*d*x + a*d)/
(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d) + 1/2*(B
^2*b^2*d^2*i*x^2*log(e)^2 + 2*(a*b*d^2*i*log(e) + (i*log(e)^2 - i*log(e))*
b^2*c*d)*B^2*x + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x + (2*a*b*c*d*i - a
^2*d^2*i)*B^2)*log(b*x + a)^2 + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x + B
^2*b^2*c^2*i)*log(d*x + c)^2 + 2*(B^2*b^2*d^2*i*x^2*log(e) + ((2*i*log(e)
- i)*b^2*c*d + a*b*d^2*i)*B^2*x + ((2*i*log(e) - i)*a*b*c*d - (i*log(e) -
i)*a^2*d^2)*B^2)*log(b*x + a) - 2*(B^2*b^2*d^2*i*x^2*log(e) + ((2*i*log(e)
 - i)*b^2*c*d + a*b*d^2*i)*B^2*x + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x
+ (2*a*b*c*d*i - a^2*d^2*i)*B^2)*log(b*x + a))*log(d*x + c))/(b^2*d)
```

Giac [F]

$$\int (ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (dix + ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2 dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (c\,i + d\,i\,x)\left(A + B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

Reduce [F]

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{i\left(-2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{bd\,x^2+adx+bcx+ac}dx\right)a^2b^2d^3+4\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{bd\,x^2+adx+bcx+ac}dx\right)a\,b^3c\,d^2-2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)x}{bd\,x^2+adx+bcx+ac}dx\right)b^4c^2d\right.}{}$$

input
```
int((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
(i*( - 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x
**2),x)*a**2*b**2*d**3 + 4*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d
*x + b*c*x + b*d*x**2),x)*a*b**3*c*d**2 - 2*int((log((a*e + b*e*x)/(c + d*
x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**4*c**2*d - 2*log(c + d*x)*a*
*3*d**2 + 4*log(c + d*x)*a**2*b*c*d + 2*log(c + d*x)*a**2*b*d**2 - 2*log(c
 + d*x)*a*b**2*c**2 - 4*log(c + d*x)*a*b**2*c*d + 2*log(c + d*x)*b**3*c**2
 + log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c*d + 2*log((a*e + b*e*x)/(c + d
*x))**2*b**3*c*d*x + log((a*e + b*e*x)/(c + d*x))**2*b**3*d**2*x**2 - 2*lo
g((a*e + b*e*x)/(c + d*x))*a**3*d**2 + 4*log((a*e + b*e*x)/(c + d*x))*a**2
*b*c*d + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d**2 + 4*log((a*e + b*e*x)/
(c + d*x))*a*b**2*c*d*x - 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*c*d + 2*lo
g((a*e + b*e*x)/(c + d*x))*a*b**2*d**2*x**2 + 2*log((a*e + b*e*x)/(c + d*x
))*a*b**2*d**2*x - 2*log((a*e + b*e*x)/(c + d*x))*b**3*c*d*x + 2*a**2*b*c*
d*x + a**2*b*d**2*x**2 + 2*a**2*b*d**2*x - 2*a*b**2*c*d*x))/(2*b*d)
```

3.59 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

Optimal result . . . 728
Mathematica [B] (verified) . . . 729
Rubi [A] (verified) . . . 730
Maple [F] . . . 735
Fricas [F] . . . 735
Sympy [F] . . . 735
Maxima [F] . . . 736
Giac [F] . . . 736
Mupad [F(-1)] . . . 737
Reduce [F] . . . 737

Optimal result

Integrand size = 40, antiderivative size = 286

$$\begin{aligned}
&\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx \\
&= \frac{2B(bc-ad)i\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g} \\
&\quad + \frac{di(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g} \\
&\quad - \frac{(bc-ad)i\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g} \\
&\quad + \frac{2B^2(bc-ad)i\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2g} \\
&\quad + \frac{2B(bc-ad)i\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g} \\
&\quad + \frac{2B^2(bc-ad)i\,\mathrm{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}
\end{aligned}$$

output

```
2*B*(-a*d+b*c)*i*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/
g+d*i*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2/g-(-a*d+b*c)*i*(A+B*ln(e*(
b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^2/g+2*B^2*(-a*d+b*c)*i*poly
log(2,d*(b*x+a)/b/(d*x+c))/b^2/g+2*B*(-a*d+b*c)*i*(A+B*ln(e*(b*x+a)/(d*x+c
)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/g+2*B^2*(-a*d+b*c)*i*polylog(3,b*(d
*x+c)/d/(b*x+a))/b^2/g
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1214 vs. $2(286) = 572$.

Time = 1.38 (sec) , antiderivative size = 1214, normalized size of antiderivative = 4.24

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag + bgx}\,dx = \text{Too large to display}$$

input

```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*
g*x),x]
```

output

```
(i*(3*A^2*b*d*x + 3*A^2*(b*c - a*d)*Log[a + b*x] - 3*A*B*(a*d*Log[a/b + x]
^2 - 2*a*d*Log[a/b + x]*(1 + Log[a + b*x]) + 2*(-(b*c) + a*d + Log[c/d + x
]*(b*c + a*d*Log[a + b*x] - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)]) + (-(b*
d*x) + a*d*Log[a + b*x])*Log[(e*(a + b*x))/(c + d*x)]) - 2*a*d*PolyLog[2,
(b*(c + d*x))/(b*c - a*d)]) + 3*A*b*B*c*(Log[a/b + x]^2 - 2*Log[a + b*x]*(
Log[a/b + x] - Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)]) - 2*(Log[c/d +
 x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*
d)])) - B^2*(a*d*Log[a/b + x]^3 - 3*d*(2*b*x - 2*(a + b*x)*Log[a/b + x] +
(a + b*x)*Log[a/b + x]^2) - 3*b*(2*d*x - 2*(c + d*x)*Log[c/d + x] + (c + d
*x)*Log[c/d + x]^2) - 3*d*(b*x - a*Log[a + b*x])*(-Log[a/b + x] + Log[c/d
+ x] + Log[(e*(a + b*x))/(c + d*x)])^2 + 6*(a*d + 2*b*d*x - b*d*x*Log[c/d
+ x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(a + b*x)) + d*(a + b*x)*Log[c
/d + x] + (b*c - a*d)*Log[(b*(c + d*x))/(b*c - a*d)]) + (b*c - a*d)*PolyLo
g[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 3*(Log[a/b + x] - Log[c/d + x] - Log
[(e*(a + b*x))/(c + d*x)])*(-2*b*c + 2*a*d - 2*d*(a + b*x)*Log[a/b + x] +
a*d*Log[a/b + x]^2 + 2*Log[c/d + x]*(b*(c + d*x) - a*d*Log[(d*(a + b*x))/(
-(b*c) + a*d)]) - 2*a*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 3*a*d*(Lo
g[a/b + x]^2*(Log[c/d + x] - Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Log[a/b +
 x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*PolyLog[3, (d*(a + b*x))/
(-(b*c) + a*d)]) + 3*a*d*(Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*...
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2962, 2789, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{ag+bgx}\,dx$$

$$\downarrow 2962$$

$$\frac{i(bc-ad)\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g}$$

$$\downarrow 2789$$

$$\frac{i(bc-ad)\left(\frac{d\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$\downarrow$ 2755

$$\frac{i(bc-ad)\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2B\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$\downarrow$ 2754

$$\frac{i(bc-ad)\left(\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2B\left(\frac{B\int \frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a}{b(c}\right.}{}\right.}{b}\right.}{b}\right.}{g}$$

$\downarrow$ 2779

$$i(bc-ad)\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2B\left(\frac{B\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}\right)}{b}\right)}{b}+\frac{2B\int-}{}\right)$$

$$g$$

$$\downarrow 2821$$

$$i(bc-ad)\left(\frac{2B\left(\frac{\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-B\int\frac{(c+dx)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}$$

$$g$$

$$\downarrow 2838$$

$$i(bc-ad)\left(\frac{2B\left(\frac{\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-B\int\frac{(c+dx)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}$$

$$g$$

$$\downarrow 7143$$

$$\frac{i(bc-ad)\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2B\left(-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}-\frac{B\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d}\right)}{b}\right)}{b}+\frac{2B\left(\mathrm{PolyLog}\left(2,\frac{b(}{d(}\right.\right.}{g}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x),x
]
```

output
```
((b*c - a*d)*i*((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(b*
(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (2*B*(-(((A + B*Log[(e*(a + b*x
))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (B*PolyLog[2, (d
*(a + b*x))/(b*(c + d*x))])/d))/b))/b + (-(((A + B*Log[(e*(a + b*x))/(c +
d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (2*B*((A + B*Log[(e*(a
 + b*x))/(c + d*x)])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + B*PolyLog[3
, (b*(c + d*x))/(d*(a + b*x))]))/b)/b))/g
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2779

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))  Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d  Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d  Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)  Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(dix+ci)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{bgx+ag}dx$$

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

output
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}dx = \int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algori
thm="fricas")
```

output
```
integral((A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log((b*e*x + a*e)/(d
*x + c))^2 + 2*(A*B*d*i*x + A*B*c*i)*log((b*e*x + a*e)/(d*x + c)))/(b*g*x
+ a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}dx$$
$$= \frac{i\left(\int \frac{A^2c}{a+bx}dx + \int \frac{A^2dx}{a+bx}dx + \int \frac{B^2c\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a+bx}dx + \int \frac{2ABc\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}dx + \int \frac{B^2dx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a+bx}dx\right.}{g}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g),x)
```

output
```
i*(Integral(A**2*c/(a + b*x), x) + Integral(A**2*d*x/(a + b*x), x) + Integ
ral(B**2*c*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x), x) + Integra
l(2*A*B*c*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Integral(B*
*2*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(2*
A*B*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algori
thm="maxima")
```

output
```
A^2*d*i*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + A^2*c*i*log(b*g*x + a*g)/(b*g
) + (B^2*b*d*i*x + (b*c*i - a*d*i)*B^2*log(b*x + a))*log(d*x + c)^2/(b^2*g
) - integrate(-(B^2*b^2*c^2*i*log(e)^2 + 2*A*B*b^2*c^2*i*log(e) + (B^2*b^2
*d^2*i*log(e)^2 + 2*A*B*b^2*d^2*i*log(e))*x^2 + (B^2*b^2*d^2*i*x^2 + 2*B^2
*b^2*c*d*i*x + B^2*b^2*c^2*i)*log(b*x + a)^2 + 2*(B^2*b^2*c*d*i*log(e)^2 +
 2*A*B*b^2*c*d*i*log(e))*x + 2*(B^2*b^2*c^2*i*log(e) + A*B*b^2*c^2*i + (B^
2*b^2*d^2*i*log(e) + A*B*b^2*d^2*i)*x^2 + 2*(B^2*b^2*c*d*i*log(e) + A*B*b^
2*c*d*i)*x)*log(b*x + a) - 2*(B^2*b^2*c^2*i*log(e) + A*B*b^2*c^2*i + ((i*l
og(e) + i)*B^2*b^2*d^2 + A*B*b^2*d^2*i)*x^2 + (2*A*B*b^2*c*d*i + (2*b^2*c*
d*i*log(e) + a*b*d^2*i)*B^2)*x + (B^2*b^2*d^2*i*x^2 + (3*b^2*c*d*i - a*b*d
^2*i)*B^2*x + (b^2*c^2*i + a*b*c*d*i - a^2*d^2*i)*B^2)*log(b*x + a))*log(d
*x + c))/(b^3*d*g*x^2 + a*b^2*c*g + (b^3*c*g + a*b^2*d*g)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algori
thm="giac")
```

output
```
integrate((d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x),x
)
```

output
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x),
x)
```

Reduce [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{i\left(\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2}{bx+a}dx\right) b^4c + 2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{bx+a}dx\right) a\,b^3c + \left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2 x}{bx+a}dx\right) b^4d + 2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right) x}{bx+a}dx\right)}{b^2g}$$

input
```
int((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

output

```
(i*(int(log((a*e + b*e*x)/(c + d*x))**2/(a + b*x),x)*b**4*c + 2*int(log((a
*e + b*e*x)/(c + d*x))/(a + b*x),x)*a*b**3*c + int((log((a*e + b*e*x)/(c +
 d*x))**2*x)/(a + b*x),x)*b**4*d + 2*int((log((a*e + b*e*x)/(c + d*x))*x)/
(a + b*x),x)*a*b**3*d - log(a + b*x)*a**3*d + log(a + b*x)*a**2*b*c + a**2
*b*d*x))/(b**2*g)
```

3.60 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result . . . 739
Mathematica [B] (verified) . . . 740
Rubi [A] (verified) . . . 741
Maple [F] . . . 744
Fricas [F] . . . 744
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Maxima [F] . . . 745
Giac [F] . . . 746
Mupad [F(-1)] . . . 746
Reduce [F] . . . 746

Optimal result

Integrand size = 40, antiderivative size = 241

$$\begin{aligned}
&\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx \\
&= -\frac{2B^2i(c+dx)}{bg^2(a+bx)} - \frac{2Bi(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)} \\
&\quad - \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} \\
&\quad + \frac{2Bdi\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{2B^2di\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}
\end{aligned}$$

output

```
-2*B^2*i*(d*x+c)/b/g^2/(b*x+a)-2*B*i*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b
/g^2/(b*x+a)-i*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b/g^2/(b*x+a)-d*i*(A+
B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^2/g^2+2*B*d*i*(A+B*
ln(e*(b*x+a)/(d*x+c)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/g^2+2*B^2*d*i*po
lylog(3,b*(d*x+c)/d/(b*x+a))/b^2/g^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1407 vs. $2(241) = 482$.

Time = 2.12 (sec) , antiderivative size = 1407, normalized size of antiderivative = 5.84

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^2}\,dx = \text{Too large to display}$$

input

```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*
g*x)^2,x]
```

output

```
(i*((3*A^2*(-(b*c) + a*d))/(a + b*x) + 3*A^2*d*Log[a + b*x] - (6*A*b*B*c*(
-(d*(a + b*x)*Log[c/d + x]) + d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) + a*d)
] + (b*c - a*d)*(1 + Log[(e*(a + b*x))/(c + d*x)])))/((b*c - a*d)*(a + b*x
)) + (3*b*B^2*c*(-2*b*c + 2*a*d - 2*d*(a + b*x)*Log[a + b*x] - 2*(b*c - a*
d)*Log[(e*(a + b*x))/(c + d*x)] - 2*d*(a + b*x)*Log[a + b*x]*Log[(e*(a + b
*x))/(c + d*x)] - (b*c - a*d)*Log[(e*(a + b*x))/(c + d*x)]^2 + 2*d*(a + b*
x)*Log[c + d*x] - 2*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c - a*
d)/(b*c + b*d*x)] + d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c
+ d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + d*(a
 + b*x)*(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a
*d)])))/((b*c - a*d)*(a + b*x)) + 3*A*B*d*(Log[a/b + x]^2 - 2*Log[a/b + x]
*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[a
 + b*x]*((a*d)/(b*c - a*d) + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])
+ 2*a*((a + b*x)^(-1) + Log[(e*(a + b*x))/(c + d*x)]/(a + b*x) + (d*Log[c
+ d*x])/(-(b*c) + a*d)) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + (B^2*
d*((b*c - a*d)*(a + b*x)*Log[a/b + x]^3 + 3*a*(b*c - a*d)*(2 + 2*Log[a/b +
 x] + Log[a/b + x]^2) + 3*(b*c - a*d)*(a + (a + b*x)*Log[a + b*x])*(-Log[a
/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])^2 + 3*a*(d*(a + b*x
)*Log[a/b + x]^2 + 2*((-(b*c) + a*d)*Log[c/d + x] + d*(a + b*x)*(Log[a ...
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {2962, 2780, 2742, 2741, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^2}\,dx$$

$$\downarrow 2962$$

$$\frac{i\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g^2}$$

$$\downarrow 2780$$

$$\frac{i\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^2}$$

$$\downarrow 2742$$

$$\frac{i\left(\frac{2B\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^2}$$

$$\downarrow 2741$$

$$\frac{i\left(\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{2B\left(-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}}{b}\right)}{g^2}$$

$$\downarrow 2779$$

$$i\left(\frac{d\left(\frac{2B\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}\right)}{b}+\frac{2B\left(-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right.}{a+bx}\right.}{g^2}$$

$$\downarrow 2821$$

$$i\left(\frac{d\left(\frac{2B\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-B\int\frac{(c+dx)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}\right)}{b}+\frac{2B}{g^2}$$

$$\downarrow 7143$$

$$i\left(\frac{d\left(\frac{2B\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+B\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}\right)}{b}+\frac{2B\left(-\frac{(c+dx)\left(B\log\right.\right.}{g^2}$$

input

```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^2
,x]
```

output

```
(i*((-(((c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x)) + 2*B
*(-((B*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/(a + b*x)))/b + (d*(-(((A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1
 - (b*(c + d*x))/(d*(a + b*x))])/b) + (2*B*((A + B*Log[(e*(a + b*x))/(c + d
*x)])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + B*PolyLog[3, (b*(c + d*x))
/(d*(a + b*x))]))/b))/b))/g^2
```

Defintions of rubi rules used

rule 2741

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2779

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2821

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(dix+ci)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(bgx+ag)^2}dx$$

input

```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

output

```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}dx = \int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}dx$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algo
rithm="fricas")
```

output

```
integral((A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log((b*e*x + a*e)/(d
*x + c))^2 + 2*(A*B*d*i*x + A*B*c*i)*log((b*e*x + a*e)/(d*x + c)))/(b^2*g^
2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algo
rithm="maxima")
```

output
```
A^2*d*i*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - 2*A*B*c*i*(
log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1/(b^2*g^2*x
+ a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2*c
 - a*b*d)*g^2)) - A^2*c*i/(b^2*g^2*x + a*b*g^2) - ((b*c*i - a*d*i)*B^2 - (
B^2*b*d*i*x + B^2*a*d*i)*log(b*x + a))*log(d*x + c)^2/(b^3*g^2*x + a*b^2*g
^2) - integrate(-(B^2*b^2*c^2*i*log(e)^2 + (B^2*b^2*d^2*i*log(e)^2 + 2*A*B
*b^2*d^2*i*log(e))*x^2 + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x + B^2*b^2*
c^2*i)*log(b*x + a)^2 + 2*(B^2*b^2*c*d*i*log(e)^2 + A*B*b^2*c*d*i*log(e))*
x + 2*(B^2*b^2*c^2*i*log(e) + (B^2*b^2*d^2*i*log(e) + A*B*b^2*d^2*i)*x^2 +
 (2*B^2*b^2*c*d*i*log(e) + A*B*b^2*c*d*i)*x)*log(b*x + a) - 2*((b^2*c^2*i*
log(e) - a*b*c*d*i + a^2*d^2*i)*B^2 + (B^2*b^2*d^2*i*log(e) + A*B*b^2*d^2*
i)*x^2 + (A*B*b^2*c*d*i + ((2*i*log(e) - i)*b^2*c*d + a*b*d^2*i)*B^2)*x +
(2*B^2*b^2*d^2*i*x^2 + 2*(b^2*c*d*i + a*b*d^2*i)*B^2*x + (b^2*c^2*i + a^2*
d^2*i)*B^2)*log(b*x + a))*log(d*x + c))/(b^4*d*g^2*x^3 + a^2*b^2*c*g^2 + (
b^4*c*g^2 + 2*a*b^3*d*g^2)*x^2 + (2*a*b^3*c*g^2 + a^2*b^2*d*g^2)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algorithm="giac")
```

output
```
integrate((d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(c\,i+d\,i\,x)\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(a\,g+b\,g\,x)^2}\,dx$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^2,x)
```

output
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^2, x)
```

Reduce [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

input
```
int((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

output

```
(i*(int((log((a*e + b*e*x)/(c + d*x))**2*x)/(a**2 + 2*a*b*x + b**2*x**2),x
)*a**3*b**4*d**2 - int((log((a*e + b*e*x)/(c + d*x))**2*x)/(a**2 + 2*a*b*x
 + b**2*x**2),x)*a**2*b**5*c*d + int((log((a*e + b*e*x)/(c + d*x))**2*x)/(
a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**5*d**2*x - int((log((a*e + b*e*x)/(
c + d*x))**2*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a*b**6*c*d*x + 2*int((log(
(a*e + b*e*x)/(c + d*x))*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**4*b**3*d**2
 - 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*
a**3*b**4*c*d + 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a**2 + 2*a*b*x + b
**2*x**2),x)*a**3*b**4*d**2*x - 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*
*2 + 2*a*b*x + b**2*x**2),x)*a**2*b**5*c*d*x + log(a + b*x)*a**5*d**2 - lo
g(a + b*x)*a**4*b*c*d + log(a + b*x)*a**4*b*d**2*x - log(a + b*x)*a**3*b**
2*c*d*x + 2*log(a + b*x)*a**2*b**3*c**2 + 2*log(a + b*x)*a*b**4*c**2*x + 2
*log(a + b*x)*a*b**4*c**2 + 2*log(a + b*x)*b**5*c**2*x - 2*log(c + d*x)*a*
*2*b**3*c**2 - 2*log(c + d*x)*a*b**4*c**2*x - 2*log(c + d*x)*a*b**4*c**2 -
 2*log(c + d*x)*b**5*c**2*x + log((a*e + b*e*x)/(c + d*x))**2*a*b**4*c**2
+ log((a*e + b*e*x)/(c + d*x))**2*a*b**4*c*d*x + 2*log((a*e + b*e*x)/(c +
d*x))*a**2*b**3*c*d*x - 2*log((a*e + b*e*x)/(c + d*x))*a*b**4*c**2*x + 2*l
og((a*e + b*e*x)/(c + d*x))*a*b**4*c*d*x - 2*log((a*e + b*e*x)/(c + d*x))*
b**5*c**2*x - a**4*b*d**2*x + 2*a**3*b**2*c*d*x - a**2*b**3*c**2*x + 2*a**
2*b**3*c*d*x - 2*a*b**4*c**2*x + 2*a*b**4*c*d*x - 2*b**5*c**2*x))/(a*b*...
```

3.61 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result . . . 748
Mathematica [C] (verified) . . . 749
Rubi [A] (verified) . . . 750
Maple [B] (verified) . . . 751
Fricas [B] (verification not implemented) . . . 753
Sympy [B] (verification not implemented) . . . 753
Maxima [B] (verification not implemented) . . . 755
Giac [A] (verification not implemented) . . . 756
Mupad [B] (verification not implemented) . . . 756
Reduce [B] (verification not implemented) . . . 757

Optimal result

Integrand size = 40, antiderivative size = 141

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{B^2 i(c+dx)^2}{4(bc-ad)g^3(a+bx)^2} - \frac{Bi(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)g^3(a+bx)^2} - \frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)g^3(a+bx)^2}$$

output

```
-1/4*B^2*i*(d*x+c)^2/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*B*i*(d*x+c)^2*(A+B*ln(e*
(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*i*(d*x+c)^2*(A+B*ln(e*(b*x+
a)/(d*x+c)))^2/(-a*d+b*c)/g^3/(b*x+a)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.98 (sec) , antiderivative size = 765, normalized size of antiderivative = 5.43

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{i\left(2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-4d(-bc+ad)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+4Bd(a+bx\right.}{}$$

input

```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*
g*x)^3,x]
```

output

```
-1/4*(i*(2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 4*d*(-(b
*c) + a*d)*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 4*B*d*(a + b
*x)*(2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d*(a + b*x)*Lo
g[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*d*(a + b*x)*(A + B*Log
[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*B*(b*c - a*d + d*(a + b*x)*Log
[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a
 + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(
-(b*c) + a*d)]) + B*d*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Lo
g[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + B*(
2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d*(-(b*c) + a*d)*
(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 4*d^2*(a + b*x)^2*Log[a +
 b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d^2*(a + b*x)^2*(A + B*Log[
(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 4*B*d*(a + b*x)*(b*c - a*d + d*(a
 + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*((b*c - a*d)^2 + 2*d*
(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x
)^2*Log[c + d*x]) + 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Lo
g[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]
) - 2*B*d^2*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*
x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b^2*(b*c -
 a*d)*g^3*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2962, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{i\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3(bc-ad)}$$

$$\downarrow 2742$$

$$\frac{i\left(B\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}\right)}{g^3(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{i\left(B\left(-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}\right)-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}\right)}{g^3(bc-ad)}$$

input

```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^3
,x]
```

output

```
(i*(-1/2*((c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x)^2
+ B*(-1/4*(B*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[(e*(a + b*
x))/(c + d*x)]))/(2*(a + b*x)^2))))/((b*c - a*d)*g^3)
```

Defintions of rubi rules used

rule 2741

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(135) = 270$.

Time = 1.47 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.34

method	result
parts	$\frac{iA^2\left(-\frac{-da+bc}{2b^2(bx+a)^2}-\frac{d}{b^2(bx+a)}\right)}{g^3}-\frac{iB^2e^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}\right)}{g^3(da-bc)}-\frac{2iA}{}$
norman	$\frac{B^2cdix\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(da-bc)}+\frac{ciBd(2A+B)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(da-bc)}-\frac{2A^2aid+2A^2bci+2adiBA+2bciBA+adiB^2+bciB^2}{4gb^2}-\frac{\left(2A^2id+2diBA+c\right.}{2gb}$ $(bx+a)^2$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{id^2eA^2}{2(da-bc)^2g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{2id^2eAB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}\right)}{(da-bc)^2g^3}+\frac{id^2eB^2\left(-\frac{\ln\left(\frac{b}{2}\right.\right.}{2\left(\frac{b}{d}\right.}\right.}{d^2}$
default	$-\frac{e(da-bc)\left(-\frac{id^2eA^2}{2(da-bc)^2g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{2id^2eAB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}\right)}{(da-bc)^2g^3}+\frac{id^2eB^2\left(-\frac{\ln\left(\frac{b}{2}\right.\right.}{2\left(\frac{b}{d}\right.}\right.}{d^2}$
risch	$\frac{iA^2da}{2g^3b^2(bx+a)^2}-\frac{iA^2c}{2g^3b(bx+a)^2}-\frac{iA^2d}{g^3b^2(bx+a)}+\frac{iB^2e^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2g^3(da-bc)\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{ebc}{d(dx+c)}\right)^2}+\frac{iB^2e^2\ln\left(\frac{be}{d}+\frac{(da}{d(d}\right.}{2g^3(da-bc)\left(\frac{be}{d}+\frac{ea}{dx+c}-\right.}$
parallelrisch	$-\frac{2A^2a^2b^2d^3i+B^2a^2b^2d^3i-B^2b^4c^2di+4A^2xab^3d^3i-4A^2xb^4cd^2i+2B^2xab^3d^3i-2B^2xb^4cd^2i-2B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4}{}$
orering	Expression too large to display

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x,method=_RETU
RNVERBOSE)
```

output
```
i*A^2/g^3*(-1/2*(-a*d+b*c)/b^2/(b*x+a)^2-d/b^2/(b*x+a))-i*B^2/g^3/(a*d-b*c
)*e^2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/
4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2*i*A*B/g^3/(a*d-b*c)*e^2*(-1/2/(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(135) = 270$.

Time = 0.09 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.05

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{2\left((2A^2+2AB+B^2)b^2cd-(2A^2+2AB+B^2)abd^2\right)ix+2\left(B^2b^2d^2ix^2+2B^2b^2cdix+B^2b^2c^2i\right)\log}{4\left((b^5c}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algo
rithm="fricas")
```

output
```
-1/4*(2*((2*A^2 + 2*A*B + B^2)*b^2*c*d - (2*A^2 + 2*A*B + B^2)*a*b*d^2)*i*
x + 2*(B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x + B^2*b^2*c^2*i)*log((b*e*x +
 a*e)/(d*x + c))^2 + ((2*A^2 + 2*A*B + B^2)*b^2*c^2 - (2*A^2 + 2*A*B + B^2
)*a^2*d^2)*i + 2*((2*A*B + B^2)*b^2*d^2*i*x^2 + 2*(2*A*B + B^2)*b^2*c*d*i*
x + (2*A*B + B^2)*b^2*c^2*i)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c - a*b^4
*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(122) = 244$.

Time = 5.15 (sec) , antiderivative size = 714, normalized size of antiderivative = 5.06

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$
$$-\frac{Bd^2i(2A+B)\log\left(x+\frac{2ABad^3i+2ABbcd^2i+B^2ad^3i+B^2bcd^2i-\frac{Ba^2d^4i(2A+B)}{ad-bc}+\frac{2Babcd^3i(2A+B)}{ad-bc}-\frac{Bb^2c^2d^2i(2A+B)}{ad-bc}}{4ABbd^3i+2B^2bd^3i}\right)}{2b^2g^3(ad-bc)}$$
$$+\frac{Bd^2i(2A+B)\log\left(x+\frac{2ABad^3i+2ABbcd^2i+B^2ad^3i+B^2bcd^2i+\frac{Ba^2d^4i(2A+B)}{ad-bc}-\frac{2Babcd^3i(2A+B)}{ad-bc}+\frac{Bb^2c^2d^2i(2A+B)}{ad-bc}}{4ABbd^3i+2B^2bd^3i}\right)}{2b^2g^3(ad-bc)}$$
$$+\frac{(B^2c^2i+2B^2cdix+B^2d^2ix^2)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^3dg^3-2a^2bcg^3+4a^2bdg^3x-4ab^2cg^3x+2ab^2dg^3x^2-2b^3cg^3x^2}$$
$$+\frac{-2A^2adi-2A^2bci-2ABadi-2ABbci-B^2adi-B^2bci+x(-4A^2bdi-4ABbdi-2B^2bdi)}{4a^2b^2g^3+8ab^3g^3x+4b^4g^3x^2}$$
$$+\frac{(-2ABadi-2ABbci-4ABbdix-B^2adi-B^2bci-2B^2bdix)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^2b^2g^3+4ab^3g^3x+2b^4g^3x^2}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3,x)
```

output
```
-B*d**2*i*(2*A + B)*log(x + (2*A*B*a*d**3*i + 2*A*B*b*c*d**2*i + B**2*a*d*
**3*i + B**2*b*c*d**2*i - B*a**2*d**4*i*(2*A + B)/(a*d - b*c) + 2*B*a*b*c*d
**3*i*(2*A + B)/(a*d - b*c) - B*b**2*c**2*d**2*i*(2*A + B)/(a*d - b*c))/(4
*A*B*b*d**3*i + 2*B**2*b*d**3*i))/(2*b**2*g**3*(a*d - b*c)) + B*d**2*i*(2*
A + B)*log(x + (2*A*B*a*d**3*i + 2*A*B*b*c*d**2*i + B**2*a*d**3*i + B**2*b
*c*d**2*i + B*a**2*d**4*i*(2*A + B)/(a*d - b*c) - 2*B*a*b*c*d**3*i*(2*A +
B)/(a*d - b*c) + B*b**2*c**2*d**2*i*(2*A + B)/(a*d - b*c))/(4*A*B*b*d**3*i
 + 2*B**2*b*d**3*i))/(2*b**2*g**3*(a*d - b*c)) + (B**2*c**2*i + 2*B**2*c*d
*i*x + B**2*d**2*i*x**2)*log(e*(a + b*x)/(c + d*x))**2/(2*a**3*d*g**3 - 2*
a**2*b*c*g**3 + 4*a**2*b*d*g**3*x - 4*a*b**2*c*g**3*x + 2*a*b**2*d*g**3*x*
*2 - 2*b**3*c*g**3*x**2) + (-2*A**2*a*d*i - 2*A**2*b*c*i - 2*A*B*a*d*i - 2
*A*B*b*c*i - B**2*a*d*i - B**2*b*c*i + x*(-4*A**2*b*d*i - 4*A*B*b*d*i - 2*
B**2*b*d*i))/(4*a**2*b**2*g**3 + 8*a*b**3*g**3*x + 4*b**4*g**3*x**2) + (-2
*A*B*a*d*i - 2*A*B*b*c*i - 4*A*B*b*d*i*x - B**2*a*d*i - B**2*b*c*i - 2*B**
2*b*d*i*x)*log(e*(a + b*x)/(c + d*x))/(2*a**2*b**2*g**3 + 4*a*b**3*g**3*x
+ 2*b**4*g**3*x**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1987 vs. $2(135) = 270$.

Time = 0.17 (sec) , antiderivative size = 1987, normalized size of antiderivative = 14.09

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^3}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algo
rithm="maxima")
```

output

```
-1/2*(2*b*x + a)*B^2*d*i*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^4*g^3*x
^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + 1/4*(2*((2*b*d*x - b*c + 3*a*d)/((b^4*
c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*
d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2
*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3))*log(b*e*x/(d*
x + c) + a*e/(d*x + c)) - (b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^
2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x +
 a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*
b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d
^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c))/(
a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b
^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 +
 a^3*b^2*d^2*g^3)*x))*B^2*c*i - 1/4*(2*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*
b*d)*x)/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*
b^3*c - a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a
*b^3*c*d + a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2
- 2*a*b^3*c*d + a^2*b^2*d^2)*g^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
(7*a*b^2*c^2 - 8*a^2*b*c*d + a^3*d^2 - 2*(2*a^2*b*c*d - a^3*d^2 + (2*b^3*c
*d - a*b^2*d^2)*x^2 + 2*(2*a*b^2*c*d - a^2*b*d^2)*x)*log(b*x + a)^2 - 2*(2
*a^2*b*c*d - a^3*d^2 + (2*b^3*c*d - a*b^2*d^2)*x^2 + 2*(2*a*b^2*c*d - a...
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.46

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$
$$-\frac{1}{4}\left(\frac{2\,(dx+c)^2B^2e^3i\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)^2g^3}+\frac{2\,(2\,ABe^3i+B^2e^3i)(dx+c)^2\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^2g^3}+\frac{(2\,A^2e^3i+2\,ABe^3i}{(bex+\right.$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algo
rithm="giac")
```

output
```
-1/4*(2*(d*x + c)^2*B^2*e^3*i*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e
)^2*g^3) + 2*(2*A*B*e^3*i + B^2*e^3*i)*(d*x + c)^2*log((b*e*x + a*e)/(d*x
 + c))/((b*e*x + a*e)^2*g^3) + (2*A^2*e^3*i + 2*A*B*e^3*i + B^2*e^3*i)*(d*x
  + c)^2/((b*e*x + a*e)^2*g^3))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((
b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 28.20 (sec) , antiderivative size = 469, normalized size of antiderivative = 3.33

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$
$$-\frac{x\,(2\,b\,d\,i\,A^2+2\,b\,d\,i\,A\,B+b\,d\,i\,B^2)+A^2\,a\,d\,i+A^2\,b\,c\,i+\frac{B^2\,a\,d\,i}{2}+\frac{B^2\,b\,c\,i}{2}+A\,B\,a\,d\,i+A\,B\,b\,c\,i}{2\,a^2\,b^2\,g^3+4\,a\,b^3\,g^3\,x+2\,b^4\,g^3\,x^2}$$
$$-\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{\frac{B^2\,c\,i}{2\,b^2\,g^3}+\frac{B^2\,a\,d\,i}{2\,b^3\,g^3}+\frac{B^2\,d\,i\,x}{b^2\,g^3}}{2\,a\,x+b\,x^2+\frac{a^2}{b}}-\frac{B^2\,d^2\,i}{2\,b^2\,g^3\,(a\,d-b\,c)}\right)$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(x\left(\frac{B^2\,i}{b^2\,g^3}+\frac{2\,A\,B\,i}{b^2\,g^3}\right)+\frac{A\,B\,a\,i}{b^3\,g^3}+\frac{B\,i\,(A\,b\,c-B\,a\,d+B\,b\,c)}{b^3\,d\,g^3}+\frac{B^2\,d^2\,i\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{2\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{2\,b\,d^2}\right)}{b^2\,g^3\,(a\,d-b\,c)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}}$$
$$-\frac{B\,d^2\,i\,\mathrm{atan}\left(\frac{\left(\frac{2\,c\,b^3\,g^3+2\,a\,d\,b^2\,g^3}{2\,b^2\,g^3}+2\,b\,d\,x\right)1\mathrm{i}}{a\,d-b\,c}\right)\,(2\,A+B)\,1\mathrm{i}}{b^2\,g^3\,(a\,d-b\,c)}$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^3
,x)
```

output
```
- (x*(2*A^2*b*d*i + B^2*b*d*i + 2*A*B*b*d*i) + A^2*a*d*i + A^2*b*c*i + (B^
2*a*d*i)/2 + (B^2*b*c*i)/2 + A*B*a*d*i + A*B*b*c*i)/(2*a^2*b^2*g^3 + 2*b^4
*g^3*x^2 + 4*a*b^3*g^3*x) - log((e*(a + b*x))/(c + d*x))^2*(((B^2*c*i)/(2*
b^2*g^3) + (B^2*a*d*i)/(2*b^3*g^3) + (B^2*d*i*x)/(b^2*g^3))/(2*a*x + b*x^2
 + a^2/b) - (B^2*d^2*i)/(2*b^2*g^3*(a*d - b*c))) - (log((e*(a + b*x))/(c +
 d*x))*(x*((B^2*i)/(b^2*g^3) + (2*A*B*i)/(b^2*g^3)) + (A*B*a*i)/(b^3*g^3)
+ (B*i*(A*b*c - B*a*d + B*b*c))/(b^3*d*g^3) + (B^2*d^2*i*((2*a^2*d^2 + b^2
*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(b^2*g^3*(a*d -
b*c))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - (B*d^2*i*atan((((2*b^3*c*g^3
 + 2*a*b^2*d*g^3)/(2*b^2*g^3) + 2*b*d*x)*1i)/(a*d - b*c))*(2*A + B)*1i)/(b
^2*g^3*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 684, normalized size of antiderivative = 4.85

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{i\left(2\log\left(\frac{bex+ae}{dx+c}\right)^2 a\,b^3c^2+4\log\left(\frac{bex+ae}{dx+c}\right)a^2b^2c^2-2a^4cd+2a^3b\,c^2+2a^3b\,d^2x^2+2\log\left(\frac{bex+ae}{dx+c}\right)^2 a\,b^3d^2x^2-2\right.}{}$$

input
```
int((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x)
```

output

```
(i*(4*log(a + b*x)*a**3*b*c*d + 8*log(a + b*x)*a**2*b**2*c*d*x + 2*log(a +
 b*x)*a**2*b**2*c*d + 4*log(a + b*x)*a*b**3*c*d*x**2 + 4*log(a + b*x)*a*b*
*3*c*d*x + 2*log(a + b*x)*b**4*c*d*x**2 - 4*log(c + d*x)*a**3*b*c*d - 8*lo
g(c + d*x)*a**2*b**2*c*d*x - 2*log(c + d*x)*a**2*b**2*c*d - 4*log(c + d*x)
*a*b**3*c*d*x**2 - 4*log(c + d*x)*a*b**3*c*d*x - 2*log(c + d*x)*b**4*c*d*x
**2 + 2*log((a*e + b*e*x)/(c + d*x))**2*a*b**3*c**2 + 4*log((a*e + b*e*x)/
(c + d*x))**2*a*b**3*c*d*x + 2*log((a*e + b*e*x)/(c + d*x))**2*a*b**3*d**2
*x**2 - 4*log((a*e + b*e*x)/(c + d*x))*a**3*b*c*d + 4*log((a*e + b*e*x)/(c
 + d*x))*a**2*b**2*c**2 - 2*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*c*d + 4
*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*d**2*x**2 + 2*log((a*e + b*e*x)/(c
 + d*x))*a*b**3*c**2 - 4*log((a*e + b*e*x)/(c + d*x))*a*b**3*c*d*x**2 + 2*
log((a*e + b*e*x)/(c + d*x))*a*b**3*d**2*x**2 - 2*log((a*e + b*e*x)/(c + d
*x))*b**4*c*d*x**2 - 2*a**4*c*d + 2*a**3*b*c**2 - 2*a**3*b*c*d + 2*a**3*b*
d**2*x**2 + 2*a**2*b**2*c**2 - 2*a**2*b**2*c*d*x**2 - a**2*b**2*c*d + 2*a*
*2*b**2*d**2*x**2 + a*b**3*c**2 - 2*a*b**3*c*d*x**2 + a*b**3*d**2*x**2 - b
**4*c*d*x**2))/(4*a*b*g**3*(a**3*d - a**2*b*c + 2*a**2*b*d*x - 2*a*b**2*c*
x + a*b**2*d*x**2 - b**3*c*x**2))
```

3.62 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result 759
Mathematica [C] (verified) 760
Rubi [A] (verified) 761
Maple [B] (verified) 763
Fricas [B] (verification not implemented) 764
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Mupad [B] (verification not implemented) 768
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Optimal result

Integrand size = 40, antiderivative size = 287

$$\begin{aligned}\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx &= \frac{B^2di(c+dx)^2}{4(bc-ad)^2g^4(a+bx)^2} \\ &\quad - \frac{2bB^2i(c+dx)^3}{27(bc-ad)^2g^4(a+bx)^3} \\ &\quad + \frac{Bdi(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} \\ &\quad - \frac{2bBi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^2g^4(a+bx)^3} \\ &\quad + \frac{di(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^2g^4(a+bx)^2} \\ &\quad - \frac{bi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^2g^4(a+bx)^3}\end{aligned}$$

output

```
1/4*B^2*d*i*(d*x+c)^2/(-a*d+b*c)^2/g^4/(b*x+a)^2-2/27*b*B^2*i*(d*x+c)^3/(-
a*d+b*c)^2/g^4/(b*x+a)^3+1/2*B*d*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(
-a*d+b*c)^2/g^4/(b*x+a)^2-2/9*b*B*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/
(-a*d+b*c)^2/g^4/(b*x+a)^3+1/2*d*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2
/(-a*d+b*c)^2/g^4/(b*x+a)^2-1/3*b*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^
2/(-a*d+b*c)^2/g^4/(b*x+a)^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.06 (sec) , antiderivative size = 1032, normalized size of antiderivative = 3.60

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input

```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*
g*x)^4,x]
```

output

```
-1/108*(i*(36*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 54*d*
(b*c - a*d)^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 2*B*(12*A
*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3 - 18*A*d*(b*c - a*d)^2*(a + b*x) - 15*B
*d*(b*c - a*d)^2*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(
b*c - a*d)*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*(a +
 b*x)^3*Log[a + b*x] - 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a
*d)^3*Log[(e*(a + b*x))/(c + d*x)] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e
*(a + b*x))/(c + d*x)] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[(e*(a + b*x)
)/(c + d*x)] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*
x)] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*(a + b*x)^3*Log[c + d*x
] + 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] -
36*B*d^3*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] - 18*B*d^3*
(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(b*(c +
 d*x))/(b*c - a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c
) + a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) +
27*B*d*(a + b*x)*(2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4
*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 4*d^2*(
a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d^2*(a +
b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 4*B*d*(a + b*x)
*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*...
```

Rubi [A] (verified)

Time $= 0.45$ (sec) , antiderivative size $= 218$, normalized size of antiderivative $= 0.76$, number of steps used $= 4$, number of rules used $= 3$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used $= \{2962, 2795, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^4}\,dx$$

$$\downarrow 2962$$

$$\frac{i\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)^2}$$

$$\downarrow 2795$$

$$\frac{i\int\left(\frac{b(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^4}-\frac{d(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3}\right)d\frac{a+bx}{c+dx}}{g^4(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{i\left(-\frac{b(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3}-\frac{2bB(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}+\frac{Bd(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}\right.}{g^4(bc-ad)^2}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^4
,x]
```

output
```
(i*((B^2*d*(c + d*x)^2)/(4*(a + b*x)^2) - (2*b*B^2*(c + d*x)^3)/(27*(a + b
*x)^3) + (B*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*
x)^2) - (2*b*B*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(a + b
*x)^3) + (d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(a + b*
x)^2) - (b*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*(a + b*x
)^3)))/((b*c - a*d)^2*g^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. 2(275) = 550.

Time = 1.66 (sec) , antiderivative size = 643, normalized size of antiderivative = 2.24

method	result
parts	$\frac{iA^2\left(-\frac{d}{2b^2(bx+a)^2}-\frac{-da+bc}{3b^2(bx+a)^3}\right)}{g^4} - \frac{iB^2(da-bc)^2e^2\left(\frac{d^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(da-bc)}{d(dx+c)}\right.}\right.}{(da-bc)^4}\right.}{g}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{id^2e^2A^2b}{3(da-bc)^3g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{id^3eA^2}{2(da-bc)^3g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{2id^2e^2ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{}{9\left(\frac{be}{d}+\right.}\right.}{(da-bc)^3g^4}\right.}{}$
default	$-\frac{e(da-bc)\left(\frac{id^2e^2A^2b}{3(da-bc)^3g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{id^3eA^2}{2(da-bc)^3g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{2id^2e^2ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{}{9\left(\frac{be}{d}+\right.}\right.}{(da-bc)^3g^4}\right.}{}$
norman	$-\frac{18A^2a^2bd^2i+18A^2ab^2cdi-36A^2b^3c^2i+30ABa^2bd^2i+30ABab^2cdi-24ABb^3c^2i+19B^2a^2bd^2i+19B^2ab^2cdi-8B^2b^3c^2i}{108gb^3(da-bc)} - \frac{\left(18\right.}{}$
parallelrisch	$-\frac{-54ABab^5c^2d^2i+108ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^6c^2d^2i-108B^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)ab^5cd^3i-108ABxab^5cd^3i-108AB\ln\left(\frac{e(bx}{dx}\right.}{}$
orering	Expression too large to display
risch	Expression too large to display

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x,method=_RETU
RNVERBOSE)
```

output
```
i*A^2/g^4*(-1/2*d/b^2/(b*x+a)^2-1/3*(-a*d+b*c)/b^2/(b*x+a)^3)-i*B^2/g^4/d^
3*(a*d-b*c)^2*e^2*(d^4/(a*d-b*c)^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-d^3/(a*d-b
*c)^4*b*e*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))-2*i*A*B/g^4/d^3*(a*d-b*c)^2*e^2*(
d^4/(a*d-b*c)^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e
/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-d^3/(a*d-b*c)^4*b*e*(-1/3
/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/
d+(a*d-b*c)*e/d/(d*x+c))^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(275) = 550$.

Time = 0.09 (sec) , antiderivative size = 601, normalized size of antiderivative = 2.09

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \frac{6\left((6\,AB+5\,B^2)b^3cd^2-(6\,AB+5\,B^2)ab^2d^3\right)ix^2-3\left((18\,A^2+6\,AB-B^2)b^3c^2d-18\left(2\,A^2+2\,AB+\right.\right.}{}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algo
rithm="fricas")
```

output

```
1/108*(6*((6*A*B + 5*B^2)*b^3*c*d^2 - (6*A*B + 5*B^2)*a*b^2*d^3)*i*x^2 - 3
*((18*A^2 + 6*A*B - B^2)*b^3*c^2*d - 18*(2*A^2 + 2*A*B + B^2)*a*b^2*c*d^2
+ (18*A^2 + 30*A*B + 19*B^2)*a^2*b*d^3)*i*x + 18*(B^2*b^3*d^3*i*x^3 + 3*B^
2*a*b^2*d^3*i*x^2 - 3*(B^2*b^3*c^2*d - 2*B^2*a*b^2*c*d^2)*i*x - (2*B^2*b^3
*c^3 - 3*B^2*a*b^2*c^2*d)*i)*log((b*e*x + a*e)/(d*x + c))^2 - (4*(9*A^2 +
6*A*B + 2*B^2)*b^3*c^3 - 27*(2*A^2 + 2*A*B + B^2)*a*b^2*c^2*d + (18*A^2 +
30*A*B + 19*B^2)*a^3*d^3)*i + 6*((6*A*B + 5*B^2)*b^3*d^3*i*x^3 + 3*(2*B^2*
b^3*c*d^2 + 3*(2*A*B + B^2)*a*b^2*d^3)*i*x^2 - 3*((6*A*B + B^2)*b^3*c^2*d
- 6*(2*A*B + B^2)*a*b^2*c*d^2)*i*x - (4*(3*A*B + B^2)*b^3*c^3 - 9*(2*A*B +
 B^2)*a*b^2*c^2*d)*i)*log((b*e*x + a*e)/(d*x + c)))/((b^7*c^2 - 2*a*b^6*c*
d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4
*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2
- 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1387 vs. $2(267) = 534$.

Time $= 10.72$ (sec) , antiderivative size $= 1387$, normalized size of antiderivative $= 4.83$

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^4}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4,x)
```

output

```
-B*d**3*i*(6*A + 5*B)*log(x + (6*A*B*a*d**4*i + 6*A*B*b*c*d**3*i + 5*B**2*
a*d**4*i + 5*B**2*b*c*d**3*i - B*a**3*d**6*i*(6*A + 5*B)/(a*d - b*c)**2 +
3*B*a**2*b*c*d**5*i*(6*A + 5*B)/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**4*i*(6
*A + 5*B)/(a*d - b*c)**2 + B*b**3*c**3*d**3*i*(6*A + 5*B)/(a*d - b*c)**2)/
(12*A*B*b*d**4*i + 10*B**2*b*d**4*i))/(18*b**2*g**4*(a*d - b*c)**2) + B*d*
*3*i*(6*A + 5*B)*log(x + (6*A*B*a*d**4*i + 6*A*B*b*c*d**3*i + 5*B**2*a*d**
4*i + 5*B**2*b*c*d**3*i + B*a**3*d**6*i*(6*A + 5*B)/(a*d - b*c)**2 - 3*B*a
**2*b*c*d**5*i*(6*A + 5*B)/(a*d - b*c)**2 + 3*B*a*b**2*c**2*d**4*i*(6*A +
5*B)/(a*d - b*c)**2 - B*b**3*c**3*d**3*i*(6*A + 5*B)/(a*d - b*c)**2)/(12*A
*B*b*d**4*i + 10*B**2*b*d**4*i))/(18*b**2*g**4*(a*d - b*c)**2) + (3*B**2*a
*c**2*d*i + 6*B**2*a*c*d**2*i*x + 3*B**2*a*d**3*i*x**2 - 2*B**2*b*c**3*i -
 3*B**2*b*c**2*d*i*x + B**2*b*d**3*i*x**3)*log(e*(a + b*x)/(c + d*x))**2/(
6*a**5*d**2*g**4 - 12*a**4*b*c*d*g**4 + 18*a**4*b*d**2*g**4*x + 6*a**3*b**
2*c**2*g**4 - 36*a**3*b**2*c*d*g**4*x + 18*a**3*b**2*d**2*g**4*x**2 + 18*a
**2*b**3*c**2*g**4*x - 36*a**2*b**3*c*d*g**4*x**2 + 6*a**2*b**3*d**2*g**4*
x**3 + 18*a*b**4*c**2*g**4*x**2 - 12*a*b**4*c*d*g**4*x**3 + 6*b**5*c**2*g*
*4*x**3) + (-6*A*B*a**2*d**2*i - 6*A*B*a*b*c*d*i - 18*A*B*a*b*d**2*i*x + 1
2*A*B*b**2*c**2*i + 18*A*B*b**2*c*d*i*x - 5*B**2*a**2*d**2*i - 5*B**2*a*b*
c*d*i - 15*B**2*a*b*d**2*i*x + 4*B**2*b**2*c**2*i + 3*B**2*b**2*c*d*i*x -
6*B**2*b**2*d**2*i*x**2)*log(e*(a + b*x)/(c + d*x))/(18*a**4*b**2*d*g**...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3282 vs. $2(275) = 550$.

Time $= 0.27$ (sec) , antiderivative size $= 3282$, normalized size of antiderivative $= 11.44$

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^4}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algo
rithm="maxima")
```

output

```
-1/6*(3*b*x + a)*B^2*d*i*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^5*g^4*x
^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/54*(6*((6*b^2*d^
2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((
b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*
d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g
^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)
/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log
(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4))*l
og(b*e*x/(d*x + c) + a*e/(d*x + c)) + (4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^
2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3
+ 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*(b^3*d^3*
x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 - 3*(5*b^3
*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b^2*d^3*
x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*a*b^2
*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2
+ 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*g^4 -
3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*g^4 -
 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a*b^6
*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)*x^
2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^...
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.59

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{1}{108}\left(\frac{18\left(2\,B^2be^4i-\frac{3\,(bex+ae)B^2de^3i}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^3bcg^4}{(dx+c)^3}-\frac{(bex+ae)^3adg^4}{(dx+c)^3}}+\frac{6\left(12\,ABbe^4i+4\,B^2be^4i-\frac{18\,(bex+ae)ABde^3i}{dx+c}-\frac{9(}{}\right.\right.$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algo
rithm="giac")
```

output

```
-1/108*(18*(2*B^2*b*e^4*i - 3*(b*e*x + a*e)*B^2*d*e^3*i/(d*x + c))*log((b*
e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)^3*b*c*g^4/(d*x + c)^3 - (b*e*x + a*
e)^3*a*d*g^4/(d*x + c)^3) + 6*(12*A*B*b*e^4*i + 4*B^2*b*e^4*i - 18*(b*e*x
+ a*e)*A*B*d*e^3*i/(d*x + c) - 9*(b*e*x + a*e)*B^2*d*e^3*i/(d*x + c))*log(
(b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4/(d*x + c)^3 - (b*e*x + a
*e)^3*a*d*g^4/(d*x + c)^3) + (36*A^2*b*e^4*i + 24*A*B*b*e^4*i + 8*B^2*b*e^
4*i - 54*(b*e*x + a*e)*A^2*d*e^3*i/(d*x + c) - 54*(b*e*x + a*e)*A*B*d*e^3*
i/(d*x + c) - 27*(b*e*x + a*e)*B^2*d*e^3*i/(d*x + c))/((b*e*x + a*e)^3*b*c
*g^4/(d*x + c)^3 - (b*e*x + a*e)^3*a*d*g^4/(d*x + c)^3))*(b*c/((b*c*e - a*
d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 29.66 (sec) , antiderivative size = 955, normalized size of antiderivative = 3.33

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input

```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^4
,x)
```

output

```
- log((e*(a + b*x))/(c + d*x))^2*(((B^2*c*i)/(3*b^2*g^4) + (B^2*a*d*i)/(6*
b^3*g^4) + (B^2*d*i*x)/(2*b^2*g^4))/(3*a^2*x + a^3/b + b^2*x^3 + 3*a*b*x^2
) - (B^2*d^3*i)/(6*b^2*g^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((18*A^2*a^
2*d^2*i - 36*A^2*b^2*c^2*i + 19*B^2*a^2*d^2*i - 8*B^2*b^2*c^2*i + 30*A*B*a
^2*d^2*i - 24*A*B*b^2*c^2*i + 18*A^2*a*b*c*d*i + 19*B^2*a*b*c*d*i + 30*A*B
*a*b*c*d*i)/(6*(a*d - b*c)) + (x^2*(5*B^2*b^2*d^2*i + 6*A*B*b^2*d^2*i))/(a
*d - b*c) + (x*(18*A^2*a*b*d^2*i + 19*B^2*a*b*d^2*i - 18*A^2*b^2*c*d*i + B
^2*b^2*c*d*i + 30*A*B*a*b*d^2*i - 6*A*B*b^2*c*d*i))/(2*(a*d - b*c)))/(18*a
^3*b^2*g^4 + 18*b^5*g^4*x^3 + 54*a^2*b^3*g^4*x + 54*a*b^4*g^4*x^2) - (log(
(e*(a + b*x))/(c + d*x))*(x*((A*B*i)/(b^2*g^4) + (B^2*d^3*i*(b*((3*a^2*d^2
 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^2*d^
2 + b^2*c^2 - 4*a*b*c*d)/(3*d^3) + (2*a*(a*d - b*c))/(3*d^2)))/(3*b^2*g^4*
(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) + (A*B*a*i)/(3*b^3*g^4) + (B*i*(2*A*b*c
- B*a*d + B*b*c))/(3*b^3*d*g^4) + (B^2*d^3*i*(a*((3*a^2*d^2 + b^2*c^2 - 4*
a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^3*d^3 - b^3*c^3 + 4
*a*b^2*c^2*d - 6*a^2*b*c*d^2)/(3*b*d^4)))/(3*b^2*g^4*(a^2*d^2 + b^2*c^2 -
2*a*b*c*d)) - (B^2*d^3*i*x^2*((b^2*c - a*b*d)/(3*d^2) - (2*b*(a*d - b*c))/
(3*d^2)))/(3*b^2*g^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))))/((3*a^2*x)/d + a^3
/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B*d^3*i*atan(((2*b*d*x - (18*b^4*
c^2*g^4 - 18*a^2*b^2*d^2*g^4)/(18*b^2*g^4*(a*d - b*c)))*1i)/(a*d - b*c)...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1481, normalized size of antiderivative = 5.16

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x)
```

output

```
(i*(36*log(a + b*x)*a**5*b*d**3 + 108*log(a + b*x)*a**4*b**2*d**3*x + 18*l
og(a + b*x)*a**4*b**2*d**3 + 12*log(a + b*x)*a**3*b**3*c*d**2 + 108*log(a
+ b*x)*a**3*b**3*d**3*x**2 + 54*log(a + b*x)*a**3*b**3*d**3*x + 36*log(a +
 b*x)*a**2*b**4*c*d**2*x + 36*log(a + b*x)*a**2*b**4*d**3*x**3 + 54*log(a
+ b*x)*a**2*b**4*d**3*x**2 + 36*log(a + b*x)*a*b**5*c*d**2*x**2 + 18*log(a
 + b*x)*a*b**5*d**3*x**3 + 12*log(a + b*x)*b**6*c*d**2*x**3 - 36*log(c + d
*x)*a**5*b*d**3 - 108*log(c + d*x)*a**4*b**2*d**3*x - 18*log(c + d*x)*a**4
*b**2*d**3 - 12*log(c + d*x)*a**3*b**3*c*d**2 - 108*log(c + d*x)*a**3*b**3
*d**3*x**2 - 54*log(c + d*x)*a**3*b**3*d**3*x - 36*log(c + d*x)*a**2*b**4*
c*d**2*x - 36*log(c + d*x)*a**2*b**4*d**3*x**3 - 54*log(c + d*x)*a**2*b**4
*d**3*x**2 - 36*log(c + d*x)*a*b**5*c*d**2*x**2 - 18*log(c + d*x)*a*b**5*d
**3*x**3 - 12*log(c + d*x)*b**6*c*d**2*x**3 + 54*log((a*e + b*e*x)/(c + d*
x))**2*a**2*b**4*c**2*d + 108*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**4*c*
d**2*x + 54*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**4*d**3*x**2 - 36*log((
a*e + b*e*x)/(c + d*x))**2*a*b**5*c**3 - 54*log((a*e + b*e*x)/(c + d*x))**
2*a*b**5*c**2*d*x + 18*log((a*e + b*e*x)/(c + d*x))**2*a*b**5*d**3*x**3 -
36*log((a*e + b*e*x)/(c + d*x))*a**5*b*d**3 - 108*log((a*e + b*e*x)/(c + d
*x))*a**4*b**2*d**3*x - 18*log((a*e + b*e*x)/(c + d*x))*a**4*b**2*d**3 + 1
08*log((a*e + b*e*x)/(c + d*x))*a**3*b**3*c**2*d + 216*log((a*e + b*e*x)/(
c + d*x))*a**3*b**3*c*d**2*x - 12*log((a*e + b*e*x)/(c + d*x))*a**3*b**...
```

3.63 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result 772
Mathematica [C] (verified) 773
Rubi [A] (verified) 774
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Giac [A] (verification not implemented) 780
Mupad [B] (verification not implemented) 780
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Optimal result

Integrand size = 40, antiderivative size = 445

$$\begin{aligned}\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = &-\frac{B^2d^2i(c+dx)^2}{4(bc-ad)^3g^5(a+bx)^2}\\ &+\frac{4bB^2di(c+dx)^3}{27(bc-ad)^3g^5(a+bx)^3}\\ &-\frac{b^2B^2i(c+dx)^4}{32(bc-ad)^3g^5(a+bx)^4}\\ &-\frac{Bd^2i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^5(a+bx)^2}\\ &+\frac{4bBdi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^3g^5(a+bx)^3}\\ &-\frac{b^2Bi(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^3g^5(a+bx)^4}\\ &-\frac{d^2i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3g^5(a+bx)^2}\\ &+\frac{2bdi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^3g^5(a+bx)^3}\\ &-\frac{b^2i(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^3g^5(a+bx)^4}\end{aligned}$$

output

```
-1/4*B^2*d^2*i*(d*x+c)^2/(-a*d+b*c)^3/g^5/(b*x+a)^2+4/27*b*B^2*d*i*(d*x+c)
^3/(-a*d+b*c)^3/g^5/(b*x+a)^3-1/32*b^2*B^2*i*(d*x+c)^4/(-a*d+b*c)^3/g^5/(b
*x+a)^4-1/2*B*d^2*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^5
/(b*x+a)^2+4/9*b*B*d*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/
g^5/(b*x+a)^3-1/8*b^2*B*i*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)
^3/g^5/(b*x+a)^4-1/2*d^2*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b
*c)^3/g^5/(b*x+a)^2+2/3*b*d*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*
d+b*c)^3/g^5/(b*x+a)^3-1/4*b^2*i*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(
-a*d+b*c)^3/g^5/(b*x+a)^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.21 (sec) , antiderivative size = 1255, normalized size of antiderivative = 2.82

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*
g*x)^5,x]
```

output

```
-1/864*(i*(216*(b*c - a*d)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 288*
d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 16*B
*d*(a + b*x)*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3 - 18*A*d*(b*c - a*d)^
2*(a + b*x) - 15*B*d*(b*c - a*d)^2*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b
*x)^2 + 66*B*d^2*(b*c - a*d)*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*
x] + 66*B*d^3*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*(a + b*x)^3*Log[a + b*x]
^2 + 12*B*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)] - 18*B*d*(b*c - a*d)^
2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^2*(b*c - a*d)*(a + b*x)^
2*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*
(a + b*x))/(c + d*x)] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*(a +
b*x)^3*Log[c + d*x] + 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d
)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)]*Log[c
+ d*x] - 18*B*d^3*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log[a
+ b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (d
*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x)
)/(b*c - a*d)]) + 3*B*(36*A*(b*c - a*d)^4 + 9*B*(b*c - a*d)^4 + 48*A*d*(-(
b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c) + a*d)^3*(a + b*x) + 72*A*d^2*(b*
c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c - a*d)^2*(a + b*x)^2 + 144*A*d^3*(-
(b*c) + a*d)*(a + b*x)^3 + 300*B*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 144*A*d^
4*(a + b*x)^4*Log[a + b*x] - 300*B*d^4*(a + b*x)^4*Log[a + b*x] + 72*B*...
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.75, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^5}\,dx$$

$$\downarrow 2962$$

$$\frac{i\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^3}$$

$$\downarrow 2795$$

$$\frac{i\int\left(\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^5}{(a+bx)^5}-\frac{2bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^4}{(a+bx)^4}+\frac{d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^3}{(a+bx)^3}\right)d\frac{a+bx}{c+dx}}{g^5(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{i\left(-\frac{b^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4(a+bx)^4}-\frac{b^2B(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{8(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}-\frac{Bd^2(c+dx)^2\left(B\log\left(\right.\right.\right.}{2(a+bx)}$$

$$g^5($$

input

```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^5
,x]
```

output
```
(i*(-1/4*(B^2*d^2*(c + d*x)^2)/(a + b*x)^2 + (4*b*B^2*d*(c + d*x)^3)/(27*(
a + b*x)^3) - (b^2*B^2*(c + d*x)^4)/(32*(a + b*x)^4) - (B*d^2*(c + d*x)^2*
(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^2) + (4*b*B*d*(c + d*x)
^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(a + b*x)^3) - (b^2*B*(c + d*x
)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(8*(a + b*x)^4) - (d^2*(c + d*x)
^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(a + b*x)^2) + (2*b*d*(c + d
*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*(a + b*x)^3) - (b^2*(c +
d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*(a + b*x)^4)))/((b*c - a
*d)^3*g^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(427) = 854$.

Time = 2.36 (sec) , antiderivative size = 926, normalized size of antiderivative = 2.08

method	result
parts	$\frac{i A^2\left(-\frac{-da+bc}{4b^2(bx+a)^4}-\frac{d}{3b^2(bx+a)^3}\right)}{g^5} - \frac{i B^2(da-bc)^2e^2\left(\frac{d^5\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(da-bc)}{d(dx+c}\right.\right.\right.}{(da-bc)^5}$
derivativedivides	Expression too large to display
default	Expression too large to display
orering	Expression too large to display
norman	Expression too large to display
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x,method=_RETU
RNVERBOSE)
```

output
```
i*A^2/g^5*(-1/4*(-a*d+b*c)/b^2/(b*x+a)^4-1/3*d/b^2/(b*x+a)^3)-i*B^2/g^5/d^
3*(a*d-b*c)^2*e^2*(d^5/(a*d-b*c)^5*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2*d^4/(a*d
-b*c)^5*b*e*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)+d^3/(a*d-b*c)^5*b^2*e^2*(-1/4/(b
*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/8/(b*e/d
+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/32/(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^4))-2*i*A*B/g^5/d^3*(a*d-b*c)^2*e^2*(d^5/(a*d-b*c)^5*(-
1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b
*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2*d^4/(a*d-b*c)^5*b*e*(-1/3/(b*e/d+(a*d-b*c
)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/
(d*x+c))^3)+d^3/(a*d-b*c)^5*b^2*e^2*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 985 vs. $2(427) = 854$.

Time = 0.10 (sec) , antiderivative size = 985, normalized size of antiderivative = 2.21

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algo
rithm="fricas")
```

output

```
-1/864*(12*((12*A*B + 13*B^2)*b^4*c*d^3 - (12*A*B + 13*B^2)*a*b^3*d^4)*i*x
^3 - 6*((12*A*B + B^2)*b^4*c^2*d^2 - 16*(6*A*B + 5*B^2)*a*b^3*c*d^3 + (84*
A*B + 79*B^2)*a^2*b^2*d^4)*i*x^2 + 4*((72*A^2 + 12*A*B - 5*B^2)*b^4*c^3*d
- 12*(18*A^2 + 6*A*B - B^2)*a*b^3*c^2*d^2 + 108*(2*A^2 + 2*A*B + B^2)*a^2*
b^2*c*d^3 - (72*A^2 + 156*A*B + 115*B^2)*a^3*b*d^4)*i*x + 72*(B^2*b^4*d^4*
i*x^4 + 4*B^2*a*b^3*d^4*i*x^3 + 6*B^2*a^2*b^2*d^4*i*x^2 + 4*(B^2*b^4*c^3*d
 - 3*B^2*a*b^3*c^2*d^2 + 3*B^2*a^2*b^2*c*d^3)*i*x + (3*B^2*b^4*c^4 - 8*B^2
*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2)*i)*log((b*e*x + a*e)/(d*x + c))^2 +
(27*(8*A^2 + 4*A*B + B^2)*b^4*c^4 - 64*(9*A^2 + 6*A*B + 2*B^2)*a*b^3*c^3*d
 + 216*(2*A^2 + 2*A*B + B^2)*a^2*b^2*c^2*d^2 - (72*A^2 + 156*A*B + 115*B^2
)*a^4*d^4)*i + 12*((12*A*B + 13*B^2)*b^4*d^4*i*x^4 + 4*(3*B^2*b^4*c*d^3 +
2*(6*A*B + 5*B^2)*a*b^3*d^4)*i*x^3 - 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^
3 - 6*(2*A*B + B^2)*a^2*b^2*d^4)*i*x^2 + 4*((12*A*B + B^2)*b^4*c^3*d - 6*(
6*A*B + B^2)*a*b^3*c^2*d^2 + 18*(2*A*B + B^2)*a^2*b^2*c*d^3)*i*x + (9*(4*A
*B + B^2)*b^4*c^4 - 32*(3*A*B + B^2)*a*b^3*c^3*d + 36*(2*A*B + B^2)*a^2*b^
2*c^2*d^2)*i)*log((b*e*x + a*e)/(d*x + c)))/((b^9*c^3 - 3*a*b^8*c^2*d + 3*
a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*
a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d +
3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d
+ 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2251 vs. $2(416) = 832$.

Time = 90.49 (sec) , antiderivative size = 2251, normalized size of antiderivative = 5.06

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**5,x)
```

output

```
-B*d**4*i*(12*A + 13*B)*log(x + (12*A*B*a*d**5*i + 12*A*B*b*c*d**4*i + 13*
B**2*a*d**5*i + 13*B**2*b*c*d**4*i - B*a**4*d**8*i*(12*A + 13*B)/(a*d - b*
c)**3 + 4*B*a**3*b*c*d**7*i*(12*A + 13*B)/(a*d - b*c)**3 - 6*B*a**2*b**2*c
**2*d**6*i*(12*A + 13*B)/(a*d - b*c)**3 + 4*B*a*b**3*c**3*d**5*i*(12*A + 1
3*B)/(a*d - b*c)**3 - B*b**4*c**4*d**4*i*(12*A + 13*B)/(a*d - b*c)**3)/(24
*A*B*b*d**5*i + 26*B**2*b*d**5*i))/(72*b**2*g**5*(a*d - b*c)**3) + B*d**4*
i*(12*A + 13*B)*log(x + (12*A*B*a*d**5*i + 12*A*B*b*c*d**4*i + 13*B**2*a*d
**5*i + 13*B**2*b*c*d**4*i + B*a**4*d**8*i*(12*A + 13*B)/(a*d - b*c)**3 -
4*B*a**3*b*c*d**7*i*(12*A + 13*B)/(a*d - b*c)**3 + 6*B*a**2*b**2*c**2*d**6
*i*(12*A + 13*B)/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**5*i*(12*A + 13*B)/(a*
d - b*c)**3 + B*b**4*c**4*d**4*i*(12*A + 13*B)/(a*d - b*c)**3)/(24*A*B*b*d
**5*i + 26*B**2*b*d**5*i))/(72*b**2*g**5*(a*d - b*c)**3) + (6*B**2*a**2*c*
*2*d**2*i + 12*B**2*a**2*c*d**3*i*x + 6*B**2*a**2*d**4*i*x**2 - 8*B**2*a*b
*c**3*d*i - 12*B**2*a*b*c**2*d**2*i*x + 4*B**2*a*b*d**4*i*x**3 + 3*B**2*b*
*2*c**4*i + 4*B**2*b**2*c**3*d*i*x + B**2*b**2*d**4*i*x**4)*log(e*(a + b*x
)/(c + d*x))**2/(12*a**7*d**3*g**5 - 36*a**6*b*c*d**2*g**5 + 48*a**6*b*d**
3*g**5*x + 36*a**5*b**2*c**2*d*g**5 - 144*a**5*b**2*c*d**2*g**5*x + 72*a**
5*b**2*d**3*g**5*x**2 - 12*a**4*b**3*c**3*g**5 + 144*a**4*b**3*c**2*d*g**5
*x - 216*a**4*b**3*c*d**2*g**5*x**2 + 48*a**4*b**3*d**3*g**5*x**3 - 48*a**
3*b**4*c**3*g**5*x + 216*a**3*b**4*c**2*d*g**5*x**2 - 144*a**3*b**4*c*d...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4808 vs. $2(427) = 854$.

Time $= 0.41$ (sec) , antiderivative size $= 4808$, normalized size of antiderivative $= 10.80$

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algo
rithm="maxima")
```

output

```
-1/12*(4*b*x + a)*B^2*d*i*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^6*g^5*
x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5)
 + 1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^
2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*
c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3
*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4
*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a
^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 -
 a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a
^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3
*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c
^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)
)*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216
*a^2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*
d^4)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72
*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*
d^4)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^
2 + 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^
2*d^2 + 324*a^2*b^2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*
d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 1...
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 744, normalized size of antiderivative = 1.67

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algo
rithm="giac")
```

output
```
-1/864*(72*(3*B^2*b^2*e^5*i - 8*(b*e*x + a*e)*B^2*b*d*e^4*i/(d*x + c) + 6*
(b*e*x + a*e)^2*B^2*d^2*e^3*i/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))^2/
((b*e*x + a*e)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*e*x + a*e)^4*a*b*c*d*g^5/(
d*x + c)^4 + (b*e*x + a*e)^4*a^2*d^2*g^5/(d*x + c)^4) + 12*(36*A*B*b^2*e^5
*i + 9*B^2*b^2*e^5*i - 96*(b*e*x + a*e)*A*B*b*d*e^4*i/(d*x + c) - 32*(b*e*
x + a*e)*B^2*b*d*e^4*i/(d*x + c) + 72*(b*e*x + a*e)^2*A*B*d^2*e^3*i/(d*x +
 c)^2 + 36*(b*e*x + a*e)^2*B^2*d^2*e^3*i/(d*x + c)^2)*log((b*e*x + a*e)/(d
*x + c))/((b*e*x + a*e)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*e*x + a*e)^4*a*b*
c*d*g^5/(d*x + c)^4 + (b*e*x + a*e)^4*a^2*d^2*g^5/(d*x + c)^4) + (216*A^2*
b^2*e^5*i + 108*A*B*b^2*e^5*i + 27*B^2*b^2*e^5*i - 576*(b*e*x + a*e)*A^2*b
*d*e^4*i/(d*x + c) - 384*(b*e*x + a*e)*A*B*b*d*e^4*i/(d*x + c) - 128*(b*e*
x + a*e)*B^2*b*d*e^4*i/(d*x + c) + 432*(b*e*x + a*e)^2*A^2*d^2*e^3*i/(d*x
 + c)^2 + 432*(b*e*x + a*e)^2*A*B*d^2*e^3*i/(d*x + c)^2 + 216*(b*e*x + a*e)
^2*B^2*d^2*e^3*i/(d*x + c)^2)/((b*e*x + a*e)^4*b^2*c^2*g^5/(d*x + c)^4 - 2
*(b*e*x + a*e)^4*a*b*c*d*g^5/(d*x + c)^4 + (b*e*x + a*e)^4*a^2*d^2*g^5/(d*
x + c)^4))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c
- a*d)))
```

Mupad [B] (verification not implemented)

Time = 33.25 (sec) , antiderivative size = 1870, normalized size of antiderivative = 4.20

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^5
,x)
```

output

```
((72*A^2*a^3*d^3*i + 216*A^2*b^3*c^3*i + 115*B^2*a^3*d^3*i + 27*B^2*b^3*c^
3*i + 156*A*B*a^3*d^3*i + 108*A*B*b^3*c^3*i - 360*A^2*a*b^2*c^2*d*i + 72*A
^2*a^2*b*c*d^2*i - 101*B^2*a*b^2*c^2*d*i + 115*B^2*a^2*b*c*d^2*i - 276*A*B
*a*b^2*c^2*d*i + 156*A*B*a^2*b*c*d^2*i)/(12*(a*d - b*c)) + (x^2*(79*B^2*a*
b^2*d^3*i - B^2*b^3*c*d^2*i + 84*A*B*a*b^2*d^3*i - 12*A*B*b^3*c*d^2*i))/(2
*(a*d - b*c)) + (x*(72*A^2*a^2*b*d^3*i + 115*B^2*a^2*b*d^3*i + 72*A^2*b^3*
c^2*d*i - 5*B^2*b^3*c^2*d*i + 156*A*B*a^2*b*d^3*i + 12*A*B*b^3*c^2*d*i - 1
44*A^2*a*b^2*c*d^2*i + 7*B^2*a*b^2*c*d^2*i - 60*A*B*a*b^2*c*d^2*i))/(3*(a*
d - b*c)) + (d*x^3*(13*B^2*b^3*d^2*i + 12*A*B*b^3*d^2*i))/(a*d - b*c))/(x*
(288*a^3*b^4*c*g^5 - 288*a^4*b^3*d*g^5) - x^3*(288*a^2*b^5*d*g^5 - 288*a*b
^6*c*g^5) + x^4*(72*b^7*c*g^5 - 72*a*b^6*d*g^5) + x^2*(432*a^2*b^5*c*g^5 -
 432*a^3*b^4*d*g^5) + 72*a^4*b^3*c*g^5 - 72*a^5*b^2*d*g^5) - log((e*(a + b
*x))/(c + d*x))^2*(((B^2*c*i)/(4*b^2*g^5) + (B^2*a*d*i)/(12*b^3*g^5) + (B^
2*d*i*x)/(3*b^2*g^5))/(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x
^3) - (B^2*d^4*i)/(12*b^2*g^5*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2))) - (log((e*(a + b*x))/(c + d*x))*(x*((2*A*B*i)/(3*b^2*g^5) + (B^2
*d^4*i*(b*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c
))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(12
*b*d^4)) + a*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d -
b*c))/(4*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d ...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 2224, normalized size of antiderivative = 5.00

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x)
```

output

```
(i*(144*log(a + b*x)*a**6*b*d**4 + 576*log(a + b*x)*a**5*b**2*d**4*x + 120
*log(a + b*x)*a**5*b**2*d**4 + 36*log(a + b*x)*a**4*b**3*c*d**3 + 864*log(
a + b*x)*a**4*b**3*d**4*x**2 + 480*log(a + b*x)*a**4*b**3*d**4*x + 144*log
(a + b*x)*a**3*b**4*c*d**3*x + 576*log(a + b*x)*a**3*b**4*d**4*x**3 + 720*
log(a + b*x)*a**3*b**4*d**4*x**2 + 216*log(a + b*x)*a**2*b**5*c*d**3*x**2
+ 144*log(a + b*x)*a**2*b**5*d**4*x**4 + 480*log(a + b*x)*a**2*b**5*d**4*x
**3 + 144*log(a + b*x)*a*b**6*c*d**3*x**3 + 120*log(a + b*x)*a*b**6*d**4*x
**4 + 36*log(a + b*x)*b**7*c*d**3*x**4 - 144*log(c + d*x)*a**6*b*d**4 - 57
6*log(c + d*x)*a**5*b**2*d**4*x - 120*log(c + d*x)*a**5*b**2*d**4 - 36*log
(c + d*x)*a**4*b**3*c*d**3 - 864*log(c + d*x)*a**4*b**3*d**4*x**2 - 480*lo
g(c + d*x)*a**4*b**3*d**4*x - 144*log(c + d*x)*a**3*b**4*c*d**3*x - 576*lo
g(c + d*x)*a**3*b**4*d**4*x**3 - 720*log(c + d*x)*a**3*b**4*d**4*x**2 - 21
6*log(c + d*x)*a**2*b**5*c*d**3*x**2 - 144*log(c + d*x)*a**2*b**5*d**4*x**
4 - 480*log(c + d*x)*a**2*b**5*d**4*x**3 - 144*log(c + d*x)*a*b**6*c*d**3*
x**3 - 120*log(c + d*x)*a*b**6*d**4*x**4 - 36*log(c + d*x)*b**7*c*d**3*x**
4 + 432*log((a*e + b*e*x)/(c + d*x))**2*a**3*b**4*c**2*d**2 + 864*log((a*e
 + b*e*x)/(c + d*x))**2*a**3*b**4*c*d**3*x + 432*log((a*e + b*e*x)/(c + d*
x))**2*a**3*b**4*d**4*x**2 - 576*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**5
*c**3*d - 864*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**5*c**2*d**2*x + 288*
log((a*e + b*e*x)/(c + d*x))**2*a**2*b**5*d**4*x**3 + 216*log((a*e + b*...
```

3.64 $\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result 784
Mathematica [B] (verified) 785
Rubi [A] (verified) 786
Maple [F] 797
Fricas [F] 797
Sympy [F(-1)] 798
Maxima [B] (verification not implemented) 798
Giac [F] 799
Mupad [F(-1)] 800
Reduce [F] 800

Optimal result

Integrand size = 42, antiderivative size = 711

$$\begin{aligned}
&\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx \\
&= \frac{3B^2(bc-ad)^5g^3i^2x}{20b^2d^3} + \frac{B^2(bc-ad)^2g^3i^2(a+bx)^4}{60b^3} - \frac{3B^2(bc-ad)^4g^3i^2(c+dx)^2}{40bd^4} \\
&\quad + \frac{B^2(bc-ad)^3g^3i^2(c+dx)^3}{60d^4} - \frac{B(bc-ad)^3g^3i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{90b^3d} \\
&\quad - \frac{B(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^3} \\
&\quad - \frac{B(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^2} \\
&\quad + \frac{(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{60b^3} \\
&\quad + \frac{(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{15b^2} \\
&\quad + \frac{g^3i^2(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b} \\
&\quad + \frac{B(bc-ad)^4g^3i^2(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^2} \\
&\quad - \frac{B(bc-ad)^5g^3i^2(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^3} \\
&\quad - \frac{B(bc-ad)^6g^3i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^4} \\
&\quad - \frac{B^2(bc-ad)^6g^3i^2\log(c+dx)}{20b^3d^4} - \frac{B^2(bc-ad)^6g^3i^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{30b^3d^4}
\end{aligned}$$

output

```
3/20*B^2*(-a*d+b*c)^5*g^3*i^2*x/b^2/d^3+1/60*B^2*(-a*d+b*c)^2*g^3*i^2*(b*x
+a)^4/b^3-3/40*B^2*(-a*d+b*c)^4*g^3*i^2*(d*x+c)^2/b/d^4+1/60*B^2*(-a*d+b*c
)^3*g^3*i^2*(d*x+c)^3/d^4-1/90*B*(-a*d+b*c)^3*g^3*i^2*(b*x+a)^3*(A+B*ln(e*
(b*x+a)/(d*x+c)))/b^3/d-1/20*B*(-a*d+b*c)^2*g^3*i^2*(b*x+a)^4*(A+B*ln(e*(b
*x+a)/(d*x+c)))/b^3-1/15*B*(-a*d+b*c)*g^3*i^2*(b*x+a)^4*(d*x+c)*(A+B*ln(e*
(b*x+a)/(d*x+c)))/b^2+1/60*(-a*d+b*c)^2*g^3*i^2*(b*x+a)^4*(A+B*ln(e*(b*x+a
)/(d*x+c)))^2/b^3+1/15*(-a*d+b*c)*g^3*i^2*(b*x+a)^4*(d*x+c)*(A+B*ln(e*(b*x
+a)/(d*x+c)))^2/b^2+1/6*g^3*i^2*(b*x+a)^4*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x
+c)))^2/b+1/180*B*(-a*d+b*c)^4*g^3*i^2*(b*x+a)^2*(3*A+B+3*B*ln(e*(b*x+a)/(
d*x+c)))/b^3/d^2-1/180*B*(-a*d+b*c)^5*g^3*i^2*(b*x+a)*(6*A+5*B+6*B*ln(e*(b
*x+a)/(d*x+c)))/b^3/d^3-1/180*B*(-a*d+b*c)^6*g^3*i^2*ln((-a*d+b*c)/b/(d*x+
c))*(6*A+11*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b^3/d^4-1/20*B^2*(-a*d+b*c)^6*g^3
*i^2*ln(d*x+c)/b^3/d^4-1/30*B^2*(-a*d+b*c)^6*g^3*i^2*polylog(2,d*(b*x+a)/b
/(d*x+c))/b^3/d^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1559 vs. $2(711) = 1422$.

Time $= 1.36$ (sec) , antiderivative size $= 1559$, normalized size of antiderivative $= 2.19$

$$\int (ag + bgx)^3 (ci + dix)^2 \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right)^2 dx = \text{Too large to display}$$

input

```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^2,x]
```

output

```
(g^3*i^2*(15*(b*c - a*d)^2*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2 + 24*d*(b*c - a*d)*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
+ 10*d^2*(a + b*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (5*B*(b*c -
a*d)^3*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a
+ b*x))/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b
*x))/(c + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])
- 6*B*(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a +
 b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*
c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c)
+ a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a
*d)])))/d^4 + (2*B*(b*c - a*d)^2*(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c -
 a*d)^3*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 12*d^2*(b*c - a*d)^2*(a +
 b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)
^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*
(a + b*x))/(c + d*x)]) - 24*B*(b*c - a*d)^4*Log[c + d*x] - 24*(b*c - a*d)^
4*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 4*B*(b*c - a*d)^2*(2
*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(
b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d
^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*(b*...
```

Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 963, normalized size of antiderivative = 1.35, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {2962, 2783, 2782, 27, 87, 49, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\downarrow 2962$$

$$g^3i^2(bc-ad)^6\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$g^3 i^2 (bc -$$

$$ad)^6 \left(-\frac{B\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{3b} + \frac{\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{3b} + \frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right.}$$

$$\downarrow 2782$$

$$g^3 i^2 (bc -$$

$$ad)^6 \left(-\frac{B\left(-B\int \frac{(a+bx)^3\left(5b-\frac{d(a+bx)}{c+dx}\right)}{20b^2(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx} + \frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}\right)}{3b} + \frac{\int (a+\right.$$

$$\downarrow 27$$

$$g^3 i^2 (bc -$$

$$ad)^6 \left(-\frac{B\left(-\frac{B\int \frac{(a+bx)^3\left(5b-\frac{d(a+bx)}{c+dx}\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{20b^2} + \frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}\right)}{3b} + \frac{\int \frac{(a+bx)^3\left(A\right.}{(c+dx\right.}$$

$$\downarrow 87$$

$$g^3 i^2 (bc -$$

$$ad)^6 \left(-\frac{B\left(-\frac{B\left(\int \frac{(a+bx)^3}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx} + \frac{(a+bx)^4}{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{20b^2} + \frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right.\right.}{5b(c+dx)^4\left(b-\frac{d(a+b}{c+dx}\right.}$$

$$\downarrow 49$$

$$g^3i^2(bc-$$

$$ad)^6\left(-\frac{B\left(-\frac{B\left(\int\left(\frac{b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{1}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}+\frac{(a+bx)^4}{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{20b^2}+\frac{(a+}{20}\right.}{3b}\right.$$

$\downarrow$ 2009

$$g^3i^2(bc-$$

$$ad)^6\left(\frac{\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{3b}-\frac{B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{B\left(\frac{}{3d^4\left(b-\right.}\right.}{}\right.}{}\right.$$

$\downarrow$ 2783

$$g^3i^2(bc-$$

$$ad)^6\left(\frac{-\frac{2B\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}}{3b}-\frac{B\left(\frac{(}{}\right.}{}\right.$$

$\downarrow$ 2773

$$ad)^6\left(\frac{g^3i^2(bc-}{3b}\right.$$

$$\left.-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int\frac{(a+bx)^3}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}\right)}{5b}+\frac{\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{(a+bx)^4\left(B\,l}{5b(c+dx)}\right)$$

$$\downarrow 49$$

$$ad)^6\left(\frac{g^3i^2(bc-}{3b}\right.$$

$$\left.-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int\left(\frac{b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{1}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}}{4b}\right)}{5b}+\frac{\int}{}\right)$$

$$\downarrow 2009$$

$$ad)^6\left(\frac{g^3i^2(bc-}{3b}\right.$$

$$\left.\frac{\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{b^3}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3b^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{3b}{d^4\left(b-\frac{d(a+}{c+}\right.\right.\right.}{4b}\right.}{5b}$$

$$\downarrow 2781$$

$$g^3 i^2 (bc -$$

$$ad)^6 \left(\frac{\frac{(a+bx)^4 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B \int \frac{(a+bx)^3 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{2b}}{5b} - \frac{2B \left(\frac{(a+bx)^4 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B \left(\frac{b^3}{3d^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^3} \right.}{} \right.}{3b}$$

$$\downarrow 2784$$

$$(bc -$$

$$ad)^6 g^3 i^2 \left(\frac{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{6b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B \left(\frac{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{20b^2(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{5b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{B \left(\right.}{(c+} \right.}{} \right.$$

$$\downarrow 2784$$

$$(bc-$$

$$ad)^6 g^3 i^2 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{B\left(\right.}{(c+} \right.$$

$$\downarrow 2784$$

$$(bc-$$

$$ad)^6 g^3 i^2 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{B\left(-\right.}{(c+} \right.$$

$$\downarrow 2754$$

$$(bc-$$

$$ad)^6 g^3 i^2 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{B\left(\right.}{(c+}\right.$$

↓ 2838

$$(bc -$$

$$ad)^6 g^3 i^2 \left(\frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{6b(c+dx)^4\left(b - \frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{B\left(\frac{}{(c+}\right.}{}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
,x]
```

output

```
(b*c - a*d)^6*g^3*i^2*(((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(6*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^6) - (B*(((a + b*x)^4*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]))/(5*b*(c + d*x)^4*(b - (d*(a + b*x))/(c
+ d*x))^5) + ((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(20*b^2*(c
 + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) - (B*((a + b*x)^4/((c + d*x)^4*
(b - (d*(a + b*x))/(c + d*x))^4) + b^3/(3*d^4*(b - (d*(a + b*x))/(c + d*x)
)^3) - (3*b^2)/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (3*b)/(d^4*(b - (
d*(a + b*x))/(c + d*x))) + Log[b - (d*(a + b*x))/(c + d*x)]/d^4))/(20*b^2)
))/(3*b) + (((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(5*b*(c +
 d*x)^4*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*(((a + b*x)^4*(A + B*Log[(
e*(a + b*x))/(c + d*x)]))/(4*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4
) - (B*(b^3/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^3) - (3*b^2)/(2*d^4*(b -
(d*(a + b*x))/(c + d*x))^2) + (3*b)/(d^4*(b - (d*(a + b*x))/(c + d*x))) +
Log[b - (d*(a + b*x))/(c + d*x)]/d^4))/(4*b)))/(5*b) + (((a + b*x)^4*(A +
B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*b*(c + d*x)^4*(b - (d*(a + b*x))/(c
+ d*x))^4) - (B*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*d*(
c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a + b*x)^2*(3*A + B + 3*B
*Log[(e*(a + b*x))/(c + d*x)]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c + d
*x))^2) - (((a + b*x)*(6*A + 5*B + 6*B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(
c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((6*A + 11*B + 6*B*Log[(e*...
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)^3 (dix+ci)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3 (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)^3 (dix+ci)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="fricas")
```

output
```
integral(A^2*b^3*d^2*g^3*i^2*x^5 + A^2*a^3*c^2*g^3*i^2 + (2*A^2*b^3*c*d +
3*A^2*a*b^2*d^2)*g^3*i^2*x^4 + (A^2*b^3*c^2 + 6*A^2*a*b^2*c*d + 3*A^2*a^2*
b*d^2)*g^3*i^2*x^3 + (3*A^2*a*b^2*c^2 + 6*A^2*a^2*b*c*d + A^2*a^3*d^2)*g^3
*i^2*x^2 + (3*A^2*a^2*b*c^2 + 2*A^2*a^3*c*d)*g^3*i^2*x + (B^2*b^3*d^2*g^3*
i^2*x^5 + B^2*a^3*c^2*g^3*i^2 + (2*B^2*b^3*c*d + 3*B^2*a*b^2*d^2)*g^3*i^2*
x^4 + (B^2*b^3*c^2 + 6*B^2*a*b^2*c*d + 3*B^2*a^2*b*d^2)*g^3*i^2*x^3 + (3*B
^2*a*b^2*c^2 + 6*B^2*a^2*b*c*d + B^2*a^3*d^2)*g^3*i^2*x^2 + (3*B^2*a^2*b*c
^2 + 2*B^2*a^3*c*d)*g^3*i^2*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3
*d^2*g^3*i^2*x^5 + A*B*a^3*c^2*g^3*i^2 + (2*A*B*b^3*c*d + 3*A*B*a*b^2*d^2)
*g^3*i^2*x^4 + (A*B*b^3*c^2 + 6*A*B*a*b^2*c*d + 3*A*B*a^2*b*d^2)*g^3*i^2*x
^3 + (3*A*B*a*b^2*c^2 + 6*A*B*a^2*b*c*d + A*B*a^3*d^2)*g^3*i^2*x^2 + (3*A*
B*a^2*b*c^2 + 2*A*B*a^3*c*d)*g^3*i^2*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5178 vs. $2(680) = 1360$.

Time = 0.22 (sec) , antiderivative size = 5178, normalized size of antiderivative = 7.28

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="maxima")
```

output
```
1/6*A^2*b^3*d^2*g^3*i^2*x^6 + 2/5*A^2*b^3*c*d*g^3*i^2*x^5 + 3/5*A^2*a*b^2*
d^2*g^3*i^2*x^5 + 1/4*A^2*b^3*c^2*g^3*i^2*x^4 + 3/2*A^2*a*b^2*c*d*g^3*i^2*
x^4 + 3/4*A^2*a^2*b*d^2*g^3*i^2*x^4 + A^2*a*b^2*c^2*g^3*i^2*x^3 + 2*A^2*a^
2*b*c*d*g^3*i^2*x^3 + 1/3*A^2*a^3*d^2*g^3*i^2*x^3 + 3/2*A^2*a^2*b*c^2*g^3*
i^2*x^2 + A^2*a^3*c*d*g^3*i^2*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x +
c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^3*c^2*g^3*i^2 + 3*(x^2*lo
g(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x +
c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^2*b*c^2*g^3*i^2 + (2*x^3*log(b*e*x/(d*
x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3
- ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*b^2
*c^2*g^3*i^2 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*lo
g(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 -
 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b
^3*c^2*g^3*i^2 + 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x
 + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^3*c*d*g^3*i^
2 + 2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3
 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^
2)*x)/(b^2*d^2))*A*B*a^2*b*c*d*g^3*i^2 + 1/2*(6*x^4*log(b*e*x/(d*x + c) +
a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3
*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - ...
```

Giac [F]

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (bgx+ag)^3(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="giac")
```

output
```
integrate((b*g*x + a*g)^3*(d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) +
A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^3\,(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2
,x)
```

output
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2
, x)
```

Reduce [F]

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
(g**3*( - 12*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b
*d*x**2),x)*a**6*b**2*d**7 + 72*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c
+ a*d*x + b*c*x + b*d*x**2),x)*a**5*b**3*c*d**6 - 180*int((log((a*e + b*e*
x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**4*b**4*c**2*d**5 +
 240*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2)
,x)*a**3*b**5*c**3*d**4 - 180*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c +
a*d*x + b*c*x + b*d*x**2),x)*a**2*b**6*c**4*d**3 + 72*int((log((a*e + b*e*
x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**7*c**5*d**2 - 12
*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*
b**8*c**6*d - 12*log(c + d*x)*a**7*d**6 + 72*log(c + d*x)*a**6*b*c*d**5 -
4*log(c + d*x)*a**6*b*d**6 - 180*log(c + d*x)*a**5*b**2*c**2*d**4 + 24*log
(c + d*x)*a**5*b**2*c*d**5 + 240*log(c + d*x)*a**4*b**3*c**3*d**3 - 60*log
(c + d*x)*a**4*b**3*c**2*d**4 - 180*log(c + d*x)*a**3*b**4*c**4*d**2 + 80*
log(c + d*x)*a**3*b**4*c**3*d**3 + 72*log(c + d*x)*a**2*b**5*c**5*d - 60*l
og(c + d*x)*a**2*b**5*c**4*d**2 - 12*log(c + d*x)*a*b**6*c**6 + 24*log(c +
 d*x)*a*b**6*c**5*d - 4*log(c + d*x)*b**7*c**6 + 6*log((a*e + b*e*x)/(c +
d*x))**2*a**5*b**2*c*d**5 - 30*log((a*e + b*e*x)/(c + d*x))**2*a**4*b**3*c
**2*d**4 - 60*log((a*e + b*e*x)/(c + d*x))**2*a**3*b**4*c**3*d**3 - 360*lo
g((a*e + b*e*x)/(c + d*x))**2*a**3*b**4*c**2*d**4*x - 360*log((a*e + b*e*x
)/(c + d*x))**2*a**3*b**4*c*d**5*x**2 - 120*log((a*e + b*e*x)/(c + d*x)...
```

3.65 $\int (ag+bgx)^2(ci+dix)^2 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result 803
Mathematica [A] (verified) 804
Rubi [A] (verified) 805
Maple [F] 816
Fricas [F] 816
Sympy [F(-1)] 817
Maxima [B] (verification not implemented) 817
Giac [F] 818
Mupad [F(-1)] 819
Reduce [F] 819

Optimal result

Integrand size = 42, antiderivative size = 761

$$
\begin{aligned}
&\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= -\frac{B^2(bc-ad)^4g^2i^2x}{10b^2d^2} - \frac{B^2(bc-ad)^3g^2i^2(c+dx)^2}{20bd^3} + \frac{B^2(bc-ad)^2g^2i^2(c+dx)^3}{30d^3} \\
&\quad + \frac{B^2(bc-ad)^5g^2i^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3} - \frac{B(bc-ad)^3g^2i^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3d} \\
&\quad - \frac{B(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^3} \\
&\quad - \frac{B(bc-ad)^3g^2i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5bd^3} \\
&\quad + \frac{4B(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3} \\
&\quad - \frac{bB(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^3} \\
&\quad + \frac{(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{30b^3} \\
&\quad + \frac{(bc-ad)g^2i^2(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2} \\
&\quad + \frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} \\
&\quad + \frac{B(bc-ad)^4g^2i^2(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3d^2} \\
&\quad + \frac{B(bc-ad)^5g^2i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3d^3} \\
&\quad + \frac{B^2(bc-ad)^5g^2i^2\log(c+dx)}{10b^3d^3} + \frac{B^2(bc-ad)^5g^2i^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{15b^3d^3}
\end{aligned}
$$

output

```
-1/10*B^2*(-a*d+b*c)^4*g^2*i^2*x/b^2/d^2-1/20*B^2*(-a*d+b*c)^3*g^2*i^2*(d*
x+c)^2/b/d^3+1/30*B^2*(-a*d+b*c)^2*g^2*i^2*(d*x+c)^3/d^3+1/30*B^2*(-a*d+b*
c)^5*g^2*i^2*ln((b*x+a)/(d*x+c))/b^3/d^3-1/30*B*(-a*d+b*c)^3*g^2*i^2*(b*x+
a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/d-1/15*B*(-a*d+b*c)^2*g^2*i^2*(b*x+a)
^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3-1/5*B*(-a*d+b*c)^3*g^2*i^2*(d*x+c)^2*(A
+B*ln(e*(b*x+a)/(d*x+c)))/b/d^3+4/15*B*(-a*d+b*c)^2*g^2*i^2*(d*x+c)^3*(A+B
*ln(e*(b*x+a)/(d*x+c)))/d^3-1/10*b*B*(-a*d+b*c)*g^2*i^2*(d*x+c)^4*(A+B*ln(
e*(b*x+a)/(d*x+c)))/d^3+1/30*(-a*d+b*c)^2*g^2*i^2*(b*x+a)^3*(A+B*ln(e*(b*x
+a)/(d*x+c)))^2/b^3+1/10*(-a*d+b*c)*g^2*i^2*(b*x+a)^3*(d*x+c)*(A+B*ln(e*(b
*x+a)/(d*x+c)))^2/b^2+1/5*g^2*i^2*(b*x+a)^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d
*x+c)))^2/b+1/30*B*(-a*d+b*c)^4*g^2*i^2*(b*x+a)*(2*A+B+2*B*ln(e*(b*x+a)/(d
*x+c)))/b^3/d^2+1/30*B*(-a*d+b*c)^5*g^2*i^2*ln((-a*d+b*c)/b/(d*x+c))*(2*A+
3*B+2*B*ln(e*(b*x+a)/(d*x+c)))/b^3/d^3+1/10*B^2*(-a*d+b*c)^5*g^2*i^2*ln(d*
x+c)/b^3/d^3+1/15*B^2*(-a*d+b*c)^5*g^2*i^2*polylog(2,d*(b*x+a)/b/(d*x+c))/
b^3/d^3
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 1194, normalized size of antiderivative = 1.57

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input

```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^2,x]
```

output

```
(g^2*i^2*(20*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d
*x)])^2 + 30*d^4*(b*c - a*d)*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x
)])^2 + 12*d^5*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 20*B*(
b*c - a*d)^3*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(
a + b*x))/(c + d*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
) - 2*B*(b*c - a*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*
x))/(c + d*x)])*Log[c + d*x] + B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c
 + d*x]) + B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c +
 d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - 10*B*(b*
c - a*d)^2*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e
*(a + b*x))/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x
)]) - 6*B*(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a +
b*x))/(c + d*x)])*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*
(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (
-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b
*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c
 - a*d)])) + B*(b*c - a*d)*(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^
3*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x)^
2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*...
```

Rubi [A] (verified)

Time = 2.10 (sec) , antiderivative size = 852, normalized size of antiderivative = 1.12, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2962, 2783, 2782, 27, 1195, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\downarrow 2962$$

$$g^2i^2(bc-ad)^5\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(-\frac{2B \int \frac{(a+bx)^2 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{2 \int \frac{(a+bx)^2 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{(a+bx)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{5b(c+dx)^3 \left(b-\frac{d(a+b}{c+d}\right.} \right.$$

$$\downarrow 2782$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(-\frac{2B \left(-B \int \frac{(c+dx)\left(b^2-\frac{4d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{12d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx} + \frac{b^2\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2b\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{B \log\left(\frac{e(a}{2d^3\left(b-\frac{d}{}\right.}\right.}{5b} \right.$$

$$\downarrow 27$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(-\frac{2B \left(-\frac{B \int \frac{(c+dx)\left(b^2-\frac{4d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{12d^3} + \frac{b^2\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2b\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{B \log\left(\frac{e(a+bx}{2d^3\left(b-\frac{d(a+b}{c+d}\right.}\right.}{5b} \right.$$

$$\downarrow 1195$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(-\frac{2B \left(-\frac{B \int \left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)} + \frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{5d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{3bd}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{c+dx}{b^2(a+bx)} \right) d\frac{a+bx}{c+dx}}{12d^3} + \frac{b^2\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} \right.}{5b} \right.$$

$$\downarrow 2009$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(\frac{2 \int \frac{(a+bx)^2 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} - \frac{2B \left(\frac{b^2 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2b \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^2} \right.$$

$$\downarrow 2783$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(\frac{2 \left(-\frac{B \int \frac{(a+bx)^2 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{2b} + \frac{\int \frac{(a+bx)^2 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{4b} + \frac{(a+bx)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} \right)}{5b} - \right.$$

$$\downarrow 2773$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(\frac{2 \left(-\frac{B \left(\frac{(a+bx)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{B \int \frac{(a+bx)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^3} d\frac{a+bx}{c+dx}}{3b} \right)}{2b} + \frac{\int \frac{(a+bx)^2 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{4b} + \frac{(a+bx)^3 \left(\right.}{4b(c+} \right.$$

$$\downarrow 49$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(\frac{2 \left(-\frac{B \left(\frac{(a+bx)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A \right)}{3b(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{B \int \left(\frac{b^2}{d^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2b}{d^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^2} + \frac{1}{d^2 \left(b-\frac{d(a+bx)}{c+dx}\right)} \right) d\frac{a+bx}{c+dx}}{3b} \right)}{2b} + \int \frac{(a+bx)^2 \left(A+B\log \right.}{(c+dx)^2 \left(b-\right.} \right.}{5b}$$

$$\downarrow 2009$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(\frac{2 \left(\frac{\int \frac{(a+bx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{4b} - \frac{B \left(\frac{(a+bx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A \right)}{3b(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{B \left(\frac{b^2}{2d^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{2b}{d^3 \left(b-\frac{d(a+bx)}{c+dx}\right)} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right.}{d^3} \right.}{3b} \right)}{2b} \right.}{5b}$$

$\downarrow$ 2781

$$g^2 i^2 (bc - ad)^5 \left(\frac{2\left(\frac{\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} d\frac{a+bx}{c+dx}}{3b}}{4b} - \frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{B\left(\frac{b^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{}\right.}{2b} \right.}{5b} \right.$$

$\downarrow$ 2784

$$g^2 i^2 (bc -$$

$$ad)^5 \left(\frac{2\left(\frac{\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{\int \frac{(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2} d\frac{a+bx}{c+dx}}{2d}\right)}{3b}}{4b} - \frac{B\left(\frac{(a+bx)^3\left(B\right.}{3b(c+dx}\right.}{5b} \right.$$

$\downarrow$ 2784

$$(bc-ad)^5 g^2 i^2 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{5b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2} \right.$$

↓ 2754

$$(bc -$$

$$ad)^5 g^2 i^2 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{5b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right.}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\right.}{} \right.$$

$\downarrow$ 2838

$$(bc-ad)^5 g^2 i^2 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{5b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2} \right.$$

input

```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
,x]
```

output
```
(b*c - a*d)^5*g^2*i^2*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(5*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*((b^2*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]))/(4*d^3*(b - (d*(a + b*x))/(c + d*x))^4) - (2*
b*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*d^3*(b - (d*(a + b*x))/(c + d*x
))^3) + (A + B*Log[(e*(a + b*x))/(c + d*x)])/(2*d^3*(b - (d*(a + b*x))/(c
 + d*x))^2) - (B*(b/(b - (d*(a + b*x))/(c + d*x))^3 - 5/(2*(b - (d*(a + b*x
))/(c + d*x))^2) + 1/(b*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c
 + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(12*d^3)))/(5*b) + (2
*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*b*(c + d*x)^3*(b
 - (d*(a + b*x))/(c + d*x))^4) - (B*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))
/(c + d*x)]))/(3*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (B*(b^2/
(2*d^3*(b - (d*(a + b*x))/(c + d*x))^2) - (2*b)/(d^3*(b - (d*(a + b*x))/(c
 + d*x))) - Log[b - (d*(a + b*x))/(c + d*x)]/d^3))/(3*b)))/(2*b) + (((a +
b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*b*(c + d*x)^3*(b - (d*(a
 + b*x))/(c + d*x))^3) - (2*B*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c +
d*x)]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(2
*A + B + 2*B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c + d*x)*(b - (d*(a + b*x)
)/(c + d*x))) - (-(((2*A + 3*B + 2*B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 -
 (d*(a + b*x))/(b*(c + d*x))])/d) - (2*B*PolyLog[2, (d*(a + b*x))/(b*(c +
d*x))])/d)/d)/(2*d)))/(3*b))/(4*b)))/(5*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)^2 (dix+ci)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2 (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)^2 (dix+ci)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="fricas")
```

output
```
integral(A^2*b^2*d^2*g^2*i^2*x^4 + A^2*a^2*c^2*g^2*i^2 + 2*(A^2*b^2*c*d +
A^2*a*b*d^2)*g^2*i^2*x^3 + (A^2*b^2*c^2 + 4*A^2*a*b*c*d + A^2*a^2*d^2)*g^2
*i^2*x^2 + 2*(A^2*a*b*c^2 + A^2*a^2*c*d)*g^2*i^2*x + (B^2*b^2*d^2*g^2*i^2*
x^4 + B^2*a^2*c^2*g^2*i^2 + 2*(B^2*b^2*c*d + B^2*a*b*d^2)*g^2*i^2*x^3 + (B
^2*b^2*c^2 + 4*B^2*a*b*c*d + B^2*a^2*d^2)*g^2*i^2*x^2 + 2*(B^2*a*b*c^2 + B
^2*a^2*c*d)*g^2*i^2*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^2*d^2*g^2
*i^2*x^4 + A*B*a^2*c^2*g^2*i^2 + 2*(A*B*b^2*c*d + A*B*a*b*d^2)*g^2*i^2*x^3
 + (A*B*b^2*c^2 + 4*A*B*a*b*c*d + A*B*a^2*d^2)*g^2*i^2*x^2 + 2*(A*B*a*b*c^
2 + A*B*a^2*c*d)*g^2*i^2*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3656 vs. $2(728) = 1456$.

Time $= 0.19$ (sec) , antiderivative size $= 3656$, normalized size of antiderivative $= 4.80$

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="maxima")
```

output

```
1/5*A^2*b^2*d^2*g^2*i^2*x^5 + 1/2*A^2*b^2*c*d*g^2*i^2*x^4 + 1/2*A^2*a*b*d^
2*g^2*i^2*x^4 + 1/3*A^2*b^2*c^2*g^2*i^2*x^3 + 4/3*A^2*a*b*c*d*g^2*i^2*x^3
+ 1/3*A^2*a^2*d^2*g^2*i^2*x^3 + A^2*a*b*c^2*g^2*i^2*x^2 + A^2*a^2*c*d*g^2*
i^2*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c
*log(d*x + c)/d)*A*B*a^2*c^2*g^2*i^2 + 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d
*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*
d))*A*B*a*b*c^2*g^2*i^2 + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))
+ 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x
^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*c^2*g^2*i^2 + 2*(x^2*log(
b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)
/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^2*c*d*g^2*i^2 + 4/3*(2*x^3*log(b*e*x/(d*
x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3
- ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*b*c
*d*g^2*i^2 + 1/6*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b
*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*
(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^2*
c*d*g^2*i^2 + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(
b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*
c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*d^2*g^2*i^2 + 1/6*(6*x^4*log(b*e*x/(d
*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/...
```

Giac [F]

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)^2(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input

```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="giac")
```

output

```
integrate((b*g*x + a*g)^2*(d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) +
A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2
,x)
```

output
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2
, x)
```

Reduce [F]

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
(g**2*( - 4*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a**5*b**2*d**6 + 20*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c +
 a*d*x + b*c*x + b*d*x**2),x)*a**4*b**3*c*d**5 - 40*int((log((a*e + b*e*x)
/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**4*c**2*d**4 + 4
0*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)
*a**2*b**5*c**3*d**3 - 20*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*
x + b*c*x + b*d*x**2),x)*a*b**6*c**4*d**2 + 4*int((log((a*e + b*e*x)/(c +
d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**7*c**5*d - 4*log(c + d*x)*
a**6*d**5 + 20*log(c + d*x)*a**5*b*c*d**4 - 40*log(c + d*x)*a**4*b**2*c**2
*d**3 + 40*log(c + d*x)*a**3*b**3*c**3*d**2 - 20*log(c + d*x)*a**2*b**4*c*
*4*d + 4*log(c + d*x)*a*b**5*c**5 + 2*log((a*e + b*e*x)/(c + d*x))**2*a**4
*b**2*c*d**4 - 8*log((a*e + b*e*x)/(c + d*x))**2*a**3*b**3*c**2*d**3 - 8*l
og((a*e + b*e*x)/(c + d*x))**2*a**2*b**4*c**3*d**2 - 60*log((a*e + b*e*x)/
(c + d*x))**2*a**2*b**4*c**2*d**3*x - 60*log((a*e + b*e*x)/(c + d*x))**2*a
**2*b**4*c*d**4*x**2 - 20*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**4*d**5*x
**3 + 2*log((a*e + b*e*x)/(c + d*x))**2*a*b**5*c**4*d - 60*log((a*e + b*e*
x)/(c + d*x))**2*a*b**5*c**2*d**3*x**2 - 80*log((a*e + b*e*x)/(c + d*x))**
2*a*b**5*c*d**4*x**3 - 30*log((a*e + b*e*x)/(c + d*x))**2*a*b**5*d**5*x**4
 - 20*log((a*e + b*e*x)/(c + d*x))**2*b**6*c**2*d**3*x**3 - 30*log((a*e +
b*e*x)/(c + d*x))**2*b**6*c*d**4*x**4 - 12*log((a*e + b*e*x)/(c + d*x))...
```

3.66 $\int (ag+bgx)(ci+dix)^2 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result 822
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Reduce [F] 834

Optimal result

Integrand size = 40, antiderivative size = 589

$$
\begin{aligned}
&\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx \\
&= \frac{B^2(bc-ad)^3gi^2x}{12b^2d} + \frac{B^2(bc-ad)^2gi^2(c+dx)^2}{12bd^2} - \frac{B^2(bc-ad)^4gi^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2} \\
&\quad - \frac{B(bc-ad)^3gi^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3d} \\
&\quad - \frac{B(bc-ad)^2gi^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3} \\
&\quad + \frac{B(bc-ad)^2gi^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4bd^2} \\
&\quad - \frac{B(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^2} \\
&\quad + \frac{(bc-ad)^2gi^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{12b^3} \\
&\quad + \frac{(bc-ad)gi^2(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b^2} \\
&\quad + \frac{gi^2(a+bx)^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} \\
&\quad - \frac{B(bc-ad)^4gi^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3d^2} \\
&\quad - \frac{B^2(bc-ad)^4gi^2\log(c+dx)}{4b^3d^2} - \frac{B^2(bc-ad)^4gi^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{6b^3d^2}
\end{aligned}
$$

output

```
1/12*B^2*(-a*d+b*c)^3*g*i^2*x/b^2/d+1/12*B^2*(-a*d+b*c)^2*g*i^2*(d*x+c)^2/
b/d^2-1/12*B^2*(-a*d+b*c)^4*g*i^2*ln((b*x+a)/(d*x+c))/b^3/d^2-1/6*B*(-a*d+
b*c)^3*g*i^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/d-1/6*B*(-a*d+b*c)^2*
g*i^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3+1/4*B*(-a*d+b*c)^2*g*i^2*(
d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-1/6*B*(-a*d+b*c)*g*i^2*(d*x+c)^
3*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2+1/12*(-a*d+b*c)^2*g*i^2*(b*x+a)^2*(A+B*l
n(e*(b*x+a)/(d*x+c)))^2/b^3+1/6*(-a*d+b*c)*g*i^2*(b*x+a)^2*(d*x+c)*(A+B*ln
(e*(b*x+a)/(d*x+c)))^2/b^2+1/4*g*i^2*(b*x+a)^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)
/(d*x+c)))^2/b-1/6*B*(-a*d+b*c)^4*g*i^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B+B*ln
(e*(b*x+a)/(d*x+c)))/b^3/d^2-1/4*B^2*(-a*d+b*c)^4*g*i^2*ln(d*x+c)/b^3/d^2-
1/6*B^2*(-a*d+b*c)^4*g*i^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^2
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 677, normalized size of antiderivative = 1.15

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$

$$= \frac{gi^2\left(-4(bc-ad)(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+3b(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{4B(bc-ad)^2\left(2A\right.}{}\right.}{}$$

input

```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)
])^2,x]
```

output

```
(g*i^2*(-4*(b*c - a*d)*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
+ 3*b*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (4*B*(b*c - a*d
)^2*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*(b*d*x + (b*c - a*d)*Log[a + b*
x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + b^2*(c +
d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*(b*c - a*d)^2*Log[a + b*x]
*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b*c - a*d)^2*Log[c + d*x] - B
*(b*c - a*d)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*
d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3 - (B*(b*c - a*d)*
(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*(b*d*x + (b*c - a*d)*Log[a +
b*x]) - B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c - a*
d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c +
d*x)] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
 + 2*b^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c - a*d)^
3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c - a*d)^3*Lo
g[c + d*x] - 3*B*(b*c - a*d)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
 d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3))
/(12*d^2)
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2962, 2783, 2782, 27, 86, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\downarrow 2962$$

$$gi^2(bc-ad)^4\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$gi^2(bc-ad)^4\left(-\frac{B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right.}{4b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right.$$

$$\downarrow 2782$$

$$gi^2(bc-ad)^4\left(-\frac{B\left(-B\int-\frac{(c+dx)\left(b-\frac{3d(a+bx)}{c+dx}\right)}{6d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{2b}\right.$$

$$\downarrow 27$$

$$gi^2(bc-ad)^4\left(-\frac{B\left(\frac{B\int\frac{(c+dx)\left(b-\frac{3d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{6d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d}{2b}\right.$$

$$\downarrow 86$$

$$gi^2(bc-ad)^4\left(-\frac{B\left(\frac{B\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{6d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right.}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}\right.$$

$$\downarrow 2009$$

$$ad)^4\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}-\frac{gi^2(bc-\quad B\left(-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+b}{c+d}\right.\right.\right.\right.}{2b}\right.$$

$$\downarrow 2783$$

$$ad)^4\left(\frac{-\frac{2B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{gi^2(bc-\quad B\left(-\right.}{2b}\right.$$

$$\downarrow 2773$$

$$ad)^4\left(\frac{-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{a+bx}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2b}\right)}{3b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+bx)^2\left(B\log\right.}{3b(c+dx)^2\left(\right.}\right.$$

$$\downarrow 49$$

$$gi^2(bc-$$

$$ad)^4\left(\frac{-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\left(\frac{b}{d\left(\frac{d(a+bx)}{c+dx}-b\right)^2}+\frac{1}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{2b}\right)}{3b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\cdots}{2b}\right.$$

$\downarrow$ 2009

$$gi^2(bc-$$

$$ad)^4\left(\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^2}\right)}{2b}\right)}{3b}+\frac{(a+bx)^2(\cdots}{3b(c+\cdots}}{2b}\right.$$

$\downarrow$ 2781

$$gi^2(bc-$$

$$ad)^4\left(\frac{\frac{\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{3b}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\cdots\right.}{2b}\right.}{3b}}{2b}\right.$$

$\downarrow$ 2784

$$gi^2(bc-$$

$$ad)^4\left(\frac{\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{b}}{3b}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.\right.}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}$$

$$\Big\downarrow 2754$$

$$gi^2(bc-$$

$$ad)^4\left(\frac{\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{B\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{d}\right)}{b}}{3b}$$

$$\Big\downarrow 2838$$

$$gi^2(bc-$$

$$ad)^4\left(-\frac{B\left(-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{1}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right)}{6d^2}\right)}{2b}$$

input
```
Int[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x
]
```

output
```
(b*c - a*d)^4*g*i^2*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/
(4*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^4) - (B*((b*(A + B*Log[(e*(
a + b*x))/(c + d*x)]))/(3*d^2*(b - (d*(a + b*x))/(c + d*x))^3) - (A + B*Lo
g[(e*(a + b*x))/(c + d*x)])/(2*d^2*(b - (d*(a + b*x))/(c + d*x))^2) + (B*(
-(b - (d*(a + b*x))/(c + d*x))^(-2) + 1/(b*(b - (d*(a + b*x))/(c + d*x)))
+ Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(6
*d^2)))/(2*b) + (((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*b
*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*(((a + b*x)^2*(A + B*
Log[(e*(a + b*x))/(c + d*x)]))/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*
x))^2) - (B*(b/(d^2*(b - (d*(a + b*x))/(c + d*x))) + Log[b - (d*(a + b*x))
/(c + d*x)]/d^2))/(2*b)))/(3*b) + (((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(
c + d*x)])^2)/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(((a
+ b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c + d*x)*(b - (d*(a + b*x
))/(c + d*x))) - (-(((A + B + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(
a + b*x))/(b*(c + d*x))])/d) - (B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])
/d)/d))/b)/(3*b))/(2*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)\,(dix+ci)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$=\int (bgx+ag)(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="fricas")
```

output
```
integral(A^2*b*d^2*g*i^2*x^3 + A^2*a*c^2*g*i^2 + (2*A^2*b*c*d + A^2*a*d^2)
*g*i^2*x^2 + (A^2*b*c^2 + 2*A^2*a*c*d)*g*i^2*x + (B^2*b*d^2*g*i^2*x^3 + B^
2*a*c^2*g*i^2 + (2*B^2*b*c*d + B^2*a*d^2)*g*i^2*x^2 + (B^2*b*c^2 + 2*B^2*a
*c*d)*g*i^2*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b*d^2*g*i^2*x^3 + A
*B*a*c^2*g*i^2 + (2*A*B*b*c*d + A*B*a*d^2)*g*i^2*x^2 + (A*B*b*c^2 + 2*A*B*
a*c*d)*g*i^2*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2259 vs. $2(562) = 1124$.

Time $= 0.17$ (sec) , antiderivative size $= 2259$, normalized size of antiderivative $= 3.84$

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="maxima")
```

output

```
1/4*A^2*b*d^2*g*i^2*x^4 + 2/3*A^2*b*c*d*g*i^2*x^3 + 1/3*A^2*a*d^2*g*i^2*x^
3 + 1/2*A^2*b*c^2*g*i^2*x^2 + A^2*a*c*d*g*i^2*x^2 + 2*(x*log(b*e*x/(d*x +
c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a*c^2*g*i^2
 + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*
log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*b*c^2*g*i^2 + 2*(x^2*log(b*e*x
/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2
- (b*c - a*d)*x/(b*d))*A*B*a*c*d*g*i^2 + 2/3*(2*x^3*log(b*e*x/(d*x + c) +
a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c
*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b*c*d*g*i^2 +
1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 -
 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)
*x)/(b^2*d^2))*A*B*a*d^2*g*i^2 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2
- a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x
)/(b^3*d^3))*A*B*b*d^2*g*i^2 + A^2*a*c^2*g*i^2*x - 1/12*(7*a^2*b*c^2*d^2*g
*i^2 - 2*a^3*c*d^3*g*i^2 - (2*g*i^2*log(e) - g*i^2)*b^3*c^4 + 2*(4*g*i^2*l
og(e) - 3*g*i^2)*a*b^2*c^3*d)*B^2*log(d*x + c)/(b^2*d^2) + 1/6*(b^4*c^4*g*
i^2 - 4*a*b^3*c^3*d*g*i^2 + 6*a^2*b^2*c^2*d^2*g*i^2 - 4*a^3*b*c*d^3*g*i^2
+ a^4*d^4*g*i^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(
-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^2) + 1/12*(3*B^2*b^4*d^4*g*i^2*...
```

Giac [F]

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="giac")
```

output

```
integrate((b*g*x + a*g)*(d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)
^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)\,(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x
)
```

output
```
int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,
x)
```

Reduce [F]

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
(g*( - 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x
**2),x)*a**4*b**2*d**5 + 8*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d
*x + b*c*x + b*d*x**2),x)*a**3*b**3*c*d**4 - 12*int((log((a*e + b*e*x)/(c
+ d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**4*c**2*d**3 + 8*int
((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b*
*5*c**3*d**2 - 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x
 + b*d*x**2),x)*b**6*c**4*d - 2*log(c + d*x)*a**5*d**4 + 8*log(c + d*x)*a*
*4*b*c*d**3 + log(c + d*x)*a**4*b*d**4 - 12*log(c + d*x)*a**3*b**2*c**2*d*
*2 - 4*log(c + d*x)*a**3*b**2*c*d**3 + 8*log(c + d*x)*a**2*b**3*c**3*d + 6
*log(c + d*x)*a**2*b**3*c**2*d**2 - 2*log(c + d*x)*a*b**4*c**4 - 4*log(c +
 d*x)*a*b**4*c**3*d + log(c + d*x)*b**5*c**4 + log((a*e + b*e*x)/(c + d*x)
)**2*a**3*b**2*c*d**3 - 3*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**3*c**2*d
**2 - log((a*e + b*e*x)/(c + d*x))**2*a*b**4*c**3*d - 12*log((a*e + b*e*x)
/(c + d*x))**2*a*b**4*c**2*d**2*x - 12*log((a*e + b*e*x)/(c + d*x))**2*a*b
**4*c*d**3*x**2 - 4*log((a*e + b*e*x)/(c + d*x))**2*a*b**4*d**4*x**3 - 6*l
og((a*e + b*e*x)/(c + d*x))**2*b**5*c**2*d**2*x**2 - 8*log((a*e + b*e*x)/(
c + d*x))**2*b**5*c*d**3*x**3 - 3*log((a*e + b*e*x)/(c + d*x))**2*b**5*d**
4*x**4 - 2*log((a*e + b*e*x)/(c + d*x))*a**5*d**4 + 8*log((a*e + b*e*x)/(c
 + d*x))*a**4*b*c*d**3 + log((a*e + b*e*x)/(c + d*x))*a**4*b*d**4 - 12*log
((a*e + b*e*x)/(c + d*x))*a**3*b**2*c**2*d**2 - 2*log((a*e + b*e*x)/(c ...
```

3.67 $\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result . . . 836
Mathematica [A] (verified) . . . 837
Rubi [A] (verified) . . . 837
Maple [F] . . . 844
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Sympy [F(-1)] . . . 845
Maxima [B] (verification not implemented) . . . 845
Giac [F] . . . 846
Mupad [F(-1)] . . . 847
Reduce [F] . . . 847

Optimal result

Integrand size = 32, antiderivative size = 334

$$
\begin{aligned}
&\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^2i^2x}{3b^2} + \frac{B^2(bc-ad)^3i^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d} \\
&\quad - \frac{2B(bc-ad)^2i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^3} \\
&\quad - \frac{B(bc-ad)i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd} \\
&\quad + \frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d} + \frac{B^2(bc-ad)^3i^2\log(c+dx)}{b^3d} \\
&\quad + \frac{2B(bc-ad)^3i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d} \\
&\quad - \frac{2B^2(bc-ad)^3i^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d}
\end{aligned}
$$

output
```
1/3*B^2*(-a*d+b*c)^2*i^2*x/b^2+1/3*B^2*(-a*d+b*c)^3*i^2*ln((b*x+a)/(d*x+c)
)/b^3/d-2/3*B*(-a*d+b*c)^2*i^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3-1/3
*B*(-a*d+b*c)*i^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/3*i^2*(d*x+c
)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d+B^2*(-a*d+b*c)^3*i^2*ln(d*x+c)/b^3/d+2
/3*B*(-a*d+b*c)^3*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a)
)/b^3/d-2/3*B^2*(-a*d+b*c)^3*i^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/d
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.86

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{i^2\left((c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B(bc-ad)\left(2Abd(bc-ad)x-B(bc-ad)(bdx+(bc-ad)\log(a+bx))+2Bd(bc-ad)(a+bx)\log\right.}{}\right.}{}$$

input
```
Integrate[(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(i^2*((c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*
(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*(b*d*x + (b*c - a*d)*Log[a + b*x])
 + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + b^2*(c + d*x)
^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*(b*c - a*d)^2*Log[a + b*x]*(A
 + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b*c - a*d)^2*Log[c + d*x] - B*(b*
c - a*d)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)])
 - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3))/(3*d)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2952, 2756, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\downarrow 2952$$

$$i^2(bc-ad)^3 \int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}$$

$$\downarrow 2756$$

$$i^2(bc-ad)^3 \left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3} d\frac{a+bx}{c+dx}}{3d}\right)$$

$$\downarrow 2789$$

$$i^2(bc-ad)^3 \left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left(\frac{d\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3} d\frac{a+bx}{c+dx}}{b} + \frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2} d\frac{a+bx}{c+dx}}{b}\right)}{3d}\right)$$

$$\downarrow 2756$$

$$i^2(bc-ad)^3 \left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{B\int \frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2} d\frac{a+bx}{c+dx}}{2d}\right)}{b} + \frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}}{b}\right)}{3d}\right)$$

$$\downarrow 54$$

$$ad)^3\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\int\frac{(c}{}\right.$$

$$i^2(bc-$$

$$\downarrow 2009$$

$$ad)^3\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+}{c+}\right.\right.\right.\right.$$

$$i^2(bc-$$

$$\downarrow 2789$$

$$ad)^3\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{i^2(bc-}{3d}\right.$$

$$\left.2B\left(\frac{d\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right.\right.$$

$$\downarrow 2751$$

$$ad)^3\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{i^2(bc-}{3d}\right.$$

$$\left.2B\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{B\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+}{c+}}{b}\right.$$

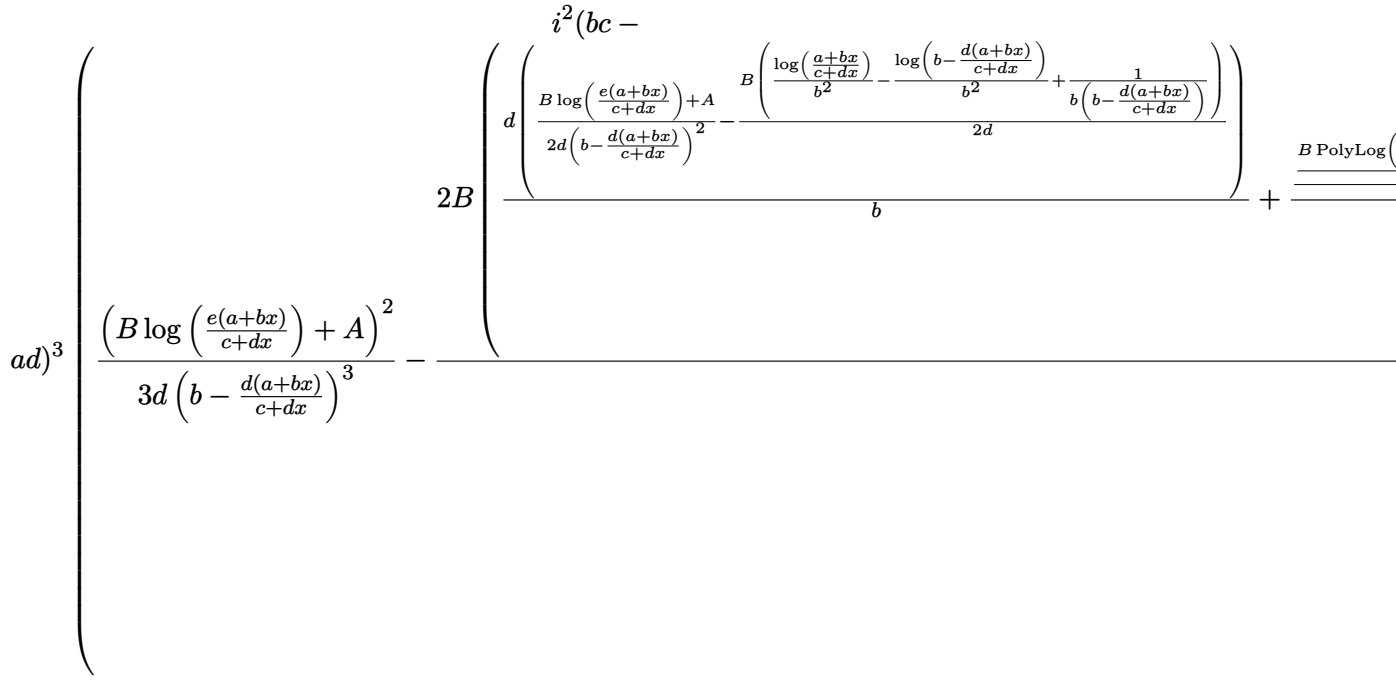
$$\downarrow 16$$

$$ad)^3\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{i^2(bc-}{\frac{\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}}{b}\right)}{3d}\right)$$

$$\downarrow 2779$$

$$ad)^3\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{i^2(bc-}{\frac{\frac{B\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\right.\right.\right.}{b(c+dx)\left(b\right.}\right.}{b}\right)$$

$$\downarrow 2838$$



input
```
Int[(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)^3*i^2*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(3*d*(b - (d*(a
+ b*x))/(c + d*x))^3) - (2*B*((d*((A + B*Log[(e*(a + b*x))/(c + d*x)])/(2*
d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(1/(b*(b - (d*(a + b*x))/(c + d*x)
)) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))
/(2*d)))/b + ((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b*(c +
 d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*Log[b - (d*(a + b*x))/(c + d*x)]
)/(b*d)))/b + (-(((A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x
))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b)/b)/
b))/(3*d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (dix + ci)^2 \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input

```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ci + dix)^2 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (dix + ci)^2 \left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2 dx$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="frica
s")
```

output

```
integral(A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x^
2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B
*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c))
, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. $2(319) = 638$.

Time = 0.14 (sec) , antiderivative size = 1202, normalized size of antiderivative = 3.60

$$\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxim
a")
```

output

```
1/3*A^2*d^2*i^2*x^3 + A^2*c*d*i^2*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*c^2*i^2 + 2*(x^2*log(b*
e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d
^2 - (b*c - a*d)*x/(b*d))*A*B*c*d*i^2 + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*d^2*i^2 + A^2*c
^2*i^2*x - 1/3*(5*a*b*c^2*d*i^2 - 2*a^2*c*d^2*i^2 + (2*i^2*log(e) - 3*i^2)
*b^2*c^3)*B^2*log(d*x + c)/(b^2*d) - 2/3*(b^3*c^3*i^2 - 3*a*b^2*c^2*d*i^2
+ 3*a^2*b*c*d^2*i^2 - a^3*d^3*i^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c -
a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d) + 1/3*(B^2*b^3*
d^3*i^2*x^3*log(e)^2 + (a*b^2*d^3*i^2*log(e) + (3*i^2*log(e)^2 - i^2*log(e
))*b^3*c*d^2)*B^2*x^2 + ((3*i^2*log(e)^2 - 4*i^2*log(e) + i^2)*b^3*c^2*d +
 2*(3*i^2*log(e) - i^2)*a*b^2*c*d^2 - (2*i^2*log(e) - i^2)*a^2*b*d^3)*B^2*
x + (B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x
 + (3*a*b^2*c^2*d*i^2 - 3*a^2*b*c*d^2*i^2 + a^3*d^3*i^2)*B^2)*log(b*x + a)
^2 + (B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*
x + B^2*b^3*c^3*i^2)*log(d*x + c)^2 + (2*B^2*b^3*d^3*i^2*x^3*log(e) + (a*b
^2*d^3*i^2 + (6*i^2*log(e) - i^2)*b^3*c*d^2)*B^2*x^2 + 2*(3*a*b^2*c*d^2*i^
2 - a^2*b*d^3*i^2 + (3*i^2*log(e) - 2*i^2)*b^3*c^2*d)*B^2*x + (2*(3*i^2*lo
g(e) - 2*i^2)*a*b^2*c^2*d - (6*i^2*log(e) - 7*i^2)*a^2*b*c*d^2 + (2*i^2...
```

Giac [F]

$$\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (dix+ci)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac"
)
```

output

```
integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (c\,i+d\,i\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

Reduce [F]

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
( - 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2
),x)*a**3*b**2*d**4 + 6*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x
+ b*c*x + b*d*x**2),x)*a**2*b**3*c*d**3 - 6*int((log((a*e + b*e*x)/(c + d*
x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**4*c**2*d**2 + 2*int((log((
a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**5*c**3*d
 - 2*log(a + b*x)*a**4*d**3 + 6*log(a + b*x)*a**3*b*c*d**2 + 3*log(a + b*x
)*a**3*b*d**3 - 6*log(a + b*x)*a**2*b**2*c**2*d - 9*log(a + b*x)*a**2*b**2
*c*d**2 + 2*log(a + b*x)*a*b**3*c**3 + 9*log(a + b*x)*a*b**3*c**2*d - 3*lo
g(a + b*x)*b**4*c**3 + log((a*e + b*e*x)/(c + d*x))**2*a**2*b**2*c*d**2 -
2*log((a*e + b*e*x)/(c + d*x))**2*a*b**3*c**2*d - 3*log((a*e + b*e*x)/(c +
 d*x))**2*b**4*c**2*d*x - 3*log((a*e + b*e*x)/(c + d*x))**2*b**4*c*d**2*x*
*2 - log((a*e + b*e*x)/(c + d*x))**2*b**4*d**3*x**3 + 2*log((a*e + b*e*x)/
(c + d*x))*a**2*b**2*c*d**2 + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*d**
3*x - 2*log((a*e + b*e*x)/(c + d*x))*a*b**3*c**3 - 6*log((a*e + b*e*x)/(c
+ d*x))*a*b**3*c**2*d*x - 5*log((a*e + b*e*x)/(c + d*x))*a*b**3*c**2*d - 6
*log((a*e + b*e*x)/(c + d*x))*a*b**3*c*d**2*x**2 - 6*log((a*e + b*e*x)/(c
+ d*x))*a*b**3*c*d**2*x - 2*log((a*e + b*e*x)/(c + d*x))*a*b**3*d**3*x**3
- log((a*e + b*e*x)/(c + d*x))*a*b**3*d**3*x**2 + 3*log((a*e + b*e*x)/(c +
 d*x))*b**4*c**3 + 4*log((a*e + b*e*x)/(c + d*x))*b**4*c**2*d*x + log((a*e
 + b*e*x)/(c + d*x))*b**4*c*d**2*x**2 + 2*a**3*b*d**3*x - 3*a**2*b**2*c...
```

3.68 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

Optimal result 850
Mathematica [B] (verified) 851
Rubi [A] (verified) 852
Maple [F] 860
Fricas [F] 860
Sympy [F] 861
Maxima [F] 861
Giac [F] 862
Mupad [F(-1)] 863
Reduce [F] 863

Optimal result

Integrand size = 42, antiderivative size = 535

$$
\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx \\
&= -\frac{Bd(bc-ad)i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g} \\
&\quad + \frac{2B(bc-ad)^2i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g} \\
&\quad + \frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g} \\
&\quad + \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg} + \frac{B^2(bc-ad)^2i^2\log(c+dx)}{b^3g} \\
&\quad + \frac{B(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad - \frac{(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad + \frac{2B^2(bc-ad)^2i^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g} - \frac{B^2(bc-ad)^2i^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad + \frac{2B(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad + \frac{2B^2(bc-ad)^2i^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}
\end{aligned}
$$

output

```
-B*d*(-a*d+b*c)*i^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/g+2*B*(-a*d+b*
c)^2*i^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/g+d*(-a*
d+b*c)*i^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^3/g+1/2*i^2*(d*x+c)^2*(
A+B*ln(e*(b*x+a)/(d*x+c)))^2/b/g+B^2*(-a*d+b*c)^2*i^2*ln(d*x+c)/b^3/g+B*(-
a*d+b*c)^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g
-(-a*d+b*c)^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/
b^3/g+2*B^2*(-a*d+b*c)^2*i^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/g-B^2*(-a*
d+b*c)^2*i^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g+2*B*(-a*d+b*c)^2*i^2*(A+
B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g+2*B^2*(-a*d+
b*c)^2*i^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^3/g
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2615 vs. $2(535) = 1070$.

Time $= 4.37$ (sec) , antiderivative size $= 2615$, normalized size of antiderivative $= 4.89$

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \text{Result too large to show}$$

input

```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x),x]
```

output

```
(i^2*(12*A^2*b*d*(2*b*c - a*d)*x + 6*A^2*b^2*d^2*x^2 + 12*A^2*(b*c - a*d)^
2*Log[a + b*x] - 24*A*b*B*c*(a*d*Log[a/b + x]^2 - 2*a*d*Log[a/b + x]*(1 +
Log[a + b*x]) + 2*(-(b*c) + a*d + Log[c/d + x]*(b*c + a*d*Log[a + b*x] - a
*d*Log[(d*(a + b*x))/(-(b*c) + a*d)]) + (-(b*d*x) + a*d*Log[a + b*x])*Log[
(e*(a + b*x))/(c + d*x)]) - 2*a*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) +
 12*A*b^2*B*c^2*(Log[a/b + x]^2 - 2*Log[a + b*x]*(Log[a/b + x] - Log[c/d +
 x] - Log[(e*(a + b*x))/(c + d*x)]) - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-
(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 6*A*B*(-4*a*d^2*
(a + b*x)*(-1 + Log[a/b + x]) + 2*a^2*d^2*Log[a/b + x]^2 + 4*a*b*d*(c + d*
x)*(-1 + Log[c/d + x]) + d^2*(b*x*(2*a - b*x) + 2*b^2*x^2*Log[a/b + x] - 2
*a^2*Log[a + b*x]) - 2*d^2*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x])*(Log[a/
b + x] - Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)]) + b^2*(d*x*(-2*c + d
*x) - 2*d^2*x^2*Log[c/d + x] + 2*c^2*Log[c + d*x]) - 4*a^2*d^2*(Log[c/d +
x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d
)])) - 8*b*B^2*c*(a*d*Log[a/b + x]^3 - 3*d*(2*b*x - 2*(a + b*x)*Log[a/b +
x] + (a + b*x)*Log[a/b + x]^2) - 3*b*(2*d*x - 2*(c + d*x)*Log[c/d + x] + (
c + d*x)*Log[c/d + x]^2) - 3*d*(b*x - a*Log[a + b*x])*(-Log[a/b + x] + Log
[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])^2 + 6*(a*d + 2*b*d*x - b*d*x*Log
[c/d + x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(a + b*x)) + d*(a + b*x)*
Log[c/d + x] + (b*c - a*d)*Log[(b*(c + d*x))/(b*c - a*d)]) + (b*c - a*d...
```

Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 501, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2962, 2789, 2756, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{ag+bgx}\,dx$$

$$\downarrow \text{2962}$$

$$\frac{i^2(bc-ad)^2\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{g}$$

$$\downarrow \text{2789}$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\int\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\downarrow 2756$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\downarrow 2789$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{d\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{d}\right)}{b}+\frac{d\int\frac{\left(A+B\log\left(\frac{e(a}{c}\right.\right.}{\left(b-\frac{d(a+b}{c+d}\right.}}{g}$$

$$\downarrow 2751$$

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{B\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{d}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+b}{c+d}}{b}\right)}{b}\right)}{g}$$

$$\downarrow 16$$

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}\right)}{d}\right)}{b}\right)}{g}$$

$$\downarrow 2755$$

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}\right)}{d}\right)}{b}\right)}{g}$$

$$\downarrow 2754$$

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}\right)}{d}\right)}{b}\right)$$

$$\downarrow 2779$$

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\frac{B\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\right.\right.}{b(c+dx)\left(b-\right.}\right.}{}\right.}{d}\right.}{b}\right.$$

$\downarrow$ 2821

$$i^2(bc-ad)^2\left(\frac{\frac{\frac{2B\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-B\int\frac{(c+dx)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right.}{b}}{b}}{b}\right.$$

$\downarrow$ 2838

$$i^2(bc-ad)^2\left(\frac{\frac{\frac{2B\left(\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-B\int\frac{(c+dx)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right.}{b}}{b}}{b}\right.$$

$$\Big\downarrow \; 7143$$

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\frac{B\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right.}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}\right.}{b}\right.}{d}\right.}{b}\right.$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
,x]
```

output
```
((b*c - a*d)^2*i^2*((d*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(2*d*(b - (
d*(a + b*x))/(c + d*x))^2) - (B*((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(
c + d*x)]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*Log[b - (d*(a
 + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[(e*(a + b*x))/(c + d*x)])*
Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(d*
(a + b*x))])/b)/b))/d))/b + ((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c +
d*x)])^2)/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (2*B*(-(((A + B*Lo
g[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (B*
PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d))/b))/b + (-(((A + B*Log[(e*(a
+ b*x))/(c + d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (2*B*((A
+ B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))]
+ B*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/b)/b)/b))/g
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(dix+ci)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{bgx+ag}dx$$

input

```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

output

```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}dx$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algo
rithm="fricas")
```

output
```
integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x
^2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*
B*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)
))/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$

$$= \frac{i^2\left(\int \frac{A^2c^2}{a+bx}\,dx + \int \frac{A^2d^2x^2}{a+bx}\,dx + \int \frac{B^2c^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a+bx}\,dx + \int \frac{2ABc^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx + \int \frac{2A^2cdx}{a+bx}\,dx + \int \frac{B^2d}{}\right.}{}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g),x)
```

output
```
i**2*(Integral(A**2*c**2/(a + b*x), x) + Integral(A**2*d**2*x**2/(a + b*x)
, x) + Integral(B**2*c**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x
), x) + Integral(2*A*B*c**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x)
, x) + Integral(2*A**2*c*d*x/(a + b*x), x) + Integral(B**2*d**2*x**2*log(a
*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(2*A*B*d**2*x**
2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Integral(2*B**2*c*d
*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(4*A*B*
c*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algo
rithm="maxima")
```

output

```
2*A^2*c*d*i^2*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + 1/2*A^2*d^2*i^2*(2*a^2*
log(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A^2*c^2*i^2*log(b*g*x +
a*g)/(b*g) + 1/2*(B^2*b^2*d^2*i^2*x^2 + 2*(2*b^2*c*d*i^2 - a*b*d^2*i^2)*B^
2*x + 2*(b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*B^2*log(b*x + a))*log(
d*x + c)^2/(b^3*g) - integrate(-(B^2*b^3*c^3*i^2*log(e)^2 + 2*A*B*b^3*c^3*
i^2*log(e) + (B^2*b^3*d^3*i^2*log(e)^2 + 2*A*B*b^3*d^3*i^2*log(e))*x^3 + 3
*(B^2*b^3*c*d^2*i^2*log(e)^2 + 2*A*B*b^3*c*d^2*i^2*log(e))*x^2 + (B^2*b^3*
d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x + B^2*b^3*c^
3*i^2)*log(b*x + a)^2 + 3*(B^2*b^3*c^2*d*i^2*log(e)^2 + 2*A*B*b^3*c^2*d*i^
2*log(e))*x + 2*(B^2*b^3*c^3*i^2*log(e) + A*B*b^3*c^3*i^2 + (B^2*b^3*d^3*i
^2*log(e) + A*B*b^3*d^3*i^2)*x^3 + 3*(B^2*b^3*c*d^2*i^2*log(e) + A*B*b^3*c
*d^2*i^2)*x^2 + 3*(B^2*b^3*c^2*d*i^2*log(e) + A*B*b^3*c^2*d*i^2)*x)*log(b*
x + a) - (2*B^2*b^3*c^3*i^2*log(e) + 2*A*B*b^3*c^3*i^2 + (2*A*B*b^3*d^3*i^
2 + (2*i^2*log(e) + i^2)*B^2*b^3*d^3)*x^3 + (6*A*B*b^3*c*d^2*i^2 - (a*b^2*
d^3*i^2 - 2*(3*i^2*log(e) + 2*i^2)*b^3*c*d^2)*B^2)*x^2 + 2*(3*A*B*b^3*c^2*
d*i^2 + (3*b^3*c^2*d*i^2*log(e) + 2*a*b^2*c*d^2*i^2 - a^2*b*d^3*i^2)*B^2)*
x + 2*(B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + (4*b^3*c^2*d*i^2 -
2*a*b^2*c*d^2*i^2 + a^2*b*d^3*i^2)*B^2*x + (b^3*c^3*i^2 + a*b^2*c^2*d*i^2
- 2*a^2*b*c*d^2*i^2 + a^3*d^3*i^2)*B^2)*log(b*x + a))*log(d*x + c))/(b^4*d
*g*x^2 + a*b^3*c*g + (b^4*c*g + a*b^3*d*g)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algo
rithm="giac")
```

output

```
integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*
g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{-2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2}{bx+a}dx\right)b^5c^2 - 4\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{bx+a}dx\right)a\,b^4c^2 - 2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2x^2}{bx+a}dx\right)b^5d^2 - 4\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2}{bx+a}\right.}{}$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

output
```
( - 2*int(log((a*e + b*e*x)/(c + d*x))**2/(a + b*x),x)*b**5*c**2 - 4*int(l
og((a*e + b*e*x)/(c + d*x))/(a + b*x),x)*a*b**4*c**2 - 2*int((log((a*e + b
*e*x)/(c + d*x))**2*x**2)/(a + b*x),x)*b**5*d**2 - 4*int((log((a*e + b*e*x
)/(c + d*x))**2*x)/(a + b*x),x)*b**5*c*d - 4*int((log((a*e + b*e*x)/(c + d
*x))*x**2)/(a + b*x),x)*a*b**4*d**2 - 8*int((log((a*e + b*e*x)/(c + d*x))*
x)/(a + b*x),x)*a*b**4*c*d - 2*log(a + b*x)*a**4*d**2 + 4*log(a + b*x)*a**
3*b*c*d - 2*log(a + b*x)*a**2*b**2*c**2 + 2*a**3*b*d**2*x - 4*a**2*b**2*c*
d*x - a**2*b**2*d**2*x**2)/(2*b**3*g)
```

3.69 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result 865
Mathematica [B] (verified) 866
Rubi [A] (verified) 867
Maple [F] 869
Fricas [F] 869
Sympy [F(-1)] 869
Maxima [F] 870
Giac [F] 871
Mupad [F(-1)] 871
Reduce [F] 871

Optimal result

Integrand size = 42, antiderivative size = 442

$$
\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx \\
&= -\frac{2B^2(bc-ad)i^2(c+dx)}{b^2g^2(a+bx)} - \frac{2B(bc-ad)i^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^2(a+bx)} \\
&\quad + \frac{2Bd(bc-ad)i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2} \\
&\quad + \frac{d^2i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^2} \\
&\quad - \frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g^2(a+bx)} \\
&\quad - \frac{2d(bc-ad)i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&\quad + \frac{2B^2d(bc-ad)i^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g^2} \\
&\quad + \frac{4Bd(bc-ad)i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&\quad + \frac{4B^2d(bc-ad)i^2\,\mathrm{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}
\end{aligned}
$$

output

```
-2*B^2*(-a*d+b*c)*i^2*(d*x+c)/b^2/g^2/(b*x+a)-2*B*(-a*d+b*c)*i^2*(d*x+c)*(
A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g^2/(b*x+a)+2*B*d*(-a*d+b*c)*i^2*ln((-a*d+b
*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/g^2+d^2*i^2*(b*x+a)*(A+B*ln
(e*(b*x+a)/(d*x+c)))^2/b^3/g^2-(-a*d+b*c)*i^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d
*x+c)))^2/b^2/g^2/(b*x+a)-2*d*(-a*d+b*c)*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2
*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^2+2*B^2*d*(-a*d+b*c)*i^2*polylog(2,d*(b*x
+a)/b/(d*x+c))/b^3/g^2+4*B*d*(-a*d+b*c)*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*po
lylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g^2+4*B^2*d*(-a*d+b*c)*i^2*polylog(3,b*(d
*x+c)/d/(b*x+a))/b^3/g^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2649 vs. $2(442) = 884$.

Time $= 4.61$ (sec) , antiderivative size $= 2649$, normalized size of antiderivative $= 5.99$

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Result too large to show}$$

input

```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^2,x]
```

output

```
(i^2*(3*A^2*b*d^2*x - (3*A^2*(b*c - a*d)^2)/(a + b*x) + 6*A^2*d*(b*c - a*d
)*Log[a + b*x] - (6*A*b^2*B*c^2*(-(d*(a + b*x)*Log[c/d + x]) + d*(a + b*x)
*Log[(d*(a + b*x))/(-(b*c) + a*d)] + (b*c - a*d)*(1 + Log[(e*(a + b*x))/(c
 + d*x)])))/((b*c - a*d)*(a + b*x)) + (3*b^2*B^2*c^2*(-2*b*c + 2*a*d - 2*d
*(a + b*x)*Log[a + b*x] - 2*(b*c - a*d)*Log[(e*(a + b*x))/(c + d*x)] - 2*d
*(a + b*x)*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - (b*c - a*d)*Log[(e*
(a + b*x))/(c + d*x)]^2 + 2*d*(a + b*x)*Log[c + d*x] - 2*d*(a + b*x)*Log[(
e*(a + b*x))/(c + d*x)]*Log[(b*c - a*d)/(b*c + b*d*x)] + d*(a + b*x)*(Log[
a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2,
(d*(a + b*x))/(-(b*c) + a*d)]) + d*(a + b*x)*(Log[(b*c - a*d)/(b*c + b*d*x
)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c + b*d*x)])
- 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/((b*c - a*d)*(a + b*x)) + 6*A
*b*B*c*d*(Log[a/b + x]^2 - 2*Log[a/b + x]*Log[a + b*x] - 2*Log[c/d + x]*Lo
g[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[a + b*x]*((a*d)/(b*c - a*d) + Log[
c/d + x] + Log[(e*(a + b*x))/(c + d*x)]) + 2*a*((a + b*x)^(-1) + Log[(e*(a
 + b*x))/(c + d*x)]/(a + b*x) + (d*Log[c + d*x])/(-(b*c) + a*d)) - 2*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)]) + 6*A*B*d^2*((a + b*x)*(-1 + Log[a/b + x
]) - a*Log[a/b + x]^2 - (a^2*(1 + Log[a/b + x]))/(a + b*x) - b*(c/d + x)*(
-1 + Log[c/d + x]) + (a^2*Log[c/d + x])/(a + b*x) + (b*x - a^2/(a + b*x) -
 2*a*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c...
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^2}\,dx$$

$$\downarrow 2962$$

$$\frac{i^2(bc-ad)\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g^2}$$

$$\downarrow 2795$$

$$\frac{i^2(bc-ad)\int \left(\frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2(a+bx)^2}+\frac{2d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right)d\frac{a+bx}{c+dx}}{g^2}$$

$$\downarrow 2009$$

$$i^2(bc-ad)\left(\frac{d^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^3(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{4Bd\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3}+\frac{2Bd\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{b^3}$$

input

```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^2,x]
```

output
```
((b*c - a*d)*i^2*((-2*B^2*(c + d*x))/(b^2*(a + b*x)) - (2*B*(c + d*x)*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*(a + b*x)) - ((c + d*x)*(A + B*Log[
(e*(a + b*x))/(c + d*x)])^2)/(b^2*(a + b*x)) + (d^2*(a + b*x)*(A + B*Log[(
e*(a + b*x))/(c + d*x)])^2)/(b^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)))
 + (2*B*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c
 + d*x))])/b^3 - (2*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (b*(c
 + d*x))/(d*(a + b*x))])/b^3 + (2*B^2*d*PolyLog[2, (d*(a + b*x))/(b*(c + d*
x))])/b^3 + (4*B*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c +
 d*x))/(d*(a + b*x))])/b^3 + (4*B^2*d*PolyLog[3, (b*(c + d*x))/(d*(a + b*x
))])/b^3))/g^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int \frac{(dix+ci)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(bgx+ag)^2}dx$$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

output
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, al
gorithm="fricas")
```

output
```
integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x
^2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*
B*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)
))/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, al
gorithm="maxima")
```

output

```
-A^2*(a^2/(b^4*g^2*x + a*b^3*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^
2))*d^2*i^2 + 2*A^2*c*d*i^2*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2
*g^2)) - 2*A*B*c^2*i^2*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x +
a*b*g^2) + 1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2)
- d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - A^2*c^2*i^2/(b^2*g^2*x + a*b*g^2
) + (B^2*b^2*d^2*i^2*x^2 + B^2*a*b*d^2*i^2*x - (b^2*c^2*i^2 - 2*a*b*c*d*i^
2 + a^2*d^2*i^2)*B^2 + 2*((b^2*c*d*i^2 - a*b*d^2*i^2)*B^2*x + (a*b*c*d*i^2
 - a^2*d^2*i^2)*B^2)*log(b*x + a))*log(d*x + c)^2/(b^4*g^2*x + a*b^3*g^2)
- integrate(-(B^2*b^3*c^3*i^2*log(e)^2 + (B^2*b^3*d^3*i^2*log(e)^2 + 2*A*B
*b^3*d^3*i^2*log(e))*x^3 + 3*(B^2*b^3*c*d^2*i^2*log(e)^2 + 2*A*B*b^3*c*d^2
*i^2*log(e))*x^2 + (B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*
b^3*c^2*d*i^2*x + B^2*b^3*c^3*i^2)*log(b*x + a)^2 + (3*B^2*b^3*c^2*d*i^2*l
og(e)^2 + 4*A*B*b^3*c^2*d*i^2*log(e))*x + 2*(B^2*b^3*c^3*i^2*log(e) + (B^2
*b^3*d^3*i^2*log(e) + A*B*b^3*d^3*i^2)*x^3 + 3*(B^2*b^3*c*d^2*i^2*log(e) +
 A*B*b^3*c*d^2*i^2)*x^2 + (3*B^2*b^3*c^2*d*i^2*log(e) + 2*A*B*b^3*c^2*d*i^
2)*x)*log(b*x + a) - 2*((A*B*b^3*d^3*i^2 + (i^2*log(e) + i^2)*B^2*b^3*d^3)
*x^3 + (b^3*c^3*i^2*log(e) - a*b^2*c^2*d*i^2 + 2*a^2*b*c*d^2*i^2 - a^3*d^3
*i^2)*B^2 + (3*A*B*b^3*c*d^2*i^2 + (3*b^3*c*d^2*i^2*log(e) + 2*a*b^2*d^3*i
^2)*B^2)*x^2 + (2*A*B*b^3*c^2*d*i^2 + (2*a*b^2*c*d^2*i^2 + (3*i^2*log(e) -
 i^2)*b^3*c^2*d)*B^2)*x + (B^2*b^3*d^3*i^2*x^3 + (5*b^3*c*d^2*i^2 - 2*a...
```

Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, al
gorithm="giac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*
g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(a\,g+b\,g\,x)^2}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^2,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^2, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

output

```
( - int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(a**2 + 2*a*b*x + b**2*x**2
),x)*a**3*b**5*d**3 + int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(a**2 + 2
*a*b*x + b**2*x**2),x)*a**2*b**6*c*d**2 - int((log((a*e + b*e*x)/(c + d*x)
)**2*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**6*d**3*x + int((log((a*
e + b*e*x)/(c + d*x))**2*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a*b**7*c*d*
*2*x - 2*int((log((a*e + b*e*x)/(c + d*x))**2*x)/(a**2 + 2*a*b*x + b**2*x*
*2),x)*a**3*b**5*c*d**2 + 2*int((log((a*e + b*e*x)/(c + d*x))**2*x)/(a**2
+ 2*a*b*x + b**2*x**2),x)*a**2*b**6*c**2*d - 2*int((log((a*e + b*e*x)/(c +
 d*x))**2*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**6*c*d**2*x + 2*int((l
og((a*e + b*e*x)/(c + d*x))**2*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a*b**7*c
**2*d*x - 2*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(a**2 + 2*a*b*x + b**2
*x**2),x)*a**4*b**4*d**3 + 2*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(a**2
 + 2*a*b*x + b**2*x**2),x)*a**3*b**5*c*d**2 - 2*int((log((a*e + b*e*x)/(c
+ d*x))*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**3*b**5*d**3*x + 2*int((lo
g((a*e + b*e*x)/(c + d*x))*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**6
*c*d**2*x - 4*int((log((a*e + b*e*x)/(c + d*x))*x)/(a**2 + 2*a*b*x + b**2*
x**2),x)*a**4*b**4*c*d**2 + 4*int((log((a*e + b*e*x)/(c + d*x))*x)/(a**2 +
 2*a*b*x + b**2*x**2),x)*a**3*b**5*c**2*d - 4*int((log((a*e + b*e*x)/(c +
d*x))*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**3*b**5*c*d**2*x + 4*int((log((
a*e + b*e*x)/(c + d*x))*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**6*c*...
```

3.70 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result . . . 873
Mathematica [B] (verified) . . . 874
Rubi [A] (verified) . . . 875
Maple [F] . . . 880
Fricas [F] . . . 881
Sympy [F] . . . 881
Maxima [F] . . . 882
Giac [F] . . . 883
Mupad [F(-1)] . . . 884
Reduce [F] . . . 884

Optimal result

Integrand size = 42, antiderivative size = 387

$$\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx \\
&= -\frac{2B^2di^2(c+dx)}{b^2g^3(a+bx)} - \frac{B^2i^2(c+dx)^2}{4bg^3(a+bx)^2} - \frac{2Bdi^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^3(a+bx)} \\
&\quad - \frac{Bi^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2} \\
&\quad - \frac{di^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g^3(a+bx)} - \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a+bx)^2} \\
&\quad - \frac{d^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} \\
&\quad + \frac{2Bd^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} \\
&\quad + \frac{2B^2d^2i^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}
\end{aligned}$$

output

```
-2*B^2*d*i^2*(d*x+c)/b^2/g^3/(b*x+a)-1/4*B^2*i^2*(d*x+c)^2/b/g^3/(b*x+a)^2
-2*B*d*i^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g^3/(b*x+a)-1/2*B*i^2*(
d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g^3/(b*x+a)^2-d*i^2*(d*x+c)*(A+B*ln
(e*(b*x+a)/(d*x+c)))^2/b^2/g^3/(b*x+a)-1/2*i^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)
/(d*x+c)))^2/b/g^3/(b*x+a)^2-d^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*
(d*x+c)/d/(b*x+a))/b^3/g^3+2*B*d^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog
(2,b*(d*x+c)/d/(b*x+a))/b^3/g^3+2*B^2*d^2*i^2*polylog(3,b*(d*x+c)/d/(b*x+a
))/b^3/g^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3426 vs. $2(387) = 774$.

Time $= 7.38$ (sec) , antiderivative size $= 3426$, normalized size of antiderivative $= 8.85$

$$\int \frac{(ci + dix)^2 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^3}\, dx = \text{Result too large to show}$$

input

```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^3,x]
```

output

```
(i^2*((-6*A^2*(b*c - a*d)^2)/(a + b*x)^2 + (24*A^2*d*(-(b*c) + a*d))/(a +
b*x) + 12*A^2*d^2*Log[a + b*x] - (6*A*b^2*B*c^2*(b^2*c^2 - 4*a*b*c*d + a^2
*d^2 - 2*b^2*c*d*x - 2*a*b*d^2*x - 2*b^2*d^2*x^2 + 2*d^2*(a + b*x)^2*Log[c
/d + x] - 2*d^2*(a + b*x)^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*b^2*c^2*
Log[(e*(a + b*x))/(c + d*x)] - 4*a*b*c*d*Log[(e*(a + b*x))/(c + d*x)] + 2*
a^2*d^2*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^2*(a + b*x)^2) - (12*A
*b*B*c*d*(3*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2 + 4*b^3*c^2*x - 6*a*b^2*c*d*
x + 2*a^2*b*d^2*x - 2*d*(-2*b*c + a*d)*(a + b*x)^2*Log[a + b*x] + 2*(b*c -
 a*d)^2*(a + 2*b*x)*Log[(e*(a + b*x))/(c + d*x)] - 4*a^2*b*c*d*Log[c + d*x
] + 2*a^3*d^2*Log[c + d*x] - 8*a*b^2*c*d*x*Log[c + d*x] + 4*a^2*b*d^2*x*Lo
g[c + d*x] - 4*b^3*c*d*x^2*Log[c + d*x] + 2*a*b^2*d^2*x^2*Log[c + d*x]))/(
(b*c - a*d)^2*(a + b*x)^2) + (3*b^2*B^2*c^2*(-(b*c - a*d)^2 + 6*d*(b*c - a
*d)*(a + b*x) + 6*d^2*(a + b*x)^2*Log[a + b*x] - 2*d^2*(a + b*x)^2*Log[a +
 b*x]^2 - 2*(b*c - a*d)^2*Log[(e*(a + b*x))/(c + d*x)] + 4*d*(b*c - a*d)*(
a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 4*d^2*(a + b*x)^2*Log[a + b*x]*Log
[(e*(a + b*x))/(c + d*x)] - 2*(b*c - a*d)^2*Log[(e*(a + b*x))/(c + d*x)]^2
 - 6*d^2*(a + b*x)^2*Log[c + d*x] + 4*d^2*(a + b*x)^2*Log[a + b*x]*Log[(b*
(c + d*x))/(b*c - a*d)] - 4*d^2*(a + b*x)^2*Log[(d*(a + b*x))/(-(b*c) + a*
d)]*Log[(b*c - a*d)/(b*c + b*d*x)] + 4*d^2*(a + b*x)^2*Log[(e*(a + b*x))/(
c + d*x)]*Log[(b*c - a*d)/(b*c + b*d*x)] - 2*d^2*(a + b*x)^2*Log[(b*c -...
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2962, 2780, 2742, 2741, 2780, 2742, 2741, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{i^2 \int \frac{(c+dx)^3 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)} d\frac{a+bx}{c+dx}}{g^3}$$

$$\downarrow 2780$$

$$\frac{i^2\left(\frac{\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^3}$$

$$\downarrow 2742$$

$$\frac{i^2\left(\frac{B\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}}{b}+\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^3}$$

$$\downarrow 2741$$

$$\frac{i^2\left(\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{B\left(-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}\right)-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}}{b}\right)}{g^3}$$

$$\downarrow 2780$$

$$\frac{i^2\left(\frac{d\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{B\left(-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}\right)}{b}\right)}{g^3}$$

$$\downarrow 2742$$

$$i^2\left(\frac{d\left(\frac{2B\int\frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}+\frac{d\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{B\left(-\frac{(c+dx)^2\left(\right.\right.}{}\right.$$

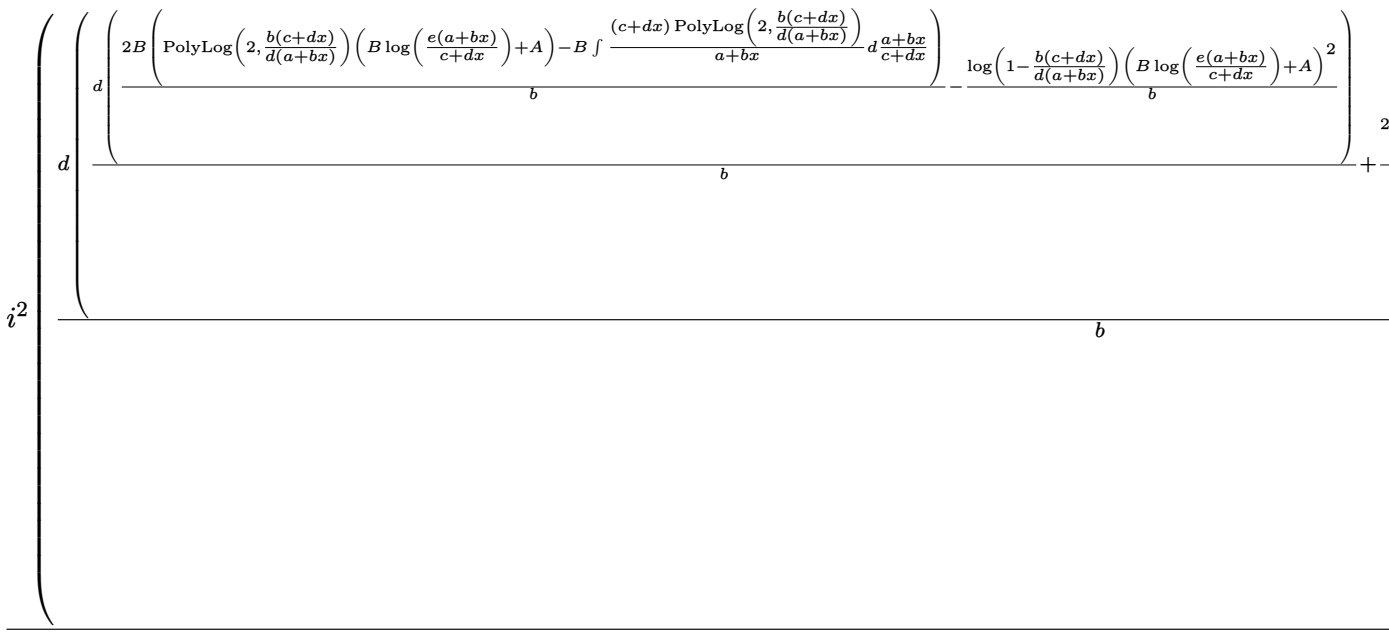
$$\Big\downarrow 2741$$

$$i^2\left(\frac{d\left(\frac{d\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}+\frac{2B\left(-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}}{b}\right)}{b}+\frac{B\left(-\frac{(c+}{}\right.$$

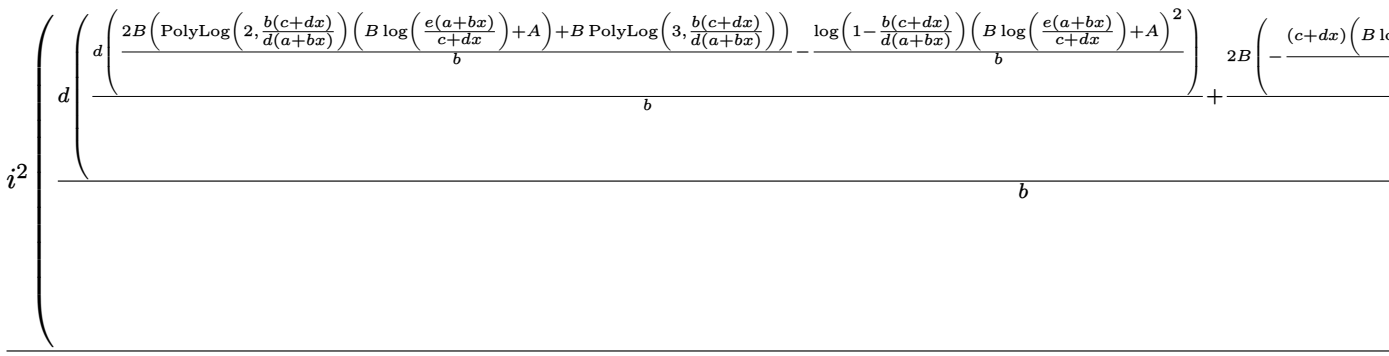
$$\Big\downarrow 2779$$

$$i^2\left(\frac{d\left(\frac{d\left(\frac{2B\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}\right)}{b}+\frac{2B\left(-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.\right.}{a+bx}\right.$$

$$\Big\downarrow 2821$$



↓ 7143



input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^3,x]
```

output
```
(i^2*((-1/2*((c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x)
^2 + B*(-1/4*(B*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]))/(2*(a + b*x)^2)))/b + (d*((-(((c + d*x)*(A + B*Log[(e*
(a + b*x))/(c + d*x)])^2)/(a + b*x)) + 2*B*(-((B*(c + d*x))/(a + b*x)) - (
(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)))/b + (d*(-(((A
+ B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/
b) + (2*B*((A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c + d*x))/(
d*(a + b*x))] + B*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/b))/b))/b))/g^
3
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2821

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)  Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(dix+ci)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(bgx+ag)^3}dx$$

input

```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x)
```

output

```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, al
gorithm="fricas")
```

output
```
integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x
^2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*
B*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)
))/(b^3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{i^2\left(\int \frac{A^2c^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{A^2d^2x^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{B^2c^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{2ABc^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+d}\right.}{a^3+3a^2bx+3ab^2x^2+b^3}\right.}{}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3,x)
```

output

```
i**2*(Integral(A**2*c**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),
x) + Integral(A**2*d**2*x**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**
3), x) + Integral(B**2*c**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a**3
+ 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*c**2*log(a*
e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x
**3), x) + Integral(2*A**2*c*d*x/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3
*x**3), x) + Integral(B**2*d**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))*
*2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*d*
*2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2
*x**2 + b**3*x**3), x) + Integral(2*B**2*c*d*x*log(a*e/(c + d*x) + b*e*x/(
c + d*x))**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integra
l(4*A*B*c*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*
a*b**2*x**2 + b**3*x**3), x))/g**3
```

Maxima [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, al
gorithm="maxima")
```

output

```
-A*B*c*d*i^2*(2*(2*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g^3*
x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + (3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d
)*x)/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3
*c - a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^
3*c*d + a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2
*a*b^3*c*d + a^2*b^2*d^2)*g^3)) + 1/2*A^2*d^2*i^2*((4*a*b*x + 3*a^2)/(b^5*
g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3)) + 1/2*A
*B*c^2*i^2*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*
c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c)
+ a*e/(d*x + c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x
 + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3
*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - (2*b*x + a)*A^2*c*d*i^2/(b^4*g^3*x
^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - 1/2*A^2*c^2*i^2/(b^3*g^3*x^2 + 2*a*b^2
*g^3*x + a^2*b*g^3) - 1/2*(4*(b^2*c*d*i^2 - a*b*d^2*i^2)*B^2*x + (b^2*c^2*
i^2 + 2*a*b*c*d*i^2 - 3*a^2*d^2*i^2)*B^2 - 2*(B^2*b^2*d^2*i^2*x^2 + 2*B^2*
a*b*d^2*i^2*x + B^2*a^2*d^2*i^2)*log(b*x + a))*log(d*x + c)^2/(b^5*g^3*x^2
 + 2*a*b^4*g^3*x + a^2*b^3*g^3) - integrate(-(3*B^2*b^3*c^2*d*i^2*x*log(e)
^2 + B^2*b^3*c^3*i^2*log(e)^2 + (B^2*b^3*d^3*i^2*log(e)^2 + 2*A*B*b^3*d^3*
i^2*log(e))*x^3 + (3*B^2*b^3*c*d^2*i^2*log(e)^2 + 2*A*B*b^3*c*d^2*i^2*log(
e))*x^2 + (B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^...
```

Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, al
gorithm="giac")
```

output

```
integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*
g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(a\,g+b\,g\,x)^3}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^3,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^3, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \text{too large to display}$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x)
```

output

```
( - 4*int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(a**3 + 3*a**2*b*x + 3*a*
b**2*x**2 + b**3*x**3),x)*a**5*b**5*d**4 + 8*int((log((a*e + b*e*x)/(c + d
*x))**2*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**4*b**6
*c*d**3 - 8*int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(a**3 + 3*a**2*b*x
+ 3*a*b**2*x**2 + b**3*x**3),x)*a**4*b**6*d**4*x - 4*int((log((a*e + b*e*x
)/(c + d*x))**2*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a
**3*b**7*c**2*d**2 + 16*int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(a**3 +
 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**3*b**7*c*d**3*x - 4*int((lo
g((a*e + b*e*x)/(c + d*x))**2*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b
**3*x**3),x)*a**3*b**7*d**4*x**2 - 8*int((log((a*e + b*e*x)/(c + d*x))**2*
x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**2*b**8*c**2*d*
*2*x + 8*int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(a**3 + 3*a**2*b*x + 3
*a*b**2*x**2 + b**3*x**3),x)*a**2*b**8*c*d**3*x**2 - 4*int((log((a*e + b*e
*x)/(c + d*x))**2*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)
*a*b**9*c**2*d**2*x**2 - 8*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(a**3 +
 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**6*b**4*d**4 + 16*int((log((
a*e + b*e*x)/(c + d*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x*
*3),x)*a**5*b**5*c*d**3 - 16*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(a**3
 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**5*b**5*d**4*x - 8*int((lo
g((a*e + b*e*x)/(c + d*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b...
```

3.71 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result 886
Mathematica [C] (verified) 887
Rubi [A] (verified) 888
Maple [B] (verified) 889
Fricas [B] (verification not implemented) 891
Sympy [B] (verification not implemented) 891
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Giac [A] (verification not implemented) 893
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Reduce [B] (verification not implemented) 895

Optimal result

Integrand size = 42, antiderivative size = 147

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{2B^2i^2(c+dx)^3}{27(bc-ad)g^4(a+bx)^3} - \frac{2Bi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)g^4(a+bx)^3} - \frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)g^4(a+bx)^3}$$

output

```
-2/27*B^2*i^2*(d*x+c)^3/(-a*d+b*c)/g^4/(b*x+a)^3-2/9*B*i^2*(d*x+c)^3*(A+B*
ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^4/(b*x+a)^3-1/3*i^2*(d*x+c)^3*(A+B*ln(
e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)/g^4/(b*x+a)^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.03 (sec) , antiderivative size = 1352, normalized size of antiderivative = 9.20

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input

```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^4,x]
```

output

```
-1/54*(i^2*(18*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 54*d
*(b*c - a*d)^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 54*d^2*(
-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + B*(12*A
*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3 - 18*A*d*(b*c - a*d)^2*(a + b*x) - 15*B
*d*(b*c - a*d)^2*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(
b*c - a*d)*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*(a +
 b*x)^3*Log[a + b*x] - 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a
*d)^3*Log[(e*(a + b*x))/(c + d*x)] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e
*(a + b*x))/(c + d*x)] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[(e*(a + b*x)
)/(c + d*x)] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*
x)] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*(a + b*x)^3*Log[c + d*x
] + 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] -
36*B*d^3*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] - 18*B*d^3*
(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(b*(c +
 d*x))/(b*c - a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c
) + a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) +
54*B*d^2*(a + b*x)^2*(2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) +
 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*d*(a
+ b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*B*(b*c - a*d
+ d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*d*(a + b*x)*...
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^4}\,dx$$

$$\downarrow 2962$$

$$\frac{i^2\int \frac{(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)}$$

$$\downarrow 2742$$

$$\frac{i^2\left(\frac{2}{3}B\int \frac{(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3}\right)}{g^4(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{i^2\left(\frac{2}{3}B\left(-\frac{(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}-\frac{B(c+dx)^3}{9(a+bx)^3}\right)-\frac{(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3}\right)}{g^4(bc-ad)}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^4,x]
```

output
```
(i^2*(-1/3*((c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x)^
3 + (2*B*(-1/9*(B*(c + d*x)^3)/(a + b*x)^3 - ((c + d*x)^3*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/(3*(a + b*x)^3)))/3))/((b*c - a*d)*g^4)
```

Defintions of rubi rules used

rule 2741

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(141) = 282$.

Time = 1.72 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.50

method	result
derivativedivides	$-\frac{e(da-bc)\left(-\frac{i^2d^2e^2A^2}{3(da-bc)^2g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}+\frac{2i^2d^2e^2AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}\right)}{(da-bc)^2g^4}+\frac{i^2d^2e^2B^2\left(-}{}\right.}{d^2}$
default	$-\frac{e(da-bc)\left(-\frac{i^2d^2e^2A^2}{3(da-bc)^2g^4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}+\frac{2i^2d^2e^2AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}\right)}{(da-bc)^2g^4}+\frac{i^2d^2e^2B^2\left(-}{}\right.}{d^2}$
parts	$\frac{i^2A^2\left(\frac{d(da-bc)}{b^3(bx+a)^2}-\frac{d^2}{b^3(bx+a)}-\frac{a^2d^2-2acdb+c^2b^2}{3b^3(bx+a)^3}\right)}{g^4}-\frac{i^2B^2e^3\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{9\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{2}{27\left(\frac{be}{d}+\frac{(d}{d(}\right.}{g^4(da-bc)}$
risch	$\frac{i^2A^2d^2a}{g^4b^3(bx+a)^2}-\frac{i^2A^2dc}{g^4b^2(bx+a)^2}-\frac{i^2A^2d^2}{g^4b^3(bx+a)}-\frac{i^2A^2a^2d^2}{3g^4b^3(bx+a)^3}+\frac{2i^2A^2acd}{3g^4b^2(bx+a)^3}-\frac{i^2A^2c^2}{3g^4b(bx+a)^3}+\frac{i^2B^2e}{3g^4(da-b}$
norman	$\frac{B^2cd^2i^2x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(da-bc)}+\frac{B^2c^2di^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(da-bc)}-\frac{9A^2a^2d^2i^2+9A^2abcdi^2+9A^2b^2c^2i^2+6ABa^2d^2i^2+6ABabcdi^2+6ABi}{27gb^3}$
parallelrisch	$-\frac{27A^2x^2ab^4d^4i^2-27A^2x^2b^5cd^3i^2+6B^2x^2ab^4d^4i^2-6B^2x^2b^5cd^3i^2-9B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b^5c^3di^2+27A^2xa^2b^3d^4i^2-}{}$
orering	Expression too large to display

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x,method=_RE
TURNVERBOSE)
```

output
```
-1/d^2*e*(a*d-b*c)*(-1/3*i^2*d^2*e^2/(a*d-b*c)^2/g^4*A^2/(b*e/d+(a*d-b*c)*
e/d/(d*x+c))^3+2*i^2*d^2*e^2/(a*d-b*c)^2/g^4*A*B*(-1/3/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^3)+i^2*d^2*e^2/(a*d-b*c)^2/g^4*B^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(141) = 282$.

Time $= 0.09$ (sec) , antiderivative size $= 444$, normalized size of antiderivative $= 3.02$

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{3\left((9A^2+6AB+2B^2)b^3cd^2-(9A^2+6AB+2B^2)ab^2d^3\right)i^2x^2+3\left((9A^2+6AB+2B^2)b^3c^2d-(9\right.}{}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, al
gorithm="fricas")
```

output
```
-1/27*(3*((9*A^2 + 6*A*B + 2*B^2)*b^3*c*d^2 - (9*A^2 + 6*A*B + 2*B^2)*a*b^
2*d^3)*i^2*x^2 + 3*((9*A^2 + 6*A*B + 2*B^2)*b^3*c^2*d - (9*A^2 + 6*A*B + 2
*B^2)*a^2*b*d^3)*i^2*x + ((9*A^2 + 6*A*B + 2*B^2)*b^3*c^3 - (9*A^2 + 6*A*B
 + 2*B^2)*a^3*d^3)*i^2 + 9*(B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2
+ 3*B^2*b^3*c^2*d*i^2*x + B^2*b^3*c^3*i^2)*log((b*e*x + a*e)/(d*x + c))^2
+ 6*((3*A*B + B^2)*b^3*d^3*i^2*x^3 + 3*(3*A*B + B^2)*b^3*c*d^2*i^2*x^2 + 3
*(3*A*B + B^2)*b^3*c^2*d*i^2*x + (3*A*B + B^2)*b^3*c^3*i^2)*log((b*e*x + a
*e)/(d*x + c)))/((b^7*c - a*b^6*d)*g^4*x^3 + 3*(a*b^6*c - a^2*b^5*d)*g^4*x
^2 + 3*(a^2*b^5*c - a^3*b^4*d)*g^4*x + (a^3*b^4*c - a^4*b^3*d)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1182 vs. $2(131) = 262$.

Time $= 27.70$ (sec) , antiderivative size $= 1182$, normalized size of antiderivative $= 8.04$

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4,x)
```

output

```
-2*B*d**3*i**2*(3*A + B)*log(x + (6*A*B*a*d**4*i**2 + 6*A*B*b*c*d**3*i**2
+ 2*B**2*a*d**4*i**2 + 2*B**2*b*c*d**3*i**2 - 2*B*a**2*d**5*i**2*(3*A + B)
/(a*d - b*c) + 4*B*a*b*c*d**4*i**2*(3*A + B)/(a*d - b*c) - 2*B*b**2*c**2*d
**3*i**2*(3*A + B)/(a*d - b*c))/(12*A*B*b*d**4*i**2 + 4*B**2*b*d**4*i**2))
/(9*b**3*g**4*(a*d - b*c)) + 2*B*d**3*i**2*(3*A + B)*log(x + (6*A*B*a*d**4
*i**2 + 6*A*B*b*c*d**3*i**2 + 2*B**2*a*d**4*i**2 + 2*B**2*b*c*d**3*i**2 +
2*B*a**2*d**5*i**2*(3*A + B)/(a*d - b*c) - 4*B*a*b*c*d**4*i**2*(3*A + B)/(
a*d - b*c) + 2*B*b**2*c**2*d**3*i**2*(3*A + B)/(a*d - b*c))/(12*A*B*b*d**4
*i**2 + 4*B**2*b*d**4*i**2))/(9*b**3*g**4*(a*d - b*c)) + (B**2*c**3*i**2 +
 3*B**2*c**2*d*i**2*x + 3*B**2*c*d**2*i**2*x**2 + B**2*d**3*i**2*x**3)*log
(e*(a + b*x)/(c + d*x))**2/(3*a**4*d*g**4 - 3*a**3*b*c*g**4 + 9*a**3*b*d*g
**4*x - 9*a**2*b**2*c*g**4*x + 9*a**2*b**2*d*g**4*x**2 - 9*a*b**3*c*g**4*x
**2 + 3*a*b**3*d*g**4*x**3 - 3*b**4*c*g**4*x**3) + (-9*A**2*a**2*d**2*i**2
 - 9*A**2*a*b*c*d*i**2 - 9*A**2*b**2*c**2*i**2 - 6*A*B*a**2*d**2*i**2 - 6*
A*B*a*b*c*d*i**2 - 6*A*B*b**2*c**2*i**2 - 2*B**2*a**2*d**2*i**2 - 2*B**2*a
*b*c*d*i**2 - 2*B**2*b**2*c**2*i**2 + x**2*(-27*A**2*b**2*d**2*i**2 - 18*A
*B*b**2*d**2*i**2 - 6*B**2*b**2*d**2*i**2) + x*(-27*A**2*a*b*d**2*i**2 - 2
7*A**2*b**2*c*d*i**2 - 18*A*B*a*b*d**2*i**2 - 18*A*B*b**2*c*d*i**2 - 6*B**
2*a*b*d**2*i**2 - 6*B**2*b**2*c*d*i**2))/(27*a**3*b**3*g**4 + 81*a**2*b**4
*g**4*x + 81*a*b**5*g**4*x**2 + 27*b**6*g**4*x**3) + (-6*A*B*a**2*d**2*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5532 vs. $2(141) = 282$.

Time $= 0.41$ (sec) , antiderivative size $= 5532$, normalized size of antiderivative $= 37.63$

$$\int \frac{(ci + dix)^2 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^4}\, dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, al
gorithm="maxima")
```

output

```
-1/3*(3*b*x + a)*B^2*c*d*i^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^5*g
^4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/3*(3*b^2*x^2
 + 3*a*b*x + a^2)*B^2*d^2*i^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^6*
g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3*g^4) - 1/54*(6*((6*b
^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*
x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b
^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d
^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x
 + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^
3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^
4))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (4*b^3*c^3 - 27*a*b^2*c^2*d + 1
08*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3
*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*(b^3
*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 - 3*(
5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b^2
*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*
a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3
*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*g
^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*
g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + ...
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.49

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx =$$
$$-\frac{1}{27}\left(\frac{9\,(dx+c)^3B^2e^4i^2\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)^3g^4}+\frac{6\,(3\,ABe^4i^2+B^2e^4i^2)(dx+c)^3\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^3g^4}+\frac{(9\,A^2e^4i^2+6\,A\right.$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, al
gorithm="giac")
```

output
```
-1/27*(9*(d*x + c)^3*B^2*e^4*i^2*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x +
a*e)^3*g^4) + 6*(3*A*B*e^4*i^2 + B^2*e^4*i^2)*(d*x + c)^3*log((b*e*x + a*e
)/(d*x + c))/((b*e*x + a*e)^3*g^4) + (9*A^2*e^4*i^2 + 6*A*B*e^4*i^2 + 2*B^
2*e^4*i^2)*(d*x + c)^3/((b*e*x + a*e)^3*g^4))*(b*c/((b*c*e - a*d*e)*(b*c -
 a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 29.44 (sec) , antiderivative size = 1153, normalized size of antiderivative = 7.84

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^4,x)
```

output
```
- (x^2*(9*A^2*b^2*d^2*i^2 + 2*B^2*b^2*d^2*i^2 + 6*A*B*b^2*d^2*i^2) + x*(9*
A^2*a*b*d^2*i^2 + 2*B^2*a*b*d^2*i^2 + 9*A^2*b^2*c*d*i^2 + 2*B^2*b^2*c*d*i^
2 + 6*A*B*a*b*d^2*i^2 + 6*A*B*b^2*c*d*i^2) + 3*A^2*a^2*d^2*i^2 + 3*A^2*b^2
*c^2*i^2 + (2*B^2*a^2*d^2*i^2)/3 + (2*B^2*b^2*c^2*i^2)/3 + 2*A*B*a^2*d^2*i
^2 + 2*A*B*b^2*c^2*i^2 + 3*A^2*a*b*c*d*i^2 + (2*B^2*a*b*c*d*i^2)/3 + 2*A*B
*a*b*c*d*i^2)/(9*a^3*b^3*g^4 + 9*b^6*g^4*x^3 + 27*a^2*b^4*g^4*x + 27*a*b^5
*g^4*x^2) - log((e*(a + b*x))/(c + d*x))^2*((x*(b*((B^2*c*d*i^2)/(3*b^3*g^
4) + (B^2*a*d^2*i^2)/(3*b^4*g^4)) + (2*B^2*c*d*i^2)/(3*b^2*g^4) + (2*B^2*a
*d^2*i^2)/(3*b^3*g^4)) + a*((B^2*c*d*i^2)/(3*b^3*g^4) + (B^2*a*d^2*i^2)/(3
*b^4*g^4)) + (B^2*c^2*i^2)/(3*b^2*g^4) + (B^2*d^2*i^2*x^2)/(b^2*g^4))/(3*a
^2*x + a^3/b + b^2*x^3 + 3*a*b*x^2) - (B^2*d^3*i^2)/(3*b^3*g^4*(a*d - b*c)
)) - (log((e*(a + b*x))/(c + d*x))*(x*(b*((B*i^2*(2*A*b*c - B*a*d + B*b*c)
)/(3*b^4*g^4) + (2*A*B*a*d*i^2)/(3*b^4*g^4)) + (2*B*i^2*(2*A*b*c - B*a*d +
 B*b*c))/(3*b^3*g^4) + (2*B^2*d^3*i^2*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d
)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^2*d^2 + b^2*c^2 - 4*a*b*c*
d)/(3*d^3) + (2*a*(a*d - b*c))/(3*d^2)))/(3*b^3*g^4*(a*d - b*c)) + (4*A*B*
a*d*i^2)/(3*b^3*g^4)) + x^2*((2*A*B*d*i^2)/(b^2*g^4) - (2*B^2*d^3*i^2*((b^
2*c - a*b*d)/(3*d^2) - (2*b*(a*d - b*c))/(3*d^2)))/(3*b^3*g^4*(a*d - b*c))
) + a*((B*i^2*(2*A*b*c - B*a*d + B*b*c))/(3*b^4*g^4) + (2*A*B*a*d*i^2)/(3*
b^4*g^4)) + (2*B*i^2*(A*b^2*c^2 - B*a^2*d^2 + B*a*b*c*d))/(3*b^4*d*g^4)...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1067, normalized size of antiderivative = 7.26

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x)
```

output

```
( - 18*log(a + b*x)*a**4*b*c*d**2 - 54*log(a + b*x)*a**3*b**2*c*d**2*x - 6
*log(a + b*x)*a**3*b**2*c*d**2 - 54*log(a + b*x)*a**2*b**3*c*d**2*x**2 - 1
8*log(a + b*x)*a**2*b**3*c*d**2*x - 18*log(a + b*x)*a*b**4*c*d**2*x**3 - 1
8*log(a + b*x)*a*b**4*c*d**2*x**2 - 6*log(a + b*x)*b**5*c*d**2*x**3 + 18*l
og(c + d*x)*a**4*b*c*d**2 + 54*log(c + d*x)*a**3*b**2*c*d**2*x + 6*log(c +
 d*x)*a**3*b**2*c*d**2 + 54*log(c + d*x)*a**2*b**3*c*d**2*x**2 + 18*log(c
+ d*x)*a**2*b**3*c*d**2*x + 18*log(c + d*x)*a*b**4*c*d**2*x**3 + 18*log(c
+ d*x)*a*b**4*c*d**2*x**2 + 6*log(c + d*x)*b**5*c*d**2*x**3 - 9*log((a*e +
 b*e*x)/(c + d*x))**2*a*b**4*c**3 - 27*log((a*e + b*e*x)/(c + d*x))**2*a*b
**4*c**2*d*x - 27*log((a*e + b*e*x)/(c + d*x))**2*a*b**4*c*d**2*x**2 - 9*l
og((a*e + b*e*x)/(c + d*x))**2*a*b**4*d**3*x**3 + 18*log((a*e + b*e*x)/(c
+ d*x))*a**4*b*c*d**2 + 54*log((a*e + b*e*x)/(c + d*x))*a**3*b**2*c*d**2*x
 + 6*log((a*e + b*e*x)/(c + d*x))*a**3*b**2*c*d**2 - 18*log((a*e + b*e*x)/
(c + d*x))*a**2*b**3*c**3 - 54*log((a*e + b*e*x)/(c + d*x))*a**2*b**3*c**2
*d*x + 18*log((a*e + b*e*x)/(c + d*x))*a**2*b**3*c*d**2*x - 18*log((a*e +
b*e*x)/(c + d*x))*a**2*b**3*d**3*x**3 - 6*log((a*e + b*e*x)/(c + d*x))*a*b
**4*c**3 - 18*log((a*e + b*e*x)/(c + d*x))*a*b**4*c**2*d*x + 18*log((a*e +
 b*e*x)/(c + d*x))*a*b**4*c*d**2*x**3 - 6*log((a*e + b*e*x)/(c + d*x))*a*b
**4*d**3*x**3 + 6*log((a*e + b*e*x)/(c + d*x))*b**5*c*d**2*x**3 + 9*a**5*c
*d**2 + 27*a**4*b*c*d**2*x + 6*a**4*b*c*d**2 - 9*a**3*b**2*c**3 - 27*a*...
```

3.72 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result 896
Mathematica [C] (verified) 897
Rubi [A] (verified) 898
Maple [B] (verified) 900
Fricas [B] (verification not implemented) 901
Sympy [B] (verification not implemented) 902
Maxima [B] (verification not implemented) 903
Giac [A] (verification not implemented) 904
Mupad [B] (verification not implemented) 905
Reduce [B] (verification not implemented) 906

Optimal result

Integrand size = 42, antiderivative size = 299

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx &= \frac{2B^2di^2(c+dx)^3}{27(bc-ad)^2g^5(a+bx)^3}\\ &\quad-\frac{bB^2i^2(c+dx)^4}{32(bc-ad)^2g^5(a+bx)^4}\\ &\quad+\frac{2Bdi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^2g^5(a+bx)^3}\\ &\quad-\frac{bBi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^2g^5(a+bx)^4}\\ &\quad+\frac{di^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^2g^5(a+bx)^3}\\ &\quad-\frac{bi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^2g^5(a+bx)^4}\end{aligned}$$

output

```
2/27*B^2*d*i^2*(d*x+c)^3/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/32*b*B^2*i^2*(d*x+c)
^4/(-a*d+b*c)^2/g^5/(b*x+a)^4+2/9*B*d*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x
+c)))/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/8*b*B*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(
d*x+c)))/(-a*d+b*c)^2/g^5/(b*x+a)^4+1/3*d*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/
(d*x+c)))^2/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/4*b*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+
a)/(d*x+c)))^2/(-a*d+b*c)^2/g^5/(b*x+a)^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 3.01 (sec) , antiderivative size = 1703, normalized size of antiderivative = 5.70

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^5,x]
```

output

```
-1/864*(i^2*(216*(b*c - a*d)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 57
6*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 43
2*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 3
2*B*d*(a + b*x)*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3 - 18*A*d*(b*c - a*
d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a
+ b*x)^2 + 66*B*d^2*(b*c - a*d)*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a +
 b*x] + 66*B*d^3*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*(a + b*x)^3*Log[a + b
*x]^2 + 12*B*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)] - 18*B*d*(b*c - a*
d)^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^2*(b*c - a*d)*(a + b*
x)^2*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[
(e*(a + b*x))/(c + d*x)] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*(a
 + b*x)^3*Log[c + d*x] + 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) +
a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)]*Log
[c + d*x] - 18*B*d^3*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log
[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2,
 (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d
*x))/(b*c - a*d)]) + 3*B*(36*A*(b*c - a*d)^4 + 9*B*(b*c - a*d)^4 + 48*A*d*
(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c) + a*d)^3*(a + b*x) + 72*A*d^2*
(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c - a*d)^2*(a + b*x)^2 + 144*A*d^3
*(-(b*c) + a*d)*(a + b*x)^3 + 300*B*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 14...
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^5}\,dx$$

$$\downarrow 2962$$

$$\frac{i^2\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^2}$$

$$\downarrow 2795$$

$$\frac{i^2\int\left(\frac{b(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^5}-\frac{d(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^4}\right)d\frac{a+bx}{c+dx}}{g^5(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{i^2\left(-\frac{b(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4(a+bx)^4}-\frac{bB(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{8(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3}+\frac{2Bd(c+dx)^3\left(B\log\left(\frac{e(}{9(a+bx)^3}\right.\right.\right.}{g^5(bc-ad)^2}$$

input

```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^5,x]
```

output

```
(i^2*((2*B^2*d*(c + d*x)^3)/(27*(a + b*x)^3) - (b*B^2*(c + d*x)^4)/(32*(a
 + b*x)^4) + (2*B*d*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(a
 + b*x)^3) - (b*B*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(8*(a
 + b*x)^4) + (d*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*(a +
 b*x)^3) - (b*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*(a +
b*x)^4)))/((b*c - a*d)^2*g^5)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. 2(287) = 574.

Time = 2.43 (sec) , antiderivative size = 685, normalized size of antiderivative = 2.29

method	result
parts	$\frac{i^2A^2\left(-\frac{d^2}{2b^3(bx+a)^2}-\frac{a^2d^2-2acdb+c^2b^2}{4b^3(bx+a)^4}+\frac{2d(da-bc)}{3b^3(bx+a)^3}\right)}{g^5}-\frac{i^2B^2(da-bc)^3e^3\left(\frac{d^5\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{2\ln\left(\frac{be}{d}+\frac{(da-}{d(dx}\right.\right.}{9\left(\frac{be}{d}+\frac{(da-bc}{d(dx+}\right.}}{(da-bc)^5}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{i^2d^2e^3A^2b}{4(da-bc)^3g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{i^2d^3e^2A^2}{3(da-bc)^3g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{2i^2d^2e^3ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{}{16\left(\frac{be}{d}\right.}\right.}{(da-bc)^3g^5}$
default	$-\frac{e(da-bc)\left(\frac{i^2d^2e^3A^2b}{4(da-bc)^3g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{i^2d^3e^2A^2}{3(da-bc)^3g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}-\frac{2i^2d^2e^3ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{}{16\left(\frac{be}{d}\right.}\right.}{(da-bc)^3g^5}$
norman	Expression too large to display
orering	Expression too large to display
parallelrisch	Expression too large to display
risch	Expression too large to display

input

```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x,method=_RE
TURNVERBOSE)
```

output

```
i^2*A^2/g^5*(-1/2*d^2/b^3/(b*x+a)^2-1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b
*x+a)^4+2/3*d*(a*d-b*c)/b^3/(b*x+a)^3)-i^2*B^2/g^5/d^4*(a*d-b*c)^3*e^3*(d^
5/(a*d-b*c)^5*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d
/(d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-d^4/(a*d-b*c)^5*b*e*(-1/4/(b*e
/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/8/(b*e/d+(
a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/32/(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^4))-2*i^2*A*B/g^5/d^4*(a*d-b*c)^3*e^3*(d^5/(a*d-b*c)^5*(-
1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b
*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-d^4/(a*d-b*c)^5*b*e*(-1/4/(b*e/d+(a*d-b*c)*
e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(287) = 574$.

Time = 0.10 (sec) , antiderivative size = 837, normalized size of antiderivative = 2.80

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, al
gorithm="fricas")
```

output

```
1/864*(12*((12*A*B + 7*B^2)*b^4*c*d^3 - (12*A*B + 7*B^2)*a*b^3*d^4)*i^2*x^
3 - 6*((72*A^2 + 12*A*B - 5*B^2)*b^4*c^2*d^2 - 16*(9*A^2 + 6*A*B + 2*B^2)*
a*b^3*c*d^3 + (72*A^2 + 84*A*B + 37*B^2)*a^2*b^2*d^4)*i^2*x^2 - 4*((144*A^
2 + 60*A*B + 11*B^2)*b^4*c^3*d - 24*(9*A^2 + 6*A*B + 2*B^2)*a*b^3*c^2*d^2
+ (72*A^2 + 84*A*B + 37*B^2)*a^3*b*d^4)*i^2*x - (27*(8*A^2 + 4*A*B + B^2)*
b^4*c^4 - 32*(9*A^2 + 6*A*B + 2*B^2)*a*b^3*c^3*d + (72*A^2 + 84*A*B + 37*B
^2)*a^4*d^4)*i^2 + 72*(B^2*b^4*d^4*i^2*x^4 + 4*B^2*a*b^3*d^4*i^2*x^3 - 6*(
B^2*b^4*c^2*d^2 - 2*B^2*a*b^3*c*d^3)*i^2*x^2 - 4*(2*B^2*b^4*c^3*d - 3*B^2*
a*b^3*c^2*d^2)*i^2*x - (3*B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d)*i^2)*log((b*e*x
 + a*e)/(d*x + c))^2 + 12*((12*A*B + 7*B^2)*b^4*d^4*i^2*x^4 + 4*(3*B^2*b^4
*c*d^3 + 4*(3*A*B + B^2)*a*b^3*d^4)*i^2*x^3 - 6*((12*A*B + B^2)*b^4*c^2*d^
2 - 8*(3*A*B + B^2)*a*b^3*c*d^3)*i^2*x^2 - 4*((24*A*B + 5*B^2)*b^4*c^3*d -
 12*(3*A*B + B^2)*a*b^3*c^2*d^2)*i^2*x - (9*(4*A*B + B^2)*b^4*c^4 - 16*(3*
A*B + B^2)*a*b^3*c^3*d)*i^2)*log((b*e*x + a*e)/(d*x + c)))/((b^9*c^2 - 2*a
*b^8*c*d + a^2*b^7*d^2)*g^5*x^4 + 4*(a*b^8*c^2 - 2*a^2*b^7*c*d + a^3*b^6*d
^2)*g^5*x^3 + 6*(a^2*b^7*c^2 - 2*a^3*b^6*c*d + a^4*b^5*d^2)*g^5*x^2 + 4*(a
^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*g^5*x + (a^4*b^5*c^2 - 2*a^5*b^4
*c*d + a^6*b^3*d^2)*g^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2055 vs. $2(277) = 554$.

Time $= 54.50$ (sec) , antiderivative size $= 2055$, normalized size of antiderivative $= 6.87$

$$\int \frac{(ci + dix)^2 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^5}\, dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**5,x)
```

output

```
-B*d**4*i**2*(12*A + 7*B)*log(x + (12*A*B*a*d**5*i**2 + 12*A*B*b*c*d**4*i*
*2 + 7*B**2*a*d**5*i**2 + 7*B**2*b*c*d**4*i**2 - B*a**3*d**7*i**2*(12*A +
7*B)/(a*d - b*c)**2 + 3*B*a**2*b*c*d**6*i**2*(12*A + 7*B)/(a*d - b*c)**2 -
 3*B*a*b**2*c**2*d**5*i**2*(12*A + 7*B)/(a*d - b*c)**2 + B*b**3*c**3*d**4*
i**2*(12*A + 7*B)/(a*d - b*c)**2)/(24*A*B*b*d**5*i**2 + 14*B**2*b*d**5*i**
2))/(72*b**3*g**5*(a*d - b*c)**2) + B*d**4*i**2*(12*A + 7*B)*log(x + (12*A
*B*a*d**5*i**2 + 12*A*B*b*c*d**4*i**2 + 7*B**2*a*d**5*i**2 + 7*B**2*b*c*d*
*4*i**2 + B*a**3*d**7*i**2*(12*A + 7*B)/(a*d - b*c)**2 - 3*B*a**2*b*c*d**6
*i**2*(12*A + 7*B)/(a*d - b*c)**2 + 3*B*a*b**2*c**2*d**5*i**2*(12*A + 7*B)
/(a*d - b*c)**2 - B*b**3*c**3*d**4*i**2*(12*A + 7*B)/(a*d - b*c)**2)/(24*A
*B*b*d**5*i**2 + 14*B**2*b*d**5*i**2))/(72*b**3*g**5*(a*d - b*c)**2) + (4*
B**2*a*c**3*d*i**2 + 12*B**2*a*c**2*d**2*i**2*x + 12*B**2*a*c*d**3*i**2*x*
*2 + 4*B**2*a*d**4*i**2*x**3 - 3*B**2*b*c**4*i**2 - 8*B**2*b*c**3*d*i**2*x
 - 6*B**2*b*c**2*d**2*i**2*x**2 + B**2*b*d**4*i**2*x**4)*log(e*(a + b*x)/(
c + d*x))**2/(12*a**6*d**2*g**5 - 24*a**5*b*c*d*g**5 + 48*a**5*b*d**2*g**5
*x + 12*a**4*b**2*c**2*g**5 - 96*a**4*b**2*c*d*g**5*x + 72*a**4*b**2*d**2*
g**5*x**2 + 48*a**3*b**3*c**2*g**5*x - 144*a**3*b**3*c*d*g**5*x**2 + 48*a*
*3*b**3*d**2*g**5*x**3 + 72*a**2*b**4*c**2*g**5*x**2 - 96*a**2*b**4*c*d*g*
*5*x**3 + 12*a**2*b**4*d**2*g**5*x**4 + 48*a*b**5*c**2*g**5*x**3 - 24*a*b*
*5*c*d*g**5*x**4 + 12*b**6*c**2*g**5*x**4) + (-12*A*B*a**3*d**3*i**2 - ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8031 vs. $2(287) = 574$.

Time $= 0.61$ (sec) , antiderivative size $= 8031$, normalized size of antiderivative $= 26.86$

$$\int \frac{(ci + dix)^2 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^5}\, dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, al
gorithm="maxima")
```

output

```
-1/6*(4*b*x + a)*B^2*c*d*i^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^6*g
^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g
^5) - 1/12*(6*b^2*x^2 + 4*a*b*x + a^2)*B^2*d^2*i^2*log(b*e*x/(d*x + c) + a
*e/(d*x + c))^2/(b^7*g^5*x^4 + 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*a^3
*b^4*g^5*x + a^4*b^3*g^5) + 1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*
b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2
+ 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*
d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*
d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^
2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*
c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^
2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 -
4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12
*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^
2*c*d^3 + a^4*b*d^4)*g^5))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - (9*b^4*c
^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4
- 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 +
163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x
^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d
^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c)^2 - ...
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.60

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{1}{864}\left(\frac{72\left(3\,B^2be^5i^2-\frac{4\,(bex+ae)B^2de^4i^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^4bcg^5}{(dx+c)^4}-\frac{(bex+ae)^4adg^5}{(dx+c)^4}}+\frac{12\left(36\,ABbe^5i^2+9\,B^2be^5i^2-\frac{48\,(bex+ae)ABde^4i^2}{dx+c}\right.}{\frac{(bex+ae)^4bcg^5}{(dx+c)^4}-\frac{(bex+a}{(dx}\right.$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, al
gorithm="giac")
```

output

```
-1/864*(72*(3*B^2*b*e^5*i^2 - 4*(b*e*x + a*e)*B^2*d*e^4*i^2/(d*x + c))*log
((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)^4*b*c*g^5/(d*x + c)^4 - (b*e*x
+ a*e)^4*a*d*g^5/(d*x + c)^4) + 12*(36*A*B*b*e^5*i^2 + 9*B^2*b*e^5*i^2 - 4
8*(b*e*x + a*e)*A*B*d*e^4*i^2/(d*x + c) - 16*(b*e*x + a*e)*B^2*d*e^4*i^2/(
d*x + c))*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^4*b*c*g^5/(d*x + c)^
4 - (b*e*x + a*e)^4*a*d*g^5/(d*x + c)^4) + (216*A^2*b*e^5*i^2 + 108*A*B*b*
e^5*i^2 + 27*B^2*b*e^5*i^2 - 288*(b*e*x + a*e)*A^2*d*e^4*i^2/(d*x + c) - 1
92*(b*e*x + a*e)*A*B*d*e^4*i^2/(d*x + c) - 64*(b*e*x + a*e)*B^2*d*e^4*i^2/
(d*x + c))/((b*e*x + a*e)^4*b*c*g^5/(d*x + c)^4 - (b*e*x + a*e)^4*a*d*g^5/
(d*x + c)^4))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b
*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 32.74 (sec) , antiderivative size = 1940, normalized size of antiderivative = 6.49

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^5,x)
```

output

```
- log((e*(a + b*x))/(c + d*x))^2*((x*(b*((B^2*c*d*i^2)/(6*b^3*g^5) + (B^2*
a*d^2*i^2)/(12*b^4*g^5)) + (B^2*c*d*i^2)/(2*b^2*g^5) + (B^2*a*d^2*i^2)/(4*
b^3*g^5)) + a*((B^2*c*d*i^2)/(6*b^3*g^5) + (B^2*a*d^2*i^2)/(12*b^4*g^5)) +
 (B^2*c^2*i^2)/(4*b^2*g^5) + (B^2*d^2*i^2*x^2)/(2*b^2*g^5))/(4*a^3*x + a^4
/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3) - (B^2*d^4*i^2)/(12*b^3*g^5*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((72*A^2*a^3*d^3*i^2 - 216*A^2*b^3*c^3*i^2
 + 37*B^2*a^3*d^3*i^2 - 27*B^2*b^3*c^3*i^2 + 84*A*B*a^3*d^3*i^2 - 108*A*B*
b^3*c^3*i^2 + 72*A^2*a*b^2*c^2*d*i^2 + 72*A^2*a^2*b*c*d^2*i^2 + 37*B^2*a*b
^2*c^2*d*i^2 + 37*B^2*a^2*b*c*d^2*i^2 + 84*A*B*a*b^2*c^2*d*i^2 + 84*A*B*a^
2*b*c*d^2*i^2)/(12*(a*d - b*c)) + (x^3*(7*B^2*b^3*d^3*i^2 + 12*A*B*b^3*d^3
*i^2))/(a*d - b*c) + (x*(72*A^2*a^2*b*d^3*i^2 + 37*B^2*a^2*b*d^3*i^2 - 144
*A^2*b^3*c^2*d*i^2 - 11*B^2*b^3*c^2*d*i^2 + 72*A^2*a*b^2*c*d^2*i^2 + 37*B^
2*a*b^2*c*d^2*i^2 + 84*A*B*a^2*b*d^3*i^2 - 60*A*B*b^3*c^2*d*i^2 + 84*A*B*a
*b^2*c*d^2*i^2))/(3*(a*d - b*c)) + (x^2*(72*A^2*a*b^2*d^3*i^2 + 37*B^2*a*b
^2*d^3*i^2 - 72*A^2*b^3*c*d^2*i^2 + 5*B^2*b^3*c*d^2*i^2 + 84*A*B*a*b^2*d^3
*i^2 - 12*A*B*b^3*c*d^2*i^2))/(2*(a*d - b*c)))/(72*a^4*b^3*g^5 + 72*b^7*g^
5*x^4 + 288*a^3*b^4*g^5*x + 288*a*b^6*g^5*x^3 + 432*a^2*b^5*g^5*x^2) - (lo
g((e*(a + b*x))/(c + d*x))*(x^2*((A*B*d*i^2)/(b^2*g^5) + (B^2*d^4*i^2*(b*(
b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2
)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d - b*c))/(2*d^2...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 2107, normalized size of antiderivative = 7.05

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x)
```

output

```
( - 144*log(a + b*x)*a**6*b*d**4 - 576*log(a + b*x)*a**5*b**2*d**4*x - 48*
log(a + b*x)*a**5*b**2*d**4 - 36*log(a + b*x)*a**4*b**3*c*d**3 - 864*log(a
 + b*x)*a**4*b**3*d**4*x**2 - 192*log(a + b*x)*a**4*b**3*d**4*x - 144*log(
a + b*x)*a**3*b**4*c*d**3*x - 576*log(a + b*x)*a**3*b**4*d**4*x**3 - 288*l
og(a + b*x)*a**3*b**4*d**4*x**2 - 216*log(a + b*x)*a**2*b**5*c*d**3*x**2 -
 144*log(a + b*x)*a**2*b**5*d**4*x**4 - 192*log(a + b*x)*a**2*b**5*d**4*x*
*3 - 144*log(a + b*x)*a*b**6*c*d**3*x**3 - 48*log(a + b*x)*a*b**6*d**4*x**
4 - 36*log(a + b*x)*b**7*c*d**3*x**4 + 144*log(c + d*x)*a**6*b*d**4 + 576*
log(c + d*x)*a**5*b**2*d**4*x + 48*log(c + d*x)*a**5*b**2*d**4 + 36*log(c
+ d*x)*a**4*b**3*c*d**3 + 864*log(c + d*x)*a**4*b**3*d**4*x**2 + 192*log(c
 + d*x)*a**4*b**3*d**4*x + 144*log(c + d*x)*a**3*b**4*c*d**3*x + 576*log(c
 + d*x)*a**3*b**4*d**4*x**3 + 288*log(c + d*x)*a**3*b**4*d**4*x**2 + 216*l
og(c + d*x)*a**2*b**5*c*d**3*x**2 + 144*log(c + d*x)*a**2*b**5*d**4*x**4 +
 192*log(c + d*x)*a**2*b**5*d**4*x**3 + 144*log(c + d*x)*a*b**6*c*d**3*x**
3 + 48*log(c + d*x)*a*b**6*d**4*x**4 + 36*log(c + d*x)*b**7*c*d**3*x**4 -
288*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**5*c**3*d - 864*log((a*e + b*e*
x)/(c + d*x))**2*a**2*b**5*c**2*d**2*x - 864*log((a*e + b*e*x)/(c + d*x))*
*2*a**2*b**5*c*d**3*x**2 - 288*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**5*d
**4*x**3 + 216*log((a*e + b*e*x)/(c + d*x))**2*a*b**6*c**4 + 576*log((a*e
+ b*e*x)/(c + d*x))**2*a*b**6*c**3*d*x + 432*log((a*e + b*e*x)/(c + d*x...
```

3.73 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx$

Optimal result 909
Mathematica [C] (verified) 910
Rubi [A] (verified) 911
Maple [B] (verified) 912
Fricas [B] (verification not implemented) 913
Sympy [F(-1)] 914
Maxima [B] (verification not implemented) 915
Giac [A] (verification not implemented) 916
Mupad [B] (verification not implemented) 916
Reduce [B] (verification not implemented) 917

Optimal result

Integrand size = 42, antiderivative size = 463

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx &= -\frac{2B^2d^2i^2(c+dx)^3}{27(bc-ad)^3g^6(a+bx)^3}\\ &+\frac{bB^2di^2(c+dx)^4}{16(bc-ad)^3g^6(a+bx)^4}\\ &-\frac{2b^2B^2i^2(c+dx)^5}{125(bc-ad)^3g^6(a+bx)^5}\\ &-\frac{2Bd^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^3g^6(a+bx)^3}\\ &+\frac{bBdi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3g^6(a+bx)^4}\\ &-\frac{2b^2Bi^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{25(bc-ad)^3g^6(a+bx)^5}\\ &-\frac{d^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^3g^6(a+bx)^3}\\ &+\frac{bdi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3g^6(a+bx)^4}\\ &-\frac{b^2i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5(bc-ad)^3g^6(a+bx)^5}\end{aligned}$$

output

```
-2/27*B^2*d^2*i^2*(d*x+c)^3/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/16*b*B^2*d*i^2*(d
*x+c)^4/(-a*d+b*c)^3/g^6/(b*x+a)^4-2/125*b^2*B^2*i^2*(d*x+c)^5/(-a*d+b*c)^
3/g^6/(b*x+a)^5-2/9*B*d^2*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+
b*c)^3/g^6/(b*x+a)^3+1/4*b*B*d*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(
-a*d+b*c)^3/g^6/(b*x+a)^4-2/25*b^2*B*i^2*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+
c)))/(-a*d+b*c)^3/g^6/(b*x+a)^5-1/3*d^2*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d
*x+c)))^2/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/2*b*d*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+
a)/(d*x+c)))^2/(-a*d+b*c)^3/g^6/(b*x+a)^4-1/5*b^2*i^2*(d*x+c)^5*(A+B*ln(e*
(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^6/(b*x+a)^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 3.20 (sec) , antiderivative size = 2010, normalized size of antiderivative = 4.34

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Result too large to show}$$

input

```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^6,x]
```

output

```
-1/54000*(i^2*(10800*(b*c - a*d)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
+ 27000*d*(b*c - a*d)^4*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 -
 18000*d^2*(-(b*c) + a*d)^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)
])^2 + 1000*B*d^2*(a + b*x)^2*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3 - 18
*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*(a + b*x) + 36*A*d^2*(
b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*(a + b*x)^2 + 36*A*d^3*(a +
b*x)^3*Log[a + b*x] + 66*B*d^3*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*(a + b*
x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)] - 18
*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^2*(b*c
- a*d)*(a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^3*(a + b*x)^3*Log
[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - 36*A*d^3*(a + b*x)^3*Log[c + d*x]
 - 66*B*d^3*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*
x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[(e*(a + b*x))/
(c + d*x)]*Log[c + d*x] - 18*B*d^3*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*(
a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*(a + b*x
)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*(a + b*x)^3*PolyLo
g[2, (b*(c + d*x))/(b*c - a*d)]) + 375*B*d*(a + b*x)*(36*A*(b*c - a*d)^4 +
 9*B*(b*c - a*d)^4 + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c) +
a*d)^3*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c - a*
d)^2*(a + b*x)^2 + 144*A*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 300*B*d^3*(-(...
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^6}\,dx$$

$$\downarrow 2962$$

$$\frac{i^2\int \frac{(c+dx)^6\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^6}d\frac{a+bx}{c+dx}}{g^6(bc-ad)^3}$$

$$\downarrow 2795$$

$$\frac{i^2\int\left(\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^6}{(a+bx)^6}-\frac{2bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^5}{(a+bx)^5}+\frac{d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^4}{(a+bx)^4}\right)d\frac{a+bx}{c+dx}}{g^6(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{i^2\left(-\frac{b^2(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5(a+bx)^5}-\frac{2b^2B(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{25(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3}-\frac{2Bd^2(c+dx)^3\left(B\log\right.}{9(a+}\right.}{g^6}$$

input

```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^6,x]
```

output
```
(i^2*((-2*B^2*d^2*(c + d*x)^3)/(27*(a + b*x)^3) + (b*B^2*d*(c + d*x)^4)/(1
6*(a + b*x)^4) - (2*b^2*B^2*(c + d*x)^5)/(125*(a + b*x)^5) - (2*B*d^2*(c +
 d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(a + b*x)^3) + (b*B*d*(c
+ d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*(a + b*x)^4) - (2*b^2*B*
(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(25*(a + b*x)^5) - (d^2*
(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*(a + b*x)^3) + (b*d
*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(a + b*x)^4) - (b^
2*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(5*(a + b*x)^5)))/((
b*c - a*d)^3*g^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(445) = 890$.

Time = 2.51 (sec) , antiderivative size = 968, normalized size of antiderivative = 2.09

method	result	size
parts	Expression too large to display	968
derivativedivides	Expression too large to display	1082
default	Expression too large to display	1082
orering	Expression too large to display	1819
norman	Expression too large to display	1977
parallelrisch	Expression too large to display	2248
risch	Expression too large to display	5135

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x,method=_RE
TURNVERBOSE)
```

output
```
i^2*A^2/g^6*(-1/5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^5+1/2*d*(a*d-b*c
)/b^3/(b*x+a)^4-1/3*d^2/b^3/(b*x+a)^3)-i^2*B^2/g^6/d^4*(a*d-b*c)^3*e^3*(d^
6/(a*d-b*c)^6*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d
/(d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-2*d^5/(a*d-b*c)^6*b*e*(-1/4/(b
*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/8/(b*e/d
+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/32/(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^4)+d^4/(a*d-b*c)^6*b^2*e^2*(-1/5/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/25/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/125/(b*e/d+(a*d-b*c)*e/d/(d*x+c))
^5))-2*i^2*A*B/g^6/d^4*(a*d-b*c)^3*e^3*(d^6/(a*d-b*c)^6*(-1/3/(b*e/d+(a*d-
b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^3)-2*d^5/(a*d-b*c)^6*b*e*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4
*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4)+d^4
/(a*d-b*c)^6*b^2*e^2*(-1/5/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5*ln(b*e/d+(a*d-b
*c)*e/d/(d*x+c))-1/25/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1323 vs. $2(445) = 890$.

Time = 0.11 (sec) , antiderivative size = 1323, normalized size of antiderivative = 2.86

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, al
gorithm="fricas")
```

output
```
-1/54000*(60*((60*A*B + 47*B^2)*b^5*c*d^4 - (60*A*B + 47*B^2)*a*b^4*d^5)*i
^2*x^4 - 30*((60*A*B - 13*B^2)*b^5*c^2*d^3 - 50*(12*A*B + 7*B^2)*a*b^4*c*d
^4 + 3*(180*A*B + 121*B^2)*a^2*b^3*d^5)*i^2*x^3 + 10*(2*(900*A^2 + 60*A*B
- 43*B^2)*b^5*c^3*d^2 - 75*(72*A^2 + 12*A*B - 5*B^2)*a*b^4*c^2*d^3 + 600*(
9*A^2 + 6*A*B + 2*B^2)*a^2*b^3*c*d^4 - (1800*A^2 + 2820*A*B + 1489*B^2)*a^
3*b^2*d^5)*i^2*x^2 + 5*(27*(200*A^2 + 60*A*B + 7*B^2)*b^5*c^4*d - 100*(144
*A^2 + 60*A*B + 11*B^2)*a*b^4*c^3*d^2 + 1200*(9*A^2 + 6*A*B + 2*B^2)*a^2*b
^3*c^2*d^3 - (1800*A^2 + 2820*A*B + 1489*B^2)*a^4*b*d^5)*i^2*x + (432*(25*
A^2 + 10*A*B + 2*B^2)*b^5*c^5 - 3375*(8*A^2 + 4*A*B + B^2)*a*b^4*c^4*d + 2
000*(9*A^2 + 6*A*B + 2*B^2)*a^2*b^3*c^3*d^2 - (1800*A^2 + 2820*A*B + 1489*
B^2)*a^5*d^5)*i^2 + 1800*(B^2*b^5*d^5*i^2*x^5 + 5*B^2*a*b^4*d^5*i^2*x^4 +
10*B^2*a^2*b^3*d^5*i^2*x^3 + 10*(B^2*b^5*c^3*d^2 - 3*B^2*a*b^4*c^2*d^3 + 3
*B^2*a^2*b^3*c*d^4)*i^2*x^2 + 5*(3*B^2*b^5*c^4*d - 8*B^2*a*b^4*c^3*d^2 + 6
*B^2*a^2*b^3*c^2*d^3)*i^2*x + (6*B^2*b^5*c^5 - 15*B^2*a*b^4*c^4*d + 10*B^2
*a^2*b^3*c^3*d^2)*i^2)*log((b*e*x + a*e)/(d*x + c))^2 + 60*((60*A*B + 47*B
^2)*b^5*d^5*i^2*x^5 + 5*(12*B^2*b^5*c*d^4 + 5*(12*A*B + 7*B^2)*a*b^4*d^5)*
i^2*x^4 - 10*(3*B^2*b^5*c^2*d^3 - 30*B^2*a*b^4*c*d^4 - 20*(3*A*B + B^2)*a^
2*b^3*d^5)*i^2*x^3 + 10*(2*(30*A*B + B^2)*b^5*c^3*d^2 - 15*(12*A*B + B^2)*
a*b^4*c^2*d^3 + 60*(3*A*B + B^2)*a^2*b^3*c*d^4)*i^2*x^2 + 5*(9*(20*A*B + 3
*B^2)*b^5*c^4*d - 20*(24*A*B + 5*B^2)*a*b^4*c^3*d^2 + 120*(3*A*B + B^2)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**6,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10880 vs. $2(445) = 890$.

Time $= 0.90$ (sec) , antiderivative size $= 10880$, normalized size of antiderivative $= 23.50$

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, al
gorithm="maxima")
```

output

```
-1/10*(5*b*x + a)*B^2*c*d*i^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^7*
g^6*x^5 + 5*a*b^6*g^6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4*g^6*x^2 + 5*a^
4*b^3*g^6*x + a^5*b^2*g^6) - 1/30*(10*b^2*x^2 + 5*a*b*x + a^2)*B^2*d^2*i^2
*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^8*g^6*x^5 + 5*a*b^7*g^6*x^4 + 1
0*a^2*b^6*g^6*x^3 + 10*a^3*b^5*g^6*x^2 + 5*a^4*b^4*g^6*x + a^5*b^3*g^6) -
1/9000*(60*((60*b^4*d^4*x^4 + 12*b^4*c^4 - 63*a*b^3*c^3*d + 137*a^2*b^2*c^
2*d^2 - 163*a^3*b*c*d^3 + 137*a^4*d^4 - 30*(b^4*c*d^3 - 9*a*b^3*d^4)*x^3 +
 10*(2*b^4*c^2*d^2 - 13*a*b^3*c*d^3 + 47*a^2*b^2*d^4)*x^2 - 5*(3*b^4*c^3*d
 - 17*a*b^3*c^2*d^2 + 43*a^2*b^2*c*d^3 - 77*a^3*b*d^4)*x)/((b^10*c^4 - 4*a
*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*g^6*x^5 +
5*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^5
*b^5*d^4)*g^6*x^4 + 10*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2
- 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*g^6*x^3 + 10*(a^3*b^7*c^4 - 4*a^4*b^6*c^3
*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*g^6*x^2 + 5*(a^4*b
^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d
^4)*g^6*x + (a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2
*c*d^3 + a^9*b*d^4)*g^6) + 60*d^5*log(b*x + a)/((b^6*c^5 - 5*a*b^5*c^4*d +
 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^
6) - 60*d^5*log(d*x + c)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 -
10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6))*log(b*e*x/(d*x ...
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.68

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, al
gorithm="giac")
```

output
```
-1/54000*(1800*(6*B^2*b^2*e^6*i^2 - 15*(b*e*x + a*e)*B^2*b*d*e^5*i^2/(d*x
 + c) + 10*(b*e*x + a*e)^2*B^2*d^2*e^4*i^2/(d*x + c)^2)*log((b*e*x + a*e)/(
d*x + c))^2/((b*e*x + a*e)^5*b^2*c^2*g^6/(d*x + c)^5 - 2*(b*e*x + a*e)^5*a
*b*c*d*g^6/(d*x + c)^5 + (b*e*x + a*e)^5*a^2*d^2*g^6/(d*x + c)^5) + 60*(36
0*A*B*b^2*e^6*i^2 + 72*B^2*b^2*e^6*i^2 - 900*(b*e*x + a*e)*A*B*b*d*e^5*i^2
/(d*x + c) - 225*(b*e*x + a*e)*B^2*b*d*e^5*i^2/(d*x + c) + 600*(b*e*x + a*
e)^2*A*B*d^2*e^4*i^2/(d*x + c)^2 + 200*(b*e*x + a*e)^2*B^2*d^2*e^4*i^2/(d*
x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^5*b^2*c^2*g^6/(d*x +
 c)^5 - 2*(b*e*x + a*e)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*e*x + a*e)^5*a^2*d^
2*g^6/(d*x + c)^5) + (10800*A^2*b^2*e^6*i^2 + 4320*A*B*b^2*e^6*i^2 + 864*B
^2*b^2*e^6*i^2 - 27000*(b*e*x + a*e)*A^2*b*d*e^5*i^2/(d*x + c) - 13500*(b*
e*x + a*e)*A*B*b*d*e^5*i^2/(d*x + c) - 3375*(b*e*x + a*e)*B^2*b*d*e^5*i^2/
(d*x + c) + 18000*(b*e*x + a*e)^2*A^2*d^2*e^4*i^2/(d*x + c)^2 + 12000*(b*e
*x + a*e)^2*A*B*d^2*e^4*i^2/(d*x + c)^2 + 4000*(b*e*x + a*e)^2*B^2*d^2*e^4
*i^2/(d*x + c)^2)/((b*e*x + a*e)^5*b^2*c^2*g^6/(d*x + c)^5 - 2*(b*e*x + a*
e)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*e*x + a*e)^5*a^2*d^2*g^6/(d*x + c)^5))*(
b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 35.88 (sec) , antiderivative size = 3434, normalized size of antiderivative = 7.42

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^6,x)
```

output

```
((1800*A^2*a^4*d^4*i^2 + 10800*A^2*b^4*c^4*i^2 + 1489*B^2*a^4*d^4*i^2 + 86
4*B^2*b^4*c^4*i^2 + 2820*A*B*a^4*d^4*i^2 + 4320*A*B*b^4*c^4*i^2 - 16200*A^
2*a*b^3*c^3*d*i^2 + 1800*A^2*a^3*b*c*d^3*i^2 - 2511*B^2*a*b^3*c^3*d*i^2 +
1489*B^2*a^3*b*c*d^3*i^2 + 1800*A^2*a^2*b^2*c^2*d^2*i^2 + 1489*B^2*a^2*b^2
*c^2*d^2*i^2 + 2820*A*B*a^2*b^2*c^2*d^2*i^2 - 9180*A*B*a*b^3*c^3*d*i^2 + 2
820*A*B*a^3*b*c*d^3*i^2)/(60*(a*d - b*c)) + (x^3*(363*B^2*a*b^3*d^4*i^2 +
13*B^2*b^4*c*d^3*i^2 + 540*A*B*a*b^3*d^4*i^2 - 60*A*B*b^4*c*d^3*i^2))/(2*(
a*d - b*c)) + (x*(1800*A^2*a^3*b*d^4*i^2 + 1489*B^2*a^3*b*d^4*i^2 + 5400*A
^2*b^4*c^3*d*i^2 + 189*B^2*b^4*c^3*d*i^2 - 9000*A^2*a*b^3*c^2*d^2*i^2 + 18
00*A^2*a^2*b^2*c*d^3*i^2 - 911*B^2*a*b^3*c^2*d^2*i^2 + 1489*B^2*a^2*b^2*c*
d^3*i^2 + 2820*A*B*a^3*b*d^4*i^2 + 1620*A*B*b^4*c^3*d*i^2 - 4380*A*B*a*b^3
*c^2*d^2*i^2 + 2820*A*B*a^2*b^2*c*d^3*i^2))/(12*(a*d - b*c)) + (x^2*(1800*
A^2*a^2*b^2*d^4*i^2 + 1489*B^2*a^2*b^2*d^4*i^2 + 1800*A^2*b^4*c^2*d^2*i^2
- 86*B^2*b^4*c^2*d^2*i^2 - 3600*A^2*a*b^3*c*d^3*i^2 + 289*B^2*a*b^3*c*d^3*
i^2 + 2820*A*B*a^2*b^2*d^4*i^2 + 120*A*B*b^4*c^2*d^2*i^2 - 780*A*B*a*b^3*c
*d^3*i^2))/(6*(a*d - b*c)) + (d*x^4*(47*B^2*b^4*d^3*i^2 + 60*A*B*b^4*d^3*i
^2))/(a*d - b*c))/(x*(4500*a^4*b^5*c*g^6 - 4500*a^5*b^4*d*g^6) - x^4*(4500
*a^2*b^7*d*g^6 - 4500*a*b^8*c*g^6) + x^5*(900*b^9*c*g^6 - 900*a*b^8*d*g^6)
 + x^2*(9000*a^3*b^6*c*g^6 - 9000*a^4*b^5*d*g^6) + x^3*(9000*a^2*b^7*c*g^6
 - 9000*a^3*b^6*d*g^6) + 900*a^5*b^4*c*g^6 - 900*a^6*b^3*d*g^6) - log((...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3022, normalized size of antiderivative = 6.53

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x)
```

output

```
( - 3600*log(a + b*x)*a**7*b*d**5 - 18000*log(a + b*x)*a**6*b**2*d**5*x -
2100*log(a + b*x)*a**6*b**2*d**5 - 720*log(a + b*x)*a**5*b**3*c*d**4 - 360
00*log(a + b*x)*a**5*b**3*d**5*x**2 - 10500*log(a + b*x)*a**5*b**3*d**5*x
- 3600*log(a + b*x)*a**4*b**4*c*d**4*x - 36000*log(a + b*x)*a**4*b**4*d**5
*x**3 - 21000*log(a + b*x)*a**4*b**4*d**5*x**2 - 7200*log(a + b*x)*a**3*b*
*5*c*d**4*x**2 - 18000*log(a + b*x)*a**3*b**5*d**5*x**4 - 21000*log(a + b*
x)*a**3*b**5*d**5*x**3 - 7200*log(a + b*x)*a**2*b**6*c*d**4*x**3 - 3600*lo
g(a + b*x)*a**2*b**6*d**5*x**5 - 10500*log(a + b*x)*a**2*b**6*d**5*x**4 -
3600*log(a + b*x)*a*b**7*c*d**4*x**4 - 2100*log(a + b*x)*a*b**7*d**5*x**5
- 720*log(a + b*x)*b**8*c*d**4*x**5 + 3600*log(c + d*x)*a**7*b*d**5 + 1800
0*log(c + d*x)*a**6*b**2*d**5*x + 2100*log(c + d*x)*a**6*b**2*d**5 + 720*l
og(c + d*x)*a**5*b**3*c*d**4 + 36000*log(c + d*x)*a**5*b**3*d**5*x**2 + 10
500*log(c + d*x)*a**5*b**3*d**5*x + 3600*log(c + d*x)*a**4*b**4*c*d**4*x +
 36000*log(c + d*x)*a**4*b**4*d**5*x**3 + 21000*log(c + d*x)*a**4*b**4*d**
5*x**2 + 7200*log(c + d*x)*a**3*b**5*c*d**4*x**2 + 18000*log(c + d*x)*a**3
*b**5*d**5*x**4 + 21000*log(c + d*x)*a**3*b**5*d**5*x**3 + 7200*log(c + d*
x)*a**2*b**6*c*d**4*x**3 + 3600*log(c + d*x)*a**2*b**6*d**5*x**5 + 10500*l
og(c + d*x)*a**2*b**6*d**5*x**4 + 3600*log(c + d*x)*a*b**7*c*d**4*x**4 + 2
100*log(c + d*x)*a*b**7*d**5*x**5 + 720*log(c + d*x)*b**8*c*d**4*x**5 - 18
000*log((a*e + b*e*x)/(c + d*x))**2*a**3*b**5*c**3*d**2 - 54000*log((a*...
```

3.74 $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result 919
Mathematica [B] (verified) 920
Rubi [A] (verified) 921
Maple [F] 938
Fricas [F] 939
Sympy [F(-1)] 939
Maxima [B] (verification not implemented) 940
Giac [F] 941
Mupad [F(-1)] 941
Reduce [F] 942

Optimal result

Integrand size = 42, antiderivative size = 1089

$$\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

output

```
5/84*B^2*(-a*d+b*c)^6*g^3*i^3*x/b^3/d^3+1/140*B^2*(-a*d+b*c)^3*g^3*i^3*(b*
x+a)^4/b^4-29/840*B^2*(-a*d+b*c)^5*g^3*i^3*(d*x+c)^2/b^2/d^4+47/1260*B^2*(
-a*d+b*c)^4*g^3*i^3*(d*x+c)^3/b/d^4-13/420*B^2*(-a*d+b*c)^3*g^3*i^3*(d*x+c
)^4/d^4+1/105*b*B^2*(-a*d+b*c)^2*g^3*i^3*(d*x+c)^5/d^4-1/210*B^2*(-a*d+b*c
)^7*g^3*i^3*ln((b*x+a)/(d*x+c))/b^4/d^4-1/210*B*(-a*d+b*c)^4*g^3*i^3*(b*x+
a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/d-3/140*B*(-a*d+b*c)^3*g^3*i^3*(b*x+a
)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4-1/35*B*(-a*d+b*c)^2*g^3*i^3*(b*x+a)^4*
(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3+2/21*B*(-a*d+b*c)^4*g^3*i^3*(d*x+c
)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d^4-3/14*B*(-a*d+b*c)^3*g^3*i^3*(d*x+c)^
4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4+6/35*b*B*(-a*d+b*c)^2*g^3*i^3*(d*x+c)^5*
(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4-1/21*b^2*B*(-a*d+b*c)*g^3*i^3*(d*x+c)^6*(A
+B*ln(e*(b*x+a)/(d*x+c)))/d^4+1/140*(-a*d+b*c)^3*g^3*i^3*(b*x+a)^4*(A+B*ln
(e*(b*x+a)/(d*x+c)))^2/b^4+1/35*(-a*d+b*c)^2*g^3*i^3*(b*x+a)^4*(d*x+c)*(A+
B*ln(e*(b*x+a)/(d*x+c)))^2/b^3+1/14*(-a*d+b*c)*g^3*i^3*(b*x+a)^4*(d*x+c)^2
*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2+1/7*g^3*i^3*(b*x+a)^4*(d*x+c)^3*(A+B*ln
(e*(b*x+a)/(d*x+c)))^2/b+1/420*B*(-a*d+b*c)^5*g^3*i^3*(b*x+a)^2*(3*A+B+3*B
*ln(e*(b*x+a)/(d*x+c)))/b^4/d^2-1/420*B*(-a*d+b*c)^6*g^3*i^3*(b*x+a)*(6*A+
5*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b^4/d^3-1/420*B*(-a*d+b*c)^7*g^3*i^3*ln((-a
*d+b*c)/b/(d*x+c))*(6*A+11*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b^4/d^4-11/420*B^2
*(-a*d+b*c)^7*g^3*i^3*ln(d*x+c)/b^4/d^4-1/70*B^2*(-a*d+b*c)^7*g^3*i^3*p...
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2330 vs. $2(1089) = 2178$.

Time = 3.36 (sec) , antiderivative size = 2330, normalized size of antiderivative = 2.14

$$\int (ag + bgx)^3 (ci + dix)^3 \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right)^2 dx = \text{Result too large to show}$$

input

```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^2,x]
```

output

```
(g^3*i^3*(35*(b*c - a*d)^3*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2 + 84*d*(b*c - a*d)^2*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^
2 + 70*d^2*(b*c - a*d)*(a + b*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
+ 20*d^3*(a + b*x)^7*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (35*B*(b*c -
 a*d)^4*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a
 + b*x))/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a +
b*x))/(c + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])
 - 6*B*(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x
))/(c + d*x)])*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a
+ b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b
*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c)
 + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)])))/(3*d^4) + (7*B*(b*c - a*d)^3*(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(
b*c - a*d)^3*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 12*d^2*(b*c - a*d)^2
*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 8*d^3*(b*c - a*d)*(a +
 b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*d^4*(a + b*x)^4*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]) - 24*B*(b*c - a*d)^4*Log[c + d*x] - 24*(b*c -
a*d)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 4*B*(b*c - a*d)
^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x])
+ B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)...
```

Rubi [A] (verified)

Time = 3.51 (sec) , antiderivative size = 1398, normalized size of antiderivative = 1.28, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {2962, 2783, 2782, 27, 2123, 2009, 2783, 2782, 27, 87, 49, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\downarrow 2962$$

$$g^3i^3(bc-ad)^7\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^8}d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$g^3 i^3 (bc -$$

$$ad)^7 \left(-\frac{2B\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^7} d\frac{a+bx}{c+dx}}{7b} + \frac{3\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^7} d\frac{a+bx}{c+dx}}{7b} + \frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right.\right.}{7b(c+dx)^4\left(b-\frac{d(a+b}{c+d}\right.}$$

$$\downarrow 2782$$

$$g^3 i^3 (bc -$$

$$ad)^7 \left(-\frac{2B\left(-B\int -\frac{(c+dx)\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{15d^2(a+bx)^2b}{(c+dx)^2}-\frac{20d^3(a+bx)^3}{(c+dx)^3}\right)}{60d^4(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx} + \frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right.\right.}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}$$

$$\downarrow 27$$

$$g^3 i^3 (bc -$$

$$ad)^7 \left(-\frac{2B\left(\frac{B\int \frac{(c+dx)\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{15d^2(a+bx)^2b}{(c+dx)^2}-\frac{20d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{60d^4} + \frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} +$$

$$\downarrow 2123$$

$$g^3 i^3 (bc -$$

$$ad)^7 \left(-\frac{2B\left(\frac{B\int \left(-\frac{10db^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6} + \frac{26db}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{19d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3 b} + \frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2 b^2} + \frac{c+dx}{(a+bx)b^3} + \frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)b^3} \right) d}{60d^4}$$

$$\downarrow 2009$$

$$g^3 i^3 (bc -$$

$$ad)^7 \left(\frac{3 \int \frac{(a+bx)^3 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^7} d\frac{a+bx}{c+dx}}{7b} - \frac{2B \left(\frac{b^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3b^2 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3b \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{4d^4 \left(b-\frac{d(a+bx)}{c+dx}\right.} \right.$$

$$\downarrow 2783$$

$$g^3 i^3 (bc -$$

$$ad)^7 \left(\frac{3 \left(-\frac{B \int \frac{(a+bx)^3 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{3b} + \frac{\int \frac{(a+bx)^3 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{3b} + \frac{(a+bx)^4 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{6b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} \right)}{7b} - \right.$$

$$\downarrow 2782$$

$$(bc -$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B \left(\frac{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) b^3}{6d^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) b^2}{5d^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B \log\left(\frac{e(a-}{4d^4 \left(b-\frac{d(a+b}{c+d}\right.} \right.} \right.$$

$$\downarrow 27$$

$$g^3 i^3 (bc -$$

$$ad)^7 \left(3 \left(\frac{B \left(- \frac{B \int \frac{(a+bx)^3 \left(5b - \frac{d(a+bx)}{c+dx}\right)}{(c+dx)^3 \left(b - \frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{20b^2} + \frac{(a+bx)^4 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{20b^2 (c+dx)^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^4} + \frac{(a+bx)^4 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b(c+dx)^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^5} \right)}{3b} + \frac{\int \frac{(a+bx)^3 \left(A+B \log\left($$

$$\downarrow 87$$

$$(bc -$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B \left(\frac{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) b^3}{6d^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) b^2}{5d^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B \log\left(\frac{e(a+b$$

$$\downarrow 49$$

$$(bc-$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a-}{c+}\right.\right.\right.$$

$$\downarrow 2009$$

$$(bc-$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a-}{c+}\right.\right.\right.$$

$$\downarrow 2783$$

$$(bc-$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a-}{c+}\right.\right.\right.$$

$$\downarrow 2773$$

$$(bc-$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a-}{c+}\right.\right.\right.$$

$$\downarrow 49$$

$$(bc-$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a-}{c+}\right.\right.\right.$$

$$\downarrow 2009$$

$$(bc-$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a-}{c+}\right.\right.\right.$$

$$\downarrow 2781$$

$$(bc -$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b - \frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a\cdots}{c\cdots}\right.\right.}{4d^4\left(b-\frac{d(a+b\cdots}{c+d\cdots}\right.}\right.}{}\right.$$

$$\downarrow 2784$$

$$(bc-$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a-}{c+}\right.\right.\right.$$

$$\downarrow 2784$$

$$(bc -$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a-}{c+}\right.\right.}{4d^4\left(b-\frac{d(a+b}{c+d}\right.}\right.}{}\right.$$

$\downarrow$ 2784

$$(bc-ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \right.$$

2754

$$(bc -$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b - \frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a-}{c+}\right.\right.}{4d^4\left(b-\frac{d(a+b}{c+d}\right.}\right.}{}\right.$$

$$\downarrow 2838$$

$$(bc-ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \right.$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
,x]
```

output
```
(b*c - a*d)^7*g^3*i^3*(((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(7*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^7) - (2*B*((b^3*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]))/(6*d^4*(b - (d*(a + b*x))/(c + d*x))^6) - (3*
b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*d^4*(b - (d*(a + b*x))/(c + d
*x))^5) + (3*b*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*d^4*(b - (d*(a + b
*x))/(c + d*x))^4) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(3*d^4*(b - (d*(
a + b*x))/(c + d*x))^3) + (B*((-2*b^2)/(b - (d*(a + b*x))/(c + d*x))^5 + (
13*b)/(2*(b - (d*(a + b*x))/(c + d*x))^4) - 19/(3*(b - (d*(a + b*x))/(c +
d*x))^3) + 1/(2*b*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^2*(b - (d*(a + b
*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^3 - Log[b - (d*(a + b*x))/(c
 + d*x)]/b^3))/(60*d^4)))/(7*b) + (3*(((a + b*x)^4*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])^2)/(6*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^6) - (B*((
(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*b*(c + d*x)^4*(b - (d
*(a + b*x))/(c + d*x))^5) + ((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(20*b^2*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) - (B*((a + b*x)^
4/((c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) + b^3/(3*d^4*(b - (d*(a +
b*x))/(c + d*x))^3) - (3*b^2)/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (3
*b)/(d^4*(b - (d*(a + b*x))/(c + d*x))) + Log[b - (d*(a + b*x))/(c + d*x)]
/d^4))/(20*b^2)))/(3*b) + (((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)
])^2)/(5*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*(((a + b...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))  Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2754

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))  Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))  Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])  u, x] - Simp[b*n  Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)^3 (dix+ci)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)^3(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="fricas")
```

output
```
integral(A^2*b^3*d^3*g^3*i^3*x^6 + A^2*a^3*c^3*g^3*i^3 + 3*(A^2*b^3*c*d^2
+ A^2*a*b^2*d^3)*g^3*i^3*x^5 + 3*(A^2*b^3*c^2*d + 3*A^2*a*b^2*c*d^2 + A^2*
a^2*b*d^3)*g^3*i^3*x^4 + (A^2*b^3*c^3 + 9*A^2*a*b^2*c^2*d + 9*A^2*a^2*b*c*
d^2 + A^2*a^3*d^3)*g^3*i^3*x^3 + 3*(A^2*a*b^2*c^3 + 3*A^2*a^2*b*c^2*d + A^
2*a^3*c*d^2)*g^3*i^3*x^2 + 3*(A^2*a^2*b*c^3 + A^2*a^3*c^2*d)*g^3*i^3*x + (
B^2*b^3*d^3*g^3*i^3*x^6 + B^2*a^3*c^3*g^3*i^3 + 3*(B^2*b^3*c*d^2 + B^2*a*b
^2*d^3)*g^3*i^3*x^5 + 3*(B^2*b^3*c^2*d + 3*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3
)*g^3*i^3*x^4 + (B^2*b^3*c^3 + 9*B^2*a*b^2*c^2*d + 9*B^2*a^2*b*c*d^2 + B^2
*a^3*d^3)*g^3*i^3*x^3 + 3*(B^2*a*b^2*c^3 + 3*B^2*a^2*b*c^2*d + B^2*a^3*c*d
^2)*g^3*i^3*x^2 + 3*(B^2*a^2*b*c^3 + B^2*a^3*c^2*d)*g^3*i^3*x)*log((b*e*x
+ a*e)/(d*x + c))^2 + 2*(A*B*b^3*d^3*g^3*i^3*x^6 + A*B*a^3*c^3*g^3*i^3 + 3
*(A*B*b^3*c*d^2 + A*B*a*b^2*d^3)*g^3*i^3*x^5 + 3*(A*B*b^3*c^2*d + 3*A*B*a*
b^2*c*d^2 + A*B*a^2*b*d^3)*g^3*i^3*x^4 + (A*B*b^3*c^3 + 9*A*B*a*b^2*c^2*d
+ 9*A*B*a^2*b*c*d^2 + A*B*a^3*d^3)*g^3*i^3*x^3 + 3*(A*B*a*b^2*c^3 + 3*A*B*
a^2*b*c^2*d + A*B*a^3*c*d^2)*g^3*i^3*x^2 + 3*(A*B*a^2*b*c^3 + A*B*a^3*c^2*
d)*g^3*i^3*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6921 vs. $2(1042) = 2084$.

Time $= 0.25$ (sec) , antiderivative size $= 6921$, normalized size of antiderivative $= 6.36$

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="maxima")
```

output
```
1/7*A^2*b^3*d^3*g^3*i^3*x^7 + 1/2*A^2*b^3*c*d^2*g^3*i^3*x^6 + 1/2*A^2*a*b^
2*d^3*g^3*i^3*x^6 + 3/5*A^2*b^3*c^2*d*g^3*i^3*x^5 + 9/5*A^2*a*b^2*c*d^2*g^
3*i^3*x^5 + 3/5*A^2*a^2*b*d^3*g^3*i^3*x^5 + 1/4*A^2*b^3*c^3*g^3*i^3*x^4 +
9/4*A^2*a*b^2*c^2*d*g^3*i^3*x^4 + 9/4*A^2*a^2*b*c*d^2*g^3*i^3*x^4 + 1/4*A^
2*a^3*d^3*g^3*i^3*x^4 + A^2*a*b^2*c^3*g^3*i^3*x^3 + 3*A^2*a^2*b*c^2*d*g^3*
i^3*x^3 + A^2*a^3*c*d^2*g^3*i^3*x^3 + 3/2*A^2*a^2*b*c^3*g^3*i^3*x^2 + 3/2*
A^2*a^3*c^2*d*g^3*i^3*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*
log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^3*c^3*g^3*i^3 + 3*(x^2*log(b*e*x/
(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 -
 (b*c - a*d)*x/(b*d))*A*B*a^2*b*c^3*g^3*i^3 + (2*x^3*log(b*e*x/(d*x + c) +
 a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*
c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*b^2*c^3*g^3
*i^3 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x +
a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*
c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^3*c^3*g
^3*i^3 + 3*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^
2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^3*c^2*d*g^3*i^3 + 3*
(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c
^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/
(b^2*d^2))*A*B*a^2*b*c^2*d*g^3*i^3 + 3/4*(6*x^4*log(b*e*x/(d*x + c) + a...
```

Giac [F]

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$=\int (bgx+ag)^3(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="giac")
```

output
```
integrate((b*g*x + a*g)^3*(d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) +
A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)^3\,(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2
,x)
```

output
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2
, x)
```

Reduce [F]

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
(g**3*i*(36*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a**7*b**2*d**8 - 252*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c
+ a*d*x + b*c*x + b*d*x**2),x)*a**6*b**3*c*d**7 + 756*int((log((a*e + b*e*
x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**5*b**4*c**2*d**6 -
 1260*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2
),x)*a**4*b**5*c**3*d**5 + 1260*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c
+ a*d*x + b*c*x + b*d*x**2),x)*a**3*b**6*c**4*d**4 - 756*int((log((a*e + b
*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**7*c**5*d**
3 + 252*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x*
*2),x)*a*b**8*c**6*d**2 - 36*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a
*d*x + b*c*x + b*d*x**2),x)*b**9*c**7*d + 36*log(a + b*x)*a**8*d**7 - 252*
log(a + b*x)*a**7*b*c*d**6 + 756*log(a + b*x)*a**6*b**2*c**2*d**5 - 1260*l
og(a + b*x)*a**5*b**3*c**3*d**4 + 1260*log(a + b*x)*a**4*b**4*c**4*d**3 -
756*log(a + b*x)*a**3*b**5*c**5*d**2 + 252*log(a + b*x)*a**2*b**6*c**6*d -
 36*log(a + b*x)*a*b**7*c**7 - 18*log((a*e + b*e*x)/(c + d*x))**2*a**6*b**
2*c*d**6 + 108*log((a*e + b*e*x)/(c + d*x))**2*a**5*b**3*c**2*d**5 - 270*l
og((a*e + b*e*x)/(c + d*x))**2*a**4*b**4*c**3*d**4 - 270*log((a*e + b*e*x)
/(c + d*x))**2*a**3*b**5*c**4*d**3 - 2520*log((a*e + b*e*x)/(c + d*x))**2*
a**3*b**5*c**3*d**4*x - 3780*log((a*e + b*e*x)/(c + d*x))**2*a**3*b**5*c**
2*d**5*x**2 - 2520*log((a*e + b*e*x)/(c + d*x))**2*a**3*b**5*c*d**6*x**...
```

3.75 $\int (ag+bgx)^2(ci+dix)^3 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result 944
Mathematica [A] (verified) 945
Rubi [A] (verified) 946
Maple [F] 960
Fricas [F] 960
Sympy [F(-1)] 961
Maxima [B] (verification not implemented) 961
Giac [F] 962
Mupad [F(-1)] 963
Reduce [F] 963

Optimal result

Integrand size = 42, antiderivative size = 908

$$
\begin{aligned}
&\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= -\frac{7B^2(bc-ad)^5g^2i^3x}{180b^3d^2} - \frac{7B^2(bc-ad)^4g^2i^3(c+dx)^2}{360b^2d^3} \\
&\quad - \frac{B^2(bc-ad)^3g^2i^3(c+dx)^3}{60bd^3} + \frac{B^2(bc-ad)^2g^2i^3(c+dx)^4}{60d^3} \\
&\quad + \frac{B^2(bc-ad)^6g^2i^3\log\left(\frac{a+bx}{c+dx}\right)}{36b^4d^3} - \frac{B(bc-ad)^4g^2i^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^4d} \\
&\quad - \frac{B(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^4} \\
&\quad - \frac{B(bc-ad)^4g^2i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^2d^3} \\
&\quad + \frac{B(bc-ad)^3g^2i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{45bd^3} \\
&\quad + \frac{7B(bc-ad)^2g^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60d^3} \\
&\quad - \frac{bB(bc-ad)g^2i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3} \\
&\quad + \frac{(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{60b^4} \\
&\quad + \frac{(bc-ad)^2g^2i^3(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^3} \\
&\quad + \frac{(bc-ad)g^2i^3(a+bx)^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2} \\
&\quad + \frac{g^2i^3(a+bx)^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b} \\
&\quad + \frac{B(bc-ad)^5g^2i^3(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^4d^2} \\
&\quad + \frac{B(bc-ad)^6g^2i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^4d^3} \\
&\quad + \frac{11B^2(bc-ad)^6g^2i^3\log(c+dx)}{180b^4d^3} + \frac{B^2(bc-ad)^6g^2i^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{30b^4d^3}
\end{aligned}
$$

output

```
-7/180*B^2*(-a*d+b*c)^5*g^2*i^3*x/b^3/d^2-7/360*B^2*(-a*d+b*c)^4*g^2*i^3*(
d*x+c)^2/b^2/d^3-1/60*B^2*(-a*d+b*c)^3*g^2*i^3*(d*x+c)^3/b/d^3+1/60*B^2*(-
a*d+b*c)^2*g^2*i^3*(d*x+c)^4/d^3+1/36*B^2*(-a*d+b*c)^6*g^2*i^3*ln((b*x+a)/
(d*x+c))/b^4/d^3-1/60*B*(-a*d+b*c)^4*g^2*i^3*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(
d*x+c)))/b^4/d-1/30*B*(-a*d+b*c)^3*g^2*i^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*
x+c)))/b^4-1/10*B*(-a*d+b*c)^4*g^2*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)
))/b^2/d^3+1/45*B*(-a*d+b*c)^3*g^2*i^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)
))/b/d^3+7/60*B*(-a*d+b*c)^2*g^2*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))
/d^3-1/15*b*B*(-a*d+b*c)*g^2*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3
+1/60*(-a*d+b*c)^3*g^2*i^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^4+1/2
0*(-a*d+b*c)^2*g^2*i^3*(b*x+a)^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^3
+1/10*(-a*d+b*c)*g^2*i^3*(b*x+a)^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2
/b^2+1/6*g^2*i^3*(b*x+a)^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/60*
B*(-a*d+b*c)^5*g^2*i^3*(b*x+a)*(2*A+B+2*B*ln(e*(b*x+a)/(d*x+c)))/b^4/d^2+1
/60*B*(-a*d+b*c)^6*g^2*i^3*ln((-a*d+b*c)/b/(d*x+c))*(2*A+3*B+2*B*ln(e*(b*x
+a)/(d*x+c)))/b^4/d^3+11/180*B^2*(-a*d+b*c)^6*g^2*i^3*ln(d*x+c)/b^4/d^3+1/
30*B^2*(-a*d+b*c)^6*g^2*i^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^3
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 1555, normalized size of antiderivative = 1.71

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input

```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^2,x]
```

output

```
(g^2*i^3*(15*(b*c - a*d)^2*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2 - 24*b*(b*c - a*d)*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
+ 10*b^2*(c + d*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (5*B*(b*c -
a*d)^3*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*(b*d*x + (b*c - a*d)*L
og[a + b*x]) - B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b
*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x)
)/(c + d*x)] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c +
 d*x)]) + 2*b^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c
- a*d)^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c - a*
d)^3*Log[c + d*x] - 3*B*(b*c - a*d)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[
(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))
)/b^4 + (2*B*(b*c - a*d)^2*(24*A*b*d*(b*c - a*d)^3*x - 12*B*(b*c - a*d)^3*
(b*d*x + (b*c - a*d)*Log[a + b*x]) - 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*
x + b^2*(c + d*x)^2 + 2*(b*c - a*d)^2*Log[a + b*x]) - B*(b*c - a*d)*(6*b*d
*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(
b*c - a*d)^3*Log[a + b*x]) + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e*(a + b*
x))/(c + d*x)] + 12*b^2*(b*c - a*d)^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))
/(c + d*x)]) + 8*b^3*(b*c - a*d)*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c +
 d*x)]) + 6*b^4*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 24*(b*c
 - a*d)^4*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 24*B*(b*c...
```

Rubi [A] (verified)

Time = 3.04 (sec) , antiderivative size = 1213, normalized size of antiderivative = 1.34, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2962, 2783, 2782, 27, 1195, 2009, 2783, 2782, 27, 1195, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\downarrow 2962$$

$$g^2i^3(bc-ad)^6\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(-\frac{B\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{3b} + \frac{\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{2b} + \frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right.}$$

$$\downarrow 2782$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(-\frac{B\left(-B\int \frac{(c+dx)\left(b^2-\frac{5d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{30d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx} + \frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\log\left(\frac{e(a+}{c+d}\right.}{3b}$$

$$\downarrow 27$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(-\frac{B\left(-\frac{B\int \frac{(c+dx)\left(b^2-\frac{5d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{30d^3} + \frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right.}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right.}}{3b}$$

$$\downarrow 1195$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(-\frac{B\left(-\frac{B\int \left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)} + \frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} + \frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{9d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{6bd}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{c+dx}{b^3(a+bx)} \right) d\frac{a+bx}{c+dx}}{30d^3} + \frac{b^2\left(B\log\right.}{5d^3\left(\right.}}{3b}$$

$$\downarrow 2009$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(\frac{\int \frac{(a+bx)^2 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{2b} - \frac{B\left(\frac{b^2\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{b\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \right.$$

$$\downarrow 2783$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(\frac{-\frac{2B \int \frac{(a+bx)^2 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{2 \int \frac{(a+bx)^2 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{(a+bx)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^5}}{2b} - \frac{B\left(\right.}{}$$

$$\downarrow 2782$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(\frac{2B\left(-B \int \frac{(c+dx)\left(b^2 - \frac{4d(a+bx)b}{c+dx} + \frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{12d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx} + \frac{b^2\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2b\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right)}{5b} + \right.$$

$$\downarrow 27$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(\frac{2B\left(-\frac{B\int \frac{(c+dx)\left(b^2-\frac{4d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{12d^3} + \frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2} \right)}{5b} + \right.$$

$$\left. \frac{}{2b} \right.$$

$$\downarrow 1195$$

$$(bc -$$

$$ad)^6 g^2 i^3 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} \right.}{} \right.$$

$$\downarrow 2009$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(\frac{2\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} - \frac{2B\left(\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} \right.}{} \right.}{5b} \right.$$

$$\left. \frac{}{2b} \right.$$

$$\downarrow 2783$$

$$(bc -$$

$$ad)^6 g^2 i^3 \left(\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3 \left(b - \frac{d(a+bx)}{c+dx}\right)^6} - \frac{B \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} \right.$$

$$\downarrow 2773$$

$$(bc -$$

$$ad)^6 g^2 i^3 \left(\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3 \left(b - \frac{d(a+bx)}{c+dx}\right)^6} - \frac{B \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} \right.$$

$$\downarrow 49$$

$$(bc-ad)^6 g^2 i^3 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} \right.$$

↓ 2009

$$(bc -$$

$$ad)^6 g^2 i^3 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right.}{} \right.$$

$\downarrow$ 2781

$$(bc-ad)^6 g^2 i^3 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} \right.$$

↓ 2784

$$(bc-ad)^6 g^2 i^3 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} \right.$$

↓ 2784

$$(bc-ad)^6 g^2 i^3 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} \right.$$

↓ 2754

$$(bc-ad)^6 g^2 i^3 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} \right.$$

↓ 2838

$$(bc-ad)^6 g^2 i^3 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} \right.$$

input

```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
,x]
```

output
```
(b*c - a*d)^6*g^2*i^3*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(6*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^6) - (B*((b^2*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/(5*d^3*(b - (d*(a + b*x))/(c + d*x))^5) - (b*(A
 + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^4
) + (A + B*Log[(e*(a + b*x))/(c + d*x)])/(3*d^3*(b - (d*(a + b*x))/(c + d*
x))^3) - (B*((3*b)/(2*(b - (d*(a + b*x))/(c + d*x))^4) - 3/(b - (d*(a + b*
x))/(c + d*x))^3 + 1/(2*b*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^2*(b - (
d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^3 - Log[b - (d*(a + 
b*x))/(c + d*x)]/b^3))/(30*d^3)))/(3*b) + (((a + b*x)^3*(A + B*Log[(e*(a + 
 b*x))/(c + d*x)])^2)/(5*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^5) - 
(2*B*((b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*d^3*(b - (d*(a + b*x))
/(c + d*x))^4) - (2*b*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*d^3*(b - (d
*(a + b*x))/(c + d*x))^3) + (A + B*Log[(e*(a + b*x))/(c + d*x)])/(2*d^3*(b
 - (d*(a + b*x))/(c + d*x))^2) - (B*(b/(b - (d*(a + b*x))/(c + d*x))^3 - 5
/(2*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b*(b - (d*(a + b*x))/(c + d*x)))
 + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(
12*d^3)))/(5*b) + (2*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)
/(4*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^4) - (B*(((a + b*x)^3*(A + 
 B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + 
 d*x))^3) - (B*(b^2/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^2) - (2*b)/(d^...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)^2 (dix+ci)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2 (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)^2 (dix+ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="fricas")
```

output
```
integral(A^2*b^2*d^3*g^2*i^3*x^5 + A^2*a^2*c^3*g^2*i^3 + (3*A^2*b^2*c*d^2
+ 2*A^2*a*b*d^3)*g^2*i^3*x^4 + (3*A^2*b^2*c^2*d + 6*A^2*a*b*c*d^2 + A^2*a^
2*d^3)*g^2*i^3*x^3 + (A^2*b^2*c^3 + 6*A^2*a*b*c^2*d + 3*A^2*a^2*c*d^2)*g^2
*i^3*x^2 + (2*A^2*a*b*c^3 + 3*A^2*a^2*c^2*d)*g^2*i^3*x + (B^2*b^2*d^3*g^2*
i^3*x^5 + B^2*a^2*c^3*g^2*i^3 + (3*B^2*b^2*c*d^2 + 2*B^2*a*b*d^3)*g^2*i^3*
x^4 + (3*B^2*b^2*c^2*d + 6*B^2*a*b*c*d^2 + B^2*a^2*d^3)*g^2*i^3*x^3 + (B^2
*b^2*c^3 + 6*B^2*a*b*c^2*d + 3*B^2*a^2*c*d^2)*g^2*i^3*x^2 + (2*B^2*a*b*c^3
 + 3*B^2*a^2*c^2*d)*g^2*i^3*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^2
*d^3*g^2*i^3*x^5 + A*B*a^2*c^3*g^2*i^3 + (3*A*B*b^2*c*d^2 + 2*A*B*a*b*d^3)
*g^2*i^3*x^4 + (3*A*B*b^2*c^2*d + 6*A*B*a*b*c*d^2 + A*B*a^2*d^3)*g^2*i^3*x
^3 + (A*B*b^2*c^3 + 6*A*B*a*b*c^2*d + 3*A*B*a^2*c*d^2)*g^2*i^3*x^2 + (2*A*
B*a*b*c^3 + 3*A*B*a^2*c^2*d)*g^2*i^3*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5196 vs. $2(869) = 1738$.

Time $= 0.20$ (sec) , antiderivative size $= 5196$, normalized size of antiderivative $= 5.72$

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="maxima")
```

output
```
1/6*A^2*b^2*d^3*g^2*i^3*x^6 + 3/5*A^2*b^2*c*d^2*g^2*i^3*x^5 + 2/5*A^2*a*b*
d^3*g^2*i^3*x^5 + 3/4*A^2*b^2*c^2*d*g^2*i^3*x^4 + 3/2*A^2*a*b*c*d^2*g^2*i^
3*x^4 + 1/4*A^2*a^2*d^3*g^2*i^3*x^4 + 1/3*A^2*b^2*c^3*g^2*i^3*x^3 + 2*A^2*
a*b*c^2*d*g^2*i^3*x^3 + A^2*a^2*c*d^2*g^2*i^3*x^3 + A^2*a*b*c^3*g^2*i^3*x^
2 + 3/2*A^2*a^2*c^2*d*g^2*i^3*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x +
c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^2*c^3*g^2*i^3 + 2*(x^2*lo
g(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x +
c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a*b*c^3*g^2*i^3 + 1/3*(2*x^3*log(b*e*x/(
d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^
3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2
*c^3*g^2*i^3 + 3*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x +
 a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^2*c^2*d*g^2*i^
3 + 2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3
 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^
2)*x)/(b^2*d^2))*A*B*a*b*c^2*d*g^2*i^3 + 1/4*(6*x^4*log(b*e*x/(d*x + c) +
a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3
*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3
*d^3)*x)/(b^3*d^3))*A*B*b^2*c^2*d*g^2*i^3 + (2*x^3*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*c*d^2*g^...
```

Giac [F]

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)^2(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="giac")
```

output
```
integrate((b*g*x + a*g)^2*(d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) +
A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2
,x)
```

output
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2
, x)
```

Reduce [F]

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
(g**2*i*(12*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a**6*b**2*d**7 - 72*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c +
 a*d*x + b*c*x + b*d*x**2),x)*a**5*b**3*c*d**6 + 180*int((log((a*e + b*e*x
)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**4*b**4*c**2*d**5 -
240*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),
x)*a**3*b**5*c**3*d**4 + 180*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a
*d*x + b*c*x + b*d*x**2),x)*a**2*b**6*c**4*d**3 - 72*int((log((a*e + b*e*x
)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**7*c**5*d**2 + 12*
int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b
**8*c**6*d + 12*log(a + b*x)*a**7*d**6 - 72*log(a + b*x)*a**6*b*c*d**5 - 4
*log(a + b*x)*a**6*b*d**6 + 180*log(a + b*x)*a**5*b**2*c**2*d**4 + 24*log(
a + b*x)*a**5*b**2*c*d**5 - 240*log(a + b*x)*a**4*b**3*c**3*d**3 - 60*log(
a + b*x)*a**4*b**3*c**2*d**4 + 180*log(a + b*x)*a**3*b**4*c**4*d**2 + 80*l
og(a + b*x)*a**3*b**4*c**3*d**3 - 72*log(a + b*x)*a**2*b**5*c**5*d - 60*lo
g(a + b*x)*a**2*b**5*c**4*d**2 + 12*log(a + b*x)*a*b**6*c**6 + 24*log(a +
b*x)*a*b**6*c**5*d - 4*log(a + b*x)*b**7*c**6 - 6*log((a*e + b*e*x)/(c + d
*x))**2*a**5*b**2*c*d**5 + 30*log((a*e + b*e*x)/(c + d*x))**2*a**4*b**3*c*
*2*d**4 - 60*log((a*e + b*e*x)/(c + d*x))**2*a**3*b**4*c**3*d**3 - 30*log(
(a*e + b*e*x)/(c + d*x))**2*a**2*b**5*c**4*d**2 - 360*log((a*e + b*e*x)/(c
 + d*x))**2*a**2*b**5*c**3*d**3*x - 540*log((a*e + b*e*x)/(c + d*x))**2...
```

3.76 $\int (ag+bgx)(ci+dix)^3 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result 966
Mathematica [A] (verified) 967
Rubi [A] (verified) 968
Maple [F] 982
Fricas [F] 982
Sympy [F(-1)] 983
Maxima [B] (verification not implemented) 983
Giac [F] 984
Mupad [F(-1)] 985
Reduce [F] 985

Optimal result

Integrand size = 40, antiderivative size = 730

$$
\begin{aligned}
&\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^4gi^3x}{60b^3d} + \frac{B^2(bc-ad)^3gi^3(c+dx)^2}{30b^2d^2} + \frac{B^2(bc-ad)^2gi^3(c+dx)^3}{30bd^2} \\
&\quad - \frac{B^2(bc-ad)^5gi^3\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d^2} - \frac{B(bc-ad)^4gi^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4d} \\
&\quad - \frac{B(bc-ad)^3gi^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4} \\
&\quad + \frac{3B(bc-ad)^3gi^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^2d^2} \\
&\quad + \frac{B(bc-ad)^2gi^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^2} \\
&\quad - \frac{B(bc-ad)gi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^2} \\
&\quad + \frac{(bc-ad)^3gi^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^4} \\
&\quad + \frac{(bc-ad)^2gi^3(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^3} \\
&\quad + \frac{3(bc-ad)gi^3(a+bx)^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2} \\
&\quad + \frac{gi^3(a+bx)^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} \\
&\quad - \frac{B(bc-ad)^5gi^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4d^2} \\
&\quad - \frac{11B^2(bc-ad)^5gi^3\log(c+dx)}{60b^4d^2} - \frac{B^2(bc-ad)^5gi^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{10b^4d^2}
\end{aligned}
$$

output

```
1/60*B^2*(-a*d+b*c)^4*g*i^3*x/b^3/d+1/30*B^2*(-a*d+b*c)^3*g*i^3*(d*x+c)^2/
b^2/d^2+1/30*B^2*(-a*d+b*c)^2*g*i^3*(d*x+c)^3/b/d^2-1/12*B^2*(-a*d+b*c)^5*
g*i^3*ln((b*x+a)/(d*x+c))/b^4/d^2-1/10*B*(-a*d+b*c)^4*g*i^3*(b*x+a)*(A+B*l
n(e*(b*x+a)/(d*x+c)))/b^4/d-1/10*B*(-a*d+b*c)^3*g*i^3*(b*x+a)^2*(A+B*ln(e*
(b*x+a)/(d*x+c)))/b^4+3/20*B*(-a*d+b*c)^3*g*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a
)/(d*x+c)))/b^2/d^2+1/30*B*(-a*d+b*c)^2*g*i^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/
(d*x+c)))/b/d^2-1/10*B*(-a*d+b*c)*g*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c
)))/d^2+1/20*(-a*d+b*c)^3*g*i^3*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^
4+1/10*(-a*d+b*c)^2*g*i^3*(b*x+a)^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/
b^3+3/20*(-a*d+b*c)*g*i^3*(b*x+a)^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^
2/b^2+1/5*g*i^3*(b*x+a)^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b-1/10*B
*(-a*d+b*c)^5*g*i^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B+B*ln(e*(b*x+a)/(d*x+c)))
/b^4/d^2-11/60*B^2*(-a*d+b*c)^5*g*i^3*ln(d*x+c)/b^4/d^2-1/10*B^2*(-a*d+b*c
)^5*g*i^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^2
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 901, normalized size of antiderivative = 1.23

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$

$$= \frac{gi^3\left(-5(bc-ad)(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+4b(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{5B(bc-ad)^2\left(6A\right.}{}\right.}{}$$

input

```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)
])^2,x]
```

output

```
(g*i^3*(-5*(b*c - a*d)*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
+ 4*b*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (5*B*(b*c - a*d
)^2*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*(b*d*x + (b*c - a*d)*Log[
a + b*x]) - B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c
 - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(
c + d*x)] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)]) + 2*b^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c - a
*d)^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c - a*d)^
3*Log[c + d*x] - 3*B*(b*c - a*d)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*
(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(
3*b^4) - (B*(b*c - a*d)*(24*A*b*d*(b*c - a*d)^3*x - 12*B*(b*c - a*d)^3*(b*
d*x + (b*c - a*d)*Log[a + b*x]) - 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*x +
 b^2*(c + d*x)^2 + 2*(b*c - a*d)^2*Log[a + b*x]) - B*(b*c - a*d)*(6*b*d*(b
*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c
 - a*d)^3*Log[a + b*x]) + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e*(a + b*x))
/(c + d*x)] + 12*b^2*(b*c - a*d)^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c
 + d*x)]) + 8*b^3*(b*c - a*d)*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)]) + 6*b^4*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 24*(b*c -
a*d)^4*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 24*B*(b*c - a*d
)^4*Log[c + d*x] - 12*B*(b*c - a*d)^4*(Log[a + b*x]*(Log[a + b*x] - 2*L...
```

Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 959, normalized size of antiderivative = 1.31, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.475$, Rules used = {2962, 2783, 2782, 27, 86, 2009, 2783, 2782, 27, 86, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\downarrow 2962$$

$$gi^3(bc-ad)^5\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$gi^3(bc-$$

$$ad)^5\left(-\frac{2B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{3\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right.}$$

$$\downarrow 2782$$

$$gi^3(bc-$$

$$ad)^5\left(-\frac{2B\left(-B\int-\frac{(c+dx)\left(b-\frac{4d(a+bx)}{c+dx}\right)}{12d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{5b}+\frac{3\int\frac{(a+bx)\left(A+B\log\right.}{(c+dx)\left(b-\frac{d}{5b}\right.}$$

$$\downarrow 27$$

$$gi^3(bc-$$

$$ad)^5\left(-\frac{2B\left(\frac{B\int\frac{(c+dx)\left(b-\frac{4d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{12d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{5b}+\frac{3\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}}{5b}$$

$$\downarrow 86$$

$$gi^3(bc-$$

$$ad)^5\left(-\frac{2B\left(\frac{B\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a+bx}{c+dx}}{12d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(\right.}{}\right.}{5b}$$

$$\downarrow 2009$$

$$ad)^5\left(\frac{3\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}-\frac{gi^3(bc-\quad 2B\left(-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\frac{d}{c}\right.\right.}{b^3}\right.\right.}{5b}\right.$$

$$\downarrow 2783$$

$$ad)^5\left(\frac{3\left(-\frac{B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{5b}-\frac{gi^3(bc-\quad 2B}{}\right.$$

$$\downarrow 2782$$

$$ad)^5\left(\frac{3\left(-\frac{B\left(-B\int-\frac{(c+dx)\left(b-\frac{3d(a+bx)}{c+dx}\right)}{6d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d}{2b}\right.}{5b}\right.$$

$$\downarrow 27$$

$$gi^3(bc-$$

$$ad)^5\left(\frac{3\left(-\frac{B\left(\frac{B\int\frac{(c+dx)\left(b-\frac{3d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{6d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}-\right.}{5b}\right.$$

$$\downarrow 86$$

$$gi^3(bc-$$

$$ad)^5\left(\frac{3\left(-\frac{B\left(\frac{B\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{6d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}-\right.}{5b}\right.$$

$$\downarrow 2009$$

$$gi^3(bc-ad)^5\left(\frac{3\left(\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}-\frac{B\left(-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{\cdots}{b(\cdots}\right.}{6d^2\cdots}\right.}{2b}\right.}{5b}\right.$$

$$\Big\downarrow \ 2783$$

$$gi^3(bc-ad)^5\left(\frac{3\left(\frac{-\frac{2B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{2b}-\frac{B\left(-\frac{B\log\left(\cdots\right.}{2d^2\left(b\cdots\right.}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow \ 2773$$

$$(bc-$$

$$ad)^5 gi^3 \left(\frac{(a+bx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3} - \frac{\log\left(b-\right.\right.}{} \right.}{5b} \right.$$

$$\downarrow 49$$

$$(bc -$$

$$ad)^5 gi^3 \left(\frac{(a+bx)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b(c+dx)^2 \left(b - \frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3} - \frac{\log\left(b-\right.}{}\right.}{}\right.}{5b}\right.$$

$\downarrow$ 2009

$$(bc -$$

$$ad)^5 gi^3 \left(\frac{(a+bx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3} - \frac{\log\left(b-\right.}{}\right.}{}\right.}{5b}\right.$$

$\downarrow$ 2781

$$(bc-$$

$$ad)^5 gi^3 \left(\frac{(a+bx)^2 \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B \left(-\frac{A+B \log \left(\frac{e(a+bx)}{c+dx}\right)}{3d^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\left(\frac{\log \left(\frac{a+bx}{c+dx}\right)}{b^3} - \frac{\log \left(b-\right.\right.}{5b} \right.$$

↓ 2784

$$(bc -$$

$$ad)^5 gi^3 \left(\frac{(a+bx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3} - \frac{\log\left(b-\right.}{}\right.}{}\right.}{5b}\right.$$

$\downarrow$ 2754

$$(bc-$$

$$ad)^5 gi^3 \left(\frac{(a+bx)^2 \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3} - \frac{\log\left(b-\right.\right.}{} \right.}{5b} \right.$$

↓ 2838

$$(bc-$$

$$ad)^5 gi^3 \left(\frac{(a+bx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3} - \frac{\log\left(b-\right.\right.}{} \right.}{5b} \right.$$

input

```
Int[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x
]
```

output

```
(b*c - a*d)^5*g*i^3*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/
(5*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*((b*(A + B*Log[(e
*(a + b*x))/(c + d*x)]))/(4*d^2*(b - (d*(a + b*x))/(c + d*x))^4) - (A + B*
Log[(e*(a + b*x))/(c + d*x)])/(3*d^2*(b - (d*(a + b*x))/(c + d*x))^3) + (B
*(-(b - (d*(a + b*x))/(c + d*x))^(-3) + 1/(2*b*(b - (d*(a + b*x))/(c + d*x
))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b
^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(12*d^2)))/(5*b) + (3*(((a + b
*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*b*(c + d*x)^2*(b - (d*(a
+ b*x))/(c + d*x))^4) - (B*((b*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*d^
2*(b - (d*(a + b*x))/(c + d*x))^3) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/
(2*d^2*(b - (d*(a + b*x))/(c + d*x))^2) + (B*(-(b - (d*(a + b*x))/(c + d*x
))^(-2) + 1/(b*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b
^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(6*d^2)))/(2*b) + (((a + b*x)^
2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*b*(c + d*x)^2*(b - (d*(a + b*
x))/(c + d*x))^3) - (2*B*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(b/(d^2*(b - (d*
(a + b*x))/(c + d*x))) + Log[b - (d*(a + b*x))/(c + d*x)]/d^2))/(2*b)))/(3
*b) + (((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*b*(c + d*x)
^2*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(((a + b*x)*(A + B*Log[(e*(a + b*
x))/(c + d*x)]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((A +...
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 86

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)\,(dix+ci)^3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$=\int (bgx+ag)(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="fricas")
```

output
```
integral(A^2*b*d^3*g*i^3*x^4 + A^2*a*c^3*g*i^3 + (3*A^2*b*c*d^2 + A^2*a*d^
3)*g*i^3*x^3 + 3*(A^2*b*c^2*d + A^2*a*c*d^2)*g*i^3*x^2 + (A^2*b*c^3 + 3*A^
2*a*c^2*d)*g*i^3*x + (B^2*b*d^3*g*i^3*x^4 + B^2*a*c^3*g*i^3 + (3*B^2*b*c*d
^2 + B^2*a*d^3)*g*i^3*x^3 + 3*(B^2*b*c^2*d + B^2*a*c*d^2)*g*i^3*x^2 + (B^2
*b*c^3 + 3*B^2*a*c^2*d)*g*i^3*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b
*d^3*g*i^3*x^4 + A*B*a*c^3*g*i^3 + (3*A*B*b*c*d^2 + A*B*a*d^3)*g*i^3*x^3 +
 3*(A*B*b*c^2*d + A*B*a*c*d^2)*g*i^3*x^2 + (A*B*b*c^3 + 3*A*B*a*c^2*d)*g*i
^3*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3218 vs. $2(697) = 1394$.

Time $= 0.18$ (sec) , antiderivative size $= 3218$, normalized size of antiderivative $= 4.41$

$$\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="maxima")
```

output

```
1/5*A^2*b*d^3*g*i^3*x^5 + 3/4*A^2*b*c*d^2*g*i^3*x^4 + 1/4*A^2*a*d^3*g*i^3*
x^4 + A^2*b*c^2*d*g*i^3*x^3 + A^2*a*c*d^2*g*i^3*x^3 + 1/2*A^2*b*c^3*g*i^3*
x^2 + 3/2*A^2*a*c^2*d*g*i^3*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)
) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a*c^3*g*i^3 + (x^2*log(b*e*x/
(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 -
 (b*c - a*d)*x/(b*d))*A*B*b*c^3*g*i^3 + 3*(x^2*log(b*e*x/(d*x + c) + a*e/(
d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b
*d))*A*B*a*c^2*d*g*i^3 + (2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a
^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 -
2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b*c^2*d*g*i^3 + (2*x^3*log(b*e*x/(
d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^
3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*c
*d^2*g*i^3 + 1/4*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b
*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*
(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b*c*
d^2*g*i^3 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b
*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*
(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*d^
3*g*i^3 + 1/30*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b
*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 ...
```

Giac [F]

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="giac")
```

output

```
integrate((b*g*x + a*g)*(d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)
^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)\,(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x
)
```

output
```
int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,
x)
```

Reduce [F]

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
(g*i*(6*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x*
*2),x)*a**5*b**2*d**6 - 30*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d
*x + b*c*x + b*d*x**2),x)*a**4*b**3*c*d**5 + 60*int((log((a*e + b*e*x)/(c
+ d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**4*c**2*d**4 - 60*in
t((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**
2*b**5*c**3*d**3 + 30*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x +
b*c*x + b*d*x**2),x)*a*b**6*c**4*d**2 - 6*int((log((a*e + b*e*x)/(c + d*x)
)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**7*c**5*d + 6*log(a + b*x)*a**6
*d**5 - 30*log(a + b*x)*a**5*b*c*d**4 - 5*log(a + b*x)*a**5*b*d**5 + 60*lo
g(a + b*x)*a**4*b**2*c**2*d**3 + 25*log(a + b*x)*a**4*b**2*c*d**4 - 60*log
(a + b*x)*a**3*b**3*c**3*d**2 - 50*log(a + b*x)*a**3*b**3*c**2*d**3 + 30*l
og(a + b*x)*a**2*b**4*c**4*d + 50*log(a + b*x)*a**2*b**4*c**3*d**2 - 6*log
(a + b*x)*a*b**5*c**5 - 25*log(a + b*x)*a*b**5*c**4*d + 5*log(a + b*x)*b**
6*c**5 - 3*log((a*e + b*e*x)/(c + d*x))**2*a**4*b**2*c*d**4 + 12*log((a*e
+ b*e*x)/(c + d*x))**2*a**3*b**3*c**2*d**3 - 18*log((a*e + b*e*x)/(c + d*x
))**2*a**2*b**4*c**3*d**2 - 3*log((a*e + b*e*x)/(c + d*x))**2*a*b**5*c**4*
d - 60*log((a*e + b*e*x)/(c + d*x))**2*a*b**5*c**3*d**2*x - 90*log((a*e +
b*e*x)/(c + d*x))**2*a*b**5*c**2*d**3*x**2 - 60*log((a*e + b*e*x)/(c + d*x
))**2*a*b**5*c*d**4*x**3 - 15*log((a*e + b*e*x)/(c + d*x))**2*a*b**5*d**5*
x**4 - 30*log((a*e + b*e*x)/(c + d*x))**2*b**6*c**3*d**2*x**2 - 60*log(...
```

3.77 $\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result . . . 987
Mathematica [A] (verified) . . . 988
Rubi [A] (verified) . . . 989
Maple [F] . . . 997
Fricas [F] . . . 998
Sympy [F(-1)] . . . 998
Maxima [B] (verification not implemented) . . . 999
Giac [F] . . . 1000
Mupad [F(-1)] . . . 1000
Reduce [F] . . . 1000

Optimal result

Integrand size = 32, antiderivative size = 420

$$
\begin{aligned}
&\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= \frac{5B^2(bc-ad)^3i^3x}{12b^3} + \frac{B^2(bc-ad)^2i^3(c+dx)^2}{12b^2d} + \frac{5B^2(bc-ad)^4i^3\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d} \\
&\quad - \frac{B(bc-ad)^3i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^4} \\
&\quad - \frac{B(bc-ad)^2i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b^2d} \\
&\quad - \frac{B(bc-ad)i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd} \\
&\quad + \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4d} + \frac{11B^2(bc-ad)^4i^3\log(c+dx)}{12b^4d} \\
&\quad + \frac{B(bc-ad)^4i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d} \\
&\quad - \frac{B^2(bc-ad)^4i^3\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d}
\end{aligned}
$$

output
```
5/12*B^2*(-a*d+b*c)^3*i^3*x/b^3+1/12*B^2*(-a*d+b*c)^2*i^3*(d*x+c)^2/b^2/d+
5/12*B^2*(-a*d+b*c)^4*i^3*ln((b*x+a)/(d*x+c))/b^4/d-1/2*B*(-a*d+b*c)^3*i^3
*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4-1/4*B*(-a*d+b*c)^2*i^3*(d*x+c)^2*
(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d-1/6*B*(-a*d+b*c)*i^3*(d*x+c)^3*(A+B*ln(e
*(b*x+a)/(d*x+c)))/b/d+1/4*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d+1
1/12*B^2*(-a*d+b*c)^4*i^3*ln(d*x+c)/b^4/d+1/2*B*(-a*d+b*c)^4*i^3*(A+B*ln(e
*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/d-1/2*B^2*(-a*d+b*c)^4*i^
3*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/d
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.93

$$\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{i^3\left((c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B(bc-ad)\left(6Abd(bc-ad)^2x-3B(bc-ad)^2(bdx+(bc-ad)\log(a+bx))-B(bc-ad)(2bd(bc-\right.\right.}{}$$

input
```
Integrate[(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(i^3*((c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*
(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*(b*d*x + (b*c - a*d)*Log[a +
b*x]) - B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c - a*
d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c +
d*x)] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
 + 2*b^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c - a*d)^
3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c - a*d)^3*Lo
g[c + d*x] - 3*B*(b*c - a*d)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
 d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(3*b^
4)))/(4*d)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.23, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {2952, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\downarrow 2952$$

$$i^3(bc-ad)^4 \int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}$$

$$\downarrow 2756$$

$$i^3(bc-ad)^4 \left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{2d}\right)$$

$$\downarrow 2789$$

$$ad)^4 \left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B\left(\frac{d\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{b} + \frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3} d\frac{a+bx}{c+dx}}{b}\right)}{2d}\right) i^3(bc-$$

$$\downarrow 2756$$

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d}{b}\right)}{2d}\right)i^3(bc-$$

$$\downarrow 54$$

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{c+dx}{b^3(a+bx)}\right)}{3d}\right)}{b}\right)}{2d}\right)i^3(bc-$$

$$\downarrow 2009$$

$$
\begin{aligned}
&ad)^4 \left(\frac{\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{i^3(bc- \right. \\
&\qquad \left. B\left(\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3} d\frac{a+bx}{c+dx}}{b} + \frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right.}{b^3} \right.}{b} \right.}{2d} \right.
\end{aligned}
$$

$$\downarrow 2789$$

$$
\begin{aligned}
&ad)^4 \left(\frac{\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{i^3(bc- \right. \\
&\qquad \left. B\left(\frac{d\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3} d\frac{a+bx}{c+dx}}{b} + \frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2} d\frac{a+bx}{c+dx}}{b} + \frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} \right.}{b} \right.}{2d} \right.
\end{aligned}
$$

$$\downarrow 2756$$

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{i^3(bc-\qquad B\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+b}{c+d}}{b}\right.\right.\right.$$

$$\downarrow 54$$

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{i^3(bc-\qquad B\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)}{(a+\right.\right.\right.$$

$$\downarrow 2009$$

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{i^3(bc-}{B\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}\right.}{2d}\right.}{b}\right.}{b}\right.$$

$$\downarrow 2789$$

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{i^3(bc-}{B\left(\frac{\frac{d\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\right.}{b}\right.}{b}\right.$$

$$\downarrow 2751$$

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{i^3(bc-}{\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{B\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}\right.$$

$$\downarrow 16$$

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{i^3(bc-}{\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}+}{b}\right.$$

$$\downarrow 2779$$

$$i^3(bc-$$

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{\frac{\frac{B\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e}{\cdots}\right.\right.}{b(c+dx)\left(b-\cdots\right.}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow \; 2838$$

$$i^3(bc-$$

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{\frac{B\operatorname{PolyLog}\left(2,\frac{b(\cdots}{d(\cdots}\right.}{b}}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)^4*i^3*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(4*d*(b - (d*(a
+ b*x))/(c + d*x))^4) - (B*((d*((A + B*Log[(e*(a + b*x))/(c + d*x)])/(3*d*
(b - (d*(a + b*x))/(c + d*x))^3) - (B*(1/(2*b*(b - (d*(a + b*x))/(c + d*x)
)^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^
3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(3*d)))/b + ((d*((A + B*Log[(e*
(a + b*x))/(c + d*x)])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(1/(b*(b
 - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(
a + b*x))/(c + d*x)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*Log[(e*(a +
b*x))/(c + d*x)]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*Log[b
- (d*(a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[(e*(a + b*x))/(c +
d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*
x))/(d*(a + b*x))])/b)/b)/b)/b))/(2*d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
 - Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [F]

$$\int (dix + ci)^3 \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (dix+ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

output
```
integral(A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*c^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2*c^3*i^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1789 vs. $2(399) = 798$.

Time = 0.14 (sec) , antiderivative size = 1789, normalized size of antiderivative = 4.26

$$\int (ci + dix)^3 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxim
a")
```

output

```
1/4*A^2*d^3*i^3*x^4 + A^2*c*d^2*i^3*x^3 + 3/2*A^2*c^2*d*i^3*x^2 + 2*(x*log
(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A
*B*c^3*i^3 + 3*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a
)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*c^2*d*i^3 + (2*x^3
*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log
(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d
^2))*A*B*c*d^2*i^3 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*
a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)
*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))
*A*B*d^3*i^3 + A^2*c^3*i^3*x - 1/12*(26*a*b^2*c^3*d*i^3 - 21*a^2*b*c^2*d^2
*i^3 + 6*a^3*c*d^3*i^3 + (6*i^3*log(e) - 11*i^3)*b^3*c^4)*B^2*log(d*x + c)
/(b^3*d) - 1/2*(b^4*c^4*i^3 - 4*a*b^3*c^3*d*i^3 + 6*a^2*b^2*c^2*d^2*i^3 -
4*a^3*b*c*d^3*i^3 + a^4*d^4*i^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*
d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d) + 1/12*(3*B^2*b^4
*d^4*i^3*x^4*log(e)^2 + 2*(a*b^3*d^4*i^3*log(e) + (6*i^3*log(e)^2 - i^3*lo
g(e))*b^4*c*d^3)*B^2*x^3 + ((18*i^3*log(e)^2 - 9*i^3*log(e) + i^3)*b^4*c^2
*d^2 + 2*(6*i^3*log(e) - i^3)*a*b^3*c*d^3 - (3*i^3*log(e) - i^3)*a^2*b^2*d
^4)*B^2*x^2 + ((12*i^3*log(e)^2 - 18*i^3*log(e) + 7*i^3)*b^4*c^3*d + (36*i
^3*log(e) - 19*i^3)*a*b^3*c^2*d^2 - (24*i^3*log(e) - 17*i^3)*a^2*b^2*c*d^3
 + (6*i^3*log(e) - 5*i^3)*a^3*b*d^4)*B^2*x + 3*(B^2*b^4*d^4*i^3*x^4 + 4...
```

Giac [F]

$$\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (dix+ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac"
)
```

output
```
integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (c\,i+d\,i\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

Reduce [F]

$$\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x)
```

output

```
(i*(6*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2
),x)*a**4*b**2*d**5 - 24*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x
 + b*c*x + b*d*x**2),x)*a**3*b**3*c*d**4 + 36*int((log((a*e + b*e*x)/(c +
d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**4*c**2*d**3 - 24*int(
(log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**
5*c**3*d**2 + 6*int((log((a*e + b*e*x)/(c + d*x))*x)/(a*c + a*d*x + b*c*x
+ b*d*x**2),x)*b**6*c**4*d + 6*log(c + d*x)*a**5*d**4 - 24*log(c + d*x)*a*
*4*b*c*d**3 - 11*log(c + d*x)*a**4*b*d**4 + 36*log(c + d*x)*a**3*b**2*c**2
*d**2 + 44*log(c + d*x)*a**3*b**2*c*d**3 - 24*log(c + d*x)*a**2*b**3*c**3*
d - 66*log(c + d*x)*a**2*b**3*c**2*d**2 + 6*log(c + d*x)*a*b**4*c**4 + 44*
log(c + d*x)*a*b**4*c**3*d - 11*log(c + d*x)*b**5*c**4 - 3*log((a*e + b*e*
x)/(c + d*x))**2*a**3*b**2*c*d**3 + 9*log((a*e + b*e*x)/(c + d*x))**2*a**2
*b**3*c**2*d**2 - 9*log((a*e + b*e*x)/(c + d*x))**2*a*b**4*c**3*d - 12*log
((a*e + b*e*x)/(c + d*x))**2*b**5*c**3*d*x - 18*log((a*e + b*e*x)/(c + d*x
))**2*b**5*c**2*d**2*x**2 - 12*log((a*e + b*e*x)/(c + d*x))**2*b**5*c*d**3
*x**3 - 3*log((a*e + b*e*x)/(c + d*x))**2*b**5*d**4*x**4 + 6*log((a*e + b*
e*x)/(c + d*x))*a**5*d**4 - 24*log((a*e + b*e*x)/(c + d*x))*a**4*b*c*d**3
- 11*log((a*e + b*e*x)/(c + d*x))*a**4*b*d**4 + 36*log((a*e + b*e*x)/(c +
d*x))*a**3*b**2*c**2*d**2 + 38*log((a*e + b*e*x)/(c + d*x))*a**3*b**2*c*d*
*3 - 6*log((a*e + b*e*x)/(c + d*x))*a**3*b**2*d**4*x - 24*log((a*e + b*...
```

3.78 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

Optimal result 1003
Mathematica [B] (verified) 1004
Rubi [A] (verified) 1004
Maple [F] 1021
Fricas [F] 1022
Sympy [F] 1022
Maxima [F] 1023
Giac [F] 1024
Mupad [F(-1)] 1025
Reduce [F] 1025

Optimal result

Integrand size = 42, antiderivative size = 712

$$
\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx \\
&= \frac{B^2d(bc-ad)^2i^3x}{3b^3g} + \frac{B^2(bc-ad)^3i^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^4g} \\
&\quad - \frac{5Bd(bc-ad)^2i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^4g} \\
&\quad - \frac{B(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2g} \\
&\quad + \frac{2B(bc-ad)^3i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g} \\
&\quad + \frac{d(bc-ad)^2i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g} \\
&\quad + \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g} \\
&\quad + \frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg} + \frac{2B^2(bc-ad)^3i^3\log(c+dx)}{b^4g} \\
&\quad + \frac{5B(bc-ad)^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g} \\
&\quad - \frac{(bc-ad)^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} \\
&\quad + \frac{2B^2(bc-ad)^3i^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g} - \frac{5B^2(bc-ad)^3i^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g} \\
&\quad + \frac{2B(bc-ad)^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} \\
&\quad + \frac{2B^2(bc-ad)^3i^3\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}
\end{aligned}
$$

output
```
1/3*B^2*d*(-a*d+b*c)^2*i^3*x/b^3/g+1/3*B^2*(-a*d+b*c)^3*i^3*ln((b*x+a)/(d*
x+c))/b^4/g-5/3*B*d*(-a*d+b*c)^2*i^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b
^4/g-1/3*B*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g+2*B*
(-a*d+b*c)^3*i^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/
g+d*(-a*d+b*c)^2*i^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^4/g+1/2*(-a*d
+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2/g+1/3*i^3*(d*x+c)^3*
(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b/g+2*B^2*(-a*d+b*c)^3*i^3*ln(d*x+c)/b^4/g+5
/3*B*(-a*d+b*c)^3*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a)
)/b^4/g-(-a*d+b*c)^3*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b
*x+a))/b^4/g+2*B^2*(-a*d+b*c)^3*i^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/g-5
/3*B^2*(-a*d+b*c)^3*i^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g+2*B*(-a*d+b*c
)^3*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g+2
*B^2*(-a*d+b*c)^3*i^3*polylog(3,b*(d*x+c)/d/(b*x+a))/b^4/g
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5044 vs. $2(712) = 1424$.

Time = 6.87 (sec) , antiderivative size = 5044, normalized size of antiderivative = 7.08

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + 
b*g*x),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.15 (sec) , antiderivative size = 857, normalized size of antiderivative = 1.20, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2962, 2789, 2756, 2789, 2756, 54, 2009, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{ag+bgx}\,dx$$

$$\downarrow 2962$$

$$\frac{i^3(bc-ad)^3\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{g}$$

$$\downarrow 2789$$

$$\frac{i^3(bc-ad)^3\left(\frac{d\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\downarrow 2756$$

$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

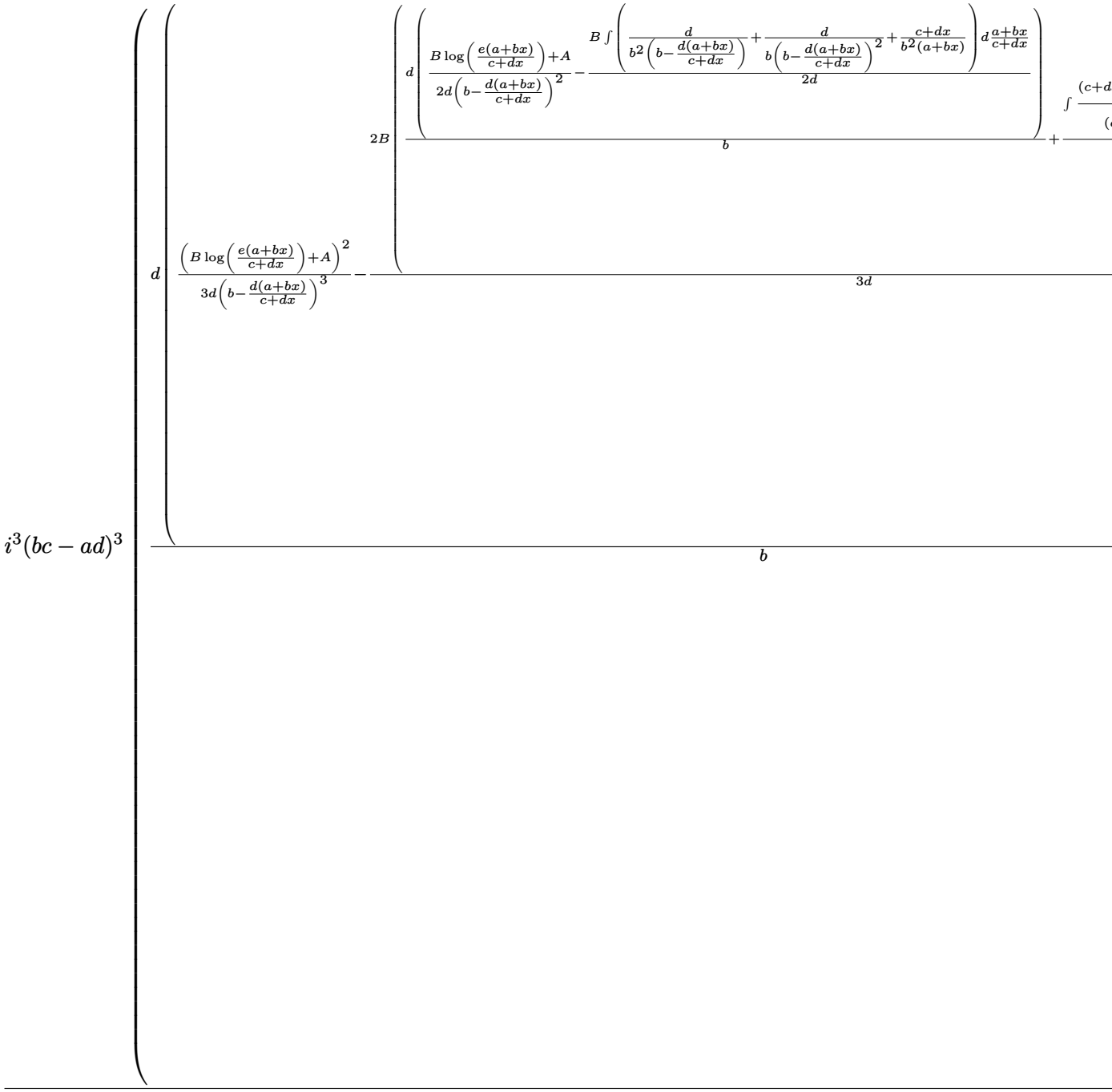
$$\downarrow 2789$$

$$i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{d\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{3d}\right)}{b}+\frac{d\int\frac{\left(A+B\log\left(\frac{e}{}\right.\right.}{\left(b-\frac{d(a+}{c+}\right.}\right)}{g}$$

$$\downarrow 2756$$

$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a}{c}}{b}\right)}{3d}\right)}{b}\right)}{g}$$

↓ 54



↓ 2009

$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx}{c+dx}\right.}{b^2}\right.}{2d}\right.}{b}\right.}{3d}\right.}{b}\right.}{g}$$

$\downarrow$ 2789

$$i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{d\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right.}{3d}\right.}{b}\right.$$

↓ 2751

$$i^3(bc-ad)^3\left(d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{B\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{3d}\right)}{b}\right)$$

↓ 16

$$i^3(bc-ad)^3\left(d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}\right)}{3d}\right)}{b}$$

↓ 2755

$$\frac{i^3(bc-ad)^3\left(d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}\right)}{3d}\right)}{b}$$

↓ 2754

$$(bc-ad)^3 i^3 \left(\frac{d \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left(\frac{d\left(\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)} \right)}{2d} \right)}{b} + \frac{d\left(\frac{(a+bx)\left(A+\right.}{b(c+dx)} \right.}{3d} \right.}{b} \right.$$

↓ 2779

$$(bc-ad)^3 i^3 \left(d \left(d \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left(d\left(\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)} \right)}{2d} \right) \right)}{b} + \frac{d\left(\frac{(a+bx)\left(A+\right.}{b(c+dx)} \right.}{b} \right.$$

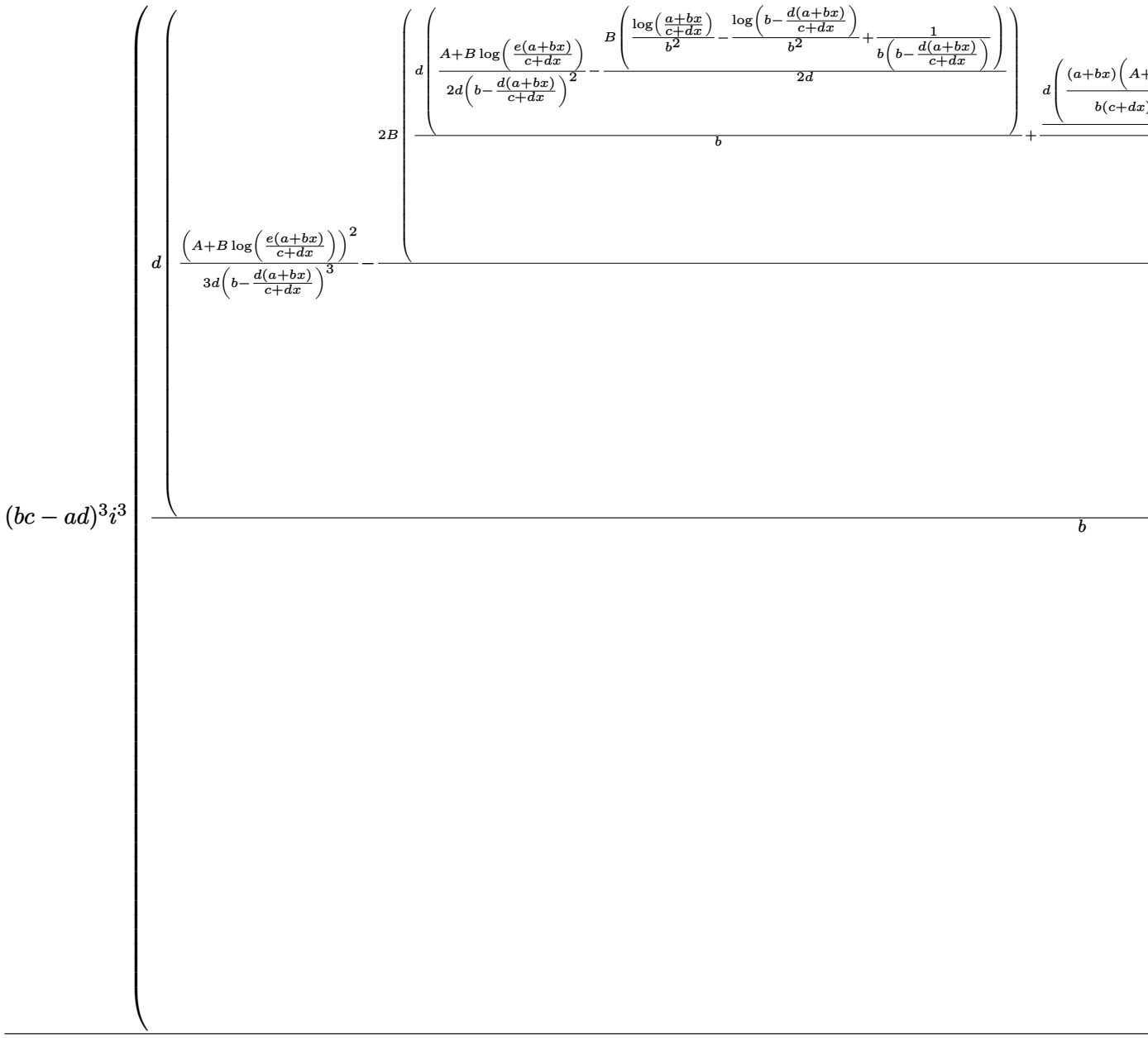
↓ 2821

$$(bc-ad)^3 i^3 \left(\frac{d\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left(\frac{d\left(\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b} + \frac{d\left(\frac{(a+bx)\left(A+\right.}{b(c+dx)}\right.}{}\right.}{b} \right.$$

$\downarrow$ 2838

$$(bc-ad)^3 i^3 \left(d \left(d \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left(d\left(\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)} \right)}{2d} \right) \right)}{b} + \frac{d\left(\frac{(a+bx)\left(A+\right.}{b(c+dx)} \right.}{b} \right.$$

↓ 7143



input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
,x]
```

output

```
((b*c - a*d)^3*i^3*((d*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(3*d*(b - (
d*(a + b*x))/(c + d*x))^3) - (2*B*((d*((A + B*Log[(e*(a + b*x))/(c + d*x)]
)/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(1/(b*(b - (d*(a + b*x))/(c +
 d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/
b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b
*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*Log[b - (d*(a + b*x))/(c +
d*x)])/(b*d)))/b + (-(((A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c
+ d*x))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b
)/b)/b))/(3*d)))/b + ((d*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(2*d*(b -
 (d*(a + b*x))/(c + d*x))^2) - (B*((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))
/(c + d*x)]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*Log[b - (d*
(a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[(e*(a + b*x))/(c + d*x)]
)*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(
d*(a + b*x))])/b)/b))/d))/b + ((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c
+ d*x)])^2)/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (2*B*(-(((A + B*
Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (
B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d))/b))/b + (-(((A + B*Log[(e*(
a + b*x))/(c + d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (2*B*((
A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))
] + B*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/b)/b)/b)/b))/g
```

Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)  Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(dix+ci)^3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{bgx+ag}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algo
rithm="fricas")
```

output
```
integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*
c^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2
*c^3*i^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^
2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log((b*e*x + a*e)/(d*x + c)))
/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{i^3\left(\int \frac{A^2c^3}{a+bx}\,dx + \int \frac{A^2d^3x^3}{a+bx}\,dx + \int \frac{B^2c^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a+bx}\,dx + \int \frac{2ABc^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx + \int \frac{3A^2cd^2x^2}{a+bx}\,dx + \int \frac{3A}{a}\right.}{}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g),x)
```

output

```
i**3*(Integral(A**2*c**3/(a + b*x), x) + Integral(A**2*d**3*x**3/(a + b*x)
, x) + Integral(B**2*c**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x
), x) + Integral(2*A*B*c**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x)
, x) + Integral(3*A**2*c*d**2*x**2/(a + b*x), x) + Integral(3*A**2*c**2*d*
x/(a + b*x), x) + Integral(B**2*d**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d
*x))**2/(a + b*x), x) + Integral(2*A*B*d**3*x**3*log(a*e/(c + d*x) + b*e*x
/(c + d*x))/(a + b*x), x) + Integral(3*B**2*c*d**2*x**2*log(a*e/(c + d*x)
+ b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(3*B**2*c**2*d*x*log(a*e/(c
+ d*x) + b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(6*A*B*c*d**2*x**2*lo
g(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Integral(6*A*B*c**2*d*x
*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algo
rithm="maxima")
```

output

```
3*A^2*c^2*d*i^3*(x/(b*g) - a*log(b*x + a)/(b^2*g)) - 1/6*A^2*d^3*i^3*(6*a^
3*log(b*x + a)/(b^4*g) - (2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/(b^3*g)) + 3/2*
A^2*c*d^2*i^3*(2*a^2*log(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A^2
*c^3*i^3*log(b*g*x + a*g)/(b*g) + 1/6*(2*B^2*b^3*d^3*i^3*x^3 + 3*(3*b^3*c*
d^2*i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 6*(3*b^3*c^2*d*i^3 - 3*a*b^2*c*d^2*i^3
+ a^2*b*d^3*i^3)*B^2*x + 6*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*d^
2*i^3 - a^3*d^3*i^3)*B^2*log(b*x + a))*log(d*x + c)^2/(b^4*g) - integrate(
-1/3*(3*B^2*b^4*c^4*i^3*log(e)^2 + 6*A*B*b^4*c^4*i^3*log(e) + 3*(B^2*b^4*d
^4*i^3*log(e)^2 + 2*A*B*b^4*d^4*i^3*log(e))*x^4 + 12*(B^2*b^4*c*d^3*i^3*lo
g(e)^2 + 2*A*B*b^4*c*d^3*i^3*log(e))*x^3 + 18*(B^2*b^4*c^2*d^2*i^3*log(e)^
2 + 2*A*B*b^4*c^2*d^2*i^3*log(e))*x^2 + 3*(B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4
*c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B^2*b
^4*c^4*i^3)*log(b*x + a)^2 + 12*(B^2*b^4*c^3*d*i^3*log(e)^2 + 2*A*B*b^4*c^
3*d*i^3*log(e))*x + 6*(B^2*b^4*c^4*i^3*log(e) + A*B*b^4*c^4*i^3 + (B^2*b^4
*d^4*i^3*log(e) + A*B*b^4*d^4*i^3)*x^4 + 4*(B^2*b^4*c*d^3*i^3*log(e) + A*B
*b^4*c*d^3*i^3)*x^3 + 6*(B^2*b^4*c^2*d^2*i^3*log(e) + A*B*b^4*c^2*d^2*i^3)
*x^2 + 4*(B^2*b^4*c^3*d*i^3*log(e) + A*B*b^4*c^3*d*i^3)*x)*log(b*x + a) -
(6*B^2*b^4*c^4*i^3*log(e) + 6*A*B*b^4*c^4*i^3 + 2*(3*A*B*b^4*d^4*i^3 + (3*
i^3*log(e) + i^3)*B^2*b^4*d^4)*x^4 + (24*A*B*b^4*c*d^3*i^3 - (a*b^3*d^4*i^
3 - 3*(8*i^3*log(e) + 3*i^3)*b^4*c*d^3)*B^2)*x^3 + 3*(12*A*B*b^4*c^2*d^...
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algo
rithm="giac")
```

output

```
integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*
g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{i\left(-6\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2}{bx+a}dx\right)b^6c^3 - 12\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{bx+a}dx\right)a\,b^5c^3 - 6\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2x^3}{bx+a}dx\right)b^6d^3 - 18\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2x^2}{bx+a}dx\right)b^6c\,d^2 - 18\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2x}{bx+a}dx\right)b^6c^2d - 12\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2x^3}{bx+a}dx\right)a\,b^5d^3 - 36\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2x^2}{bx+a}dx\right)a\,b^5c\,d^2 - 36\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2x}{bx+a}dx\right)a\,b^5c^2d + 6\log(a+bx)a^5d^3 - 18\log(a+bx)a^4bcd^2 + 18\log(a+bx)a^3b^2c^2d - 6\log(a+bx)a^2b^3c^3 - 6a^4bd^3x + 18a^3b^2cd^2x + 3a^3b^2d^3x^2 - 18a^2b^3c^2dx - 9a^2b^3cd^2x^2 - 2a^2b^3d^3x^3\right)}{6b^4g}$$

input
```
int((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

output
```
(i*( - 6*int(log((a*e + b*e*x)/(c + d*x))**2/(a + b*x),x)*b**6*c**3 - 12*i
nt(log((a*e + b*e*x)/(c + d*x))/(a + b*x),x)*a*b**5*c**3 - 6*int((log((a*e
 + b*e*x)/(c + d*x))**2*x**3)/(a + b*x),x)*b**6*d**3 - 18*int((log((a*e +
b*e*x)/(c + d*x))**2*x**2)/(a + b*x),x)*b**6*c*d**2 - 18*int((log((a*e + b
*e*x)/(c + d*x))**2*x)/(a + b*x),x)*b**6*c**2*d - 12*int((log((a*e + b*e*x
)/(c + d*x))*x**3)/(a + b*x),x)*a*b**5*d**3 - 36*int((log((a*e + b*e*x)/(c
 + d*x))*x**2)/(a + b*x),x)*a*b**5*c*d**2 - 36*int((log((a*e + b*e*x)/(c +
 d*x))*x)/(a + b*x),x)*a*b**5*c**2*d + 6*log(a + b*x)*a**5*d**3 - 18*log(a
 + b*x)*a**4*b*c*d**2 + 18*log(a + b*x)*a**3*b**2*c**2*d - 6*log(a + b*x)*
a**2*b**3*c**3 - 6*a**4*b*d**3*x + 18*a**3*b**2*c*d**2*x + 3*a**3*b**2*d**
3*x**2 - 18*a**2*b**3*c**2*d*x - 9*a**2*b**3*c*d**2*x**2 - 2*a**2*b**3*d**
3*x**3))/(6*b**4*g)
```

3.79 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result 1027
Mathematica [B] (verified) 1028
Rubi [A] (verified) 1028
Maple [F] 1030
Fricas [F] 1031
Sympy [F(-1)] 1031
Maxima [F] 1031
Giac [F] 1032
Mupad [F(-1)] 1033
Reduce [F] 1033

Optimal result

Integrand size = 42, antiderivative size = 692

$$
\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx \\
&= -\frac{2B^2(bc-ad)^2i^3(c+dx)}{b^3g^2(a+bx)} - \frac{Bd^2(bc-ad)i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^2} \\
&\quad - \frac{2B(bc-ad)^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2(a+bx)} \\
&\quad + \frac{4Bd(bc-ad)^2i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^2} \\
&\quad + \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g^2} \\
&\quad - \frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^2(a+bx)} \\
&\quad + \frac{di^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g^2} + \frac{B^2d(bc-ad)^2i^3\log(c+dx)}{b^4g^2} \\
&\quad + \frac{Bd(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} \\
&\quad - \frac{3d(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} \\
&\quad + \frac{4B^2d(bc-ad)^2i^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^2} - \frac{B^2d(bc-ad)^2i^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} \\
&\quad + \frac{6Bd(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} \\
&\quad + \frac{6B^2d(bc-ad)^2i^3\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}
\end{aligned}
$$

output

```
-2*B^2*(-a*d+b*c)^2*i^3*(d*x+c)/b^3/g^2/(b*x+a)-B*d^2*(-a*d+b*c)*i^3*(b*x+
a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/g^2-2*B*(-a*d+b*c)^2*i^3*(d*x+c)*(A+B*l
n(e*(b*x+a)/(d*x+c)))/b^3/g^2/(b*x+a)+4*B*d*(-a*d+b*c)^2*i^3*ln((-a*d+b*c)
/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/g^2+2*d^2*(-a*d+b*c)*i^3*(b*x+
a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^4/g^2-(-a*d+b*c)^2*i^3*(d*x+c)*(A+B*ln(
e*(b*x+a)/(d*x+c)))^2/b^3/g^2/(b*x+a)+1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a
)/(d*x+c)))^2/b^2/g^2+B^2*d*(-a*d+b*c)^2*i^3*ln(d*x+c)/b^4/g^2+B*d*(-a*d+b
*c)^2*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^2-3*
d*(-a*d+b*c)^2*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))
/b^4/g^2+4*B^2*d*(-a*d+b*c)^2*i^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/g^2-B
^2*d*(-a*d+b*c)^2*i^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^2+6*B*d*(-a*d+b
*c)^2*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g
^2+6*B^2*d*(-a*d+b*c)^2*i^3*polylog(3,b*(d*x+c)/d/(b*x+a))/b^4/g^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5266 vs. $2(692) = 1384$.

Time = 7.58 (sec) , antiderivative size = 5266, normalized size of antiderivative = 7.61

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Result too large to show}$$

input

```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^2,x]
```

output

```
Result too large to show
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 577, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^2}\,dx$$

$\Big\downarrow$ 2962

$$\frac{i^3(bc-ad)^2\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{g^2}$$

$\Big\downarrow$ 2795

$$\frac{i^3(bc-ad)^2\int \left(\frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3(a+bx)^2}+\frac{3d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{2d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d^2\left(A+B\log\left(\frac{e(a}{c-}\right.\right.}{b^2\left(b-\frac{d(a+b}{c+dx}\right.}\right.}{g^2}$$

$\Big\downarrow$ 2009

$$i^3(bc-ad)^2\left(-\frac{Bd^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{2d^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{6Bd\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{b^4}\right.$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^2,x]
```

output
```
((b*c - a*d)^2*i^3*((-2*B^2*(c + d*x))/(b^3*(a + b*x)) - (2*B*(c + d*x)*(A
 + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^3*(a + b*x)) - (B*d^2*(a + b*x)*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]))/(b^4*(c + d*x)*(b - (d*(a + b*x))/(c +
d*x))) - ((c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(b^3*(a + b*x)
) + (d*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*b^2*(b - (d*(a + b*x))/(
c + d*x))^2) + (2*d^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(b
^4*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (B^2*d*Log[b - (d*(a + b*x))
/(c + d*x)])/b^4 + (4*B*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*
(a + b*x))/(b*(c + d*x))])/b^4 + (B*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])
*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b^4 - (3*d*(A + B*Log[(e*(a + b*x))
/(c + d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b^4 + (4*B^2*d*PolyLo
g[2, (d*(a + b*x))/(b*(c + d*x))])/b^4 - (B^2*d*PolyLog[2, (b*(c + d*x))/(
d*(a + b*x))])/b^4 + (6*B*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2
, (b*(c + d*x))/(d*(a + b*x))])/b^4 + (6*B^2*d*PolyLog[3, (b*(c + d*x))/(d
*(a + b*x))])/b^4))/g^2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(bgx+ag)^2}dx$$

input

```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

output

```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, al
gorithm="fricas")
```

output
```
integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*
c^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2
*c^3*i^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^
2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log((b*e*x + a*e)/(d*x + c)))
/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, al
gorithm="maxima")
```

output

```
-3*A^2*(a^2/(b^4*g^2*x + a*b^3*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*
g^2))*c*d^2*i^3 + 1/2*(2*a^3/(b^5*g^2*x + a*b^4*g^2) + 6*a^2*log(b*x + a)/
(b^4*g^2) + (b*x^2 - 4*a*x)/(b^3*g^2))*A^2*d^3*i^3 + 3*A^2*c^2*d*i^3*(a/(b
^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - 2*A*B*c^3*i^3*(log(b*e*x
/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1/(b^2*g^2*x + a*b*g^2
) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)
*g^2)) - A^2*c^3*i^3/(b^2*g^2*x + a*b*g^2) + 1/2*(B^2*b^3*d^3*i^3*x^3 + 3*
(2*b^3*c*d^2*i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 2*(3*a*b^2*c*d^2*i^3 - 2*a^2*b
*d^3*i^3)*B^2*x - 2*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*d^2*i^3 -
 a^3*d^3*i^3)*B^2 + 6*((b^3*c^2*d*i^3 - 2*a*b^2*c*d^2*i^3 + a^2*b*d^3*i^3)
*B^2*x + (a*b^2*c^2*d*i^3 - 2*a^2*b*c*d^2*i^3 + a^3*d^3*i^3)*B^2)*log(b*x
+ a))*log(d*x + c)^2/(b^5*g^2*x + a*b^4*g^2) - integrate(-(B^2*b^4*c^4*i^3
*log(e)^2 + (B^2*b^4*d^4*i^3*log(e)^2 + 2*A*B*b^4*d^4*i^3*log(e))*x^4 + 4*
(B^2*b^4*c*d^3*i^3*log(e)^2 + 2*A*B*b^4*c*d^3*i^3*log(e))*x^3 + 6*(B^2*b^4
*c^2*d^2*i^3*log(e)^2 + 2*A*B*b^4*c^2*d^2*i^3*log(e))*x^2 + (B^2*b^4*d^4*i
^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c
^3*d*i^3*x + B^2*b^4*c^4*i^3)*log(b*x + a)^2 + 2*(2*B^2*b^4*c^3*d*i^3*log(
e)^2 + 3*A*B*b^4*c^3*d*i^3*log(e))*x + 2*(B^2*b^4*c^4*i^3*log(e) + (B^2*b^
4*d^4*i^3*log(e) + A*B*b^4*d^4*i^3)*x^4 + 4*(B^2*b^4*c*d^3*i^3*log(e) + A*
B*b^4*c*d^3*i^3)*x^3 + 6*(B^2*b^4*c^2*d^2*i^3*log(e) + A*B*b^4*c^2*d^2*...
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, al
gorithm="giac")
```

output

```
integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*
g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(a\,g+b\,g\,x)^2}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^2,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^2, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \text{too large to display}$$

input
```
int((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

output

```
(i*( - 2*int((log((a*e + b*e*x)/(c + d*x))**2*x**3)/(a**2 + 2*a*b*x + b**2
*x**2),x)*a**3*b**6*d**4 + 2*int((log((a*e + b*e*x)/(c + d*x))**2*x**3)/(a
**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**7*c*d**3 - 2*int((log((a*e + b*e*x)/
(c + d*x))**2*x**3)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**7*d**4*x + 2*i
nt((log((a*e + b*e*x)/(c + d*x))**2*x**3)/(a**2 + 2*a*b*x + b**2*x**2),x)*
a*b**8*c*d**3*x - 6*int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(a**2 + 2*a
*b*x + b**2*x**2),x)*a**3*b**6*c*d**3 + 6*int((log((a*e + b*e*x)/(c + d*x)
)**2*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**7*c**2*d**2 - 6*int((lo
g((a*e + b*e*x)/(c + d*x))**2*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b
**7*c*d**3*x + 6*int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(a**2 + 2*a*b*
x + b**2*x**2),x)*a*b**8*c**2*d**2*x - 6*int((log((a*e + b*e*x)/(c + d*x))
**2*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**3*b**6*c**2*d**2 + 6*int((log((a
*e + b*e*x)/(c + d*x))**2*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**7*c**
3*d - 6*int((log((a*e + b*e*x)/(c + d*x))**2*x)/(a**2 + 2*a*b*x + b**2*x**
2),x)*a**2*b**7*c**2*d**2*x + 6*int((log((a*e + b*e*x)/(c + d*x))**2*x)/(a
**2 + 2*a*b*x + b**2*x**2),x)*a*b**8*c**3*d*x - 4*int((log((a*e + b*e*x)/(
c + d*x))*x**3)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**4*b**5*d**4 + 4*int((lo
g((a*e + b*e*x)/(c + d*x))*x**3)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**3*b**6
*c*d**3 - 4*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(a**2 + 2*a*b*x + b**2
*x**2),x)*a**3*b**6*d**4*x + 4*int((log((a*e + b*e*x)/(c + d*x))*x**3)/...
```

3.80 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result 1036
Mathematica [B] (verified) 1037
Rubi [A] (verified) 1037
Maple [F] 1039
Fricas [F] 1040
Sympy [F] 1040
Maxima [F] 1041
Giac [F] 1042
Mupad [F(-1)] 1043
Reduce [F] 1043

Optimal result

Integrand size = 42, antiderivative size = 604

$$
\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx \\
&= -\frac{4B^2d(bc-ad)i^3(c+dx)}{b^3g^3(a+bx)} - \frac{B^2(bc-ad)i^3(c+dx)^2}{4b^2g^3(a+bx)^2} \\
&\quad - \frac{4Bd(bc-ad)i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^3(a+bx)} \\
&\quad - \frac{B(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^3(a+bx)^2} \\
&\quad + \frac{2Bd^2(bc-ad)i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^3} \\
&\quad + \frac{d^3i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g^3} \\
&\quad - \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^3(a+bx)} \\
&\quad - \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g^3(a+bx)^2} \\
&\quad - \frac{3d^2(bc-ad)i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} \\
&\quad + \frac{2B^2d^2(bc-ad)i^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^3} \\
&\quad + \frac{6Bd^2(bc-ad)i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} \\
&\quad + \frac{6B^2d^2(bc-ad)i^3\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}
\end{aligned}
$$

output
```
-4*B^2*d*(-a*d+b*c)*i^3*(d*x+c)/b^3/g^3/(b*x+a)-1/4*B^2*(-a*d+b*c)*i^3*(d*
x+c)^2/b^2/g^3/(b*x+a)^2-4*B*d*(-a*d+b*c)*i^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d
*x+c)))/b^3/g^3/(b*x+a)-1/2*B*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(
d*x+c)))/b^2/g^3/(b*x+a)^2+2*B*d^2*(-a*d+b*c)*i^3*ln((-a*d+b*c)/b/(d*x+c))
*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/g^3+d^3*i^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*
x+c)))^2/b^4/g^3-2*d*(-a*d+b*c)*i^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/
b^3/g^3/(b*x+a)-1/2*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2
/b^2/g^3/(b*x+a)^2-3*d^2*(-a*d+b*c)*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1
-b*(d*x+c)/d/(b*x+a))/b^4/g^3+2*B^2*d^2*(-a*d+b*c)*i^3*polylog(2,d*(b*x+a)
/b/(d*x+c))/b^4/g^3+6*B*d^2*(-a*d+b*c)*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*pol
ylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^3+6*B^2*d^2*(-a*d+b*c)*i^3*polylog(3,b*(
d*x+c)/d/(b*x+a))/b^4/g^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5989 vs. $2(604) = 1208$.

Time $= 7.63$ (sec) , antiderivative size $= 5989$, normalized size of antiderivative $= 9.92$

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + 
b*g*x)^3,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time $= 0.82$ (sec) , antiderivative size $= 486$, normalized size of antiderivative $= 0.80$, number of steps used $= 4$, number of rules used $= 3$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used $= \{2962, 2795, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{i^3(bc-ad)\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g^3}$$

$$\downarrow 2795$$

$$\frac{i^3(bc-ad)\int \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 d^3}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 d^2}{b^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 d}{b^3(a+bx)^2}+\frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2(a+}\right.}{g^3}$$

$$\downarrow 2009$$

$$i^3(bc-ad)\left(\frac{d^3(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{6Bd^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4}+\frac{2Bd^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+}{c+}\right.\right.$$

input

```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^3,x]
```

output

```
((b*c - a*d)*i^3*((-4*B^2*d*(c + d*x))/(b^3*(a + b*x)) - (B^2*(c + d*x)^2)
/(4*b^2*(a + b*x)^2) - (4*B*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)
]))/(b^3*(a + b*x)) - (B*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))
/(2*b^2*(a + b*x)^2) - (2*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])
^2)/(b^3*(a + b*x)) - ((c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)
/(2*b^2*(a + b*x)^2) + (d^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])
^2)/(b^4*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (2*B*d^2*(A + B*Log[(e
*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/b^4 - (3*d^2
*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x)
)])/b^4 + (2*B^2*d^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/b^4 + (6*B*d
^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c + d*x))/(d*(a + b
*x))])/b^4 + (6*B^2*d^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/b^4))/g^3
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(bgx+ag)^3}dx$$

input

```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x)
```

output

```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, al
gorithm="fricas")
```

output
```
integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*
c^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2
*c^3*i^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^
2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log((b*e*x + a*e)/(d*x + c)))
/(b^3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{i^3\left(\int \frac{A^2c^3}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{A^2d^3x^3}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{B^2c^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{2ABc^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+d}\right.}{a^3+3a^2bx+3ab^2x^2+b^3}\right.}{}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3,x)
```

output

```
i**3*(Integral(A**2*c**3/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),
x) + Integral(A**2*d**3*x**3/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**
3), x) + Integral(B**2*c**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a**3
+ 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*c**3*log(a*
e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x
**3), x) + Integral(3*A**2*c*d**2*x**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2
+ b**3*x**3), x) + Integral(3*A**2*c**2*d*x/(a**3 + 3*a**2*b*x + 3*a*b**2*
x**2 + b**3*x**3), x) + Integral(B**2*d**3*x**3*log(a*e/(c + d*x) + b*e*x/
(c + d*x))**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integr
al(2*A*B*d**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x
 + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(3*B**2*c*d**2*x**2*log(a*e/(c
 + d*x) + b*e*x/(c + d*x))**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x*
*3), x) + Integral(3*B**2*c**2*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2
/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(6*A*B*c*d*
*2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2
*x**2 + b**3*x**3), x) + Integral(6*A*B*c**2*d*x*log(a*e/(c + d*x) + b*e*x
/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x))/g**3
```

Maxima [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, al
gorithm="maxima")
```

output

```
-3/2*A*B*c^2*d*i^3*(2*(2*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^
4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + (3*a*b*c - a^2*d + 2*(2*b^2*c -
 a*b*d)*x)/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a
^2*b^3*c - a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 -
2*a*b^3*c*d + a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c
^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3)) - 1/2*A^2*d^3*i^3*((6*a^2*b*x + 5*a^
3)/(b^6*g^3*x^2 + 2*a*b^5*g^3*x + a^2*b^4*g^3) - 2*x/(b^3*g^3) + 6*a*log(b
*x + a)/(b^4*g^3)) + 3/2*A^2*c*d^2*i^3*((4*a*b*x + 3*a^2)/(b^5*g^3*x^2 + 2
*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3)) + 1/2*A*B*c^3*i^3*
((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2
*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x
+ c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3
*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*
b^2*c*d + a^2*b*d^2)*g^3)) - 3/2*(2*b*x + a)*A^2*c^2*d*i^3/(b^4*g^3*x^2 +
2*a*b^3*g^3*x + a^2*b^2*g^3) - 1/2*A^2*c^3*i^3/(b^3*g^3*x^2 + 2*a*b^2*g^3*
x + a^2*b*g^3) + 1/2*(2*B^2*b^3*d^3*i^3*x^3 + 4*B^2*a*b^2*d^3*i^3*x^2 - 2*
(3*b^3*c^2*d*i^3 - 6*a*b^2*c*d^2*i^3 + 2*a^2*b*d^3*i^3)*B^2*x - (b^3*c^3*i
^3 + 3*a*b^2*c^2*d*i^3 - 9*a^2*b*c*d^2*i^3 + 5*a^3*d^3*i^3)*B^2 + 6*((b^3*
c*d^2*i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 2*(a*b^2*c*d^2*i^3 - a^2*b*d^3*i^3)*B
^2*x + (a^2*b*c*d^2*i^3 - a^3*d^3*i^3)*B^2)*log(b*x + a))*log(d*x + c)^...
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, al
gorithm="giac")
```

output

```
integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*
g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(a\,g+b\,g\,x)^3}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^3,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^3, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \text{too large to display}$$

input
```
int((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x)
```

output

```
(i*( - 4*int((log((a*e + b*e*x)/(c + d*x))**2*x**3)/(a**3 + 3*a**2*b*x + 3
*a*b**2*x**2 + b**3*x**3),x)*a**5*b**6*d**5 + 8*int((log((a*e + b*e*x)/(c
+ d*x))**2*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**4*b
**7*c*d**4 - 8*int((log((a*e + b*e*x)/(c + d*x))**2*x**3)/(a**3 + 3*a**2*b
*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**4*b**7*d**5*x - 4*int((log((a*e + b*
e*x)/(c + d*x))**2*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x
)*a**3*b**8*c**2*d**3 + 16*int((log((a*e + b*e*x)/(c + d*x))**2*x**3)/(a**
3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**3*b**8*c*d**4*x - 4*int(
(log((a*e + b*e*x)/(c + d*x))**2*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2
+ b**3*x**3),x)*a**3*b**8*d**5*x**2 - 8*int((log((a*e + b*e*x)/(c + d*x))*
*2*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**2*b**9*c**2
*d**3*x + 8*int((log((a*e + b*e*x)/(c + d*x))**2*x**3)/(a**3 + 3*a**2*b*x
+ 3*a*b**2*x**2 + b**3*x**3),x)*a**2*b**9*c*d**4*x**2 - 4*int((log((a*e +
b*e*x)/(c + d*x))**2*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)
,x)*a*b**10*c**2*d**3*x**2 - 12*int((log((a*e + b*e*x)/(c + d*x))**2*x**2)
/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**5*b**6*c*d**4 + 24*
int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x
**2 + b**3*x**3),x)*a**4*b**7*c**2*d**3 - 24*int((log((a*e + b*e*x)/(c + d
*x))**2*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**4*b**7
*c*d**4*x - 12*int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(a**3 + 3*a**...
```

3.81 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result 1045
Mathematica [C] (verified) 1046
Rubi [A] (verified) 1047
Maple [B] (verified) 1048
Fricas [B] (verification not implemented) 1050
Sympy [F(-1)] 1050
Maxima [B] (verification not implemented) 1051
Giac [A] (verification not implemented) 1052
Mupad [B] (verification not implemented) 1052
Reduce [B] (verification not implemented) 1053

Optimal result

Integrand size = 42, antiderivative size = 147

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{B^2i^3(c+dx)^4}{32(bc-ad)g^5(a+bx)^4} - \frac{Bi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)g^5(a+bx)^4} - \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)g^5(a+bx)^4}$$

output

```
-1/32*B^2*i^3*(d*x+c)^4/(-a*d+b*c)/g^5/(b*x+a)^4-1/8*B*i^3*(d*x+c)^4*(A+B*
ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^5/(b*x+a)^4-1/4*i^3*(d*x+c)^4*(A+B*ln(
e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)/g^5/(b*x+a)^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.30 (sec) , antiderivative size = 2401, normalized size of antiderivative = 16.33

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Result too large to show}$$

input

```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^5,x]
```

output

```
-1/4*((b*c - a*d)^3*i^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(b^4*g^5*(
a + b*x)^4) - (d*(b*c - a*d)^2*i^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)
/(b^4*g^5*(a + b*x)^3) - (3*d^2*(b*c - a*d)*i^3*(A + B*Log[(e*(a + b*x))/(
c + d*x)])^2)/(2*b^4*g^5*(a + b*x)^2) - (d^3*i^3*(A + B*Log[(e*(a + b*x))/
(c + d*x)])^2)/(b^4*g^5*(a + b*x)) + (B*(b*c - a*d)^4*i^3*(-1/4*A/((b*c -
a*d)*(a + b*x)^4) - B/(16*(b*c - a*d)*(a + b*x)^4) + (A*d)/(3*(b*c - a*d)^
2*(a + b*x)^3) + (7*B*d)/(36*(b*c - a*d)^2*(a + b*x)^3) - (A*d^2)/(2*(b*c
- a*d)^3*(a + b*x)^2) - (13*B*d^2)/(24*(b*c - a*d)^3*(a + b*x)^2) + (A*d^3
)/((b*c - a*d)^4*(a + b*x)) + (25*B*d^3)/(12*(b*c - a*d)^4*(a + b*x)) + (A
*d^4*Log[a + b*x])/(b*c - a*d)^5 + (25*B*d^4*Log[a + b*x])/(12*(b*c - a*d)
^5) - (B*d^4*Log[a + b*x]^2)/(2*(b*c - a*d)^5) - (B*Log[(e*(a + b*x))/(c +
 d*x)])/(4*(b*c - a*d)*(a + b*x)^4) + (B*d*Log[(e*(a + b*x))/(c + d*x)])/(
3*(b*c - a*d)^2*(a + b*x)^3) - (B*d^2*Log[(e*(a + b*x))/(c + d*x)])/(2*(b*
c - a*d)^3*(a + b*x)^2) + (B*d^3*Log[(e*(a + b*x))/(c + d*x)])/((b*c - a*d
)^4*(a + b*x)) + (B*d^4*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)])/(b*c -
a*d)^5 - (A*d^4*Log[c + d*x])/(b*c - a*d)^5 - (25*B*d^4*Log[c + d*x])/(12*
(b*c - a*d)^5) + (B*d^4*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b
*c - a*d)^5 - (B*d^4*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x])/(b*c - a*d
)^5 - (B*d^4*Log[c + d*x]^2)/(2*(b*c - a*d)^5) + (B*d^4*Log[a + b*x]*Log[(
b*(c + d*x))/(b*c - a*d)])/(b*c - a*d)^5 + (B*d^4*PolyLog[2, -((d*(a + ...
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^5}\,dx$$

$$\downarrow 2962$$

$$\frac{i^3\int \frac{(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^5}\,d\frac{a+bx}{c+dx}}{g^5(bc-ad)}$$

$$\downarrow 2742$$

$$\frac{i^3\left(\frac{1}{2}B\int \frac{(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^5}\,d\frac{a+bx}{c+dx}-\frac{(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4(a+bx)^4}\right)}{g^5(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{i^3\left(\frac{1}{2}B\left(-\frac{(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}-\frac{B(c+dx)^4}{16(a+bx)^4}\right)-\frac{(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4(a+bx)^4}\right)}{g^5(bc-ad)}$$

input

```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^5,x]
```

output

```
(i^3*(-1/4*((c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x)^
4 + (B*(-1/16*(B*(c + d*x)^4)/(a + b*x)^4 - ((c + d*x)^4*(A + B*Log[(e*(a
+ b*x))/(c + d*x)]))/(4*(a + b*x)^4)))/2))/((b*c - a*d)*g^5)
```

Defintions of rubi rules used

rule 2741

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(141) = 282$.

Time = 2.36 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.50

method	result
derivativedivides	$-\frac{e(da-bc)\left(-\frac{i^3d^2e^3A^2}{4(da-bc)^2g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}+\frac{2i^3d^2e^3AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{1}{16\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}\right)}{(da-bc)^2g^5}+\frac{i^3d^2e^3B^2\left(-\right.}{}\right.}{d^2}$
default	$-\frac{e(da-bc)\left(-\frac{i^3d^2e^3A^2}{4(da-bc)^2g^5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}+\frac{2i^3d^2e^3AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\frac{1}{16\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}\right)}{(da-bc)^2g^5}+\frac{i^3d^2e^3B^2\left(-\right.}{}\right.}{d^2}$
parts	$\frac{i^3A^2\left(\frac{3d^2(da-bc)}{2b^4(bx+a)^2}-\frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{4b^4(bx+a)^4}-\frac{d^3}{b^4(bx+a)}-\frac{d\left(a^2d^2-2acdb+c^2b^2\right)}{b^4(bx+a)^3}\right)}{g^5}-\frac{i^3B^2e^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)}{d(dx+c}\right.\right.}{4\left(\frac{be}{d}+\frac{(da-bc)}{d(dx+c}\right.\right.}$
risch	$\frac{3i^3A^2d^3a}{2g^5b^4(bx+a)^2}-\frac{3i^3A^2d^2c}{2g^5b^3(bx+a)^2}+\frac{i^3A^2a^3d^3}{4g^5b^4(bx+a)^4}-\frac{3i^3A^2a^2cd^2}{4g^5b^3(bx+a)^4}+\frac{3i^3A^2ac^2d}{4g^5b^2(bx+a)^4}-\frac{i^3A^2c^3}{4g^5b(bx+a)^4}-\frac{i^3A}{g^5b^4(}$
norman	$\frac{B^2cd^3i^3x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{(da-bc)g}+\frac{B^2c^3di^3x\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(da-bc)}+\frac{\left(8i^3A^2c^3+4i^3c^3BA+i^3c^3B^2\right)x}{8ga}+\frac{3\left(8A^2ac^2di^3+8A^2bc^3i^3+4ABac^2\right.}{}$
parallelrisch	$\frac{8A^2x^4a^6cd^4i^3-8A^2x^4a^2b^4c^5i^3+B^2x^4a^6cd^4i^3-B^2x^4a^2b^4c^5i^3+32A^2x^3a^6c^2d^3i^3-32A^2x^3a^3b^3c^5i^3+4B^2x^3a^6c^2d^3i^3}{}$
orering	Expression too large to display

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x,method=_RE
TURNVERBOSE)
```

output
```
-1/d^2*e*(a*d-b*c)*(-1/4*i^3*d^2*e^3/(a*d-b*c)^2/g^5*A^2/(b*e/d+(a*d-b*c)*
e/d/(d*x+c))^4+2*i^3*d^2*e^3/(a*d-b*c)^2/g^5*A*B*(-1/4/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*
x+c))^4)+i^3*d^2*e^3/(a*d-b*c)^2/g^5*B^2*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c
))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/8/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4
*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/32/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(141) = 282$.

Time = 0.10 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.80

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{4\left((8A^2+4AB+B^2)b^4cd^3-(8A^2+4AB+B^2)ab^3d^4\right)i^3x^3+6\left((8A^2+4AB+B^2)b^4c^2d^2-(8A^2\right.}{}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, al
gorithm="fricas")
```

output
```
-1/32*(4*((8*A^2 + 4*A*B + B^2)*b^4*c*d^3 - (8*A^2 + 4*A*B + B^2)*a*b^3*d^
4)*i^3*x^3 + 6*((8*A^2 + 4*A*B + B^2)*b^4*c^2*d^2 - (8*A^2 + 4*A*B + B^2)*
a^2*b^2*d^4)*i^3*x^2 + 4*((8*A^2 + 4*A*B + B^2)*b^4*c^3*d - (8*A^2 + 4*A*B
 + B^2)*a^3*b*d^4)*i^3*x + ((8*A^2 + 4*A*B + B^2)*b^4*c^4 - (8*A^2 + 4*A*B
 + B^2)*a^4*d^4)*i^3 + 8*(B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 +
6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B^2*b^4*c^4*i^3)*log((
b*e*x + a*e)/(d*x + c))^2 + 4*((4*A*B + B^2)*b^4*d^4*i^3*x^4 + 4*(4*A*B +
B^2)*b^4*c*d^3*i^3*x^3 + 6*(4*A*B + B^2)*b^4*c^2*d^2*i^3*x^2 + 4*(4*A*B +
B^2)*b^4*c^3*d*i^3*x + (4*A*B + B^2)*b^4*c^4*i^3)*log((b*e*x + a*e)/(d*x +
 c)))/((b^9*c - a*b^8*d)*g^5*x^4 + 4*(a*b^8*c - a^2*b^7*d)*g^5*x^3 + 6*(a^
2*b^7*c - a^3*b^6*d)*g^5*x^2 + 4*(a^3*b^6*c - a^4*b^5*d)*g^5*x + (a^4*b^5*
c - a^5*b^4*d)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11688 vs. $2(141) = 282$.

Time = 0.84 (sec) , antiderivative size = 11688, normalized size of antiderivative = 79.51

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, al
gorithm="maxima")
```

output

```
-1/4*(4*b*x + a)*B^2*c^2*d*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^6
*g^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2
*g^5) - 1/4*(6*b^2*x^2 + 4*a*b*x + a^2)*B^2*c*d^2*i^3*log(b*e*x/(d*x + c)
+ a*e/(d*x + c))^2/(b^7*g^5*x^4 + 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*
a^3*b^4*g^5*x + a^4*b^3*g^5) - 1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x +
a^3)*B^2*d^3*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^8*g^5*x^4 + 4*a
*b^7*g^5*x^3 + 6*a^2*b^6*g^5*x^2 + 4*a^3*b^5*g^5*x + a^4*b^4*g^5) + 1/288*
(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^
3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 1
3*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)
*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)
*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^
3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*
d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)
*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2
- 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*
b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(b*e
*x/(d*x + c) + a*e/(d*x + c)) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*
c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3
+ 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*...
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.48

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx =$$
$$-\frac{1}{32}\left(\frac{8\,(dx+c)^4B^2e^5i^3\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)^4g^5}+\frac{4\,(4\,ABe^5i^3+B^2e^5i^3)(dx+c)^4\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^4g^5}+\frac{(8\,A^2e^5i^3+4\,A\right.$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, al
gorithm="giac")
```

output
```
-1/32*(8*(d*x + c)^4*B^2*e^5*i^3*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x +
a*e)^4*g^5) + 4*(4*A*B*e^5*i^3 + B^2*e^5*i^3)*(d*x + c)^4*log((b*e*x + a*e
)/(d*x + c))/((b*e*x + a*e)^4*g^5) + (8*A^2*e^5*i^3 + 4*A*B*e^5*i^3 + B^2*
e^5*i^3)*(d*x + c)^4/((b*e*x + a*e)^4*g^5))*(b*c/((b*c*e - a*d*e)*(b*c - a
*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 31.53 (sec) , antiderivative size = 1565, normalized size of antiderivative = 10.65

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^5,x)
```

output

```
-(24*A^2*a^4*d^4*i^3 - 24*A^2*b^4*c^4*i^3 + 3*B^2*a^4*d^4*i^3 - 3*B^2*b^4*
c^4*i^3 + 12*A*B*a^4*d^4*i^3 - 12*A*B*b^4*c^4*i^3 - 24*B^2*b^4*c^4*i^3*log
((e*(a + b*x))/(c + d*x))^2 + B^2*a^4*d^4*i^3*atan((a*d*1i + b*c*1i + b*d*
x*2i)/(a*d - b*c))*24i + 12*B^2*a^4*d^4*i^3*log((e*(a + b*x))/(c + d*x)) -
 12*B^2*b^4*c^4*i^3*log((e*(a + b*x))/(c + d*x)) - 24*B^2*b^4*d^4*i^3*x^4*
log((e*(a + b*x))/(c + d*x))^2 + 96*A^2*a^3*b*d^4*i^3*x + 12*B^2*a^3*b*d^4
*i^3*x - 96*A^2*b^4*c^3*d*i^3*x - 12*B^2*b^4*c^3*d*i^3*x + B^2*b^4*d^4*i^3
*x^4*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*24i + A*B*a^4*d^4*i^3*
atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*96i + 96*A^2*a*b^3*d^4*i^3*
x^3 + 12*B^2*a*b^3*d^4*i^3*x^3 - 96*A^2*b^4*c*d^3*i^3*x^3 - 12*B^2*b^4*c*d
^3*i^3*x^3 + 48*A*B*a^4*d^4*i^3*log((e*(a + b*x))/(c + d*x)) - 48*A*B*b^4*
c^4*i^3*log((e*(a + b*x))/(c + d*x)) + 144*A^2*a^2*b^2*d^4*i^3*x^2 + 18*B^
2*a^2*b^2*d^4*i^3*x^2 - 144*A^2*b^4*c^2*d^2*i^3*x^2 - 18*B^2*b^4*c^2*d^2*i
^3*x^2 + 48*A*B*a^3*b*d^4*i^3*x - 48*A*B*b^4*c^3*d*i^3*x + 48*B^2*a*b^3*d^
4*i^3*x^3*log((e*(a + b*x))/(c + d*x)) - 96*B^2*b^4*c^3*d*i^3*x*log((e*(a
+ b*x))/(c + d*x))^2 - 48*B^2*b^4*c*d^3*i^3*x^3*log((e*(a + b*x))/(c + d*x
)) + A*B*b^4*d^4*i^3*x^4*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*96
i + B^2*a^3*b*d^4*i^3*x*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*96i
 + 48*A*B*a*b^3*d^4*i^3*x^3 - 48*A*B*b^4*c*d^3*i^3*x^3 + 72*B^2*a^2*b^2*d^
4*i^3*x^2*log((e*(a + b*x))/(c + d*x)) - 72*B^2*b^4*c^2*d^2*i^3*x^2*log...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1432, normalized size of antiderivative = 9.74

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x)
```

output

```
(i*( - 16*log(a + b*x)*a**5*b*c*d**3 - 64*log(a + b*x)*a**4*b**2*c*d**3*x
- 4*log(a + b*x)*a**4*b**2*c*d**3 - 96*log(a + b*x)*a**3*b**3*c*d**3*x**2
- 16*log(a + b*x)*a**3*b**3*c*d**3*x - 64*log(a + b*x)*a**2*b**4*c*d**3*x*
*3 - 24*log(a + b*x)*a**2*b**4*c*d**3*x**2 - 16*log(a + b*x)*a*b**5*c*d**3
*x**4 - 16*log(a + b*x)*a*b**5*c*d**3*x**3 - 4*log(a + b*x)*b**6*c*d**3*x*
*4 + 16*log(c + d*x)*a**5*b*c*d**3 + 64*log(c + d*x)*a**4*b**2*c*d**3*x +
4*log(c + d*x)*a**4*b**2*c*d**3 + 96*log(c + d*x)*a**3*b**3*c*d**3*x**2 +
16*log(c + d*x)*a**3*b**3*c*d**3*x + 64*log(c + d*x)*a**2*b**4*c*d**3*x**3
 + 24*log(c + d*x)*a**2*b**4*c*d**3*x**2 + 16*log(c + d*x)*a*b**5*c*d**3*x
**4 + 16*log(c + d*x)*a*b**5*c*d**3*x**3 + 4*log(c + d*x)*b**6*c*d**3*x**4
 - 8*log((a*e + b*e*x)/(c + d*x))**2*a*b**5*c**4 - 32*log((a*e + b*e*x)/(c
 + d*x))**2*a*b**5*c**3*d*x - 48*log((a*e + b*e*x)/(c + d*x))**2*a*b**5*c*
*2*d**2*x**2 - 32*log((a*e + b*e*x)/(c + d*x))**2*a*b**5*c*d**3*x**3 - 8*l
og((a*e + b*e*x)/(c + d*x))**2*a*b**5*d**4*x**4 + 16*log((a*e + b*e*x)/(c
+ d*x))*a**5*b*c*d**3 + 64*log((a*e + b*e*x)/(c + d*x))*a**4*b**2*c*d**3*x
 + 4*log((a*e + b*e*x)/(c + d*x))*a**4*b**2*c*d**3 + 96*log((a*e + b*e*x)/
(c + d*x))*a**3*b**3*c*d**3*x**2 + 16*log((a*e + b*e*x)/(c + d*x))*a**3*b*
*3*c*d**3*x - 16*log((a*e + b*e*x)/(c + d*x))*a**2*b**4*c**4 - 64*log((a*e
 + b*e*x)/(c + d*x))*a**2*b**4*c**3*d*x - 96*log((a*e + b*e*x)/(c + d*x))*
a**2*b**4*c**2*d**2*x**2 + 24*log((a*e + b*e*x)/(c + d*x))*a**2*b**4*c*...
```

3.82 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx$

Optimal result 1055
Mathematica [C] (verified) 1056
Rubi [A] (verified) 1057
Maple [B] (verified) 1059
Fricas [B] (verification not implemented) 1060
Sympy [F(-1)] 1061
Maxima [B] (verification not implemented) 1062
Giac [A] (verification not implemented) 1063
Mupad [B] (verification not implemented) 1063
Reduce [B] (verification not implemented) 1064

Optimal result

Integrand size = 42, antiderivative size = 299

$$\begin{aligned}\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx &= \frac{B^2di^3(c+dx)^4}{32(bc-ad)^2g^6(a+bx)^4}\\ &\quad-\frac{2bB^2i^3(c+dx)^5}{125(bc-ad)^2g^6(a+bx)^5}\\ &\quad+\frac{Bdi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^2g^6(a+bx)^4}\\ &\quad-\frac{2bBi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{25(bc-ad)^2g^6(a+bx)^5}\\ &\quad+\frac{di^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^2g^6(a+bx)^4}\\ &\quad-\frac{bi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5(bc-ad)^2g^6(a+bx)^5}\end{aligned}$$

output

```
1/32*B^2*d*i^3*(d*x+c)^4/(-a*d+b*c)^2/g^6/(b*x+a)^4-2/125*b*B^2*i^3*(d*x+c
)^5/(-a*d+b*c)^2/g^6/(b*x+a)^5+1/8*B*d*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*
x+c)))/(-a*d+b*c)^2/g^6/(b*x+a)^4-2/25*b*B*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)
/(d*x+c)))/(-a*d+b*c)^2/g^6/(b*x+a)^5+1/4*d*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a
)/(d*x+c)))^2/(-a*d+b*c)^2/g^6/(b*x+a)^4-1/5*b*i^3*(d*x+c)^5*(A+B*ln(e*(b*
x+a)/(d*x+c)))^2/(-a*d+b*c)^2/g^6/(b*x+a)^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 5.02 (sec) , antiderivative size = 2456, normalized size of antiderivative = 8.21

$$\int \frac{(ci+dix)^3 \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Result too large to show}$$

input

```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^6,x]
```

output

```
-1/36000*(i^3*(7200*(b*c - a*d)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 +
 27000*d*(b*c - a*d)^4*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 -
36000*d^2*(-(b*c) + a*d)^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2 + 18000*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^2 + 2000*B*d^2*(a + b*x)^2*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3 -
18*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*(a + b*x) + 36*A*d^2
*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*(a + b*x)^2 + 36*A*d^3*(a
+ b*x)^3*Log[a + b*x] + 66*B*d^3*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*(a +
b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)] -
18*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^2*(b*
c - a*d)*(a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^3*(a + b*x)^3*L
og[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - 36*A*d^3*(a + b*x)^3*Log[c + d*
x] - 66*B*d^3*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*(a + b*x)^3*Log[(d*(a +
b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[(e*(a + b*x)
)/(c + d*x)]*Log[c + d*x] - 18*B*d^3*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3
*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*(a + b
*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*(a + b*x)^3*Poly
Log[2, (b*(c + d*x))/(b*c - a*d)]) + 375*B*d*(a + b*x)*(36*A*(b*c - a*d)^4
 + 9*B*(b*c - a*d)^4 + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c)
+ a*d)^3*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c...
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^6}\,dx$$

$$\downarrow 2962$$

$$\frac{i^3 \int \frac{(c+dx)^6 \left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^6} d\frac{a+bx}{c+dx}}{g^6(bc-ad)^2}$$

$$\downarrow 2795$$

$$\frac{i^3\int\left(\frac{b(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^6}-\frac{d(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^5}\right)d\frac{a+bx}{c+dx}}{g^6(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{i^3\left(-\frac{b(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5(a+bx)^5}-\frac{2bB(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{25(a+bx)^5}+\frac{d(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4(a+bx)^4}+\frac{Bd(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{8(a+bx)^4}\right.}{g^6(bc-ad)^2}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^6,x]
```

output
```
(i^3*((B^2*d*(c + d*x)^4)/(32*(a + b*x)^4) - (2*b*B^2*(c + d*x)^5)/(125*(a
 + b*x)^5) + (B*d*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(8*(a
+ b*x)^4) - (2*b*B*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(25*(
a + b*x)^5) + (d*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*(a
 + b*x)^4) - (b*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(5*(a
+ b*x)^5)))/((b*c - a*d)^2*g^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. 2(287) = 574.

Time = 2.50 (sec) , antiderivative size = 722, normalized size of antiderivative = 2.41

method	result
derivativedivides	$-\dfrac{e(da-bc)\left(\dfrac{i^3d^2e^4A^2b}{5(da-bc)^3g^6\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^5}-\dfrac{i^3d^3e^3A^2}{4(da-bc)^3g^6\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\dfrac{2i^3d^2e^4ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^5}-\frac{25\left(\frac{be}{d}\right.\right.}{(da-bc)^3g^6}$
default	$-\dfrac{e(da-bc)\left(\dfrac{i^3d^2e^4A^2b}{5(da-bc)^3g^6\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^5}-\dfrac{i^3d^3e^3A^2}{4(da-bc)^3g^6\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^4}-\dfrac{2i^3d^2e^4ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{5\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^5}-\frac{25\left(\frac{be}{d}\right.\right.}{(da-bc)^3g^6}$
parts	$\dfrac{i^3A^2\left(-\frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{5b^4(bx+a)^5}-\frac{d^3}{2b^4(bx+a)^2}-\frac{3d\left(a^2d^2-2acdb+c^2b^2\right)}{4b^4(bx+a)^4}+\frac{d^2(da-bc)}{b^4(bx+a)^3}\right)}{g^6}-\dfrac{i^3B^2(da-bc)^4e^4\left(\frac{d^6}{}\right.}{}$
norman	Expression too large to display
orering	Expression too large to display
parallelrisch	Expression too large to display
risch	Expression too large to display

input

```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x,method=_RE
TURNVERBOSE)
```

output

```
-1/d^2*e*(a*d-b*c)*(1/5*i^3*d^2*e^4/(a*d-b*c)^3/g^6*A^2*b/(b*e/d+(a*d-b*c)
*e/d/(d*x+c))^5-1/4*i^3*d^3*e^3/(a*d-b*c)^3/g^6*A^2/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^4-2*i^3*d^2*e^4/(a*d-b*c)^3/g^6*A*B*b*(-1/5/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/25/(b*e/d+(a*d-b*c)*e/d/(d*x+c
))^5)+2*i^3*d^3*e^3/(a*d-b*c)^3/g^6*A*B*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4)-
i^3*d^2*e^4/(a*d-b*c)^3/g^6*B^2*b*(-1/5/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5*ln
(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/25/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-2/125/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5)+i^3*d^3*
e^3/(a*d-b*c)^3/g^6*B^2*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))^2-1/8/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*
c)*e/d/(d*x+c))-1/32/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1045 vs. $2(287) = 574$.

Time = 0.10 (sec) , antiderivative size = 1045, normalized size of antiderivative = 3.49

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, al
gorithm="fricas")
```

output

```
1/4000*(20*((20*A*B + 9*B^2)*b^5*c*d^4 - (20*A*B + 9*B^2)*a*b^4*d^5)*i^3*x
^4 - 10*((200*A^2 + 20*A*B - 11*B^2)*b^5*c^2*d^3 - 50*(8*A^2 + 4*A*B + B^2
)*a*b^4*c*d^4 + (200*A^2 + 180*A*B + 61*B^2)*a^2*b^3*d^5)*i^3*x^3 - 10*(2*
(200*A^2 + 60*A*B + 7*B^2)*b^5*c^3*d^2 - 75*(8*A^2 + 4*A*B + B^2)*a*b^4*c^
2*d^3 + (200*A^2 + 180*A*B + 61*B^2)*a^3*b^2*d^5)*i^3*x^2 - 5*((600*A^2 +
220*A*B + 39*B^2)*b^5*c^4*d - 100*(8*A^2 + 4*A*B + B^2)*a*b^4*c^3*d^2 + (2
00*A^2 + 180*A*B + 61*B^2)*a^4*b*d^5)*i^3*x - (32*(25*A^2 + 10*A*B + 2*B^2
)*b^5*c^5 - 125*(8*A^2 + 4*A*B + B^2)*a*b^4*c^4*d + (200*A^2 + 180*A*B + 6
1*B^2)*a^5*d^5)*i^3 + 200*(B^2*b^5*d^5*i^3*x^5 + 5*B^2*a*b^4*d^5*i^3*x^4 -
 10*(B^2*b^5*c^2*d^3 - 2*B^2*a*b^4*c*d^4)*i^3*x^3 - 10*(2*B^2*b^5*c^3*d^2
- 3*B^2*a*b^4*c^2*d^3)*i^3*x^2 - 5*(3*B^2*b^5*c^4*d - 4*B^2*a*b^4*c^3*d^2)
*i^3*x - (4*B^2*b^5*c^5 - 5*B^2*a*b^4*c^4*d)*i^3)*log((b*e*x + a*e)/(d*x +
 c))^2 + 20*((20*A*B + 9*B^2)*b^5*d^5*i^3*x^5 + 5*(4*B^2*b^5*c*d^4 + 5*(4*
A*B + B^2)*a*b^4*d^5)*i^3*x^4 - 10*((20*A*B + B^2)*b^5*c^2*d^3 - 10*(4*A*B
 + B^2)*a*b^4*c*d^4)*i^3*x^3 - 10*(2*(20*A*B + 3*B^2)*b^5*c^3*d^2 - 15*(4*
A*B + B^2)*a*b^4*c^2*d^3)*i^3*x^2 - 5*((60*A*B + 11*B^2)*b^5*c^4*d - 20*(4
*A*B + B^2)*a*b^4*c^3*d^2)*i^3*x - (16*(5*A*B + B^2)*b^5*c^5 - 25*(4*A*B +
 B^2)*a*b^4*c^4*d)*i^3)*log((b*e*x + a*e)/(d*x + c)))/((b^11*c^2 - 2*a*b^1
0*c*d + a^2*b^9*d^2)*g^6*x^5 + 5*(a*b^10*c^2 - 2*a^2*b^9*c*d + a^3*b^8*d^2
)*g^6*x^4 + 10*(a^2*b^9*c^2 - 2*a^3*b^8*c*d + a^4*b^7*d^2)*g^6*x^3 + 10...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Timed out}$$

input

```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**6,x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15765 vs. $2(287) = 574$.

Time = 1.26 (sec) , antiderivative size = 15765, normalized size of antiderivative = 52.73

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, al
gorithm="maxima")
```

output
```
-3/20*(5*b*x + a)*B^2*c^2*d*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^
7*g^6*x^5 + 5*a*b^6*g^6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4*g^6*x^2 + 5*
a^4*b^3*g^6*x + a^5*b^2*g^6) - 1/10*(10*b^2*x^2 + 5*a*b*x + a^2)*B^2*c*d^2
*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^8*g^6*x^5 + 5*a*b^7*g^6*x^4
 + 10*a^2*b^6*g^6*x^3 + 10*a^3*b^5*g^6*x^2 + 5*a^4*b^4*g^6*x + a^5*b^3*g^6
) - 1/20*(10*b^3*x^3 + 10*a*b^2*x^2 + 5*a^2*b*x + a^3)*B^2*d^3*i^3*log(b*e
*x/(d*x + c) + a*e/(d*x + c))^2/(b^9*g^6*x^5 + 5*a*b^8*g^6*x^4 + 10*a^2*b^
7*g^6*x^3 + 10*a^3*b^6*g^6*x^2 + 5*a^4*b^5*g^6*x + a^5*b^4*g^6) - 1/9000*(
60*((60*b^4*d^4*x^4 + 12*b^4*c^4 - 63*a*b^3*c^3*d + 137*a^2*b^2*c^2*d^2 -
163*a^3*b*c*d^3 + 137*a^4*d^4 - 30*(b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 10*(2*b
^4*c^2*d^2 - 13*a*b^3*c*d^3 + 47*a^2*b^2*d^4)*x^2 - 5*(3*b^4*c^3*d - 17*a*
b^3*c^2*d^2 + 43*a^2*b^2*c*d^3 - 77*a^3*b*d^4)*x)/((b^10*c^4 - 4*a*b^9*c^3
*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*g^6*x^5 + 5*(a*b^9
*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4
)*g^6*x^4 + 10*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 - 4*a^5*
b^5*c*d^3 + a^6*b^4*d^4)*g^6*x^3 + 10*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*a
^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*g^6*x^2 + 5*(a^4*b^6*c^4 -
 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4)*g^6*
x + (a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2*c*d^3 +
 a^9*b*d^4)*g^6) + 60*d^5*log(b*x + a)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a...
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.60

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx =$$

$$-\frac{1}{4000}\left(\frac{200\left(4\,B^2be^6i^3-\frac{5\,(bex+ae)B^2de^5i^3}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^5bcg^6}{(dx+c)^5}-\frac{(bex+ae)^5adg^6}{(dx+c)^5}}+\frac{20\left(80\,ABbe^6i^3+16\,B^2be^6i^3-\frac{100\,(bex+ae)A}{dx+c}\right.}{\frac{(bex+ae)^5bcg^6}{(dx+c)^5}-\frac{(b}{}}\right.$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, al
gorithm="giac")
```

output
```
-1/4000*(200*(4*B^2*b*e^6*i^3 - 5*(b*e*x + a*e)*B^2*d*e^5*i^3/(d*x + c))*l
og((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)^5*b*c*g^6/(d*x + c)^5 - (b*e*
x + a*e)^5*a*d*g^6/(d*x + c)^5) + 20*(80*A*B*b*e^6*i^3 + 16*B^2*b*e^6*i^3
- 100*(b*e*x + a*e)*A*B*d*e^5*i^3/(d*x + c) - 25*(b*e*x + a*e)*B^2*d*e^5*i
^3/(d*x + c))*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^5*b*c*g^6/(d*x +
 c)^5 - (b*e*x + a*e)^5*a*d*g^6/(d*x + c)^5) + (800*A^2*b*e^6*i^3 + 320*A*
B*b*e^6*i^3 + 64*B^2*b*e^6*i^3 - 1000*(b*e*x + a*e)*A^2*d*e^5*i^3/(d*x + c
) - 500*(b*e*x + a*e)*A*B*d*e^5*i^3/(d*x + c) - 125*(b*e*x + a*e)*B^2*d*e^
5*i^3/(d*x + c))/((b*e*x + a*e)^5*b*c*g^6/(d*x + c)^5 - (b*e*x + a*e)^5*a*
d*g^6/(d*x + c)^5))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d
*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 35.59 (sec) , antiderivative size = 3720, normalized size of antiderivative = 12.44

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^6,x)
```

output

```
- log((e*(a + b*x))/(c + d*x))^2*((x*(a*(b*((B^2*a*d^3*i^3)/(20*b^5*g^6) +
 (B^2*c*d^2*i^3)/(10*b^4*g^6)) + (3*B^2*a*d^3*i^3)/(20*b^4*g^6) + (3*B^2*c
*d^2*i^3)/(10*b^3*g^6)) + b*(a*((B^2*a*d^3*i^3)/(20*b^5*g^6) + (B^2*c*d^2*
i^3)/(10*b^4*g^6)) + (3*B^2*c^2*d*i^3)/(20*b^3*g^6)) + (3*B^2*c^2*d*i^3)/(
5*b^2*g^6)) + x^2*(b*(b*((B^2*a*d^3*i^3)/(20*b^5*g^6) + (B^2*c*d^2*i^3)/(1
0*b^4*g^6)) + (3*B^2*a*d^3*i^3)/(20*b^4*g^6) + (3*B^2*c*d^2*i^3)/(10*b^3*g
^6)) + (3*B^2*a*d^3*i^3)/(10*b^3*g^6) + (3*B^2*c*d^2*i^3)/(5*b^2*g^6)) + a
*(a*((B^2*a*d^3*i^3)/(20*b^5*g^6) + (B^2*c*d^2*i^3)/(10*b^4*g^6)) + (3*B^2
*c^2*d*i^3)/(20*b^3*g^6)) + (B^2*c^3*i^3)/(5*b^2*g^6) + (B^2*d^3*i^3*x^3)/
(2*b^2*g^6))/(5*a^4*x + a^5/b + b^4*x^5 + 10*a^3*b*x^2 + 5*a*b^3*x^4 + 10*
a^2*b^2*x^3) - (B^2*d^5*i^3)/(20*b^4*g^6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))
 - ((200*A^2*a^4*d^4*i^3 - 800*A^2*b^4*c^4*i^3 + 61*B^2*a^4*d^4*i^3 - 64*B
^2*b^4*c^4*i^3 + 180*A*B*a^4*d^4*i^3 - 320*A*B*b^4*c^4*i^3 + 200*A^2*a*b^3
*c^3*d*i^3 + 200*A^2*a^3*b*c*d^3*i^3 + 61*B^2*a*b^3*c^3*d*i^3 + 61*B^2*a^3
*b*c*d^3*i^3 + 200*A^2*a^2*b^2*c^2*d^2*i^3 + 61*B^2*a^2*b^2*c^2*d^2*i^3 +
180*A*B*a^2*b^2*c^2*d^2*i^3 + 180*A*B*a*b^3*c^3*d*i^3 + 180*A*B*a^3*b*c*d^
3*i^3)/(20*(a*d - b*c)) + (x^4*(9*B^2*b^4*d^4*i^3 + 20*A*B*b^4*d^4*i^3))/(
a*d - b*c) + (x^3*(200*A^2*a*b^3*d^4*i^3 + 61*B^2*a*b^3*d^4*i^3 - 200*A^2*
b^4*c*d^3*i^3 + 11*B^2*b^4*c*d^3*i^3 + 180*A*B*a*b^3*d^4*i^3 - 20*A*B*b^4*
c*d^3*i^3))/(2*(a*d - b*c)) + (x*(200*A^2*a^3*b*d^4*i^3 + 61*B^2*a^3*b*...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2741, normalized size of antiderivative = 9.17

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x)
```

output

```
(i*( - 400*log(a + b*x)*a**7*b*d**5 - 2000*log(a + b*x)*a**6*b**2*d**5*x -
 100*log(a + b*x)*a**6*b**2*d**5 - 80*log(a + b*x)*a**5*b**3*c*d**4 - 4000
*log(a + b*x)*a**5*b**3*d**5*x**2 - 500*log(a + b*x)*a**5*b**3*d**5*x - 40
0*log(a + b*x)*a**4*b**4*c*d**4*x - 4000*log(a + b*x)*a**4*b**4*d**5*x**3
 - 1000*log(a + b*x)*a**4*b**4*d**5*x**2 - 800*log(a + b*x)*a**3*b**5*c*d**
4*x**2 - 2000*log(a + b*x)*a**3*b**5*d**5*x**4 - 1000*log(a + b*x)*a**3*b*
*5*d**5*x**3 - 800*log(a + b*x)*a**2*b**6*c*d**4*x**3 - 400*log(a + b*x)*a
**2*b**6*d**5*x**5 - 500*log(a + b*x)*a**2*b**6*d**5*x**4 - 400*log(a + b*
x)*a*b**7*c*d**4*x**4 - 100*log(a + b*x)*a*b**7*d**5*x**5 - 80*log(a + b*x
)*b**8*c*d**4*x**5 + 400*log(c + d*x)*a**7*b*d**5 + 2000*log(c + d*x)*a**6
*b**2*d**5*x + 100*log(c + d*x)*a**6*b**2*d**5 + 80*log(c + d*x)*a**5*b**3
*c*d**4 + 4000*log(c + d*x)*a**5*b**3*d**5*x**2 + 500*log(c + d*x)*a**5*b*
*3*d**5*x + 400*log(c + d*x)*a**4*b**4*c*d**4*x + 4000*log(c + d*x)*a**4*b
**4*d**5*x**3 + 1000*log(c + d*x)*a**4*b**4*d**5*x**2 + 800*log(c + d*x)*a
**3*b**5*c*d**4*x**2 + 2000*log(c + d*x)*a**3*b**5*d**5*x**4 + 1000*log(c
+ d*x)*a**3*b**5*d**5*x**3 + 800*log(c + d*x)*a**2*b**6*c*d**4*x**3 + 400*
log(c + d*x)*a**2*b**6*d**5*x**5 + 500*log(c + d*x)*a**2*b**6*d**5*x**4 +
400*log(c + d*x)*a*b**7*c*d**4*x**4 + 100*log(c + d*x)*a*b**7*d**5*x**5 +
80*log(c + d*x)*b**8*c*d**4*x**5 - 1000*log((a*e + b*e*x)/(c + d*x))**2*a*
*2*b**6*c**4*d - 4000*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**6*c**3*d*...
```

3.83 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx$

Optimal result 1067
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Rubi [A] (verified) 1069
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Reduce [B] (verification not implemented) 1076

Optimal result

Integrand size = 42, antiderivative size = 463

$$\begin{aligned}\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx &= -\frac{B^2d^2i^3(c+dx)^4}{32(bc-ad)^3g^7(a+bx)^4}\\ &+\frac{4bB^2di^3(c+dx)^5}{125(bc-ad)^3g^7(a+bx)^5}\\ &-\frac{b^2B^2i^3(c+dx)^6}{108(bc-ad)^3g^7(a+bx)^6}\\ &-\frac{Bd^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^3g^7(a+bx)^4}\\ &+\frac{4bBdi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{25(bc-ad)^3g^7(a+bx)^5}\\ &-\frac{b^2Bi^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{18(bc-ad)^3g^7(a+bx)^6}\\ &-\frac{d^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^3g^7(a+bx)^4}\\ &+\frac{2bdi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5(bc-ad)^3g^7(a+bx)^5}\\ &-\frac{b^2i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6(bc-ad)^3g^7(a+bx)^6}\end{aligned}$$

output

```
-1/32*B^2*d^2*i^3*(d*x+c)^4/(-a*d+b*c)^3/g^7/(b*x+a)^4+4/125*b*B^2*d*i^3*(
d*x+c)^5/(-a*d+b*c)^3/g^7/(b*x+a)^5-1/108*b^2*B^2*i^3*(d*x+c)^6/(-a*d+b*c)
^3/g^7/(b*x+a)^6-1/8*B*d^2*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d
+b*c)^3/g^7/(b*x+a)^4+4/25*b*B*d*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))
/(-a*d+b*c)^3/g^7/(b*x+a)^5-1/18*b^2*B*i^3*(d*x+c)^6*(A+B*ln(e*(b*x+a)/(d*
x+c)))/(-a*d+b*c)^3/g^7/(b*x+a)^6-1/4*d^2*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/
(d*x+c)))^2/(-a*d+b*c)^3/g^7/(b*x+a)^4+2/5*b*d*i^3*(d*x+c)^5*(A+B*ln(e*(b*
x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^7/(b*x+a)^5-1/6*b^2*i^3*(d*x+c)^6*(A+B*ln(
e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^7/(b*x+a)^6
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 5.83 (sec) , antiderivative size = 2606, normalized size of antiderivative = 5.63

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx = \text{Result too large to show}$$

input

```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^7,x]
```

output

```
(i^3*(-6000*A*B*(b*c - a*d)^6 - 1000*B^2*(b*c - a*d)^6 + 25920*a*A*B*d*(-(
b*c) + a*d)^5 + 5184*a*B^2*d*(-(b*c) + a*d)^5 - 25920*A*b*B*d*(b*c - a*d)^
5*x - 5184*b*B^2*d*(b*c - a*d)^5*x + 32400*a*A*B*d^2*(b*c - a*d)^4*(a + b*
x) + 14580*a*B^2*d^2*(b*c - a*d)^4*(a + b*x) + 7200*A*B*d*(b*c - a*d)^5*(a
 + b*x) + 2640*B^2*d*(b*c - a*d)^5*(a + b*x) + 32400*A*b*B*d^2*(b*c - a*d)
^4*x*(a + b*x) + 14580*b*B^2*d^2*(b*c - a*d)^4*x*(a + b*x) - 49500*A*B*d^2
*(b*c - a*d)^4*(a + b*x)^2 - 15675*B^2*d^2*(b*c - a*d)^4*(a + b*x)^2 + 432
00*a*A*B*d^3*(-(b*c) + a*d)^3*(a + b*x)^2 + 33840*a*B^2*d^3*(-(b*c) + a*d)
^3*(a + b*x)^2 - 43200*A*b*B*d^3*(b*c - a*d)^3*x*(a + b*x)^2 - 33840*b*B^2
*d^3*(b*c - a*d)^3*x*(a + b*x)^2 + 64800*a*A*B*d^4*(b*c - a*d)^2*(a + b*x)
^3 + 83160*a*B^2*d^4*(b*c - a*d)^2*(a + b*x)^3 + 42000*A*B*d^3*(b*c - a*d)
^3*(a + b*x)^3 + 34900*B^2*d^3*(b*c - a*d)^3*(a + b*x)^3 + 64800*A*b*B*d^4
*(b*c - a*d)^2*x*(a + b*x)^3 + 83160*b*B^2*d^4*(b*c - a*d)^2*x*(a + b*x)^3
 - 63000*A*B*d^4*(b*c - a*d)^2*(a + b*x)^4 - 83850*B^2*d^4*(b*c - a*d)^2*(
a + b*x)^4 + 129600*a*A*B*d^5*(-(b*c) + a*d)*(a + b*x)^4 + 295920*a*B^2*d^
5*(-(b*c) + a*d)*(a + b*x)^4 - 129600*A*b*B*d^5*(b*c - a*d)*x*(a + b*x)^4
- 295920*b*B^2*d^5*(b*c - a*d)*x*(a + b*x)^4 + 126000*A*B*d^5*(b*c - a*d)*
(a + b*x)^5 + 293700*B^2*d^5*(b*c - a*d)*(a + b*x)^5 - 129600*a*A*B*d^6*(a
 + b*x)^5*Log[a + b*x] - 295920*a*B^2*d^6*(a + b*x)^5*Log[a + b*x] - 12960
0*A*b*B*d^6*x*(a + b*x)^5*Log[a + b*x] - 295920*b*B^2*d^6*x*(a + b*x)^5...
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^7}\,dx$$

$$\downarrow 2962$$

$$\frac{i^3\int \frac{(c+dx)^7\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^7}\,d\frac{a+bx}{c+dx}}{g^7(bc-ad)^3}$$

$$\downarrow 2795$$

$$\frac{i^3\int\left(\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^7}{(a+bx)^7}-\frac{2bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^6}{(a+bx)^6}+\frac{d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^5}{(a+bx)^5}\right)d\frac{a+bx}{c+dx}}{g^7(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{i^3\left(-\frac{b^2(c+dx)^6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{6(a+bx)^6}-\frac{b^2B(c+dx)^6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{18(a+bx)^6}-\frac{d^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4(a+bx)^4}-\frac{Bd^2(c+dx)^4\left(B\log(\right.}{8(a+bx}\right.}{g^7(}$$

input

```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^7,x]
```

output
```
(i^3*(-1/32*(B^2*d^2*(c + d*x)^4)/(a + b*x)^4 + (4*b*B^2*d*(c + d*x)^5)/(1
25*(a + b*x)^5) - (b^2*B^2*(c + d*x)^6)/(108*(a + b*x)^6) - (B*d^2*(c + d*
x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(8*(a + b*x)^4) + (4*b*B*d*(c +
 d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(25*(a + b*x)^5) - (b^2*B*(c
 + d*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(18*(a + b*x)^6) - (d^2*(c
 + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*(a + b*x)^4) + (2*b*d
*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(5*(a + b*x)^5) - (b^
2*(c + d*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(6*(a + b*x)^6)))/((
b*c - a*d)^3*g^7)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. $2(445) = 890$.

Time = 3.20 (sec) , antiderivative size = 1019, normalized size of antiderivative = 2.20

method	result	size
parts	Expression too large to display	1019
derivativedivides	Expression too large to display	1082
default	Expression too large to display	1082
orering	Expression too large to display	2152
norman	Expression too large to display	2555
parallelrisch	Expression too large to display	2803
risch	Expression too large to display	6548

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^7,x,method=_RE
TURNVERBOSE)
```

output
```
i^3*A^2/g^7*(-3/5*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^5-1/6*(-a^3*d^
3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^6+3/4*d^2*(a*d-b*c)/b^4
/(b*x+a)^4-1/3*d^3/b^4/(b*x+a)^3)-i^3*B^2/g^7/d^5*(a*d-b*c)^4*e^4*(d^7/(a*
d-b*c)^7*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^2-1/8/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))
-1/32/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4)-2*d^6/(a*d-b*c)^7*b*e*(-1/5/(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/25/(b*e/d+(a*
d-b*c)*e/d/(d*x+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/125/(b*e/d+(a*d-b*
c)*e/d/(d*x+c))^5)+d^5/(a*d-b*c)^7*b^2*e^2*(-1/6/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^6*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/18/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^6*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/108/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^6)
)-2*i^3*A*B/g^7/d^5*(a*d-b*c)^4*e^4*(d^7/(a*d-b*c)^7*(-1/4/(b*e/d+(a*d-b*c
)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d
/(d*x+c))^4)-2*d^6/(a*d-b*c)^7*b*e*(-1/5/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/25/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5)+d^5/(
a*d-b*c)^7*b^2*e^2*(-1/6/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^6*ln(b*e/d+(a*d-b*c
)*e/d/(d*x+c))-1/36/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^6))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1610 vs. $2(445) = 890$.

Time $= 0.12$ (sec) , antiderivative size $= 1610$, normalized size of antiderivative $= 3.48$

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^7,x, al
gorithm="fricas")
```

output

```
-1/108000*(60*((60*A*B + 37*B^2)*b^6*c*d^5 - (60*A*B + 37*B^2)*a*b^5*d^6)*
i^3*x^5 - 30*((60*A*B - 23*B^2)*b^6*c^2*d^4 - 36*(20*A*B + 9*B^2)*a*b^5*c*
d^5 + (660*A*B + 347*B^2)*a^2*b^4*d^6)*i^3*x^4 + 20*((1800*A^2 + 60*A*B -
53*B^2)*b^6*c^3*d^3 - 27*(200*A^2 + 20*A*B - 11*B^2)*a*b^5*c^2*d^4 + 675*(
8*A^2 + 4*A*B + B^2)*a^2*b^4*c*d^5 - (1800*A^2 + 2220*A*B + 919*B^2)*a^3*b
^3*d^6)*i^3*x^3 + 15*((5400*A^2 + 1140*A*B + 73*B^2)*b^6*c^4*d^2 - 72*(200
*A^2 + 60*A*B + 7*B^2)*a*b^5*c^3*d^3 + 1350*(8*A^2 + 4*A*B + B^2)*a^2*b^4*
c^2*d^4 - (1800*A^2 + 2220*A*B + 919*B^2)*a^4*b^2*d^6)*i^3*x^2 + 6*(8*(135
0*A^2 + 390*A*B + 53*B^2)*b^6*c^5*d - 45*(600*A^2 + 220*A*B + 39*B^2)*a*b^
5*c^4*d^2 + 2250*(8*A^2 + 4*A*B + B^2)*a^2*b^4*c^3*d^3 - (1800*A^2 + 2220*
A*B + 919*B^2)*a^5*b*d^6)*i^3*x + (1000*(18*A^2 + 6*A*B + B^2)*b^6*c^6 - 1
728*(25*A^2 + 10*A*B + 2*B^2)*a*b^5*c^5*d + 3375*(8*A^2 + 4*A*B + B^2)*a^2
*b^4*c^4*d^2 - (1800*A^2 + 2220*A*B + 919*B^2)*a^6*d^6)*i^3 + 1800*(B^2*b^
6*d^6*i^3*x^6 + 6*B^2*a*b^5*d^6*i^3*x^5 + 15*B^2*a^2*b^4*d^6*i^3*x^4 + 20*
(B^2*b^6*c^3*d^3 - 3*B^2*a*b^5*c^2*d^4 + 3*B^2*a^2*b^4*c*d^5)*i^3*x^3 + 15
*(3*B^2*b^6*c^4*d^2 - 8*B^2*a*b^5*c^3*d^3 + 6*B^2*a^2*b^4*c^2*d^4)*i^3*x^2
 + 6*(6*B^2*b^6*c^5*d - 15*B^2*a*b^5*c^4*d^2 + 10*B^2*a^2*b^4*c^3*d^3)*i^3
*x + (10*B^2*b^6*c^6 - 24*B^2*a*b^5*c^5*d + 15*B^2*a^2*b^4*c^4*d^2)*i^3)*l
og((b*e*x + a*e)/(d*x + c))^2 + 60*((60*A*B + 37*B^2)*b^6*d^6*i^3*x^6 + 6*
(10*B^2*b^6*c*d^5 + 3*(20*A*B + 9*B^2)*a*b^5*d^6)*i^3*x^5 - 15*(2*B^2*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**7,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20330 vs. $2(445) = 890$.

Time = 1.83 (sec) , antiderivative size = 20330, normalized size of antiderivative = 43.91

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^7,x, al
gorithm="maxima")
```

output

```
-1/10*(6*b*x + a)*B^2*c^2*d*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^
8*g^7*x^6 + 6*a*b^7*g^7*x^5 + 15*a^2*b^6*g^7*x^4 + 20*a^3*b^5*g^7*x^3 + 15
*a^4*b^4*g^7*x^2 + 6*a^5*b^3*g^7*x + a^6*b^2*g^7) - 1/20*(15*b^2*x^2 + 6*a
*b*x + a^2)*B^2*c*d^2*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^9*g^7*
x^6 + 6*a*b^8*g^7*x^5 + 15*a^2*b^7*g^7*x^4 + 20*a^3*b^6*g^7*x^3 + 15*a^4*b
^5*g^7*x^2 + 6*a^5*b^4*g^7*x + a^6*b^3*g^7) - 1/60*(20*b^3*x^3 + 15*a*b^2*
x^2 + 6*a^2*b*x + a^3)*B^2*d^3*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/
(b^10*g^7*x^6 + 6*a*b^9*g^7*x^5 + 15*a^2*b^8*g^7*x^4 + 20*a^3*b^7*g^7*x^3
+ 15*a^4*b^6*g^7*x^2 + 6*a^5*b^5*g^7*x + a^6*b^4*g^7) + 1/10800*(60*((60*b
^5*d^5*x^5 - 10*b^5*c^5 + 62*a*b^4*c^4*d - 163*a^2*b^3*c^3*d^2 + 237*a^3*b
^2*c^2*d^3 - 213*a^4*b*c*d^4 + 147*a^5*d^5 - 30*(b^5*c*d^4 - 11*a*b^4*d^5)
*x^4 + 20*(b^5*c^2*d^3 - 8*a*b^4*c*d^4 + 37*a^2*b^3*d^5)*x^3 - 15*(b^5*c^3
*d^2 - 7*a*b^4*c^2*d^3 + 23*a^2*b^3*c*d^4 - 57*a^3*b^2*d^5)*x^2 + 6*(2*b^5
*c^4*d - 13*a*b^4*c^3*d^2 + 37*a^2*b^3*c^2*d^3 - 63*a^3*b^2*c*d^4 + 87*a^4
*b*d^5)*x)/((b^12*c^5 - 5*a*b^11*c^4*d + 10*a^2*b^10*c^3*d^2 - 10*a^3*b^9*
c^2*d^3 + 5*a^4*b^8*c*d^4 - a^5*b^7*d^5)*g^7*x^6 + 6*(a*b^11*c^5 - 5*a^2*b
^10*c^4*d + 10*a^3*b^9*c^3*d^2 - 10*a^4*b^8*c^2*d^3 + 5*a^5*b^7*c*d^4 - a^
6*b^6*d^5)*g^7*x^5 + 15*(a^2*b^10*c^5 - 5*a^3*b^9*c^4*d + 10*a^4*b^8*c^3*d
^2 - 10*a^5*b^7*c^2*d^3 + 5*a^6*b^6*c*d^4 - a^7*b^5*d^5)*g^7*x^4 + 20*(a^3
*b^9*c^5 - 5*a^4*b^8*c^4*d + 10*a^5*b^7*c^3*d^2 - 10*a^6*b^6*c^2*d^3 + ...
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.68

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^7,x, al
gorithm="giac")
```

output

```
-1/108000*(1800*(10*B^2*b^2*e^7*i^3 - 24*(b*e*x + a*e)*B^2*b*d*e^6*i^3/(d*
x + c) + 15*(b*e*x + a*e)^2*B^2*d^2*e^5*i^3/(d*x + c)^2)*log((b*e*x + a*e)
/(d*x + c))^2/((b*e*x + a*e)^6*b^2*c^2*g^7/(d*x + c)^6 - 2*(b*e*x + a*e)^6
*a*b*c*d*g^7/(d*x + c)^6 + (b*e*x + a*e)^6*a^2*d^2*g^7/(d*x + c)^6) + 60*(
600*A*B*b^2*e^7*i^3 + 100*B^2*b^2*e^7*i^3 - 1440*(b*e*x + a*e)*A*B*b*d*e^6
*i^3/(d*x + c) - 288*(b*e*x + a*e)*B^2*b*d*e^6*i^3/(d*x + c) + 900*(b*e*x
+ a*e)^2*A*B*d^2*e^5*i^3/(d*x + c)^2 + 225*(b*e*x + a*e)^2*B^2*d^2*e^5*i^3
/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^6*b^2*c^2*g^7/(d
*x + c)^6 - 2*(b*e*x + a*e)^6*a*b*c*d*g^7/(d*x + c)^6 + (b*e*x + a*e)^6*a^
2*d^2*g^7/(d*x + c)^6) + (18000*A^2*b^2*e^7*i^3 + 6000*A*B*b^2*e^7*i^3 + 1
000*B^2*b^2*e^7*i^3 - 43200*(b*e*x + a*e)*A^2*b*d*e^6*i^3/(d*x + c) - 1728
0*(b*e*x + a*e)*A*B*b*d*e^6*i^3/(d*x + c) - 3456*(b*e*x + a*e)*B^2*b*d*e^6
*i^3/(d*x + c) + 27000*(b*e*x + a*e)^2*A^2*d^2*e^5*i^3/(d*x + c)^2 + 13500
*(b*e*x + a*e)^2*A*B*d^2*e^5*i^3/(d*x + c)^2 + 3375*(b*e*x + a*e)^2*B^2*d^
2*e^5*i^3/(d*x + c)^2)/((b*e*x + a*e)^6*b^2*c^2*g^7/(d*x + c)^6 - 2*(b*e*x
 + a*e)^6*a*b*c*d*g^7/(d*x + c)^6 + (b*e*x + a*e)^6*a^2*d^2*g^7/(d*x + c)^
6))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d))
)
```

Mupad [B] (verification not implemented)

Time = 36.44 (sec) , antiderivative size = 6275, normalized size of antiderivative = 13.55

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx = \text{Too large to display}$$

input

```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^7,x)
```

output

```
((1800*A^2*a^5*d^5*i^3 + 18000*A^2*b^5*c^5*i^3 + 919*B^2*a^5*d^5*i^3 + 100
0*B^2*b^5*c^5*i^3 + 2220*A*B*a^5*d^5*i^3 + 6000*A*B*b^5*c^5*i^3 - 25200*A^
2*a*b^4*c^4*d*i^3 + 1800*A^2*a^4*b*c*d^4*i^3 - 2456*B^2*a*b^4*c^4*d*i^3 +
919*B^2*a^4*b*c*d^4*i^3 + 1800*A^2*a^2*b^3*c^3*d^2*i^3 + 1800*A^2*a^3*b^2*
c^2*d^3*i^3 + 919*B^2*a^2*b^3*c^3*d^2*i^3 + 919*B^2*a^3*b^2*c^2*d^3*i^3 +
2220*A*B*a^2*b^3*c^3*d^2*i^3 + 2220*A*B*a^3*b^2*c^2*d^3*i^3 - 11280*A*B*a*
b^4*c^4*d*i^3 + 2220*A*B*a^4*b*c*d^4*i^3)/(60*(a*d - b*c)) + (x^4*(347*B^2
*a*b^4*d^5*i^3 + 23*B^2*b^5*c*d^4*i^3 + 660*A*B*a*b^4*d^5*i^3 - 60*A*B*b^5
*c*d^4*i^3))/(2*(a*d - b*c)) + (x^2*(1800*A^2*a^3*b^2*d^5*i^3 + 919*B^2*a^
3*b^2*d^5*i^3 + 5400*A^2*b^5*c^3*d^2*i^3 + 73*B^2*b^5*c^3*d^2*i^3 - 9000*A
^2*a*b^4*c^2*d^3*i^3 + 1800*A^2*a^2*b^3*c*d^4*i^3 - 431*B^2*a*b^4*c^2*d^3*
i^3 + 919*B^2*a^2*b^3*c*d^4*i^3 + 2220*A*B*a^3*b^2*d^5*i^3 + 1140*A*B*b^5*
c^3*d^2*i^3 - 3180*A*B*a*b^4*c^2*d^3*i^3 + 2220*A*B*a^2*b^3*c*d^4*i^3))/(4
*(a*d - b*c)) + (x^3*(1800*A^2*a^2*b^3*d^5*i^3 + 919*B^2*a^2*b^3*d^5*i^3 +
 1800*A^2*b^5*c^2*d^3*i^3 - 53*B^2*b^5*c^2*d^3*i^3 - 3600*A^2*a*b^4*c*d^4*
i^3 + 244*B^2*a*b^4*c*d^4*i^3 + 2220*A*B*a^2*b^3*d^5*i^3 + 60*A*B*b^5*c^2*
d^3*i^3 - 480*A*B*a*b^4*c*d^4*i^3))/(3*(a*d - b*c)) + (x*(1800*A^2*a^4*b*d
^5*i^3 + 919*B^2*a^4*b*d^5*i^3 + 10800*A^2*b^5*c^4*d*i^3 + 424*B^2*b^5*c^4
*d*i^3 - 16200*A^2*a*b^4*c^3*d^2*i^3 + 1800*A^2*a^3*b^2*c*d^4*i^3 - 1331*B
^2*a*b^4*c^3*d^2*i^3 + 919*B^2*a^3*b^2*c*d^4*i^3 + 2220*A*B*a^4*b*d^5*i...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 3828, normalized size of antiderivative = 8.27

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^7,x)
```

output

```
(i*( - 3600*log(a + b*x)*a**8*b*d**6 - 21600*log(a + b*x)*a**7*b**2*d**6*x
 - 1620*log(a + b*x)*a**7*b**2*d**6 - 600*log(a + b*x)*a**6*b**3*c*d**5 -
54000*log(a + b*x)*a**6*b**3*d**6*x**2 - 9720*log(a + b*x)*a**6*b**3*d**6*
x - 3600*log(a + b*x)*a**5*b**4*c*d**5*x - 72000*log(a + b*x)*a**5*b**4*d*
*6*x**3 - 24300*log(a + b*x)*a**5*b**4*d**6*x**2 - 9000*log(a + b*x)*a**4*
b**5*c*d**5*x**2 - 54000*log(a + b*x)*a**4*b**5*d**6*x**4 - 32400*log(a +
b*x)*a**4*b**5*d**6*x**3 - 12000*log(a + b*x)*a**3*b**6*c*d**5*x**3 - 2160
0*log(a + b*x)*a**3*b**6*d**6*x**5 - 24300*log(a + b*x)*a**3*b**6*d**6*x**
4 - 9000*log(a + b*x)*a**2*b**7*c*d**5*x**4 - 3600*log(a + b*x)*a**2*b**7*
d**6*x**6 - 9720*log(a + b*x)*a**2*b**7*d**6*x**5 - 3600*log(a + b*x)*a*b*
*8*c*d**5*x**5 - 1620*log(a + b*x)*a*b**8*d**6*x**6 - 600*log(a + b*x)*b**
9*c*d**5*x**6 + 3600*log(c + d*x)*a**8*b*d**6 + 21600*log(c + d*x)*a**7*b*
*2*d**6*x + 1620*log(c + d*x)*a**7*b**2*d**6 + 600*log(c + d*x)*a**6*b**3*
c*d**5 + 54000*log(c + d*x)*a**6*b**3*d**6*x**2 + 9720*log(c + d*x)*a**6*b
**3*d**6*x + 3600*log(c + d*x)*a**5*b**4*c*d**5*x + 72000*log(c + d*x)*a**
5*b**4*d**6*x**3 + 24300*log(c + d*x)*a**5*b**4*d**6*x**2 + 9000*log(c + d
*x)*a**4*b**5*c*d**5*x**2 + 54000*log(c + d*x)*a**4*b**5*d**6*x**4 + 32400
*log(c + d*x)*a**4*b**5*d**6*x**3 + 12000*log(c + d*x)*a**3*b**6*c*d**5*x*
*3 + 21600*log(c + d*x)*a**3*b**6*d**6*x**5 + 24300*log(c + d*x)*a**3*b**6
*d**6*x**4 + 9000*log(c + d*x)*a**2*b**7*c*d**5*x**4 + 3600*log(c + d*x...
```

3.84 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$

Optimal result 1079
Mathematica [A] (verified) 1080
Rubi [A] (verified) 1081
Maple [F] 1083
Fricas [F] 1084
Sympy [F] 1084
Maxima [F] 1085
Giac [F] 1086
Mupad [F(-1)] 1087
Reduce [F] 1087

Optimal result

Integrand size = 42, antiderivative size = 718

$$
\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx \\
&= \frac{bB^2(bc-ad)^2g^3x}{3d^3i} + \frac{B^2(bc-ad)^3g^3\log\left(\frac{a+bx}{c+dx}\right)}{3d^4i} \\
&\quad + \frac{7B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^3i} \\
&\quad - \frac{b^2B(bc-ad)g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^4i} \\
&\quad + \frac{6B(bc-ad)^3g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i} \\
&\quad + \frac{3(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i} \\
&\quad - \frac{3b^2(bc-ad)g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^4i} \\
&\quad + \frac{b^3g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d^4i} \\
&\quad + \frac{(bc-ad)^3g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i} - \frac{2B^2(bc-ad)^3g^3\log(c+dx)}{d^4i} \\
&\quad - \frac{7B(bc-ad)^3g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i} \\
&\quad + \frac{6B^2(bc-ad)^3g^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} \\
&\quad + \frac{2B(bc-ad)^3g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} \\
&\quad + \frac{7B^2(bc-ad)^3g^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i} - \frac{2B^2(bc-ad)^3g^3\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}
\end{aligned}
$$

output

```
1/3*b*B^2*(-a*d+b*c)^2*g^3*x/d^3/i+1/3*B^2*(-a*d+b*c)^3*g^3*ln((b*x+a)/(d*
x+c))/d^4/i+7/3*B*(-a*d+b*c)^2*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3
/i-1/3*b^2*B*(-a*d+b*c)*g^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4/i+6*
B*(-a*d+b*c)^3*g^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^
4/i+3*(-a*d+b*c)^2*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i-3/2*b^2
*(-a*d+b*c)*g^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i+1/3*b^3*g^3*
(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i+(-a*d+b*c)^3*g^3*ln((-a*d+b*
c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i-2*B^2*(-a*d+b*c)^3*g^3*l
n(d*x+c)/d^4/i-7/3*B*(-a*d+b*c)^3*g^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(
d*x+c)/d/(b*x+a))/d^4/i+6*B^2*(-a*d+b*c)^3*g^3*polylog(2,d*(b*x+a)/b/(d*x+
c))/d^4/i+2*B*(-a*d+b*c)^3*g^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,d*(b*
x+a)/b/(d*x+c))/d^4/i+7/3*B^2*(-a*d+b*c)^3*g^3*polylog(2,b*(d*x+c)/d/(b*x+
a))/d^4/i-2*B^2*(-a*d+b*c)^3*g^3*polylog(3,d*(b*x+a)/b/(d*x+c))/d^4/i
```

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 1020, normalized size of antiderivative = 1.42

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \text{Too large to display}$$

input

```
Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i +
d*i*x),x]
```

output

```
(g^3*(6*b*d*(b*c - a*d)^2*x*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 3*d^2
*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 2*d^3
*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 6*A^2*(b*c - a*d)^3*
Log[c + d*x] + 12*A*B*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c
 - a*d)/(b*c + b*d*x)] + 6*B^2*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)]^2
*Log[(b*c - a*d)/(b*c + b*d*x)] - 6*A*B*(b*c - a*d)^3*(Log[(b*c - a*d)/(b*
c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c +
b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 6*B*(b*c - a*d)^2*(2
*a*d*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b*c*(A + B*Log[
(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - a*B*d*(Log[a + b*x]*(Log[a + b*x]
 - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
+ a*d)]) + b*B*c*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log
[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 3*B*(b*c - a*d)^2*
(2*A*b*d*x + 2*B*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 2*B*(b*c - a*d
)*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c
+ d*x] + B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x
])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 2*B*(b*c - a
*d)*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))
/(c + d*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(
b*c - a*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c...
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 629, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{ci+dix}\,dx$$

$$\downarrow 2962$$

$$\frac{g^3(bc-ad)^3 \int \frac{(a+bx)^3 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{i}$$

$$\downarrow 2795$$

$$\frac{g^3(bc-ad)^3\int\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)d\frac{a-}{c-}}{i}$$

$$\downarrow 2009$$

$$\frac{g^3(bc-ad)^3\left(\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{b^2B\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{2B\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\right.\right.}{d^4}$$

input

```
Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)
,x]
```

output

```
((b*c - a*d)^3*g^3*((b*B^2)/(3*d^4*(b - (d*(a + b*x))/(c + d*x))) + (B^2*L
og[(a + b*x)/(c + d*x)])/(3*d^4) - (b^2*B*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (7*B*(a + b*x)*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/(3*d^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)))
 + (b^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*d^4*(b - (d*(a + b*x))/
(c + d*x))^3) - (3*b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*d^4*(b -
 (d*(a + b*x))/(c + d*x))^2) + (3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c +
d*x)])^2)/(d^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (2*B^2*Log[b - (
d*(a + b*x))/(c + d*x)])/d^4 + (6*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*L
og[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 + ((A + B*Log[(e*(a + b*x))/(c +
d*x)])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 - (7*B*(A + B*Log[(e*(a
 + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(3*d^4) + (6*B^
2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^4 + (2*B*(A + B*Log[(e*(a + b
*x))/(c + d*x)])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^4 + (7*B^2*Pol
yLog[2, (b*(c + d*x))/(d*(a + b*x))])/(3*d^4) - (2*B^2*PolyLog[3, (d*(a +
b*x))/(b*(c + d*x))])/d^4))/i
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{dix+ci}dx$$

input

```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

output

```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algo
rithm="fricas")
```

output
```
integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*
a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2
*a^3*g^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^
2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((b*e*x + a*e)/(d*x + c)))
/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{g^3\left(\int \frac{A^2a^3}{c+dx}\,dx + \int \frac{A^2b^3x^3}{c+dx}\,dx + \int \frac{B^2a^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c+dx}\,dx + \int \frac{2ABa^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx + \int \frac{3A^2ab^2x^2}{c+dx}\,dx + \int \frac{3}{}\right.}{}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i),x)
```

output

```
g**3*(Integral(A**2*a**3/(c + d*x), x) + Integral(A**2*b**3*x**3/(c + d*x)
, x) + Integral(B**2*a**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x
), x) + Integral(2*A*B*a**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x)
, x) + Integral(3*A**2*a*b**2*x**2/(c + d*x), x) + Integral(3*A**2*a**2*b*
x/(c + d*x), x) + Integral(B**2*b**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d
*x))**2/(c + d*x), x) + Integral(2*A*B*b**3*x**3*log(a*e/(c + d*x) + b*e*x
/(c + d*x))/(c + d*x), x) + Integral(3*B**2*a*b**2*x**2*log(a*e/(c + d*x)
+ b*e*x/(c + d*x))**2/(c + d*x), x) + Integral(3*B**2*a**2*b*x*log(a*e/(c
+ d*x) + b*e*x/(c + d*x))**2/(c + d*x), x) + Integral(6*A*B*a*b**2*x**2*lo
g(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Integral(6*A*B*a**2*b*x
*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x))/i
```

Maxima [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input

```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algo
rithm="maxima")
```

output

```
3*A^2*a^2*b*g^3*(x/(d*i) - c*log(d*x + c)/(d^2*i)) - 1/6*A^2*b^3*g^3*(6*c^
3*log(d*x + c)/(d^4*i) - (2*d^2*x^3 - 3*c*d*x^2 + 6*c^2*x)/(d^3*i)) + 3/2*
A^2*a*b^2*g^3*(2*c^2*log(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A^2
*a^3*g^3*log(d*i*x + c*i)/(d*i) - 1/6*(2*(b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3
+ 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*B^2*log(d*x + c)^3 - (2*B^2*b^3*d^3*g^3
*x^3 - 3*(b^3*c*d^2*g^3 - 3*a*b^2*d^3*g^3)*B^2*x^2 + 6*(b^3*c^2*d*g^3 - 3*
a*b^2*c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B^2*x)*log(d*x + c)^2)/(d^4*i) - integr
ate(-1/3*(3*B^2*a^3*d^2*g^3*log(e)^2 + 6*A*B*a^3*d^2*g^3*log(e) + 3*(B^2*b
^3*d^2*g^3*log(e)^2 + 2*A*B*b^3*d^2*g^3*log(e))*x^3 + 9*(B^2*a*b^2*d^2*g^3
*log(e)^2 + 2*A*B*a*b^2*d^2*g^3*log(e))*x^2 + 3*(B^2*b^3*d^2*g^3*x^3 + 3*B
^2*a*b^2*d^2*g^3*x^2 + 3*B^2*a^2*b*d^2*g^3*x + B^2*a^3*d^2*g^3)*log(b*x +
a)^2 + 9*(B^2*a^2*b*d^2*g^3*log(e)^2 + 2*A*B*a^2*b*d^2*g^3*log(e))*x + 6*(
B^2*a^3*d^2*g^3*log(e) + A*B*a^3*d^2*g^3 + (B^2*b^3*d^2*g^3*log(e) + A*B*b
^3*d^2*g^3)*x^3 + 3*(B^2*a*b^2*d^2*g^3*log(e) + A*B*a*b^2*d^2*g^3)*x^2 + 3
*(B^2*a^2*b*d^2*g^3*log(e) + A*B*a^2*b*d^2*g^3)*x)*log(b*x + a) - (6*B^2*a
^3*d^2*g^3*log(e) + 6*A*B*a^3*d^2*g^3 + 2*(3*A*B*b^3*d^2*g^3 + (3*g^3*log(
e) + g^3)*B^2*b^3*d^2)*x^3 + 3*(6*A*B*a*b^2*d^2*g^3 - (b^3*c*d*g^3 - 3*(2*
g^3*log(e) + g^3)*a*b^2*d^2)*B^2)*x^2 + 6*(3*A*B*a^2*b*d^2*g^3 + (b^3*c^2*
g^3 - 3*a*b^2*c*d*g^3 + 3*(g^3*log(e) + g^3)*a^2*b*d^2)*B^2)*x + 6*(B^2*b^
3*d^2*g^3*x^3 + 3*B^2*a*b^2*d^2*g^3*x^2 + 3*B^2*a^2*b*d^2*g^3*x + B^2*a...
```

Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input

```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algo
rithm="giac")
```

output

```
integrate((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*
i), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{g^3 i\left(-6\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2}{dx+c}dx\right)a^3b^2d^4 - 12\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{dx+c}dx\right)a^4b\,d^4 - 6\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2 x^3}{dx+c}dx\right)b^5d^4 - 18\left(\int$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

output
```
(g**3*i*( - 6*int(log((a*e + b*e*x)/(c + d*x))**2/(c + d*x),x)*a**3*b**2*d
**4 - 12*int(log((a*e + b*e*x)/(c + d*x))/(c + d*x),x)*a**4*b*d**4 - 6*int
((log((a*e + b*e*x)/(c + d*x))**2*x**3)/(c + d*x),x)*b**5*d**4 - 18*int((l
og((a*e + b*e*x)/(c + d*x))**2*x**2)/(c + d*x),x)*a*b**4*d**4 - 18*int((lo
g((a*e + b*e*x)/(c + d*x))**2*x)/(c + d*x),x)*a**2*b**3*d**4 - 12*int((log
((a*e + b*e*x)/(c + d*x))*x**3)/(c + d*x),x)*a*b**4*d**4 - 36*int((log((a*
e + b*e*x)/(c + d*x))*x**2)/(c + d*x),x)*a**2*b**3*d**4 - 36*int((log((a*e
 + b*e*x)/(c + d*x))*x)/(c + d*x),x)*a**3*b**2*d**4 - 6*log(c + d*x)*a**5*
d**3 + 18*log(c + d*x)*a**4*b*c*d**2 - 18*log(c + d*x)*a**3*b**2*c**2*d +
6*log(c + d*x)*a**2*b**3*c**3 - 18*a**4*b*d**3*x + 18*a**3*b**2*c*d**2*x -
 9*a**3*b**2*d**3*x**2 - 6*a**2*b**3*c**2*d*x + 3*a**2*b**3*c*d**2*x**2 -
2*a**2*b**3*d**3*x**3))/(6*d**4)
```

3.85 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$

Optimal result 1089
Mathematica [A] (verified) 1090
Rubi [A] (verified) 1091
Maple [F] 1093
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Sympy [F] 1093
Maxima [F] 1094
Giac [F] 1095
Mupad [F(-1)] 1095
Reduce [F] 1095

Optimal result

Integrand size = 42, antiderivative size = 536

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx \\
&= -\frac{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i} \\
&\quad -\frac{4B(bc-ad)^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i} \\
&\quad -\frac{2(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i} \\
&\quad +\frac{b^2g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^3i} \\
&\quad -\frac{(bc-ad)^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i}+\frac{B^2(bc-ad)^2g^2\log(c+dx)}{d^3i} \\
&\quad +\frac{B(bc-ad)^2g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i} \\
&\quad -\frac{4B^2(bc-ad)^2g^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i} \\
&\quad -\frac{2B(bc-ad)^2g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i} \\
&\quad -\frac{B^2(bc-ad)^2g^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i}+\frac{2B^2(bc-ad)^2g^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}
\end{aligned}$$

output

```
-B*(-a*d+b*c)*g^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2/i-4*B*(-a*d+b*c)
^2*g^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3/i-2*(-a*d+
b*c)*g^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i+1/2*b^2*g^2*(d*x+c)^2
*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i-(-a*d+b*c)^2*g^2*ln((-a*d+b*c)/b/(d*x
+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i+B^2*(-a*d+b*c)^2*g^2*ln(d*x+c)/d^
3/i+B*(-a*d+b*c)^2*g^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a
))/d^3/i-4*B^2*(-a*d+b*c)^2*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i-2*B*(
-a*d+b*c)^2*g^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))
/d^3/i-B^2*(-a*d+b*c)^2*g^2*polylog(2,b*(d*x+c)/d/(b*x+a))/d^3/i+2*B^2*(-a
*d+b*c)^2*g^2*polylog(3,d*(b*x+a)/b/(d*x+c))/d^3/i
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.36

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{g^2\left(-2bd(bc-ad)x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+2A^2(bc-ad)^2\log(\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i +
d*i*x),x]
```

output
```
(g^2*(-2*b*d*(b*c - a*d)*x*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + d^2*(a
 + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 2*A^2*(b*c - a*d)^2*Log
[c + d*x] - 4*A*B*(b*c - a*d)^2*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c - a*
d)/(b*c + b*d*x)] - 2*B^2*(b*c - a*d)^2*Log[(e*(a + b*x))/(c + d*x)]^2*Log
[(b*c - a*d)/(b*c + b*d*x)] + 2*A*B*(b*c - a*d)^2*(Log[(b*c - a*d)/(b*c +
b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c + b*d*
x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 2*B*(b*c - a*d)*(2*a*d*L
og[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b*c*(A + B*Log[(e*(a
 + b*x))/(c + d*x)])*Log[c + d*x] - a*B*d*(Log[a + b*x]*(Log[a + b*x] - 2*L
og[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)
]) + b*B*c*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d
*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - B*(b*c - a*d)*(2*A*b*d*x
 + 2*B*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 2*B*(b*c - a*d)*Log[c +
d*x] - 2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + B
*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
 d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - 4*B^2*(b*c - a*d)^2*(L
og[(e*(a + b*x))/(c + d*x)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - Poly
Log[3, (d*(a + b*x))/(b*(c + d*x))])))/(2*d^3*i)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{ci+dix}\,dx$$

$$\downarrow 2962$$

$$\frac{g^2(bc-ad)^2\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{i}$$

$$\downarrow 2795$$

$$\frac{g^2(bc-ad)^2\int\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)d\frac{a+bx}{c+dx}}{i}$$

$$\downarrow 2009$$

$$g^2(bc-ad)^2\left(\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3}-\frac{4B\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3}\right.$$

input

```
Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)
,x]
```

output
```
((b*c - a*d)^2*g^2*(-((B*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(
d^2*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)))) + (b^2*(A + B*Log[(e*(a + b*
x))/(c + d*x)])^2)/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^2) - (2*(a + b*x)*
(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^2*(c + d*x)*(b - (d*(a + b*x))/
(c + d*x))) - (B^2*Log[b - (d*(a + b*x))/(c + d*x)])/d^3 - (4*B*(A + B*Log
[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 - ((A
 + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])
/d^3 + (B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a
 + b*x))])/d^3 - (4*B^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^3 - (2*
B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (d*(a + b*x))/(b*(c + d*
x))])/d^3 - (B^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/d^3 + (2*B^2*Pol
yLog[3, (d*(a + b*x))/(b*(c + d*x))])/d^3))/i
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{dix+ci}dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algo
rithm="fricas")
```

output
```
integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x
^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*
B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)
))/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}dx$$
$$= \frac{g^2\left(\int \frac{A^2a^2}{c+dx}\,dx + \int \frac{A^2b^2x^2}{c+dx}\,dx + \int \frac{B^2a^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c+dx}\,dx + \int \frac{2ABa^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx + \int \frac{2A^2abx}{c+dx}\,dx + \int \frac{B^2}{}\right.}{}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i),x)
```

output

```
g**2*(Integral(A**2*a**2/(c + d*x), x) + Integral(A**2*b**2*x**2/(c + d*x)
, x) + Integral(B**2*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x
), x) + Integral(2*A*B*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x)
, x) + Integral(2*A**2*a*b*x/(c + d*x), x) + Integral(B**2*b**2*x**2*log(a
*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x), x) + Integral(2*A*B*b**2*x**
2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Integral(2*B**2*a*b
*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x), x) + Integral(4*A*B*
a*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x))/i
```

Maxima [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algo
rithm="maxima")
```

output

```
2*A^2*a*b*g^2*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + 1/2*A^2*b^2*g^2*(2*c^2*
log(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A^2*a^2*g^2*log(d*i*x + 
c*i)/(d*i) + 1/6*(2*(b^2*c^2*g^2 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)*B^2*log(d*
x + c)^3 + 3*(B^2*b^2*d^2*g^2*x^2 - 2*(b^2*c*d*g^2 - 2*a*b*d^2*g^2)*B^2*x)
*log(d*x + c)^2)/(d^3*i) - integrate(-(B^2*a^2*d*g^2*log(e)^2 + 2*A*B*a^2*
d*g^2*log(e) + (B^2*b^2*d*g^2*log(e)^2 + 2*A*B*b^2*d*g^2*log(e))*x^2 + (B^
2*b^2*d*g^2*x^2 + 2*B^2*a*b*d*g^2*x + B^2*a^2*d*g^2)*log(b*x + a)^2 + 2*(B
^2*a*b*d*g^2*log(e)^2 + 2*A*B*a*b*d*g^2*log(e))*x + 2*(B^2*a^2*d*g^2*log(e
) + A*B*a^2*d*g^2 + (B^2*b^2*d*g^2*log(e) + A*B*b^2*d*g^2)*x^2 + 2*(B^2*a*
b*d*g^2*log(e) + A*B*a*b*d*g^2)*x)*log(b*x + a) - (2*B^2*a^2*d*g^2*log(e)
+ 2*A*B*a^2*d*g^2 + (2*A*B*b^2*d*g^2 + (2*g^2*log(e) + g^2)*B^2*b^2*d)*x^2
 + 2*(2*A*B*a*b*d*g^2 - (b^2*c*g^2 - 2*(g^2*log(e) + g^2)*a*b*d)*B^2)*x + 
2*(B^2*b^2*d*g^2*x^2 + 2*B^2*a*b*d*g^2*x + B^2*a^2*d*g^2)*log(b*x + a))*lo
g(d*x + c))/(d^2*i*x + c*d*i), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*i), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x),x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x), x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{g^2 i\left(-2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2}{dx+c}dx\right)a^2b^2d^3-4\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{dx+c}dx\right)a^3b\,d^3-2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2x^2}{dx+c}dx\right)b^4d^3-4\left(\int \frac{\log(}{}\right.\right.}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

output
```
(g**2*i*( - 2*int(log((a*e + b*e*x)/(c + d*x))**2/(c + d*x),x)*a**2*b**2*d
**3 - 4*int(log((a*e + b*e*x)/(c + d*x))/(c + d*x),x)*a**3*b*d**3 - 2*int(
(log((a*e + b*e*x)/(c + d*x))**2*x**2)/(c + d*x),x)*b**4*d**3 - 4*int((log
((a*e + b*e*x)/(c + d*x))**2*x)/(c + d*x),x)*a*b**3*d**3 - 4*int((log((a*e
 + b*e*x)/(c + d*x))*x**2)/(c + d*x),x)*a*b**3*d**3 - 8*int((log((a*e + b*
e*x)/(c + d*x))*x)/(c + d*x),x)*a**2*b**2*d**3 - 2*log(c + d*x)*a**4*d**2
+ 4*log(c + d*x)*a**3*b*c*d - 2*log(c + d*x)*a**2*b**2*c**2 - 4*a**3*b*d**
2*x + 2*a**2*b**2*c*d*x - a**2*b**2*d**2*x**2))/(2*d**3)
```

3.86 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$

Optimal result . . . 1097
Mathematica [B] (verified) . . . 1098
Rubi [A] (verified) . . . 1099
Maple [F] . . . 1101
Fricas [F] . . . 1101
Sympy [F] . . . 1102
Maxima [F] . . . 1102
Giac [F] . . . 1103
Mupad [F(-1)] . . . 1103
Reduce [F] . . . 1103

Optimal result

Integrand size = 40, antiderivative size = 283

$$\begin{aligned}
&\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx \\
&= \frac{2B(bc-ad)g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i} \\
&\quad + \frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di} \\
&\quad + \frac{(bc-ad)g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i} \\
&\quad + \frac{2B^2(bc-ad)g\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i} \\
&\quad + \frac{2B(bc-ad)g\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i} \\
&\quad - \frac{2B^2(bc-ad)g\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}
\end{aligned}$$

output

```
2*B*(-a*d+b*c)*g*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2/
i+g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d/i+(-a*d+b*c)*g*ln((-a*d+b*c)/b
/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i+2*B^2*(-a*d+b*c)*g*polylog(2
,d*(b*x+a)/b/(d*x+c))/d^2/i+2*B*(-a*d+b*c)*g*(A+B*ln(e*(b*x+a)/(d*x+c)))*p
olylog(2,d*(b*x+a)/b/(d*x+c))/d^2/i-2*B^2*(-a*d+b*c)*g*polylog(3,d*(b*x+a)
/b/(d*x+c))/d^2/i
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1227 vs. $2(283) = 566$.

Time $= 1.17$ (sec) , antiderivative size $= 1227$, normalized size of antiderivative $= 4.34$

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \text{Too large to display}$$

input

```
Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*
i*x),x]
```

output

```
(g*(3*A^2*b*d*x - 3*A^2*(b*c - a*d)*Log[c + d*x] - 3*a*A*B*d*(Log[c/d + x]
^2 + 2*(Log[a/b + x] - Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)])*Log[c
+ d*x] - 2*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)])) - 3*A*B*(-2*d*(a + b*x)*(-1 + Log[a/b + x]) + 2
*b*(c + d*x)*(-1 + Log[c/d + x]) - b*c*Log[c/d + x]^2 + 2*b*(Log[a/b + x]
- Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)])*(d*x - c*Log[c + d*x]) + 2*
b*c*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x)
)/(-(b*c) + a*d)])) + a*B^2*d*(Log[c/d + x]^3 + 3*Log[c/d + x]^2*(-Log[a/b
 + x] + Log[(d*(a + b*x))/(-(b*c) + a*d)]) + 3*(-Log[a/b + x] + Log[c/d +
x] + Log[(e*(a + b*x))/(c + d*x)])^2*Log[c + d*x] + 3*Log[a/b + x]^2*Log[(
b*(c + d*x))/(b*c - a*d)] + 6*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c
) + a*d)] + 3*(Log[a/b + x] - Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)])
*(Log[c/d + x]^2 - 2*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLo
g[2, (d*(a + b*x))/(-(b*c) + a*d)])) + 6*Log[c/d + x]*PolyLog[2, (b*(c + d
*x))/(b*c - a*d)] - 6*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*PolyLog
[3, (b*(c + d*x))/(b*c - a*d)]) + B^2*(3*d*(2*b*x - 2*(a + b*x)*Log[a/b +
x] + (a + b*x)*Log[a/b + x]^2) - b*c*Log[c/d + x]^3 + 3*b*(2*d*x - 2*(c +
d*x)*Log[c/d + x] + (c + d*x)*Log[c/d + x]^2) + 3*b*(-Log[a/b + x] + Log[c
/d + x] + Log[(e*(a + b*x))/(c + d*x)])^2*(d*x - c*Log[c + d*x]) - 6*(a*d
+ 2*b*d*x - b*d*x*Log[c/d + x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(...
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{ci+dix}\,dx$$

$$\downarrow 2962$$

$$\frac{g(bc-ad)\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i}$$

$$\downarrow 2795$$

$$\frac{g(bc-ad)\int\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}+\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d\left(\frac{d(a+bx)}{c+dx}-b\right)^2}\right)d\frac{a+bx}{c+dx}}{i}$$

$$\downarrow 2009$$

$$\frac{g(bc-ad)\left(\frac{2B\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^2}+\frac{2B\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^2}+\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^2}\right.}{i}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x),x
]
```

output
```
((b*c - a*d)*g*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d*(c +
 d*x)*(b - (d*(a + b*x))/(c + d*x))) + (2*B*(A + B*Log[(e*(a + b*x))/(c +
d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 + ((A + B*Log[(e*(a + b*x
))/(c + d*x)])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 + (2*B^2*PolyLo
g[2, (d*(a + b*x))/(b*(c + d*x))])/d^2 + (2*B*(A + B*Log[(e*(a + b*x))/(c
+ d*x)])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^2 - (2*B^2*PolyLog[3,
(d*(a + b*x))/(b*(c + d*x))])/d^2))/i
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{dix+ci}dx$$

input

```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

output

```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}dx = \int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}dx$$

input

```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algori
thm="fricas")
```

output

```
integral((A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b*e*x + a*e)/(d
*x + c))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b*e*x + a*e)/(d*x + c)))/(d*i*x
+ c*i), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$

$$= \frac{g\left(\int \frac{A^2a}{c+dx}\,dx + \int \frac{A^2bx}{c+dx}\,dx + \int \frac{B^2a\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c+dx}\,dx + \int \frac{2ABa\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx + \int \frac{B^2bx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c+dx}\,d\right.}{i}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i),x)
```

output
```
g*(Integral(A**2*a/(c + d*x), x) + Integral(A**2*b*x/(c + d*x), x) + Integ
ral(B**2*a*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x), x) + Integra
l(2*A*B*a*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Integral(B*
*2*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x), x) + Integral(2*
A*B*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x))/i
```

Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algori
thm="maxima")
```

output
```
A^2*b*g*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + A^2*a*g*log(d*i*x + c*i)/(d*i
) + 1/3*(3*B^2*b*d*g*x*log(d*x + c)^2 - (b*c*g - a*d*g)*B^2*log(d*x + c)^3
)/(d^2*i) - integrate(-(B^2*a*g*log(e)^2 + 2*A*B*a*g*log(e) + (B^2*b*g*x +
 B^2*a*g)*log(b*x + a)^2 + (B^2*b*g*log(e)^2 + 2*A*B*b*g*log(e))*x + 2*(B^
2*a*g*log(e) + A*B*a*g + (B^2*b*g*log(e) + A*B*b*g)*x)*log(b*x + a) - 2*(B
^2*a*g*log(e) + A*B*a*g + ((g*log(e) + g)*B^2*b + A*B*b*g)*x + (B^2*b*g*x
+ B^2*a*g)*log(b*x + a))*log(d*x + c))/(d*i*x + c*i), x)
```

Giac [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*i), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x),x)
```

output
```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x), x)
```

Reduce [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{gi\left(-\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2}{dx+c}dx\right)a\,b^2d^2 - 2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{dx+c}dx\right)a^2b\,d^2 - \left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2 x}{dx+c}dx\right)b^3d^2 - 2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right.}{dx+c}\right.\right.}{d^2}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

output
```
(g*i*( - int(log((a*e + b*e*x)/(c + d*x))**2/(c + d*x),x)*a*b**2*d**2 - 2*
int(log((a*e + b*e*x)/(c + d*x))/(c + d*x),x)*a**2*b*d**2 - int((log((a*e
+ b*e*x)/(c + d*x))**2*x)/(c + d*x),x)*b**3*d**2 - 2*int((log((a*e + b*e*x
)/(c + d*x))*x)/(c + d*x),x)*a*b**2*d**2 - log(c + d*x)*a**3*d + log(c + d
*x)*a**2*b*c - a**2*b*d*x))/d**2
```

3.87 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$

Optimal result . . . 1105
Mathematica [A] (verified) . . . 1106
Rubi [A] (verified) . . . 1106
Maple [B] (verified) . . . 1108
Fricas [F] . . . 1110
Sympy [F] . . . 1110
Maxima [F] . . . 1111
Giac [F] . . . 1111
Mupad [F(-1)] . . . 1111
Reduce [F] . . . 1112

Optimal result

Integrand size = 32, antiderivative size = 127

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di} - \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{di} + \frac{2B^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{di}$$

output

```
-ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d/i-2*B*(A+B*ln(e*
(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/d/i+2*B^2*polylog(3,d*(b*
x+a)/b/(d*x+c))/d/i
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.98

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$
$$=\frac{A^2\log(c+dx)+2AB\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log\left(\frac{bc-ad}{bc+bdx}\right)-2AB\log\left(\frac{e(a+bx)}{c+dx}\right)\log\left(\frac{bc-ad}{bc+bdx}\right)-B^2\log^2\left(\frac{e(a+bx)}{c+dx}\right)\log$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x),x]
```

output
```
(A^2*Log[c + d*x] + 2*A*B*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[(b*c - a*d
)/(b*c + b*d*x)] - 2*A*B*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c - a*d)/(b*c
 + b*d*x)] - B^2*Log[(e*(a + b*x))/(c + d*x)]^2*Log[(b*c - a*d)/(b*c + b*d
*x)] + A*B*Log[(b*c - a*d)/(b*c + b*d*x)]^2 - 2*B^2*Log[(e*(a + b*x))/(c +
 d*x)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*A*B*PolyLog[2, (b*(c +
d*x))/(b*c - a*d)] + 2*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(d*i)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2952, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{ci+dix}\,dx$$
$$\downarrow 2952$$
$$\frac{\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b-\frac{d(a+bx)}{c+dx}}\,d\frac{a+bx}{c+dx}}{i}$$
$$\downarrow 2754$$

$$\frac{\frac{2B\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d} - \frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d}}{i}$$

$$\downarrow 2821$$

$$\frac{\frac{2B\left(B\int \frac{(c+dx)\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx} - \mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)\right)}{d} - \frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d}}{i}$$

$$\downarrow 7143$$

$$\frac{\frac{2B\left(B\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right) - \mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)\right)}{d} - \frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d}}{i}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x),x]
```

output
```
(-(((A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (d*(a + b*x))/(b*(c + d
*x))])/d) + (2*B*(-((A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (d*(a
+ b*x))/(b*(c + d*x))]) + B*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))]))/d)/i
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)  Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2952

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(127) = 254$.

Time = 3.11 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.66

method	result
parts	$\frac{A^2\ln(dx+c)}{id} - \frac{B^2\left(\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be}\right)+2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\text{polylog}\left(2,\frac{d\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right.\right.\right.}{id}$
risch	$\frac{A^2\ln(dx+c)}{id} - \frac{B^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be}\right)}{di} - \frac{2B^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\text{polylog}\left(2,\frac{d\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right.\right.}{di}$
derivativedivides	$e(da-bc)\left(\frac{dA^2\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{ie(da-bc)}+\frac{dB^2\left(\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be}\right)+2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right.}{ie(da-bc)}\right.$
default	$e(da-bc)\left(\frac{dA^2\ln\left(be-\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)d\right)}{ie(da-bc)}+\frac{dB^2\left(\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{be}\right)+2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right.}{ie(da-bc)}\right.$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x,method=_RETURNVERBOSE)
```

output
```
A^2/i*ln(d*x+c)/d-B^2/i/d*(ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(1-1/b/e*d*
(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*polylog(2
,1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-2*polylog(3,1/b/e*d*(b*e/d+(a*d-b*
c)*e/d/(d*x+c))))+2*A*B/i*(-dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b
/e)/d-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d
-b*e)/b/e)/d)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algorithm="fricas"
)
```

output
```
integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*
x + c)) + A^2)/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \frac{\int \frac{A^2}{c+dx}\,dx + \int \frac{B^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c+dx}\,dx + \int \frac{2AB\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx}{i}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i),x)
```

output
```
(Integral(A**2/(c + d*x), x) + Integral(B**2*log(a*e/(c + d*x) + b*e*x/(c
+ d*x))**2/(c + d*x), x) + Integral(2*A*B*log(a*e/(c + d*x) + b*e*x/(c + d
*x))/(c + d*x), x))/i
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algorithm="maxima"
)
```

output
```
1/3*B^2*log(d*x + c)^3/(d*i) + A^2*log(d*i*x + c*i)/(d*i) - integrate(-(B^
2*log(b*x + a)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*log(e) + A*B)*log(
b*x + a) - 2*(B^2*log(b*x + a) + B^2*log(e) + A*B)*log(d*x + c))/(d*i*x +
c*i), x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*i), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(c*i + d*i*x),x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(c*i + d*i*x), x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \frac{i\left(-\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)^2}{dx+c}dx\right)b^2d-2\left(\int \frac{\log\left(\frac{bex+ae}{dx+c}\right)}{dx+c}dx\right)abd-\log(dx+c)\,a^2\right)}{d}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

output
```
(i*( - int(log((a*e + b*e*x)/(c + d*x))**2/(c + d*x),x)*b**2*d - 2*int(log
((a*e + b*e*x)/(c + d*x))/(c + d*x),x)*a*b*d - log(c + d*x)*a**2))/d
```

3.88 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$

Optimal result 1113
Mathematica [A] (verified) 1113
Rubi [A] (warning: unable to verify) 1114
Maple [B] (verified) 1115
Fricas [B] (verification not implemented) 1116
Sympy [B] (verification not implemented) 1117
Maxima [B] (verification not implemented) 1118
Giac [B] (verification not implemented) 1119
Mupad [B] (verification not implemented) 1119
Reduce [B] (verification not implemented) 1120

Optimal result

Integrand size = 42, antiderivative size = 44

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)gi}$$

output
```
1/3*(A+B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)/g/i
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.80

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = \frac{3A^2\log\left(\frac{e(a+bx)}{c+dx}\right)+3AB\log^2\left(\frac{e(a+bx)}{c+dx}\right)+B^2\log^3\left(\frac{e(a+bx)}{c+dx}\right)}{3bcgi-3adgi}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i
*x)),x]
```

output
```
(3*A^2*Log[(e*(a + b*x))/(c + d*x)] + 3*A*B*Log[(e*(a + b*x))/(c + d*x)]^2
 + B^2*Log[(e*(a + b*x))/(c + d*x)]^3)/(3*b*c*g*i - 3*a*d*g*i)
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)(ci+dix)}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx} d\frac{a+bx}{c+dx}}{gi(bc-ad)}$$

$$\downarrow 2739$$

$$\frac{\int \frac{(a+bx)^2}{(c+dx)^2} d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{Bgi(bc-ad)}$$

$$\downarrow 15$$

$$\frac{(a+bx)^3}{3Bgi(c+dx)^3(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i*x)),x
]
```

output
```
(a + b*x)^3/(3*B*(b*c - a*d)*g*i*(c + d*x)^3)
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(42) = 84$.

Time = 1.50 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.41

method	result	size
parallelrisch	$-\frac{B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^3b^2d^2+3AB\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b^2d^2+3A^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2d^2}{3b^2d^2gi(da-bc)}$	106
norman	$-\frac{B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^3}{3gi(da-bc)}-\frac{A^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi(da-bc)}-\frac{AB\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{gi(da-bc)}$	113
parts	$\frac{A^2\left(\frac{\ln(dx+c)}{da-bc}-\frac{\ln(bx+a)}{da-bc}\right)}{gi}-\frac{B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^3}{3gi(da-bc)}-\frac{AB\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{gi(da-bc)}$	123
risch	$\frac{A^2\ln(dx+c)}{gi(da-bc)}-\frac{A^2\ln(bx+a)}{gi(da-bc)}-\frac{B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^3}{3gi(da-bc)}-\frac{AB\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{gi(da-bc)}$	130
derivativedivides	$-\frac{e(da-bc)\left(\frac{d^2A^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei(da-bc)^2g}+\frac{d^2AB\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{ei(da-bc)^2g}+\frac{d^2B^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}{3ei(da-bc)^2g}\right)}{d^2}$	182
default	$-\frac{e(da-bc)\left(\frac{d^2A^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei(da-bc)^2g}+\frac{d^2AB\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{ei(da-bc)^2g}+\frac{d^2B^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}{3ei(da-bc)^2g}\right)}{d^2}$	182

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(B^2*ln(e*(b*x+a)/(d*x+c))^3*b^2*d^2+3*A*B*ln(e*(b*x+a)/(d*x+c))^2*b^2*d^2+3*A^2*ln(e*(b*x+a)/(d*x+c))*b^2*d^2)/b^2/d^2/g/i/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(42) = 84$.

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.98

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = \frac{B^2\log\left(\frac{bex+ae}{dx+c}\right)^3+3\,AB\log\left(\frac{bex+ae}{dx+c}\right)^2+3\,A^2\log\left(\frac{bex+ae}{dx+c}\right)}{3\,(bc-ad)gi}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i),x, algorithm="fricas")
```

output
```
1/3*(B^2*log((b*e*x + a*e)/(d*x + c))^3 + 3*A*B*log((b*e*x + a*e)/(d*x + c
))^2 + 3*A^2*log((b*e*x + a*e)/(d*x + c)))/((b*c - a*d)*g*i)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(31) = 62$.

Time = 0.35 (sec) , antiderivative size = 206, normalized size of antiderivative = 4.68

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = A^2\left(\frac{\log\left(x + \frac{-\frac{a^2d^2}{ad-bc} + \frac{2abcd}{ad-bc} + ad - \frac{b^2c^2}{ad-bc} + bc}{2bd}\right)}{gi\,(ad-bc)} - \frac{\log\left(x + \frac{\frac{a^2d^2}{ad-bc} - \frac{2abcd}{ad-bc} + ad + \frac{b^2c^2}{ad-bc} + bc}{2bd}\right)}{gi\,(ad-bc)}\right) - \frac{AB\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{adgi - bcgi} - \frac{B^2\log\left(\frac{e(a+bx)}{c+dx}\right)^3}{3adgi - 3bcgi}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)/(d*i*x+c*i),x)
```

output
```
A**2*(log(x + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2
*c**2/(a*d - b*c) + b*c)/(2*b*d))/(g*i*(a*d - b*c)) - log(x + (a**2*d**2/(
a*d - b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2
*b*d))/(g*i*(a*d - b*c))) - A*B*log(e*(a + b*x)/(c + d*x))**2/(a*d*g*i - b
*c*g*i) - B**2*log(e*(a + b*x)/(c + d*x))**3/(3*a*d*g*i - 3*b*c*g*i)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(42) = 84$.

Time $= 0.06$ (sec) , antiderivative size $= 397$, normalized size of antiderivative $= 9.02$

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$$

$$= B^2\left(\frac{\log(bx+a)}{(bc-ad)gi} - \frac{\log(dx+c)}{(bc-ad)gi}\right)\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right)^2$$

$$+ 2AB\left(\frac{\log(bx+a)}{(bc-ad)gi} - \frac{\log(dx+c)}{(bc-ad)gi}\right)\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right)$$

$$- \frac{1}{3}B^2\left(\frac{3\left(\log(bx+a)^2 - 2\log(bx+a)\log(dx+c) + \log(dx+c)^2\right)\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right)}{bcgi - adgi} - \frac{\log(bx+a)^3}{}\right.$$

$$+ A^2\left(\frac{\log(bx+a)}{(bc-ad)gi} - \frac{\log(dx+c)}{(bc-ad)gi}\right)$$

$$- \frac{\left(\log(bx+a)^2 - 2\log(bx+a)\log(dx+c) + \log(dx+c)^2\right)AB}{bcgi - adgi}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i),x, algori
thm="maxima")
```

output

```
B^2*(log(b*x + a)/((b*c - a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))*log(
b*e*x/(d*x + c) + a*e/(d*x + c))^2 + 2*A*B*(log(b*x + a)/((b*c - a*d)*g*i)
 - log(d*x + c)/((b*c - a*d)*g*i))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) -
1/3*B^2*(3*(log(b*x + a)^2 - 2*log(b*x + a)*log(d*x + c) + log(d*x + c)^2)
*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b*c*g*i - a*d*g*i) - (log(b*x + a)^
3 - 3*log(b*x + a)^2*log(d*x + c) + 3*log(b*x + a)*log(d*x + c)^2 - log(d*
x + c)^3)/(b*c*g*i - a*d*g*i)) + A^2*(log(b*x + a)/((b*c - a*d)*g*i) - log
(d*x + c)/((b*c - a*d)*g*i)) - (log(b*x + a)^2 - 2*log(b*x + a)*log(d*x +
c) + log(d*x + c)^2)*A*B/(b*c*g*i - a*d*g*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(42) = 84$.

Time $= 0.15$ (sec) , antiderivative size $= 132$, normalized size of antiderivative $= 3.00$

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$$
$$= \frac{\left(B^2 e\log\left(\frac{bex+ae}{dx+c}\right)^3 + 3\,ABe\log\left(\frac{bex+ae}{dx+c}\right)^2 + 3\,A^2 e\log\left(\frac{bex+ae}{dx+c}\right)\right)\left(\frac{bc}{(bce-ade)(bc-ad)} - \frac{ad}{(bce-ade)(bc-ad)}\right)}{3\,gi}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i),x, algori
thm="giac")
```

output
```
1/3*(B^2*e*log((b*e*x + a*e)/(d*x + c))^3 + 3*A*B*e*log((b*e*x + a*e)/(d*x
 + c))^2 + 3*A^2*e*log((b*e*x + a*e)/(d*x + c)))*(b*c/((b*c*e - a*d*e)*(b*
c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(g*i)
```

Mupad [B] (verification not implemented)

Time $= 27.62$ (sec) , antiderivative size $= 96$, normalized size of antiderivative $= 2.18$

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$$
$$= -\frac{-6\mathrm{i}\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,A^2 + 3\,A\,B\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2 + B^2\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^3}{3\,g\,i\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)*(c*i + d*i*x)),x
)
```

output
```
-(B^2*log((e*(a + b*x))/(c + d*x))^3 - A^2*atan((a*d*1i + b*c*1i + b*d*x*2
i)/(a*d - b*c))*6i + 3*A*B*log((e*(a + b*x))/(c + d*x))^2)/(3*g*i*(a*d - b
*c))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.93

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$$
$$=\frac{i\left(3\log(bx+a)\,a^2-3\log(dx+c)\,a^2+\log\left(\frac{bex+ae}{dx+c}\right)^3 b^2+3\log\left(\frac{bex+ae}{dx+c}\right)^2 ab\right)}{3g\,(ad-bc)}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i),x)
```

output
```
(i*(3*log(a + b*x)*a**2 - 3*log(c + d*x)*a**2 + log((a*e + b*e*x)/(c + d*x
))**3*b**2 + 3*log((a*e + b*e*x)/(c + d*x))**2*a*b))/(3*g*(a*d - b*c))
```

3.89 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$

Optimal result . . . 1121
Mathematica [A] (verified) . . . 1122
Rubi [A] (verified) . . . 1122
Maple [B] (verified) . . . 1124
Fricas [A] (verification not implemented) . . . 1125
Sympy [B] (verification not implemented) . . . 1125
Maxima [B] (verification not implemented) . . . 1127
Giac [F] . . . 1128
Mupad [B] (verification not implemented) . . . 1129
Reduce [B] (verification not implemented) . . . 1130

Optimal result

Integrand size = 42, antiderivative size = 183

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{2bB^2(c+dx)}{(bc-ad)^2g^2i(a+bx)} - \frac{2bB(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2g^2i(a+bx)} - \frac{d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^2g^2i}$$

output

```
-2*b*B^2*(d*x+c)/(-a*d+b*c)^2/g^2/i/(b*x+a)-2*b*B*(d*x+c)*(A+B*ln(e*(b*x+a
)/(d*x+c)))/(-a*d+b*c)^2/g^2/i/(b*x+a)-b*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)
))^2/(-a*d+b*c)^2/g^2/i/(b*x+a)-1/3*d*(A+B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*
d+b*c)^2/g^2/i
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.02

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx =$$
$$-\frac{3(A^2+2AB+2B^2)\,d(a+bx)\log(a+bx)+6B(A+B)(bc-ad)\log\left(\frac{e(a+bx)}{c+dx}\right)+3B(aAd+Abdx+}{3(bc}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d
*i*x)),x]
```

output
```
-1/3*(3*(A^2 + 2*A*B + 2*B^2)*d*(a + b*x)*Log[a + b*x] + 6*B*(A + B)*(b*c
- a*d)*Log[(e*(a + b*x))/(c + d*x)] + 3*B*(a*A*d + A*b*d*x + b*B*(c + d*x)
)*Log[(e*(a + b*x))/(c + d*x)]^2 + B^2*d*(a + b*x)*Log[(e*(a + b*x))/(c +
d*x)]^3 + 3*(A^2 + 2*A*B + 2*B^2)*(b*c - a*d - d*(a + b*x)*Log[c + d*x]))/
((b*c - a*d)^2*g^2*i*(a + b*x))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$$
$$\downarrow 2962$$
$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2}\,d\frac{a+bx}{c+dx}}{g^2i(bc-ad)^2}$$
$$\downarrow 2795$$

$$\frac{\int \left(\frac{b(c+dx)^2 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2} - \frac{d(c+dx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx} \right) d\frac{a+bx}{c+dx}}{g^2 i (bc-ad)^2}$$

$$\downarrow \text{2009}$$

$$\frac{-\frac{d\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^3}{3B} - \frac{b(c+dx)\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx} - \frac{2bB(c+dx)\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx} - \frac{2bB^2(c+dx)}{a+bx}}{g^2 i (bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d*i*x))
,x]
```

output
```
((-2*b*B^2*(c + d*x))/(a + b*x) - (2*b*B*(c + d*x)*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]))/(a + b*x) - (b*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)
])^2)/(a + b*x) - (d*(A + B*Log[(e*(a + b*x))/(c + d*x)])^3)/(3*B))/((b*c
- a*d)^2*g^2*i)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(181) = 362$.

Time = 1.80 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.19

method	result
norman	$\frac{-\frac{\left(A^2ad+2ABbc+2B^2bc\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2acdb+c^2b^2\right)}-\frac{B(Ada+Bbc)\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{gi\left(a^2d^2-2acdb+c^2b^2\right)}-\frac{B^2ad\ln\left(\frac{e(bx+a)}{dx+c}\right)^3}{3gi\left(a^2d^2-2acdb+c^2b^2\right)}-\frac{b\left(A^2d+2ABd+2B^2d\right)x\ln\left(\right.}{gi\left(a^2d^2-2acdb+c^2\right.}}{g(bx+a)}$
parts	$\frac{A^2\left(\frac{d\ln(dx+c)}{(da-bc)^2}+\frac{1}{(bx+a)(da-bc)}-\frac{d\ln(bx+a)}{(da-bc)^2}\right)}{g^2i}-\frac{B^2\left(\frac{d^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}{3(da-bc)^2}-\frac{dbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{2\ln\left(\frac{be}{d}+\frac{(da}{d(}\right.}{\frac{be}{d}+\frac{(da-}{d(dx}}\right.}{(da-bc)^2}\right.}{g^2id}$
parallelrisch	$-\frac{3AB\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a^4c^2d+3A^2x\,a^3b\,c^2d-6ABx\,a^2b^2c^3+6AB\ln\left(\frac{e(bx+a)}{dx+c}\right)a^3b\,c^3+6B^2x\,a^3b\,c^2d+3ABx\ln\left(\frac{e(bx+a}{dx+c}\right.}{}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{d^2A^2b}{i(da-bc)^3g^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{d^3A^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei(da-bc)^3g^2}-\frac{2d^2ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)}{i(da-bc)^3g^2}+\right.}{}$
default	$-\frac{e(da-bc)\left(\frac{d^2A^2b}{i(da-bc)^3g^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{d^3A^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei(da-bc)^3g^2}-\frac{2d^2ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}}\right)}{i(da-bc)^3g^2}+\right.}{}$
risch	$\frac{A^2d\ln(dx+c)}{g^2i(da-bc)^2}+\frac{A^2}{g^2i(bx+a)(da-bc)}-\frac{A^2d\ln(bx+a)}{g^2i(da-bc)^2}-\frac{B^2d\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}{3g^2i(da-bc)^2}-\frac{B^2be\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{g^2i(da-bc)^2\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{}{d(}\right.}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x,method=_RETU
RNVERBOSE)
```

output
```
(-(A^2*a*d+2*A*B*b*c+2*B^2*b*c)/g/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+
a)/(d*x+c))-B*(A*a*d+B*b*c)/g/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a)/(
d*x+c))^2-1/3*B^2*a*d/g/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a)/(d*x+c)
)^3-1/g/i*b*(A^2*d+2*A*B*d+2*B^2*d)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*
x+a)/(d*x+c))-(A^2+2*A*B+2*B^2)*b/g/i/a/(a*d-b*c)*x-1/3*b*B^2*d/g/i/(a^2*d
^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^3-B*b*d*(A+B)/g/i/(a^2*d^2-2
*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^2)/g/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.26

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx =$$
$$-\frac{(B^2bdx + B^2ad)\log\left(\frac{bex+ae}{dx+c}\right)^3 + 3\,(A^2 + 2\,AB + 2\,B^2)bc - 3\,(A^2 + 2\,AB + 2\,B^2)ad + 3\,(B^2bc + ABa}{3\,((b^3c^2 - 2\,ab^2cd + a^2bd^2)g^2ix}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x, algo
rithm="fricas")
```

output
```
-1/3*((B^2*b*d*x + B^2*a*d)*log((b*e*x + a*e)/(d*x + c))^3 + 3*(A^2 + 2*A*
B + 2*B^2)*b*c - 3*(A^2 + 2*A*B + 2*B^2)*a*d + 3*(B^2*b*c + A*B*a*d + (A*B
 + B^2)*b*d*x)*log((b*e*x + a*e)/(d*x + c))^2 + 3*(A^2*a*d + (A^2 + 2*A*B
+ 2*B^2)*b*d*x + 2*(A*B + B^2)*b*c)*log((b*e*x + a*e)/(d*x + c)))/((b^3*c^
2 - 2*a*b^2*c*d + a^2*b*d^2)*g^2*i*x + (a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)
*g^2*i)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(158) = 316$.

Time = 0.72 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.96

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$$

$$= -\frac{B^2 d\log\left(\frac{e(a+bx)}{c+dx}\right)^3}{3a^2d^2g^2i-6abcdg^2i+3b^2c^2g^2i} + \frac{(2AB+2B^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{a^2dg^2i-abcg^2i+abdg^2ix-b^2cg^2ix}$$

$$+\left(A^2+2AB+2B^2\right)\left(\frac{d\log\left(x+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{g^2i\left(ad-bc\right)^2}\right.$$

$$\left.-\frac{d\log\left(x+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{g^2i\left(ad-bc\right)^2}\right.$$

$$\left.+\frac{1}{a^2dg^2i-abcg^2i+x\left(abdg^2i-b^2cg^2i\right)}\right)$$

$$+\frac{(-ABad-ABbdx-B^2bc-B^2bdx)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{a^3d^2g^2i-2a^2bcdg^2i+a^2bd^2g^2ix+ab^2c^2g^2i-2ab^2cdg^2ix+b^3c^2g^2ix}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2/(d*i*x+c*i),x)
```

output

```
-B**2*d*log(e*(a + b*x)/(c + d*x))**3/(3*a**2*d**2*g**2*i - 6*a*b*c*d*g**2
*i + 3*b**2*c**2*g**2*i) + (2*A*B + 2*B**2)*log(e*(a + b*x)/(c + d*x))/(a*
*2*d*g**2*i - a*b*c*g**2*i + a*b*d*g**2*i*x - b**2*c*g**2*i*x) + (A**2 + 2
*A*B + 2*B**2)*(d*log(x + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*
d - b*c)**2 - 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*
d - b*c)**2 + b*c*d)/(2*b*d**2))/(g**2*i*(a*d - b*c)**2) - d*log(x + (a**3
*d**4/(a*d - b*c)**2 - 3*a**2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2
/(a*d - b*c)**2 + a*d**2 - b**3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))
/(g**2*i*(a*d - b*c)**2) + 1/(a**2*d*g**2*i - a*b*c*g**2*i + x*(a*b*d*g**2
*i - b**2*c*g**2*i))) + (-A*B*a*d - A*B*b*d*x - B**2*b*c - B**2*b*d*x)*log
(e*(a + b*x)/(c + d*x))**2/(a**3*d**2*g**2*i - 2*a**2*b*c*d*g**2*i + a**2*
b*d**2*g**2*i*x + a*b**2*c**2*g**2*i - 2*a*b**2*c*d*g**2*i*x + b**3*c**2*g
**2*i*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. $2(181) = 362$.

Time $= 0.10$ (sec) , antiderivative size $= 1008$, normalized size of antiderivative $= 5.51$

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^2(ci + dix)}\, dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x, algo
rithm="maxima")
```

output

```
-B^2*(1/((b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)
/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*g^2*i))*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2 - 2*A*B*(1
/((b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*g^2*i))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 1/3*B^2*(3*((b*d*
x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d - 2
*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))
*log(d*x + c))*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(a*b^2*c^2*g^2*i - 2*a
^2*b*c*d*g^2*i + a^3*d^2*g^2*i + (b^3*c^2*g^2*i - 2*a*b^2*c*d*g^2*i + a^2*
b*d^2*g^2*i)*x) - ((b*d*x + a*d)*log(b*x + a)^3 - (b*d*x + a*d)*log(d*x +
c)^3 - 3*(b*d*x + a*d)*log(b*x + a)^2 - 3*(b*d*x + a*d - (b*d*x + a*d)*log
(b*x + a))*log(d*x + c)^2 + 6*b*c - 6*a*d + 6*(b*d*x + a*d)*log(b*x + a) -
 3*(2*b*d*x + (b*d*x + a*d)*log(b*x + a)^2 + 2*a*d - 2*(b*d*x + a*d)*log(b
*x + a))*log(d*x + c))/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*i + a^3*d^2*g^2*
i + (b^3*c^2*g^2*i - 2*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*x)) - A^2*(1/((b
^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((b^2*c^2
- 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*g^2*i)) + ((b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)
^2 - 2*b*c + 2*a*d - 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b...
```

Giac [F]

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2(dix+ci)}\,dx$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x, algo
rithm="giac")
```

output

```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/((b*g*x + a*g)^2*(d*i*x + c
*i)), x)
```

Mupad [B] (verification not implemented)

Time = 28.04 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.29

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$$
$$= \frac{A^2+2AB+2B^2}{(ad-bc)\,(ag^2i+bg^2ix)} - \ln\left(\frac{e(a+bx)}{c+dx}\right)^2\left(\frac{Bd(A+B)}{g^2i\,(a^2d^2-2abcd+b^2c^2)} - \frac{B^2(ad-bc)}{bdg^2i\,\left(\frac{x}{d}+\frac{a}{bd}\right)\,(a^2d^2-2abcd+b^2c^2)}\right)$$
$$- \frac{B^2d\ln\left(\frac{e(a+bx)}{c+dx}\right)^3}{3g^2i\,(a^2d^2-2abcd+b^2c^2)} + \frac{2B\,\ln\left(\frac{e(a+bx)}{c+dx}\right)\,(ad-bc)\,(A+B)}{bdg^2i\,\left(\frac{x}{d}+\frac{a}{bd}\right)\,(a^2d^2-2abcd+b^2c^2)}$$
$$+ \frac{d\,\mathrm{atan}\left(\frac{d\left(2bdx+\frac{a^2d^2g^2i-b^2c^2g^2i}{g^2i(ad-bc)}\right)(A^2+2AB+2B^2)\,1\mathrm{i}}{(ad-bc)(dA^2+2dAB+2dB^2)}\right)\,(A^2+2AB+2B^2)\,2\mathrm{i}}{g^2i(ad-bc)^2}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^2*(c*i + d*i*x))
,x)
```

output
```
(A^2 + 2*B^2 + 2*A*B)/((a*d - b*c)*(a*g^2*i + b*g^2*i*x)) - log((e*(a + b*
x))/(c + d*x))^2*((B*d*(A + B))/(g^2*i*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) -
(B^2*(a*d - b*c))/(b*d*g^2*i*(x/d + a/(b*d))*(a^2*d^2 + b^2*c^2 - 2*a*b*c*
d))) - (B^2*d*log((e*(a + b*x))/(c + d*x))^3)/(3*g^2*i*(a^2*d^2 + b^2*c^2
- 2*a*b*c*d)) + (d*atan((d*(2*b*d*x + (a^2*d^2*g^2*i - b^2*c^2*g^2*i)/(g^2
*i*(a*d - b*c)))*(A^2 + 2*B^2 + 2*A*B)*1i)/((a*d - b*c)*(A^2*d + 2*B^2*d +
 2*A*B*d)))*(A^2 + 2*B^2 + 2*A*B)*2i)/(g^2*i*(a*d - b*c)^2) + (2*B*log((e*
(a + b*x))/(c + d*x))*(a*d - b*c)*(A + B))/(b*d*g^2*i*(x/d + a/(b*d))*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 537, normalized size of antiderivative = 2.93

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$$
$$=\frac{i\Big(3\log(bx+a)\,a^4d-3\log(dx+c)\,a^4d-6b^4cx+3\log(bx+a)\,a^3bdx+6\log(bx+a)\,ab^3cx-3\log(dx+$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x)
```

output
```
(i*(3*log(a + b*x)*a**4*d + 3*log(a + b*x)*a**3*b*d*x + 6*log(a + b*x)*a**
2*b**2*c + 6*log(a + b*x)*a*b**3*c*x + 6*log(a + b*x)*a*b**3*c + 6*log(a +
 b*x)*b**4*c*x - 3*log(c + d*x)*a**4*d - 3*log(c + d*x)*a**3*b*d*x - 6*log
(c + d*x)*a**2*b**2*c - 6*log(c + d*x)*a*b**3*c*x - 6*log(c + d*x)*a*b**3*
c - 6*log(c + d*x)*b**4*c*x + log((a*e + b*e*x)/(c + d*x))**3*a**2*b**2*d
 + log((a*e + b*e*x)/(c + d*x))**3*a*b**3*d*x + 3*log((a*e + b*e*x)/(c + d*
x))**2*a**3*b*d + 3*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**2*d*x + 3*log(
(a*e + b*e*x)/(c + d*x))**2*a*b**3*c + 3*log((a*e + b*e*x)/(c + d*x))**2*a
*b**3*d*x + 6*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*d*x - 6*log((a*e + b*
e*x)/(c + d*x))*a*b**3*c*x + 6*log((a*e + b*e*x)/(c + d*x))*a*b**3*d*x - 6
*log((a*e + b*e*x)/(c + d*x))*b**4*c*x + 3*a**3*b*d*x - 3*a**2*b**2*c*x +
6*a**2*b**2*d*x - 6*a*b**3*c*x + 6*a*b**3*d*x - 6*b**4*c*x))/(3*a*g**2*(a*
*3*d**2 - 2*a**2*b*c*d + a**2*b*d**2*x + a*b**2*c**2 - 2*a*b**2*c*d*x + b*
*3*c**2*x))
```

3.90 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$

Optimal result 1131
Mathematica [A] (verified) 1132
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Reduce [B] (verification not implemented) 1140

Optimal result

Integrand size = 42, antiderivative size = 343

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = {}& \frac{4bB^2d(c+dx)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2B^2(c+dx)^2}{4(bc-ad)^3g^3i(a+bx)^2} \\ & + \frac{4bBd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i(a+bx)} \\ & - \frac{b^2B(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2} \\ & + \frac{2bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^3i(a+bx)} \\ & - \frac{b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3g^3i(a+bx)^2} \\ & + \frac{d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^3g^3i}\end{aligned}$$

output
```
4*b*B^2*d*(d*x+c)/(-a*d+b*c)^3/g^3/i/(b*x+a)-1/4*b^2*B^2*(d*x+c)^2/(-a*d+b
*c)^3/g^3/i/(b*x+a)^2+4*b*B*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*
c)^3/g^3/i/(b*x+a)-1/2*b^2*B*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b
*c)^3/g^3/i/(b*x+a)^2+2*b*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*
c)^3/g^3/i/(b*x+a)-1/2*b^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b
*c)^3/g^3/i/(b*x+a)^2+1/3*d^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)^3
/g^3/i
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.93

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$$
$$= \frac{-3(2A^2+2AB+B^2)\,(bc-ad)^2+6(2A^2+6AB+7B^2)\,d(bc-ad)(a+bx)+6(2A^2+6AB+7B^2)\,d^2$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d
*i*x)),x]
```

output
```
(-3*(2*A^2 + 2*A*B + B^2)*(b*c - a*d)^2 + 6*(2*A^2 + 6*A*B + 7*B^2)*d*(b*c
 - a*d)*(a + b*x) + 6*(2*A^2 + 6*A*B + 7*B^2)*d^2*(a + b*x)^2*Log[a + b*x]
 - 6*B*(b*c - a*d)*(-6*a*A*d - 7*a*B*d + b*B*(c - 6*d*x) + 2*A*b*(c - 2*d*
x))*Log[(e*(a + b*x))/(c + d*x)] - 6*B*(-2*a^2*A*d^2 - 4*a*b*d*(A*d*x + B*
(c + d*x)) + b^2*(-2*A*d^2*x^2 + B*(c^2 - 2*c*d*x - 3*d^2*x^2)))*Log[(e*(a
 + b*x))/(c + d*x)]^2 + 4*B^2*d^2*(a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)]
^3 - 6*(2*A^2 + 6*A*B + 7*B^2)*d^2*(a + b*x)^2*Log[c + d*x])/(12*(b*c - a*
d)^3*g^3*i*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3} d\frac{a+bx}{c+dx}}{g^3 i(bc-ad)^3}$$

$$\downarrow 2795$$

$$\frac{\int \left(\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^3}{(a+bx)^3} - \frac{2bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^2}{(a+bx)^2} + \frac{d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)}{a+bx}\right) d\frac{a+bx}{c+dx}}{g^3 i(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2} - \frac{b^2B(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2} + \frac{d^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^3}{3B} + \frac{2bd(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}}{g^3 i(bc-ad)^3}$$

input

```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d*i*x))
,x]
```

output

```
((4*b*B^2*d*(c + d*x))/(a + b*x) - (b^2*B^2*(c + d*x)^2)/(4*(a + b*x)^2) +
 (4*b*B*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) - (b^2
*B*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^2) + (2*
b*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x) - (b^2*(c
+ d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(a + b*x)^2) + (d^2*(A
 + B*Log[(e*(a + b*x))/(c + d*x)])^3)/(3*B))/((b*c - a*d)^3*g^3*i)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(335) = 670$.

Time = 2.23 (sec) , antiderivative size = 767, normalized size of antiderivative = 2.24

method	result
parts	$\frac{A^2\left(\frac{d^2\ln(dx+c)}{(da-bc)^3}+\frac{1}{2(da-bc)(bx+a)^2}+\frac{d}{(da-bc)^2(bx+a)}-\frac{d^2\ln(bx+a)}{(da-bc)^3}\right)}{g^3 i}-\frac{B^2\left(\frac{d^3\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}{3(da-bc)^3}-\frac{2d^2be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(}{}\right.}{\frac{be}{d}+\frac{(}{}}\right.}{}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{d^2eA^2b^2}{2i(da-bc)^4g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{2d^3A^2b}{i(da-bc)^4g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{d^4A^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei(da-bc)^4g^3}+\frac{2d^2eAB\,b^2\left(\right.}{}\right.}{}$
default	$-\frac{e(da-bc)\left(-\frac{d^2eA^2b^2}{2i(da-bc)^4g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}+\frac{2d^3A^2b}{i(da-bc)^4g^3\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}+\frac{d^4A^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei(da-bc)^4g^3}+\frac{2d^2eAB\,b^2\left(\right.}{}\right.}{}$
norman	$\frac{6A^2a\,b^2d-2A^2b^3c+14ABa\,b^2d-2AB\,b^3c+15B^2a\,b^2d-B^2b^3c}{4gi\,b^2\left(a^2d^2-2acdb+c^2b^2\right)}-\frac{\left(2A^2a^2d^2+8ABabcd-2AB\,b^2c^2+8B^2abcd-B^2b^2c^2\right)\ln\left(\frac{e(bx+a}{dx+c}\right.}{2ig\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$
risch	$\frac{A^2d^2\ln(dx+c)}{g^3i(da-bc)^3}+\frac{A^2}{2g^3i(da-bc)(bx+a)^2}+\frac{A^2d}{g^3i(da-bc)^2(bx+a)}-\frac{A^2d^2\ln(bx+a)}{g^3i(da-bc)^3}-\frac{B^2d^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^3}{3g^3i(da-bc)^3}-$
parallelrisch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x,method=_RETU
RNVERBOSE)
```

output

```
A^2/g^3/i*(d^2/(a*d-b*c)^3*ln(d*x+c)+1/2/(a*d-b*c)/(b*x+a)^2+d/(a*d-b*c)^2
/(b*x+a)-d^2/(a*d-b*c)^3*ln(b*x+a))-B^2/g^3/i/d*(1/3*d^3/(a*d-b*c)^3*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))^3-2*d^2/(a*d-b*c)^3*b*e*(-1/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c
))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+d/(a*d
-b*c)^3*b^2*e^2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))-2*A*B/g^3/i/d*(1/2*d^3/(a*d-
b*c)^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*d^2/(a*d-b*c)^3*b*e*(-1/(b*e/d+
(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*
e/d/(d*x+c)))+d/(a*d-b*c)^3*b^2*e^2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.57

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx =$$

$$-\frac{3\,(2\,A^2+2\,AB+B^2)b^2c^2-24\,(A^2+2\,AB+2\,B^2)abcd+3\,(6\,A^2+14\,AB+15\,B^2)a^2d^2-4\,(B^2b^2d^2}{}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x, algo
rithm="fricas")
```

output

```
-1/12*(3*(2*A^2 + 2*A*B + B^2)*b^2*c^2 - 24*(A^2 + 2*A*B + 2*B^2)*a*b*c*d
+ 3*(6*A^2 + 14*A*B + 15*B^2)*a^2*d^2 - 4*(B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2
*x + B^2*a^2*d^2)*log((b*e*x + a*e)/(d*x + c))^3 - 6*((2*A*B + 3*B^2)*b^2*
d^2*x^2 - B^2*b^2*c^2 + 4*B^2*a*b*c*d + 2*A*B*a^2*d^2 + 2*(B^2*b^2*c*d + 2
*(A*B + B^2)*a*b*d^2)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 6*((2*A^2 + 6*A*
B + 7*B^2)*b^2*c*d - (2*A^2 + 6*A*B + 7*B^2)*a*b*d^2)*x - 6*((2*A^2 + 6*A*
B + 7*B^2)*b^2*d^2*x^2 + 2*A^2*a^2*d^2 - (2*A*B + B^2)*b^2*c^2 + 8*(A*B +
B^2)*a*b*c*d + 2*((2*A*B + 3*B^2)*b^2*c*d + 2*(A^2 + 2*A*B + 2*B^2)*a*b*d^
2)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*
c*d^2 - a^3*b^2*d^3)*g^3*i*x^2 + 2*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^
2*c*d^2 - a^4*b*d^3)*g^3*i*x + (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*
d^2 - a^5*d^3)*g^3*i)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1488 vs. $2(303) = 606$.

Time = 4.17 (sec) , antiderivative size = 1488, normalized size of antiderivative = 4.34

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3/(d*i*x+c*i),x)
```

output

```
-B**2*d**2*log(e*(a + b*x)/(c + d*x))**3/(3*a**3*d**3*g**3*i - 9*a**2*b*c*
d**2*g**3*i + 9*a*b**2*c**2*d*g**3*i - 3*b**3*c**3*g**3*i) + d**2*(2*A**2
+ 6*A*B + 7*B**2)*log(x + (2*A**2*a*d**3 + 2*A**2*b*c*d**2 + 6*A*B*a*d**3
+ 6*A*B*b*c*d**2 + 7*B**2*a*d**3 + 7*B**2*b*c*d**2 - a**4*d**6*(2*A**2 + 6
*A*B + 7*B**2)/(a*d - b*c)**3 + 4*a**3*b*c*d**5*(2*A**2 + 6*A*B + 7*B**2)/
(a*d - b*c)**3 - 6*a**2*b**2*c**2*d**4*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*
c)**3 + 4*a*b**3*c**3*d**3*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*c)**3 - b**4
*c**4*d**2*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*c)**3)/(4*A**2*b*d**3 + 12*A
*B*b*d**3 + 14*B**2*b*d**3))/(2*g**3*i*(a*d - b*c)**3) - d**2*(2*A**2 + 6*
A*B + 7*B**2)*log(x + (2*A**2*a*d**3 + 2*A**2*b*c*d**2 + 6*A*B*a*d**3 + 6*
A*B*b*c*d**2 + 7*B**2*a*d**3 + 7*B**2*b*c*d**2 + a**4*d**6*(2*A**2 + 6*A*B
 + 7*B**2)/(a*d - b*c)**3 - 4*a**3*b*c*d**5*(2*A**2 + 6*A*B + 7*B**2)/(a*d
 - b*c)**3 + 6*a**2*b**2*c**2*d**4*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*c)**
3 - 4*a*b**3*c**3*d**3*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*c)**3 + b**4*c**
4*d**2*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*c)**3)/(4*A**2*b*d**3 + 12*A*B*b
*d**3 + 14*B**2*b*d**3))/(2*g**3*i*(a*d - b*c)**3) + (6*A*B*a*d - 2*A*B*b*
c + 4*A*B*b*d*x + 7*B**2*a*d - B**2*b*c + 6*B**2*b*d*x)*log(e*(a + b*x)/(c
 + d*x))/(2*a**4*d**2*g**3*i - 4*a**3*b*c*d*g**3*i + 4*a**3*b*d**2*g**3*i*
x + 2*a**2*b**2*c**2*g**3*i - 8*a**2*b**2*c*d*g**3*i*x + 2*a**2*b**2*d**2*
g**3*i*x**2 + 4*a*b**3*c**2*g**3*i*x - 4*a*b**3*c*d*g**3*i*x**2 + 2*b**...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2115 vs. $2(335) = 670$.

Time $= 0.19$ (sec) , antiderivative size $= 2115$, normalized size of antiderivative $= 6.17$

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x, algo
rithm="maxima")
```

output
```
1/2*B^2*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^
3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a^2*b^2*c^2
 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*
c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(b*e*x/(d*x + c) + a*e/
(d*x + c))^2 + A*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*
b^2*d^2)*g^3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (
a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(b*e*x/(d*x
+ c) + a*e/(d*x + c)) - 1/12*B^2*(6*(b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(
b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a
*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*
x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x
 + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d
*x + c))*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(a^2*b^3*c^3*g^3*i - 3*a^3*b
^2*c^2*d*g^3*i + 3*a^4*b*c*d^2*g^3*i - a^5*d^3*g^3*i + (b^5*c^3*g^3*i - 3*
a*b^4*c^2*d*g^3*i + 3*a^2*b^3*c*d^2*g^3*i - a^3*b^2*d^3*g^3*i)*x^2 + 2*(a*
b^4*c^3*g^3*i - 3*a^2*b^3*c^2*d*g^3*i + 3*a^3*b^2*c*d^2*g^3*i - a^4*b*d^3*
g^3*i)*x) + (3*b^2*c^2 - 48*a*b*c*d + 45*a^2*d^2 - 4*(b^2*d^2*x^2 + 2*a...
```

Giac [A] (verification not implemented)

Time = 57.21 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.62

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx =$$
$$-\frac{1}{4}\left(\frac{2\,(dx+c)^2B^2e^3\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)^2g^3i}+\frac{2\,(2\,ABe^3+B^2e^3)(dx+c)^2\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^2g^3i}+\frac{(2\,A^2e^3+2\,ABe^3+B$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x, algo
rithm="giac")
```

output
```
-1/4*(2*(d*x + c)^2*B^2*e^3*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)^
2*g^3*i) + 2*(2*A*B*e^3 + B^2*e^3)*(d*x + c)^2*log((b*e*x + a*e)/(d*x + c)
)/((b*e*x + a*e)^2*g^3*i) + (2*A^2*e^3 + 2*A*B*e^3 + B^2*e^3)*(d*x + c)^2/
((b*e*x + a*e)^2*g^3*i))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e
- a*d*e)*(b*c - a*d)))^2
```

Mupad [B] (verification not implemented)

Time = 29.85 (sec) , antiderivative size = 981, normalized size of antiderivative = 2.86

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^3*(c*i + d*i*x))
,x)
```

output

```
log((e*(a + b*x))/(c + d*x))^2*(((B^2*d^2*((2*a^2*d^2 + b^2*c^2 - 3*a*b*c*
d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(g^3*i*(a^3*d^3 - b^3*c^3 + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B^2*x*(a*d - b*c))/(g^3*i*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d)
 - (B*d^2*(2*A + 3*B))/(2*g^3*i*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*
b*c*d^2))) - ((6*A^2*a*d - 2*A^2*b*c + 15*B^2*a*d - B^2*b*c + 14*A*B*a*d -
 2*A*B*b*c)/(2*(a*d - b*c)) + (x*(2*A^2*b*d + 7*B^2*b*d + 6*A*B*b*d))/(a*d
 - b*c))/(x^2*(2*b^3*c*g^3*i - 2*a*b^2*d*g^3*i) + x*(4*a*b^2*c*g^3*i - 4*a
^2*b*d*g^3*i) - 2*a^3*d*g^3*i + 2*a^2*b*c*g^3*i) + (log((e*(a + b*x))/(c +
 d*x))*((B*d^2*((2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*
c))/(2*b*d^2))*(2*A + 3*B))/(g^3*i*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*
a^2*b*c*d^2)) - B^2/(b*d*g^3*i*(a*d - b*c)) + (B*x*(2*A + 3*B)*(a*d - b*c)
)/(g^3*i*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((b*x^2)/d
 + a^2/(b*d) + (2*a*x)/d) - (B^2*d^2*log((e*(a + b*x))/(c + d*x))^3)/(3*g^
3*i*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d^2*atan((d^2*
(A^2 + (7*B^2)/2 + 3*A*B)*(2*a^3*d^3*g^3*i + 2*b^3*c^3*g^3*i - 2*a*b^2*c^2
*d*g^3*i - 2*a^2*b*c*d^2*g^3*i)*1i)/(g^3*i*(a*d - b*c)^3*(2*A^2*d^2 + 7*B^
2*d^2 + 6*A*B*d^2)) + (b*d^3*x*(a^2*d^2*g^3*i + b^2*c^2*g^3*i - 2*a*b*c*d*
g^3*i)*(A^2 + (7*B^2)/2 + 3*A*B)*4i)/(g^3*i*(a*d - b*c)^3*(2*A^2*d^2 + 7*B
^2*d^2 + 6*A*B*d^2)))*(A^2 + (7*B^2)/2 + 3*A*B)*2i)/(g^3*i*(a*d - b*c)^...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1426, normalized size of antiderivative = 4.16

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x)
```

output

```
(i*(12*log(a + b*x)*a**5*d**2 + 24*log(a + b*x)*a**4*b*d**2*x + 24*log(a +
 b*x)*a**4*b*d**2 + 12*log(a + b*x)*a**3*b**2*c*d + 12*log(a + b*x)*a**3*b
**2*d**2*x**2 + 48*log(a + b*x)*a**3*b**2*d**2*x + 24*log(a + b*x)*a**3*b*
*2*d**2 + 24*log(a + b*x)*a**2*b**3*c*d*x + 18*log(a + b*x)*a**2*b**3*c*d
+ 24*log(a + b*x)*a**2*b**3*d**2*x**2 + 48*log(a + b*x)*a**2*b**3*d**2*x +
 12*log(a + b*x)*a*b**4*c*d*x**2 + 36*log(a + b*x)*a*b**4*c*d*x + 24*log(a
 + b*x)*a*b**4*d**2*x**2 + 18*log(a + b*x)*b**5*c*d*x**2 - 12*log(c + d*x)
*a**5*d**2 - 24*log(c + d*x)*a**4*b*d**2*x - 24*log(c + d*x)*a**4*b*d**2 -
 12*log(c + d*x)*a**3*b**2*c*d - 12*log(c + d*x)*a**3*b**2*d**2*x**2 - 48*
log(c + d*x)*a**3*b**2*d**2*x - 24*log(c + d*x)*a**3*b**2*d**2 - 24*log(c
+ d*x)*a**2*b**3*c*d*x - 18*log(c + d*x)*a**2*b**3*c*d - 24*log(c + d*x)*a
**2*b**3*d**2*x**2 - 48*log(c + d*x)*a**2*b**3*d**2*x - 12*log(c + d*x)*a*
b**4*c*d*x**2 - 36*log(c + d*x)*a*b**4*c*d*x - 24*log(c + d*x)*a*b**4*d**2
*x**2 - 18*log(c + d*x)*b**5*c*d*x**2 + 4*log((a*e + b*e*x)/(c + d*x))**3*
a**3*b**2*d**2 + 8*log((a*e + b*e*x)/(c + d*x))**3*a**2*b**3*d**2*x + 4*lo
g((a*e + b*e*x)/(c + d*x))**3*a*b**4*d**2*x**2 + 12*log((a*e + b*e*x)/(c +
 d*x))**2*a**4*b*d**2 + 24*log((a*e + b*e*x)/(c + d*x))**2*a**3*b**2*d**2*
x + 24*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**3*c*d + 12*log((a*e + b*e*x
)/(c + d*x))**2*a**2*b**3*d**2*x**2 + 24*log((a*e + b*e*x)/(c + d*x))**2*a
**2*b**3*d**2*x - 6*log((a*e + b*e*x)/(c + d*x))**2*a*b**4*c**2 + 12*lo...
```

3.91 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx$

Optimal result 1143
Mathematica [A] (verified) 1144
Rubi [A] (verified) 1144
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Fricas [A] (verification not implemented) 1147
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Giac [A] (verification not implemented) 1150
Mupad [B] (verification not implemented) 1151
Reduce [B] (verification not implemented) 1152

Optimal result

Integrand size = 42, antiderivative size = 507

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = &-\frac{6bB^2d^2(c+dx)}{(bc-ad)^4g^4i(a+bx)}+\frac{3b^2B^2d(c+dx)^2}{4(bc-ad)^4g^4i(a+bx)^2}\\ &-\frac{2b^3B^2(c+dx)^3}{27(bc-ad)^4g^4i(a+bx)^3}\\ &-\frac{6bBd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i(a+bx)}\\ &+\frac{3b^2Bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2}\\ &-\frac{2b^3B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^4g^4i(a+bx)^3}\\ &-\frac{3bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^4i(a+bx)}\\ &+\frac{3b^2d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^4i(a+bx)^2}\\ &-\frac{b^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^4g^4i(a+bx)^3}\\ &-\frac{d^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^4g^4i}\end{aligned}$$

output

```
-6*b*B^2*d^2*(d*x+c)/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/4*b^2*B^2*d*(d*x+c)^2/(-
a*d+b*c)^4/g^4/i/(b*x+a)^2-2/27*b^3*B^2*(d*x+c)^3/(-a*d+b*c)^4/g^4/i/(b*x+
a)^3-6*b*B*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i/(b*x
+a)+3/2*b^2*B*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i/(
b*x+a)^2-2/9*b^3*B*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/
i/(b*x+a)^3-3*b*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^4
/i/(b*x+a)+3/2*b^2*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/
g^4/i/(b*x+a)^2-1/3*b^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)
^4/g^4/i/(b*x+a)^3-1/3*d^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)^4/g^
4/i
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 442, normalized size of antiderivative = 0.87

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx =$$

$$-\frac{4(9A^2+6AB+2B^2)\,(bc-ad)^3-3(18A^2+30AB+19B^2)\,d(bc-ad)^2(a+bx)-6(18A^2+66AB+$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d
*i*x)),x]
```

output
```
-1/108*(4*(9*A^2 + 6*A*B + 2*B^2)*(b*c - a*d)^3 - 3*(18*A^2 + 30*A*B + 19*
B^2)*d*(b*c - a*d)^2*(a + b*x) - 6*(18*A^2 + 66*A*B + 85*B^2)*d^2*(-(b*c)
+ a*d)*(a + b*x)^2 + 6*(18*A^2 + 66*A*B + 85*B^2)*d^3*(a + b*x)^3*Log[a +
b*x] + 6*B*(b*c - a*d)*(4*(3*A + B)*(b*c - a*d)^2 + 3*(6*A + 5*B)*d*(-(b*c
) + a*d)*(a + b*x) + 6*(6*A + 11*B)*d^2*(a + b*x)^2)*Log[(e*(a + b*x))/(c
+ d*x)] + 18*B*(6*a^3*A*d^3 + 18*a^2*b*d^2*(A*d*x + B*(c + d*x)) + 9*a*b^2
*d*(2*A*d^2*x^2 + B*(-c^2 + 2*c*d*x + 3*d^2*x^2)) + b^3*(6*A*d^3*x^3 + B*(
2*c^3 - 3*c^2*d*x + 6*c*d^2*x^2 + 11*d^3*x^3)))*Log[(e*(a + b*x))/(c + d*x
)]^2 + 36*B^2*d^3*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)]^3 - 6*(18*A^2 +
 66*A*B + 85*B^2)*d^3*(a + b*x)^3*Log[c + d*x])/((b*c - a*d)^4*g^4*i*(a +
b*x)^3)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^4(ci+dix)}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4i(bc-ad)^4}$$

$$\downarrow 2795$$

$$\frac{\int\left(\frac{b^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^4}{(a+bx)^4}-\frac{3b^2d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^3}{(a+bx)^3}+\frac{3bd^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^2}{(a+bx)^2}-\frac{d^3\left(A+B\log\left(\frac{e(a+}{a+b}\right.\right.}{g^4i(bc-ad)^4}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3}-\frac{2b^3B(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}+\frac{3b^2Bd(c+dx)^2\left(B\log\right.}{2(a+bx}$$

input

```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d*i*x))
,x]
```

output

```
((-6*b*B^2*d^2*(c + d*x))/(a + b*x) + (3*b^2*B^2*d*(c + d*x)^2)/(4*(a + b*
x)^2) - (2*b^3*B^2*(c + d*x)^3)/(27*(a + b*x)^3) - (6*b*B*d^2*(c + d*x)*(A
 + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) + (3*b^2*B*d*(c + d*x)^2*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^2) - (2*b^3*B*(c + d*x)^3*
(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(a + b*x)^3) - (3*b*d^2*(c + d*x)
*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x) + (3*b^2*d*(c + d*x)^2*
(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(a + b*x)^2) - (b^3*(c + d*x)^3
*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*(a + b*x)^3) - (d^3*(A + B*Log
[(e*(a + b*x))/(c + d*x)])^3)/(3*B))/((b*c - a*d)^4*g^4*i)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1075 vs. $2(493) = 986$.

Time = 2.64 (sec) , antiderivative size = 1076, normalized size of antiderivative = 2.12

method	result	size
parts	Expression too large to display	1076
derivativedivides	Expression too large to display	1244
default	Expression too large to display	1244
risch	Expression too large to display	1422
norman	Expression too large to display	1693
parallelrisch	Expression too large to display	1967

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x,method=_RETU
RNVERBOSE)
```

output

```
A^2/g^4/i*(d^3/(a*d-b*c)^4*ln(d*x+c)+1/3/(a*d-b*c)/(b*x+a)^3+1/2*d/(a*d-b*
c)^2/(b*x+a)^2+d^2/(a*d-b*c)^3/(b*x+a)-d^3/(a*d-b*c)^4*ln(b*x+a))-B^2/g^4/
i/d*(1/3*d^4/(a*d-b*c)^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3-3*d^3/(a*d-b*c)
^4*b*e*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2
-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+
(a*d-b*c)*e/d/(d*x+c)))+3*d^2/(a*d-b*c)^4*b^2*e^2*(-1/2/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^2)-d/(a*d-b*c)^4*b^3*e^3*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d
-b*c)*e/d/(d*x+c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))-2*A*B/g^4/i/d*(1
/2*d^4/(a*d-b*c)^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-3*d^3/(a*d-b*c)^4*b*e
*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/
d+(a*d-b*c)*e/d/(d*x+c)))+3*d^2/(a*d-b*c)^4*b^2*e^2*(-1/2/(b*e/d+(a*d-b*c)
*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2)-d/(a*d-b*c)^4*b^3*e^3*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(
b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 940, normalized size of antiderivative = 1.85

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x, algo
rithm="fricas")
```

output

```
-1/108*(4*(9*A^2 + 6*A*B + 2*B^2)*b^3*c^3 - 81*(2*A^2 + 2*A*B + B^2)*a*b^2
*c^2*d + 324*(A^2 + 2*A*B + 2*B^2)*a^2*b*c*d^2 - (198*A^2 + 510*A*B + 575*
B^2)*a^3*d^3 + 36*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3
*x + B^2*a^3*d^3)*log((b*e*x + a*e)/(d*x + c))^3 + 6*((18*A^2 + 66*A*B + 8
5*B^2)*b^3*c*d^2 - (18*A^2 + 66*A*B + 85*B^2)*a*b^2*d^3)*x^2 + 18*((6*A*B
+ 11*B^2)*b^3*d^3*x^3 + 2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c
*d^2 + 6*A*B*a^3*d^3 + 3*(2*B^2*b^3*c*d^2 + 3*(2*A*B + 3*B^2)*a*b^2*d^3)*x
^2 - 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 6*(A*B + B^2)*a^2*b*d^3)*x)*lo
g((b*e*x + a*e)/(d*x + c))^2 - 3*((18*A^2 + 30*A*B + 19*B^2)*b^3*c^2*d - 5
4*(2*A^2 + 6*A*B + 7*B^2)*a*b^2*c*d^2 + (90*A^2 + 294*A*B + 359*B^2)*a^2*b
*d^3)*x + 6*((18*A^2 + 66*A*B + 85*B^2)*b^3*d^3*x^3 + 18*A^2*a^3*d^3 + 4*(
3*A*B + B^2)*b^3*c^3 - 27*(2*A*B + B^2)*a*b^2*c^2*d + 108*(A*B + B^2)*a^2*
b*c*d^2 + 3*(2*(6*A*B + 11*B^2)*b^3*c*d^2 + 9*(2*A^2 + 6*A*B + 7*B^2)*a*b^
2*d^3)*x^2 - 3*((6*A*B + 5*B^2)*b^3*c^2*d - 18*(2*A*B + 3*B^2)*a*b^2*c*d^2
 - 18*(A^2 + 2*A*B + 2*B^2)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c)))/((
b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^
4)*g^4*i*x^3 + 3*(a*b^6*c^4 - 4*a^2*b^5*c^3*d + 6*a^3*b^4*c^2*d^2 - 4*a^4*
b^3*c*d^3 + a^5*b^2*d^4)*g^4*i*x^2 + 3*(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*
a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^3 + a^6*b*d^4)*g^4*i*x + (a^3*b^4*c^4 - 4*
a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*g^4*i)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2388 vs. $2(459) = 918$.

Time $= 19.09$ (sec) , antiderivative size $= 2388$, normalized size of antiderivative $= 4.71$

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4/(d*i*x+c*i),x)
```

output

```
-B**2*d**3*log(e*(a + b*x)/(c + d*x))**3/(3*a**4*d**4*g**4*i - 12*a**3*b*c
*d**3*g**4*i + 18*a**2*b**2*c**2*d**2*g**4*i - 12*a*b**3*c**3*d*g**4*i + 3
*b**4*c**4*g**4*i) + d**3*(18*A**2 + 66*A*B + 85*B**2)*log(x + (18*A**2*a*
d**4 + 18*A**2*b*c*d**3 + 66*A*B*a*d**4 + 66*A*B*b*c*d**3 + 85*B**2*a*d**4
 + 85*B**2*b*c*d**3 - a**5*d**8*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**
4 + 5*a**4*b*c*d**7*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 - 10*a**3*
b**2*c**2*d**6*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 + 10*a**2*b**3*
c**3*d**5*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 - 5*a*b**4*c**4*d**4
*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 + b**5*c**5*d**3*(18*A**2 + 6
6*A*B + 85*B**2)/(a*d - b*c)**4)/(36*A**2*b*d**4 + 132*A*B*b*d**4 + 170*B*
*2*b*d**4))/(18*g**4*i*(a*d - b*c)**4) - d**3*(18*A**2 + 66*A*B + 85*B**2)
*log(x + (18*A**2*a*d**4 + 18*A**2*b*c*d**3 + 66*A*B*a*d**4 + 66*A*B*b*c*d
**3 + 85*B**2*a*d**4 + 85*B**2*b*c*d**3 + a**5*d**8*(18*A**2 + 66*A*B + 85
*B**2)/(a*d - b*c)**4 - 5*a**4*b*c*d**7*(18*A**2 + 66*A*B + 85*B**2)/(a*d
- b*c)**4 + 10*a**3*b**2*c**2*d**6*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c
)**4 - 10*a**2*b**3*c**3*d**5*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4
+ 5*a*b**4*c**4*d**4*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 - b**5*c*
*5*d**3*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4)/(36*A**2*b*d**4 + 132
*A*B*b*d**4 + 170*B**2*b*d**4))/(18*g**4*i*(a*d - b*c)**4) + (66*A*B*a**2*
d**2 - 42*A*B*a*b*c*d + 90*A*B*a*b*d**2*x + 12*A*B*b**2*c**2 - 18*A*B*b...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3434 vs. $2(493) = 986$.

Time $= 0.33$ (sec) , antiderivative size $= 3434$, normalized size of antiderivative $= 6.77$

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x, algo
rithm="maxima")
```

output

```
-1/6*B^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
 - 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3
)*g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d
^3)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b
*d^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*
g^4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x + c)/((b^4*c^4 - 4*a*b^3
*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i))*log(b*e*x/(d
*x + c) + a*e/(d*x + c))^2 - 1/3*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c
*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3
*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d +
 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2
*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d
 + 3*a^5*b*c*d^2 - a^6*d^3)*g^4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^
3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(
d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*g^4*i))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/108*B^2*(6*(4*b^3
*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b
^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)
*log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a...
```

Giac [A] (verification not implemented)

Time = 72.17 (sec) , antiderivative size = 451, normalized size of antiderivative = 0.89

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx =$$

$$-\frac{1}{108}\left(\frac{18\left(2\,B^2be^4-\frac{3\,(bex+ae)B^2de^3}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^3bcg^4i}{(dx+c)^3}-\frac{(bex+ae)^3adg^4i}{(dx+c)^3}}+\frac{6\left(12\,ABbe^4+4\,B^2be^4-\frac{18\,(bex+ae)ABde^3}{dx+c}-\frac{9\,(bex+}{d}\right.\right.$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x, algo
rithm="giac")
```

output

```
-1/108*(18*(2*B^2*b*e^4 - 3*(b*e*x + a*e)*B^2*d*e^3/(d*x + c))*log((b*e*x
+ a*e)/(d*x + c))^2/((b*e*x + a*e)^3*b*c*g^4*i/(d*x + c)^3 - (b*e*x + a*e)
^3*a*d*g^4*i/(d*x + c)^3) + 6*(12*A*B*b*e^4 + 4*B^2*b*e^4 - 18*(b*e*x + a*
e)*A*B*d*e^3/(d*x + c) - 9*(b*e*x + a*e)*B^2*d*e^3/(d*x + c))*log((b*e*x +
 a*e)/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4*i/(d*x + c)^3 - (b*e*x + a*e)^3*
a*d*g^4*i/(d*x + c)^3) + (36*A^2*b*e^4 + 24*A*B*b*e^4 + 8*B^2*b*e^4 - 54*(
b*e*x + a*e)*A^2*d*e^3/(d*x + c) - 54*(b*e*x + a*e)*A*B*d*e^3/(d*x + c) -
27*(b*e*x + a*e)*B^2*d*e^3/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4*i/(d*x + c)
^3 - (b*e*x + a*e)^3*a*d*g^4*i/(d*x + c)^3))*(b*c/((b*c*e - a*d*e)*(b*c -
a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [B] (verification not implemented)

Time = 33.83 (sec) , antiderivative size = 1882, normalized size of antiderivative = 3.71

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input

```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^4*(c*i + d*i*x))
,x)
```

output

```
log((e*(a + b*x))/(c + d*x))^2*(((B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b
*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^3*d^3 - b^3*c^3 + 4*a*
b^2*c^2*d - 6*a^2*b*c*d^2)/(3*b*d^4)))/(g^4*i*(a^4*d^4 + b^4*c^4 + 6*a^2*b
^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d^3*x^2*((b^2*c - a*b*
d)/(3*d^2) - (2*b*(a*d - b*c))/(3*d^2)))/(g^4*i*(a^4*d^4 + b^4*c^4 + 6*a^2
*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^3*x*(b*((3*a^2*d^2
 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^2*d^
2 + b^2*c^2 - 4*a*b*c*d)/(3*d^3) + (2*a*(a*d - b*c))/(3*d^2)))/(g^4*i*(a^4
*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))/((3*
a^2*x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (d^3*(11*B^2 + 6*A*B
))/(6*g^4*i*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3
*b*c*d^3))) + ((198*A^2*a^2*d^2 + 36*A^2*b^2*c^2 + 575*B^2*a^2*d^2 + 8*B^2
*b^2*c^2 + 510*A*B*a^2*d^2 + 24*A*B*b^2*c^2 - 126*A^2*a*b*c*d - 73*B^2*a*b
*c*d - 138*A*B*a*b*c*d)/(6*(a*d - b*c)) + (x*(90*A^2*a*b*d^2 + 359*B^2*a*b
*d^2 - 18*A^2*b^2*c*d - 19*B^2*b^2*c*d + 294*A*B*a*b*d^2 - 30*A*B*b^2*c*d)
)/(2*(a*d - b*c)) + (d*x^2*(18*A^2*b^2*d + 85*B^2*b^2*d + 66*A*B*b^2*d))/(
a*d - b*c))/(x*(54*a^4*b*d^2*g^4*i + 54*a^2*b^3*c^2*g^4*i - 108*a^3*b^2*c*
d*g^4*i) + x^2*(54*a*b^4*c^2*g^4*i + 54*a^3*b^2*d^2*g^4*i - 108*a^2*b^3*c*
d*g^4*i) + x^3*(18*b^5*c^2*g^4*i + 18*a^2*b^3*d^2*g^4*i - 36*a*b^4*c*d*g^4
*i) + 18*a^5*d^2*g^4*i + 18*a^3*b^2*c^2*g^4*i - 36*a^4*b*c*d*g^4*i) - (...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 2371, normalized size of antiderivative = 4.68

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x)
```

output

```
(i*(108*log(a + b*x)*a**6*d**3 + 324*log(a + b*x)*a**5*b*d**3*x + 324*log(
a + b*x)*a**5*b*d**3 + 72*log(a + b*x)*a**4*b**2*c*d**2 + 324*log(a + b*x)
*a**4*b**2*d**3*x**2 + 972*log(a + b*x)*a**4*b**2*d**3*x + 378*log(a + b*x
)*a**4*b**2*d**3 + 216*log(a + b*x)*a**3*b**3*c*d**2*x + 132*log(a + b*x)*
a**3*b**3*c*d**2 + 108*log(a + b*x)*a**3*b**3*d**3*x**3 + 972*log(a + b*x)
*a**3*b**3*d**3*x**2 + 1134*log(a + b*x)*a**3*b**3*d**3*x + 216*log(a + b*
x)*a**2*b**4*c*d**2*x**2 + 396*log(a + b*x)*a**2*b**4*c*d**2*x + 324*log(a
 + b*x)*a**2*b**4*d**3*x**3 + 1134*log(a + b*x)*a**2*b**4*d**3*x**2 + 72*l
og(a + b*x)*a*b**5*c*d**2*x**3 + 396*log(a + b*x)*a*b**5*c*d**2*x**2 + 378
*log(a + b*x)*a*b**5*d**3*x**3 + 132*log(a + b*x)*b**6*c*d**2*x**3 - 108*l
og(c + d*x)*a**6*d**3 - 324*log(c + d*x)*a**5*b*d**3*x - 324*log(c + d*x)*
a**5*b*d**3 - 72*log(c + d*x)*a**4*b**2*c*d**2 - 324*log(c + d*x)*a**4*b**
2*d**3*x**2 - 972*log(c + d*x)*a**4*b**2*d**3*x - 378*log(c + d*x)*a**4*b*
*2*d**3 - 216*log(c + d*x)*a**3*b**3*c*d**2*x - 132*log(c + d*x)*a**3*b**3
*c*d**2 - 108*log(c + d*x)*a**3*b**3*d**3*x**3 - 972*log(c + d*x)*a**3*b**
3*d**3*x**2 - 1134*log(c + d*x)*a**3*b**3*d**3*x - 216*log(c + d*x)*a**2*b
**4*c*d**2*x**2 - 396*log(c + d*x)*a**2*b**4*c*d**2*x - 324*log(c + d*x)*a
**2*b**4*d**3*x**3 - 1134*log(c + d*x)*a**2*b**4*d**3*x**2 - 72*log(c + d*
x)*a*b**5*c*d**2*x**3 - 396*log(c + d*x)*a*b**5*c*d**2*x**2 - 378*log(c +
d*x)*a*b**5*d**3*x**3 - 132*log(c + d*x)*b**6*c*d**2*x**3 + 36*log((a*e...
```

3.92 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$

Optimal result 1155
Mathematica [B] (verified) 1156
Rubi [A] (verified) 1157
Maple [F] 1159
Fricas [F] 1159
Sympy [F(-1)] 1159
Maxima [F] 1160
Giac [F] 1161
Mupad [F(-1)] 1161
Reduce [F] 1161

Optimal result

Integrand size = 42, antiderivative size = 722

$$
\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx \\
&= \frac{2AB(bc-ad)^2g^3(a+bx)}{d^3i^2(c+dx)} - \frac{2B^2(bc-ad)^2g^3(a+bx)}{d^3i^2(c+dx)} \\
&\quad + \frac{2B^2(bc-ad)^2g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^2(c+dx)} \\
&\quad - \frac{bB(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^2} \\
&\quad - \frac{6bB(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i^2} \\
&\quad - \frac{3b(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2} \\
&\quad - \frac{(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2(c+dx)} \\
&\quad + \frac{b^3g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^4i^2} \\
&\quad - \frac{3b(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i^2} \\
&\quad + \frac{bB^2(bc-ad)^2g^3\log(c+dx)}{d^4i^2} \\
&\quad + \frac{bB(bc-ad)^2g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2} \\
&\quad - \frac{6bB^2(bc-ad)^2g^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2} \\
&\quad - \frac{6bB(bc-ad)^2g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2} \\
&\quad - \frac{bB^2(bc-ad)^2g^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2} + \frac{6bB^2(bc-ad)^2g^3\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}
\end{aligned}
$$

output

```
2*A*B*(-a*d+b*c)^2*g^3*(b*x+a)/d^3/i^2/(d*x+c)-2*B^2*(-a*d+b*c)^2*g^3*(b*x
+a)/d^3/i^2/(d*x+c)+2*B^2*(-a*d+b*c)^2*g^3*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d
^3/i^2/(d*x+c)-b*B*(-a*d+b*c)*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3/
i^2-6*b*B*(-a*d+b*c)^2*g^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x
+c)))/d^4/i^2-3*b*(-a*d+b*c)*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3
/i^2-(-a*d+b*c)^2*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i^2/(d*x+c
)+1/2*b^3*g^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i^2-3*b*(-a*d+b*
c)^2*g^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i^2+b*
B^2*(-a*d+b*c)^2*g^3*ln(d*x+c)/d^4/i^2+b*B*(-a*d+b*c)^2*g^3*(A+B*ln(e*(b*x
+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/d^4/i^2-6*b*B^2*(-a*d+b*c)^2*g^3*p
olylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^2-6*b*B*(-a*d+b*c)^2*g^3*(A+B*ln(e*(b*
x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^2-b*B^2*(-a*d+b*c)^2*g
^3*polylog(2,b*(d*x+c)/d/(b*x+a))/d^4/i^2+6*b*B^2*(-a*d+b*c)^2*g^3*polylog
(3,d*(b*x+a)/b/(d*x+c))/d^4/i^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5199 vs. $2(722) = 1444$.

Time = 8.08 (sec) , antiderivative size = 5199, normalized size of antiderivative = 7.20

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \text{Result too large to show}$$

input

```
Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i +
d*i*x)^2,x]
```

output

```
Result too large to show
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 592, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ci+dix)^2}\,dx$$

$$\downarrow 2962$$

$$\frac{g^3(bc-ad)^2\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{i^2}$$

$$\downarrow 2795$$

$$\frac{g^3(bc-ad)^2\int \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3}\right)d\frac{a}{c}}{i^2}$$

$$\downarrow 2009$$

$$g^3(bc-ad)^2\left(\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{6bB\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^4}-\frac{3b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^4}\right.$$

input

```
Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)
^2,x]
```

output

```
((b*c - a*d)^2*g^3*((2*A*B*(a + b*x))/(d^3*(c + d*x)) - (2*B^2*(a + b*x))/
(d^3*(c + d*x)) + (2*B^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d^3*(c +
 d*x)) - (b*B*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d^3*(c + d*
x)*(b - (d*(a + b*x))/(c + d*x))) - ((a + b*x)*(A + B*Log[(e*(a + b*x))/(c
 + d*x)])^2)/(d^3*(c + d*x)) + (b^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) - (3*b*(a + b*x)*(A + B*Log[(e*(
a + b*x))/(c + d*x)])^2)/(d^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (
b*B^2*Log[b - (d*(a + b*x))/(c + d*x)])/d^4 - (6*b*B*(A + B*Log[(e*(a + b*
x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 - (3*b*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 +
(b*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*
x))])/d^4 - (6*b*B^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^4 - (6*b*B
*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x
))])/d^4 - (b*B^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/d^4 + (6*b*B^2*
PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/d^4))/i^2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(dix+ci)^2}dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}dx = \int \frac{(bgx+ag)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, al
gorithm="fricas")
```

output
```
integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*
a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2
*a^3*g^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^
2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((b*e*x + a*e)/(d*x + c)))
/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, al
gorithm="maxima")
```

output
```
1/2*(2*c^3/(d^5*i^2*x + c*d^4*i^2) + 6*c^2*log(d*x + c)/(d^4*i^2) + (d*x^2
 - 4*c*x)/(d^3*i^2))*A^2*b^3*g^3 - 3*A^2*a*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2
) - x/(d^2*i^2) + 2*c*log(d*x + c)/(d^3*i^2))*g^3 + 3*A^2*a^2*b*g^3*(c/(d^
3*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2)) - 2*A*B*a^3*g^3*(log(b*e*x/
(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x + c*d*i^2) - 1/(d^2*i^2*x + c*d*i^2)
 - b*log(b*x + a)/((b*c*d - a*d^2)*i^2) + b*log(d*x + c)/((b*c*d - a*d^2)*
i^2)) - A^2*a^3*g^3/(d^2*i^2*x + c*d*i^2) + 1/2*(2*((b^3*c^2*d*g^3 - 2*a*b
^2*c*d^2*g^3 + a^2*b*d^3*g^3)*B^2*x + (b^3*c^3*g^3 - 2*a*b^2*c^2*d*g^3 + a
^2*b*c*d^2*g^3)*B^2)*log(d*x + c)^3 + (B^2*b^3*d^3*g^3*x^3 - 3*(b^3*c*d^2*
g^3 - 2*a*b^2*d^3*g^3)*B^2*x^2 - 2*(2*b^3*c^2*d*g^3 - 3*a*b^2*c*d^2*g^3)*B
^2*x + 2*(b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^
3)*B^2)*log(d*x + c)^2)/(d^5*i^2*x + c*d^4*i^2) - integrate(-(B^2*a^3*d^3*
g^3*log(e)^2 + (B^2*b^3*d^3*g^3*log(e)^2 + 2*A*B*b^3*d^3*g^3*log(e))*x^3 +
 3*(B^2*a*b^2*d^3*g^3*log(e)^2 + 2*A*B*a*b^2*d^3*g^3*log(e))*x^2 + (B^2*b^
3*d^3*g^3*x^3 + 3*B^2*a*b^2*d^3*g^3*x^2 + 3*B^2*a^2*b*d^3*g^3*x + B^2*a^3*
d^3*g^3)*log(b*x + a)^2 + 3*(B^2*a^2*b*d^3*g^3*log(e)^2 + 2*A*B*a^2*b*d^3*
g^3*log(e))*x + 2*(B^2*a^3*d^3*g^3*log(e) + (B^2*b^3*d^3*g^3*log(e) + A*B*
b^3*d^3*g^3)*x^3 + 3*(B^2*a*b^2*d^3*g^3*log(e) + A*B*a*b^2*d^3*g^3)*x^2 +
3*(B^2*a^2*b*d^3*g^3*log(e) + A*B*a^2*b*d^3*g^3)*x)*log(b*x + a) - ((2*A*B
*b^3*d^3*g^3 + (2*g^3*log(e) + g^3)*B^2*b^3*d^3)*x^3 + 2*(b^3*c^3*g^3 -...
```

Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, al
gorithm="giac")
```

output
```
integrate((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*
i)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(c\,i+d\,i\,x)^2}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
^2,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
^2, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

output

```
(g**3*( - 2*int((log((a*e + b*e*x)/(c + d*x))**2*x**3)/(c**2 + 2*c*d*x + d
**2*x**2),x)*a*b**5*c**2*d**5 - 2*int((log((a*e + b*e*x)/(c + d*x))**2*x**
3)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**5*c*d**6*x + 2*int((log((a*e + b*e
*x)/(c + d*x))**2*x**3)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**6*c**3*d**4 + 2
*int((log((a*e + b*e*x)/(c + d*x))**2*x**3)/(c**2 + 2*c*d*x + d**2*x**2),x
)*b**6*c**2*d**5*x - 6*int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(c**2 +
2*c*d*x + d**2*x**2),x)*a**2*b**4*c**2*d**5 - 6*int((log((a*e + b*e*x)/(c
+ d*x))**2*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b**4*c*d**6*x + 6*in
t((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*a
*b**5*c**3*d**4 + 6*int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(c**2 + 2*c
*d*x + d**2*x**2),x)*a*b**5*c**2*d**5*x - 6*int((log((a*e + b*e*x)/(c + d*
x))**2*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**3*b**3*c**2*d**5 - 6*int((log
((a*e + b*e*x)/(c + d*x))**2*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**3*b**3*
c*d**6*x + 6*int((log((a*e + b*e*x)/(c + d*x))**2*x)/(c**2 + 2*c*d*x + d**
2*x**2),x)*a**2*b**4*c**3*d**4 + 6*int((log((a*e + b*e*x)/(c + d*x))**2*x)
/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b**4*c**2*d**5*x - 4*int((log((a*e +
 b*e*x)/(c + d*x))*x**3)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b**4*c**2*d*
*5 - 4*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(c**2 + 2*c*d*x + d**2*x**2
),x)*a**2*b**4*c*d**6*x + 4*int((log((a*e + b*e*x)/(c + d*x))*x**3)/(c**2
+ 2*c*d*x + d**2*x**2),x)*a*b**5*c**3*d**4 + 4*int((log((a*e + b*e*x)/(...
```

3.93 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$

Optimal result 1164
Mathematica [B] (verified) 1165
Rubi [A] (verified) 1166
Maple [F] 1168
Fricas [F] 1168
Sympy [F(-1)] 1168
Maxima [F] 1169
Giac [F] 1169
Mupad [F(-1)] 1170
Reduce [F] 1170

Optimal result

Integrand size = 42, antiderivative size = 469

$$
\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx \\
&= -\frac{2AB(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)} + \frac{2B^2(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)} \\
&\quad - \frac{2B^2(bc-ad)g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^2(c+dx)} \\
&\quad + \frac{2bB(bc-ad)g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^2} \\
&\quad + \frac{bg^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2} \\
&\quad + \frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2(c+dx)} \\
&\quad + \frac{2b(bc-ad)g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2} \\
&\quad + \frac{2bB^2(bc-ad)g^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} \\
&\quad + \frac{4bB(bc-ad)g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} \\
&\quad - \frac{4bB^2(bc-ad)g^2\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}
\end{aligned}
$$

output

```
-2*A*B*(-a*d+b*c)*g^2*(b*x+a)/d^2/i^2/(d*x+c)+2*B^2*(-a*d+b*c)*g^2*(b*x+a)
/d^2/i^2/(d*x+c)-2*B^2*(-a*d+b*c)*g^2*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d^2/i^
2/(d*x+c)+2*b*B*(-a*d+b*c)*g^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/
(d*x+c)))/d^3/i^2+b*g^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i^2+(-a*
d+b*c)*g^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i^2/(d*x+c)+2*b*(-a*d
+b*c)*g^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i^2+2
*b*B^2*(-a*d+b*c)*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^2+4*b*B*(-a*d+b
*c)*g^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^2
-4*b*B^2*(-a*d+b*c)*g^2*polylog(3,d*(b*x+a)/b/(d*x+c))/d^3/i^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2622 vs. $2(469) = 938$.

Time = 6.49 (sec) , antiderivative size = 2622, normalized size of antiderivative = 5.59

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \text{Result too large to show}$$

input

```
Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i +
d*i*x)^2,x]
```

output

```
(g^2*(3*A^2*b^2*d*x - (3*A^2*(b*c - a*d)^2)/(c + d*x) + 6*A^2*b*(-(b*c) +
a*d)*Log[c + d*x] + (6*a^2*A*B*d^2*(b*c - a*d + b*(c + d*x)*Log[a/b + x] +
 (-(b*c) + a*d)*Log[(e*(a + b*x))/(c + d*x)] - b*c*Log[(b*(c + d*x))/(b*c
- a*d)] - b*d*x*Log[(b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)*(c + d*x)) +
 6*a*A*b*B*d*(-Log[c/d + x]^2 + 2*Log[c/d + x]*Log[c + d*x] + 2*(-(c/(c +
d*x)) + (b*c*Log[a + b*x])/(-(b*c) + a*d) + (b*c*Log[c + d*x])/(b*c - a*d)
 - Log[a/b + x]*Log[c + d*x] + Log[(e*(a + b*x))/(c + d*x)]*(c/(c + d*x) +
 Log[c + d*x]) + Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)]) + 2*PolyLog[
2, (d*(a + b*x))/(-(b*c) + a*d)]) + 6*A*b^2*B*(d*(a/b + x)*(-1 + Log[a/b +
 x]) - (c^2*Log[a/b + x])/(c + d*x) - (c + d*x)*(-1 + Log[c/d + x]) + c*Lo
g[c/d + x]^2 + (c^2*(1 + Log[c/d + x]))/(c + d*x) + (b*c^2*(Log[a + b*x] -
 Log[c + d*x]))/(b*c - a*d) + (-Log[a/b + x] + Log[c/d + x] + Log[(e*(a +
b*x))/(c + d*x)])*(d*x - c^2/(c + d*x) - 2*c*Log[c + d*x]) - 2*c*(Log[a/b
+ x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a
*d)])) - (3*a^2*B^2*d^2*(2*b*c - 2*a*d + 2*b*(c + d*x)*Log[a + b*x] - 2*(b
*c - a*d)*Log[(e*(a + b*x))/(c + d*x)] - 2*b*(c + d*x)*Log[a + b*x]*Log[(e
*(a + b*x))/(c + d*x)] + (b*c - a*d)*Log[(e*(a + b*x))/(c + d*x)]^2 - 2*b*
(c + d*x)*Log[c + d*x] - 2*b*(c + d*x)*Log[(e*(a + b*x))/(c + d*x)]*Log[(b
*c - a*d)/(b*c + b*d*x)] + b*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log
[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d...
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ci+dix)^2}\,dx$$

$$\downarrow 2962$$

$$\frac{g^2(bc-ad)\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i^2}$$

$$\downarrow 2795$$

$$\frac{g^2(bc-ad)\int\left(\frac{2b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)}+\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2}+\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)^2}\right)d\frac{a+bx}{c+dx}}{i^2}$$

$$\downarrow 2009$$

$$g^2(bc-ad)\left(\frac{4bB\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3}+\frac{2bB\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3}+\frac{2b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\right.}{d^3}$$

input

```
Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)
^2,x]
```

output
```
((b*c - a*d)*g^2*((-2*A*B*(a + b*x))/(d^2*(c + d*x)) + (2*B^2*(a + b*x))/(
d^2*(c + d*x)) - (2*B^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d^2*(c +
d*x)) + ((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^2*(c + d*x))
 + (b*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^2*(c + d*x)*(b
- (d*(a + b*x))/(c + d*x))) + (2*b*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*
Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 + (2*b*(A + B*Log[(e*(a + b*x))/
(c + d*x)])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 + (2*b*B^2*PolyLog
[2, (d*(a + b*x))/(b*(c + d*x))])/d^3 + (4*b*B*(A + B*Log[(e*(a + b*x))/(c
 + d*x)])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^3 - (4*b*B^2*PolyLog[
3, (d*(a + b*x))/(b*(c + d*x))])/d^3))/i^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(dix+ci)^2}dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, al
gorithm="fricas")
```

output
```
integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x
^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*
B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)
))/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, al
gorithm="maxima")
```

output
```
-A^2*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2) - x/(d^2*i^2) + 2*c*log(d*x + c)/(d^
3*i^2))*g^2 + 2*A^2*a*b*g^2*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2
*i^2)) - 2*A*B*a^2*g^2*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x +
c*d*i^2) - 1/(d^2*i^2*x + c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^2)
+ b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) - A^2*a^2*g^2/(d^2*i^2*x + c*d*i^2
) - 1/3*(2*((b^2*c*d*g^2 - a*b*d^2*g^2)*B^2*x + (b^2*c^2*g^2 - a*b*c*d*g^2
)*B^2)*log(d*x + c)^3 - 3*(B^2*b^2*d^2*g^2*x^2 + B^2*b^2*c*d*g^2*x - (b^2*
c^2*g^2 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)*B^2)*log(d*x + c)^2)/(d^4*i^2*x + c
*d^3*i^2) - integrate(-(B^2*a^2*d^2*g^2*log(e)^2 + (B^2*b^2*d^2*g^2*log(e)
^2 + 2*A*B*b^2*d^2*g^2*log(e))*x^2 + (B^2*b^2*d^2*g^2*x^2 + 2*B^2*a*b*d^2*
g^2*x + B^2*a^2*d^2*g^2)*log(b*x + a)^2 + 2*(B^2*a*b*d^2*g^2*log(e)^2 + 2*
A*B*a*b*d^2*g^2*log(e))*x + 2*(B^2*a^2*d^2*g^2*log(e) + (B^2*b^2*d^2*g^2*l
og(e) + A*B*b^2*d^2*g^2)*x^2 + 2*(B^2*a*b*d^2*g^2*log(e) + A*B*a*b*d^2*g^2
)*x)*log(b*x + a) + 2*((b^2*c^2*g^2 - 2*a*b*c*d*g^2 - (g^2*log(e) - g^2)*a
^2*d^2)*B^2 - (A*B*b^2*d^2*g^2 + (g^2*log(e) + g^2)*B^2*b^2*d^2)*x^2 - (2*
A*B*a*b*d^2*g^2 + (2*a*b*d^2*g^2*log(e) + b^2*c*d*g^2)*B^2)*x - (B^2*b^2*d
^2*g^2*x^2 + 2*B^2*a*b*d^2*g^2*x + B^2*a^2*d^2*g^2)*log(b*x + a))*log(d*x
+ c))/(d^4*i^2*x^2 + 2*c*d^3*i^2*x + c^2*d^2*i^2), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, al
gorithm="giac")
```

output
```
integrate((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*
i)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(c\,i+d\,i\,x)^2}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
^2,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
^2, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

output

```
(g**2*( - int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(c**2 + 2*c*d*x + d**
2*x**2),x)*a*b**4*c**2*d**4 - int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(
c**2 + 2*c*d*x + d**2*x**2),x)*a*b**4*c*d**5*x + int((log((a*e + b*e*x)/(c
 + d*x))**2*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**5*c**3*d**3 + int((lo
g((a*e + b*e*x)/(c + d*x))**2*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**5*c
**2*d**4*x - 2*int((log((a*e + b*e*x)/(c + d*x))**2*x)/(c**2 + 2*c*d*x + d
**2*x**2),x)*a**2*b**3*c**2*d**4 - 2*int((log((a*e + b*e*x)/(c + d*x))**2*
x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b**3*c*d**5*x + 2*int((log((a*e +
b*e*x)/(c + d*x))**2*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**4*c**3*d**3 +
 2*int((log((a*e + b*e*x)/(c + d*x))**2*x)/(c**2 + 2*c*d*x + d**2*x**2),x)
*a*b**4*c**2*d**4*x - 2*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(c**2 + 2*
c*d*x + d**2*x**2),x)*a**2*b**3*c**2*d**4 - 2*int((log((a*e + b*e*x)/(c +
d*x))*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b**3*c*d**5*x + 2*int((lo
g((a*e + b*e*x)/(c + d*x))*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**4*c*
*3*d**3 + 2*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(c**2 + 2*c*d*x + d**2
*x**2),x)*a*b**4*c**2*d**4*x - 4*int((log((a*e + b*e*x)/(c + d*x))*x)/(c**
2 + 2*c*d*x + d**2*x**2),x)*a**3*b**2*c**2*d**4 - 4*int((log((a*e + b*e*x)
/(c + d*x))*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**3*b**2*c*d**5*x + 4*int(
(log((a*e + b*e*x)/(c + d*x))*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b**3
*c**3*d**3 + 4*int((log((a*e + b*e*x)/(c + d*x))*x)/(c**2 + 2*c*d*x + d...
```

3.94 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$

Optimal result . . . 1172
Mathematica [B] (verified) . . . 1173
Rubi [A] (verified) . . . 1174
Maple [F] . . . 1175
Fricas [F] . . . 1176
Sympy [F(-1)] . . . 1176
Maxima [F] . . . 1176
Giac [F] . . . 1177
Mupad [F(-1)] . . . 1177
Reduce [F] . . . 1178

Optimal result

Integrand size = 40, antiderivative size = 261

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= \frac{2ABg(a+bx)}{di^2(c+dx)} - \frac{2B^2g(a+bx)}{di^2(c+dx)} + \frac{2B^2g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{di^2(c+dx)}$$
$$- \frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di^2(c+dx)} - \frac{bg\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2}$$
$$- \frac{2bBg\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2} + \frac{2bB^2g\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}$$

output

```
2*A*B*g*(b*x+a)/d/i^2/(d*x+c)-2*B^2*g*(b*x+a)/d/i^2/(d*x+c)+2*B^2*g*(b*x+a
)*ln(e*(b*x+a)/(d*x+c))/d/i^2/(d*x+c)-g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c))
)^2/d/i^2/(d*x+c)-b*g*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))
^2/d^2/i^2-2*b*B*g*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+
c))/d^2/i^2+2*b*B^2*g*polylog(3,d*(b*x+a)/b/(d*x+c))/d^2/i^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1412 vs. $2(261) = 522$.

Time = 1.83 (sec) , antiderivative size = 1412, normalized size of antiderivative = 5.41

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*
i*x)^2,x]
```

output

```
(g*((3*A^2*(b*c - a*d))/(c + d*x) + 3*A^2*b*Log[c + d*x] + (6*a*A*B*d*(b*c
 - a*d + b*(c + d*x)*Log[a/b + x] + (-(b*c) + a*d)*Log[(e*(a + b*x))/(c +
d*x)] - b*c*Log[(b*(c + d*x))/(b*c - a*d)] - b*d*x*Log[(b*(c + d*x))/(b*c
- a*d)]))/((b*c - a*d)*(c + d*x)) + 3*A*b*B*(-Log[c/d + x]^2 + 2*Log[c/d +
 x]*Log[c + d*x] + 2*(-(c/(c + d*x)) + (b*c*Log[a + b*x])/(-(b*c) + a*d) +
 (b*c*Log[c + d*x])/(b*c - a*d) - Log[a/b + x]*Log[c + d*x] + Log[(e*(a +
b*x))/(c + d*x)]*(c/(c + d*x) + Log[c + d*x]) + Log[a/b + x]*Log[(b*(c + d
*x))/(b*c - a*d)]) + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - (3*a*B^
2*d*(2*b*c - 2*a*d + 2*b*(c + d*x)*Log[a + b*x] - 2*(b*c - a*d)*Log[(e*(a
+ b*x))/(c + d*x)] - 2*b*(c + d*x)*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x
)] + (b*c - a*d)*Log[(e*(a + b*x))/(c + d*x)]^2 - 2*b*(c + d*x)*Log[c + d*
x] - 2*b*(c + d*x)*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c - a*d)/(b*c + b*d
*x)] + b*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c
- a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*(c + d*x)*(Log[
(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c
 - a*d)/(b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/((b*c
 - a*d)*(c + d*x)) + (b*B^2*((b*c - a*d)*(c + d*x)*Log[c/d + x]^3 + 3*c*(b
*c - a*d)*(2 + 2*Log[c/d + x] + Log[c/d + x]^2) + 3*(b*c - a*d)*(-Log[a/b
+ x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])^2*(c + (c + d*x)*Log[c
 + d*x]) + 3*c*Log[a/b + x]*(-(d*(a + b*x)*Log[a/b + x]) + 2*b*(c + d*x...
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ci+dix)^2}\,dx$$

$$\downarrow 2962$$

$$\frac{g\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{i^2}$$

$$\downarrow 2795$$

$$\frac{g\int\left(-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d}-\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{i^2}$$

$$\downarrow 2009$$

$$\frac{g\left(-\frac{2bB\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^2}-\frac{b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^2}-\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d(c+dx)}+\frac{2A}{d}\right.}{i^2}$$

input

```
Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)^2
,x]
```

output

```
(g*((2*A*B*(a + b*x))/(d*(c + d*x)) - (2*B^2*(a + b*x))/(d*(c + d*x)) + (2
*B^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d*(c + d*x)) - ((a + b*x)*(A
 + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d*(c + d*x)) - (b*(A + B*Log[(e*(a
+ b*x))/(c + d*x)])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 - (2*b*B*(
A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))
])/d^2 + (2*b*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/d^2))/i^2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(dix+ci)^2}dx$$

input

```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

output

```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algo
rithm="fricas")
```

output
```
integral((A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b*e*x + a*e)/(d
*x + c))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b*e*x + a*e)/(d*x + c)))/(d^2*i^
2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algo
rithm="maxima")
```

output
```
A^2*b*g*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2)) - 2*A*B*a*g*(
log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x + c*d*i^2) - 1/(d^2*i^2*x
+ c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^2) + b*log(d*x + c)/((b*c*d
 - a*d^2)*i^2)) - A^2*a*g/(d^2*i^2*x + c*d*i^2) + 1/3*(3*(b*c*g - a*d*g)*B
^2*log(d*x + c)^2 + (B^2*b*d*g*x + B^2*b*c*g)*log(d*x + c)^3)/(d^3*i^2*x +
 c*d^2*i^2) - integrate(-(B^2*a*d*g*log(e)^2 + (B^2*b*d*g*x + B^2*a*d*g)*l
og(b*x + a)^2 + (B^2*b*d*g*log(e)^2 + 2*A*B*b*d*g*log(e))*x + 2*(B^2*a*d*g
*log(e) + (B^2*b*d*g*log(e) + A*B*b*d*g)*x)*log(b*x + a) - 2*(((g*log(e) -
 g)*a*d + b*c*g)*B^2 + (B^2*b*d*g*log(e) + A*B*b*d*g)*x + (B^2*b*d*g*x + B
^2*a*d*g)*log(b*x + a))*log(d*x + c))/(d^3*i^2*x^2 + 2*c*d^2*i^2*x + c^2*d
*i^2), x)
```

Giac [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algo
rithm="giac")
```

output
```
integrate((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*i)
^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(a\,g+b\,g\,x)\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(c\,i+d\,i\,x)^2}\,dx$$

input
```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)^2
,x)
```

output
```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)^2
, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

output
```
(g*( - int((log((a*e + b*e*x)/(c + d*x))**2*x)/(c**2 + 2*c*d*x + d**2*x**2
),x)*a*b**3*c**2*d**3 - int((log((a*e + b*e*x)/(c + d*x))**2*x)/(c**2 + 2*
c*d*x + d**2*x**2),x)*a*b**3*c*d**4*x + int((log((a*e + b*e*x)/(c + d*x))*
*2*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**4*c**3*d**2 + int((log((a*e + b*e
*x)/(c + d*x))**2*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**4*c**2*d**3*x - 2*
int((log((a*e + b*e*x)/(c + d*x))*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*
b**2*c**2*d**3 - 2*int((log((a*e + b*e*x)/(c + d*x))*x)/(c**2 + 2*c*d*x +
d**2*x**2),x)*a**2*b**2*c*d**4*x + 2*int((log((a*e + b*e*x)/(c + d*x))*x)/
(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**3*c**3*d**2 + 2*int((log((a*e + b*e*x
)/(c + d*x))*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**3*c**2*d**3*x + 2*log
(a + b*x)*a**3*b*c*d**2 + 2*log(a + b*x)*a**3*b*d**3*x - 2*log(a + b*x)*a*
*2*b**2*c*d**2 - 2*log(a + b*x)*a**2*b**2*d**3*x - log(c + d*x)*a**3*b*c**
2*d - log(c + d*x)*a**3*b*c*d**2*x - 2*log(c + d*x)*a**3*b*c*d**2 - 2*log(
c + d*x)*a**3*b*d**3*x + log(c + d*x)*a**2*b**2*c**3 + log(c + d*x)*a**2*b
**2*c**2*d*x + 2*log(c + d*x)*a**2*b**2*c*d**2 + 2*log(c + d*x)*a**2*b**2*
d**3*x + log((a*e + b*e*x)/(c + d*x))**2*a**2*b**2*c*d**2 + log((a*e + b*e
*x)/(c + d*x))**2*a*b**3*c*d**2*x - 2*log((a*e + b*e*x)/(c + d*x))*a**3*b*
d**3*x + 2*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*c*d**2*x + 2*log((a*e +
b*e*x)/(c + d*x))*a**2*b**2*d**3*x - 2*log((a*e + b*e*x)/(c + d*x))*a*b**3
*c*d**2*x - a**4*d**3*x + 2*a**3*b*c*d**2*x + 2*a**3*b*d**3*x - a**2*b*...
```

3.95 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$

Optimal result 1179
Mathematica [C] (verified) 1180
Rubi [A] (verified) 1180
Maple [A] (verified) 1182
Fricas [A] (verification not implemented) 1183
Sympy [B] (verification not implemented) 1183
Maxima [B] (verification not implemented) 1184
Giac [A] (verification not implemented) 1185
Mupad [B] (verification not implemented) 1186
Reduce [B] (verification not implemented) 1186

Optimal result

Integrand size = 32, antiderivative size = 152

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = -\frac{2AB(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{2B^2(a+bx)}{(bc-ad)i^2(c+dx)} - \frac{2B^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)i^2(c+dx)} + \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)i^2(c+dx)}$$

output

```
-2*A*B*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)+2*B^2*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)
-2*B^2*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)/i^2/(d*x+c)+(b*x+a)*(A+B*l
n(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)/i^2/(d*x+c)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.07

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= \frac{-\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + \frac{B\left(2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2b(c+dx)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x)^2,x]
```

output
```
(-(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*(b*c - a*d)*(A + B*Log[(e
*(a + b*x))/(c + d*x)]) + 2*b*(c + d*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*
x))/(c + d*x)]) - 2*b*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c
 + d*x] - 2*B*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c +
d*x]) - b*B*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b
*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*(c + d*x)*
((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*Pol
yLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(d*i^2*(c + d*x))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2952, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ci+dix)^2}\,dx$$
$$\downarrow 2952$$

$$\frac{\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2733$$

$$\frac{\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx}-2B\int\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2009$$

$$\frac{\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx}-2B\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}-\frac{B(a+bx)}{c+dx}\right)}{i^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x)^2,x]
```

output
```
(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c + d*x) - 2*B*((A*(a
 + b*x))/(c + d*x) - (B*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[(e*(a + b*
x))/(c + d*x)])/(c + d*x)))/((b*c - a*d)*i^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.20

method	result
norman	$\frac{\left(A^2-2AB+2B^2\right)x}{ic}-\frac{B^2a\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{i(da-bc)}-\frac{B^2bx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{(da-bc)i}-\frac{2aB(A-B)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{i(da-bc)}-\frac{2Bb(A-B)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{i(da-bc)}$ $i(dx+c)$
parts	$-\frac{A^2}{i^2(dx+c)d}-\frac{B^2\left(\frac{\ln\left(\frac{e(bx+a)}{dx+c}\right)^2e(bx+a)}{dx+c}-\frac{2\ln\left(\frac{e(bx+a)}{dx+c}\right)e(bx+a)}{dx+c}+\frac{2e(bx+a)}{dx+c}\right)}{i^2e(da-bc)}-\frac{2AB\left(\frac{\ln\left(\frac{e(bx+a)}{dx+c}\right)e(bx+a)}{dx+c}-\frac{e(}{}\right.}{i^2e(da-bc)}$
parallelrisch	$-\frac{2B^2abd^3-2B^2b^2cd^2+A^2abd^3-A^2b^2cd^2-2ABabd^3+2ABb^2cd^2+B^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b^2d^3-2B^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2$ $i^2(dx+c)bd^3(da-$
derivativedivides	$-\frac{e(da-bc)\left(\frac{d^2A^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{(da-bc)^2e^2i^2}+\frac{2d^2AB\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-\frac{(da-bc)e}{d(dx+c)}-\frac{be}{d}\right)}{(da-bc)^2e^2i^2}+\frac{d^2B^2\left(\left(\frac{be}{d}+\frac{(da-bc)}{d(dx+c}\right.\right.}{d^2}$
default	$-\frac{e(da-bc)\left(\frac{d^2A^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{(da-bc)^2e^2i^2}+\frac{2d^2AB\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-\frac{(da-bc)e}{d(dx+c)}-\frac{be}{d}\right)}{(da-bc)^2e^2i^2}+\frac{d^2B^2\left(\left(\frac{be}{d}+\frac{(da-bc)}{d(dx+c}\right.\right.}{d^2}$
risch	$-\frac{A^2}{i^2(dx+c)d}-\frac{B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2bx}{i^2(da-bc)(dx+c)}-\frac{B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a}{i^2(da-bc)(dx+c)}+\frac{2B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)bx}{i^2(da-bc)(dx+c)}+\frac{2B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a}{i^2(da-bc)(dx+c)}-\frac{}{i^2(d}$
orering	$-\frac{(dx+c)\left(8b^2dx^2+15xabd+b^2cx+7a^2d+abc\right)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(a^2d^2-2acdb+c^2b^2)(dix+ci)^2}-\frac{(bx+a)(dx+c)^2(7bdx+6da+bc)\left(\frac{2\left(A+B\ln\right.\right.}{}\right.$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x,method=_RETURNVERBOSE)
```

output
```
((A^2-2*A*B+2*B^2)/i/c*x-B^2*a/i/(a*d-b*c)*ln(e*(b*x+a)/(d*x+c))^2-B^2*b/(
a*d-b*c)/i*x*ln(e*(b*x+a)/(d*x+c))^2-2*a*B*(A-B)/i/(a*d-b*c)*ln(e*(b*x+a)/
(d*x+c))-2*B*b*(A-B)/i/(a*d-b*c)*x*ln(e*(b*x+a)/(d*x+c)))/i/(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.02

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = -\frac{(A^2-2\,AB+2\,B^2)bc-(A^2-2\,AB+2\,B^2)ad-(B^2bdx+B^2ad)\log\left(\frac{bex+ae}{dx+c}\right)^2-2\left((AB-B^2)bdx\right.}{(bcd^2-ad^3)i^2x+(bc^2d-acd^2)i^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algorithm="fricas")
```

output
```
-((A^2 - 2*A*B + 2*B^2)*b*c - (A^2 - 2*A*B + 2*B^2)*a*d - (B^2*b*d*x + B^2*a*d)*log((b*e*x + a*e)/(d*x + c))^2 - 2*((A*B - B^2)*b*d*x + (A*B - B^2)*a*d)*log((b*e*x + a*e)/(d*x + c)))/((b*c*d^2 - a*d^3)*i^2*x + (b*c^2*d - a*c*d^2)*i^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(128) = 256.

Time = 1.15 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.84

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= \frac{2Bb(A-B)\log\left(x+\frac{2ABabd+2ABb^2c-2B^2abd-2B^2b^2c-\frac{2Ba^2bd^2(A-B)}{ad-bc}+\frac{4Bab^2cd(A-B)}{ad-bc}-\frac{2Bb^3c^2(A-B)}{ad-bc}}{4ABb^2d-4B^2b^2d}\right)}{di^2\,(ad-bc)}$$
$$- \frac{2Bb(A-B)\log\left(x+\frac{2ABabd+2ABb^2c-2B^2abd-2B^2b^2c+\frac{2Ba^2bd^2(A-B)}{ad-bc}-\frac{4Bab^2cd(A-B)}{ad-bc}+\frac{2Bb^3c^2(A-B)}{ad-bc}}{4ABb^2d-4B^2b^2d}\right)}{di^2\,(ad-bc)}$$
$$+ \frac{(-2AB+2B^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{cdi^2+d^2i^2x}$$
$$+ \frac{(-B^2a-B^2bx)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{acdi^2+ad^2i^2x-bc^2i^2-bcdi^2x} + \frac{-A^2+2AB-2B^2}{cdi^2+d^2i^2x}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**2,x)
```

output
```
2*B*b*(A - B)*log(x + (2*A*B*a*b*d + 2*A*B*b**2*c - 2*B**2*a*b*d - 2*B**2*
b**2*c - 2*B*a**2*b*d**2*(A - B)/(a*d - b*c) + 4*B*a*b**2*c*d*(A - B)/(a*d
 - b*c) - 2*B*b**3*c**2*(A - B)/(a*d - b*c))/(4*A*B*b**2*d - 4*B**2*b**2*d
))/(d*i**2*(a*d - b*c)) - 2*B*b*(A - B)*log(x + (2*A*B*a*b*d + 2*A*B*b**2*
c - 2*B**2*a*b*d - 2*B**2*b**2*c + 2*B*a**2*b*d**2*(A - B)/(a*d - b*c) - 4
*B*a*b**2*c*d*(A - B)/(a*d - b*c) + 2*B*b**3*c**2*(A - B)/(a*d - b*c))/(4*
A*B*b**2*d - 4*B**2*b**2*d))/(d*i**2*(a*d - b*c)) + (-2*A*B + 2*B**2)*log(
e*(a + b*x)/(c + d*x))/(c*d*i**2 + d**2*i**2*x) + (-B**2*a - B**2*b*x)*log
(e*(a + b*x)/(c + d*x))**2/(a*c*d*i**2 + a*d**2*i**2*x - b*c**2*i**2 - b*c
*d*i**2*x) + (-A**2 + 2*A*B - 2*B**2)/(c*d*i**2 + d**2*i**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(152) = 304$.

Time $= 0.06$ (sec) , antiderivative size $= 416$, normalized size of antiderivative $= 2.74$

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$$

$$= \left(2\left(\frac{1}{d^2i^2x + cdi^2} + \frac{b\log(bx+a)}{(bcd-ad^2)i^2} - \frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right)\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) - \frac{(bdx+bc)\log(bx+a)^2}{}\right.$$

$$- 2AB\left(\frac{\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right)}{d^2i^2x + cdi^2} - \frac{1}{d^2i^2x + cdi^2} - \frac{b\log(bx+a)}{(bcd-ad^2)i^2} + \frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right)$$

$$- \frac{B^2\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right)^2}{d^2i^2x + cdi^2} - \frac{A^2}{d^2i^2x + cdi^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algorithm="maxim
a")
```

output
```
(2*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2) - b*log
(d*x + c)/((b*c*d - a*d^2)*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - ((
b*d*x + b*c)*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d
 + 2*(b*d*x + b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*x +
 a))*log(d*x + c))/(b*c^2*d*i^2 - a*c*d^2*i^2 + (b*c*d^2*i^2 - a*d^3*i^2)*
x))*B^2 - 2*A*B*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x + c*d*i^2
) - 1/(d^2*i^2*x + c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^2) + b*log
(d*x + c)/((b*c*d - a*d^2)*i^2)) - B^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)
)^2/(d^2*i^2*x + c*d*i^2) - A^2/(d^2*i^2*x + c*d*i^2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.14

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= \left(\frac{(bex+ae)B^2\log\left(\frac{bex+ae}{dx+c}\right)^2}{(dx+c)i^2} + \frac{2\,(bex+ae)(AB-B^2)\log\left(\frac{bex+ae}{dx+c}\right)}{(dx+c)i^2} + \frac{(bex+ae)(A^2-2\,AB+2\,B^2)}{(dx+c)i^2}\right)$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algorithm="giac"
)
```

output
```
((b*e*x + a*e)*B^2*log((b*e*x + a*e)/(d*x + c))^2/((d*x + c)*i^2) + 2*(b*e
*x + a*e)*(A*B - B^2)*log((b*e*x + a*e)/(d*x + c))/((d*x + c)*i^2) + (b*e*
x + a*e)*(A^2 - 2*A*B + 2*B^2)/((d*x + c)*i^2))*(b*c/((b*c*e - a*d*e)*(b*c
 - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 27.36 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.46

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{2\,B^2}{b\,d^2\,i^2}-\frac{2\,A\,B}{b\,d^2\,i^2}\right)}{\frac{x}{b}+\frac{c}{b\,d}} - \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{B^2}{d^2\,i^2\,\left(x+\frac{c}{d}\right)}+\frac{B^2\,b}{d\,i^2\,(a\,d-b\,c)}\right) - \frac{A^2-2\,A\,B+2\,B^2}{x\,d^2\,i^2+c\,d\,i^2} + \frac{B\,b\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{a\,d^2\,i^2+b\,c\,d\,i^2}{d\,i^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A-B)\,4\mathrm{i}}{d\,i^2\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(c*i + d*i*x)^2,x)
```

output
```
(log((e*(a + b*x))/(c + d*x))*((2*B^2)/(b*d^2*i^2) - (2*A*B)/(b*d^2*i^2)))
/(x/b + c/(b*d)) - log((e*(a + b*x))/(c + d*x))^2*(B^2/(d^2*i^2*(x + c/d))
 + (B^2*b)/(d*i^2*(a*d - b*c))) - (A^2 + 2*B^2 - 2*A*B)/(d^2*i^2*x + c*d*i
^2) + (B*b*atan(((2*b*d*x + (a*d^2*i^2 + b*c*d*i^2)/(d*i^2))*1i)/(a*d - b*
c))*(A - B)*4i)/(d*i^2*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.18

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= \frac{2\log(bx+a)\,a^2bc+2\log(bx+a)\,a^2bdx-2\log(bx+a)\,ab^2c-2\log(bx+a)\,ab^2dx-2\log(dx+c)\,a^2bc}$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

output

```
(2*log(a + b*x)*a**2*b*c + 2*log(a + b*x)*a**2*b*d*x - 2*log(a + b*x)*a*b*
*2*c - 2*log(a + b*x)*a*b**2*d*x - 2*log(c + d*x)*a**2*b*c - 2*log(c + d*x
)*a**2*b*d*x + 2*log(c + d*x)*a*b**2*c + 2*log(c + d*x)*a*b**2*d*x + log((
a*e + b*e*x)/(c + d*x))**2*a*b**2*c + log((a*e + b*e*x)/(c + d*x))**2*b**3
*c*x - 2*log((a*e + b*e*x)/(c + d*x))*a**2*b*d*x + 2*log((a*e + b*e*x)/(c
+ d*x))*a*b**2*c*x + 2*log((a*e + b*e*x)/(c + d*x))*a*b**2*d*x - 2*log((a*
e + b*e*x)/(c + d*x))*b**3*c*x - a**3*d*x + a**2*b*c*x + 2*a**2*b*d*x - 2*
a*b**2*c*x - 2*a*b**2*d*x + 2*b**3*c*x)/(c*(a*c*d + a*d**2*x - b*c**2 - b*
c*d*x))
```

3.96 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$

Optimal result 1188
Mathematica [A] (verified) 1189
Rubi [A] (warning: unable to verify) 1189
Maple [A] (verified) 1192
Fricas [A] (verification not implemented) 1193
Sympy [B] (verification not implemented) 1193
Maxima [B] (verification not implemented) 1195
Giac [A] (verification not implemented) 1196
Mupad [B] (verification not implemented) 1197
Reduce [B] (verification not implemented) 1198

Optimal result

Integrand size = 42, antiderivative size = 214

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx &= \frac{2ABd(a+bx)}{(bc-ad)^2gi^2(c+dx)} - \frac{2B^2d(a+bx)}{(bc-ad)^2gi^2(c+dx)} \\ &\quad + \frac{2B^2d(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^2gi^2(c+dx)} \\ &\quad - \frac{d(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2gi^2(c+dx)} \\ &\quad + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^2gi^2}\end{aligned}$$

output

```
2*A*B*d*(b*x+a)/(-a*d+b*c)^2/g/i^2/(d*x+c)-2*B^2*d*(b*x+a)/(-a*d+b*c)^2/g/
i^2/(d*x+c)+2*B^2*d*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)^2/g/i^2/(d*x+
c)-d*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^2/g/i^2/(d*x+c)+1/3*
b*(A+B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)^2/g/i^2
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.87

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$

$$= \frac{3b(A^2 - 2AB + 2B^2)(c+dx)\log(a+bx) + 6(A-B)B(bc-ad)\log\left(\frac{e(a+bx)}{c+dx}\right) + 3B(-Bd(a+bx) + A$$

$$3(bc -$$

input

```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i
*x)^2),x]
```

output

```
(3*b*(A^2 - 2*A*B + 2*B^2)*(c + d*x)*Log[a + b*x] + 6*(A - B)*B*(b*c - a*d
)*Log[(e*(a + b*x))/(c + d*x)] + 3*B*(-(B*d*(a + b*x)) + A*b*(c + d*x))*Lo
g[(e*(a + b*x))/(c + d*x)]^2 + b*B^2*(c + d*x)*Log[(e*(a + b*x))/(c + d*x)
]^3 - 3*(A^2 - 2*A*B + 2*B^2)*(-(b*c) + a*d + b*(c + d*x)*Log[c + d*x]))/(
3*(b*c - a*d)^2*g*i^2*(c + d*x))
```

Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2962, 2788, 2733, 2009, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)\left(b - \frac{d(a+bx)}{c+dx}\right)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}\,d\frac{a+bx}{c+dx}}{gi^2(bc-ad)^2}$$

$$\downarrow 2788$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx} - d\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 d\frac{a+bx}{c+dx}}{gi^2(bc-ad)^2}$$

$$\downarrow 2733$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx} - d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx} - 2B\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d\frac{a+bx}{c+dx}\right)}{gi^2(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx} - d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx} - 2B\left(\frac{A(a+bx)}{c+dx} + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx} - \frac{B(a+bx)}{c+dx}\right.\right.}{gi^2(bc-ad)^2}$$

$$\downarrow 2739$$

$$\frac{b\int \frac{(a+bx)^2}{(c+dx)^2}d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B} - d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx} - 2B\left(\frac{A(a+bx)}{c+dx} + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx} - \frac{B(a+bx)}{c+dx}\right)\right)$$
$$gi^2(bc-ad)^2$$

$$\downarrow 15$$

$$\frac{\frac{b(a+bx)^3}{3B(c+dx)^3} - d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx} - 2B\left(\frac{A(a+bx)}{c+dx} + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx} - \frac{B(a+bx)}{c+dx}\right)\right)}{gi^2(bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i*x)^2)
,x]
```

output
```
((b*(a + b*x)^3)/(3*B*(c + d*x)^3) - d*(((a + b*x)*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])^2)/(c + d*x) - 2*B*((A*(a + b*x))/(c + d*x) - (B*(a + b*x))/
(c + d*x) + (B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(c + d*x))))/((b*c
- a*d)^2*g*i^2)
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2739

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2788

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.86

method	result
norman	$\frac{\left(A^2bc-2ABad+2B^2ad\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{B(Abc-Bad)\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{gi\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{d\left(A^2b-2ABb+2B^2b\right)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{\left(A^2-2AB+2B^2\right)}{gic(da-bc)}$
parallelrisch	$\frac{-3A^2ab^2d^4+3A^2b^3cd^3-6B^2ab^2d^4+6B^2b^3cd^3-3B^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b^3d^4+B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^3b^3cd^3+3A^2x\ln\left(\frac{e(bx+c}{dx+c}\right.$
parts	$\frac{A^2\left(-\frac{1}{(da-bc)(dx+c)}-\frac{b\ln(dx+c)}{(da-bc)^2}+\frac{b\ln(bx+a)}{(da-bc)^2}\right)}{gi^2}-\frac{B^2\left(\frac{d\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2-2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}\right.\right.\right.}{da-bc}}{gi^2(da-bc)}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{d^2A^2b\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei^2(da-bc)^3g}+\frac{d^3A^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e^2i^2(da-bc)^3g}-\frac{d^2ABb\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{ei^2(da-bc)^3g}+\frac{2d^3AB\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right.}{e^2}\right.}$
default	$-\frac{e(da-bc)\left(-\frac{d^2A^2b\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{ei^2(da-bc)^3g}+\frac{d^3A^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e^2i^2(da-bc)^3g}-\frac{d^2ABb\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{ei^2(da-bc)^3g}+\frac{2d^3AB\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\right.}{e^2}\right.}$
risch	$-\frac{A^2}{gi^2(da-bc)(dx+c)}-\frac{A^2b\ln(dx+c)}{gi^2(da-bc)^2}+\frac{A^2b\ln(bx+a)}{gi^2(da-bc)^2}-\frac{B^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2b}{gi^2(da-bc)^2}-\frac{B^2d\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2a}{gi^2(da-bc)^2(dx+c)}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x,method=_RETU
RNVERBOSE)
```

output
```
((A^2*b*c-2*A*B*a*d+2*B^2*a*d)/g/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a
)/(d*x+c))+B*(A*b*c-B*a*d)/g/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a)/(d
*x+c))^2+1/g/i*d*(A^2*b-2*A*B*b+2*B^2*b)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(
e*(b*x+a)/(d*x+c))+(A^2-2*A*B+2*B^2)*d/g/i/c/(a*d-b*c)*x+b*d*B*(A-B)/g/i/(
a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^2+1/3*B^2*b*c/g/i/(a^2*
d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a)/(d*x+c))^3+1/3*b*B^2*d/g/i/(a^2*d^2-2*
a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^3)/i/(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.10

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$

$$= \frac{(B^2bdx + B^2bc)\log\left(\frac{bex+ae}{dx+c}\right)^3 + 3\,(A^2 - 2\,AB + 2\,B^2)bc - 3\,(A^2 - 2\,AB + 2\,B^2)ad + 3\,(ABbc - B^2ad -}{3\,((b^2c^2d - 2\,abcd^2 + a^2d^3)gi^2x +}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algo
rithm="fricas")
```

output
```
1/3*((B^2*b*d*x + B^2*b*c)*log((b*e*x + a*e)/(d*x + c))^3 + 3*(A^2 - 2*A*B
 + 2*B^2)*b*c - 3*(A^2 - 2*A*B + 2*B^2)*a*d + 3*(A*B*b*c - B^2*a*d + (A*B
 - B^2)*b*d*x)*log((b*e*x + a*e)/(d*x + c))^2 + 3*(A^2*b*c + (A^2 - 2*A*B +
 2*B^2)*b*d*x - 2*(A*B - B^2)*a*d)*log((b*e*x + a*e)/(d*x + c)))/((b^2*c^2
*d - 2*a*b*c*d^2 + a^2*d^3)*g*i^2*x + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*
g*i^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(185) = 370.

Time = 0.72 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.52

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$

$$= \frac{B^2 b\log\left(\frac{e(a+bx)}{c+dx}\right)^3}{3a^2d^2gi^2-6abcdgi^2+3b^2c^2gi^2} + \frac{(-2AB+2B^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{acdgi^2+ad^2gi^2x-bc^2gi^2-bcdgi^2x}$$

$$+\left(A^2-2AB+2B^2\right)\left(-\frac{b\log\left(x+\frac{-\frac{a^3bd^3}{(ad-bc)^2}+\frac{3a^2b^2cd^2}{(ad-bc)^2}-\frac{3ab^3c^2d}{(ad-bc)^2}+abd+\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{gi^2(ad-bc)^2}\right.$$

$$+\frac{b\log\left(x+\frac{\frac{a^3bd^3}{(ad-bc)^2}-\frac{3a^2b^2cd^2}{(ad-bc)^2}+\frac{3ab^3c^2d}{(ad-bc)^2}+abd-\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{gi^2(ad-bc)^2}$$

$$\left.-\frac{1}{acdgi^2-bc^2gi^2+x\left(ad^2gi^2-bcdgi^2\right)}\right)$$

$$+\frac{(ABbc+ABbdx-B^2ad-B^2bdx)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{a^2cd^2gi^2+a^2d^3gi^2x-2abc^2dgi^2-2abcd^2gi^2x+b^2c^3gi^2+b^2c^2dgi^2x}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)/(d*i*x+c*i)**2,x)
```

output

```
B**2*b*log(e*(a + b*x)/(c + d*x))**3/(3*a**2*d**2*g*i**2 - 6*a*b*c*d*g*i**
2 + 3*b**2*c**2*g*i**2) + (-2*A*B + 2*B**2)*log(e*(a + b*x)/(c + d*x))/(a*
c*d*g*i**2 + a*d**2*g*i**2*x - b*c**2*g*i**2 - b*c*d*g*i**2*x) + (A**2 - 2
*A*B + 2*B**2)*(-b*log(x + (-a**3*b*d**3/(a*d - b*c)**2 + 3*a**2*b**2*c*d*
*2/(a*d - b*c)**2 - 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d + b**4*c**3/(a*
d - b*c)**2 + b**2*c)/(2*b**2*d))/(g*i**2*(a*d - b*c)**2) + b*log(x + (a**
3*b*d**3/(a*d - b*c)**2 - 3*a**2*b**2*c*d**2/(a*d - b*c)**2 + 3*a*b**3*c**
2*d/(a*d - b*c)**2 + a*b*d - b**4*c**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d)
)/(g*i**2*(a*d - b*c)**2) - 1/(a*c*d*g*i**2 - b*c**2*g*i**2 + x*(a*d**2*g*
i**2 - b*c*d*g*i**2))) + (A*B*b*c + A*B*b*d*x - B**2*a*d - B**2*b*d*x)*log
(e*(a + b*x)/(c + d*x))**2/(a**2*c*d**2*g*i**2 + a**2*d**3*g*i**2*x - 2*a*
b*c**2*d*g*i**2 - 2*a*b*c*d**2*g*i**2*x + b**2*c**3*g*i**2 + b**2*c**2*d*g
*i**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1004 vs. $2(212) = 424$.

Time $= 0.11$ (sec) , antiderivative size $= 1004$, normalized size of antiderivative $= 4.69$

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)(ci + dix)^2}\, dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algo
rithm="maxima")
```

output

```
B^2*(1/((b*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/
((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*g*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2 + 2*A*B*(1/
((b*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*g*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/3*B^2*(3*((b*d*x
 + b*c)*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*
(b*d*x + b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*
log(d*x + c))*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*c^3*g*i^2 - 2*a*b*
c^2*d*g*i^2 + a^2*c*d^2*g*i^2 + (b^2*c^2*d*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2
*d^3*g*i^2)*x) - ((b*d*x + b*c)*log(b*x + a)^3 - (b*d*x + b*c)*log(d*x + c
)^3 + 3*(b*d*x + b*c)*log(b*x + a)^2 + 3*(b*d*x + b*c + (b*d*x + b*c)*log(
b*x + a))*log(d*x + c)^2 + 6*b*c - 6*a*d + 6*(b*d*x + b*c)*log(b*x + a) -
3*(2*b*d*x + (b*d*x + b*c)*log(b*x + a)^2 + 2*b*c + 2*(b*d*x + b*c)*log(b*
x + a))*log(d*x + c))/(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 + a^2*c*d^2*g*i^2
 + (b^2*c^2*d*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x)) + A^2*(1/((b*
c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b^2*c^2 -
 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*g*i^2)) - ((b*d*x + b*c)*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^
2 + 2*b*c - 2*a*d + 2*(b*d*x + b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*...
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.58

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{1}{3}\left(\frac{B^2be\log\left(\frac{bex+ae}{dx+c}\right)^3}{bcgi^2-adgi^2} + \frac{3\,A^2be\log\left(\frac{bex+ae}{dx+c}\right)}{bcgi^2-adgi^2} + 3\left(\frac{ABbe}{bcgi^2-adgi^2} - \frac{(bex+ae)B^2d}{(bcgi^2-adgi^2)(dx+c)}\right)\log\left(\frac{bex+}{dx+}\right.\right.$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algo
rithm="giac")
```

output

```
1/3*(B^2*b*e*log((b*e*x + a*e)/(d*x + c))^3/(b*c*g*i^2 - a*d*g*i^2) + 3*A^
2*b*e*log((b*e*x + a*e)/(d*x + c))/(b*c*g*i^2 - a*d*g*i^2) + 3*(A*B*b*e/(b
*c*g*i^2 - a*d*g*i^2) - (b*e*x + a*e)*B^2*d/((b*c*g*i^2 - a*d*g*i^2)*(d*x
+ c)))*log((b*e*x + a*e)/(d*x + c))^2 - 6*(A*B*d - B^2*d)*(b*e*x + a*e)*lo
g((b*e*x + a*e)/(d*x + c))/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)) - 3*(A^2*d
- 2*A*B*d + 2*B^2*d)*(b*e*x + a*e)/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)))*(b
*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 28.06 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.98

$$
\begin{aligned}
&\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx \\
&= \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{B\,b\,(A-B)}{g\,i^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right. \\
&\qquad \left. -\frac{B^2\,(a\,d-b\,c)}{b\,d\,g\,i^2\,\left(\frac{x}{b}+\frac{c}{b\,d}\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)-\frac{A^2-2\,A\,B+2\,B^2}{(a\,d-b\,c)\,(c\,g\,i^2+d\,g\,i^2\,x)} \\
&\quad +\frac{B^2\,b\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^3}{3\,g\,i^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{2\,B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,(A-B)\,(a\,d-b\,c)}{b\,d\,g\,i^2\,\left(\frac{x}{b}+\frac{c}{b\,d}\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)} \\
&\quad -\frac{b\,\mathrm{atan}\left(\frac{b\left(2\,b\,d\,x+\frac{a^2\,d^2\,g\,i^2-b^2\,c^2\,g\,i^2}{g\,i^2\,(a\,d-b\,c)}\right)\,\left(A^2-2\,A\,B+2\,B^2\right)\,1\mathrm{i}}{(a\,d-b\,c)\,\left(b\,A^2-2\,b\,A\,B+2\,b\,B^2\right)}\right)\,\left(A^2-2\,A\,B+2\,B^2\right)\,2\mathrm{i}}{g\,i^2\,(a\,d-b\,c)^2}
\end{aligned}
$$

input

```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)*(c*i + d*i*x)^2)
,x)
```

output
```
log((e*(a + b*x))/(c + d*x))^2*((B*b*(A - B))/(g*i^2*(a^2*d^2 + b^2*c^2 -
2*a*b*c*d)) - (B^2*(a*d - b*c))/(b*d*g*i^2*(x/b + c/(b*d))*(a^2*d^2 + b^2*
c^2 - 2*a*b*c*d))) - (A^2 + 2*B^2 - 2*A*B)/((a*d - b*c)*(c*g*i^2 + d*g*i^2
*x)) + (B^2*b*log((e*(a + b*x))/(c + d*x))^3)/(3*g*i^2*(a^2*d^2 + b^2*c^2
- 2*a*b*c*d)) - (b*atan((b*(2*b*d*x + (a^2*d^2*g*i^2 - b^2*c^2*g*i^2)/(g*i
^2*(a*d - b*c)))*(A^2 + 2*B^2 - 2*A*B)*1i)/((a*d - b*c)*(A^2*b + 2*B^2*b -
 2*A*B*b)))*(A^2 + 2*B^2 - 2*A*B)*2i)/(g*i^2*(a*d - b*c)^2) - (2*B*log((e*
(a + b*x))/(c + d*x))*(A - B)*(a*d - b*c))/(b*d*g*i^2*(x/b + c/(b*d))*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.64

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{-3\log\left(\frac{bex+ae}{dx+c}\right)^2 a\,b^2cdx + 6\log(bx+a)\,a^2b\,d^2x - 6\log(bx+a)\,a\,b^2cd - 6\log(bx+a)\,a\,b^2d^2x - 6\log(dx$$

input
```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x)
```

output
```
( - 3*log(a + b*x)*a**2*b*c**2 - 3*log(a + b*x)*a**2*b*c*d*x + 6*log(a + b
*x)*a**2*b*c*d + 6*log(a + b*x)*a**2*b*d**2*x - 6*log(a + b*x)*a*b**2*c*d
- 6*log(a + b*x)*a*b**2*d**2*x + 3*log(c + d*x)*a**2*b*c**2 + 3*log(c + d*
x)*a**2*b*c*d*x - 6*log(c + d*x)*a**2*b*c*d - 6*log(c + d*x)*a**2*b*d**2*x
 + 6*log(c + d*x)*a*b**2*c*d + 6*log(c + d*x)*a*b**2*d**2*x - log((a*e + b
*e*x)/(c + d*x))**3*b**3*c**2 - log((a*e + b*e*x)/(c + d*x))**3*b**3*c*d*x
 - 3*log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c**2 - 3*log((a*e + b*e*x)/(c
+ d*x))**2*a*b**2*c*d*x + 3*log((a*e + b*e*x)/(c + d*x))**2*a*b**2*c*d + 3
*log((a*e + b*e*x)/(c + d*x))**2*b**3*c*d*x - 6*log((a*e + b*e*x)/(c + d*x
))*a**2*b*d**2*x + 6*log((a*e + b*e*x)/(c + d*x))*a*b**2*c*d*x + 6*log((a*
e + b*e*x)/(c + d*x))*a*b**2*d**2*x - 6*log((a*e + b*e*x)/(c + d*x))*b**3*
c*d*x - 3*a**3*d**2*x + 3*a**2*b*c*d*x + 6*a**2*b*d**2*x - 6*a*b**2*c*d*x
- 6*a*b**2*d**2*x + 6*b**3*c*d*x)/(3*c*g*(a**2*c*d**2 + a**2*d**3*x - 2*a*
b*c**2*d - 2*a*b*c*d**2*x + b**2*c**3 + b**2*c**2*d*x))
```

3.97 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx$

Optimal result 1199
Mathematica [A] (verified) 1200
Rubi [A] (verified) 1201
Maple [B] (verified) 1202
Fricas [A] (verification not implemented) 1204
Sympy [B] (verification not implemented) 1204
Maxima [B] (verification not implemented) 1205
Giac [A] (verification not implemented) 1206
Mupad [B] (verification not implemented) 1207
Reduce [B] (verification not implemented) 1208

Optimal result

Integrand size = 42, antiderivative size = 365

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = &-\frac{2ABd^2(a+bx)}{(bc-ad)^3g^2i^2(c+dx)}+\frac{2B^2d^2(a+bx)}{(bc-ad)^3g^2i^2(c+dx)}\\ &-\frac{2b^2B^2(c+dx)}{(bc-ad)^3g^2i^2(a+bx)}\\ &-\frac{2B^2d^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^3g^2i^2(c+dx)}\\ &-\frac{2b^2B(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2(a+bx)}\\ &+\frac{d^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^2i^2(c+dx)}\\ &-\frac{b^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^2i^2(a+bx)}\\ &-\frac{2bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^3g^2i^2}\end{aligned}$$

output

```
-2*A*B*d^2*(b*x+a)/(-a*d+b*c)^3/g^2/i^2/(d*x+c)+2*B^2*d^2*(b*x+a)/(-a*d+b*
c)^3/g^2/i^2/(d*x+c)-2*b^2*B^2*(d*x+c)/(-a*d+b*c)^3/g^2/i^2/(b*x+a)-2*B^2*
d^2*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)^3/g^2/i^2/(d*x+c)-2*b^2*B*(d*
x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^2/i^2/(b*x+a)+d^2*(b*x+a)*
(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^2/i^2/(d*x+c)-b^2*(d*x+c)*(A+
B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^2/i^2/(b*x+a)-2/3*b*d*(A+B*ln(e*
(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)^3/g^2/i^2
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.84

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx =$$

$$-\frac{-3(A^2-2AB+2B^2)\,d(-bc+ad)(a+bx)+3b(A^2+2AB+2B^2)\,(bc-ad)(c+dx)+6b(A^2+2B^2$$

input

```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d
*i*x)^2),x]
```

output

```
-1/3*(-3*(A^2 - 2*A*B + 2*B^2)*d*(-(b*c) + a*d)*(a + b*x) + 3*b*(A^2 + 2*A
*B + 2*B^2)*(b*c - a*d)*(c + d*x) + 6*b*(A^2 + 2*B^2)*d*(a + b*x)*(c + d*x
)*Log[a + b*x] + 6*B*(b*c - a*d)*(A*b*c + b*B*c + a*A*d - a*B*d + 2*A*b*d*
x)*Log[(e*(a + b*x))/(c + d*x)] + 3*B*(-(a^2*B*d^2) + 2*a*b*d*(-(B*d*x) +
A*(c + d*x)) + b^2*(2*A*d*x*(c + d*x) + B*c*(c + 2*d*x)))*Log[(e*(a + b*x)
)/(c + d*x)]^2 + 2*b*B^2*d*(a + b*x)*(c + d*x)*Log[(e*(a + b*x))/(c + d*x)
]^3 - 6*b*(A^2 + 2*B^2)*d*(a + b*x)*(c + d*x)*Log[c + d*x])/((b*c - a*d)^3
*g^2*i^2*(a + b*x)*(c + d*x))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2} d\frac{a+bx}{c+dx}}{g^2i^2(bc-ad)^3}$$

$$\downarrow 2795$$

$$\frac{\int \left(d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2}-\frac{2bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}\right) d\frac{a+bx}{c+dx}}{g^2i^2(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}-\frac{2b^2B(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}+\frac{d^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx}-\frac{2ABd^2(a+bx)}{c+dx}-\frac{2bd\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{c+dx}}{g^2i^2(bc-ad)^3}$$

input

```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d*i*x)^
2),x]
```

output

```
((-2*A*B*d^2*(a + b*x))/(c + d*x) + (2*B^2*d^2*(a + b*x))/(c + d*x) - (2*b
^2*B^2*(c + d*x))/(a + b*x) - (2*B^2*d^2*(a + b*x)*Log[(e*(a + b*x))/(c +
d*x)])/(c + d*x) - (2*b^2*B*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)/(a + b*x) + (d^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c +
d*x) - (b^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x) -
(2*b*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])^3)/(3*B))/((b*c - a*d)^3*g^2*i
^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(363) = 726$.

Time = 2.00 (sec) , antiderivative size = 766, normalized size of antiderivative = 2.10

method	result
parts	$\frac{A^2\left(-\frac{d}{(da-bc)^2(dx+c)}-\frac{2db\ln(dx+c)}{(da-bc)^3}-\frac{b}{(da-bc)^2(bx+a)}+\frac{2db\ln(bx+a)}{(da-bc)^3}\right)}{g^2i^2}-\frac{B^2\left(\frac{d^2\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\right.}{}\right.}{}$
derivativedivides	$-\frac{e(da-bc)\left(-\frac{d^2A^2b^2}{i^2(da-bc)^4g^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}-\frac{2d^3A^2b\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e\,i^2(da-bc)^4g^2}+\frac{d^4A^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e^2i^2(da-bc)^4g^2}+\frac{2d^2ABb^2\left(-\frac{\ln\left(\frac{be}{d}+\right.}{\frac{be}{d}+}\right.}{}$
default	$-\frac{e(da-bc)\left(-\frac{d^2A^2b^2}{i^2(da-bc)^4g^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}-\frac{2d^3A^2b\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e\,i^2(da-bc)^4g^2}+\frac{d^4A^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e^2i^2(da-bc)^4g^2}+\frac{2d^2ABb^2\left(-\frac{\ln\left(\frac{be}{d}+\right.}{\frac{be}{d}+}\right.}{}$
norman	$\frac{\left(2A^2abcd-2AB\,a^2d^2+2AB\,b^2c^2+2B^2a^2d^2+2B^2b^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{B\left(2Aabcd-B\,a^2d^2+B\,b^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{gi\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{\left(2A^2ab\,d\right.}{}$
parallelrisch	Expression too large to display
risch	Expression too large to display

input

```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,method=_RE
TURNVERBOSE)
```

output

```
1/g^2*A^2/i^2*(-d/(a*d-b*c)^2/(d*x+c)-2*d/(a*d-b*c)^3*b*ln(d*x+c)-b/(a*d-b
*c)^2/(b*x+a)+2*d/(a*d-b*c)^3*b*ln(b*x+a))-B^2/g^2/i^2/(a*d-b*c)/e*(d^2/(a
*d-b*c)^2*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2
-2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*(a*d-b*
c)*e/d/(d*x+c)+2*b*e/d)-2/3/(a*d-b*c)^2*b*d*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))^3+1/(a*d-b*c)^2*b^2*e^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e
/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))-2*A*B/g^2/i^2/(a*d-b*c)/e*(d
^2/(a*d-b*c)^2*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)-1/(a*d-b*c)^2*b*d*e*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^2+1/(a*d-b*c)^2*b^2*e^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.41

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx$$
$$= \frac{12\,ABabcd - 3\,(A^2 + 2\,AB + 2\,B^2)b^2c^2 + 3\,(A^2 - 2\,AB + 2\,B^2)a^2d^2 - 2\,(B^2b^2d^2x^2 + B^2abcd + (B^2b^2c$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, al
gorithm="fricas")
```

output
```
1/3*(12*A*B*a*b*c*d - 3*(A^2 + 2*A*B + 2*B^2)*b^2*c^2 + 3*(A^2 - 2*A*B + 2
*B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + B^2*a*b*c*d + (B^2*b^2*c*d + B^2*a*b*
d^2)*x)*log((b*e*x + a*e)/(d*x + c))^3 - 3*(2*A*B*b^2*d^2*x^2 + B^2*b^2*c^
2 + 2*A*B*a*b*c*d - B^2*a^2*d^2 + 2*((A*B + B^2)*b^2*c*d + (A*B - B^2)*a*b
*d^2)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 6*((A^2 + 2*B^2)*b^2*c*d - (A^2
+ 2*B^2)*a*b*d^2)*x - 6*((A^2 + 2*B^2)*b^2*d^2*x^2 + A^2*a*b*c*d + (A*B +
B^2)*b^2*c^2 - (A*B - B^2)*a^2*d^2 + ((A^2 + 2*A*B + 2*B^2)*b^2*c*d + (A^2
 - 2*A*B + 2*B^2)*a*b*d^2)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^4*c^3*d -
3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*g^2*i^2*x^2 + (b^4*c^4 - 2*
a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*g^2*i^2*x + (a*b^3*c^4 - 3*a^2*b^2*
c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*g^2*i^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1404 vs. $2(335) = 670$.

Time = 3.06 (sec) , antiderivative size = 1404, normalized size of antiderivative = 3.85

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2/(d*i*x+c*i)**2,x)
```

output

```
2*B**2*b*d*log(e*(a + b*x)/(c + d*x))**3/(3*a**3*d**3*g**2*i**2 - 9*a**2*b
*c*d**2*g**2*i**2 + 9*a*b**2*c**2*d*g**2*i**2 - 3*b**3*c**3*g**2*i**2) - 2
*b*d*(A**2 + 2*B**2)*log(x + (2*A**2*a*b*d**2 + 2*A**2*b**2*c*d + 4*B**2*a
*b*d**2 + 4*B**2*b**2*c*d - 2*a**4*b*d**5*(A**2 + 2*B**2)/(a*d - b*c)**3 +
 8*a**3*b**2*c*d**4*(A**2 + 2*B**2)/(a*d - b*c)**3 - 12*a**2*b**3*c**2*d**
3*(A**2 + 2*B**2)/(a*d - b*c)**3 + 8*a*b**4*c**3*d**2*(A**2 + 2*B**2)/(a*d
 - b*c)**3 - 2*b**5*c**4*d*(A**2 + 2*B**2)/(a*d - b*c)**3)/(4*A**2*b**2*d*
*2 + 8*B**2*b**2*d**2))/(g**2*i**2*(a*d - b*c)**3) + 2*b*d*(A**2 + 2*B**2)
*log(x + (2*A**2*a*b*d**2 + 2*A**2*b**2*c*d + 4*B**2*a*b*d**2 + 4*B**2*b**
2*c*d + 2*a**4*b*d**5*(A**2 + 2*B**2)/(a*d - b*c)**3 - 8*a**3*b**2*c*d**4*
(A**2 + 2*B**2)/(a*d - b*c)**3 + 12*a**2*b**3*c**2*d**3*(A**2 + 2*B**2)/(a
*d - b*c)**3 - 8*a*b**4*c**3*d**2*(A**2 + 2*B**2)/(a*d - b*c)**3 + 2*b**5*
c**4*d*(A**2 + 2*B**2)/(a*d - b*c)**3)/(4*A**2*b**2*d**2 + 8*B**2*b**2*d**
2))/(g**2*i**2*(a*d - b*c)**3) + (-2*A*B*a*d - 2*A*B*b*c - 4*A*B*b*d*x + 2
*B**2*a*d - 2*B**2*b*c)*log(e*(a + b*x)/(c + d*x))/(a**3*c*d**2*g**2*i**2
+ a**3*d**3*g**2*i**2*x - 2*a**2*b*c**2*d*g**2*i**2 - a**2*b*c*d**2*g**2*i
**2*x + a**2*b*d**3*g**2*i**2*x**2 + a*b**2*c**3*g**2*i**2 - a*b**2*c**2*d
*g**2*i**2*x - 2*a*b**2*c*d**2*g**2*i**2*x**2 + b**3*c**3*g**2*i**2*x + b*
*3*c**2*d*g**2*i**2*x**2) + (2*A*B*a*b*c*d + 2*A*B*a*b*d**2*x + 2*A*B*b**2
*c*d*x + 2*A*B*b**2*d**2*x**2 - B**2*a**2*d**2 - 2*B**2*a*b*d**2*x + B*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1995 vs. $2(363) = 726$.

Time $= 0.17$ (sec) , antiderivative size $= 1995$, normalized size of antiderivative $= 5.47$

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^2(ci + dix)^2}\, dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, al
gorithm="maxima")
```

output

```
-B^2*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i
^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*g^2*i^2*x + (a*b^
2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d*log(b*x + a)/((b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2) - 2*b*d*log(d*x + c)/
((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2))*log(b*e*x/(
d*x + c) + a*e/(d*x + c))^2 - 2*A*B*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2
*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d
^2 + a^3*d^3)*g^2*i^2*x + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2)
 + 2*b*d*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*g^2*i^2) - 2*b*d*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*g^2*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 2/3*B^2*(3*(b^2
*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*
x)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(
b*x + a)*log(d*x + c) - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*lo
g(d*x + c)^2)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(a*b^3*c^4*g^2*i^2 - 3*
a^2*b^2*c^3*d*g^2*i^2 + 3*a^3*b*c^2*d^2*g^2*i^2 - a^4*c*d^3*g^2*i^2 + (b^4
*c^3*d*g^2*i^2 - 3*a*b^3*c^2*d^2*g^2*i^2 + 3*a^2*b^2*c*d^3*g^2*i^2 - a^3*b
*d^4*g^2*i^2)*x^2 + (b^4*c^4*g^2*i^2 - 2*a*b^3*c^3*d*g^2*i^2 + 2*a^3*b*c*d
^3*g^2*i^2 - a^4*d^4*g^2*i^2)*x) + (3*b^2*c^2 - 3*a^2*d^2 + (b^2*d^2*x^2 +
 a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)^3 + 3*(b^2*d^2*x^2 + a*b...
```

Giac [A] (verification not implemented)

Time = 57.73 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.56

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx =$$
$$-\left(\frac{(dx+c)B^2e^2\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)g^2i^2} + \frac{2\,(ABe^2+B^2e^2)(dx+c)\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)g^2i^2} + \frac{(A^2e^2+2\,ABe^2+2\,B^2e^2)(d\right.$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, al
gorithm="giac")
```

output
```
-((d*x + c)*B^2*e^2*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)*g^2*i^2)
 + 2*(A*B*e^2 + B^2*e^2)*(d*x + c)*log((b*e*x + a*e)/(d*x + c))/((b*e*x +
a*e)*g^2*i^2) + (A^2*e^2 + 2*A*B*e^2 + 2*B^2*e^2)*(d*x + c)/((b*e*x + a*e)
*g^2*i^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c
- a*d)))^2
```

Mupad [B] (verification not implemented)

Time = 28.81 (sec) , antiderivative size = 731, normalized size of antiderivative = 2.00

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \frac{2B^2bd\ln\left(\frac{e(a+bx)}{c+dx}\right)^3}{3g^2i^2(ad-bc)^3}$$
$$-\frac{\frac{A^2ad+A^2bc+2B^2ad+2B^2bc-2ABad+2ABbc}{ad-bc}+\frac{2x\left(bdA^2+2bdB^2\right)}{ad-bc}}{x\left(a^2d^2g^2i^2-b^2c^2g^2i^2\right)+x^2\left(abd^2g^2i^2-b^2cdg^2i^2\right)-abc^2g^2i^2+a^2cdg^2i^2}$$
$$-\frac{\ln\left(\frac{e(a+bx)}{c+dx}\right)\left(\frac{2\left(B^2bc-B^2ad+ABad+ABbc\right)}{g^2i^2\left(a^2bd^3-2ab^2cd^2+b^3c^2d\right)}+\frac{4ABx}{g^2i^2\left(a^2d^2-2abcd+b^2c^2\right)}\right)}{x^2+\frac{x(ad+bc)}{bd}+\frac{ac}{bd}}$$
$$-\ln\left(\frac{e(a+bx)}{c+dx}\right)^2\left(\frac{\frac{B^2(ad+bc)}{g^2i^2\left(a^2bd^3-2ab^2cd^2+b^3c^2d\right)}+\frac{2B^2x}{g^2i^2\left(a^2d^2-2abcd+b^2c^2\right)}}{x^2+\frac{x(ad+bc)}{bd}+\frac{ac}{bd}}-\frac{2ABbd}{g^2i^2(ad-bc)^3}\right)$$
$$-\frac{bd\operatorname{atan}\left(\frac{bd\left(A^2+2B^2\right)\left(\frac{a^3d^3g^2i^2-a^2bcd^2g^2i^2-ab^2c^2dg^2i^2+b^3c^3g^2i^2}{a^2d^2g^2i^2-2abcdg^2i^2+b^2c^2g^2i^2}+2bdx\right)\left(a^2d^2g^2i^2-2abcdg^2i^2+b^2c^2g^2i^2\right)2\mathrm{i}}{g^2i^2(ad-bc)^3\left(2bdA^2+4bdB^2\right)}\right)(A^2-}{g^2i^2(ad-bc)^3}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^2*(c*i + d*i*x)^
2),x)
```

output

```
(2*B^2*b*d*log((e*(a + b*x))/(c + d*x))^3)/(3*g^2*i^2*(a*d - b*c)^3) - ((A
^2*a*d + A^2*b*c + 2*B^2*a*d + 2*B^2*b*c - 2*A*B*a*d + 2*A*B*b*c)/(a*d - b
*c) + (2*x*(A^2*b*d + 2*B^2*b*d))/(a*d - b*c))/(x*(a^2*d^2*g^2*i^2 - b^2*c
^2*g^2*i^2) + x^2*(a*b*d^2*g^2*i^2 - b^2*c*d*g^2*i^2) - a*b*c^2*g^2*i^2 +
a^2*c*d*g^2*i^2) - (log((e*(a + b*x))/(c + d*x))*((2*(B^2*b*c - B^2*a*d +
A*B*a*d + A*B*b*c))/(g^2*i^2*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (4
*A*B*x)/(g^2*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))))/(x^2 + (x*(a*d + b*c))
/(b*d) + (a*c)/(b*d)) - (b*d*atan((b*d*(A^2 + 2*B^2)*((a^3*d^3*g^2*i^2 + b
^3*c^3*g^2*i^2 - a*b^2*c^2*d*g^2*i^2 - a^2*b*c*d^2*g^2*i^2)/(a^2*d^2*g^2*i
^2 + b^2*c^2*g^2*i^2 - 2*a*b*c*d*g^2*i^2) + 2*b*d*x)*(a^2*d^2*g^2*i^2 + b^
2*c^2*g^2*i^2 - 2*a*b*c*d*g^2*i^2)*2i)/(g^2*i^2*(a*d - b*c)^3*(2*A^2*b*d +
 4*B^2*b*d)))*(A^2 + 2*B^2)*4i)/(g^2*i^2*(a*d - b*c)^3) - log((e*(a + b*x)
)/(c + d*x))^2*(((B^2*(a*d + b*c))/(g^2*i^2*(a^2*b*d^3 + b^3*c^2*d - 2*a*b
^2*c*d^2)) + (2*B^2*x)/(g^2*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x^2 + (
x*(a*d + b*c))/(b*d) + (a*c)/(b*d)) - (2*A*B*b*d)/(g^2*i^2*(a*d - b*c)^3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1957, normalized size of antiderivative = 5.36

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x)
```

output

```
( - 6*log(a + b*x)*a**4*b*c*d**2 - 6*log(a + b*x)*a**4*b*d**3*x - 6*log(a
+ b*x)*a**3*b**2*c**2*d - 12*log(a + b*x)*a**3*b**2*c*d**2*x + 12*log(a +
b*x)*a**3*b**2*c*d**2 - 6*log(a + b*x)*a**3*b**2*d**3*x**2 + 12*log(a + b*
x)*a**3*b**2*d**3*x - 6*log(a + b*x)*a**2*b**3*c**2*d*x - 12*log(a + b*x)*
a**2*b**3*c**2*d - 6*log(a + b*x)*a**2*b**3*c*d**2*x**2 - 12*log(a + b*x)*
a**2*b**3*c*d**2 + 12*log(a + b*x)*a**2*b**3*d**3*x**2 - 12*log(a + b*x)*a
**2*b**3*d**3*x - 12*log(a + b*x)*a*b**4*c**2*d*x - 12*log(a + b*x)*a*b**4
*c**2*d - 12*log(a + b*x)*a*b**4*c*d**2*x**2 - 24*log(a + b*x)*a*b**4*c*d*
*2*x - 12*log(a + b*x)*a*b**4*d**3*x**2 - 12*log(a + b*x)*b**5*c**2*d*x -
12*log(a + b*x)*b**5*c*d**2*x**2 + 6*log(c + d*x)*a**4*b*c*d**2 + 6*log(c
+ d*x)*a**4*b*d**3*x + 6*log(c + d*x)*a**3*b**2*c**2*d + 12*log(c + d*x)*a
**3*b**2*c*d**2*x - 12*log(c + d*x)*a**3*b**2*c*d**2 + 6*log(c + d*x)*a**3
*b**2*d**3*x**2 - 12*log(c + d*x)*a**3*b**2*d**3*x + 6*log(c + d*x)*a**2*b
**3*c**2*d*x + 12*log(c + d*x)*a**2*b**3*c**2*d + 6*log(c + d*x)*a**2*b**3
*c*d**2*x**2 + 12*log(c + d*x)*a**2*b**3*c*d**2 - 12*log(c + d*x)*a**2*b**
3*d**3*x**2 + 12*log(c + d*x)*a**2*b**3*d**3*x + 12*log(c + d*x)*a*b**4*c*
*2*d*x + 12*log(c + d*x)*a*b**4*c**2*d + 12*log(c + d*x)*a*b**4*c*d**2*x**
2 + 24*log(c + d*x)*a*b**4*c*d**2*x + 12*log(c + d*x)*a*b**4*d**3*x**2 + 1
2*log(c + d*x)*b**5*c**2*d*x + 12*log(c + d*x)*b**5*c*d**2*x**2 - 2*log((a
*e + b*e*x)/(c + d*x))**3*a**2*b**3*c*d**2 - 2*log((a*e + b*e*x)/(c + d...
```

3.98 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx$

Optimal result 1211
Mathematica [A] (verified) 1212
Rubi [A] (verified) 1212
Maple [B] (verified) 1214
Fricas [A] (verification not implemented) 1215
Sympy [B] (verification not implemented) 1216
Maxima [B] (verification not implemented) 1217
Giac [A] (verification not implemented) 1218
Mupad [B] (verification not implemented) 1219
Reduce [B] (verification not implemented) 1220

Optimal result

Integrand size = 42, antiderivative size = 523

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx &= \frac{2ABd^3(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} - \frac{2B^2d^3(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} \\ &+ \frac{6b^2B^2d(c+dx)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3B^2(c+dx)^2}{4(bc-ad)^4g^3i^2(a+bx)^2} \\ &+ \frac{2B^2d^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^4g^3i^2(c+dx)} \\ &+ \frac{6b^2Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} \\ &- \frac{b^3B(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} \\ &- \frac{d^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^3i^2(c+dx)} \\ &+ \frac{3b^2d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^3i^2(a+bx)} \\ &- \frac{b^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^3i^2(a+bx)^2} \\ &+ \frac{bd^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{B(bc-ad)^4g^3i^2}\end{aligned}$$

output

```
2*A*B*d^3*(b*x+a)/(-a*d+b*c)^4/g^3/i^2/(d*x+c)-2*B^2*d^3*(b*x+a)/(-a*d+b*c
)^4/g^3/i^2/(d*x+c)+6*b^2*B^2*d*(d*x+c)/(-a*d+b*c)^4/g^3/i^2/(b*x+a)-1/4*b
^3*B^2*(d*x+c)^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2+2*B^2*d^3*(b*x+a)*ln(e*(b*
x+a)/(d*x+c))/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+6*b^2*B*d*(d*x+c)*(A+B*ln(e*(b*
x+a)/(d*x+c)))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)-1/2*b^3*B*(d*x+c)^2*(A+B*ln(e*
(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2-d^3*(b*x+a)*(A+B*ln(e*(b*
x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+3*b^2*d*(d*x+c)*(A+B*ln(e*(b
*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)-1/2*b^3*(d*x+c)^2*(A+B*ln(e
*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2+b*d^2*(A+B*ln(e*(b*x+a
)/(d*x+c)))^3/B/(-a*d+b*c)^4/g^3/i^2
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.89

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx$$
$$=\frac{4(A^2-2AB+2B^2)\,d^2(bc-ad)(a+bx)^2-b(2A^2+2AB+B^2)\,(bc-ad)^2(c+dx)+2b(4A^2+10AB-$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d
*i*x)^2),x]
```

output
```
(4*(A^2 - 2*A*B + 2*B^2)*d^2*(b*c - a*d)*(a + b*x)^2 - b*(2*A^2 + 2*A*B +
B^2)*(b*c - a*d)^2*(c + d*x) + 2*b*(4*A^2 + 10*A*B + 11*B^2)*d*(b*c - a*d)
*(a + b*x)*(c + d*x) + 6*b*(2*A^2 + 2*A*B + 5*B^2)*d^2*(a + b*x)^2*(c + d*
x)*Log[a + b*x] + 2*B*(b*c - a*d)*(4*(A - B)*d^2*(a + b*x)^2 - b*(2*A + B)
*(b*c - a*d)*(c + d*x) + 2*b*(4*A + 5*B)*d*(a + b*x)*(c + d*x))*Log[(e*(a
+ b*x))/(c + d*x)] - 2*B*(2*a^3*B*d^3 - 6*a^2*b*d^2*(-(B*d*x) + A*(c + d*x
)) - 6*a*b^2*d*(2*A*d*x*(c + d*x) + B*c*(c + 2*d*x)) + b^3*(-6*A*d^2*x^2*(
c + d*x) + B*(c^3 - 3*c^2*d*x - 9*c*d^2*x^2 - 3*d^3*x^3)))*Log[(e*(a + b*x
))/(c + d*x)]^2 + 4*b*B^2*d^2*(a + b*x)^2*(c + d*x)*Log[(e*(a + b*x))/(c +
 d*x)]^3 - 6*b*(2*A^2 + 2*A*B + 5*B^2)*d^2*(a + b*x)^2*(c + d*x)*Log[c + d
*x])/(4*(b*c - a*d)^4*g^3*i^2*(a + b*x)^2*(c + d*x))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3i^2(bc-ad)^4}$$

$$\downarrow 2795$$

$$\frac{\int\left(-\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2d^3+\frac{3b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2d^2}{a+bx}-\frac{3b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2d}{(a+bx)^2}+\frac{b^3(c+dx)^3\left(A+B\right.}{(a+b}}{g^3i^2(bc-ad)^4}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^3(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}-\frac{b^3B(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}+\frac{6b^2Bd(c+dx)\left(B\log\left(\frac{e}{}\right.\right.}{a+bx}$$

input

```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d*i*x)^
2),x]
```

output

```
((2*A*B*d^3*(a + b*x))/(c + d*x) - (2*B^2*d^3*(a + b*x))/(c + d*x) + (6*b^
2*B^2*d*(c + d*x))/(a + b*x) - (b^3*B^2*(c + d*x)^2)/(4*(a + b*x)^2) + (2*
B^2*d^3*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(c + d*x) + (6*b^2*B*d*(c
+ d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) - (b^3*B*(c + d*x)^
2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^2) - (d^3*(a + b*x)*(
A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c + d*x) + (3*b^2*d*(c + d*x)*(A +
 B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x) - (b^3*(c + d*x)^2*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)])^2)/(2*(a + b*x)^2) + (b*d^2*(A + B*Log[(e*(a +
 b*x))/(c + d*x)])^3)/B)/((b*c - a*d)^4*g^3*i^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(517) = 1034$.

Time = 3.48 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.07

method	result	size
parts	Expression too large to display	1080
derivativedivides	Expression too large to display	1228
default	Expression too large to display	1228
risch	Expression too large to display	1736
parallelrisch	Expression too large to display	1748
norman	Expression too large to display	1849

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,method=_RE
TURNVERBOSE)
```

output

```
A^2/g^3/i^2*(-d^2/(a*d-b*c)^3/(d*x+c)-3*d^2/(a*d-b*c)^4*b*ln(d*x+c)-1/2*b/
(a*d-b*c)^2/(b*x+a)^2+3*d^2/(a*d-b*c)^4*b*ln(b*x+a)-2*b/(a*d-b*c)^3*d/(b*x
+a))-B^2/g^3/i^2/(a*d-b*c)/e*(d^3/(a*d-b*c)^3*((b*e/d+(a*d-b*c)*e/d/(d*x+c
))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))+2*(a*d-b*c)*e/d/(d*x+c)+2*b*e/d)-1/(a*d-b*c)^3*
b*d^2*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3+3/(a*d-b*c)^3*b^2*d*e^2*(-1/(b*e
/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-
b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(
d*x+c)))-1/(a*d-b*c)^3*b^3*e^3*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))-2*A*B/g^3/i^2
/(a*d-b*c)/e*(d^3/(a*d-b*c)^3*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d
-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)-3/2/(a*d-b*c)^3*b*d^2*e*ln
(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+3/(a*d-b*c)^3*b^2*d*e^2*(-1/(b*e/d+(a*d-b*
c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*
x+c)))-1/(a*d-b*c)^3*b^3*e^3*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/
d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1005, normalized size of antiderivative = 1.92

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, al
gorithm="fricas")
```

output

```
-1/4*((2*A^2 + 2*A*B + B^2)*b^3*c^3 - 12*(A^2 + 2*A*B + 2*B^2)*a*b^2*c^2*d
 + 3*(2*A^2 + 10*A*B + 5*B^2)*a^2*b*c*d^2 + 4*(A^2 - 2*A*B + 2*B^2)*a^3*d^
3 - 4*(B^2*b^3*d^3*x^3 + B^2*a^2*b*c*d^2 + (B^2*b^3*c*d^2 + 2*B^2*a*b^2*d^
3)*x^2 + (2*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c
))^3 - 6*((2*A^2 + 2*A*B + 5*B^2)*b^3*c*d^2 - (2*A^2 + 2*A*B + 5*B^2)*a*b^
2*d^3)*x^2 - 2*(3*(2*A*B + B^2)*b^3*d^3*x^3 - B^2*b^3*c^3 + 6*B^2*a*b^2*c^
2*d + 6*A*B*a^2*b*c*d^2 - 2*B^2*a^3*d^3 + 3*(4*A*B*a*b^2*d^3 + (2*A*B + 3*
B^2)*b^3*c*d^2)*x^2 + 3*(B^2*b^3*c^2*d + 4*(A*B + B^2)*a*b^2*c*d^2 + 2*(A*
B - B^2)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 3*((2*A^2 + 6*A*B
+ 7*B^2)*b^3*c^2*d + 2*(2*A^2 - 2*A*B + 3*B^2)*a*b^2*c*d^2 - (6*A^2 + 2*A*
B + 13*B^2)*a^2*b*d^3)*x - 2*(3*(2*A^2 + 2*A*B + 5*B^2)*b^3*d^3*x^3 + 6*A^
2*a^2*b*c*d^2 - (2*A*B + B^2)*b^3*c^3 + 12*(A*B + B^2)*a*b^2*c^2*d - 4*(A*
B - B^2)*a^3*d^3 + 3*((2*A^2 + 6*A*B + 7*B^2)*b^3*c*d^2 + 4*(A^2 + 2*B^2)*
a*b^2*d^3)*x^2 + 3*((2*A*B + 3*B^2)*b^3*c^2*d + 4*(A^2 + 2*A*B + 2*B^2)*a*
b^2*c*d^2 + 2*(A^2 - 2*A*B + 2*B^2)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x +
 c)))/((b^6*c^4*d - 4*a*b^5*c^3*d^2 + 6*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*d^4
+ a^4*b^2*d^5)*g^3*i^2*x^3 + (b^6*c^5 - 2*a*b^5*c^4*d - 2*a^2*b^4*c^3*d^2
+ 8*a^3*b^3*c^2*d^3 - 7*a^4*b^2*c*d^4 + 2*a^5*b*d^5)*g^3*i^2*x^2 + (2*a*b^
5*c^5 - 7*a^2*b^4*c^4*d + 8*a^3*b^3*c^3*d^2 - 2*a^4*b^2*c^2*d^3 - 2*a^5*b*
c*d^4 + a^6*d^5)*g^3*i^2*x + (a^2*b^4*c^5 - 4*a^3*b^3*c^4*d + 6*a^4*b^2...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2683 vs. $2(483) = 966$.

Time $= 27.33$ (sec) , antiderivative size $= 2683$, normalized size of antiderivative $= 5.13$

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3/(d*i*x+c*i)**2,x)
```

output

```
B**2*b*d**2*log(e*(a + b*x)/(c + d*x))**3/(a**4*d**4*g**3*i**2 - 4*a**3*b*
c*d**3*g**3*i**2 + 6*a**2*b**2*c**2*d**2*g**3*i**2 - 4*a*b**3*c**3*d*g**3*
i**2 + b**4*c**4*g**3*i**2) - 3*b*d**2*(2*A**2 + 2*A*B + 5*B**2)*log(x + (
6*A**2*a*b*d**3 + 6*A**2*b**2*c*d**2 + 6*A*B*a*b*d**3 + 6*A*B*b**2*c*d**2
+ 15*B**2*a*b*d**3 + 15*B**2*b**2*c*d**2 - 3*a**5*b*d**7*(2*A**2 + 2*A*B +
 5*B**2)/(a*d - b*c)**4 + 15*a**4*b**2*c*d**6*(2*A**2 + 2*A*B + 5*B**2)/(a
*d - b*c)**4 - 30*a**3*b**3*c**2*d**5*(2*A**2 + 2*A*B + 5*B**2)/(a*d - b*c
)**4 + 30*a**2*b**4*c**3*d**4*(2*A**2 + 2*A*B + 5*B**2)/(a*d - b*c)**4 - 1
5*a*b**5*c**4*d**3*(2*A**2 + 2*A*B + 5*B**2)/(a*d - b*c)**4 + 3*b**6*c**5*
d**2*(2*A**2 + 2*A*B + 5*B**2)/(a*d - b*c)**4)/(12*A**2*b**2*d**3 + 12*A*B
*b**2*d**3 + 30*B**2*b**2*d**3))/(2*g**3*i**2*(a*d - b*c)**4) + 3*b*d**2*(
2*A**2 + 2*A*B + 5*B**2)*log(x + (6*A**2*a*b*d**3 + 6*A**2*b**2*c*d**2 + 6
*A*B*a*b*d**3 + 6*A*B*b**2*c*d**2 + 15*B**2*a*b*d**3 + 15*B**2*b**2*c*d**2
 + 3*a**5*b*d**7*(2*A**2 + 2*A*B + 5*B**2)/(a*d - b*c)**4 - 15*a**4*b**2*c
*d**6*(2*A**2 + 2*A*B + 5*B**2)/(a*d - b*c)**4 + 30*a**3*b**3*c**2*d**5*(2
*A**2 + 2*A*B + 5*B**2)/(a*d - b*c)**4 - 30*a**2*b**4*c**3*d**4*(2*A**2 +
2*A*B + 5*B**2)/(a*d - b*c)**4 + 15*a*b**5*c**4*d**3*(2*A**2 + 2*A*B + 5*B
**2)/(a*d - b*c)**4 - 3*b**6*c**5*d**2*(2*A**2 + 2*A*B + 5*B**2)/(a*d - b*
c)**4)/(12*A**2*b**2*d**3 + 12*A*B*b**2*d**3 + 30*B**2*b**2*d**3))/(2*g**3
*i**2*(a*d - b*c)**4) + (-4*A*B*a**2*d**2 - 10*A*B*a*b*c*d - 18*A*B*a*b...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4187 vs. $2(517) = 1034$.

Time $= 0.39$ (sec) , antiderivative size $= 4187$, normalized size of antiderivative $= 8.01$

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, al
gorithm="maxima")
```

output

```
1/2*B^2*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3
*a*b*d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4
)*g^3*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d
^3 - 2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2
*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3
*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^
4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2)
- 6*b*d^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a
^3*b*c*d^3 + a^4*d^4)*g^3*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2 + A
*B*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*b*
d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*g^3
*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 -
2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*
d^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d +
3*a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) - 6*b
*d^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*
c*d^3 + a^4*d^4)*g^3*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/4*B^2*
(2*(b^3*c^3 - 12*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3 - 6*(b^3*c*d^2 -
 a*b^2*d^3)*x^2 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d...
```

Giac [A] (verification not implemented)

Time = 71.57 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.88

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx =$$
$$-\frac{1}{4}\left(\frac{2\left(B^2be^3 - \frac{2\,(bex+ae)B^2de^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^2bcg^3i^2}{(dx+c)^2} - \frac{(bex+ae)^2adg^3i^2}{(dx+c)^2}} + \frac{2\left(2\,ABbe^3 + B^2be^3 - \frac{4\,(bex+ae)ABde^2}{dx+c} - \frac{4\,(bex+ae)B^2de^2}{dx+c}\right)}{\frac{(bex+ae)^2bcg^3i^2}{(dx+c)^2} - \frac{(bex+ae)^2adg^3i^2}{(dx+c)^2}}\right.$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, al
gorithm="giac")
```

output

```
-1/4*(2*(B^2*b*e^3 - 2*(b*e*x + a*e)*B^2*d*e^2/(d*x + c))*log((b*e*x + a*e
)/(d*x + c))^2/((b*e*x + a*e)^2*b*c*g^3*i^2/(d*x + c)^2 - (b*e*x + a*e)^2*
a*d*g^3*i^2/(d*x + c)^2) + 2*(2*A*B*b*e^3 + B^2*b*e^3 - 4*(b*e*x + a*e)*A*
B*d*e^2/(d*x + c) - 4*(b*e*x + a*e)*B^2*d*e^2/(d*x + c))*log((b*e*x + a*e)
/(d*x + c))/((b*e*x + a*e)^2*b*c*g^3*i^2/(d*x + c)^2 - (b*e*x + a*e)^2*a*d
*g^3*i^2/(d*x + c)^2) + (2*A^2*b*e^3 + 2*A*B*b*e^3 + B^2*b*e^3 - 4*(b*e*x
+ a*e)*A^2*d*e^2/(d*x + c) - 8*(b*e*x + a*e)*A*B*d*e^2/(d*x + c) - 8*(b*e*
x + a*e)*B^2*d*e^2/(d*x + c))/((b*e*x + a*e)^2*b*c*g^3*i^2/(d*x + c)^2 - (
b*e*x + a*e)^2*a*d*g^3*i^2/(d*x + c)^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)
) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [B] (verification not implemented)

Time = 32.86 (sec) , antiderivative size = 1497, normalized size of antiderivative = 2.86

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^3*(c*i + d*i*x)^
2),x)
```

output

```
(B^2*b*d^2*log((e*(a + b*x))/(c + d*x))^3)/(g^3*i^2*(a*d - b*c)^2*(a^2*d^2
 + b^2*c^2 - 2*a*b*c*d)) - ((4*A^2*a^2*d^2 - 2*A^2*b^2*c^2 + 8*B^2*a^2*d^2
 - B^2*b^2*c^2 - 8*A*B*a^2*d^2 - 2*A*B*b^2*c^2 + 10*A^2*a*b*c*d + 23*B^2*a
*b*c*d + 22*A*B*a*b*c*d)/(2*(a*d - b*c)) + (3*x^2*(2*A^2*b^2*d^2 + 5*B^2*b
^2*d^2 + 2*A*B*b^2*d^2))/(a*d - b*c) + (3*x*(6*A^2*a*b*d^2 + 13*B^2*a*b*d^
2 + 2*A^2*b^2*c*d + 7*B^2*b^2*c*d + 2*A*B*a*b*d^2 + 6*A*B*b^2*c*d))/(2*(a*
d - b*c)))/(x*(2*a^4*d^3*g^3*i^2 + 4*a*b^3*c^3*g^3*i^2 - 6*a^2*b^2*c^2*d*g
^3*i^2) + x^2*(2*b^4*c^3*g^3*i^2 + 4*a^3*b*d^3*g^3*i^2 - 6*a^2*b^2*c*d^2*g
^3*i^2) + x^3*(2*a^2*b^2*d^3*g^3*i^2 + 2*b^4*c^2*d*g^3*i^2 - 4*a*b^3*c*d^2
*g^3*i^2) + 2*a^2*b^2*c^3*g^3*i^2 + 2*a^4*c*d^2*g^3*i^2 - 4*a^3*b*c^2*d*g^
3*i^2) - (log((e*(a + b*x))/(c + d*x))*((B^2*b*c - 4*B^2*a*d + 4*A*B*a*d +
 2*A*B*b*c)/(2*g^3*i^2*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - x*((3*(B
^2 - 2*A*B))/(2*g^3*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (3*B*(2*A + B)*
(a*d + b*c))/(g^3*i^2*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) + (3*B
*a*c*(2*A + B))/(g^3*i^2*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (3
*B*b*d*x^2*(2*A + B))/(g^3*i^2*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)
)))/(b*x^3 + (a^2*c)/(b*d) + (x^2*(b^2*c + 2*a*b*d))/(b*d) + (x*(a^2*d + 2
*a*b*c))/(b*d)) - (b*d^2*atan((b*d^2*(2*A^2 + 5*B^2 + 2*A*B)*(2*a^4*d^4*g^
3*i^2 - 2*b^4*c^4*g^3*i^2 + 4*a*b^3*c^3*d*g^3*i^2 - 4*a^3*b*c*d^3*g^3*i^2)
*3i)/(2*g^3*i^2*(a*d - b*c)^4*(6*A^2*b*d^2 + 15*B^2*b*d^2 + 6*A*B*b*d^2...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3290, normalized size of antiderivative = 6.29

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x)
```

output

```
( - 24*log(a + b*x)*a**5*b*c*d**3 - 24*log(a + b*x)*a**5*b*d**4*x - 12*log
(a + b*x)*a**4*b**2*c**2*d**2 - 60*log(a + b*x)*a**4*b**2*c*d**3*x - 48*lo
g(a + b*x)*a**4*b**2*d**4*x**2 - 24*log(a + b*x)*a**3*b**3*c**2*d**2*x - 3
6*log(a + b*x)*a**3*b**3*c**2*d**2 - 48*log(a + b*x)*a**3*b**3*c*d**3*x**2
 - 36*log(a + b*x)*a**3*b**3*c*d**3*x - 48*log(a + b*x)*a**3*b**3*c*d**3 -
 24*log(a + b*x)*a**3*b**3*d**4*x**3 - 48*log(a + b*x)*a**3*b**3*d**4*x -
12*log(a + b*x)*a**2*b**4*c**2*d**2*x**2 - 72*log(a + b*x)*a**2*b**4*c**2*
d**2*x - 42*log(a + b*x)*a**2*b**4*c**2*d**2 - 12*log(a + b*x)*a**2*b**4*c
*d**3*x**3 - 72*log(a + b*x)*a**2*b**4*c*d**3*x**2 - 138*log(a + b*x)*a**2
*b**4*c*d**3*x - 96*log(a + b*x)*a**2*b**4*d**4*x**2 - 36*log(a + b*x)*a*b
**5*c**2*d**2*x**2 - 84*log(a + b*x)*a*b**5*c**2*d**2*x - 36*log(a + b*x)*
a*b**5*c*d**3*x**3 - 132*log(a + b*x)*a*b**5*c*d**3*x**2 - 48*log(a + b*x)
*a*b**5*d**4*x**3 - 42*log(a + b*x)*b**6*c**2*d**2*x**2 - 42*log(a + b*x)*
b**6*c*d**3*x**3 + 24*log(c + d*x)*a**5*b*c*d**3 + 24*log(c + d*x)*a**5*b*
d**4*x + 12*log(c + d*x)*a**4*b**2*c**2*d**2 + 60*log(c + d*x)*a**4*b**2*c
*d**3*x + 48*log(c + d*x)*a**4*b**2*d**4*x**2 + 24*log(c + d*x)*a**3*b**3*
c**2*d**2*x + 36*log(c + d*x)*a**3*b**3*c**2*d**2 + 48*log(c + d*x)*a**3*b
**3*c*d**3*x**2 + 36*log(c + d*x)*a**3*b**3*c*d**3*x + 48*log(c + d*x)*a**
3*b**3*c*d**3 + 24*log(c + d*x)*a**3*b**3*d**4*x**3 + 48*log(c + d*x)*a**3
*b**3*d**4*x + 12*log(c + d*x)*a**2*b**4*c**2*d**2*x**2 + 72*log(c + d*...
```

3.99 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx$

Optimal result 1223
Mathematica [A] (verified) 1224
Rubi [A] (verified) 1225
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Giac [A] (verification not implemented) 1231
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Reduce [B] (verification not implemented) 1232

Optimal result

Integrand size = 42, antiderivative size = 682

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = &-\frac{2ABd^4(a+bx)}{(bc-ad)^5g^4i^2(c+dx)}+\frac{2B^2d^4(a+bx)}{(bc-ad)^5g^4i^2(c+dx)}\\ &-\frac{12b^2B^2d^2(c+dx)}{(bc-ad)^5g^4i^2(a+bx)}+\frac{b^3B^2d(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2}\\ &-\frac{2b^4B^2(c+dx)^3}{27(bc-ad)^5g^4i^2(a+bx)^3}\\ &-\frac{2B^2d^4(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^5g^4i^2(c+dx)}\\ &-\frac{12b^2Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)}\\ &+\frac{2b^3Bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2}\\ &-\frac{2b^4B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^5g^4i^2(a+bx)^3}\\ &+\frac{d^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(c+dx)}\\ &-\frac{6b^2d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)}\\ &+\frac{2b^3d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)^2}\\ &-\frac{b^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^5g^4i^2(a+bx)^3}\\ &-\frac{4bd^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^5g^4i^2}\end{aligned}$$

output

```
-2*A*B*d^4*(b*x+a)/(-a*d+b*c)^5/g^4/i^2/(d*x+c)+2*B^2*d^4*(b*x+a)/(-a*d+b*
c)^5/g^4/i^2/(d*x+c)-12*b^2*B^2*d^2*(d*x+c)/(-a*d+b*c)^5/g^4/i^2/(b*x+a)+b
^3*B^2*d*(d*x+c)^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-2/27*b^4*B^2*(d*x+c)^3/(
-a*d+b*c)^5/g^4/i^2/(b*x+a)^3-2*B^2*d^4*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*
d+b*c)^5/g^4/i^2/(d*x+c)-12*b^2*B*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/
(-a*d+b*c)^5/g^4/i^2/(b*x+a)+2*b^3*B*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)
))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-2/9*b^4*B*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d
*x+c)))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3+d^4*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+
c)))^2/(-a*d+b*c)^5/g^4/i^2/(d*x+c)-6*b^2*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d
*x+c)))^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)+2*b^3*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)
/(d*x+c)))^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-1/3*b^4*(d*x+c)^3*(A+B*ln(e*(b
*x+a)/(d*x+c)))^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3-4/3*b*d^3*(A+B*ln(e*(b*x+
a)/(d*x+c)))^3/B/(-a*d+b*c)^5/g^4/i^2
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 613, normalized size of antiderivative = 0.90

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx =$$

$$-\frac{-27(A^2-2AB+2B^2)\,d^3(-bc+ad)(a+bx)^3+b(9A^2+6AB+2B^2)\,(bc-ad)^3(c+dx)-3b(9A^2-$$

input

```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d
*i*x)^2),x]
```

output

```
-1/27*(-27*(A^2 - 2*A*B + 2*B^2)*d^3*(-(b*c) + a*d)*(a + b*x)^3 + b*(9*A^2
 + 6*A*B + 2*B^2)*(b*c - a*d)^3*(c + d*x) - 3*b*(9*A^2 + 12*A*B + 7*B^2)*d
*(b*c - a*d)^2*(a + b*x)*(c + d*x) + 3*b*(27*A^2 + 78*A*B + 92*B^2)*d^2*(b
*c - a*d)*(a + b*x)^2*(c + d*x) + 6*b*(18*A^2 + 30*A*B + 55*B^2)*d^3*(a +
b*x)^3*(c + d*x)*Log[a + b*x] + 6*B*(b*c - a*d)*(9*(A - B)*d^3*(a + b*x)^3
 + b*(3*A + B)*(b*c - a*d)^2*(c + d*x) - 3*b*(3*A + 2*B)*d*(b*c - a*d)*(a
+ b*x)*(c + d*x) + 3*b*(9*A + 13*B)*d^2*(a + b*x)^2*(c + d*x))*Log[(e*(a +
 b*x))/(c + d*x)] + 9*B*(-3*a^4*B*d^4 + 12*a^3*b*d^3*(-(B*d*x) + A*(c + d*
x)) + 18*a^2*b^2*d^2*(2*A*d*x*(c + d*x) + B*c*(c + 2*d*x)) + 6*a*b^3*d*(6*
A*d^2*x^2*(c + d*x) + B*(-c^3 + 3*c^2*d*x + 9*c*d^2*x^2 + 3*d^3*x^3)) + b^
4*(12*A*d^3*x^3*(c + d*x) + B*(c^4 - 2*c^3*d*x + 6*c^2*d^2*x^2 + 22*c*d^3*
x^3 + 10*d^4*x^4)))*Log[(e*(a + b*x))/(c + d*x)]^2 + 36*b*B^2*d^3*(a + b*x
)^3*(c + d*x)*Log[(e*(a + b*x))/(c + d*x)]^3 - 6*b*(18*A^2 + 30*A*B + 55*B
^2)*d^3*(a + b*x)^3*(c + d*x)*Log[c + d*x])/((b*c - a*d)^5*g^4*i^2*(a + b*
x)^3*(c + d*x))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 475, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^4} d\frac{a+bx}{c+dx}}{g^4 i^2 (bc-ad)^5}$$

$$\downarrow 2795$$

$$\int \left(\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 d^4 - \frac{4b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 d^3}{a+bx} + \frac{6b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 d^2}{(a+bx)^2} - \frac{4b^3(c+dx)^3\left(A+B\right.}{(a+b}$$

$$g^4 i^2 (bc-ad)^5$$

$$\downarrow 2009$$

$$-\frac{b^4(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3}-\frac{2b^4B(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9(a+bx)^3}+\frac{2b^3d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(a+bx)^2}+\frac{2b^3Bd(c+dx)^2\left(B\log\right.}{(a+bx)}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d*i*x)^
2),x]
```

output
```
((-2*A*B*d^4*(a + b*x))/(c + d*x) + (2*B^2*d^4*(a + b*x))/(c + d*x) - (12*
b^2*B^2*d^2*(c + d*x))/(a + b*x) + (b^3*B^2*d*(c + d*x)^2)/(a + b*x)^2 - (
2*b^4*B^2*(c + d*x)^3)/(27*(a + b*x)^3) - (2*B^2*d^4*(a + b*x)*Log[(e*(a +
 b*x))/(c + d*x)])/(c + d*x) - (12*b^2*B*d^2*(c + d*x)*(A + B*Log[(e*(a +
b*x))/(c + d*x)]))/(a + b*x) + (2*b^3*B*d*(c + d*x)^2*(A + B*Log[(e*(a + b
*x))/(c + d*x)]))/(a + b*x)^2 - (2*b^4*B*(c + d*x)^3*(A + B*Log[(e*(a + b*
x))/(c + d*x)]))/(9*(a + b*x)^3) + (d^4*(a + b*x)*(A + B*Log[(e*(a + b*x))
/(c + d*x)])^2)/(c + d*x) - (6*b^2*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/
(c + d*x)])^2)/(a + b*x) + (2*b^3*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(
c + d*x)])^2)/(a + b*x)^2 - (b^4*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c +
 d*x)])^2)/(3*(a + b*x)^3) - (4*b*d^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])
^3)/(3*B))/((b*c - a*d)^5*g^4*i^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1385 vs. $2(674) = 1348$.

Time $= 3.59$ (sec) , antiderivative size $= 1386$, normalized size of antiderivative $= 2.03$

method	result	size
parts	Expression too large to display	1386
derivativedivides	Expression too large to display	1588
default	Expression too large to display	1588
risch	Expression too large to display	2173
parallelrisch	Expression too large to display	2680
norman	Expression too large to display	2748

input

```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,method=_RE
TURNVERBOSE)
```

output

```
A^2/g^4/i^2*(-d^3/(a*d-b*c)^4/(d*x+c)-4*d^3/(a*d-b*c)^5*b*ln(d*x+c)-1/3*b/
(a*d-b*c)^2/(b*x+a)^3+4*d^3/(a*d-b*c)^5*b*ln(b*x+a)-3*b/(a*d-b*c)^4*d^2/(b
*x+a)-b/(a*d-b*c)^3*d/(b*x+a)^2)-B^2/g^4/i^2/(a*d-b*c)/e*(d^4/(a*d-b*c)^4*
((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*(b*e/d+
(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*(a*d-b*c)*e/d/(d*
x+c)+2*b*e/d)-4/3/(a*d-b*c)^4*b*d^3*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3+6/
(a*d-b*c)^4*b^2*d^2*e^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*
c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-4/(a*d-b*c)^4*b^3*d*e^3*(-1/2/(b*
e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^2)+1/(a*d-b*c)^4*b^4*e^4*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+
c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))-2
*A*B/g^4/i^2/(a*d-b*c)/e*(d^4/(a*d-b*c)^4*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)-2/(a*d-b*c)^4*
b*d^3*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+6/(a*d-b*c)^4*b^2*d^2*e^2*(-1/(b
*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-
b*c)*e/d/(d*x+c)))-4/(a*d-b*c)^4*b^3*d*e^3*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1534 vs. $2(674) = 1348$.

Time = 0.12 (sec) , antiderivative size = 1534, normalized size of antiderivative = 2.25

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, al
gorithm="fricas")
```

output

```
-1/27*((9*A^2 + 6*A*B + 2*B^2)*b^4*c^4 - 27*(2*A^2 + 2*A*B + B^2)*a*b^3*c^
3*d + 162*(A^2 + 2*A*B + 2*B^2)*a^2*b^2*c^2*d^2 - 5*(18*A^2 + 66*A*B + 49*
B^2)*a^3*b*c*d^3 - 27*(A^2 - 2*A*B + 2*B^2)*a^4*d^4 + 6*((18*A^2 + 30*A*B
+ 55*B^2)*b^4*c*d^3 - (18*A^2 + 30*A*B + 55*B^2)*a*b^3*d^4)*x^3 + 36*(B^2*
b^4*d^4*x^4 + B^2*a^3*b*c*d^3 + (B^2*b^4*c*d^3 + 3*B^2*a*b^3*d^4)*x^3 + 3*
(B^2*a*b^3*c*d^3 + B^2*a^2*b^2*d^4)*x^2 + (3*B^2*a^2*b^2*c*d^3 + B^2*a^3*b
*d^4)*x)*log((b*e*x + a*e)/(d*x + c))^3 + 3*((18*A^2 + 66*A*B + 85*B^2)*b^
4*c^2*d^2 + 8*(9*A^2 + 6*A*B + 20*B^2)*a*b^3*c*d^3 - (90*A^2 + 114*A*B + 2
45*B^2)*a^2*b^2*d^4)*x^2 + 9*(2*(6*A*B + 5*B^2)*b^4*d^4*x^4 + B^2*b^4*c^4
- 6*B^2*a*b^3*c^3*d + 18*B^2*a^2*b^2*c^2*d^2 + 12*A*B*a^3*b*c*d^3 - 3*B^2*
a^4*d^4 + 2*((6*A*B + 11*B^2)*b^4*c*d^3 + 9*(2*A*B + B^2)*a*b^3*d^4)*x^3 +
 6*(B^2*b^4*c^2*d^2 + 6*A*B*a^2*b^2*d^4 + 3*(2*A*B + 3*B^2)*a*b^3*c*d^3)*x
^2 - 2*(B^2*b^4*c^3*d - 9*B^2*a*b^3*c^2*d^2 - 18*(A*B + B^2)*a^2*b^2*c*d^3
 - 6*(A*B - B^2)*a^3*b*d^4)*x)*log((b*e*x + a*e)/(d*x + c))^2 - ((18*A^2 +
 30*A*B + 19*B^2)*b^4*c^3*d - 81*(2*A^2 + 6*A*B + 7*B^2)*a*b^3*c^2*d^2 - 3
*(18*A^2 - 114*A*B - 29*B^2)*a^2*b^2*c*d^3 + (198*A^2 + 114*A*B + 461*B^2)
*a^3*b*d^4)*x + 6*((18*A^2 + 30*A*B + 55*B^2)*b^4*d^4*x^4 + 18*A^2*a^3*b*c
*d^3 + (3*A*B + B^2)*b^4*c^4 - 9*(2*A*B + B^2)*a*b^3*c^3*d + 54*(A*B + B^2
)*a^2*b^2*c^2*d^2 - 9*(A*B - B^2)*a^4*d^4 + ((18*A^2 + 66*A*B + 85*B^2)*b^
4*c*d^3 + 27*(2*A^2 + 2*A*B + 5*B^2)*a*b^3*d^4)*x^3 + 3*((6*A*B + 11*B^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Timed out}$$

input

```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4/(d*i*x+c*i)**2,x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6160 vs. $2(674) = 1348$.

Time $= 0.57$ (sec) , antiderivative size $= 6160$, normalized size of antiderivative $= 9.03$

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, al
gorithm="maxima")
```

output

```
-1/3*B^2*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 13*a^2*b*c*d^2 + 3*a
^3*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 8*a*b^2*c*d^2 -
11*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3
*b^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5
*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*
x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d
^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^
4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*
g^4*i^2*x + (a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c
^2*d^3 + a^7*c*d^4)*g^4*i^2) + 12*b*d^3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c
^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*
g^4*i^2) - 12*b*d^3*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^
3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2))*log(b*e*x/
(d*x + c) + a*e/(d*x + c))^2 - 2/3*A*B*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^
2*c^2*d + 13*a^2*b*c*d^2 + 3*a^3*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2
*(b^3*c^2*d - 8*a*b^2*c*d^2 - 11*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d
^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7
*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c
*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3
*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2...
```

Giac [A] (verification not implemented)

Time = 86.56 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.11

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, al
gorithm="giac")
```

output
```
-1/54*(18*(B^2*b^2*e^4 - 3*(b*e*x + a*e)*B^2*b*d*e^3/(d*x + c) + 3*(b*e*x
 + a*e)^2*B^2*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x +
 a*e)^3*b^2*c^2*g^4*i^2/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4*i^2/(d
*x + c)^3 + (b*e*x + a*e)^3*a^2*d^2*g^4*i^2/(d*x + c)^3) + 6*(6*A*B*b^2*e^
4 + 2*B^2*b^2*e^4 - 18*(b*e*x + a*e)*A*B*b*d*e^3/(d*x + c) - 9*(b*e*x + a*
e)*B^2*b*d*e^3/(d*x + c) + 18*(b*e*x + a*e)^2*A*B*d^2*e^2/(d*x + c)^2 + 18
*(b*e*x + a*e)^2*B^2*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b
*e*x + a*e)^3*b^2*c^2*g^4*i^2/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4*
i^2/(d*x + c)^3 + (b*e*x + a*e)^3*a^2*d^2*g^4*i^2/(d*x + c)^3) + (18*A^2*b
^2*e^4 + 12*A*B*b^2*e^4 + 4*B^2*b^2*e^4 - 54*(b*e*x + a*e)*A^2*b*d*e^3/(d*
x + c) - 54*(b*e*x + a*e)*A*B*b*d*e^3/(d*x + c) - 27*(b*e*x + a*e)*B^2*b*d
*e^3/(d*x + c) + 54*(b*e*x + a*e)^2*A^2*d^2*e^2/(d*x + c)^2 + 108*(b*e*x +
 a*e)^2*A*B*d^2*e^2/(d*x + c)^2 + 108*(b*e*x + a*e)^2*B^2*d^2*e^2/(d*x + c
)^2)/((b*e*x + a*e)^3*b^2*c^2*g^4*i^2/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*
c*d*g^4*i^2/(d*x + c)^3 + (b*e*x + a*e)^3*a^2*d^2*g^4*i^2/(d*x + c)^3))*(b
*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [B] (verification not implemented)

Time = 35.65 (sec) , antiderivative size = 2701, normalized size of antiderivative = 3.96

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^4*(c*i + d*i*x)^
2),x)
```

output

```
(log((e*(a + b*x))/(c + d*x))*(x^2*((4*B^2*b*d)/(g^4*i^2*(a*d - b*c)^3) -
(4*b*d^3*(b*d*((2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c
))/(2*b*d^2)) + ((a*d + b*c)*(a*d - b*c))/d^2)*(5*B^2 + 6*A*B))/(3*g^4*i^2
*(a*d - b*c)^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) + x*(
(8*(2*B^2 - 3*A*B))/(9*g^4*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (4*B^2*(
a*d + b*c))/(g^4*i^2*(a*d - b*c)^3) - (4*b*d^3*(((2*a^2*d^2 + b^2*c^2 - 3*
a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2))*(a*d + b*c) + (a*c*(a*d -
b*c))/d^2)*(5*B^2 + 6*A*B))/(3*g^4*i^2*(a*d - b*c)^2*(a^3*d^3 - b^3*c^3 +
3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (2*(B^2*b*c - 9*B^2*a*d + 9*A*B*a*d + 3
*A*B*b*c))/(9*g^4*i^2*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (4*B^2*a*
c)/(g^4*i^2*(a*d - b*c)^3) - (4*b^2*d^2*x^3*(5*B^2 + 6*A*B))/(3*g^4*i^2*(a
*d - b*c)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (4*a*b*c*
d^3*((2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^
2))*(5*B^2 + 6*A*B))/(3*g^4*i^2*(a*d - b*c)^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2))))/(b^2*x^4 + (a^3*c)/(b*d) + (x*(a^3*d + 3*a^2*b*
c))/(b*d) + (x^3*(b^3*c + 3*a*b^2*d))/(b*d) + (x^2*(3*a*b^2*c + 3*a^2*b*d)
)/(b*d)) - ((27*A^2*a^3*d^3 + 9*A^2*b^3*c^3 + 54*B^2*a^3*d^3 + 2*B^2*b^3*c
^3 - 54*A*B*a^3*d^3 + 6*A*B*b^3*c^3 - 45*A^2*a*b^2*c^2*d + 117*A^2*a^2*b*c
*d^2 - 25*B^2*a*b^2*c^2*d + 299*B^2*a^2*b*c*d^2 - 48*A*B*a*b^2*c^2*d + 276
*A*B*a^2*b*c*d^2)/(3*(a*d - b*c)) + (2*x^3*(18*A^2*b^3*d^3 + 55*B^2*b^3...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 5157, normalized size of antiderivative = 7.56

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x)
```

output

```
( - 324*log(a + b*x)*a**6*b*c*d**4 - 324*log(a + b*x)*a**6*b*d**5*x - 108*
log(a + b*x)*a**5*b**2*c**2*d**3 - 1080*log(a + b*x)*a**5*b**2*c*d**4*x -
324*log(a + b*x)*a**5*b**2*c*d**4 - 972*log(a + b*x)*a**5*b**2*d**5*x**2 -
 324*log(a + b*x)*a**5*b**2*d**5*x - 324*log(a + b*x)*a**4*b**3*c**2*d**3*
x - 396*log(a + b*x)*a**4*b**3*c**2*d**3 - 1296*log(a + b*x)*a**4*b**3*c*d
**4*x**2 - 1368*log(a + b*x)*a**4*b**3*c*d**4*x - 810*log(a + b*x)*a**4*b*
*3*c*d**4 - 972*log(a + b*x)*a**4*b**3*d**5*x**3 - 972*log(a + b*x)*a**4*b
**3*d**5*x**2 - 810*log(a + b*x)*a**4*b**3*d**5*x - 324*log(a + b*x)*a**3*
b**4*c**2*d**3*x**2 - 1188*log(a + b*x)*a**3*b**4*c**2*d**3*x - 510*log(a
+ b*x)*a**3*b**4*c**2*d**3 - 648*log(a + b*x)*a**3*b**4*c*d**4*x**3 - 2160
*log(a + b*x)*a**3*b**4*c*d**4*x**2 - 2940*log(a + b*x)*a**3*b**4*c*d**4*x
 - 324*log(a + b*x)*a**3*b**4*d**5*x**4 - 972*log(a + b*x)*a**3*b**4*d**5*
x**3 - 2430*log(a + b*x)*a**3*b**4*d**5*x**2 - 108*log(a + b*x)*a**2*b**5*
c**2*d**3*x**3 - 1188*log(a + b*x)*a**2*b**5*c**2*d**3*x**2 - 1530*log(a +
 b*x)*a**2*b**5*c**2*d**3*x - 108*log(a + b*x)*a**2*b**5*c*d**4*x**4 - 151
2*log(a + b*x)*a**2*b**5*c*d**4*x**3 - 3960*log(a + b*x)*a**2*b**5*c*d**4*
x**2 - 324*log(a + b*x)*a**2*b**5*d**5*x**4 - 2430*log(a + b*x)*a**2*b**5*
d**5*x**3 - 396*log(a + b*x)*a*b**6*c**2*d**3*x**3 - 1530*log(a + b*x)*a*b
**6*c**2*d**3*x**2 - 396*log(a + b*x)*a*b**6*c*d**4*x**4 - 2340*log(a + b*
x)*a*b**6*c*d**4*x**3 - 810*log(a + b*x)*a*b**6*d**5*x**4 - 510*log(a +...
```

3.100 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$

Optimal result 1235
Mathematica [B] (verified) 1236
Rubi [A] (verified) 1236
Maple [F] 1238
Fricas [F] 1239
Sympy [F] 1239
Maxima [F] 1240
Giac [F] 1241
Mupad [F(-1)] 1242
Reduce [F] 1242

Optimal result

Integrand size = 42, antiderivative size = 635

$$
\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx \\
&= \frac{B^2(bc-ad)g^3(a+bx)^2}{4d^2i^3(c+dx)^2} - \frac{4AbB(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)} \\
&\quad + \frac{4bB^2(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)} - \frac{4bB^2(bc-ad)g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^3(c+dx)} \\
&\quad - \frac{B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^3(c+dx)^2} \\
&\quad + \frac{2b^2B(bc-ad)g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i^3} \\
&\quad + \frac{b^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3} \\
&\quad + \frac{(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^2i^3(c+dx)^2} \\
&\quad + \frac{2b(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3(c+dx)} \\
&\quad + \frac{3b^2(bc-ad)g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i^3} \\
&\quad + \frac{2b^2B^2(bc-ad)g^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3} \\
&\quad + \frac{6b^2B(bc-ad)g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3} \\
&\quad - \frac{6b^2B^2(bc-ad)g^3\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}
\end{aligned}
$$

output
```
1/4*B^2*(-a*d+b*c)*g^3*(b*x+a)^2/d^2/i^3/(d*x+c)^2-4*A*b*B*(-a*d+b*c)*g^3*
(b*x+a)/d^3/i^3/(d*x+c)+4*b*B^2*(-a*d+b*c)*g^3*(b*x+a)/d^3/i^3/(d*x+c)-4*b
*B^2*(-a*d+b*c)*g^3*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d^3/i^3/(d*x+c)-1/2*B*(-
a*d+b*c)*g^3*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2/i^3/(d*x+c)^2+2*b^2
*B*(-a*d+b*c)*g^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4
/i^3+b^2*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i^3+1/2*(-a*d+b*c)*
g^3*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i^3/(d*x+c)^2+2*b*(-a*d+b*
c)*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i^3/(d*x+c)+3*b^2*(-a*d+b
*c)*g^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i^3+2*b
^2*B^2*(-a*d+b*c)*g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^3+6*b^2*B*(-a*d
+b*c)*g^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i
^3-6*b^2*B^2*(-a*d+b*c)*g^3*polylog(3,d*(b*x+a)/b/(d*x+c))/d^4/i^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5929 vs. $2(635) = 1270$.

Time $= 8.40$ (sec) , antiderivative size $= 5929$, normalized size of antiderivative $= 9.34$

$$\int \frac{(ag+bgx)^3 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + 
d*i*x)^3,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time $= 0.78$ (sec) , antiderivative size $= 504$, normalized size of antiderivative $= 0.79$, number of steps used $= 4$, number of rules used $= 3$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used $= \{2962, 2795, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ci+dix)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{g^3(bc-ad)\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i^3}$$

$$\downarrow 2795$$

$$\frac{g^3(bc-ad)\int \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^3}{d^3\left(\frac{d(a+bx)}{c+dx}-b\right)^2}+\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^2}{d^3\left(\frac{d(a+bx)}{c+dx}-b\right)}+\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b}{d^3}+\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2(c+dx)}\right.}{i^3}$$

$$\downarrow 2009$$

$$g^3(bc-ad)\left(\frac{6b^2B\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^4}+\frac{3b^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^4}+\frac{2b^2B\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{d^4}\right.$$

input

```
Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)
^3,x]
```

output

```
((b*c - a*d)*g^3*((B^2*(a + b*x)^2)/(4*d^2*(c + d*x)^2) - (4*A*b*B*(a + b*
x))/(d^3*(c + d*x)) + (4*b*B^2*(a + b*x))/(d^3*(c + d*x)) - (4*b*B^2*(a +
b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d^3*(c + d*x)) - (B*(a + b*x)^2*(A + B
*Log[(e*(a + b*x))/(c + d*x)]))/(2*d^2*(c + d*x)^2) + ((a + b*x)^2*(A + B*
Log[(e*(a + b*x))/(c + d*x)])^2)/(2*d^2*(c + d*x)^2) + (2*b*(a + b*x)*(A +
 B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^3*(c + d*x)) + (b^2*(a + b*x)*(A +
B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^3*(c + d*x)*(b - (d*(a + b*x))/(c +
d*x))) + (2*b^2*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x
))/(b*(c + d*x))])/d^4 + (3*b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log
[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 + (2*b^2*B^2*PolyLog[2, (d*(a + b*x
))/(b*(c + d*x))])/d^4 + (6*b^2*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Pol
yLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^4 - (6*b^2*B^2*PolyLog[3, (d*(a +
b*x))/(b*(c + d*x))])/d^4))/i^3
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(dix+ci)^3}dx$$

input

```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x)
```

output

```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, al
gorithm="fricas")
```

output
```
integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*
a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2
*a^3*g^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^
2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((b*e*x + a*e)/(d*x + c)))
/(d^3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$

$$= g^3\left(\int \frac{A^2a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{A^2b^3x^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{B^2a^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2ABa^3\log\left(\frac{ae}{c+dx}+\frac{be}{c+}\right.}{c^3+3c^2dx+3cd^2x^2+d^}\right.$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**3,x)
```

output

```
g**3*(Integral(A**2*a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(A**2*b**3*x**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**
3), x) + Integral(B**2*a**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c**3
+ 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*a**3*log(a*
e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x
**3), x) + Integral(3*A**2*a*b**2*x**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
+ d**3*x**3), x) + Integral(3*A**2*a**2*b*x/(c**3 + 3*c**2*d*x + 3*c*d**2*
x**2 + d**3*x**3), x) + Integral(B**2*b**3*x**3*log(a*e/(c + d*x) + b*e*x/
(c + d*x))**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(2*A*B*b**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x
 + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(3*B**2*a*b**2*x**2*log(a*e/(c
 + d*x) + b*e*x/(c + d*x))**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x*
*3), x) + Integral(3*B**2*a**2*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2
/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(6*A*B*a*b*
*2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2
*x**2 + d**3*x**3), x) + Integral(6*A*B*a**2*b*x*log(a*e/(c + d*x) + b*e*x
/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/i**3
```

Maxima [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input

```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, al
gorithm="maxima")
```

output

```
-3/2*A*B*a^2*b*g^3*(2*(2*d*x + c)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^
4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (b*c^2 - 3*a*c*d + 2*(b*c*d - 2
*a*d^2)*x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b
*c^3*d^2 - a*c^2*d^3)*i^3) - 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4)*i^3) + 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c
^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3)) - 1/2*A^2*b^3*g^3*((6*c^2*d*x + 5*c^
3)/(d^6*i^3*x^2 + 2*c*d^5*i^3*x + c^2*d^4*i^3) - 2*x/(d^3*i^3) + 6*c*log(d
*x + c)/(d^4*i^3)) + 1/2*A*B*a^3*g^3*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 -
a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3
) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^3*i^3*x^2 + 2*c*d^2*i^3*x +
c^2*d*i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)
- 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) + 3/2*A^2*
a*b^2*g^3*((4*c*d*x + 3*c^2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) +
 2*log(d*x + c)/(d^3*i^3)) - 3/2*(2*d*x + c)*A^2*a^2*b*g^3/(d^4*i^3*x^2 +
2*c*d^3*i^3*x + c^2*d^2*i^3) - 1/2*A^2*a^3*g^3/(d^3*i^3*x^2 + 2*c*d^2*i^3*
x + c^2*d*i^3) - 1/2*(2*((b^3*c*d^2*g^3 - a*b^2*d^3*g^3)*B^2*x^2 + 2*(b^3*
c^2*d*g^3 - a*b^2*c*d^2*g^3)*B^2*x + (b^3*c^3*g^3 - a*b^2*c^2*d*g^3)*B^2)*
log(d*x + c)^3 - (2*B^2*b^3*d^3*g^3*x^3 + 4*B^2*b^3*c*d^2*g^3*x^2 - 2*(2*b
^3*c^2*d*g^3 - 6*a*b^2*c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B^2*x - (5*b^3*c^3*g^3
 - 9*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 + a^3*d^3*g^3)*B^2)*log(d*x + ...
```

Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input

```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, al
gorithm="giac")
```

output

```
integrate((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*
i)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(c\,i+d\,i\,x)^3}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
^3,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
^3, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x)
```

output

```
(g**3*i*(4*int((log((a*e + b*e*x)/(c + d*x))**2*x**3)/(c**3 + 3*c**2*d*x +
 3*c*d**2*x**2 + d**3*x**3),x)*a**2*b**5*c**4*d**6 + 8*int((log((a*e + b*e
*x)/(c + d*x))**2*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)
*a**2*b**5*c**3*d**7*x + 4*int((log((a*e + b*e*x)/(c + d*x))**2*x**3)/(c**
3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*b**5*c**2*d**8*x**2 -
8*int((log((a*e + b*e*x)/(c + d*x))**2*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2
*x**2 + d**3*x**3),x)*a*b**6*c**5*d**5 - 16*int((log((a*e + b*e*x)/(c + d*
x))**2*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**6*c**
4*d**6*x - 8*int((log((a*e + b*e*x)/(c + d*x))**2*x**3)/(c**3 + 3*c**2*d*x
 + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**6*c**3*d**7*x**2 + 4*int((log((a*e +
 b*e*x)/(c + d*x))**2*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3
),x)*b**7*c**6*d**4 + 8*int((log((a*e + b*e*x)/(c + d*x))**2*x**3)/(c**3 +
 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**7*c**5*d**5*x + 4*int((log(
(a*e + b*e*x)/(c + d*x))**2*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3),x)*b**7*c**4*d**6*x**2 + 12*int((log((a*e + b*e*x)/(c + d*x))**2*x
**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**3*b**4*c**4*d**
6 + 24*int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(c**3 + 3*c**2*d*x + 3*c
*d**2*x**2 + d**3*x**3),x)*a**3*b**4*c**3*d**7*x + 12*int((log((a*e + b*e*
x)/(c + d*x))**2*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*
a**3*b**4*c**2*d**8*x**2 - 24*int((log((a*e + b*e*x)/(c + d*x))**2*x**2...
```

3.101 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$

Optimal result . . . 1244
Mathematica [B] (verified) . . . 1245
Rubi [A] (verified) . . . 1246
Maple [F] . . . 1248
Fricas [F] . . . 1248
Sympy [F] . . . 1249
Maxima [F] . . . 1249
Giac [F] . . . 1250
Mupad [F(-1)] . . . 1251
Reduce [F] . . . 1251

Optimal result

Integrand size = 42, antiderivative size = 410

$$
\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx \\
&= -\frac{B^2g^2(a+bx)^2}{4di^3(c+dx)^2} + \frac{2AbBg^2(a+bx)}{d^2i^3(c+dx)} - \frac{2bB^2g^2(a+bx)}{d^2i^3(c+dx)} \\
&\quad + \frac{2bB^2g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^3(c+dx)} + \frac{Bg^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^3(c+dx)^2} \\
&\quad - \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2di^3(c+dx)^2} - \frac{bg^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^3(c+dx)} \\
&\quad - \frac{b^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3} \\
&\quad - \frac{2b^2Bg^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3} \\
&\quad + \frac{2b^2B^2g^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}
\end{aligned}
$$

output

```
-1/4*B^2*g^2*(b*x+a)^2/d/i^3/(d*x+c)^2+2*A*b*B*g^2*(b*x+a)/d^2/i^3/(d*x+c)
-2*b*B^2*g^2*(b*x+a)/d^2/i^3/(d*x+c)+2*b*B^2*g^2*(b*x+a)*ln(e*(b*x+a)/(d*x
+c))/d^2/i^3/(d*x+c)+1/2*B*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i^3
/(d*x+c)^2-1/2*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d/i^3/(d*x+c)^2
-b*g^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i^3/(d*x+c)-b^2*g^2*ln((-
a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i^3-2*b^2*B*g^2*(A+B
*ln(e*(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^3+2*b^2*B^2*g
^2*polylog(3,d*(b*x+a)/b/(d*x+c))/d^3/i^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3840 vs. $2(410) = 820$.

Time $= 7.26$ (sec) , antiderivative size $= 3840$, normalized size of antiderivative $= 9.37$

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \text{Result too large to show}$$

input

```
Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i +
d*i*x)^3,x]
```

output

```
-1/2*(A^2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2)/(d^3*i^3*(c + d*x)^2) - (2*
(-(A^2*b^2*c*g^2) + a*A^2*b*d*g^2))/(d^3*i^3*(c + d*x)) + (A^2*b^2*g^2*Log
[c + d*x])/(d^3*i^3) + (2*a^2*A*B*g^2*(((c/d + x)*(2*Log[c/d + x] + 4*Log[
c/d + x]^2))/(8*(c + d*x)^3*Log[c/d + x]) + ((d*(a/b + x))/((-c + (a*d)/b)
^3*(1 - (d*(a/b + x))/(-c + (a*d)/b))) - ((d^2*(a/b + x)^2)/((-c + (a*d)/b
)^4*(1 - (d*(a/b + x))/(-c + (a*d)/b))^2) + (2*d*(a/b + x))/((-c + (a*d)/b
)^3*(1 - (d*(a/b + x))/(-c + (a*d)/b))))*Log[a/b + x] - Log[1 - (d*(a/b +
x))/(-c + (a*d)/b)]/(-c + (a*d)/b)^2)/(2*d) - (-Log[a/b + x] + Log[c/d + x
] + Log[(a*e)/(c + d*x) + (b*e*x)/(c + d*x)])/(2*d*(c + d*x)^2)))/i^3 + (4
*a*A*b*B*g^2*((1 + Log[c/d + x])/(d^2*(c + d*x)) - (c*(1 + 2*Log[c/d + x])
)/(4*d^2*(c + d*x)^2) + (-(Log[a/b + x]/(d*(c + d*x))) + ((b*Log[a + b*x])
/(b*c - a*d) - (b*Log[c + d*x])/(b*c - a*d))/d)/d - (c*(-Log[a/b + x] + (b
*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x
]))/(b*c - a*d)^2))/(2*d^2*(c + d*x)^2) - ((c + 2*d*x)*(-Log[a/b + x] + Lo
g[c/d + x] + Log[(a*e)/(c + d*x) + (b*e*x)/(c + d*x)]))/(2*d^2*(c + d*x)^2
)))/i^3 + (2*A*b^2*B*g^2*(-1/2*Log[c/d + x]^2/d^3 - (2*c*(1 + Log[c/d + x]
))/(d^3*(c + d*x)) + (c^2*(1 + 2*Log[c/d + x]))/(4*d^3*(c + d*x)^2) - (2*c
*(-(Log[a/b + x]/(d*(c + d*x))) + ((b*Log[a + b*x])/(b*c - a*d) - (b*Log[c
 + d*x])/(b*c - a*d))/d))/d^2 + (c^2*(-Log[a/b + x] + (b*(c + d*x)*(b*c -
a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]))/(b*c - a*d)...
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ci+dix)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{g^2\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{i^3}$$

$$\downarrow 2795$$

$$\frac{g^2\int\left(-\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d(c+dx)}-\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)}-\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2}\right)d\frac{a+bx}{c+dx}}{i^3}$$

$$\downarrow 2009$$

$$g^2\left(-\frac{2b^2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3}-\frac{b^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^3}-\frac{b(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^2(c+dx)}+$$

input

```
Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)
^3,x]
```

output

```
(g^2*(-1/4*(B^2*(a + b*x)^2)/(d*(c + d*x)^2) + (2*A*b*B*(a + b*x))/(d^2*(c
 + d*x)) - (2*b*B^2*(a + b*x))/(d^2*(c + d*x)) + (2*b*B^2*(a + b*x)*Log[(e
*(a + b*x))/(c + d*x)])/(d^2*(c + d*x)) + (B*(a + b*x)^2*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/(2*d*(c + d*x)^2) - ((a + b*x)^2*(A + B*Log[(e*(a + b
*x))/(c + d*x)])^2)/(2*d*(c + d*x)^2) - (b*(a + b*x)*(A + B*Log[(e*(a + b*
x))/(c + d*x)])^2)/(d^2*(c + d*x)) - (b^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 - (2*b^2*B*(A + B*Log[(e*
(a + b*x))/(c + d*x)])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^3 + (2*b
^2*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/d^3))/i^3
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(dix+ci)^3}dx$$

input

```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x)
```

output

```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}dx$$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, al
gorithm="fricas")
```

output

```
integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x
^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*
B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)
))/(d^3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$

$$= g^2\left(\int \frac{A^2a^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{A^2b^2x^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{B^2a^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2ABa^2\log\left(\frac{ae}{c+dx}+\frac{be}{c+}\right.\right.$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**3,x)
```

output
```
g**2*(Integral(A**2*a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(A**2*b**2*x**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**
3), x) + Integral(B**2*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c**3
+ 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*a**2*log(a*
e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x
**3), x) + Integral(2*A**2*a*b*x/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3), x) + Integral(B**2*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))*
*2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*b*
*2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2
*x**2 + d**3*x**3), x) + Integral(2*B**2*a*b*x*log(a*e/(c + d*x) + b*e*x/(
c + d*x))**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integra
l(4*A*B*a*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*
c*d**2*x**2 + d**3*x**3), x))/i**3
```

Maxima [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, al
gorithm="maxima")
```

output

```
-A*B*a*b*g^2*(2*(2*d*x + c)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^4*i^3*
x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2
)*x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d
^2 - a*c^2*d^3)*i^3) - 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*
a*b*c*d^3 + a^2*d^4)*i^3) + 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2
 - 2*a*b*c*d^3 + a^2*d^4)*i^3)) + 1/2*A*B*a^2*g^2*((2*b*d*x + 3*b*c - a*d)
/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a
*c^2*d^2)*i^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^3*i^3*x^2 + 2*c
*d^2*i^3*x + c^2*d*i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3
)) + 1/2*A^2*b^2*g^2*((4*c*d*x + 3*c^2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2
*d^3*i^3) + 2*log(d*x + c)/(d^3*i^3)) - (2*d*x + c)*A^2*a*b*g^2/(d^4*i^3*x
^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - 1/2*A^2*a^2*g^2/(d^3*i^3*x^2 + 2*c*d^2
*i^3*x + c^2*d*i^3) + 1/6*(2*(B^2*b^2*d^2*g^2*x^2 + 2*B^2*b^2*c*d*g^2*x +
B^2*b^2*c^2*g^2)*log(d*x + c)^3 + 3*(4*(b^2*c*d*g^2 - a*b*d^2*g^2)*B^2*x +
 (3*b^2*c^2*g^2 - 2*a*b*c*d*g^2 - a^2*d^2*g^2)*B^2)*log(d*x + c)^2)/(d^5*i
^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) - integrate(-(2*B^2*a*b*d^2*g^2*x*lo
g(e)^2 + B^2*a^2*d^2*g^2*log(e)^2 + (B^2*b^2*d^2*g^2*log(e)^2 + 2*A*B*b^2*
d^2*g^2*log(e))*x^2 + (B^2*b^2*d^2*g^2*x^2 + 2*B^2*a*b*d^2*g^2*x + B^2*a^2
*d^2*g^2)*log(b*x + a)^2 + 2*(2*B^2*a*b*d^2*g^2*x*log(e) + B^2*a^2*d^2*...
```

Giac [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, al
gorithm="giac")
```

output

```
integrate((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*
i)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(c\,i+d\,i\,x)^3}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
^3,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
^3, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x)
```

output

```
(g**2*i*(4*int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(c**3 + 3*c**2*d*x +
 3*c*d**2*x**2 + d**3*x**3),x)*a**2*b**4*c**4*d**5 + 8*int((log((a*e + b*e
*x)/(c + d*x))**2*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)
*a**2*b**4*c**3*d**6*x + 4*int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(c**
3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*b**4*c**2*d**7*x**2 -
8*int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2
*x**2 + d**3*x**3),x)*a*b**5*c**5*d**4 - 16*int((log((a*e + b*e*x)/(c + d*
x))**2*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**5*c**
4*d**5*x - 8*int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(c**3 + 3*c**2*d*x
 + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**5*c**3*d**6*x**2 + 4*int((log((a*e +
 b*e*x)/(c + d*x))**2*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3
),x)*b**6*c**6*d**3 + 8*int((log((a*e + b*e*x)/(c + d*x))**2*x**2)/(c**3 +
 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**6*c**5*d**4*x + 4*int((log(
(a*e + b*e*x)/(c + d*x))**2*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3),x)*b**6*c**4*d**5*x**2 + 8*int((log((a*e + b*e*x)/(c + d*x))*x**2)
/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**3*b**3*c**4*d**5 +
16*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x
**2 + d**3*x**3),x)*a**3*b**3*c**3*d**6*x + 8*int((log((a*e + b*e*x)/(c +
d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**3*b**3*c
**2*d**7*x**2 - 16*int((log((a*e + b*e*x)/(c + d*x))*x**2)/(c**3 + 3*c*...
```

3.102 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$

Optimal result 1253
Mathematica [C] (verified) 1254
Rubi [A] (verified) 1255
Maple [B] (verified) 1256
Fricas [B] (verification not implemented) 1258
Sympy [B] (verification not implemented) 1258
Maxima [B] (verification not implemented) 1260
Giac [A] (verification not implemented) 1261
Mupad [B] (verification not implemented) 1261
Reduce [B] (verification not implemented) 1262

Optimal result

Integrand size = 40, antiderivative size = 141

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \frac{B^2g(a+bx)^2}{4(bc-ad)i^3(c+dx)^2} - \frac{Bg(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)i^3(c+dx)^2} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)i^3(c+dx)^2}$$

output

```
1/4*B^2*g*(b*x+a)^2/(-a*d+b*c)/i^3/(d*x+c)^2-1/2*B*g*(b*x+a)^2*(A+B*ln(e*(
b*x+a)/(d*x+c)))/(-a*d+b*c)/i^3/(d*x+c)^2+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x+a
)/(d*x+c)))^2/(-a*d+b*c)/i^3/(d*x+c)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.91 (sec) , antiderivative size = 767, normalized size of antiderivative = 5.44

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{g\left(2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-4b(bc-ad)(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+4bB(c+dx)\left(2\right.\right.}{}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*
i*x)^3,x]
```

output
```
(g*(2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 4*b*(b*c - a*
d)*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 4*b*B*(c + d*x)*(2*(
b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*b*(c + d*x)*Log[a + b*
x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b*(c + d*x)*(A + B*Log[(e*(a +
 b*x))/(c + d*x)])*Log[c + d*x] - 2*B*(b*c - a*d + b*(c + d*x)*Log[a + b*x
] - b*(c + d*x)*Log[c + d*x]) - b*B*(c + d*x)*(Log[a + b*x]*(Log[a + b*x]
 - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) +
 a*d)]) + b*B*(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*
x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - B*(2*(b*c -
 a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*b*(b*c - a*d)*(c + d*x)*(
A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*b^2*(c + d*x)^2*Log[a + b*x]*(A +
B*Log[(e*(a + b*x))/(c + d*x)]) - 4*b^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)])*Log[c + d*x] - 4*b*B*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log
[a + b*x] - b*(c + d*x)*Log[c + d*x]) - B*((b*c - a*d)^2 + 2*b*(b*c - a*d)
*(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*
x]) - 2*b^2*B*(c + d*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x)
)/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^2*B*(c
 + d*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*
x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(4*d^2*(b*c - a*d)*i^3*(c
 + d*x)^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2962, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ci+dix)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{g\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{c+dx}d\frac{a+bx}{c+dx}}{i^3(bc-ad)}$$

$$\downarrow 2742$$

$$\frac{g\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(c+dx)^2}-B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{c+dx}d\frac{a+bx}{c+dx}\right)}{i^3(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{g\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(c+dx)^2}-B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(c+dx)^2}-\frac{B(a+bx)^2}{4(c+dx)^2}\right)\right)}{i^3(bc-ad)}$$

input

```
Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)^3
,x]
```

output

```
(g*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(c + d*x)^2) -
 B*(-1/4*(B*(a + b*x)^2)/(c + d*x)^2 + ((a + b*x)^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)]))/(2*(c + d*x)^2))))/((b*c - a*d)*i^3)
```

Defintions of rubi rules used

rule 2741

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(135) = 270$.

Time = 1.47 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.44

method	result
norman	$\frac{-\frac{2A^2adg+2A^2bcg-2dgaBA-2cbgBA+dga\,B^2+cbg\,B^2}{4i\,d^2}-\frac{\left(2A^2bg-2bgBA+bg\,B^2\right)x}{2id}-\frac{B^2a^2g\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2i(da-bc)}-\frac{B^2b^2gx^2\ln\left(\frac{e(b}{2(da-bc)}}{i^2(da}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{g\,d^2A^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2(da-bc)^2e^3i^3}+\frac{2g\,d^2AB\left(\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{4}\right)}{(da-bc)^2e^3i^3}+\frac{g\,d^2B^2\left(\right.}{d^2}$
default	$-\frac{e(da-bc)\left(\frac{g\,d^2A^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2(da-bc)^2e^3i^3}+\frac{2g\,d^2AB\left(\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{4}\right)}{(da-bc)^2e^3i^3}+\frac{g\,d^2B^2\left(\right.}{d^2}$
parallelrisch	$-\frac{2A^2a^2b\,d^4g-2A^2b^3c^2d^2g-2AB\,a^2b\,d^4g+2AB\,b^3c^2d^2g+B^2a^2b\,d^4g-B^2b^3c^2d^2g+2B^2xa\,b^2d^4g+8ABx\ln\left(\frac{e(bx+a)}{dx+c}\right.}{}$
parts	$\frac{g\,A^2\left(-\frac{da-bc}{2d^2(dx+c)^2}-\frac{b}{d^2(dx+c)}\right)}{i^3}-\frac{g\,B^2d\left(a\left(\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2}-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(da-b}{d(dx+}\right.\right.\right.}{}$
orering	Expression too large to display
risch	Expression too large to display

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x,method=_RETU
RNVERBOSE)
```

output
```
(-1/4*(2*A^2*a*d*g+2*A^2*b*c*g-2*A*B*a*d*g-2*A*B*b*c*g+B^2*a*d*g+B^2*b*c*g
)/i/d^2-1/2*(2*A^2*b*g-2*A*B*b*g+B^2*b*g)/i/d*x-1/2*B^2*a^2*g/i/(a*d-b*c)*
ln(e*(b*x+a)/(d*x+c))^2-1/2*B^2*b^2*g/(a*d-b*c)/i*x^2*ln(e*(b*x+a)/(d*x+c)
)^2-1/2*(2*A-B)*g*a^2*B/i/(a*d-b*c)*ln(e*(b*x+a)/(d*x+c))-B^2*a*b*g/i/(a*d
-b*c)*x*ln(e*(b*x+a)/(d*x+c))^2-1/2*b^2/i*g*B*(2*A-B)/(a*d-b*c)*x^2*ln(e*(
b*x+a)/(d*x+c))-g*a*B/i*b*(2*A-B)/(a*d-b*c)*x*ln(e*(b*x+a)/(d*x+c)))/i^2/(
d*x+c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(135) = 270$.

Time = 0.09 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.09

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx =$$

$$-\frac{2\left((2A^2-2AB+B^2)b^2cd-(2A^2-2AB+B^2)abd^2\right)gx-2\left(B^2b^2d^2gx^2+2B^2abd^2gx+B^2a^2d^2g\right)}{4((b$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algo
rithm="fricas")
```

output
```
-1/4*(2*((2*A^2 - 2*A*B + B^2)*b^2*c*d - (2*A^2 - 2*A*B + B^2)*a*b*d^2)*g*
x - 2*(B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log((b*e*x +
 a*e)/(d*x + c))^2 + ((2*A^2 - 2*A*B + B^2)*b^2*c^2 - (2*A^2 - 2*A*B + B^2
)*a^2*d^2)*g - 2*((2*A*B - B^2)*b^2*d^2*g*x^2 + 2*(2*A*B - B^2)*a*b*d^2*g*
x + (2*A*B - B^2)*a^2*d^2*g)*log((b*e*x + a*e)/(d*x + c)))/((b*c*d^4 - a*d
^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(121) = 242$.

Time = 5.17 (sec) , antiderivative size = 712, normalized size of antiderivative = 5.05

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$

$$= \frac{Bb^2g(2A-B)\log\left(x+\frac{2ABab^2dg+2ABb^3cg-B^2ab^2dg-B^2b^3cg-\frac{Ba^2b^2d^2g(2A-B)}{ad-bc}+\frac{2Bab^3cdg(2A-B)}{ad-bc}-\frac{Bb^4c^2g(2A-B)}{ad-bc}}{4ABb^3dg-2B^2b^3dg}\right)}{2d^2i^3(ad-bc)}$$

$$- \frac{Bb^2g(2A-B)\log\left(x+\frac{2ABab^2dg+2ABb^3cg-B^2ab^2dg-B^2b^3cg+\frac{Ba^2b^2d^2g(2A-B)}{ad-bc}-\frac{2Bab^3cdg(2A-B)}{ad-bc}+\frac{Bb^4c^2g(2A-B)}{ad-bc}}{4ABb^3dg-2B^2b^3dg}\right)}{2d^2i^3(ad-bc)}$$

$$+ \frac{(-B^2a^2g-2B^2abgx-B^2b^2gx^2)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2ac^2di^3+4acd^2i^3x+2ad^3i^3x^2-2bc^3i^3-4bc^2di^3x-2bcd^2i^3x^2}$$

$$+ \frac{-2A^2adg-2A^2bcg+2ABadg+2ABbcg-B^2adg-B^2bcg+x(-4A^2bdg+4ABbdg-2B^2bdg)}{4c^2d^2i^3+8cd^3i^3x+4d^4i^3x^2}$$

$$+ \frac{(-2ABadg-2ABbcg-4ABbdgx+B^2adg+B^2bcg+2B^2bdgx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2c^2d^2i^3+4cd^3i^3x+2d^4i^3x^2}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**3,x)
```

output
```
B*b**2*g*(2*A - B)*log(x + (2*A*B*a*b**2*d*g + 2*A*B*b**3*c*g - B**2*a*b**
2*d*g - B**2*b**3*c*g - B*a**2*b**2*d**2*g*(2*A - B)/(a*d - b*c) + 2*B*a*b
**3*c*d*g*(2*A - B)/(a*d - b*c) - B*b**4*c**2*g*(2*A - B)/(a*d - b*c))/(4*
A*B*b**3*d*g - 2*B**2*b**3*d*g))/(2*d**2*i**3*(a*d - b*c)) - B*b**2*g*(2*A
 - B)*log(x + (2*A*B*a*b**2*d*g + 2*A*B*b**3*c*g - B**2*a*b**2*d*g - B**2*
b**3*c*g + B*a**2*b**2*d**2*g*(2*A - B)/(a*d - b*c) - 2*B*a*b**3*c*d*g*(2*
A - B)/(a*d - b*c) + B*b**4*c**2*g*(2*A - B)/(a*d - b*c))/(4*A*B*b**3*d*g
 - 2*B**2*b**3*d*g))/(2*d**2*i**3*(a*d - b*c)) + (-B**2*a**2*g - 2*B**2*a*b
*g*x - B**2*b**2*g*x**2)*log(e*(a + b*x)/(c + d*x))**2/(2*a*c**2*d*i**3 +
4*a*c*d**2*i**3*x + 2*a*d**3*i**3*x**2 - 2*b*c**3*i**3 - 4*b*c**2*d*i**3*x
 - 2*b*c*d**2*i**3*x**2) + (-2*A**2*a*d*g - 2*A**2*b*c*g + 2*A*B*a*d*g + 2
*A*B*b*c*g - B**2*a*d*g - B**2*b*c*g + x*(-4*A**2*b*d*g + 4*A*B*b*d*g - 2*
B**2*b*d*g))/(4*c**2*d**2*i**3 + 8*c*d**3*i**3*x + 4*d**4*i**3*x**2) + (-2
*A*B*a*d*g - 2*A*B*b*c*g - 4*A*B*b*d*g*x + B**2*a*d*g + B**2*b*c*g + 2*B**
2*b*d*g*x)*log(e*(a + b*x)/(c + d*x))/(2*c**2*d**2*i**3 + 4*c*d**3*i**3*x
+ 2*d**4*i**3*x**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1966 vs. $2(135) = 270$.

Time = 0.16 (sec) , antiderivative size = 1966, normalized size of antiderivative = 13.94

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algo
rithm="maxima")
```

output

```
-1/2*(2*d*x + c)*B^2*b*g*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(d^4*i^3*x
^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) + 1/4*(2*((2*b*d*x + 3*b*c - a*d)/((b*c*
d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^
2)*i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2
*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3))*log(b*e*x/(d*
x + c) + a*e/(d*x + c)) - (7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^
2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c
^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))/(
b^2*c^4*d*i^3 - 2*a*b*c^3*d^2*i^3 + a^2*c^2*d^3*i^3 + (b^2*c^2*d^3*i^3 - 2
*a*b*c*d^4*i^3 + a^2*d^5*i^3)*x^2 + 2*(b^2*c^3*d^2*i^3 - 2*a*b*c^2*d^3*i^3
 + a^2*c*d^4*i^3)*x))*B^2*a*g + 1/4*(2*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*
d^2)*x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^
3*d^2 - a*c^2*d^3)*i^3) + 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*i^3) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*
d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) -
(b^2*c^3 - 8*a*b*c^2*d + 7*a^2*c*d^2 + 2*(b^2*c^3 - 2*a*b*c^2*d + (b^2*c*d
^2 - 2*a*b*d^3)*x^2 + 2*(b^2*c^2*d - 2*a*b*c*d^2)*x)*log(b*x + a)^2 + 2*(b
^2*c^3 - 2*a*b*c^2*d + (b^2*c*d^2 - 2*a*b*d^3)*x^2 + 2*(b^2*c^2*d - 2*a...
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.40

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(\frac{2\,(bex+ae)^2B^2g\log\left(\frac{bex+ae}{dx+c}\right)^2}{(dx+c)^2ei^3}+\frac{2\,(2\,ABg-B^2g)(bex+ae)^2\log\left(\frac{bex+ae}{dx+c}\right)}{(dx+c)^2ei^3}+\frac{(2\,A^2g-2\,ABg+B^2g}{(dx+c)^2ei^3}\right.$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algo
rithm="giac")
```

output
```
1/4*(2*(b*e*x + a*e)^2*B^2*g*log((b*e*x + a*e)/(d*x + c))^2/((d*x + c)^2*e
*i^3) + 2*(2*A*B*g - B^2*g)*(b*e*x + a*e)^2*log((b*e*x + a*e)/(d*x + c))/(
(d*x + c)^2*e*i^3) + (2*A^2*g - 2*A*B*g + B^2*g)*(b*e*x + a*e)^2/((d*x + c
)^2*e*i^3))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c
 - a*d)))
```

Mupad [B] (verification not implemented)

Time = 28.15 (sec) , antiderivative size = 474, normalized size of antiderivative = 3.36

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx =$$
$$-\frac{x\,(2\,b\,d\,g\,A^2-2\,b\,d\,g\,A\,B+b\,d\,g\,B^2)+A^2\,a\,d\,g+A^2\,b\,c\,g+\frac{B^2\,a\,d\,g}{2}+\frac{B^2\,b\,c\,g}{2}-A\,B\,a\,d\,g-A\,B\,b\,c\,g}{2\,c^2\,d^2\,i^3+4\,c\,d^3\,i^3\,x+2\,d^4\,i^3\,x^2}$$
$$-\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{\frac{B^2\,a\,g}{2\,d^2\,i^3}+\frac{B^2\,b\,c\,g}{2\,d^3\,i^3}+\frac{B^2\,b\,g\,x}{d^2\,i^3}}{2\,c\,x+d\,x^2+\frac{c^2}{d}}+\frac{B^2\,b^2\,g}{2\,d^2\,i^3\,(a\,d-b\,c)}\right)$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{A\,B\,c\,g}{d^3\,i^3}-x\left(\frac{B^2\,g}{d^2\,i^3}-\frac{2\,A\,B\,g}{d^2\,i^3}\right)+\frac{B\,g\,(A\,a\,d-B\,a\,d+B\,b\,c)}{b\,d^3\,i^3}+\frac{B^2\,b^2\,g\left(\frac{a^2\,d^2-3\,a\,b\,c\,d+2\,b^2\,c^2}{2\,b^3\,d}-\frac{c\,(a\,d-b\,c)}{2\,b^2\,d}\right)}{d^2\,i^3\,(a\,d-b\,c)}\right)}{\frac{d\,x^2}{b}+\frac{c^2}{b\,d}+\frac{2\,c\,x}{b}}$$
$$+\frac{B\,b^2\,g\,\mathrm{atan}\left(\frac{\left(\frac{2\,a\,d^3\,i^3+2\,b\,c\,d^2\,i^3}{2\,d^2\,i^3}+2\,b\,d\,x\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(2\,A-B)\,1\mathrm{i}}{d^2\,i^3\,(a\,d-b\,c)}$$

input
```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)^3
,x)
```

output
```
(B*b^2*g*atan((((2*a*d^3*i^3 + 2*b*c*d^2*i^3)/(2*d^2*i^3) + 2*b*d*x)*1i)/(
a*d - b*c))*(2*A - B)*1i)/(d^2*i^3*(a*d - b*c)) - log((e*(a + b*x))/(c + d
*x))^2*(((B^2*a*g)/(2*d^2*i^3) + (B^2*b*c*g)/(2*d^3*i^3) + (B^2*b*g*x)/(d^
2*i^3))/(2*c*x + d*x^2 + c^2/d) + (B^2*b^2*g)/(2*d^2*i^3*(a*d - b*c))) - (
log((e*(a + b*x))/(c + d*x))*((A*B*c*g)/(d^3*i^3) - x*((B^2*g)/(d^2*i^3) -
 (2*A*B*g)/(d^2*i^3)) + (B*g*(A*a*d - B*a*d + B*b*c))/(b*d^3*i^3) + (B^2*b
^2*g*((a^2*d^2 + 2*b^2*c^2 - 3*a*b*c*d)/(2*b^3*d) - (c*(a*d - b*c))/(2*b^2
*d)))/(d^2*i^3*(a*d - b*c))))/((d*x^2)/b + c^2/(b*d) + (2*c*x)/b) - (x*(2*
A^2*b*d*g + B^2*b*d*g - 2*A*B*b*d*g) + A^2*a*d*g + A^2*b*c*g + (B^2*a*d*g)
/2 + (B^2*b*c*g)/2 - A*B*a*d*g - A*B*b*c*g)/(2*c^2*d^2*i^3 + 2*d^4*i^3*x^2
 + 4*c*d^3*i^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 695, normalized size of antiderivative = 4.93

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{gi\left(4\log\left(\frac{bex+ae}{dx+c}\right)a^2b^2c^2-2a^4cd+2a^3b\,c^2+2a^3b\,d^2x^2-4\log(bx+a)\,a^2b^2d^2x^2+2\log(bx+a)\,a\,b^3d^2x^2-\right.}{}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x)
```

output

```
(g*i*( - 4*log(a + b*x)*a**2*b**2*c**2 - 8*log(a + b*x)*a**2*b**2*c*d*x -
4*log(a + b*x)*a**2*b**2*d**2*x**2 + 2*log(a + b*x)*a*b**3*c**2 + 4*log(a
+ b*x)*a*b**3*c*d*x + 2*log(a + b*x)*a*b**3*d**2*x**2 + 4*log(c + d*x)*a**
2*b**2*c**2 + 8*log(c + d*x)*a**2*b**2*c*d*x + 4*log(c + d*x)*a**2*b**2*d*
*2*x**2 - 2*log(c + d*x)*a*b**3*c**2 - 4*log(c + d*x)*a*b**3*c*d*x - 2*log
(c + d*x)*a*b**3*d**2*x**2 - 2*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**2*c
*d - 4*log((a*e + b*e*x)/(c + d*x))**2*a*b**3*c*d*x - 2*log((a*e + b*e*x)/
(c + d*x))**2*b**4*c*d*x**2 - 4*log((a*e + b*e*x)/(c + d*x))*a**3*b*c*d +
4*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*c**2 + 2*log((a*e + b*e*x)/(c + d
*x))*a**2*b**2*c*d + 4*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*d**2*x**2 -
2*log((a*e + b*e*x)/(c + d*x))*a*b**3*c**2 - 4*log((a*e + b*e*x)/(c + d*x)
)*a*b**3*c*d*x**2 - 2*log((a*e + b*e*x)/(c + d*x))*a*b**3*d**2*x**2 + 2*lo
g((a*e + b*e*x)/(c + d*x))*b**4*c*d*x**2 - 2*a**4*c*d + 2*a**3*b*c**2 + 2*
a**3*b*c*d + 2*a**3*b*d**2*x**2 - 2*a**2*b**2*c**2 - 2*a**2*b**2*c*d*x**2
- a**2*b**2*c*d - 2*a**2*b**2*d**2*x**2 + a*b**3*c**2 + 2*a*b**3*c*d*x**2
+ a*b**3*d**2*x**2 - b**4*c*d*x**2))/(4*c*d*(a*c**2*d + 2*a*c*d**2*x + a*d
**3*x**2 - b*c**3 - 2*b*c**2*d*x - b*c*d**2*x**2))
```

3.103 $\int \frac{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$

Optimal result 1264
Mathematica [C] (verified) 1265
Rubi [A] (verified) 1266
Maple [A] (verified) 1267
Fricas [A] (verification not implemented) 1269
Sympy [B] (verification not implemented) 1269
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Giac [A] (verification not implemented) 1271
Mupad [B] (verification not implemented) 1272
Reduce [B] (verification not implemented) 1273

Optimal result

Integrand size = 32, antiderivative size = 296

$$\begin{aligned}\int \frac{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx &= -\frac{B^2 d(a+bx)^2}{4(bc-ad)^2 i^3 (c+dx)^2} - \frac{2AbB(a+bx)}{(bc-ad)^2 i^3 (c+dx)} \\ &+ \frac{2bB^2(a+bx)}{(bc-ad)^2 i^3 (c+dx)} - \frac{2bB^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^2 i^3 (c+dx)} \\ &+ \frac{Bd(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2 i^3 (c+dx)^2} \\ &- \frac{d(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^2 i^3 (c+dx)^2} \\ &+ \frac{b(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2 i^3 (c+dx)}\end{aligned}$$

output
```
-1/4*B^2*d*(b*x+a)^2/(-a*d+b*c)^2/i^3/(d*x+c)^2-2*A*b*B*(b*x+a)/(-a*d+b*c)
^2/i^3/(d*x+c)+2*b*B^2*(b*x+a)/(-a*d+b*c)^2/i^3/(d*x+c)-2*b*B^2*(b*x+a)*ln
(e*(b*x+a)/(d*x+c))/(-a*d+b*c)^2/i^3/(d*x+c)+1/2*B*d*(b*x+a)^2*(A+B*ln(e*(
b*x+a)/(d*x+c)))/(-a*d+b*c)^2/i^3/(d*x+c)^2-1/2*d*(b*x+a)^2*(A+B*ln(e*(b*x
+a)/(d*x+c)))^2/(-a*d+b*c)^2/i^3/(d*x+c)^2+b*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*
x+c)))^2/(-a*d+b*c)^2/i^3/(d*x+c)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.43 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.50

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{-2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B\left(2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+4b(bc-ad)(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+4b^2(c+dx)^2\log(a+bx\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x)^3,x]
```

output
```
(-2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*(b*c - a*d)^2*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]) + 4*b*(b*c - a*d)*(c + d*x)*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]) + 4*b^2*(c + d*x)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x
))/(c + d*x)]) - 4*b^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Lo
g[c + d*x] - 4*b*B*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c
+ d*x)*Log[c + d*x]) - B*((b*c - a*d)^2 + 2*b*(b*c - a*d)*(c + d*x) + 2*b^
2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*x]) - 2*b^2*B*(c
+ d*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) -
 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^2*B*(c + d*x)^2*((2*Log
[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2,
 (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(4*d*i^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2952, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ci+dix)^3}\,dx$$

$$\downarrow 2952$$

$$\frac{\int \left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 d\frac{a+bx}{c+dx}}{i^3(bc-ad)^2}$$

$$\downarrow 2767$$

$$\frac{\int \left(b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{d(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{c+dx}\right) d\frac{a+bx}{c+dx}}{i^3(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{\frac{Bd(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(c+dx)^2}+\frac{b(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx}-\frac{d(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(c+dx)^2}-\frac{2AbB(a+bx)}{c+dx}-\frac{2bB^2(a+bx)\log}{c+}}{i^3(bc-ad)^2}$$

input

```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x)^3,x]
```

output

```
(-1/4*(B^2*d*(a + b*x)^2)/(c + d*x)^2 - (2*A*b*B*(a + b*x))/(c + d*x) + (2
*b*B^2*(a + b*x))/(c + d*x) - (2*b*B^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*
x)])/(c + d*x) + (B*d*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2
*(c + d*x)^2) - (d*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*
(c + d*x)^2) + (b*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c + d
*x))/((b*c - a*d)^2*i^3)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2952

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.63

method	result
norman	$\frac{Bb(2Abc-Bad-2Bbc)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{i\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{B^2b^2cx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{i\left(a^2d^2-2acdb+c^2b^2\right)}+\frac{\left(2A^2ad-2A^2bc-2ABad+4ABbc+B^2ad-4B^2bc\right)x}{2ci(da-bc)}-\frac{Ba(2Ada$
parts	$-\frac{A^2}{2i^3(dx+c)^2d}-\frac{B^2d\left(\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2}-\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2}+\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{4}\right.}{i^3(da}$
parallelrisch	$-\frac{-4A^2ab^2cd^4-2ABa^2bd^5-6ABb^3c^2d^3-8B^2ab^2cd^4+2A^2a^2bd^5+2A^2b^3c^2d^3+B^2a^2bd^5+7B^2b^3c^2d^3-2B^2x^2\ln($
derivativedivides	$-\frac{e(da-bc)\left(-\frac{d^2A^2b\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{(da-bc)^3e^2i^3}+\frac{d^3A^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2(da-bc)^3e^3i^3}-\frac{2d^2ABb\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-\frac{(da-bc)e}{d(dx+c)}\right.}{(da-bc)^3e^2i^3}\right.}{$
default	$-\frac{e(da-bc)\left(-\frac{d^2A^2b\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{(da-bc)^3e^2i^3}+\frac{d^3A^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2(da-bc)^3e^3i^3}-\frac{2d^2ABb\left(\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)-\frac{(da-bc)e}{d(dx+c)}\right.}{(da-bc)^3e^2i^3}\right.}{$
orering	$-\frac{(dx+c)\left(-90b^3d^2x^3-148ab^2d^2x^2-122b^3cdx^2-39a^2bd^2x-218ab^2cdx-13b^3c^2x+19a^3d^2-96a^2bcd-13ab^2c^2\right)\left(A-\right.}{8(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)(dix+ci)^3}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x,method=_RETURNVERBOSE)
```

output
```
(B/i*b*(2*A*b*c-B*a*d-2*B*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/
(d*x+c))+B^2*b^2*c/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^2
+1/2*(2*A^2*a*d-2*A^2*b*c-2*A*B*a*d+4*A*B*b*c+B^2*a*d-4*B^2*b*c)/c/i/(a*d-
b*c)*x-1/2*B*a*(2*A*a*d-4*A*b*c-B*a*d+4*B*b*c)/i/(a^2*d^2-2*a*b*c*d+b^2*c^
2)*ln(e*(b*x+a)/(d*x+c))-1/2*B^2*a*(a*d-2*b*c)/i/(a^2*d^2-2*a*b*c*d+b^2*c^
2)*ln(e*(b*x+a)/(d*x+c))^2+1/4*(2*A^2*a*d-2*A^2*b*c-2*A*B*a*d+6*A*B*b*c+B^
2*a*d-7*B^2*b*c)/c^2*d/i/(a*d-b*c)*x^2+1/2*b^2*d*B^2/i/(a^2*d^2-2*a*b*c*d+
b^2*c^2)*x^2*ln(e*(b*x+a)/(d*x+c))^2+1/2*d*B/i*b^2*(2*A-3*B)/(a^2*d^2-2*a*
b*c*d+b^2*c^2)*x^2*ln(e*(b*x+a)/(d*x+c)))/i^2/(d*x+c)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.26

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx =$$
$$-\frac{(2\,A^2 - 6\,AB + 7\,B^2)b^2c^2 - 4\,(A^2 - 2\,AB + 2\,B^2)abcd + (2\,A^2 - 2\,AB + B^2)a^2d^2 - 2\,(B^2b^2d^2x^2 + 2$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algorithm="frica
s")
```

output
```
-1/4*((2*A^2 - 6*A*B + 7*B^2)*b^2*c^2 - 4*(A^2 - 2*A*B + 2*B^2)*a*b*c*d +
(2*A^2 - 2*A*B + B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + 2*B^2*b^2*c*d*x + 2*B
^2*a*b*c*d - B^2*a^2*d^2)*log((b*e*x + a*e)/(d*x + c))^2 - 2*((2*A*B - 3*B
^2)*b^2*c*d - (2*A*B - 3*B^2)*a*b*d^2)*x - 2*((2*A*B - 3*B^2)*b^2*d^2*x^2
+ 4*(A*B - B^2)*a*b*c*d - (2*A*B - B^2)*a^2*d^2 - 2*(B^2*a*b*d^2 - 2*(A*B
- B^2)*b^2*c*d)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d
^4 + a^2*d^5)*i^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*i^3*x
+ (b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*i^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. 2(269) = 538.

Time = 2.28 (sec) , antiderivative size = 892, normalized size of antiderivative = 3.01

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**3,x)
```

output

```
-B*b**2*(2*A - 3*B)*log(x + (2*A*B*a*b**2*d + 2*A*B*b**3*c - 3*B**2*a*b**2
*d - 3*B**2*b**3*c - B*a**3*b**2*d**3*(2*A - 3*B)/(a*d - b*c)**2 + 3*B*a**
2*b**3*c*d**2*(2*A - 3*B)/(a*d - b*c)**2 - 3*B*a*b**4*c**2*d*(2*A - 3*B)/(
a*d - b*c)**2 + B*b**5*c**3*(2*A - 3*B)/(a*d - b*c)**2)/(4*A*B*b**3*d - 6*
B**2*b**3*d))/(2*d*i**3*(a*d - b*c)**2) + B*b**2*(2*A - 3*B)*log(x + (2*A*
B*a*b**2*d + 2*A*B*b**3*c - 3*B**2*a*b**2*d - 3*B**2*b**3*c + B*a**3*b**2*
d**3*(2*A - 3*B)/(a*d - b*c)**2 - 3*B*a**2*b**3*c*d**2*(2*A - 3*B)/(a*d -
b*c)**2 + 3*B*a*b**4*c**2*d*(2*A - 3*B)/(a*d - b*c)**2 - B*b**5*c**3*(2*A
- 3*B)/(a*d - b*c)**2)/(4*A*B*b**3*d - 6*B**2*b**3*d))/(2*d*i**3*(a*d - b*
c)**2) + (-B**2*a**2*d + 2*B**2*a*b*c + 2*B**2*b**2*c*x + B**2*b**2*d*x**2
)*log(e*(a + b*x)/(c + d*x))**2/(2*a**2*c**2*d**2*i**3 + 4*a**2*c*d**3*i**
3*x + 2*a**2*d**4*i**3*x**2 - 4*a*b*c**3*d*i**3 - 8*a*b*c**2*d**2*i**3*x -
 4*a*b*c*d**3*i**3*x**2 + 2*b**2*c**4*i**3 + 4*b**2*c**3*d*i**3*x + 2*b**2
*c**2*d**2*i**3*x**2) + (-2*A*B*a*d + 2*A*B*b*c + B**2*a*d - 3*B**2*b*c -
2*B**2*b*d*x)*log(e*(a + b*x)/(c + d*x))/(2*a*c**2*d**2*i**3 + 4*a*c*d**3*
i**3*x + 2*a*d**4*i**3*x**2 - 2*b*c**3*d*i**3 - 4*b*c**2*d**2*i**3*x - 2*b
*c*d**3*i**3*x**2) + (-2*A**2*a*d + 2*A**2*b*c + 2*A*B*a*d - 6*A*B*b*c - B
**2*a*d + 7*B**2*b*c + x*(-4*A*B*b*d + 6*B**2*b*d))/(4*a*c**2*d**2*i**3 -
4*b*c**3*d*i**3 + x**2*(4*a*d**4*i**3 - 4*b*c*d**3*i**3) + x*(8*a*c*d**3*i
**3 - 8*b*c**2*d**2*i**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(290) = 580$.

Time = 0.09 (sec) , antiderivative size = 848, normalized size of antiderivative = 2.86

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci + dix)^3}\, dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algorithm="maxim
a")
```

output

```
1/4*(2*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2
- a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log(b*x + a)/((b^2*c
^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*
b*c*d^2 + a^2*d^3)*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - (7*b^2*c^2
 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x +
 a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*
d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*
(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*log(b*x + a))*log(d*x + c))/(b^2*c^4*d*i^3 - 2*a*b*c^3*d^2*i^3 +
a^2*c^2*d^3*i^3 + (b^2*c^2*d^3*i^3 - 2*a*b*c*d^4*i^3 + a^2*d^5*i^3)*x^2 +
2*(b^2*c^3*d^2*i^3 - 2*a*b*c^2*d^3*i^3 + a^2*c*d^4*i^3)*x))*B^2 + 1/2*A*B*
((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d
^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x
+ c))/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) + 2*b^2*log(b*x + a)/((b^2
*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*B^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^
2/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A^2/(d^3*i^3*x^2 + 2*c*d
^2*i^3*x + c^2*d*i^3)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.25

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{1}{4}\left(2\left(\frac{2\,(bex+ae)B^2b}{(bci^3-adi^3)(dx+c)} - \frac{(bex+ae)^2B^2d}{(bcei^3-adei^3)(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2 - 2\left(\frac{(2\,ABd-B^2d)(bex+\right.\right.$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algorithm="giac"
)
```

output

```
1/4*(2*(2*(b*e*x + a*e)*B^2*b/((b*c*i^3 - a*d*i^3)*(d*x + c)) - (b*e*x + a
*e)^2*B^2*d/((b*c*e*i^3 - a*d*e*i^3)*(d*x + c)^2))*log((b*e*x + a*e)/(d*x
+ c))^2 - 2*((2*A*B*d - B^2*d)*(b*e*x + a*e)^2/((b*c*e*i^3 - a*d*e*i^3)*(d
*x + c)^2) - 4*(A*B*b - B^2*b)*(b*e*x + a*e)/((b*c*i^3 - a*d*i^3)*(d*x + c
)))*log((b*e*x + a*e)/(d*x + c)) - (2*A^2*d - 2*A*B*d + B^2*d)*(b*e*x + a*
e)^2/((b*c*e*i^3 - a*d*e*i^3)*(d*x + c)^2) + 4*(A^2*b - 2*A*B*b + 2*B^2*b)
*(b*e*x + a*e)/((b*c*i^3 - a*d*i^3)*(d*x + c)))*(b*c/((b*c*e - a*d*e)*(b*c
 - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 27.98 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.71

$$
\begin{aligned}
&\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx \\
&= -\frac{\frac{2A^2ad-2A^2bc+B^2ad-7B^2bc-2ABad+6ABbc}{2(ad-bc)} - \frac{x\left(3B^2bd-2ABbd\right)}{ad-bc}}{2c^2di^3+4cd^2i^3x+2d^3i^3x^2} \\
&\quad - \ln\left(\frac{e(a+bx)}{c+dx}\right)^2\left(\frac{B^2}{2d^2i^3\left(2cx+dx^2+\frac{c^2}{d}\right)} - \frac{B^2b^2}{2di^3\left(a^2d^2-2abcd+b^2c^2\right)}\right) \\
&\quad - \frac{\ln\left(\frac{e(a+bx)}{c+dx}\right)\left(\frac{AB}{bd^2i^3}+\frac{B^2x(ad-bc)}{di^3\left(a^2d^2-2abcd+b^2c^2\right)} - \frac{B^2b^2\left(\frac{a^2d^2-3abcd+2b^2c^2}{2b^3d}-\frac{c(ad-bc)}{2b^2d}\right)}{di^3\left(a^2d^2-2abcd+b^2c^2\right)}\right)}{\frac{dx^2}{b}+\frac{c^2}{bd}+\frac{2cx}{b}} \\
&\quad + \frac{Bb^2\operatorname{atan}\left(\frac{Bb^2\left(2bdx+\frac{a^2d^3i^3-b^2c^2di^3}{di^3(ad-bc)}\right)(2A-3B)\,1\mathrm{i}}{(ad-bc)\left(3B^2b^2-2ABb^2\right)}\right)(2A-3B)\,1\mathrm{i}}{di^3(ad-bc)^2}
\end{aligned}
$$

input

```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(c*i + d*i*x)^3,x)
```

output

```
(B*b^2*atan((B*b^2*(2*b*d*x + (a^2*d^3*i^3 - b^2*c^2*d*i^3)/(d*i^3*(a*d -
b*c)))*(2*A - 3*B)*1i)/((a*d - b*c)*(3*B^2*b^2 - 2*A*B*b^2)))*(2*A - 3*B)*
1i)/(d*i^3*(a*d - b*c)^2) - log((e*(a + b*x))/(c + d*x))^2*(B^2/(2*d^2*i^3
*(2*c*x + d*x^2 + c^2/d)) - (B^2*b^2)/(2*d*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d))) - (log((e*(a + b*x))/(c + d*x))*((A*B)/(b*d^2*i^3) + (B^2*x*(a*d -
b*c))/(d*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (B^2*b^2*((a^2*d^2 + 2*b^2
*c^2 - 3*a*b*c*d)/(2*b^3*d) - (c*(a*d - b*c))/(2*b^2*d)))/(d*i^3*(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d))))/((d*x^2)/b + c^2/(b*d) + (2*c*x)/b) - ((2*A^2*a*
d - 2*A^2*b*c + B^2*a*d - 7*B^2*b*c - 2*A*B*a*d + 6*A*B*b*c)/(2*(a*d - b*c
)) - (x*(3*B^2*b*d - 2*A*B*b*d))/(a*d - b*c))/(2*c^2*d*i^3 + 2*d^3*i^3*x^2
 + 4*c*d^2*i^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 886, normalized size of antiderivative = 2.99

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x)
```

output

```
(i*(4*log(a + b*x)*a*b**3*c**3 + 8*log(a + b*x)*a*b**3*c**2*d*x - 2*log(a
+ b*x)*a*b**3*c**2*d + 4*log(a + b*x)*a*b**3*c*d**2*x**2 - 4*log(a + b*x)*
a*b**3*c*d**2*x - 2*log(a + b*x)*a*b**3*d**3*x**2 - 4*log(a + b*x)*b**4*c*
*3 - 8*log(a + b*x)*b**4*c**2*d*x - 4*log(a + b*x)*b**4*c*d**2*x**2 - 4*lo
g(c + d*x)*a*b**3*c**3 - 8*log(c + d*x)*a*b**3*c**2*d*x + 2*log(c + d*x)*a
*b**3*c**2*d - 4*log(c + d*x)*a*b**3*c*d**2*x**2 + 4*log(c + d*x)*a*b**3*c
*d**2*x + 2*log(c + d*x)*a*b**3*d**3*x**2 + 4*log(c + d*x)*b**4*c**3 + 8*l
og(c + d*x)*b**4*c**2*d*x + 4*log(c + d*x)*b**4*c*d**2*x**2 - 2*log((a*e +
 b*e*x)/(c + d*x))**2*a**2*b**2*c*d**2 + 4*log((a*e + b*e*x)/(c + d*x))**2
*a*b**3*c**2*d + 4*log((a*e + b*e*x)/(c + d*x))**2*b**4*c**2*d*x + 2*log((
a*e + b*e*x)/(c + d*x))**2*b**4*c*d**2*x**2 - 4*log((a*e + b*e*x)/(c + d*x
))*a**3*b*c*d**2 + 8*log((a*e + b*e*x)/(c + d*x))*a**2*b**2*c**2*d + 2*log
((a*e + b*e*x)/(c + d*x))*a**2*b**2*c*d**2 - 4*log((a*e + b*e*x)/(c + d*x)
)*a*b**3*c**3 - 6*log((a*e + b*e*x)/(c + d*x))*a*b**3*c**2*d + 2*log((a*e
+ b*e*x)/(c + d*x))*a*b**3*d**3*x**2 + 4*log((a*e + b*e*x)/(c + d*x))*b**4
*c**3 - 2*log((a*e + b*e*x)/(c + d*x))*b**4*c*d**2*x**2 - 2*a**4*c*d**2 +
4*a**3*b*c**2*d + 2*a**3*b*c*d**2 - 2*a**2*b**2*c**3 - 6*a**2*b**2*c**2*d
- a**2*b**2*c*d**2 + 2*a**2*b**2*d**3*x**2 + 4*a*b**3*c**3 + 5*a*b**3*c**2
*d - 2*a*b**3*c*d**2*x**2 - 3*a*b**3*d**3*x**2 - 4*b**4*c**3 + 3*b**4*c*d*
*2*x**2))/(4*c*d*(a**2*c**2*d**2 + 2*a**2*c*d**3*x + a**2*d**4*x**2 - 2...
```

3.104 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$

Optimal result 1275
Mathematica [A] (verified) 1276
Rubi [A] (warning: unable to verify) 1277
Maple [B] (verified) 1280
Fricas [A] (verification not implemented) 1281
Sympy [B] (verification not implemented) 1282
Maxima [B] (verification not implemented) 1283
Giac [A] (verification not implemented) 1284
Mupad [B] (verification not implemented) 1284
Reduce [B] (verification not implemented) 1285

Optimal result

Integrand size = 42, antiderivative size = 375

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx &= \frac{B^2d^2(a+bx)^2}{4(bc-ad)^3gi^3(c+dx)^2}+\frac{4AbBd(a+bx)}{(bc-ad)^3gi^3(c+dx)}\\ &-\frac{4bB^2d(a+bx)}{(bc-ad)^3gi^3(c+dx)}+\frac{4bB^2d(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^3gi^3(c+dx)}\\ &-\frac{Bd^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2}\\ &+\frac{d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3gi^3(c+dx)^2}\\ &-\frac{2bd(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3gi^3(c+dx)}\\ &+\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^3gi^3}\end{aligned}$$

output
```
1/4*B^2*d^2*(b*x+a)^2/(-a*d+b*c)^3/g/i^3/(d*x+c)^2+4*A*b*B*d*(b*x+a)/(-a*d
+b*c)^3/g/i^3/(d*x+c)-4*b*B^2*d*(b*x+a)/(-a*d+b*c)^3/g/i^3/(d*x+c)+4*b*B^2
*d*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)^3/g/i^3/(d*x+c)-1/2*B*d^2*(b*x
+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g/i^3/(d*x+c)^2+1/2*d^2*(b*
x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g/i^3/(d*x+c)^2-2*b*d*(b
*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g/i^3/(d*x+c)+1/3*b^2*(A+
B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)^3/g/i^3
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.77

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \frac{\frac{3(2A^2-2AB+B^2)(bc-ad)^2}{(c+dx)^2}+\frac{6b(2A^2-6AB+7B^2)(bc-ad)}{c+dx}+6b^2(2A^2-6AB+7B^2)\log(a+bx)+\frac{6B(bc-ad)(B(-7bc+a}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i
*x)^3),x]
```

output
```
((3*(2*A^2 - 2*A*B + B^2)*(b*c - a*d)^2)/(c + d*x)^2 + (6*b*(2*A^2 - 6*A*B
 + 7*B^2)*(b*c - a*d))/(c + d*x) + 6*b^2*(2*A^2 - 6*A*B + 7*B^2)*Log[a + b
*x] + (6*B*(b*c - a*d)*(B*(-7*b*c + a*d - 6*b*d*x) + A*(6*b*c - 2*a*d + 4*
b*d*x))*Log[(e*(a + b*x))/(c + d*x)])/(c + d*x)^2 + (6*B*(2*A*b^2*(c + d*x
)^2 + B*d*(a + b*x)*(-4*b*c + a*d - 3*b*d*x))*Log[(e*(a + b*x))/(c + d*x)]
^2)/(c + d*x)^2 + 4*b^2*B^2*Log[(e*(a + b*x))/(c + d*x)]^3 - 6*b^2*(2*A^2
 - 6*A*B + 7*B^2)*Log[c + d*x])/(12*(b*c - a*d)^3*g*i^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.86 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2962, 2788, 2767, 2009, 2788, 2733, 2009, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx} d\frac{a+bx}{c+dx}}{gi^3(bc-ad)^3}$$

$$\downarrow 2788$$

$$\frac{b\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx} d\frac{a+bx}{c+dx} - d\int \left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 d\frac{a+bx}{c+dx}}{gi^3(bc-ad)^3}$$

$$\downarrow 2767$$

$$\frac{b\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx} d\frac{a+bx}{c+dx} - d\int \left(b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{d(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{c+dx}\right) d\frac{a+}{c+}}{gi^3(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{b\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx} d\frac{a+bx}{c+dx} - d\left(\frac{Bd(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(c+dx)^2} + \frac{b(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx} - \frac{d(a-}{}\right.}{gi^3(bc-ad)^3}$$

$$\downarrow 2788$$

$$\frac{b\left(b\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx} d\frac{a+bx}{c+dx} - d\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 d\frac{a+bx}{c+dx}\right) - d\left(\frac{Bd(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(c+dx)^2} + \right.}{gi^3}$$

$$\downarrow 2733$$

$$b\left(b\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx} - d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx} - 2B\int\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d\frac{a+bx}{c+dx}\right)\right) - d$$

$$\downarrow 2009$$

$$b\left(b\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx} - d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx} - 2B\left(\frac{A(a+bx)}{c+dx} + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx} - \frac{B(a+b}{c+d}\right.\right.\right.$$

$$\downarrow 2739$$

$$b\left(\frac{b\int \frac{(a+bx)^2}{(c+dx)^2}d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B} - d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx} - 2B\left(\frac{A(a+bx)}{c+dx} + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx} - \frac{B(a+bx)}{c+dx}\right)\right)\right.$$

$$\downarrow 15$$

$$b\left(\frac{b(a+bx)^3}{3B(c+dx)^3} - d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx} - 2B\left(\frac{A(a+bx)}{c+dx} + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx} - \frac{B(a+bx)}{c+dx}\right)\right)\right) - d\left(\frac{Bd(a+bx)^2\left(}{2}\right.$$

input

```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i*x)^3)
,x]
```

output

```
(-(d*(-1/4*(B^2*d*(a + b*x)^2)/(c + d*x)^2 - (2*A*b*B*(a + b*x))/(c + d*x)
 + (2*b*B^2*(a + b*x))/(c + d*x) - (2*b*B^2*(a + b*x)*Log[(e*(a + b*x))/(c
 + d*x)])/(c + d*x) + (B*d*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(2*(c + d*x)^2) - (d*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(2*(c + d*x)^2) + (b*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(
c + d*x))) + b*((b*(a + b*x)^3)/(3*B*(c + d*x)^3) - d*(((a + b*x)*(A + B*L
og[(e*(a + b*x))/(c + d*x)])^2)/(c + d*x) - 2*B*((A*(a + b*x))/(c + d*x) -
 (B*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(c +
 d*x)))))/((b*c - a*d)^3*g*i^3)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. 2(367) = 734.

Time = 2.09 (sec) , antiderivative size = 769, normalized size of antiderivative = 2.05

method	result
parts	$\frac{A^2\left(-\frac{1}{2(da-bc)(dx+c)^2}+\frac{b^2\ln(dx+c)}{(da-bc)^3}+\frac{b}{(da-bc)^2(dx+c)}-\frac{b^2\ln(bx+a)}{(da-bc)^3}\right)}{g\,i^3}-\frac{B^2d\left(\frac{d\left(\frac{\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{2}\right.}{}\right.}{}$
derivativedivides	$-\frac{e(da-bc)\left(\frac{d^2A^2b^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e\,i^3(da-bc)^4g}-\frac{2d^3A^2b\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e^2i^3(da-bc)^4g}+\frac{d^4A^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2e^3i^3(da-bc)^4g}+\frac{d^2AB\,b^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e\,i^3(da-bc)^4g}\right.}{}$
default	$-\frac{e(da-bc)\left(\frac{d^2A^2b^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e\,i^3(da-bc)^4g}-\frac{2d^3A^2b\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e^2i^3(da-bc)^4g}+\frac{d^4A^2\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)^2}{2e^3i^3(da-bc)^4g}+\frac{d^2AB\,b^2\ln\left(\frac{be}{d}+\frac{(da-bc)e}{d(dx+c)}\right)}{e\,i^3(da-bc)^4g}\right.}{}$
norman	$-\frac{2A^2a\,d^3-6A^2bc\,d^2-2ABa\,d^3+14ABbc\,d^2+B^2a\,d^3-15B^2bc\,d^2}{4ig\,d^2\left(a^2d^2-2acdb+c^2b^2\right)}-\frac{\left(2A^2b^2c^2+2AB\,a^2d^2-8ABabcd-B^2a^2d^2+8B^2abcd\right)\ln\left(\frac{e(b}{da}\right.}{}$
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x,method=_RETU
RNVERBOSE)
```

output

```
1/g*A^2/i^3*(-1/2/(a*d-b*c)/(d*x+c)^2+b^2/(a*d-b*c)^3*ln(d*x+c)+b/(a*d-b*c
)^2/(d*x+c)-b^2/(a*d-b*c)^3*ln(b*x+a))-B^2/g/i^3*d/(a*d-b*c)^2/e^2*(d/(a*d
-b*c)*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))
^2-1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+1/4
*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2/(a*d-b*c)*b*e*((b*e/d+(a*d-b*c)*e/d/(d
*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*(a*d-b*c)*e/d/(d*x+c)+2*b*e/d)+1/3/d/(a*d
-b*c)*b^2*e^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-2*A*B/g/i^3*d/(a*d-b*c)^2
/e^2*(d/(a*d-b*c)*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2/(a*d-b*c)*b*e*((b*e/d+
(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+
c)-b*e/d)+1/2/d/(a*d-b*c)*b^2*e^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.45

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \frac{3\,(6\,A^2 - 14\,AB + 15\,B^2)b^2c^2 - 24\,(A^2 - 2\,AB + 2\,B^2)abcd + 3\,(2\,A^2 - 2\,AB + B^2)a^2d^2 + 4\,(B^2b^2d^2x^2}{}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algo
rithm="fricas")
```

output

```
1/12*(3*(6*A^2 - 14*A*B + 15*B^2)*b^2*c^2 - 24*(A^2 - 2*A*B + 2*B^2)*a*b*c
*d + 3*(2*A^2 - 2*A*B + B^2)*a^2*d^2 + 4*(B^2*b^2*d^2*x^2 + 2*B^2*b^2*c*d*
x + B^2*b^2*c^2)*log((b*e*x + a*e)/(d*x + c))^3 + 6*((2*A*B - 3*B^2)*b^2*d
^2*x^2 + 2*A*B*b^2*c^2 - 4*B^2*a*b*c*d + B^2*a^2*d^2 - 2*(B^2*a*b*d^2 - 2*
(A*B - B^2)*b^2*c*d)*x)*log((b*e*x + a*e)/(d*x + c))^2 + 6*((2*A^2 - 6*A*B
 + 7*B^2)*b^2*c*d - (2*A^2 - 6*A*B + 7*B^2)*a*b*d^2)*x + 6*((2*A^2 - 6*A*B
 + 7*B^2)*b^2*d^2*x^2 + 2*A^2*b^2*c^2 - 8*(A*B - B^2)*a*b*c*d + (2*A*B - B
^2)*a^2*d^2 + 2*(2*(A^2 - 2*A*B + 2*B^2)*b^2*c*d - (2*A*B - 3*B^2)*a*b*d^2
)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2
*b*c*d^4 - a^3*d^5)*g*i^3*x^2 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c
^2*d^3 - a^3*c*d^4)*g*i^3*x + (b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 -
 a^3*c^2*d^3)*g*i^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1488 vs. $2(333) = 666$.

Time = 4.17 (sec) , antiderivative size = 1488, normalized size of antiderivative = 3.97

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)/(d*i*x+c*i)**3,x)
```

output

```
-B**2*b**2*log(e*(a + b*x)/(c + d*x))**3/(3*a**3*d**3*g*i**3 - 9*a**2*b*c*
d**2*g*i**3 + 9*a*b**2*c**2*d*g*i**3 - 3*b**3*c**3*g*i**3) + b**2*(2*A**2
 - 6*A*B + 7*B**2)*log(x + (2*A**2*a*b**2*d + 2*A**2*b**3*c - 6*A*B*a*b**2*
d - 6*A*B*b**3*c + 7*B**2*a*b**2*d + 7*B**2*b**3*c - a**4*b**2*d**4*(2*A**
2 - 6*A*B + 7*B**2)/(a*d - b*c)**3 + 4*a**3*b**3*c*d**3*(2*A**2 - 6*A*B + 
7*B**2)/(a*d - b*c)**3 - 6*a**2*b**4*c**2*d**2*(2*A**2 - 6*A*B + 7*B**2)/(
a*d - b*c)**3 + 4*a*b**5*c**3*d*(2*A**2 - 6*A*B + 7*B**2)/(a*d - b*c)**3 - 
 b**6*c**4*(2*A**2 - 6*A*B + 7*B**2)/(a*d - b*c)**3)/(4*A**2*b**3*d - 12*A
*B*b**3*d + 14*B**2*b**3*d))/(2*g*i**3*(a*d - b*c)**3) - b**2*(2*A**2 - 6*
A*B + 7*B**2)*log(x + (2*A**2*a*b**2*d + 2*A**2*b**3*c - 6*A*B*a*b**2*d - 
6*A*B*b**3*c + 7*B**2*a*b**2*d + 7*B**2*b**3*c + a**4*b**2*d**4*(2*A**2 - 
6*A*B + 7*B**2)/(a*d - b*c)**3 - 4*a**3*b**3*c*d**3*(2*A**2 - 6*A*B + 7*B*
*2)/(a*d - b*c)**3 + 6*a**2*b**4*c**2*d**2*(2*A**2 - 6*A*B + 7*B**2)/(a*d
 - b*c)**3 - 4*a*b**5*c**3*d*(2*A**2 - 6*A*B + 7*B**2)/(a*d - b*c)**3 + b**
6*c**4*(2*A**2 - 6*A*B + 7*B**2)/(a*d - b*c)**3)/(4*A**2*b**3*d - 12*A*B*b
**3*d + 14*B**2*b**3*d))/(2*g*i**3*(a*d - b*c)**3) + (-2*A*B*a*d + 6*A*B*b
*c + 4*A*B*b*d*x + B**2*a*d - 7*B**2*b*c - 6*B**2*b*d*x)*log(e*(a + b*x)/(
c + d*x))/(2*a**2*c**2*d**2*g*i**3 + 4*a**2*c*d**3*g*i**3*x + 2*a**2*d**4*
g*i**3*x**2 - 4*a*b*c**3*d*g*i**3 - 8*a*b*c**2*d**2*g*i**3*x - 4*a*b*c*d**
3*g*i**3*x**2 + 2*b**2*c**4*g*i**3 + 4*b**2*c**3*d*g*i**3*x + 2*b**2*c*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2116 vs. $2(367) = 734$.

Time $= 0.29$ (sec) , antiderivative size $= 2116$, normalized size of antiderivative $= 5.64$

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algo
rithm="maxima")
```

output

```
1/2*B^2*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g*
i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b^2*c^4 - 2
*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*
c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(b*e*x/(d*x + c) + a*e/
(d*x + c))^2 + A*B*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*g*i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (
b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(b*e*x/(d*x
+ c) + a*e/(d*x + c)) - 1/12*B^2*(6*(7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x
 + 3*b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d
*x + c))*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^3*c^5*g*i^3 - 3*a*b^2*c^4
*d*g*i^3 + 3*a^2*b*c^3*d^2*g*i^3 - a^3*c^2*d^3*g*i^3 + (b^3*c^3*d^2*g*i^3
- 3*a*b^2*c^2*d^3*g*i^3 + 3*a^2*b*c*d^4*g*i^3 - a^3*d^5*g*i^3)*x^2 + 2*(b^
3*c^4*d*g*i^3 - 3*a*b^2*c^3*d^2*g*i^3 + 3*a^2*b*c^2*d^3*g*i^3 - a^3*c*d^4*
g*i^3)*x) - (45*b^2*c^2 - 48*a*b*c*d + 3*a^2*d^2 + 4*(b^2*d^2*x^2 + 2*b...
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.79

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$
$$=\frac{1}{12}\left(\frac{4\,B^2b^2e\log\left(\frac{bex+ae}{dx+c}\right)^3}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}+\frac{12\,A^2b^2e\log\left(\frac{bex+ae}{dx+c}\right)}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}+6\left(\frac{2\,ABb^2e}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}\right.\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algo
rithm="giac")
```

output
```
1/12*(4*B^2*b^2*e*log((b*e*x + a*e)/(d*x + c))^3/(b^2*c^2*g*i^3 - 2*a*b*c*
d*g*i^3 + a^2*d^2*g*i^3) + 12*A^2*b^2*e*log((b*e*x + a*e)/(d*x + c))/(b^2*
c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3) + 6*(2*A*B*b^2*e/(b^2*c^2*g*i
^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3) - 4*(b*e*x + a*e)*B^2*b*d/((b^2*c^2*
g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)) + (b*e*x + a*e)^2*B^2*
d^2/((b^2*c^2*e*g*i^3 - 2*a*b*c*d*e*g*i^3 + a^2*d^2*e*g*i^3)*(d*x + c)^2))
*log((b*e*x + a*e)/(d*x + c))^2 + 6*((2*A*B*d^2 - B^2*d^2)*(b*e*x + a*e)^2
/((b^2*c^2*e*g*i^3 - 2*a*b*c*d*e*g*i^3 + a^2*d^2*e*g*i^3)*(d*x + c)^2) - 8
*(A*B*b*d - B^2*b*d)*(b*e*x + a*e)/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2
*d^2*g*i^3)*(d*x + c)))*log((b*e*x + a*e)/(d*x + c)) + 3*(2*A^2*d^2 - 2*A*
B*d^2 + B^2*d^2)*(b*e*x + a*e)^2/((b^2*c^2*e*g*i^3 - 2*a*b*c*d*e*g*i^3 + a
^2*d^2*e*g*i^3)*(d*x + c)^2) - 24*(A^2*b*d - 2*A*B*b*d + 2*B^2*b*d)*(b*e*x
 + a*e)/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)))*(b*
c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 30.06 (sec) , antiderivative size = 984, normalized size of antiderivative = 2.62

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)*(c*i + d*i*x)^3)
,x)
```

output

```
(b^2*atan((b^2*(A^2 + (7*B^2)/2 - 3*A*B)*(2*a^3*d^3*g*i^3 + 2*b^3*c^3*g*i^
3 - 2*a*b^2*c^2*d*g*i^3 - 2*a^2*b*c*d^2*g*i^3)*1i)/(g*i^3*(a*d - b*c)^3*(2
*A^2*b^2 + 7*B^2*b^2 - 6*A*B*b^2)) + (b^3*d*x*(a^2*d^2*g*i^3 + b^2*c^2*g*i
^3 - 2*a*b*c*d*g*i^3)*(A^2 + (7*B^2)/2 - 3*A*B)*4i)/(g*i^3*(a*d - b*c)^3*(
2*A^2*b^2 + 7*B^2*b^2 - 6*A*B*b^2)))*(A^2 + (7*B^2)/2 - 3*A*B)*2i)/(g*i^3*
(a*d - b*c)^3) - ((2*A^2*a*d - 6*A^2*b*c + B^2*a*d - 15*B^2*b*c - 2*A*B*a*
d + 14*A*B*b*c)/(2*(a*d - b*c)) - (x*(2*A^2*b*d + 7*B^2*b*d - 6*A*B*b*d))/
(a*d - b*c))/(x^2*(2*a*d^3*g*i^3 - 2*b*c*d^2*g*i^3) + x*(4*a*c*d^2*g*i^3 -
 4*b*c^2*d*g*i^3) - 2*b*c^3*g*i^3 + 2*a*c^2*d*g*i^3) - (log((e*(a + b*x))/
(c + d*x))*(B^2/(b*d*g*i^3*(a*d - b*c)) + (B*b^2*((a^2*d^2 + 2*b^2*c^2 - 3
*a*b*c*d)/(2*b^3*d) - (c*(a*d - b*c))/(2*b^2*d))*(2*A - 3*B))/(g*i^3*(a^3*
d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B*x*(2*A - 3*B)*(a*d -
b*c))/(g*i^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((d*x^
2)/b + c^2/(b*d) + (2*c*x)/b) - log((e*(a + b*x))/(c + d*x))^2*(((B^2*b^2*
((a^2*d^2 + 2*b^2*c^2 - 3*a*b*c*d)/(2*b^3*d) - (c*(a*d - b*c))/(2*b^2*d)))
/(g*i^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B^2*x*(a*d
 - b*c))/(g*i^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))/((d*
x^2)/b + c^2/(b*d) + (2*c*x)/b) + (B*b^2*(2*A - 3*B))/(2*g*i^3*(a^3*d^3 -
b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (B^2*b^2*log((e*(a + b*x))/(c
 + d*x))^3)/(3*g*i^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1457, normalized size of antiderivative = 3.89

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)(ci + dix)^3}\, dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x)
```

output

```
(i*( - 12*log(a + b*x)*a**2*b**2*c**3 - 24*log(a + b*x)*a**2*b**2*c**2*d*x
 + 12*log(a + b*x)*a**2*b**2*c**2*d - 12*log(a + b*x)*a**2*b**2*c*d**2*x**
2 + 24*log(a + b*x)*a**2*b**2*c*d**2*x + 12*log(a + b*x)*a**2*b**2*d**3*x*
*2 + 24*log(a + b*x)*a*b**3*c**3 + 48*log(a + b*x)*a*b**3*c**2*d*x - 18*lo
g(a + b*x)*a*b**3*c**2*d + 24*log(a + b*x)*a*b**3*c*d**2*x**2 - 36*log(a +
 b*x)*a*b**3*c*d**2*x - 18*log(a + b*x)*a*b**3*d**3*x**2 - 24*log(a + b*x)
*b**4*c**3 - 48*log(a + b*x)*b**4*c**2*d*x - 24*log(a + b*x)*b**4*c*d**2*x
**2 + 12*log(c + d*x)*a**2*b**2*c**3 + 24*log(c + d*x)*a**2*b**2*c**2*d*x
- 12*log(c + d*x)*a**2*b**2*c**2*d + 12*log(c + d*x)*a**2*b**2*c*d**2*x**2
 - 24*log(c + d*x)*a**2*b**2*c*d**2*x - 12*log(c + d*x)*a**2*b**2*d**3*x**
2 - 24*log(c + d*x)*a*b**3*c**3 - 48*log(c + d*x)*a*b**3*c**2*d*x + 18*log
(c + d*x)*a*b**3*c**2*d - 24*log(c + d*x)*a*b**3*c*d**2*x**2 + 36*log(c +
d*x)*a*b**3*c*d**2*x + 18*log(c + d*x)*a*b**3*d**3*x**2 + 24*log(c + d*x)*
b**4*c**3 + 48*log(c + d*x)*b**4*c**2*d*x + 24*log(c + d*x)*b**4*c*d**2*x*
*2 - 4*log((a*e + b*e*x)/(c + d*x))**3*b**4*c**3 - 8*log((a*e + b*e*x)/(c
+ d*x))**3*b**4*c**2*d*x - 4*log((a*e + b*e*x)/(c + d*x))**3*b**4*c*d**2*x
**2 - 6*log((a*e + b*e*x)/(c + d*x))**2*a**2*b**2*c*d**2 - 12*log((a*e + b
*e*x)/(c + d*x))**2*a*b**3*c**3 - 24*log((a*e + b*e*x)/(c + d*x))**2*a*b**
3*c**2*d*x + 24*log((a*e + b*e*x)/(c + d*x))**2*a*b**3*c**2*d - 12*log((a*
e + b*e*x)/(c + d*x))**2*a*b**3*c*d**2*x**2 + 12*log((a*e + b*e*x)/(c +...
```

3.105 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx$

Optimal result 1288
Mathematica [A] (verified) 1289
Rubi [A] (verified) 1289
Maple [B] (verified) 1291
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Maxima [B] (verification not implemented) 1294
Giac [F] 1295
Mupad [B] (verification not implemented) 1296
Reduce [B] (verification not implemented) 1297

Optimal result

Integrand size = 42, antiderivative size = 525

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = &-\frac{B^2d^3(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2}-\frac{6AbBd^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)}\\&+\frac{6bB^2d^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)}-\frac{2b^3B^2(c+dx)}{(bc-ad)^4g^2i^3(a+bx)}\\&-\frac{6bB^2d^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^4g^2i^3(c+dx)}\\&+\frac{Bd^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2}\\&-\frac{2b^3B(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3(a+bx)}\\&-\frac{d^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^2i^3(c+dx)^2}\\&+\frac{3bd^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^2i^3(c+dx)}\\&-\frac{b^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^2i^3(a+bx)}\\&-\frac{b^2d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{B(bc-ad)^4g^2i^3}\end{aligned}$$

output

```
-1/4*B^2*d^3*(b*x+a)^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2-6*A*b*B*d^2*(b*x+a)/
(-a*d+b*c)^4/g^2/i^3/(d*x+c)+6*b*B^2*d^2*(b*x+a)/(-a*d+b*c)^4/g^2/i^3/(d*x
+c)-2*b^3*B^2*(d*x+c)/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-6*b*B^2*d^2*(b*x+a)*ln(
e*(b*x+a)/(d*x+c))/(-a*d+b*c)^4/g^2/i^3/(d*x+c)+1/2*B*d^3*(b*x+a)^2*(A+B*l
n(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2-2*b^3*B*(d*x+c)*(A+B*
ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-1/2*d^3*(b*x+a)^2*(A+B
*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2+3*b*d^2*(b*x+a)*(
A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)-b^3*(d*x+c)*(A+B
*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-b^2*d*(A+B*ln(e*(b*
x+a)/(d*x+c)))^3/B/(-a*d+b*c)^4/g^2/i^3
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.86

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx =$$

$$-\frac{(2A^2-2AB+B^2)\,d(bc-ad)^2(a+bx)+2b(4A^2-10AB+11B^2)\,d(bc-ad)(a+bx)(c+dx)+4b^2}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d
*i*x)^3),x]
```

output
```
-1/4*((2*A^2 - 2*A*B + B^2)*d*(b*c - a*d)^2*(a + b*x) + 2*b*(4*A^2 - 10*A*
B + 11*B^2)*d*(b*c - a*d)*(a + b*x)*(c + d*x) + 4*b^2*(A^2 + 2*A*B + 2*B^2
)*(b*c - a*d)*(c + d*x)^2 + 6*b^2*(2*A^2 - 2*A*B + 5*B^2)*d*(a + b*x)*(c +
 d*x)^2*Log[a + b*x] + 2*B*(b*c - a*d)*((2*A - B)*d*(b*c - a*d)*(a + b*x)
+ 2*b*(4*A - 5*B)*d*(a + b*x)*(c + d*x) + 4*b^2*(A + B)*(c + d*x)^2)*Log[(
e*(a + b*x))/(c + d*x)] + 2*B*(a^3*B*d^3 - 3*a^2*b*B*d^2*(2*c + d*x) + 3*a
*b^2*d*(2*A*(c + d*x)^2 - B*d*x*(4*c + 3*d*x)) + b^3*(6*A*d*x*(c + d*x)^2
+ B*(2*c^3 + 6*c^2*d*x - 3*d^3*x^3)))*Log[(e*(a + b*x))/(c + d*x)]^2 + 4*b
^2*B^2*d*(a + b*x)*(c + d*x)^2*Log[(e*(a + b*x))/(c + d*x)]^3 - 6*b^2*(2*A
^2 - 2*A*B + 5*B^2)*d*(a + b*x)*(c + d*x)^2*Log[c + d*x])/((b*c - a*d)^4*g
^2*i^3*(a + b*x)*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i^3(bc-ad)^4}$$

$$\downarrow 2795$$

$$\frac{\int\left(\frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^3}{(a+bx)^2}-\frac{3d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^2}{a+bx}+3d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b-\frac{d^3(a+bx)\left(A+B\log\right.}{c+dx}\right.}{g^2i^3(bc-ad)^4}$$

$$\downarrow 2009$$

$$-\frac{b^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}-\frac{2b^3B(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{b^2d\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^3}{B}-\frac{d^3(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(c+dx)^2}$$

input

```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d*i*x)^
3),x]
```

output

```
(-1/4*(B^2*d^3*(a + b*x)^2)/(c + d*x)^2 - (6*A*b*B*d^2*(a + b*x))/(c + d*x
) + (6*b*B^2*d^2*(a + b*x))/(c + d*x) - (2*b^3*B^2*(c + d*x))/(a + b*x) -
(6*b*B^2*d^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(c + d*x) + (B*d^3*(a
 + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(c + d*x)^2) - (2*b^3*B
*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) - (d^3*(a + b*x
)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(c + d*x)^2) + (3*b*d^2*(a
+ b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c + d*x) - (b^3*(c + d*x)*
(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x) - (b^2*d*(A + B*Log[(e*(
a + b*x))/(c + d*x)])^3)/B)/((b*c - a*d)^4*g^2*i^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(519) = 1038$.

Time = 3.41 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.06

method	result	size
parts	Expression too large to display	1080
derivativedivides	Expression too large to display	1233
default	Expression too large to display	1233
parallelrisch	Expression too large to display	1762
norman	Expression too large to display	1852
risch	Expression too large to display	2114

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,method=_RE
TURNVERBOSE)
```

output

```
1/g^2*A^2/i^3*(-1/2*d/(a*d-b*c)^2/(d*x+c)^2+3*d/(a*d-b*c)^4*b^2*ln(d*x+c)+
2*d/(a*d-b*c)^3*b/(d*x+c)+b^2/(a*d-b*c)^3/(b*x+a)-3*d/(a*d-b*c)^4*b^2*ln(b
*x+a))-B^2/g^2/i^3*d/(a*d-b*c)^2/e^2*(d^2/(a*d-b*c)^2*(1/2*(b*e/d+(a*d-b*c
)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2*(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+1/4*(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^2)-3/(a*d-b*c)^2*b*d*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^2-2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/
(d*x+c))+2*(a*d-b*c)*e/d/(d*x+c)+2*b*e/d)+1/(a*d-b*c)^2*b^2*e^2*ln(b*e/d+(
a*d-b*c)*e/d/(d*x+c))^3-1/d/(a*d-b*c)^2*b^3*e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))-2*A*B/g^
2/i^3*d/(a*d-b*c)^2/e^2*(d^2/(a*d-b*c)^2*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-3
*d/(a*d-b*c)^2*b*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)+3/2/(a*d-b*c)^2*b^2*e^2*ln(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^2-1/d/(a*d-b*c)^2*b^3*e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 1008, normalized size of antiderivative = 1.92

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, al
gorithm="fricas")
```

output

```
-1/4*(4*(A^2 + 2*A*B + 2*B^2)*b^3*c^3 + 3*(2*A^2 - 10*A*B + 5*B^2)*a*b^2*c
^2*d - 12*(A^2 - 2*A*B + 2*B^2)*a^2*b*c*d^2 + (2*A^2 - 2*A*B + B^2)*a^3*d^
3 + 4*(B^2*b^3*d^3*x^3 + B^2*a*b^2*c^2*d + (2*B^2*b^3*c*d^2 + B^2*a*b^2*d^
3)*x^2 + (B^2*b^3*c^2*d + 2*B^2*a*b^2*c*d^2)*x)*log((b*e*x + a*e)/(d*x + c
))^3 + 6*((2*A^2 - 2*A*B + 5*B^2)*b^3*c*d^2 - (2*A^2 - 2*A*B + 5*B^2)*a*b^
2*d^3)*x^2 + 2*(3*(2*A*B - B^2)*b^3*d^3*x^3 + 2*B^2*b^3*c^3 + 6*A*B*a*b^2*
c^2*d - 6*B^2*a^2*b*c*d^2 + B^2*a^3*d^3 + 3*(4*A*B*b^3*c*d^2 + (2*A*B - 3*
B^2)*a*b^2*d^3)*x^2 - 3*(B^2*a^2*b*d^3 - 2*(A*B + B^2)*b^3*c^2*d - 4*(A*B
- B^2)*a*b^2*c*d^2)*x)*log((b*e*x + a*e)/(d*x + c))^2 + 3*((6*A^2 - 2*A*B
+ 13*B^2)*b^3*c^2*d - 2*(2*A^2 + 2*A*B + 3*B^2)*a*b^2*c*d^2 - (2*A^2 - 6*A
*B + 7*B^2)*a^2*b*d^3)*x + 2*(3*(2*A^2 - 2*A*B + 5*B^2)*b^3*d^3*x^3 + 6*A^
2*a*b^2*c^2*d + 4*(A*B + B^2)*b^3*c^3 - 12*(A*B - B^2)*a^2*b*c*d^2 + (2*A*
B - B^2)*a^3*d^3 + 3*(4*(A^2 + 2*B^2)*b^3*c*d^2 + (2*A^2 - 6*A*B + 7*B^2)*
a*b^2*d^3)*x^2 + 3*(2*(A^2 + 2*A*B + 2*B^2)*b^3*c^2*d + 4*(A^2 - 2*A*B + 2
*B^2)*a*b^2*c*d^2 - (2*A*B - 3*B^2)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x +
 c)))/((b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^
5 + a^4*b*d^6)*g^2*i^3*x^3 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^
3*d^3 - 2*a^3*b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*g^2*i^3*x^2 + (b^5*c^
6 - 2*a*b^4*c^5*d - 2*a^2*b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^
4 + 2*a^5*c*d^5)*g^2*i^3*x + (a*b^4*c^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2683 vs. $2(484) = 968$.

Time $= 27.49$ (sec) , antiderivative size $= 2683$, normalized size of antiderivative $= 5.11$

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2/(d*i*x+c*i)**3,x)
```

output

```
-B**2*b**2*d*log(e*(a + b*x)/(c + d*x))**3/(a**4*d**4*g**2*i**3 - 4*a**3*b
*c*d**3*g**2*i**3 + 6*a**2*b**2*c**2*d**2*g**2*i**3 - 4*a*b**3*c**3*d*g**2
*i**3 + b**4*c**4*g**2*i**3) + 3*b**2*d*(2*A**2 - 2*A*B + 5*B**2)*log(x +
(6*A**2*a*b**2*d**2 + 6*A**2*b**3*c*d - 6*A*B*a*b**2*d**2 - 6*A*B*b**3*c*d
 + 15*B**2*a*b**2*d**2 + 15*B**2*b**3*c*d - 3*a**5*b**2*d**6*(2*A**2 - 2*A
*B + 5*B**2)/(a*d - b*c)**4 + 15*a**4*b**3*c*d**5*(2*A**2 - 2*A*B + 5*B**2
)/(a*d - b*c)**4 - 30*a**3*b**4*c**2*d**4*(2*A**2 - 2*A*B + 5*B**2)/(a*d -
 b*c)**4 + 30*a**2*b**5*c**3*d**3*(2*A**2 - 2*A*B + 5*B**2)/(a*d - b*c)**4
 - 15*a*b**6*c**4*d**2*(2*A**2 - 2*A*B + 5*B**2)/(a*d - b*c)**4 + 3*b**7*c
**5*d*(2*A**2 - 2*A*B + 5*B**2)/(a*d - b*c)**4)/(12*A**2*b**3*d**2 - 12*A*
B*b**3*d**2 + 30*B**2*b**3*d**2))/(2*g**2*i**3*(a*d - b*c)**4) - 3*b**2*d*
(2*A**2 - 2*A*B + 5*B**2)*log(x + (6*A**2*a*b**2*d**2 + 6*A**2*b**3*c*d -
6*A*B*a*b**2*d**2 - 6*A*B*b**3*c*d + 15*B**2*a*b**2*d**2 + 15*B**2*b**3*c*
d + 3*a**5*b**2*d**6*(2*A**2 - 2*A*B + 5*B**2)/(a*d - b*c)**4 - 15*a**4*b*
*3*c*d**5*(2*A**2 - 2*A*B + 5*B**2)/(a*d - b*c)**4 + 30*a**3*b**4*c**2*d**
4*(2*A**2 - 2*A*B + 5*B**2)/(a*d - b*c)**4 - 30*a**2*b**5*c**3*d**3*(2*A**
2 - 2*A*B + 5*B**2)/(a*d - b*c)**4 + 15*a*b**6*c**4*d**2*(2*A**2 - 2*A*B +
 5*B**2)/(a*d - b*c)**4 - 3*b**7*c**5*d*(2*A**2 - 2*A*B + 5*B**2)/(a*d - b
*c)**4)/(12*A**2*b**3*d**2 - 12*A*B*b**3*d**2 + 30*B**2*b**3*d**2))/(2*g**
2*i**3*(a*d - b*c)**4) + (-2*A*B*a**2*d**2 + 10*A*B*a*b*c*d + 6*A*B*a*b...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4188 vs. $2(519) = 1038$.

Time $= 0.56$ (sec) , antiderivative size $= 4188$, normalized size of antiderivative $= 7.98$

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^2(ci + dix)^3}\, dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, al
gorithm="maxima")
```

output

```
-1/2*B^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d
+ a*b*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^
5)*g^2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*
b*c*d^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^
2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*
d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c
^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3)
 - 6*b^2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4)*g^2*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2 -
A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b
*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*g^
2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d
^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5
*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3
*a^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c^4 -
4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3) - 6*
b^2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4)*g^2*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/4*B^2
*(2*(4*b^3*c^3 - 15*a*b^2*c^2*d + 12*a^2*b*c*d^2 - a^3*d^3 - 6*(b^3*c*d^2
- a*b^2*d^3)*x^2 - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*...
```

Giac [F]

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2(dix+ci)^3}\,dx$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, al
gorithm="giac")
```

output

```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/((b*g*x + a*g)^2*(d*i*x + c
*i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 32.62 (sec) , antiderivative size = 1505, normalized size of antiderivative = 2.87

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^2*(c*i + d*i*x)^
3),x)
```

output

```
((4*A^2*b^2*c^2 - 2*A^2*a^2*d^2 - B^2*a^2*d^2 + 8*B^2*b^2*c^2 + 2*A*B*a^2*
d^2 + 8*A*B*b^2*c^2 + 10*A^2*a*b*c*d + 23*B^2*a*b*c*d - 22*A*B*a*b*c*d)/(2
*(a*d - b*c)) + (3*x^2*(2*A^2*b^2*d^2 + 5*B^2*b^2*d^2 - 2*A*B*b^2*d^2))/(a
*d - b*c) + (3*x*(2*A^2*a*b*d^2 + 7*B^2*a*b*d^2 + 6*A^2*b^2*c*d + 13*B^2*b
^2*c*d - 6*A*B*a*b*d^2 - 2*A*B*b^2*c*d))/(2*(a*d - b*c)))/(x*(2*b^3*c^4*g^
2*i^3 + 4*a^3*c*d^3*g^2*i^3 - 6*a^2*b*c^2*d^2*g^2*i^3) + x^2*(2*a^3*d^4*g^
2*i^3 + 4*b^3*c^3*d*g^2*i^3 - 6*a*b^2*c^2*d^2*g^2*i^3) + x^3*(2*b^3*c^2*d^
2*g^2*i^3 + 2*a^2*b*d^4*g^2*i^3 - 4*a*b^2*c*d^3*g^2*i^3) + 2*a^3*c^2*d^2*g
^2*i^3 + 2*a*b^2*c^4*g^2*i^3 - 4*a^2*b*c^3*d*g^2*i^3) - log((e*(a + b*x))/
(c + d*x))^2*((x*((3*B^2)/(2*g^2*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (3
*B^2*(a*d + b*c))/(g^2*i^3*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) +
 (B^2*(a*d + 2*b*c))/(2*g^2*i^3*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) -
 (3*B^2*a*c)/(g^2*i^3*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (3*B^
2*b*d*x^2)/(g^2*i^3*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(d*x^3 +
 (a*c^2)/(b*d) + (x^2*(a*d^2 + 2*b*c*d))/(b*d) + (x*(b*c^2 + 2*a*c*d))/(b*
d)) + (3*B*b^2*d*(2*A - B))/(2*g^2*i^3*(a*d - b*c)^2*(a^2*d^2 + b^2*c^2 -
2*a*b*c*d))) - (log((e*(a + b*x))/(c + d*x))*(x*((3*(B^2 + 2*A*B))/(2*g^2*
i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (3*B*(2*A - B)*(a*d + b*c))/(g^2*i^
3*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) + (4*B^2*b*c - B^2*a*d + 2
*A*B*a*d + 4*A*B*b*c)/(2*g^2*i^3*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 3298, normalized size of antiderivative = 6.28

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x)
```

output

```
(i*( - 12*log(a + b*x)*a**4*b**2*c**2*d**2 - 24*log(a + b*x)*a**4*b**2*c*d
**3*x - 12*log(a + b*x)*a**4*b**2*d**4*x**2 - 24*log(a + b*x)*a**3*b**3*c*
*3*d - 60*log(a + b*x)*a**3*b**3*c**2*d**2*x + 36*log(a + b*x)*a**3*b**3*c
**2*d**2 - 48*log(a + b*x)*a**3*b**3*c*d**3*x**2 + 72*log(a + b*x)*a**3*b*
*3*c*d**3*x - 12*log(a + b*x)*a**3*b**3*d**4*x**3 + 36*log(a + b*x)*a**3*b
**3*d**4*x**2 - 24*log(a + b*x)*a**2*b**4*c**3*d*x - 48*log(a + b*x)*a**2*
b**4*c**2*d**2*x**2 + 36*log(a + b*x)*a**2*b**4*c**2*d**2*x - 42*log(a + b
*x)*a**2*b**4*c**2*d**2 - 24*log(a + b*x)*a**2*b**4*c*d**3*x**3 + 72*log(a
 + b*x)*a**2*b**4*c*d**3*x**2 - 84*log(a + b*x)*a**2*b**4*c*d**3*x + 36*lo
g(a + b*x)*a**2*b**4*d**4*x**3 - 42*log(a + b*x)*a**2*b**4*d**4*x**2 - 48*
log(a + b*x)*a*b**5*c**3*d - 138*log(a + b*x)*a*b**5*c**2*d**2*x - 132*log
(a + b*x)*a*b**5*c*d**3*x**2 - 42*log(a + b*x)*a*b**5*d**4*x**3 - 48*log(a
 + b*x)*b**6*c**3*d*x - 96*log(a + b*x)*b**6*c**2*d**2*x**2 - 48*log(a + b
*x)*b**6*c*d**3*x**3 + 12*log(c + d*x)*a**4*b**2*c**2*d**2 + 24*log(c + d*
x)*a**4*b**2*c*d**3*x + 12*log(c + d*x)*a**4*b**2*d**4*x**2 + 24*log(c + d
*x)*a**3*b**3*c**3*d + 60*log(c + d*x)*a**3*b**3*c**2*d**2*x - 36*log(c + 
d*x)*a**3*b**3*c**2*d**2 + 48*log(c + d*x)*a**3*b**3*c*d**3*x**2 - 72*log(
c + d*x)*a**3*b**3*c*d**3*x + 12*log(c + d*x)*a**3*b**3*d**4*x**3 - 36*log
(c + d*x)*a**3*b**3*d**4*x**2 + 24*log(c + d*x)*a**2*b**4*c**3*d*x + 48*lo
g(c + d*x)*a**2*b**4*c**2*d**2*x**2 - 36*log(c + d*x)*a**2*b**4*c**2*d*...
```

3.106 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx$

Optimal result 1299
Mathematica [A] (verified) 1300
Rubi [A] (verified) 1301
Maple [B] (verified) 1303
Fricas [B] (verification not implemented) 1304
Sympy [B] (verification not implemented) 1305
Maxima [B] (verification not implemented) 1306
Giac [F] 1307
Mupad [B] (verification not implemented) 1308
Reduce [B] (verification not implemented) 1309

Optimal result

Integrand size = 42, antiderivative size = 685

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = {} & \frac{B^2d^4(a+bx)^2}{4(bc-ad)^5g^3i^3(c+dx)^2}+\frac{8AbBd^3(a+bx)}{(bc-ad)^5g^3i^3(c+dx)}\\ & -\frac{8bB^2d^3(a+bx)}{(bc-ad)^5g^3i^3(c+dx)}+\frac{8b^3B^2d(c+dx)}{(bc-ad)^5g^3i^3(a+bx)}\\ & -\frac{b^4B^2(c+dx)^2}{4(bc-ad)^5g^3i^3(a+bx)^2}\\ & +\frac{8bB^2d^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^5g^3i^3(c+dx)}\\ & -\frac{Bd^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2}\\ & +\frac{8b^3Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3(a+bx)}\\ & -\frac{b^4B(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2}\\ & +\frac{d^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^5g^3i^3(c+dx)^2}\\ & -\frac{4bd^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^3i^3(c+dx)}\\ & +\frac{4b^3d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^3i^3(a+bx)}\\ & -\frac{b^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^5g^3i^3(a+bx)^2}\\ & +\frac{2b^2d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{B(bc-ad)^5g^3i^3}\end{aligned}$$

output

```
1/4*B^2*d^4*(b*x+a)^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2+8*A*b*B*d^3*(b*x+a)/(
-a*d+b*c)^5/g^3/i^3/(d*x+c)-8*b*B^2*d^3*(b*x+a)/(-a*d+b*c)^5/g^3/i^3/(d*x+
c)+8*b^3*B^2*d*(d*x+c)/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/4*b^4*B^2*(d*x+c)^2/
(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+8*b*B^2*d^3*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(
-a*d+b*c)^5/g^3/i^3/(d*x+c)-1/2*B*d^4*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c))
)/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2+8*b^3*B*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+
c)))/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/2*b^4*B*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d
*x+c)))/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+1/2*d^4*(b*x+a)^2*(A+B*ln(e*(b*x+a)
/(d*x+c)))^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2-4*b*d^3*(b*x+a)*(A+B*ln(e*(b*x
+a)/(d*x+c)))^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)+4*b^3*d*(d*x+c)*(A+B*ln(e*(b*
x+a)/(d*x+c)))^2/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/2*b^4*(d*x+c)^2*(A+B*ln(e*
(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+2*b^2*d^2*(A+B*ln(e*(b*
x+a)/(d*x+c)))^3/B/(-a*d+b*c)^5/g^3/i^3
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 611, normalized size of antiderivative = 0.89

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx$$

$$=\frac{(2A^2-2AB+B^2)\,d^2(bc-ad)^2(a+bx)^2+2b(6A^2-14AB+15B^2)\,d^2(bc-ad)(a+bx)^2(c+dx)-b^2$$

input

```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d
*i*x)^3),x]
```

output

```
((2*A^2 - 2*A*B + B^2)*d^2*(b*c - a*d)^2*(a + b*x)^2 + 2*b*(6*A^2 - 14*A*B
 + 15*B^2)*d^2*(b*c - a*d)*(a + b*x)^2*(c + d*x) - b^2*(2*A^2 + 2*A*B + B^
2)*(b*c - a*d)^2*(c + d*x)^2 + 2*b^2*(6*A^2 + 14*A*B + 15*B^2)*d*(b*c - a*
d)*(a + b*x)*(c + d*x)^2 + 12*b^2*(2*A^2 + 5*B^2)*d^2*(a + b*x)^2*(c + d*x
)^2*Log[a + b*x] + 2*B*(b*c - a*d)*((2*A - B)*d^2*(b*c - a*d)*(a + b*x)^2
+ 2*b*(6*A - 7*B)*d^2*(a + b*x)^2*(c + d*x) - b^2*(2*A + B)*(b*c - a*d)*(c
 + d*x)^2 + 2*b^2*(6*A + 7*B)*d*(a + b*x)*(c + d*x)^2)*Log[(e*(a + b*x))/(
c + d*x)] - 2*B*(-(a^4*B*d^4) + 4*a^3*b*B*d^3*(2*c + d*x) - 6*a^2*b^2*d^2*
(2*A*(c + d*x)^2 - B*d*x*(4*c + 3*d*x)) + b^4*(-12*A*d^2*x^2*(c + d*x)^2 +
 B*c*(c^3 - 4*c^2*d*x - 18*c*d^2*x^2 - 12*d^3*x^3)) - 4*a*b^3*d*(6*A*d*x*(
c + d*x)^2 + B*(2*c^3 + 6*c^2*d*x - 3*d^3*x^3)))*Log[(e*(a + b*x))/(c + d*
x)]^2 + 8*b^2*B^2*d^2*(a + b*x)^2*(c + d*x)^2*Log[(e*(a + b*x))/(c + d*x)]
^3 - 12*b^2*(2*A^2 + 5*B^2)*d^2*(a + b*x)^2*(c + d*x)^2*Log[c + d*x])/(4*(
b*c - a*d)^5*g^3*i^3*(a + b*x)^2*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 478, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3}\,d\frac{a+bx}{c+dx}}{g^3i^3(bc-ad)^5}$$

$$\downarrow 2795$$

$$\frac{\int \left(\frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 b^4}{(a+bx)^3}-\frac{4d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 b^3}{(a+bx)^2}+\frac{6d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 b^2}{a+bx}-4d^3\left(A+B\log\right.\right.}{g^3i^3(bc-ad)^5}$$

$$\downarrow 2009$$

$$-\frac{b^4(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}-\frac{b^4B(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{4b^3d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}+\frac{8b^3Bd(c+dx)\left(B\log\left(\frac{e}{}\right.\right.}{a+bx}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d*i*x)^
3),x]
```

output
```
((B^2*d^4*(a + b*x)^2)/(4*(c + d*x)^2) + (8*A*b*B*d^3*(a + b*x))/(c + d*x)
 - (8*b*B^2*d^3*(a + b*x))/(c + d*x) + (8*b^3*B^2*d*(c + d*x))/(a + b*x) -
 (b^4*B^2*(c + d*x)^2)/(4*(a + b*x)^2) + (8*b*B^2*d^3*(a + b*x)*Log[(e*(a
+ b*x))/(c + d*x)])/(c + d*x) - (B*d^4*(a + b*x)^2*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]))/(2*(c + d*x)^2) + (8*b^3*B*d*(c + d*x)*(A + B*Log[(e*(a + b
*x))/(c + d*x)]))/(a + b*x) - (b^4*B*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/
(c + d*x)]))/(2*(a + b*x)^2) + (d^4*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(
c + d*x)])^2)/(2*(c + d*x)^2) - (4*b*d^3*(a + b*x)*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])^2)/(c + d*x) + (4*b^3*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(
c + d*x)])^2)/(a + b*x) - (b^4*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d
*x)])^2)/(2*(a + b*x)^2) + (2*b^2*d^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
^3)/B)/((b*c - a*d)^5*g^3*i^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. $2(673) = 1346$.

Time = 3.04 (sec) , antiderivative size = 1396, normalized size of antiderivative = 2.04

method	result	size
parts	Expression too large to display	1396
derivativedivides	Expression too large to display	1589
default	Expression too large to display	1589
risch	Expression too large to display	2582
parallelrisch	Expression too large to display	2630
norman	Expression too large to display	2758

input

```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,method=_RE
TURNVERBOSE)
```

output

```
A^2/g^3/i^3*(-1/2*d^2/(a*d-b*c)^3/(d*x+c)^2+6*d^2/(a*d-b*c)^5*b^2*ln(d*x+c
)+3*d^2/(a*d-b*c)^4*b/(d*x+c)+1/2*b^2/(a*d-b*c)^3/(b*x+a)^2-6*d^2/(a*d-b*c
)^5*b^2*ln(b*x+a)+3*b^2/(a*d-b*c)^4*d/(b*x+a))-B^2/g^3/i^3*d/(a*d-b*c)^2/e
^2*(d^3/(a*d-b*c)^3*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c
)*e/d/(d*x+c))^2-1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))+1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-4*d^2/(a*d-b*c)^3*b*e*((b*
e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*(b*e/d+(a*d
-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*(a*d-b*c)*e/d/(d*x+c)
+2*b*e/d)+2*d/(a*d-b*c)^3*b^2*e^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3-4/(a*d
-b*c)^3*b^3*e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-
2/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+1/d/(a*d-b*c)^3*b^4*e^4*(-1/2/(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c
)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/
(d*x+c))^2))-2*A*B/g^3/i^3*d/(a*d-b*c)^2/e^2*(d^3/(a*d-b*c)^3*(1/2*(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^2)-4*d^2/(a*d-b*c)^3*b*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)+3*d/(a*d-b*c)^
3*b^2*e^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-4/(a*d-b*c)^3*b^3*e^3*(-1/(b*e
/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1517 vs. $2(673) = 1346$.

Time $= 0.12$ (sec) , antiderivative size $= 1517$, normalized size of antiderivative $= 2.21$

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, al
gorithm="fricas")
```

output

```
-1/4*(60*A*B*a^2*b^2*c^2*d^2 + (2*A^2 + 2*A*B + B^2)*b^4*c^4 - 16*(A^2 + 2
*A*B + 2*B^2)*a*b^3*c^3*d + 16*(A^2 - 2*A*B + 2*B^2)*a^3*b*c*d^3 - (2*A^2
 - 2*A*B + B^2)*a^4*d^4 - 12*((2*A^2 + 5*B^2)*b^4*c*d^3 - (2*A^2 + 5*B^2)*a
*b^3*d^4)*x^3 - 8*(B^2*b^4*d^4*x^4 + B^2*a^2*b^2*c^2*d^2 + 2*(B^2*b^4*c*d^
3 + B^2*a*b^3*d^4)*x^3 + (B^2*b^4*c^2*d^2 + 4*B^2*a*b^3*c*d^3 + B^2*a^2*b^
2*d^4)*x^2 + 2*(B^2*a*b^3*c^2*d^2 + B^2*a^2*b^2*c*d^3)*x)*log((b*e*x + a*e
)/(d*x + c))^3 + 6*(8*A*B*a*b^3*c*d^3 - (6*A^2 + 4*A*B + 15*B^2)*b^4*c^2*d
^2 + (6*A^2 - 4*A*B + 15*B^2)*a^2*b^2*d^4)*x^2 - 2*(12*A*B*b^4*d^4*x^4 - B
^2*b^4*c^4 + 8*B^2*a*b^3*c^3*d + 12*A*B*a^2*b^2*c^2*d^2 - 8*B^2*a^3*b*c*d^
3 + B^2*a^4*d^4 + 12*((2*A*B + B^2)*b^4*c*d^3 + (2*A*B - B^2)*a*b^3*d^4)*x
^3 + 6*(8*A*B*a*b^3*c*d^3 + (2*A*B + 3*B^2)*b^4*c^2*d^2 + (2*A*B - 3*B^2)*
a^2*b^2*d^4)*x^2 + 4*(B^2*b^4*c^3*d - B^2*a^3*b*d^4 + 6*(A*B + B^2)*a*b^3*
c^2*d^2 + 6*(A*B - B^2)*a^2*b^2*c*d^3)*x)*log((b*e*x + a*e)/(d*x + c))^2 -
 4*((2*A^2 + 6*A*B + 7*B^2)*b^4*c^3*d + 6*(2*A^2 - A*B + 4*B^2)*a*b^3*c^2*
d^2 - 6*(2*A^2 + A*B + 4*B^2)*a^2*b^2*c*d^3 - (2*A^2 - 6*A*B + 7*B^2)*a^3*
b*d^4)*x - 2*(6*(2*A^2 + 5*B^2)*b^4*d^4*x^4 + 12*A^2*a^2*b^2*c^2*d^2 - (2*
A*B + B^2)*b^4*c^4 + 16*(A*B + B^2)*a*b^3*c^3*d - 16*(A*B - B^2)*a^3*b*c*d
^3 + (2*A*B - B^2)*a^4*d^4 + 12*((2*A^2 + 2*A*B + 5*B^2)*b^4*c*d^3 + (2*A^
2 - 2*A*B + 5*B^2)*a*b^3*d^4)*x^3 + 6*((2*A^2 + 6*A*B + 7*B^2)*b^4*c^2*d^2
 + 8*(A^2 + 2*B^2)*a*b^3*c*d^3 + (2*A^2 - 6*A*B + 7*B^2)*a^2*b^2*d^4)*x...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3720 vs. $2(638) = 1276$.

Time $= 50.01$ (sec) , antiderivative size $= 3720$, normalized size of antiderivative $= 5.43$

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3/(d*i*x+c*i)**3,x)
```

output

```
-2*B**2*b**2*d**2*log(e*(a + b*x)/(c + d*x))**3/(a**5*d**5*g**3*i**3 - 5*a
**4*b*c*d**4*g**3*i**3 + 10*a**3*b**2*c**2*d**3*g**3*i**3 - 10*a**2*b**3*c
**3*d**2*g**3*i**3 + 5*a*b**4*c**4*d*g**3*i**3 - b**5*c**5*g**3*i**3) + 3*
b**2*d**2*(2*A**2 + 5*B**2)*log(x + (6*A**2*a*b**2*d**3 + 6*A**2*b**3*c*d*
*2 + 15*B**2*a*b**2*d**3 + 15*B**2*b**3*c*d**2 - 3*a**6*b**2*d**8*(2*A**2
+ 5*B**2)/(a*d - b*c)**5 + 18*a**5*b**3*c*d**7*(2*A**2 + 5*B**2)/(a*d - b*
c)**5 - 45*a**4*b**4*c**2*d**6*(2*A**2 + 5*B**2)/(a*d - b*c)**5 + 60*a**3*
b**5*c**3*d**5*(2*A**2 + 5*B**2)/(a*d - b*c)**5 - 45*a**2*b**6*c**4*d**4*(
2*A**2 + 5*B**2)/(a*d - b*c)**5 + 18*a*b**7*c**5*d**3*(2*A**2 + 5*B**2)/(a
*d - b*c)**5 - 3*b**8*c**6*d**2*(2*A**2 + 5*B**2)/(a*d - b*c)**5)/(12*A**2
*b**3*d**3 + 30*B**2*b**3*d**3))/(g**3*i**3*(a*d - b*c)**5) - 3*b**2*d**2*
(2*A**2 + 5*B**2)*log(x + (6*A**2*a*b**2*d**3 + 6*A**2*b**3*c*d**2 + 15*B*
*2*a*b**2*d**3 + 15*B**2*b**3*c*d**2 + 3*a**6*b**2*d**8*(2*A**2 + 5*B**2)/
(a*d - b*c)**5 - 18*a**5*b**3*c*d**7*(2*A**2 + 5*B**2)/(a*d - b*c)**5 + 45
*a**4*b**4*c**2*d**6*(2*A**2 + 5*B**2)/(a*d - b*c)**5 - 60*a**3*b**5*c**3*
d**5*(2*A**2 + 5*B**2)/(a*d - b*c)**5 + 45*a**2*b**6*c**4*d**4*(2*A**2 + 5
*B**2)/(a*d - b*c)**5 - 18*a*b**7*c**5*d**3*(2*A**2 + 5*B**2)/(a*d - b*c)*
*5 + 3*b**8*c**6*d**2*(2*A**2 + 5*B**2)/(a*d - b*c)**5)/(12*A**2*b**3*d**3
 + 30*B**2*b**3*d**3))/(g**3*i**3*(a*d - b*c)**5) + (-2*A*B*a**3*d**3 + 14
*A*B*a**2*b*c*d**2 + 8*A*B*a**2*b*d**3*x + 14*A*B*a*b**2*c**2*d + 56*A*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5583 vs. $2(673) = 1346$.

Time $= 0.42$ (sec) , antiderivative size $= 5583$, normalized size of antiderivative $= 8.15$

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, al
gorithm="maxima")
```

output

```
1/2*B^2*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d
^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b
*d^3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c
*d^5 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b
^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3
+ (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 +
a^6*d^6)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2
+ 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^
6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g
^3*i^3) + 12*b^2*d^2*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c
^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d
^2*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^
2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3))*log(b*e*x/(d*x + c) + a*e/(
d*x + c))^2 + A*B*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d
^2 - a^3*d^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d
^2 + a^2*b*d^3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4
*a^3*b^3*c*d^5 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2
 + 2*a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^
3*i^3*x^3 + (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*
c^2*d^4 + a^6*d^6)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3...
```

Giac [F]

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3(dix+ci)^3}\,dx$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, al
gorithm="giac")
```

output

```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/((b*g*x + a*g)^3*(d*i*x + c
*i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 36.22 (sec) , antiderivative size = 2155, normalized size of antiderivative = 3.15

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^3*(c*i + d*i*x)^
3),x)
```

output

```
((2*x*(2*A^2*a^2*b*d^3 + 7*B^2*a^2*b*d^3 + 2*A^2*b^3*c^2*d + 7*B^2*b^3*c^2
*d + 14*A^2*a*b^2*c*d^2 + 31*B^2*a*b^2*c*d^2 - 6*A*B*a^2*b*d^3 + 6*A*B*b^3
*c^2*d))/(a*d - b*c) - (2*A^2*a^3*d^3 + 2*A^2*b^3*c^3 + B^2*a^3*d^3 + B^2*
b^3*c^3 - 2*A*B*a^3*d^3 + 2*A*B*b^3*c^3 - 14*A^2*a*b^2*c^2*d - 14*A^2*a^2*
b*c*d^2 - 31*B^2*a*b^2*c^2*d - 31*B^2*a^2*b*c*d^2 - 30*A*B*a*b^2*c^2*d + 3
0*A*B*a^2*b*c*d^2)/(2*(a*d - b*c)) + (6*x^3*(2*A^2*b^3*d^3 + 5*B^2*b^3*d^3
))/(a*d - b*c) + (3*x^2*(6*A^2*a*b^2*d^3 + 15*B^2*a*b^2*d^3 + 6*A^2*b^3*c*
d^2 + 15*B^2*b^3*c*d^2 - 4*A*B*a*b^2*d^3 + 4*A*B*b^3*c*d^2))/(a*d - b*c))/
(x^4*(2*a^3*b^2*d^5*g^3*i^3 - 2*b^5*c^3*d^2*g^3*i^3 + 6*a*b^4*c^2*d^3*g^3*
i^3 - 6*a^2*b^3*c*d^4*g^3*i^3) - x*(4*a*b^4*c^5*g^3*i^3 - 4*a^5*c*d^4*g^3*
i^3 - 8*a^2*b^3*c^4*d*g^3*i^3 + 8*a^4*b*c^2*d^3*g^3*i^3) + x^3*(4*a^4*b*d^
5*g^3*i^3 - 4*b^5*c^4*d*g^3*i^3 + 8*a*b^4*c^3*d^2*g^3*i^3 - 8*a^3*b^2*c*d^
4*g^3*i^3) + x^2*(2*a^5*d^5*g^3*i^3 - 2*b^5*c^5*g^3*i^3 - 2*a*b^4*c^4*d*g^
3*i^3 + 2*a^4*b*c*d^4*g^3*i^3 + 16*a^2*b^3*c^3*d^2*g^3*i^3 - 16*a^3*b^2*c^
2*d^3*g^3*i^3) - 2*a^2*b^3*c^5*g^3*i^3 + 2*a^5*c^2*d^3*g^3*i^3 + 6*a^3*b^2
*c^4*d*g^3*i^3 - 6*a^4*b*c^3*d^2*g^3*i^3) + log((e*(a + b*x))/(c + d*x))^2
*((x*((3*B^2*(a*d + b*c)^2)/(g^3*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) -
B^2/(g^3*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (6*B^2*a*b*c*d)/(g^3*i^3*(
a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)) - (B^2*(a*d + b*c))/(2*g^3*i^3*(a^2*b*d
^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (6*B^2*b^2*d^2*x^3)/(g^3*i^3*(a^2*d^...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 4870, normalized size of antiderivative = 7.11

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x)
```

output

```
(i*( - 24*log(a + b*x)*a**5*b**2*c**2*d**3 - 48*log(a + b*x)*a**5*b**2*c*d
**4*x - 24*log(a + b*x)*a**5*b**2*d**5*x**2 - 24*log(a + b*x)*a**4*b**3*c*
*3*d**2 - 96*log(a + b*x)*a**4*b**3*c**2*d**3*x + 24*log(a + b*x)*a**4*b**
3*c**2*d**3 - 120*log(a + b*x)*a**4*b**3*c*d**4*x**2 + 48*log(a + b*x)*a**
4*b**3*c*d**4*x - 48*log(a + b*x)*a**4*b**3*d**5*x**3 + 24*log(a + b*x)*a*
*4*b**3*d**5*x**2 - 48*log(a + b*x)*a**3*b**4*c**3*d**2*x - 24*log(a + b*x
)*a**3*b**4*c**3*d**2 - 120*log(a + b*x)*a**3*b**4*c**2*d**3*x**2 - 60*log
(a + b*x)*a**3*b**4*c**2*d**3 - 96*log(a + b*x)*a**3*b**4*c*d**4*x**3 + 72
*log(a + b*x)*a**3*b**4*c*d**4*x**2 - 120*log(a + b*x)*a**3*b**4*c*d**4*x
 - 24*log(a + b*x)*a**3*b**4*d**5*x**4 + 48*log(a + b*x)*a**3*b**4*d**5*x**
3 - 60*log(a + b*x)*a**3*b**4*d**5*x**2 - 24*log(a + b*x)*a**2*b**5*c**3*d
**2*x**2 - 48*log(a + b*x)*a**2*b**5*c**3*d**2*x - 60*log(a + b*x)*a**2*b*
*5*c**3*d**2 - 48*log(a + b*x)*a**2*b**5*c**2*d**3*x**3 - 72*log(a + b*x)*
a**2*b**5*c**2*d**3*x**2 - 240*log(a + b*x)*a**2*b**5*c**2*d**3*x - 24*log
(a + b*x)*a**2*b**5*c*d**4*x**4 - 300*log(a + b*x)*a**2*b**5*c*d**4*x**2 +
 24*log(a + b*x)*a**2*b**5*d**5*x**4 - 120*log(a + b*x)*a**2*b**5*d**5*x**
3 - 24*log(a + b*x)*a*b**6*c**3*d**2*x**2 - 120*log(a + b*x)*a*b**6*c**3*d
**2*x - 48*log(a + b*x)*a*b**6*c**2*d**3*x**3 - 300*log(a + b*x)*a*b**6*c*
*2*d**3*x**2 - 24*log(a + b*x)*a*b**6*c*d**4*x**4 - 240*log(a + b*x)*a*b**
6*c*d**4*x**3 - 60*log(a + b*x)*a*b**6*d**5*x**4 - 60*log(a + b*x)*b**7...
```

3.107 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx$

Optimal result 1311
Mathematica [A] (verified) 1312
Rubi [A] (verified) 1313
Maple [B] (verified) 1315
Fricas [B] (verification not implemented) 1316
Sympy [F(-1)] 1317
Maxima [B] (verification not implemented) 1318
Giac [F] 1319
Mupad [B] (verification not implemented) 1319
Reduce [B] (verification not implemented) 1320

Optimal result

Integrand size = 42, antiderivative size = 851

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = &-\frac{B^2d^5(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2}-\frac{10AbBd^4(a+bx)}{(bc-ad)^6g^4i^3(c+dx)}\\&+\frac{10bB^2d^4(a+bx)}{(bc-ad)^6g^4i^3(c+dx)}-\frac{20b^3B^2d^2(c+dx)}{(bc-ad)^6g^4i^3(a+bx)}\\&+\frac{5b^4B^2d(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2}\\&-\frac{2b^5B^2(c+dx)^3}{27(bc-ad)^6g^4i^3(a+bx)^3}\\&-\frac{10bB^2d^4(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^6g^4i^3(c+dx)}\\&+\frac{Bd^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2}\\&-\frac{20b^3Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(a+bx)}\\&+\frac{5b^4Bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2}\\&-\frac{2b^5B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^6g^4i^3(a+bx)^3}\\&-\frac{d^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^6g^4i^3(c+dx)^2}\\&+\frac{5bd^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^6g^4i^3(c+dx)}\\&-\frac{10b^3d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^6g^4i^3(a+bx)}\\&+\frac{5b^4d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^6g^4i^3(a+bx)^2}\\&-\frac{b^5(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^6g^4i^3(a+bx)^3}\\&-\frac{10b^2d^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^6g^4i^3}\end{aligned}$$

output

```
-1/4*B^2*d^5*(b*x+a)^2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2-10*A*b*B*d^4*(b*x+a)
/(-a*d+b*c)^6/g^4/i^3/(d*x+c)+10*b*B^2*d^4*(b*x+a)/(-a*d+b*c)^6/g^4/i^3/(d
*x+c)-20*b^3*B^2*d^2*(d*x+c)/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/4*b^4*B^2*d*(d
*x+c)^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-2/27*b^5*B^2*(d*x+c)^3/(-a*d+b*c)^6
/g^4/i^3/(b*x+a)^3-10*b*B^2*d^4*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)^6
/g^4/i^3/(d*x+c)+1/2*B*d^5*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c
)^6/g^4/i^3/(d*x+c)^2-20*b^3*B*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a
*d+b*c)^6/g^4/i^3/(b*x+a)+5/2*b^4*B*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c))
)/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-2/9*b^5*B*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*
x+c)))/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^3-1/2*d^5*(b*x+a)^2*(A+B*ln(e*(b*x+a)/
(d*x+c)))^2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2+5*b*d^4*(b*x+a)*(A+B*ln(e*(b*x+
a)/(d*x+c)))^2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)-10*b^3*d^2*(d*x+c)*(A+B*ln(e*(
b*x+a)/(d*x+c)))^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/2*b^4*d*(d*x+c)^2*(A+B*l
n(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-1/3*b^5*(d*x+c)^3*(
A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^3-10/3*b^2*d^3*(
A+B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)^6/g^4/i^3
```

Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 793, normalized size of antiderivative = 0.93

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx =$$

$$-\frac{27(2A^2-2AB+B^2)\,d^3(bc-ad)^2(a+bx)^3+54b(8A^2-18AB+19B^2)\,d^3(bc-ad)(a+bx)^3(c+dx$$

input

```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d
*i*x)^3),x]
```

output

```
-1/108*(27*(2*A^2 - 2*A*B + B^2)*d^3*(b*c - a*d)^2*(a + b*x)^3 + 54*b*(8*A
^2 - 18*A*B + 19*B^2)*d^3*(b*c - a*d)*(a + b*x)^3*(c + d*x) + 4*b^2*(9*A^2
 + 6*A*B + 2*B^2)*(b*c - a*d)^3*(c + d*x)^2 - 3*b^2*(54*A^2 + 66*A*B + 37*
B^2)*d*(b*c - a*d)^2*(a + b*x)*(c + d*x)^2 + 6*b^2*(108*A^2 + 282*A*B + 31
9*B^2)*d^2*(b*c - a*d)*(a + b*x)^2*(c + d*x)^2 + 60*b^2*(18*A^2 + 12*A*B +
 49*B^2)*d^3*(a + b*x)^3*(c + d*x)^2*Log[a + b*x] + 6*B*(b*c - a*d)*(9*(2*
A - B)*d^3*(b*c - a*d)*(a + b*x)^3 + 18*b*(8*A - 9*B)*d^3*(a + b*x)^3*(c +
 d*x) + 4*b^2*(3*A + B)*(b*c - a*d)^2*(c + d*x)^2 - 3*b^2*(18*A + 11*B)*d*
(b*c - a*d)*(a + b*x)*(c + d*x)^2 + 6*b^2*(36*A + 47*B)*d^2*(a + b*x)^2*(c
 + d*x)^2)*Log[(e*(a + b*x))/(c + d*x)] + 18*B*(3*a^5*B*d^5 - 15*a^4*b*B*d
^4*(2*c + d*x) + 30*a^3*b^2*d^3*(2*A*(c + d*x)^2 - B*d*x*(4*c + 3*d*x)) +
30*a^2*b^3*d^2*(6*A*d*x*(c + d*x)^2 + B*(2*c^3 + 6*c^2*d*x - 3*d^3*x^3)) +
 15*a*b^4*d*(12*A*d^2*x^2*(c + d*x)^2 + B*c*(-c^3 + 4*c^2*d*x + 18*c*d^2*x
^2 + 12*d^3*x^3)) + b^5*(60*A*d^3*x^3*(c + d*x)^2 + B*(2*c^5 - 5*c^4*d*x +
 20*c^3*d^2*x^2 + 110*c^2*d^3*x^3 + 100*c*d^4*x^4 + 20*d^5*x^5)))*Log[(e*(
a + b*x))/(c + d*x)]^2 + 360*b^2*B^2*d^3*(a + b*x)^3*(c + d*x)^2*Log[(e*(a
 + b*x))/(c + d*x)]^3 - 60*b^2*(18*A^2 + 12*A*B + 49*B^2)*d^3*(a + b*x)^3*
(c + d*x)^2*Log[c + d*x])/((b*c - a*d)^6*g^4*i^3*(a + b*x)^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 596, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx$$

$$\downarrow 2962$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^4}\,d\frac{a+bx}{c+dx}}{g^4i^3(bc-ad)^6}$$

$$\downarrow 2795$$

$$\int \left(\frac{(c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 b^5}{(a+bx)^4} - \frac{5d(c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 b^4}{(a+bx)^3} + \frac{10d^2(c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 b^3}{(a+bx)^2} - \frac{10d^3(c+dx)\left(A+ \right.}{g^4 i^3 (bc-ad)^6}$$

$$\downarrow 2009$$

$$-\frac{b^5(c+dx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3} - \frac{2b^5 B(c+dx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9(a+bx)^3} + \frac{5b^4 d(c+dx)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2} + \frac{5b^4 Bd(c+dx)^2\left(B\log \right.}{2(a+bx}$$

input

```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d*i*x)^
3),x]
```

output

```
(-1/4*(B^2*d^5*(a + b*x)^2)/(c + d*x)^2 - (10*A*b*B*d^4*(a + b*x))/(c + d*
x) + (10*b*B^2*d^4*(a + b*x))/(c + d*x) - (20*b^3*B^2*d^2*(c + d*x))/(a +
b*x) + (5*b^4*B^2*d*(c + d*x)^2)/(4*(a + b*x)^2) - (2*b^5*B^2*(c + d*x)^3)
/(27*(a + b*x)^3) - (10*b*B^2*d^4*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/
(c + d*x) + (B*d^5*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(c
 + d*x)^2) - (20*b^3*B*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))
/(a + b*x) + (5*b^4*B*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/
(2*(a + b*x)^2) - (2*b^5*B*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(9*(a + b*x)^3) - (d^5*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
^2)/(2*(c + d*x)^2) + (5*b*d^4*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x
)])^2)/(c + d*x) - (10*b^3*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x
)])^2)/(a + b*x) + (5*b^4*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)
])^2)/(2*(a + b*x)^2) - (b^5*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x
)])^2)/(3*(a + b*x)^3) - (10*b^2*d^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^
3)/(3*B))/((b*c - a*d)^6*g^4*i^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1707 vs. $2(831) = 1662$.

Time = 8.22 (sec) , antiderivative size = 1708, normalized size of antiderivative = 2.01

method	result	size
parts	Expression too large to display	1708
derivativedivides	Expression too large to display	1951
default	Expression too large to display	1951
risch	Expression too large to display	3021
parallelrisch	Expression too large to display	3971
norman	Expression too large to display	4027

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,method=_RE
TURNVERBOSE)
```

output

```
A^2/g^4/i^3*(-1/2*d^3/(a*d-b*c)^4/(d*x+c)^2+10*d^3/(a*d-b*c)^6*b^2*ln(d*x+
c)+4*d^3/(a*d-b*c)^5*b/(d*x+c)+1/3*b^2/(a*d-b*c)^3/(b*x+a)^3-10*d^3/(a*d-b
*c)^6*b^2*ln(b*x+a)+6*b^2/(a*d-b*c)^5*d^2/(b*x+a)+3/2*b^2/(a*d-b*c)^4*d/(b
*x+a)^2)-B^2/g^4/i^3*d/(a*d-b*c)^2/e^2*(d^4/(a*d-b*c)^4*(1/2*(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2*(b*e/d+(a*d-b*c)*
e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+1/4*(b*e/d+(a*d-b*c)*e/d/(d
*x+c))^2)-5/(a*d-b*c)^4*b*d^3*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a
*d-b*c)*e/d/(d*x+c))^2-2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))+2*(a*d-b*c)*e/d/(d*x+c)+2*b*e/d)+10/3*d^2/(a*d-b*c)^4*b^2*e^2
*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3-10*d/(a*d-b*c)^4*b^3*e^3*(-1/(b*e/d+(a*
d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
))+5/(a*d-b*c)^4*b^4*e^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a
*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b
*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-1/d/(a*d-b*c)^4*b^5*
e^5*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/27
/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))-2*A*B/g^4/i^3*d/(a*d-b*c)^2/e^2*(d^4/(a
*d-b*c)^4*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x
+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-5*d^3/(a*d-b*c)^4*b*e*((b*e/d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2257 vs. $2(831) = 1662$.

Time $= 0.16$ (sec) , antiderivative size $= 2257$, normalized size of antiderivative $= 2.65$

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, al
gorithm="fricas")
```

output

```
-1/108*(4*(9*A^2 + 6*A*B + 2*B^2)*b^5*c^5 - 135*(2*A^2 + 2*A*B + B^2)*a*b^
4*c^4*d + 1080*(A^2 + 2*A*B + 2*B^2)*a^2*b^3*c^3*d^2 - 20*(18*A^2 + 147*A*
B + 49*B^2)*a^3*b^2*c^2*d^3 - 540*(A^2 - 2*A*B + 2*B^2)*a^4*b*c*d^4 + 27*(
2*A^2 - 2*A*B + B^2)*a^5*d^5 + 60*((18*A^2 + 12*A*B + 49*B^2)*b^5*c*d^4 -
(18*A^2 + 12*A*B + 49*B^2)*a*b^4*d^5)*x^4 + 30*(3*(18*A^2 + 24*A*B + 53*B^
2)*b^5*c^2*d^3 + 2*(18*A^2 - 24*A*B + 37*B^2)*a*b^4*c*d^4 - (90*A^2 + 24*A
*B + 233*B^2)*a^2*b^3*d^5)*x^3 + 360*(B^2*b^5*d^5*x^5 + B^2*a^3*b^2*c^2*d^
3 + (2*B^2*b^5*c*d^4 + 3*B^2*a*b^4*d^5)*x^4 + (B^2*b^5*c^2*d^3 + 6*B^2*a*b
^4*c*d^4 + 3*B^2*a^2*b^3*d^5)*x^3 + (3*B^2*a*b^4*c^2*d^3 + 6*B^2*a^2*b^3*c
*d^4 + B^2*a^3*b^2*d^5)*x^2 + (3*B^2*a^2*b^3*c^2*d^3 + 2*B^2*a^3*b^2*c*d^4
)*x)*log((b*e*x + a*e)/(d*x + c))^3 + 10*(2*(18*A^2 + 66*A*B + 85*B^2)*b^5
*c^3*d^2 + 3*(126*A^2 + 84*A*B + 307*B^2)*a*b^4*c^2*d^3 - 12*(18*A^2 + 39*
A*B + 49*B^2)*a^2*b^3*c*d^4 - (198*A^2 - 84*A*B + 503*B^2)*a^3*b^2*d^5)*x^
2 + 18*(20*(3*A*B + B^2)*b^5*d^5*x^5 + 2*B^2*b^5*c^5 - 15*B^2*a*b^4*c^4*d
+ 60*B^2*a^2*b^3*c^3*d^2 + 60*A*B*a^3*b^2*c^2*d^3 - 30*B^2*a^4*b*c*d^4 + 3
*B^2*a^5*d^5 + 20*(9*A*B*a*b^4*d^5 + (6*A*B + 5*B^2)*b^5*c*d^4)*x^4 + 10*(
(6*A*B + 11*B^2)*b^5*c^2*d^3 + 18*(2*A*B + B^2)*a*b^4*c*d^4 + 9*(2*A*B - B
^2)*a^2*b^3*d^5)*x^3 + 10*(2*B^2*b^5*c^3*d^2 + 36*A*B*a^2*b^3*c*d^4 + 9*(2
*A*B + 3*B^2)*a*b^4*c^2*d^3 + 3*(2*A*B - 3*B^2)*a^3*b^2*d^5)*x^2 - 5*(B^2*
b^5*c^4*d - 12*B^2*a*b^4*c^3*d^2 + 3*B^2*a^4*b*d^5 - 36*(A*B + B^2)*a^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Timed out}$$

input

```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4/(d*i*x+c*i)**3,x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9282 vs. $2(831) = 1662$.

Time = 0.90 (sec) , antiderivative size = 9282, normalized size of antiderivative = 10.91

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, al
gorithm="maxima")
```

output

```
-1/6*B^2*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2*b^2*c^2*d^
2 + 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 10*(
2*b^4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3*d - 11*a
*b^3*c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/((b^8*c^5*d^2 - 5*a*b^7*
c^4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*
b^3*d^7)*g^4*i^3*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3
+ 10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d
^7)*g^4*i^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5
*c^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*
b*d^7)*g^4*i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25
*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 -
 a^8*d^7)*g^4*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5
*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*
d^6)*g^4*i^3*x + (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*
a^6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i^3) + 60*b^2*d^3*log
(b*x + a)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*
d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3) - 60*b^2*d^3*
log(d*x + c)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c
^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3))*log(b*e*x
/(d*x + c) + a*e/(d*x + c))^2 - 1/3*A*B*((60*b^4*d^4*x^4 + 2*b^4*c^4 - ...
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^4(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, al
gorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/((b*g*x + a*g)^4*(d*i*x + c
*i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 40.90 (sec) , antiderivative size = 3550, normalized size of antiderivative = 4.17

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^4*(c*i + d*i*x)^
3),x)
```

output

```
((36*A^2*b^4*c^4 - 54*A^2*a^4*d^4 - 27*B^2*a^4*d^4 + 8*B^2*b^4*c^4 + 54*A*
B*a^4*d^4 + 24*A*B*b^4*c^4 + 846*A^2*a^2*b^2*c^2*d^2 + 2033*B^2*a^2*b^2*c^
2*d^2 - 234*A^2*a*b^3*c^3*d + 486*A^2*a^3*b*c*d^3 - 127*B^2*a*b^3*c^3*d +
1053*B^2*a^3*b*c*d^3 - 246*A*B*a*b^3*c^3*d - 1026*A*B*a^3*b*c*d^3 + 1914*A
*B*a^2*b^2*c^2*d^2)/(6*(a*d - b*c)) + (10*x^4*(18*A^2*b^4*d^4 + 49*B^2*b^4
*d^4 + 12*A*B*b^4*d^4))/(a*d - b*c) + (5*x*(54*A^2*a^3*b*d^4 + 189*B^2*a^3
*b*d^4 - 18*A^2*b^4*c^3*d - 19*B^2*b^4*c^3*d + 198*A^2*a*b^3*c^2*d^2 + 630
*A^2*a^2*b^2*c*d^3 + 737*B^2*a*b^3*c^2*d^2 + 1445*B^2*a^2*b^2*c*d^3 - 162*
A*B*a^3*b*d^4 - 30*A*B*b^4*c^3*d + 618*A*B*a*b^3*c^2*d^2 + 150*A*B*a^2*b^2
*c*d^3))/(6*(a*d - b*c)) + (5*x^2*(198*A^2*a^2*b^2*d^4 + 503*B^2*a^2*b^2*d
^4 + 36*A^2*b^4*c^2*d^2 + 170*B^2*b^4*c^2*d^2 - 84*A*B*a^2*b^2*d^4 + 132*A
*B*b^4*c^2*d^2 + 414*A^2*a*b^3*c*d^3 + 1091*B^2*a*b^3*c*d^3 + 384*A*B*a*b^
3*c*d^3))/(3*(a*d - b*c)) + (5*x^3*(90*A^2*a*b^3*d^4 + 233*B^2*a*b^3*d^4 +
 54*A^2*b^4*c*d^3 + 159*B^2*b^4*c*d^3 + 24*A*B*a*b^3*d^4 + 72*A*B*b^4*c*d^
3))/(a*d - b*c))/(x^5*(18*a^4*b^3*d^6*g^4*i^3 + 18*b^7*c^4*d^2*g^4*i^3 - 7
2*a*b^6*c^3*d^3*g^4*i^3 - 72*a^3*b^4*c*d^5*g^4*i^3 + 108*a^2*b^5*c^2*d^4*g
^4*i^3) + x*(54*a^2*b^5*c^6*g^4*i^3 + 36*a^7*c*d^5*g^4*i^3 - 180*a^3*b^4*c
^5*d*g^4*i^3 - 90*a^6*b*c^2*d^4*g^4*i^3 + 180*a^4*b^3*c^4*d^2*g^4*i^3) + x
^2*(18*a^7*d^6*g^4*i^3 + 54*a*b^6*c^6*g^4*i^3 + 36*a^6*b*c*d^5*g^4*i^3 - 1
08*a^2*b^5*c^5*d*g^4*i^3 - 90*a^3*b^4*c^4*d^2*g^4*i^3 + 360*a^4*b^3*c^3...
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 7204, normalized size of antiderivative = 8.47

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x)
```

output

```
(i*( - 3240*log(a + b*x)*a**6*b**2*c**2*d**4 - 6480*log(a + b*x)*a**6*b**2
*c*d**5*x - 3240*log(a + b*x)*a**6*b**2*d**6*x**2 - 2160*log(a + b*x)*a**5
*b**3*c**3*d**3 - 14040*log(a + b*x)*a**5*b**3*c**2*d**4*x - 21600*log(a +
 b*x)*a**5*b**3*c*d**5*x**2 - 9720*log(a + b*x)*a**5*b**3*d**6*x**3 - 6480
*log(a + b*x)*a**4*b**4*c**3*d**3*x - 3600*log(a + b*x)*a**4*b**4*c**3*d**
3 - 22680*log(a + b*x)*a**4*b**4*c**2*d**4*x**2 - 7200*log(a + b*x)*a**4*b
**4*c**2*d**4*x - 8100*log(a + b*x)*a**4*b**4*c**2*d**4 - 25920*log(a + b*
x)*a**4*b**4*c*d**5*x**3 - 3600*log(a + b*x)*a**4*b**4*c*d**5*x**2 - 16200
*log(a + b*x)*a**4*b**4*c*d**5*x - 9720*log(a + b*x)*a**4*b**4*d**6*x**4 -
 8100*log(a + b*x)*a**4*b**4*d**6*x**2 - 6480*log(a + b*x)*a**3*b**5*c**3*
d**3*x**2 - 10800*log(a + b*x)*a**3*b**5*c**3*d**3*x - 6600*log(a + b*x)*a
**3*b**5*c**3*d**3 - 16200*log(a + b*x)*a**3*b**5*c**2*d**4*x**3 - 21600*l
og(a + b*x)*a**3*b**5*c**2*d**4*x**2 - 37500*log(a + b*x)*a**3*b**5*c**2*d
**4*x - 12960*log(a + b*x)*a**3*b**5*c*d**5*x**4 - 10800*log(a + b*x)*a**3
*b**5*c*d**5*x**3 - 55200*log(a + b*x)*a**3*b**5*c*d**5*x**2 - 3240*log(a
+ b*x)*a**3*b**5*d**6*x**5 - 24300*log(a + b*x)*a**3*b**5*d**6*x**3 - 2160
*log(a + b*x)*a**2*b**6*c**3*d**3*x**3 - 10800*log(a + b*x)*a**2*b**6*c**3
*d**3*x**2 - 19800*log(a + b*x)*a**2*b**6*c**3*d**3*x - 4320*log(a + b*x)*
a**2*b**6*c**2*d**4*x**4 - 21600*log(a + b*x)*a**2*b**6*c**2*d**4*x**3 - 6
3900*log(a + b*x)*a**2*b**6*c**2*d**4*x**2 - 2160*log(a + b*x)*a**2*b**...
```

3.108 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result 1322
Mathematica [A] (verified) 1323
Rubi [A] (verified) 1323
Maple [B] (verified) 1326
Fricas [B] (verification not implemented) 1327
Sympy [F(-1)] 1327
Maxima [B] (verification not implemented) 1328
Giac [B] (verification not implemented) 1329
Mupad [B] (verification not implemented) 1330
Reduce [B] (verification not implemented) 1331

Optimal result

Integrand size = 41, antiderivative size = 223

$$\begin{aligned}
&\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx \\
&= -\frac{B(bc-ad)^4g^3inx}{20bd^3}+\frac{B(bc-ad)^3g^3in(a+bx)^2}{40b^2d^2} \\
&\quad -\frac{B(bc-ad)^2g^3in(a+bx)^3}{60b^2d}+\frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b} \\
&\quad +\frac{(bc-ad)g^3i(a+bx)^4\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^2} \\
&\quad +\frac{B(bc-ad)^5g^3in\log(c+dx)}{20b^2d^4}
\end{aligned}$$

output

```
-1/20*B*(-a*d+b*c)^4*g^3*i*n*x/b/d^3+1/40*B*(-a*d+b*c)^3*g^3*i*n*(b*x+a)^2
/b^2/d^2-1/60*B*(-a*d+b*c)^2*g^3*i*n*(b*x+a)^3/b^2/d+1/5*g^3*i*(b*x+a)^4*(
d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b+1/20*(-a*d+b*c)*g^3*i*(b*x+a)^4*(
A-B*n+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2+1/20*B*(-a*d+b*c)^5*g^3*i*n*ln(d*x+
c)/b^2/d^4
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.21

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{g^3 i\Big(30(bc-ad)(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+24d(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{5B(bc-ad)^2n(}{}$$

input
```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]),x]
```

output
```
(g^3*i*(30*(b*c - a*d)*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
+ 24*d*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (5*B*(b*c - a*
d)^2*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(
a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/d^4 + (2*B*(b*c - a*d)*n*(12*b
*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c - a*d)*(
a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]))/d^4))/(12
0*b^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {2959, 27, 2947, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$
$$\downarrow \text{2959}$$
$$\frac{i(bc-ad)\int g^3(a+bx)^3\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{5b}+$$
$$\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}$$

$$\downarrow 27$$

$$\frac{g^3 i(bc-ad)\int (a+bx)^3\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{5b}+\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}$$

$$\downarrow 2947$$

$$\frac{g^3 i(bc-ad)\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{4b}-\frac{Bn(bc-ad)\int\frac{(a+bx)^3}{c+dx}dx}{4b}\right)}{5b}+\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}$$

$$\downarrow 49$$

$$\frac{g^3 i(bc-ad)\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{4b}-\frac{Bn(bc-ad)\int\left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{4b}\right)}{5b}+\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}$$

$$\downarrow 2009$$

$$\frac{g^3 i(bc-ad)\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{4b}-\frac{Bn(bc-ad)\left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{4b}\right)}{5b}+\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x
]
```

output
```
(g^3*i*(a + b*x)^4*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*b)
 + ((b*c - a*d)*g^3*i*(((a + b*x)^4*(A - B*n + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(4*b) - (B*(b*c - a*d)*n*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(
a + b*x)^2)/(2*d^2) + (a + b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4
))/(4*b)))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

rule 2959
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Simp[(f
 + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 2
))), x] + Simp[i*((b*c - a*d)/(b*d*(m + 2)))   Int[(f + g*x)^m*(A - B*n + B
*Log[e*((a + b*x)/(c + d*x))^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
i, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c
*i, 0] && IGtQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1194 vs. 2(211) = 422.

Time = 11.74 (sec) , antiderivative size = 1195, normalized size of antiderivative = 5.36

method	result	size
parallelrisch	Expression too large to display	1195

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RE
TURNVERBOSE)
```

output
```
1/120*(24*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^5*g^3*i*n-20*B*x^3*a*b^4*c
*d^4*g^3*i*n^2+120*A*x^3*a*b^4*c*d^4*g^3*i*n+60*B*x^2*ln(e*((b*x+a)/(d*x+c
))^n)*a^3*b^2*d^5*g^3*i*n+30*B*x*a^3*b^2*c*d^4*g^3*i*n^2-60*B*x*a^2*b^3*c^
2*d^3*g^3*i*n^2+30*B*x*a*b^4*c^3*d^2*g^3*i*n^2+30*B*ln(e*((b*x+a)/(d*x+c))
^n)*a*b^4*c^4*d*g^3*i*n-60*B*ln(b*x+a)*a^3*b^2*c^2*d^3*g^3*i*n^2+60*B*ln(b
*x+a)*a^2*b^3*c^3*d^2*g^3*i*n^2-30*B*ln(b*x+a)*a*b^4*c^4*d*g^3*i*n^2+30*B*
ln(b*x+a)*a^4*b*c*d^4*g^3*i*n^2-15*B*x^2*a^2*b^3*c*d^4*g^3*i*n^2-15*B*x^2*
a*b^4*c^2*d^3*g^3*i*n^2+180*A*x^2*a^2*b^3*c*d^4*g^3*i*n+90*B*x^4*ln(e*((b*
x+a)/(d*x+c))^n)*a*b^4*d^5*g^3*i*n+6*B*x^4*a*b^4*d^5*g^3*i*n^2-6*B*x^4*b^5
*c*d^4*g^3*i*n^2+90*A*x^4*a*b^4*d^5*g^3*i*n+30*A*x^4*b^5*c*d^4*g^3*i*n+22*
B*x^3*a^2*b^3*d^5*g^3*i*n^2-2*B*x^3*b^5*c^2*d^3*g^3*i*n^2+120*A*x^3*a^2*b^
3*d^5*g^3*i*n+27*B*x^2*a^3*b^2*d^5*g^3*i*n^2+3*B*x^2*b^5*c^3*d^2*g^3*i*n^2
+60*A*x^2*a^3*b^2*d^5*g^3*i*n+6*B*x*a^4*b*d^5*g^3*i*n^2-6*B*x*b^5*c^4*d*g^
3*i*n^2+120*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c*d^4*g^3*i*n+180*B*x^2*
ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c*d^4*g^3*i*n+120*B*x*ln(e*((b*x+a)/(d*x
+c))^n)*a^3*b^2*c*d^4*g^3*i*n+120*A*x*a^3*b^2*c*d^4*g^3*i*n+60*B*ln(e*((b*
x+a)/(d*x+c))^n)*a^3*b^2*c^2*d^3*g^3*i*n-60*B*ln(e*((b*x+a)/(d*x+c))^n)*a^
2*b^3*c^3*d^2*g^3*i*n+30*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d^4*g^3*i*n
+120*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*d^5*g^3*i*n-6*B*ln(b*x+a)*a^5
*d^5*g^3*i*n^2+24*A*x^5*b^5*d^5*g^3*i*n-6*B*ln(e*((b*x+a)/(d*x+c))^n)*b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. 2(213) = 426.

Time = 0.17 (sec) , antiderivative size = 720, normalized size of antiderivative = 3.23

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{24\,Ab^5d^5g^3ix^5+6\,(5\,Ba^4bcd^4-Ba^5d^5)g^3in\log\left(bx+a\right)+6\,(Bb^5c^5-5\,Bab^4c^4d+10\,Ba^2b^3c^3d^2-10\,B}{}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="fricas")
```

output
```
1/120*(24*A*b^5*d^5*g^3*i*x^5 + 6*(5*B*a^4*b*c*d^4 - B*a^5*d^5)*g^3*i*n*lo
g(b*x + a) + 6*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*
a^3*b^2*c^2*d^3)*g^3*i*n*log(d*x + c) - 6*((B*b^5*c*d^4 - B*a*b^4*d^5)*g^3
*i*n - 5*(A*b^5*c*d^4 + 3*A*a*b^4*d^5)*g^3*i)*x^4 - 2*((B*b^5*c^2*d^3 + 10
*B*a*b^4*c*d^4 - 11*B*a^2*b^3*d^5)*g^3*i*n - 60*(A*a*b^4*c*d^4 + A*a^2*b^3
*d^5)*g^3*i)*x^3 + 3*((B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 - 5*B*a^2*b^3*c*d
^4 + 9*B*a^3*b^2*d^5)*g^3*i*n + 20*(3*A*a^2*b^3*c*d^4 + A*a^3*b^2*d^5)*g^3
*i)*x^2 + 6*(20*A*a^3*b^2*c*d^4*g^3*i - (B*b^5*c^4*d - 5*B*a*b^4*c^3*d^2 +
 10*B*a^2*b^3*c^2*d^3 - 5*B*a^3*b^2*c*d^4 - B*a^4*b*d^5)*g^3*i*n)*x + 6*(4
*B*b^5*d^5*g^3*i*x^5 + 20*B*a^3*b^2*c*d^4*g^3*i*x + 5*(B*b^5*c*d^4 + 3*B*a
*b^4*d^5)*g^3*i*x^4 + 20*(B*a*b^4*c*d^4 + B*a^2*b^3*d^5)*g^3*i*x^3 + 10*(3
*B*a^2*b^3*c*d^4 + B*a^3*b^2*d^5)*g^3*i*x^2)*log(e) + 6*(4*B*b^5*d^5*g^3*i
*n*x^5 + 20*B*a^3*b^2*c*d^4*g^3*i*n*x + 5*(B*b^5*c*d^4 + 3*B*a*b^4*d^5)*g^
3*i*n*x^4 + 20*(B*a*b^4*c*d^4 + B*a^2*b^3*d^5)*g^3*i*n*x^3 + 10*(3*B*a^2*b
^3*c*d^4 + B*a^3*b^2*d^5)*g^3*i*n*x^2)*log((b*x + a)/(d*x + c)))/(b^2*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1118 vs. 2(213) = 426.

Time = 0.07 (sec) , antiderivative size = 1118, normalized size of antiderivative = 5.01

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="maxima")
```

output
```
1/5*B*b^3*d*g^3*i*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*b^3*d
*g^3*i*x^5 + 1/4*B*b^3*c*g^3*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ 3/4*B*a*b^2*d*g^3*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b
^3*c*g^3*i*x^4 + 3/4*A*a*b^2*d*g^3*i*x^4 + B*a*b^2*c*g^3*i*x^3*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n) + B*a^2*b*d*g^3*i*x^3*log(e*(b*x/(d*x + c) + a
/(d*x + c))^n) + A*a*b^2*c*g^3*i*x^3 + A*a^2*b*d*g^3*i*x^3 + 3/2*B*a^2*b*c
*g^3*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*B*a^3*d*g^3*i*x^2*
log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*a^2*b*c*g^3*i*x^2 + 1/2*A*a
^3*d*g^3*i*x^2 + 1/60*B*b^3*d*g^3*i*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*lo
g(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2
*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4
*d^4)) - 1/24*B*b^3*c*g^3*i*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)
/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*
(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/8*B*a*b^2*d*g^3*i*n*(6*a^4*log(b*x +
 a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3
*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*a*b^
2*c*g^3*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d -
 a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 1/2*B*a^2*b*d*g^3*i*
n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*
x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3/2*B*a^2*b*c*g^3*i*n*(a^2*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3945 vs. $2(213) = 426$.

Time $= 0.99$ (sec) , antiderivative size $= 3945$, normalized size of antiderivative $= 17.69$

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="giac")
```

output

```
-1/120*(6*(B*b^9*c^6*g^3*i*n - 6*B*a*b^8*c^5*d*g^3*i*n - 5*(b*x + a)*B*b^8
*c^6*d*g^3*i*n/(d*x + c) + 15*B*a^2*b^7*c^4*d^2*g^3*i*n + 30*(b*x + a)*B*a
*b^7*c^5*d^2*g^3*i*n/(d*x + c) + 10*(b*x + a)^2*B*b^7*c^6*d^2*g^3*i*n/(d*x
 + c)^2 - 20*B*a^3*b^6*c^3*d^3*g^3*i*n - 75*(b*x + a)*B*a^2*b^6*c^4*d^3*g^
3*i*n/(d*x + c) - 60*(b*x + a)^2*B*a*b^6*c^5*d^3*g^3*i*n/(d*x + c)^2 - 10*
(b*x + a)^3*B*b^6*c^6*d^3*g^3*i*n/(d*x + c)^3 + 15*B*a^4*b^5*c^2*d^4*g^3*i
*n + 100*(b*x + a)*B*a^3*b^5*c^3*d^4*g^3*i*n/(d*x + c) + 150*(b*x + a)^2*B
*a^2*b^5*c^4*d^4*g^3*i*n/(d*x + c)^2 + 60*(b*x + a)^3*B*a*b^5*c^5*d^4*g^3*
i*n/(d*x + c)^3 - 6*B*a^5*b^4*c*d^5*g^3*i*n - 75*(b*x + a)*B*a^4*b^4*c^2*d
^5*g^3*i*n/(d*x + c) - 200*(b*x + a)^2*B*a^3*b^4*c^3*d^5*g^3*i*n/(d*x + c)
^2 - 150*(b*x + a)^3*B*a^2*b^4*c^4*d^5*g^3*i*n/(d*x + c)^3 + B*a^6*b^3*d^6
*g^3*i*n + 30*(b*x + a)*B*a^5*b^3*c*d^6*g^3*i*n/(d*x + c) + 150*(b*x + a)^
2*B*a^4*b^3*c^2*d^6*g^3*i*n/(d*x + c)^2 + 200*(b*x + a)^3*B*a^3*b^3*c^3*d^
6*g^3*i*n/(d*x + c)^3 - 5*(b*x + a)*B*a^6*b^2*d^7*g^3*i*n/(d*x + c) - 60*(
b*x + a)^2*B*a^5*b^2*c*d^7*g^3*i*n/(d*x + c)^2 - 150*(b*x + a)^3*B*a^4*b^2
*c^2*d^7*g^3*i*n/(d*x + c)^3 + 10*(b*x + a)^2*B*a^6*b*d^8*g^3*i*n/(d*x + c
)^2 + 60*(b*x + a)^3*B*a^5*b*c*d^8*g^3*i*n/(d*x + c)^3 - 10*(b*x + a)^3*B*
a^6*d^9*g^3*i*n/(d*x + c)^3)*log((b*x + a)/(d*x + c))/(b^5*d^4 - 5*(b*x + 
a)*b^4*d^5/(d*x + c) + 10*(b*x + a)^2*b^3*d^6/(d*x + c)^2 - 10*(b*x + a)^3
*b^2*d^7/(d*x + c)^3 + 5*(b*x + a)^4*b*d^8/(d*x + c)^4 - (b*x + a)^5*d^...
```

Mupad [B] (verification not implemented)

Time = 27.08 (sec) , antiderivative size = 1237, normalized size of antiderivative = 5.55

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input

```
int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x
)
```

output

```
x*((a*c*(((20*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d*n - B
*b*c*n))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(20*b*d) - (b*g^3*i*(24*
A*a^2*d^2 + 4*A*b^2*c^2 + 3*B*a^2*d^2*n - B*b^2*c^2*n + 32*A*a*b*c*d - 2*B
*a*b*c*d*n))/(4*d) + A*a*b^2*c*g^3*i))/(b*d) - ((20*a*d + 20*b*c)*(((20*a*
d + 20*b*c)*(((20*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d*n
 - B*b*c*n))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(20*b*d) - (b*g^3*i*
(24*A*a^2*d^2 + 4*A*b^2*c^2 + 3*B*a^2*d^2*n - B*b^2*c^2*n + 32*A*a*b*c*d -
 2*B*a*b*c*d*n))/(4*d) + A*a*b^2*c*g^3*i))/(20*b*d) - (a*c*((b^2*g^3*i*(20
*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c)
)/20))/(b*d) + (a*g^3*i*(4*A*a^2*d^2 + 4*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c
^2*n + 12*A*a*b*c*d))/d))/(20*b*d) + (a^2*g^3*i*(2*A*a^2*d^2 + 12*A*b^2*c^
2 + B*a^2*d^2*n - 3*B*b^2*c^2*n + 16*A*a*b*c*d + 2*B*a*b*c*d*n))/(2*b*d))
+ x^2*(((20*a*d + 20*b*c)*(((20*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d + 10*A
*b*c + B*a*d*n - B*b*c*n))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(20*b*
d) - (b*g^3*i*(24*A*a^2*d^2 + 4*A*b^2*c^2 + 3*B*a^2*d^2*n - B*b^2*c^2*n +
32*A*a*b*c*d - 2*B*a*b*c*d*n))/(4*d) + A*a*b^2*c*g^3*i))/(40*b*d) - (a*c*(
(b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*b^2*g^3*i*(20
*a*d + 20*b*c))/20))/(2*b*d) + (a*g^3*i*(4*A*a^2*d^2 + 4*A*b^2*c^2 + B*a^2
*d^2*n - B*b^2*c^2*n + 12*A*a*b*c*d))/(2*d)) - x^3*(((20*a*d + 20*b*c)*((b
^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*b^2*g^3*i*(2...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 801, normalized size of antiderivative = 3.59

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input

```
int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output

```
(g**3*i*( - 6*log(a + b*x)*a**5*d**5*n + 30*log(a + b*x)*a**4*b*c*d**4*n -
 60*log(a + b*x)*a**3*b**2*c**2*d**3*n + 60*log(a + b*x)*a**2*b**3*c**3*d*
*2*n - 30*log(a + b*x)*a*b**4*c**4*d*n + 6*log(a + b*x)*b**5*c**5*n + 60*l
og(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**2*c**2*d**3 + 120*log(((a + b*x)
**n*e)/(c + d*x)**n)*a**3*b**2*c*d**4*x + 60*log(((a + b*x)**n*e)/(c + d*x
)**n)*a**3*b**2*d**5*x**2 - 60*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**
3*c**3*d**2 + 180*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*c*d**4*x**2
 + 120*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*d**5*x**3 + 30*log(((a
 + b*x)**n*e)/(c + d*x)**n)*a*b**4*c**4*d + 120*log(((a + b*x)**n*e)/(c +
d*x)**n)*a*b**4*c*d**4*x**3 + 90*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**4
*d**5*x**4 - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*c**5 + 30*log(((a +
 b*x)**n*e)/(c + d*x)**n)*b**5*c*d**4*x**4 + 24*log(((a + b*x)**n*e)/(c +
d*x)**n)*b**5*d**5*x**5 + 120*a**4*b*c*d**4*x + 6*a**4*b*d**5*n*x + 60*a**
4*b*d**5*x**2 + 30*a**3*b**2*c*d**4*n*x + 180*a**3*b**2*c*d**4*x**2 + 27*a
**3*b**2*d**5*n*x**2 + 120*a**3*b**2*d**5*x**3 - 60*a**2*b**3*c**2*d**3*n*
x - 15*a**2*b**3*c*d**4*n*x**2 + 120*a**2*b**3*c*d**4*x**3 + 22*a**2*b**3*
d**5*n*x**3 + 90*a**2*b**3*d**5*x**4 + 30*a*b**4*c**3*d**2*n*x - 15*a*b**4
*c**2*d**3*n*x**2 - 20*a*b**4*c*d**4*n*x**3 + 30*a*b**4*c*d**4*x**4 + 6*a*
b**4*d**5*n*x**4 + 24*a*b**4*d**5*x**5 - 6*b**5*c**4*d*n*x + 3*b**5*c**3*d
**2*n*x**2 - 2*b**5*c**2*d**3*n*x**3 - 6*b**5*c*d**4*n*x**4))/(120*b*d*...
```

3.109 $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result 1332
Mathematica [A] (verified) 1333
Rubi [A] (verified) 1333
Maple [B] (verified) 1335
Fricas [B] (verification not implemented) 1336
Sympy [F(-1)] 1337
Maxima [B] (verification not implemented) 1337
Giac [B] (verification not implemented) 1338
Mupad [B] (verification not implemented) 1340
Reduce [B] (verification not implemented) 1341

Optimal result

Integrand size = 41, antiderivative size = 190

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx \\
&= \frac{B(bc-ad)^3g^2inx}{12bd^2} - \frac{B(bc-ad)^2g^2in(a+bx)^2}{24b^2d} \\
&\quad + \frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b} \\
&\quad + \frac{(bc-ad)g^2i(a+bx)^3\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2} \\
&\quad - \frac{B(bc-ad)^4g^2in\log(c+dx)}{12b^2d^3}
\end{aligned}$$

output

```
1/12*B*(-a*d+b*c)^3*g^2*i*n*x/b/d^2-1/24*B*(-a*d+b*c)^2*g^2*i*n*(b*x+a)^2/
b^2/d+1/4*g^2*i*(b*x+a)^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b+1/12*(
-a*d+b*c)*g^2*i*(b*x+a)^3*(A-B*n+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2-1/12*B*(
-a*d+b*c)^4*g^2*i*n*ln(d*x+c)/b^2/d^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.18

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{g^2 i\Big(8(bc-ad)(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+6d(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{4B(bc-ad)^2n(2bd}{24b}$$

input
```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]),x]
```

output
```
(g^2*i*(8*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) +
 6*d*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (4*B*(b*c - a*d)
^2*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]
))/d^3 - (B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a
 + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/d^3))/(24*b
^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {2959, 27, 2947, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$
$$\downarrow \text{2959}$$
$$\frac{i(bc-ad)\int g^2(a+bx)^2\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{4b}+$$
$$\frac{g^2 i(a+bx)^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}$$

$$\downarrow 27$$

$$\frac{g^2 i(bc-ad)\int (a+bx)^2\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{4b}+\frac{g^2 i(a+bx)^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}$$

$$\downarrow 2947$$

$$\frac{g^2 i(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{3b}-\frac{Bn(bc-ad)\int\frac{(a+bx)^2}{c+dx}dx}{3b}\right)}{4b}+\frac{g^2 i(a+bx)^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}$$

$$\downarrow 49$$

$$\frac{g^2 i(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{3b}-\frac{Bn(bc-ad)\int\left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{3b}\right)}{4b}+\frac{g^2 i(a+bx)^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}$$

$$\downarrow 2009$$

$$\frac{g^2 i(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{3b}-\frac{Bn(bc-ad)\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{3b}\right)}{4b}+\frac{g^2 i(a+bx)^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}$$

input

```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x
]
```

output

```
(g^2*i*(a + b*x)^3*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b)
 + ((b*c - a*d)*g^2*i*(((a + b*x)^3*(A - B*n + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(3*b) - (B*(b*c - a*d)*n*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(
2*d) + ((b*c - a*d)^2*Log[c + d*x])/d^3))/(3*b)))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

rule 2959
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Simp[(f
 + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 2
))), x] + Simp[i*((b*c - a*d)/(b*d*(m + 2)))   Int[(f + g*x)^m*(A - B*n + B
*Log[e*((a + b*x)/(c + d*x))^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
i, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c
*i, 0] && IGtQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(180) = 360$.

Time = 4.98 (sec) , antiderivative size = 876, normalized size of antiderivative = 4.61

method	result
parallelrisch	$\frac{6Bx^4\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4d^4g^2in+8B\ln(bx+a)ab^3c^3dg^2in^2+12B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^2b^2c^2d^2g^2in-2Bb^4c^4g^2in^2-2Ba^4d^4g^2i$

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RE
TURNVERBOSE)
```

output
```
1/24*(6*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^4*g^2*i*n+8*B*ln(b*x+a)*a*b^
3*c^3*d*g^2*i*n^2+12*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^2*d^2*g^2*i*n-2
*B*b^4*c^4*g^2*i*n^2-2*B*a^4*d^4*g^2*i*n^2+24*B*x*ln(e*((b*x+a)/(d*x+c))^n
)*a^2*b^2*c*d^3*g^2*i*n+24*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c*d^3*g^2
*i*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^4*g^2*i*n-2*B*ln(b*x+a)*b^4*c^4*g
^2*i*n^2-2*B*ln(b*x+a)*a^4*d^4*g^2*i*n^2+6*A*x^4*b^4*d^4*g^2*i*n-48*A*a^2*
b^2*c^2*d^2*g^2*i*n+2*B*x^3*a*b^3*d^4*g^2*i*n^2-2*B*x^3*b^4*c*d^3*g^2*i*n^
2-8*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c^3*d*g^2*i*n+16*B*x^3*ln(e*((b*x+a)
/(d*x+c))^n)*a*b^3*d^4*g^2*i*n+8*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^3
*g^2*i*n+12*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*d^4*g^2*i*n-4*B*x^2*a*
b^3*c*d^3*g^2*i*n^2+24*A*x^2*a*b^3*c*d^3*g^2*i*n+4*B*x*a^2*b^2*c*d^3*g^2*i
*n^2-8*B*x*a*b^3*c^2*d^2*g^2*i*n^2+24*A*x*a^2*b^2*c*d^3*g^2*i*n+8*B*ln(b*x
+a)*a^3*b*c*d^3*g^2*i*n^2-12*B*ln(b*x+a)*a^2*b^2*c^2*d^2*g^2*i*n^2-11*B*a^
3*b*c*d^3*g^2*i*n^2+8*B*a^2*b^2*c^2*d^2*g^2*i*n^2+7*B*a*b^3*c^3*d*g^2*i*n^
2-36*A*a^3*b*c*d^3*g^2*i*n+16*A*x^3*a*b^3*d^4*g^2*i*n+8*A*x^3*b^4*c*d^3*g^
2*i*n+5*B*x^2*a^2*b^2*d^4*g^2*i*n^2-B*x^2*b^4*c^2*d^2*g^2*i*n^2+12*A*x^2*a
^2*b^2*d^4*g^2*i*n+2*B*x*a^3*b*d^4*g^2*i*n^2+2*B*x*b^4*c^3*d*g^2*i*n^2)/b^
2/d^3/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(182) = 364$.

Time $= 0.11$ (sec) , antiderivative size $= 529$, normalized size of antiderivative $= 2.78$

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{6\,Ab^4d^4g^2ix^4+2\left(4\,Ba^3bcd^3-Ba^4d^4\right)g^2in\log\left(bx+a\right)-2\left(Bb^4c^4-4\,Bab^3c^3d+6\,Ba^2b^2c^2d^2\right)g^2in\log($$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="fricas")
```

output
```
1/24*(6*A*b^4*d^4*g^2*i*x^4 + 2*(4*B*a^3*b*c*d^3 - B*a^4*d^4)*g^2*i*n*log(
b*x + a) - 2*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2)*g^2*i*n*l
og(d*x + c) - 2*((B*b^4*c*d^3 - B*a*b^3*d^4)*g^2*i*n - 4*(A*b^4*c*d^3 + 2*
A*a*b^3*d^4)*g^2*i)*x^3 - ((B*b^4*c^2*d^2 + 4*B*a*b^3*c*d^3 - 5*B*a^2*b^2*
d^4)*g^2*i*n - 12*(2*A*a*b^3*c*d^3 + A*a^2*b^2*d^4)*g^2*i)*x^2 + 2*(12*A*a
^2*b^2*c*d^3*g^2*i + (B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 2*B*a^2*b^2*c*d^3
+ B*a^3*b*d^4)*g^2*i*n)*x + 2*(3*B*b^4*d^4*g^2*i*x^4 + 12*B*a^2*b^2*c*d^3*
g^2*i*x + 4*(B*b^4*c*d^3 + 2*B*a*b^3*d^4)*g^2*i*x^3 + 6*(2*B*a*b^3*c*d^3 +
 B*a^2*b^2*d^4)*g^2*i*x^2)*log(e) + 2*(3*B*b^4*d^4*g^2*i*n*x^4 + 12*B*a^2*
b^2*c*d^3*g^2*i*n*x + 4*(B*b^4*c*d^3 + 2*B*a*b^3*d^4)*g^2*i*n*x^3 + 6*(2*B
*a*b^3*c*d^3 + B*a^2*b^2*d^4)*g^2*i*n*x^2)*log((b*x + a)/(d*x + c)))/(b^2*
d^3)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(182) = 364$.

Time = 0.06 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.89

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="maxima")
```

output

```
1/4*B*b^2*d*g^2*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b^2*d
*g^2*i*x^4 + 1/3*B*b^2*c*g^2*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ 2/3*B*a*b*d*g^2*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*b^2
*c*g^2*i*x^3 + 2/3*A*a*b*d*g^2*i*x^3 + B*a*b*c*g^2*i*x^2*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n) + 1/2*B*a^2*d*g^2*i*x^2*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n) + A*a*b*c*g^2*i*x^2 + 1/2*A*a^2*d*g^2*i*x^2 - 1/24*B*b^2*d*g^2*
i*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b
^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^
3*d^3)) + 1/6*B*b^2*c*g^2*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 1/
3*B*a*b*d*g^2*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*a*b*c*g^2*i*
n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) - 1/
2*B*a^2*d*g^2*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*
d)*x/(b*d)) + B*a^2*c*g^2*i*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a^
2*c*g^2*i*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a^2*c*g^2*i*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2571 vs. $2(182) = 364$.

Time $= 0.85$ (sec) , antiderivative size $= 2571$, normalized size of antiderivative $= 13.53$

$$\int (ag + bgx)^2 (ci + dix) \left(A + B \log \left(e \left(\frac{a + bx}{c + dx} \right)^n \right) \right) \, dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="giac")
```

output

```
1/24*(2*(B*b^7*c^5*g^2*i*n - 5*B*a*b^6*c^4*d*g^2*i*n - 4*(b*x + a)*B*b^6*c
^5*d*g^2*i*n/(d*x + c) + 10*B*a^2*b^5*c^3*d^2*g^2*i*n + 20*(b*x + a)*B*a*b
^5*c^4*d^2*g^2*i*n/(d*x + c) + 6*(b*x + a)^2*B*b^5*c^5*d^2*g^2*i*n/(d*x +
c)^2 - 10*B*a^3*b^4*c^2*d^3*g^2*i*n - 40*(b*x + a)*B*a^2*b^4*c^3*d^3*g^2*i
*n/(d*x + c) - 30*(b*x + a)^2*B*a*b^4*c^4*d^3*g^2*i*n/(d*x + c)^2 + 5*B*a^
4*b^3*c*d^4*g^2*i*n + 40*(b*x + a)*B*a^3*b^3*c^2*d^4*g^2*i*n/(d*x + c) + 6
0*(b*x + a)^2*B*a^2*b^3*c^3*d^4*g^2*i*n/(d*x + c)^2 - B*a^5*b^2*d^5*g^2*i*
n - 20*(b*x + a)*B*a^4*b^2*c*d^5*g^2*i*n/(d*x + c) - 60*(b*x + a)^2*B*a^3*
b^2*c^2*d^5*g^2*i*n/(d*x + c)^2 + 4*(b*x + a)*B*a^5*b*d^6*g^2*i*n/(d*x + c
) + 30*(b*x + a)^2*B*a^4*b*c*d^6*g^2*i*n/(d*x + c)^2 - 6*(b*x + a)^2*B*a^5
*d^7*g^2*i*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))/(b^4*d^3 - 4*(b*x + a)*
b^3*d^4/(d*x + c) + 6*(b*x + a)^2*b^2*d^5/(d*x + c)^2 - 4*(b*x + a)^3*b*d^
6/(d*x + c)^3 + (b*x + a)^4*d^7/(d*x + c)^4) + (B*b^8*c^5*g^2*i*n - 5*B*a*
b^7*c^4*d*g^2*i*n - 2*(b*x + a)*B*b^7*c^5*d*g^2*i*n/(d*x + c) + 10*B*a^2*b
^6*c^3*d^2*g^2*i*n + 10*(b*x + a)*B*a*b^6*c^4*d^2*g^2*i*n/(d*x + c) - (b*x
 + a)^2*B*b^6*c^5*d^2*g^2*i*n/(d*x + c)^2 - 10*B*a^3*b^5*c^2*d^3*g^2*i*n -
 20*(b*x + a)*B*a^2*b^5*c^3*d^3*g^2*i*n/(d*x + c) + 5*(b*x + a)^2*B*a*b^5*
c^4*d^3*g^2*i*n/(d*x + c)^2 + 2*(b*x + a)^3*B*b^5*c^5*d^3*g^2*i*n/(d*x + c
)^3 + 5*B*a^4*b^4*c*d^4*g^2*i*n + 20*(b*x + a)*B*a^3*b^4*c^2*d^4*g^2*i*n/(
d*x + c) - 10*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^2*i*n/(d*x + c)^2 - 10*(b...
```

Mupad [B] (verification not implemented)

Time = 26.14 (sec) , antiderivative size = 663, normalized size of antiderivative = 3.49

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$

$$= \ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(Ba^2cg^2ix+\frac{Bag^2ix^2(ad+2bc)}{2}+\frac{Bbg^2ix^3(2ad+bc)}{3}+\frac{Bb^2dg^2ix^4}{4}\right)$$
$$+x^3\left(\frac{bg^2i(12Aad+8Abc+Badn-Bbcn)}{12}-\frac{Abg^2i(12ad+12bc)}{36}\right)$$
$$+x\left(\frac{(12ad+12bc)\left(\frac{bg^2i(12Aad+8Abc+Badn-Bbcn)}{4}-\frac{Abg^2i(12ad+12bc)}{12}\right)}{12bd}-\frac{g^2i(9Aa^2d^2+3Ab^2c^2+2}{12bd}\right.$$
$$\left.-\frac{ac\left(\frac{bg^2i(12Aad+8Abc+Badn-Bbcn)}{4}-\frac{Abg^2i(12ad+12bc)}{12}\right)}{bd}+\frac{ag^2i(2Aa^2d^2+6Ab^2c^2+Ba^2d^2n-2Bb^2c^2n+12Aabcd+Babcdn)}{2bd}\right)$$
$$-x^2\left(\frac{(12ad+12bc)\left(\frac{bg^2i(12Aad+8Abc+Badn-Bbcn)}{4}-\frac{Abg^2i(12ad+12bc)}{12}\right)}{24bd}\right.$$
$$\left.-\frac{g^2i(9Aa^2d^2+3Ab^2c^2+2Ba^2d^2n-Bb^2c^2n+18Aabcd-Babcdn)}{6d}+\frac{Aabcg^2i}{2}\right)-\frac{\ln(a+bx)\,(Ba^4dg^2in-4Ba^3bcg^2in)}{12b^2}$$
$$-\frac{\ln(c+dx)\,(6Bina^2c^2d^2g^2-4Binabc^3dg^2+Binb^2c^4g^2)}{12d^3}+\frac{Ab^2dg^2ix^4}{4}$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x
)
```

output

```
log(e*((a + b*x)/(c + d*x))^n)*(B*a^2*c*g^2*i*x + (B*a*g^2*i*x^2*(a*d + 2*
b*c))/2 + (B*b*g^2*i*x^3*(2*a*d + b*c))/3 + (B*b^2*d*g^2*i*x^4)/4) + x^3*(
(b*g^2*i*(12*A*a*d + 8*A*b*c + B*a*d*n - B*b*c*n))/12 - (A*b*g^2*i*(12*a*d
 + 12*b*c))/36) + x*(((12*a*d + 12*b*c)*(((12*a*d + 12*b*c)*((b*g^2*i*(12*
A*a*d + 8*A*b*c + B*a*d*n - B*b*c*n))/4 - (A*b*g^2*i*(12*a*d + 12*b*c))/12
))/(12*b*d) - (g^2*i*(9*A*a^2*d^2 + 3*A*b^2*c^2 + 2*B*a^2*d^2*n - B*b^2*c^
2*n + 18*A*a*b*c*d - B*a*b*c*d*n))/(3*d) + A*a*b*c*g^2*i))/(12*b*d) - (a*c
*((b*g^2*i*(12*A*a*d + 8*A*b*c + B*a*d*n - B*b*c*n))/4 - (A*b*g^2*i*(12*a*
d + 12*b*c))/12))/(b*d) + (a*g^2*i*(2*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2*
n - 2*B*b^2*c^2*n + 12*A*a*b*c*d + B*a*b*c*d*n))/(2*b*d)) - x^2*(((12*a*d
+ 12*b*c)*((b*g^2*i*(12*A*a*d + 8*A*b*c + B*a*d*n - B*b*c*n))/4 - (A*b*g^2
*i*(12*a*d + 12*b*c))/12))/(24*b*d) - (g^2*i*(9*A*a^2*d^2 + 3*A*b^2*c^2 +
2*B*a^2*d^2*n - B*b^2*c^2*n + 18*A*a*b*c*d - B*a*b*c*d*n))/(6*d) + (A*a*b*
c*g^2*i)/2) - (log(a + b*x)*(B*a^4*d*g^2*i*n - 4*B*a^3*b*c*g^2*i*n))/(12*b
^2) - (log(c + d*x)*(B*b^2*c^4*g^2*i*n + 6*B*a^2*c^2*d^2*g^2*i*n - 4*B*a*b
*c^3*d*g^2*i*n))/(12*d^3) + (A*b^2*d*g^2*i*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.02

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{g^2 i\left(12\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a^2b^2c^2d^2-8\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a\,b^3c^3d+6\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b^4d^4x^4+12a^3b\,d^4x^2+16a^2b^2d$$

input

```
int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output

```
(g**2*i*( - 2*log(a + b*x)*a**4*d**4*n + 8*log(a + b*x)*a**3*b*c*d**3*n -
12*log(a + b*x)*a**2*b**2*c**2*d**2*n + 8*log(a + b*x)*a*b**3*c**3*d*n - 2
*log(a + b*x)*b**4*c**4*n + 12*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**
2*c**2*d**2 + 24*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*c*d**3*x + 1
2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*d**4*x**2 - 8*log(((a + b*x
)**n*e)/(c + d*x)**n)*a*b**3*c**3*d + 24*log(((a + b*x)**n*e)/(c + d*x)**n
)*a*b**3*c*d**3*x**2 + 16*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*d**4*x
**3 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*c**4 + 8*log(((a + b*x)**n
*e)/(c + d*x)**n)*b**4*c*d**3*x**3 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*
b**4*d**4*x**4 + 24*a**3*b*c*d**3*x + 2*a**3*b*d**4*n*x + 12*a**3*b*d**4*x
**2 + 4*a**2*b**2*c*d**3*n*x + 24*a**2*b**2*c*d**3*x**2 + 5*a**2*b**2*d**4
*n*x**2 + 16*a**2*b**2*d**4*x**3 - 8*a*b**3*c**2*d**2*n*x - 4*a*b**3*c*d**
3*n*x**2 + 8*a*b**3*c*d**3*x**3 + 2*a*b**3*d**4*n*x**3 + 6*a*b**3*d**4*x**
4 + 2*b**4*c**3*d*n*x - b**4*c**2*d**2*n*x**2 - 2*b**4*c*d**3*n*x**3))/(24
*b*d**3)
```

3.110 $\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result 1343
Mathematica [A] (verified) 1343
Rubi [A] (verified) 1344
Maple [B] (verified) 1346
Fricas [B] (verification not implemented) 1347
Sympy [B] (verification not implemented) 1348
Maxima [B] (verification not implemented) 1349
Giac [B] (verification not implemented) 1350
Mupad [B] (verification not implemented) 1351
Reduce [B] (verification not implemented) 1352

Optimal result

Integrand size = 39, antiderivative size = 149

$$\begin{aligned}&\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\&=-\frac{B(bc-ad)^2ginx}{6bd}+\frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b}\\&\quad+\frac{(bc-ad)gi(a+bx)^2\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^2}+\frac{B(bc-ad)^3gin\log(c+dx)}{6b^2d^2}\end{aligned}$$

output

```
-1/6*B*(-a*d+b*c)^2*g*i*n*x/b/d+1/3*g*i*(b*x+a)^2*(d*x+c)*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))/b+1/6*(-a*d+b*c)*g*i*(b*x+a)^2*(A-B*n+B*ln(e*((b*x+a)/(d*x
+c))^n))/b^2+1/6*B*(-a*d+b*c)^3*g*i*n*ln(d*x+c)/b^2/d^2
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.27

$$\begin{aligned}&\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\&=\frac{gi\left(-a^2Bd^2(3bc+ad)n\log(a+bx)+b\left(dx(a^2Bd^2n-b^2Bcn(c+dx)+Ab^2dx(3c+2dx)+abd(6Ac+3\right.\right.}\end{aligned}$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]),x]
```

output
```
(g*i*(-(a^2*B*d^2*(3*b*c + a*d)*n*Log[a + b*x]) + b*(d*x*(a^2*B*d^2*n - b^
2*B*c*n*(c + d*x) + A*b^2*d*x*(3*c + 2*d*x) + a*b*d*(6*A*c + 3*A*d*x + B*d
*n*x)) + B*d^2*(6*a^2*c + 3*a*b*x*(2*c + d*x) + b^2*x^2*(3*c + 2*d*x))*Log
[e*((a + b*x)/(c + d*x))^n] + B*c*(b^2*c^2 - 3*a*b*c*d + 6*a^2*d^2)*n*Log[
c + d*x])))/(6*b^2*d^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {2959, 27, 2947, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\downarrow 2959$$

$$\frac{i(bc-ad)\int g(a+bx)\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}$$

$$\downarrow 27$$

$$\frac{gi(bc-ad)\int (a+bx)\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}$$

$$\downarrow 2947$$

$$\frac{gi(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{2b}-\frac{Bn(bc-ad)\int\frac{a+bx}{c+dx}dx}{2b}\right)}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}$$

$$\downarrow 49$$

$$\frac{gi(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{2b}-\frac{Bn(bc-ad)\int\left(\frac{b}{d}+\frac{ad-bc}{d(c+dx)}\right)dx}{2b}\right)}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}$$

$$\downarrow 2009$$

$$\frac{gi(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{2b}-\frac{Bn(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\right)}{2b}\right)}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}$$

input
```
Int[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g*i*(a + b*x)^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b) +
 ((b*c - a*d)*g*i*(((a + b*x)^2*(A - B*n + B*Log[e*((a + b*x)/(c + d*x))^n
]))/(2*b) - (B*(b*c - a*d)*n*((b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2))/(
2*b)))/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

rule 2959
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Simp[(f
 + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 2
))), x] + Simp[i*((b*c - a*d)/(b*d*(m + 2)))   Int[(f + g*x)^m*(A - B*n + B
*Log[e*((a + b*x)/(c + d*x))^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
i, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c
*i, 0] && IGtQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. 2(141) = 282.

Time = 1.87 (sec) , antiderivative size = 507, normalized size of antiderivative = 3.40

method	result
parallelrisch	$\frac{2Bx^3\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3d^3gin+6Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)ab^2cd^2gin-B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3c^3gin+2Ax^3b^3d^3gin-B\ln(bx+a)a^3d^3$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETU
RNVERBOSE)
```

output
```
1/6*(2*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^3*g*i*n+6*B*x*ln(e*((b*x+a)/(
d*x+c))^n)*a*b^2*c*d^2*g*i*n-B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^3*g*i*n+2*A
*x^3*b^3*d^3*g*i*n-B*ln(b*x+a)*a^3*d^3*g*i*n^2+B*ln(b*x+a)*b^3*c^3*g*i*n^2
+B*b^3*c^3*g*i*n^2-B*a^3*d^3*g*i*n^2+3*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c
^2*d*g*i*n+3*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^3*g*i*n+3*B*x^2*ln(e*
((b*x+a)/(d*x+c))^n)*b^3*c*d^2*g*i*n+6*A*x*a*b^2*c*d^2*g*i*n+3*B*ln(b*x+a)
*a^2*b*c*d^2*g*i*n^2-3*B*ln(b*x+a)*a*b^2*c^2*d*g*i*n^2-2*B*a^2*b*c*d^2*g*i
*n^2+2*B*a*b^2*c^2*d*g*i*n^2-9*A*a^2*b*c*d^2*g*i*n-9*A*a*b^2*c^2*d*g*i*n+B
*x^2*a*b^2*d^3*g*i*n^2-B*x^2*b^3*c*d^2*g*i*n^2+3*A*x^2*a*b^2*d^3*g*i*n+3*A
*x^2*b^3*c*d^2*g*i*n+B*x*a^2*b*d^3*g*i*n^2-B*x*b^3*c^2*d*g*i*n^2)/n/b^2/d^
2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(143) = 286$.

Time $= 0.10$ (sec) , antiderivative size $= 309$, normalized size of antiderivative $= 2.07$

$$\int (ag + bgx)(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{2\,Ab^3d^3gix^3 + (3\,Ba^2bcd^2 - Ba^3d^3)gin\log(bx+a) + (Bb^3c^3 - 3\,Bab^2c^2d)gin\log(dx+c) - ((Bb^3cd^2 -}{}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algo
rithm="fricas")
```

output
```
1/6*(2*A*b^3*d^3*g*i*x^3 + (3*B*a^2*b*c*d^2 - B*a^3*d^3)*g*i*n*log(b*x + a
) + (B*b^3*c^3 - 3*B*a*b^2*c^2*d)*g*i*n*log(d*x + c) - ((B*b^3*c*d^2 - B*a
*b^2*d^3)*g*i*n - 3*(A*b^3*c*d^2 + A*a*b^2*d^3)*g*i)*x^2 + (6*A*a*b^2*c*d^
2*g*i - (B*b^3*c^2*d - B*a^2*b*d^3)*g*i*n)*x + (2*B*b^3*d^3*g*i*x^3 + 6*B*
a*b^2*c*d^2*g*i*x + 3*(B*b^3*c*d^2 + B*a*b^2*d^3)*g*i*x^2)*log(e) + (2*B*b
^3*d^3*g*i*n*x^3 + 6*B*a*b^2*c*d^2*g*i*n*x + 3*(B*b^3*c*d^2 + B*a*b^2*d^3)
*g*i*n*x^2)*log((b*x + a)/(d*x + c)))/(b^2*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(134) = 268$.

Time = 137.27 (sec) , antiderivative size = 643, normalized size of antiderivative = 4.32

$$\int (ag + bgx)(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Piecewise((a*c*g*i*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (a*g*(
A*c*i*x + A*d*i*x**2/2 + B*c**2*i*log(e*(a/(c + d*x))**n)/(2*d) + B*c*i*n*
x/2 + B*c*i*x*log(e*(a/(c + d*x))**n) + B*d*i*n*x**2/4 + B*d*i*x**2*log(e*
(a/(c + d*x))**n)/2), Eq(b, 0)), (c*i*(A*a*g*x + A*b*g*x**2/2 + B*a**2*g*l
og(e*(a/c + b*x/c)**n)/(2*b) - B*a*g*n*x/2 + B*a*g*x*log(e*(a/c + b*x/c)**
n) - B*b*g*n*x**2/4 + B*b*g*x**2*log(e*(a/c + b*x/c)**n)/2), Eq(d, 0)), (A
*a*c*g*i*x + A*a*d*g*i*x**2/2 + A*b*c*g*i*x**2/2 + A*b*d*g*i*x**3/3 - B*a*
*3*d*g*i*n*log(c/d + x)/(6*b**2) - B*a**3*d*g*i*log(e*(a/(c + d*x) + b*x/(
c + d*x))**n)/(6*b**2) + B*a**2*c*g*i*n*log(c/d + x)/(2*b) + B*a**2*c*g*i*
log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(2*b) + B*a**2*d*g*i*n*x/(6*b) - B
*a*c**2*g*i*n*log(c/d + x)/(2*d) + B*a*c*g*i*x*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n) + B*a*d*g*i*n*x**2/6 + B*a*d*g*i*x**2*log(e*(a/(c + d*x) + b*
x/(c + d*x))**n)/2 + B*b*c**3*g*i*n*log(c/d + x)/(6*d**2) - B*b*c**2*g*i*n
*x/(6*d) - B*b*c*g*i*n*x**2/6 + B*b*c*g*i*x**2*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)/2 + B*b*d*g*i*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/3,
 True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(143) = 286.

Time = 0.05 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.64

$$
\begin{aligned}
&\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx \\
&= \frac{1}{3}Bbdgix^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}Abdgix^3 \\
&\quad+\frac{1}{2}Bbcgix^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) \\
&\quad+\frac{1}{2}Badgix^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}Abcgix^2+\frac{1}{2}Aadgix^2 \\
&\quad+\frac{1}{6}Bbdgin\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right) \\
&\quad-\frac{1}{2}Bbcgin\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) \\
&\quad-\frac{1}{2}Badgin\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) \\
&\quad+Bacgin\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right) \\
&\quad+Bacgix\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aacgix
\end{aligned}
$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algo
rithm="maxima")
```

output

```
1/3*B*b*d*g*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*b*d*g*i*x
^3 + 1/2*B*b*c*g*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*B*a*d*
g*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*b*c*g*i*x^2 + 1/2*A
*a*d*g*i*x^2 + 1/6*B*b*d*g*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c
)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 1
/2*B*b*c*g*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*
x/(b*d)) - 1/2*B*a*d*g*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 +
(b*c - a*d)*x/(b*d)) + B*a*c*g*i*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) +
 B*a*c*g*i*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*c*g*i*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1276 vs. $2(143) = 286$.

Time = 0.49 (sec) , antiderivative size = 1276, normalized size of antiderivative = 8.56

$$\int (ag + bgx)(ci + dix)\left(A + B\log\left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algo
rithm="giac")
```

output

```
-1/6*((B*b^5*c^4*g*i*n - 4*B*a*b^4*c^3*d*g*i*n - 3*(b*x + a)*B*b^4*c^4*d*g
*i*n/(d*x + c) + 6*B*a^2*b^3*c^2*d^2*g*i*n + 12*(b*x + a)*B*a*b^3*c^3*d^2*
g*i*n/(d*x + c) - 4*B*a^3*b^2*c*d^3*g*i*n - 18*(b*x + a)*B*a^2*b^2*c^2*d^3
*g*i*n/(d*x + c) + B*a^4*b*d^4*g*i*n + 12*(b*x + a)*B*a^3*b*c*d^4*g*i*n/(d
*x + c) - 3*(b*x + a)*B*a^4*d^5*g*i*n/(d*x + c))*log((b*x + a)/(d*x + c))/
(b^3*d^2 - 3*(b*x + a)*b^2*d^3/(d*x + c) + 3*(b*x + a)^2*b*d^4/(d*x + c)^2
 - (b*x + a)^3*d^5/(d*x + c)^3) + ((b*x + a)*B*b^5*c^4*d*g*i*n/(d*x + c) -
 4*(b*x + a)*B*a*b^4*c^3*d^2*g*i*n/(d*x + c) - (b*x + a)^2*B*b^4*c^4*d^2*g
*i*n/(d*x + c)^2 + 6*(b*x + a)*B*a^2*b^3*c^2*d^3*g*i*n/(d*x + c) + 4*(b*x
+ a)^2*B*a*b^3*c^3*d^3*g*i*n/(d*x + c)^2 - 4*(b*x + a)*B*a^3*b^2*c*d^4*g*i
*n/(d*x + c) - 6*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g*i*n/(d*x + c)^2 + (b*x +
a)*B*a^4*b*d^5*g*i*n/(d*x + c) + 4*(b*x + a)^2*B*a^3*b*c*d^5*g*i*n/(d*x +
c)^2 - (b*x + a)^2*B*a^4*d^6*g*i*n/(d*x + c)^2 + B*b^6*c^4*g*i*log(e) - 4*
B*a*b^5*c^3*d*g*i*log(e) - 3*(b*x + a)*B*b^5*c^4*d*g*i*log(e)/(d*x + c) +
6*B*a^2*b^4*c^2*d^2*g*i*log(e) + 12*(b*x + a)*B*a*b^4*c^3*d^2*g*i*log(e)/(
d*x + c) - 4*B*a^3*b^3*c*d^3*g*i*log(e) - 18*(b*x + a)*B*a^2*b^3*c^2*d^3*g
*i*log(e)/(d*x + c) + B*a^4*b^2*d^4*g*i*log(e) + 12*(b*x + a)*B*a^3*b^2*c*
d^4*g*i*log(e)/(d*x + c) - 3*(b*x + a)*B*a^4*b*d^5*g*i*log(e)/(d*x + c) +
A*b^6*c^4*g*i - 4*A*a*b^5*c^3*d*g*i - 3*(b*x + a)*A*b^5*c^4*d*g*i/(d*x + c
) + 6*A*a^2*b^4*c^2*d^2*g*i + 12*(b*x + a)*A*a*b^4*c^3*d^2*g*i/(d*x + c...
```

Mupad [B] (verification not implemented)

Time = 25.84 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.98

$$\begin{aligned}
&\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx \\
&= \ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(\frac{B\,b\,d\,g\,i\,x^3}{3}+\frac{B\,g\,i\,(a\,d+b\,c)\,x^2}{2}+B\,a\,c\,g\,i\,x\right) \\
&\quad - x\left(\frac{\left(\frac{g\,i\,(6\,A\,a\,d+6\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3}-\frac{A\,g\,i\,(6\,a\,d+6\,b\,c)}{6}\right)(6\,a\,d+6\,b\,c)}{6\,b\,d}+A\,a\,c\,g\,i\right. \\
&\qquad\qquad \left.-\frac{g\,i\,(2\,A\,a^2\,d^2+2\,A\,b^2\,c^2+B\,a^2\,d^2\,n-B\,b^2\,c^2\,n+8\,A\,a\,b\,c\,d)}{2\,b\,d}\right) \\
&\quad + x^2\left(\frac{g\,i\,(6\,A\,a\,d+6\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{6}-\frac{A\,g\,i\,(6\,a\,d+6\,b\,c)}{12}\right) \\
&\quad - \frac{\ln(a+bx)\,(B\,a^3\,d\,g\,i\,n-3\,B\,a^2\,b\,c\,g\,i\,n)}{6\,b^2} \\
&\quad + \frac{\ln(c+dx)\,(B\,b\,c^3\,g\,i\,n-3\,B\,a\,c^2\,d\,g\,i\,n)}{6\,d^2}+\frac{A\,b\,d\,g\,i\,x^3}{3}
\end{aligned}$$

input

```
int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output

```
log(e*((a + b*x)/(c + d*x))^n)*((B*g*i*x^2*(a*d + b*c))/2 + (B*b*d*g*i*x^3
)/3 + B*a*c*g*i*x) - x*((((g*i*(6*A*a*d + 6*A*b*c + B*a*d*n - B*b*c*n))/3
- (A*g*i*(6*a*d + 6*b*c))/6)*(6*a*d + 6*b*c))/(6*b*d) + A*a*c*g*i - (g*i*(
2*A*a^2*d^2 + 2*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 8*A*a*b*c*d))/(2*b
*d)) + x^2*((g*i*(6*A*a*d + 6*A*b*c + B*a*d*n - B*b*c*n))/6 - (A*g*i*(6*a*
d + 6*b*c))/12) - (log(a + b*x)*(B*a^3*d*g*i*n - 3*B*a^2*b*c*g*i*n))/(6*b^
2) + (log(c + d*x)*(B*b*c^3*g*i*n - 3*B*a*c^2*d*g*i*n))/(6*d^2) + (A*b*d*g
*i*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.32

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$

$$=\frac{gi\Big(-\log(bx+a)\,a^3d^3n+3\log(bx+a)\,a^2bc\,d^2n-3\log(bx+a)\,a\,b^2c^2dn+\log(bx+a)\,b^3c^3n+3\log\Big(\frac{(bx}{(dx}$$

input

```
int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output

```
(g*i*( - log(a + b*x)*a**3*d**3*n + 3*log(a + b*x)*a**2*b*c*d**2*n - 3*log
(a + b*x)*a*b**2*c**2*d*n + log(a + b*x)*b**3*c**3*n + 3*log(((a + b*x)**n
*e)/(c + d*x)**n)*a*b**2*c**2*d + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b
**2*c*d**2*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**3*x**2 - log
(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c**3 + 3*log(((a + b*x)**n*e)/(c + d*
x)**n)*b**3*c*d**2*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*d**3*x
**3 + 6*a**2*b*c*d**2*x + a**2*b*d**3*n*x + 3*a**2*b*d**3*x**2 + 3*a*b**2*
c*d**2*x**2 + a*b**2*d**3*n*x**2 + 2*a*b**2*d**3*x**3 - b**3*c**2*d*n*x -
b**3*c*d**2*n*x**2))/(6*b*d**2)
```

3.111 $\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result 1353
Mathematica [A] (verified) 1353
Rubi [A] (verified) 1354
Maple [B] (verified) 1355
Fricas [B] (verification not implemented) 1356
Sympy [B] (verification not implemented) 1356
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Giac [B] (verification not implemented) 1358
Mupad [B] (verification not implemented) 1359
Reduce [B] (verification not implemented) 1359

Optimal result

Integrand size = 31, antiderivative size = 86

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= -\frac{B(bc-ad)inx}{2b} - \frac{B(bc-ad)^2in\log(a+bx)}{2b^2d} + \frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d}$$

output
```
-1/2*B*(-a*d+b*c)*i*n*x/b-1/2*B*(-a*d+b*c)^2*i*n*ln(b*x+a)/b^2/d+1/2*i*(d*
x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{i\left(-\frac{B(bc-ad)n(bdx+(bc-ad)\log(a+bx))}{b^2} + (c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{2d}$$

input
```
Integrate[(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(i*(-((B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a + b*x]))/b^2) + (c + d*x
)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(2*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\downarrow 2947$$

$$\frac{i(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bn(bc-ad)\int\frac{i^2(c+dx)}{a+bx}dx}{2di}$$

$$\downarrow 27$$

$$\frac{i(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bin(bc-ad)\int\frac{c+dx}{a+bx}dx}{2d}$$

$$\downarrow 49$$

$$\frac{i(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bin(bc-ad)\int\left(\frac{d}{b}+\frac{bc-ad}{b(a+bx)}\right)dx}{2d}$$

$$\downarrow 2009$$

$$\frac{i(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bin(bc-ad)\left(\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}\right)}{2d}$$

input
```
Int[(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
-1/2*(B*(b*c - a*d)*i*n*((d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2))/d + (i
*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(80) = 160$.

Time = 0.80 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.91

method	result
parallelrisch	$\frac{B x^2 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) b^2 d^2 i n + A x^2 b^2 d^2 i n - B \ln(bx+a) a^2 d^2 i n^2 + 2B \ln(bx+a) abcdi n^2 - B \ln(bx+a) b^2 c^2 i n^2 + 2Bx \ln\left(e\left(\frac{bx+}{dx+}\right.\right.$ 2

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*i*n+A*x^2*b^2*d^2*i*n-B*ln(b*
x+a)*a^2*d^2*i*n^2+2*B*ln(b*x+a)*a*b*c*d*i*n^2-B*ln(b*x+a)*b^2*c^2*i*n^2+2
*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*i*n+B*x*a*b*d^2*i*n^2-B*x*b^2*c*d*i
*n^2+2*A*x*b^2*c*d*i*n+B*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c^2*i*n-B*a^2*d^2*i
*n^2+B*b^2*c^2*i*n^2-3*A*a*b*c*d*i*n-2*A*b^2*c^2*i*n)/n/b^2/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(80) = 160.

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.88

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{Ab^2d^2ix^2-Bb^2c^2in\log(dx+c)+(2\,Babcd-Ba^2d^2)in\log(bx+a)+(2\,Ab^2cdi-(Bb^2cd-Babd^2)in}{2\,b^2d}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="frica
s")
```

output
```
1/2*(A*b^2*d^2*i*x^2 - B*b^2*c^2*i*n*log(d*x + c) + (2*B*a*b*c*d - B*a^2*d
^2)*i*n*log(b*x + a) + (2*A*b^2*c*d*i - (B*b^2*c*d - B*a*b*d^2)*i*n)*x + (
B*b^2*d^2*i*x^2 + 2*B*b^2*c*d*i*x)*log(e) + (B*b^2*d^2*i*n*x^2 + 2*B*b^2*c
*d*i*n*x)*log((b*x + a)/(d*x + c)))/(b^2*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(73) = 146.

Time = 21.46 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.44

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\begin{cases} cix\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ Acix+\frac{Adix^2}{2}+\frac{Bc^2i\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2d}+\frac{Bcinx}{2}+Bcix\log\left(e\left(\frac{a}{c+dx}\right)^n\right)+\frac{Bdinx^2}{4}+\frac{Bdix^2\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2} \\ ci\left(Ax+\frac{Ba\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b}-Bnx+Bx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)\right) \\ Acix+\frac{Adix^2}{2}-\frac{Ba^2din\log\left(\frac{c}{d}+x\right)}{2b^2}-\frac{Ba^2di\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{2b^2}+\frac{Bacin\log\left(\frac{c}{d}+x\right)}{b}+\frac{Baci\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{b}+\frac{Ba} \end{cases}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Piecewise((c*i*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (A*c*i*x +
 A*d*i*x**2/2 + B*c**2*i*log(e*(a/(c + d*x))**n)/(2*d) + B*c*i*n*x/2 + B*c
*i*x*log(e*(a/(c + d*x))**n) + B*d*i*n*x**2/4 + B*d*i*x**2*log(e*(a/(c + d
*x))**n)/2, Eq(b, 0)), (c*i*(A*x + B*a*log(e*(a/c + b*x/c)**n)/b - B*n*x +
 B*x*log(e*(a/c + b*x/c)**n)), Eq(d, 0)), (A*c*i*x + A*d*i*x**2/2 - B*a**2
*d*i*n*log(c/d + x)/(2*b**2) - B*a**2*d*i*log(e*(a/(c + d*x) + b*x/(c + d*
x))**n)/(2*b**2) + B*a*c*i*n*log(c/d + x)/b + B*a*c*i*log(e*(a/(c + d*x) +
 b*x/(c + d*x))**n)/b + B*a*d*i*n*x/(2*b) - B*c**2*i*n*log(c/d + x)/(2*d)
- B*c*i*n*x/2 + B*c*i*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*d*i*x*
*2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.81

$$
\begin{aligned}
&\int (ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx \\
&= \frac{1}{2}\,Bdix^2\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right) + \frac{1}{2}\,Adix^2 \\
&\quad - \frac{1}{2}\,Bdin\left(\frac{a^2\log\left(bx+a\right)}{b^2} - \frac{c^2\log\left(dx+c\right)}{d^2} + \frac{(bc-ad)x}{bd}\right) \\
&\quad + Bcin\left(\frac{a\log\left(bx+a\right)}{b} - \frac{c\log\left(dx+c\right)}{d}\right) \\
&\quad + Bcix\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right) + Acix
\end{aligned}
$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxim
a")
```

output
```
1/2*B*d*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*d*i*x^2 - 1/2
*B*d*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d
)) + B*c*i*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c*i*x*log(e*(b*x/(d
*x + c) + a/(d*x + c))^n) + A*c*i*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(80) = 160.

Time = 0.24 (sec) , antiderivative size = 580, normalized size of antiderivative = 6.74

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$

$$=\frac{1}{2}\left(\frac{\left(Bb^3c^3in-3\,Bab^2c^2din+3\,Ba^2bcd^2in-Ba^3d^3in\right)\log\left(\frac{bx+a}{dx+c}\right)}{b^2d-\frac{2\,(bx+a)bd^2}{dx+c}+\frac{(bx+a)^2d^3}{(dx+c)^2}}-\frac{Bb^4c^3in-3\,Bab^3c^2din-\frac{(bx+a)Bb^3}{dx+c}}{dx+c}\right.$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac"
)
```

output

```
1/2*((B*b^3*c^3*i*n - 3*B*a*b^2*c^2*d*i*n + 3*B*a^2*b*c*d^2*i*n - B*a^3*d^
3*i*n)*log((b*x + a)/(d*x + c))/(b^2*d - 2*(b*x + a)*b*d^2/(d*x + c) + (b*
x + a)^2*d^3/(d*x + c)^2) - (B*b^4*c^3*i*n - 3*B*a*b^3*c^2*d*i*n - (b*x +
a)*B*b^3*c^3*d*i*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*i*n + 3*(b*x + a)*B*a*b^2
*c^2*d^2*i*n/(d*x + c) - B*a^3*b*d^3*i*n - 3*(b*x + a)*B*a^2*b*c*d^3*i*n/(
d*x + c) + (b*x + a)*B*a^3*d^4*i*n/(d*x + c) - B*b^4*c^3*i*log(e) + 3*B*a*
b^3*c^2*d*i*log(e) - 3*B*a^2*b^2*c*d^2*i*log(e) + B*a^3*b*d^3*i*log(e) - A
*b^4*c^3*i + 3*A*a*b^3*c^2*d*i - 3*A*a^2*b^2*c*d^2*i + A*a^3*b*d^3*i)/(b^3
*d - 2*(b*x + a)*b^2*d^2/(d*x + c) + (b*x + a)^2*b*d^3/(d*x + c)^2) + (B*b
^3*c^3*i*n - 3*B*a*b^2*c^2*d*i*n + 3*B*a^2*b*c*d^2*i*n - B*a^3*d^3*i*n)*lo
g(-b + (b*x + a)*d/(d*x + c))/(b^2*d) - (B*b^3*c^3*i*n - 3*B*a*b^2*c^2*d*i
*n + 3*B*a^2*b*c*d^2*i*n - B*a^3*d^3*i*n)*log((b*x + a)/(d*x + c))/(b^2*d)
)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 25.64 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\begin{aligned}
&\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx \\
&= x\left(\frac{i\,(2\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,b}-\frac{A\,i\,(2\,a\,d+2\,b\,c)}{2\,b}\right) \\
&\quad+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,d\,i\,x^2}{2}+B\,c\,i\,x\right) \\
&\quad-\frac{\ln(a+b\,x)\,(B\,a^2\,d\,i\,n-2\,B\,a\,b\,c\,i\,n)}{2\,b^2}+\frac{A\,d\,i\,x^2}{2}-\frac{B\,c^2\,i\,n\,\ln(c+d\,x)}{2\,d}
\end{aligned}$$

input
```
int((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x*((i*(2*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*b) - (A*i*(2*a*d + 2*b*c
))/(2*b)) + log(e*((a + b*x)/(c + d*x))^n)*((B*d*i*x^2)/2 + B*c*i*x) - (lo
g(a + b*x)*(B*a^2*d*i*n - 2*B*a*b*c*i*n))/(2*b^2) + (A*d*i*x^2)/2 - (B*c^2
*i*n*log(c + d*x))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.26

$$\begin{aligned}
&\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx \\
&=\frac{i\left(-\log(dx+c)\,a^2d^2n+2\log(dx+c)\,abcdn-\log(dx+c)\,b^2c^2n-\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a^2d^2+2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)\right.}{2bd}
\end{aligned}$$

input
```
int((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
(i*( - log(c + d*x)*a**2*d**2*n + 2*log(c + d*x)*a*b*c*d*n - log(c + d*x)*
b**2*c**2*n - log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*d**2 + 2*log(((a + b
*x)**n*e)/(c + d*x)**n)*a*b*c*d + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**
2*c*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d**2*x**2 + 2*a*b*c*d*x
+ a*b*d**2*n*x + a*b*d**2*x**2 - b**2*c*d*n*x))/(2*b*d)
```

3.112 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$

Optimal result . . . 1360
Mathematica [A] (verified) . . . 1361
Rubi [A] (verified) . . . 1361
Maple [F] . . . 1364
Fricas [F] . . . 1365
Sympy [F] . . . 1365
Maxima [A] (verification not implemented) . . . 1366
Giac [F] . . . 1366
Mupad [F(-1)] . . . 1367
Reduce [F] . . . 1367

Optimal result

Integrand size = 41, antiderivative size = 141

$$\begin{aligned}
&\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx \\
&= \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg} \\
&\quad - \frac{(bc-ad)i\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g} \\
&\quad + \frac{B(bc-ad)in\,\mathrm{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{b^2g}
\end{aligned}$$

output

```
i*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g-(-a*d+b*c)*i*ln(-(-a*d+b*c)/
d/(b*x+a))*(A-B*n+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g+B*(-a*d+b*c)*i*n*poly
log(2,1+(-a*d+b*c)/d/(b*x+a))/b^2/g
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.22

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i\Big(B(-bc+ad)n\log^2(a+bx)+2\big(Abdx+Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+B(-bc+ad)n\log(c+dx)\big)+2}{2b}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*
g*x),x]
```

output
```
(i*(B*(-(b*c) + a*d)*n*Log[a + b*x]^2 + 2*(A*b*d*x + B*d*(a + b*x)*Log[e*(
(a + b*x)/(c + d*x))^n] + B*(-(b*c) + a*d)*n*Log[c + d*x]) + 2*(b*c - a*d)
*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(b*(c + d*x)
)/(b*c - a*d)]) + 2*B*(b*c - a*d)*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d
)]))/(2*b^2*g)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2959, 27, 2941, 2858, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{ag+bgx}\,dx$$
$$\downarrow 2959$$
$$\frac{i(bc-ad)\int \frac{A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(a+bx)}dx}{b} + \frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$
$$\downarrow 27$$

$$\frac{i(bc-ad)\int \frac{A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx}dx}{bg}+\frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\downarrow 2941$$

$$\frac{i(bc-ad)\left(\frac{Bn(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}dx}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\downarrow 2858$$

$$\frac{i(bc-ad)\left(\frac{Bn(bc-ad)\int \frac{b\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)\left(b\left(c-\frac{ad}{b}\right)+d(a+bx)\right)}d(a+bx)}{b^2}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\downarrow 27$$

$$\frac{i(bc-ad)\left(\frac{Bn(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(bc-ad+d(a+bx))}d(a+bx)}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\downarrow 2778$$

$$\frac{i(bc-ad)\left(-\frac{Bn(bc-ad)\int \frac{(a+bx)\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{bc-ad+d(a+bx)}d\frac{1}{a+bx}}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\downarrow 2005$$

$$\frac{i(bc-ad)\left(-\frac{Bn(bc-ad)\int\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{d+\frac{bc-ad}{a+bx}}d\frac{1}{a+bx}}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\downarrow 2752$$

$$\frac{i(bc-ad)\left(\frac{Bn\,\mathrm{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x),x
]
```

output
```
(i*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b*g) + ((b*c - a*d)*
i*(-((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A - B*n + B*Log[e*((a + b*x)/(c +
 d*x))^n]))/b) + (B*n*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/b))/(b*g)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
 + n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2941
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a + b*
x))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Simp[B*n*((b*c - a*d
)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f -
 a*g, 0]
```

rule 2959
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Simp[(f
 + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 2
))), x] + Simp[i*((b*c - a*d)/(b*d*(m + 2)))   Int[(f + g*x)^m*(A - B*n + B
*Log[e*((a + b*x)/(c + d*x))^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
i, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c
*i, 0] && IGtQ[m, -2]
```

Maple [F]

$$\int \frac{(dix+ci)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{bgx+ag}dx$$

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

output
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algo
rithm="fricas")
```

output
```
integral((A*d*i*x + A*c*i + (B*d*i*x + B*c*i)*log(e*((b*x + a)/(d*x + c))^
n))/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \frac{i\left(\int \frac{Ac}{a+bx}\,dx + \int \frac{Adx}{a+bx}\,dx + \int \frac{Bc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx + \int \frac{Bdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx\right)}{g}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g),x)
```

output
```
i*(Integral(A*c/(a + b*x), x) + Integral(A*d*x/(a + b*x), x) + Integral(B*
c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x) + Integral(B*d*x*l
og(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x))/g
```

Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.96

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$= Adi\left(\frac{x}{bg}-\frac{a\log(bx+a)}{b^2g}\right)-\frac{Bcin\log(dx+c)}{bg}+\frac{Aci\log(bgx+ag)}{bg}$$
$$+\frac{(bcin-adin)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^2g}$$
$$+\frac{2\,Bbdix\log(e)-(bcin-adin)B\log(bx+a)^2+2\,(bci\log(e)+(in-i\log(e))ad)B\log(bx+a)+2(}{}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algo
rithm="maxima")
```

output
```
A*d*i*(x/(b*g) - a*log(b*x + a)/(b^2*g)) - B*c*i*n*log(d*x + c)/(b*g) + A*
c*i*log(b*g*x + a*g)/(b*g) + (b*c*i*n - a*d*i*n)*(log(b*x + a)*log((b*d*x
+ a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^2*g) + 1
/2*(2*B*b*d*i*x*log(e) - (b*c*i*n - a*d*i*n)*B*log(b*x + a)^2 + 2*(b*c*i*l
og(e) + (i*n - i*log(e))*a*d)*B*log(b*x + a) + 2*(B*b*d*i*x + (b*c*i - a*d
*i)*B*log(b*x + a))*log((b*x + a)^n) - 2*(B*b*d*i*x + (b*c*i - a*d*i)*B*lo
g(b*x + a))*log((d*x + c)^n))/(b^2*g)
```

Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx=\int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algo
rithm="giac")
```

output
```
integrate((d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a*
g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x),x
)
```

output
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x),
x)
```

Reduce [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \frac{i\left(\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{bx+a}dx\right) b^3c + \left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x}{bx+a}dx\right) b^3d - \log(bx+a)\,a^2d + \log(bx+a)\,abc + abdx\right)}{b^2g}$$

input
```
int((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

output
```
(i*(int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a + b*x),x)*b**3*c + int((log(
((a + b*x)**n*e)/(c + d*x)**n)*x)/(a + b*x),x)*b**3*d - log(a + b*x)*a**2*
d + log(a + b*x)*a*b*c + a*b*d*x))/(b**2*g)
```

3.113 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$

Optimal result . . . 1368
Mathematica [A] (verified) . . . 1369
Rubi [A] (verified) . . . 1369
Maple [F] . . . 1371
Fricas [F] . . . 1372
Sympy [F(-1)] . . . 1372
Maxima [F] . . . 1372
Giac [F] . . . 1373
Mupad [F(-1)] . . . 1373
Reduce [F] . . . 1374

Optimal result

Integrand size = 41, antiderivative size = 150

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = -\frac{Bin(c+dx)}{bg^2(a+bx)} - \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{Bdin\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}$$

output

```
-B*i*n*(d*x+c)/b/g^2/(b*x+a)-i*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g
^2/(b*x+a)-d*i*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b
^2/g^2+B*d*i*n*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/g^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.26

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$$
$$= \frac{i\left(-\frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2(a+bx)}+\frac{d\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2}-\frac{Bn\left(\frac{bc-ad}{a+bx}+d\log(a+bx)-d\log(c+dx)\right)}{b^2}-\frac{Bdn\left(\log^2(a\right.}{g^2}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*
g*x)^2,x]
```

output
```
(i*(-(((b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^2*(a + b*x))
) + (d*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/b^2 - (B*n*((b
*c - a*d)/(a + b*x) + d*Log[a + b*x] - d*Log[c + d*x]))/b^2 - (B*d*n*(Log[
a + b*x]^2 - 2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[2,
-((d*(a + b*x))/(b*c - a*d))]))/(2*b^2)))/g^2
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {2961, 2780, 2741, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^2}\,dx$$
$$\downarrow \text{2961}$$
$$\frac{i\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g^2}$$
$$\downarrow \text{2780}$$

$$\frac{i\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^2}$$

$$\downarrow 2741$$

$$\frac{i\left(\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}}{b}\right)}{g^2}$$

$$\downarrow 2779$$

$$\frac{i\left(\frac{d\left(\frac{Bn\int \frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}\right)+\frac{-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}}{b}\right)}{g^2}$$

$$\downarrow 2838$$

$$\frac{i\left(\frac{d\left(\frac{Bn\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}\right)}{b}+\frac{-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}}{b}\right)}{g^2}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^2
,x]
```

output
```
(i*((-((B*n*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n]))/(a + b*x))/b + (d*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*
Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*(c + d*x))/(
d*(a + b*x))])/b))/b))/g^2
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(dix+ci)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(bgx+ag)^2}dx$$

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

output
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, al
gorithm="fricas")
```

output
```
integral((A*d*i*x + A*c*i + (B*d*i*x + B*c*i)*log(e*((b*x + a)/(d*x + c))^
n))/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, al
gorithm="maxima")
```

output
```
-B*c*i*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) -
 d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) + B*d*i*((((b*x + a)*log(b*x + a) +
 a)*log((b*x + a)^n) - ((b*x + a)*log(b*x + a) + a)*log((d*x + c)^n))/(b^3
*g^2*x + a*b^2*g^2) + integrate((b^2*d*x^2*log(e) + b^2*c*x*log(e) - a*b*c
*n + a^2*d*n - (a*b*c*n - a^2*d*n + (b^2*c*n - a*b*d*n)*x)*log(b*x + a))/(
b^4*d*g^2*x^3 + a^2*b^2*c*g^2 + (b^4*c*g^2 + 2*a*b^3*d*g^2)*x^2 + (2*a*b^3
*c*g^2 + a^2*b^2*d*g^2)*x), x)) + A*d*i*(a/(b^3*g^2*x + a*b^2*g^2) + log(b
*x + a)/(b^2*g^2)) - B*c*i*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^2*g^2
*x + a*b*g^2) - A*c*i/(b^2*g^2*x + a*b*g^2)
```

Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, al
gorithm="giac")
```

output
```
integrate((d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a*
g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(ci+dix)\left(A+B\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$$

input
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^2
,x)
```

output
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^2
, x)
```

Reduce [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$$

$$= \frac{i\left(\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x}{b^2x^2+2abx+a^2}dx\right)a^3b^3d^2 - \left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x}{b^2x^2+2abx+a^2}dx\right)a^2b^4cd + \left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x}{b^2x^2+2abx+a^2}dx\right)a^2b^4d^2x - \left(\int \frac{\log\left(\frac{(b}{b^2x^2+}\right.\right.}{}$$

input
```
int((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

output
```
(i*(int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**2 + 2*a*b*x + b**2*x**2
),x)*a**3*b**3*d**2 - int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**2 + 2
*a*b*x + b**2*x**2),x)*a**2*b**4*c*d + int((log(((a + b*x)**n*e)/(c + d*x)
**n)*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**4*d**2*x - int((log(((a +
b*x)**n*e)/(c + d*x)**n)*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a*b**5*c*d*x +
 log(a + b*x)*a**4*d**2 - log(a + b*x)*a**3*b*c*d + log(a + b*x)*a**3*b*d*
*2*x - log(a + b*x)*a**2*b**2*c*d*x + log(a + b*x)*a*b**3*c**2*n + log(a +
 b*x)*b**4*c**2*n*x - log(c + d*x)*a*b**3*c**2*n - log(c + d*x)*b**4*c**2*
n*x + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c*d*x - log(((a + b*x)**n*
e)/(c + d*x)**n)*b**4*c**2*x - a**3*b*d**2*x + 2*a**2*b**2*c*d*x - a*b**3*
c**2*x + a*b**3*c*d*n*x - b**4*c**2*n*x))/(a*b**2*g**2*(a**2*d - a*b*c + a
*b*d*x - b**2*c*x))
```

3.114 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$

Optimal result . . . 1375
Mathematica [B] (verified) . . . 1375
Rubi [A] (verified) . . . 1376
Maple [B] (verified) . . . 1377
Fricas [B] (verification not implemented) . . . 1378
Sympy [F(-1)] . . . 1378
Maxima [B] (verification not implemented) . . . 1379
Giac [A] (verification not implemented) . . . 1380
Mupad [B] (verification not implemented) . . . 1380
Reduce [B] (verification not implemented) . . . 1381

Optimal result

Integrand size = 41, antiderivative size = 89

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = -\frac{Bin(c+dx)^2}{4(bc-ad)g^3(a+bx)^2} - \frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)g^3(a+bx)^2}$$

output

```
-1/4*B*i*n*(d*x+c)^2/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*i*(d*x+c)^2*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/(-a*d+b*c)/g^3/(b*x+a)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 216 vs. 2(89) = 178.

Time = 0.20 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.43

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \frac{i\left(-\frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2(a+bx)^2} - \frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2(a+bx)} - \frac{Bdn\left(\frac{1}{a+bx}+\frac{d\log(a+bx)}{bc-ad}-\frac{d\log(c+dx)}{bc-ad}\right)}{b^2} - \frac{Bn\left(\frac{bc-ad}{(a+bx)^2}-\frac{2d}{a+bx}-\frac{2}{}\right.\right.}{g^3}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*
g*x)^3,x]
```

output
```
(i*(-1/2*((b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^2*(a + b*
x)^2) - (d*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^2*(a + b*x)) - (B*d*
n*((a + b*x)^(-1) + (d*Log[a + b*x])/(b*c - a*d) - (d*Log[c + d*x])/(b*c -
 a*d)))/b^2 - (B*n*((b*c - a*d)/(a + b*x)^2 - (2*d)/(a + b*x) - (2*d^2*Log
[a + b*x])/(b*c - a*d) + (2*d^2*Log[c + d*x])/(b*c - a*d)))/(4*b^2)))/g^3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$, Rules used = {2961, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{i\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{i\left(-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}\right)}{g^3(bc-ad)}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^3
,x]
```

output
```
(i*(-1/4*(B*n*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(2*(a + b*x)^2)))/((b*c - a*d)*g^3)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(85) = 170$.

Time = 4.73 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.66

method	result
parallelrisch	$-\frac{-4Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4cd^2in+Ba^2b^2d^3in^2-Bb^4c^2din^2+2Aa^2b^2d^3in-2Ab^4c^2din-2B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4c^2din-2Bx}{4g^3(bx+a)^2b^4dn(da-bc)}$

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,method=_RE
TURNVERBOSE)
```

output
```
-1/4*(-4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^2*i*n+B*a^2*b^2*d^3*i*n^2-B
*b^4*c^2*d*i*n^2+2*A*a^2*b^2*d^3*i*n-2*A*b^4*c^2*d*i*n-2*B*ln(e*((b*x+a)/(
d*x+c))^n)*b^4*c^2*d*i*n-2*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^3*i*n+2*B
*x*a*b^3*d^3*i*n^2-2*B*x*b^4*c*d^2*i*n^2+4*A*x*a*b^3*d^3*i*n-4*A*x*b^4*c*d
^2*i*n)/g^3/(b*x+a)^2/b^4/d/n/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(85) = 170$.

Time = 0.09 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.81

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx =$$
$$-\frac{(Bb^2c^2-Ba^2d^2)in+2\left(Ab^2c^2-Aa^2d^2\right)i+2\left((Bb^2cd-Babd^2)in+2\left(Ab^2cd-Aabd^2\right)i\right)x+2\left(2\left(Bb\right.\right.}{4\left(\left(b^5c-ab^4d\right)g^3x^2+2\left(ab^4c-a^2\right.\right.}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, al
gorithm="fricas")
```

output
```
-1/4*((B*b^2*c^2 - B*a^2*d^2)*i*n + 2*(A*b^2*c^2 - A*a^2*d^2)*i + 2*((B*b^
2*c*d - B*a*b*d^2)*i*n + 2*(A*b^2*c*d - A*a*b*d^2)*i)*x + 2*(2*(B*b^2*c*d
 - B*a*b*d^2)*i*x + (B*b^2*c^2 - B*a^2*d^2)*i)*log(e) + 2*(B*b^2*d^2*i*n*x^
2 + 2*B*b^2*c*d*i*n*x + B*b^2*c^2*i*n)*log((b*x + a)/(d*x + c)))/((b^5*c -
 a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d
)*g^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(85) = 170$.

Time = 0.06 (sec) , antiderivative size = 582, normalized size of antiderivative = 6.54

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx =$$
$$-\frac{1}{4}Bdin\left(\frac{3\,abc-a^2d+2\,(2\,b^2c-abd)x}{(b^5c-ab^4d)g^3x^2+2\,(ab^4c-a^2b^3d)g^3x+(a^2b^3c-a^3b^2d)g^3}+\frac{2\,(2\,bcd-ad^2)\log(bx+a)}{(b^4c^2-2\,ab^3cd+a^2b^2d^2)g^3}-\right.$$
$$+\frac{1}{4}Bcin\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,d^2\log(bx+a)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}-\right.$$
$$-\frac{(2\,bx+a)Bdi\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2\,(b^4g^3x^2+2\,ab^3g^3x+a^2b^2g^3)}-\frac{(2\,bx+a)Adi}{2\,(b^4g^3x^2+2\,ab^3g^3x+a^2b^2g^3)}$$
$$-\frac{Bci\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}-\frac{Aci}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, al
gorithm="maxima")
```

output
```
-1/4*B*d*i*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c - a*b^4*d)
*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3) +
2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^
3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^
2)*g^3)) + 1/4*B*c*i*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2
 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(
b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((
b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*(2*b*x + a)*B*d*i*log(e*(b*
x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3)
- 1/2*(2*b*x + a)*A*d*i/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - 1/2*
B*c*i*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x
+ a^2*b*g^3) - 1/2*A*c*i/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx =$$
$$-\frac{1}{4}\left(\frac{2\,(dx+c)^2 Bin\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^2 g^3}+\frac{(Bin+2\,Bi\log(e)+2\,Ai)(dx+c)^2}{(bx+a)^2 g^3}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, al
gorithm="giac")
```

output
```
-1/4*(2*(d*x + c)^2*B*i*n*log((b*x + a)/(d*x + c))/((b*x + a)^2*g^3) + (B*
i*n + 2*B*i*log(e) + 2*A*i)*(d*x + c)^2/((b*x + a)^2*g^3))*(b*c/(b*c - a*d
)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 26.43 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.29

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$$
$$= -\frac{x\,(2\,A\,b\,d\,i+B\,b\,d\,i\,n)+A\,a\,d\,i+A\,b\,c\,i+\frac{B\,a\,d\,i\,n}{2}+\frac{B\,b\,c\,i\,n}{2}}{2\,a^2\,b^2\,g^3+4\,a\,b^3\,g^3\,x+2\,b^4\,g^3\,x^2}$$
$$-\frac{\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(\frac{B\,c\,i}{2\,b}+\frac{B\,a\,d\,i}{2\,b^2}+\frac{B\,d\,i\,x}{b}\right)}{a^2\,g^3+2\,a\,b\,g^3\,x+b^2\,g^3\,x^2}-\frac{B\,d^2\,i\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,1\mathrm{i}}{b^2\,g^3\,(a\,d-b\,c)}$$

input
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^3
,x)
```

output
```
- (x*(2*A*b*d*i + B*b*d*i*n) + A*a*d*i + A*b*c*i + (B*a*d*i*n)/2 + (B*b*c*
i*n)/2)/(2*a^2*b^2*g^3 + 2*b^4*g^3*x^2 + 4*a*b^3*g^3*x) - (log(e*((a + b*x
)/(c + d*x))^n)*((B*c*i)/(2*b) + (B*a*d*i)/(2*b^2) + (B*d*i*x)/b))/(a^2*g^
3 + b^2*g^3*x^2 + 2*a*b*g^3*x) - (B*d^2*i*n*atan((b*c*2i + b*d*x*2i)/(a*d
- b*c) + 1i)*1i)/(b^2*g^3*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.97

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$$

$$= \frac{i\Big(2\log(bx+a)\,a^2bcdn+4\log(bx+a)\,ab^2cdnx+2\log(bx+a)\,b^3cdn\,x^2-2\log(dx+c)\,a^2bcdn-4\log($$

input
```
int((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
```

output
```
(i*(2*log(a + b*x)*a**2*b*c*d*n + 4*log(a + b*x)*a*b**2*c*d*n*x + 2*log(a
+ b*x)*b**3*c*d*n*x**2 - 2*log(c + d*x)*a**2*b*c*d*n - 4*log(c + d*x)*a*b*
*2*c*d*n*x - 2*log(c + d*x)*b**3*c*d*n*x**2 - 2*log(((a + b*x)**n*e)/(c +
d*x)**n)*a**2*b*c*d + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c**2 + 2
*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**2*x**2 - 2*log(((a + b*x)**n
*e)/(c + d*x)**n)*b**3*c*d*x**2 - 2*a**3*c*d + 2*a**2*b*c**2 - a**2*b*c*d*
n + 2*a**2*b*d**2*x**2 + a*b**2*c**2*n - 2*a*b**2*c*d*x**2 + a*b**2*d**2*n
*x**2 - b**3*c*d*n*x**2))/(4*a*b*g**3*(a**3*d - a**2*b*c + 2*a**2*b*d*x -
2*a*b**2*c*x + a*b**2*d*x**2 - b**3*c*x**2))
```

3.115 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$

Optimal result . . . 1382
Mathematica [A] (verified) . . . 1383
Rubi [A] (verified) . . . 1383
Maple [B] (verified) . . . 1385
Fricas [B] (verification not implemented) . . . 1386
Sympy [F(-1)] . . . 1387
Maxima [B] (verification not implemented) . . . 1387
Giac [A] (verification not implemented) . . . 1388
Mupad [B] (verification not implemented) . . . 1389
Reduce [B] (verification not implemented) . . . 1390

Optimal result

Integrand size = 41, antiderivative size = 181

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \frac{Bdin(c+dx)^2}{4(bc-ad)^2g^4(a+bx)^2} - \frac{bBin(c+dx)^3}{9(bc-ad)^2g^4(a+bx)^3} + \frac{di(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} - \frac{bi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^2g^4(a+bx)^3}$$

output

```
1/4*B*d*i*n*(d*x+c)^2/(-a*d+b*c)^2/g^4/(b*x+a)^2-1/9*b*B*i*n*(d*x+c)^3/(-a
*d+b*c)^2/g^4/(b*x+a)^3+1/2*d*i*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/
(-a*d+b*c)^2/g^4/(b*x+a)^2-1/3*b*i*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n
))/(-a*d+b*c)^2/g^4/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.08

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{i\left(\frac{12Abc}{(a+bx)^3}-\frac{12aAd}{(a+bx)^3}+\frac{4bBcn}{(a+bx)^3}-\frac{4aBdn}{(a+bx)^3}+\frac{18Ad}{(a+bx)^2}+\frac{3Bdn}{(a+bx)^2}-\frac{6Bd^2n}{(bc-ad)(a+bx)}-\frac{6Bd^3n\log(a+bx)}{(bc-ad)^2}+\frac{6B(2bc+ad+3}{(}\right.}{36b^2g^4}$$

input

```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*
g*x)^4,x]
```

output

```
-1/36*(i*((12*A*b*c)/(a + b*x)^3 - (12*a*A*d)/(a + b*x)^3 + (4*b*B*c*n)/(a
 + b*x)^3 - (4*a*B*d*n)/(a + b*x)^3 + (18*A*d)/(a + b*x)^2 + (3*B*d*n)/(a
+ b*x)^2 - (6*B*d^2*n)/((b*c - a*d)*(a + b*x)) - (6*B*d^3*n*Log[a + b*x])/
(b*c - a*d)^2 + (6*B*(2*b*c + a*d + 3*b*d*x)*Log[e*((a + b*x)/(c + d*x))^n
])/(a + b*x)^3 + (6*B*d^3*n*Log[c + d*x])/(b*c - a*d)^2))/(b^2*g^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {2961, 2772, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^4}\,dx$$

$$\downarrow \text{2961}$$

$$\frac{i\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)^2}$$

$$\downarrow \text{2772}$$

$$\frac{i\left(-Bn\int -\frac{(c+dx)^4\left(2b-\frac{3d(a+bx)}{c+dx}\right)}{6(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}\right)}{g^4(bc-ad)^2}$$

$$\downarrow 27$$

$$\frac{i\left(\frac{1}{6}Bn\int \frac{(c+dx)^4\left(2b-\frac{3d(a+bx)}{c+dx}\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}\right)}{g^4(bc-ad)^2}$$

$$\downarrow 53$$

$$\frac{i\left(\frac{1}{6}Bn\int \left(\frac{2b(c+dx)^4}{(a+bx)^4}-\frac{3d(c+dx)^3}{(a+bx)^3}\right)d\frac{a+bx}{c+dx}-\frac{b(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}\right)}{g^4(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{i\left(-\frac{b(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{1}{6}Bn\left(\frac{3d(c+dx)^2}{2(a+bx)^2}-\frac{2b(c+dx)^3}{3(a+bx)^3}\right)\right)}{g^4(bc-ad)^2}$$

input

```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^4
,x]
```

output

```
(i*((B*n*((3*d*(c + d*x)^2)/(2*(a + b*x)^2) - (2*b*(c + d*x)^3)/(3*(a + b*
x)^3)))/6 + (d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a +
 b*x)^2) - (b*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(a +
b*x)^3)))/((b*c - a*d)^2*g^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(173) = 346.

Time = 10.16 (sec) , antiderivative size = 455, normalized size of antiderivative = 2.51

method	result
parallelrisch	$-\frac{5Ba^3b^3d^4in^2+4Bb^6c^3din^2+6Aa^3b^3d^4in+12Ab^6c^3din-36Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)ab^5cd^3in-18Bxab^5cd^3in^2-36Axab^5c$

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,method=_RE
TURNVERBOSE)
```

output
```
-1/36*(5*B*a^3*b^3*d^4*i*n^2+4*B*b^6*c^3*d*i*n^2+6*A*a^3*b^3*d^4*i*n+12*A*
b^6*c^3*d*i*n-36*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^3*i*n-18*B*x*a*b^
5*c*d^3*i*n^2-36*A*x*a*b^5*c*d^3*i*n-18*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*
c^2*d^2*i*n-9*B*a*b^5*c^2*d^2*i*n^2-18*A*a*b^5*c^2*d^2*i*n-6*B*x^3*ln(e*((
b*x+a)/(d*x+c))^n)*b^6*d^4*i*n+6*B*x^2*a*b^5*d^4*i*n^2-6*B*x^2*b^6*c*d^3*i
*n^2+15*B*x*a^2*b^4*d^4*i*n^2+3*B*x*b^6*c^2*d^2*i*n^2+18*A*x*a^2*b^4*d^4*i
*n+18*A*x*b^6*c^2*d^2*i*n+12*B*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^3*d*i*n-18*
B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^4*i*n+18*B*x*ln(e*((b*x+a)/(d*x+c)
)^n)*b^6*c^2*d^2*i*n)/g^4/(b*x+a)^3/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^5/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(173) = 346$.

Time = 0.10 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.64

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$$
$$= \frac{6\left(Bb^3cd^2-Bab^2d^3\right)inx^2-\left(4\,Bb^3c^3-9\,Bab^2c^2d+5\,Ba^3d^3\right)in-6\left(2\,Ab^3c^3-3\,Aab^2c^2d+Aa^3d^3\right)i-3$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, al
gorithm="fricas")
```

output
```
1/36*(6*(B*b^3*c*d^2 - B*a*b^2*d^3)*i*n*x^2 - (4*B*b^3*c^3 - 9*B*a*b^2*c^2
*d + 5*B*a^3*d^3)*i*n - 6*(2*A*b^3*c^3 - 3*A*a*b^2*c^2*d + A*a^3*d^3)*i -
3*((B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*i*n + 6*(A*b^3*c^2*d -
2*A*a*b^2*c*d^2 + A*a^2*b*d^3)*i)*x - 6*(3*(B*b^3*c^2*d - 2*B*a*b^2*c*d^2
+ B*a^2*b*d^3)*i*x + (2*B*b^3*c^3 - 3*B*a*b^2*c^2*d + B*a^3*d^3)*i)*log(e)
 + 6*(B*b^3*d^3*i*n*x^3 + 3*B*a*b^2*d^3*i*n*x^2 - 3*(B*b^3*c^2*d - 2*B*a*b
^2*c*d^2)*i*n*x - (2*B*b^3*c^3 - 3*B*a*b^2*c^2*d)*i*n)*log((b*x + a)/(d*x
+ c)))/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a
^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b
^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(173) = 346$.

Time = 0.09 (sec) , antiderivative size = 945, normalized size of antiderivative = 5.22

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, al
gorithm="maxima")
```

output

```
-1/18*B*c*i*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^
2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(
a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*
b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g
^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3
*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*
d^2 - a^3*b*d^3)*g^4)) - 1/36*B*d*i*n*((5*a*b^2*c^2 - 22*a^2*b*c*d + 5*a^3
*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*a*b^2*c*d + 5*a^2
*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 -
 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a
^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4) - 6*(
3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^
2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x + c)/((b^5*c^3 - 3*a
*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4)) - 1/6*(3*b*x + a)*B*d*i*
log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^5*g^4*x^3 + 3*a*b^4*g^4*x^2 + 3*
a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/6*(3*b*x + a)*A*d*i/(b^5*g^4*x^3 + 3*a*b^
4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/3*B*c*i*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n)/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3
*b*g^4) - 1/3*A*c*i/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3
*b*g^4)
```

Giac [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.29

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{1}{36}\left(\frac{6\left(2\,Bbin-\frac{3\,(bx+a)Bdin}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^3bcg^4}{(dx+c)^3}-\frac{(bx+a)^3adg^4}{(dx+c)^3}}+\frac{4\,Bbin-\frac{9\,(bx+a)Bdin}{dx+c}+12\,Bbi\log(e)-\frac{18\,(bx+a)Bdi\log(e)}{dx+c}+}{\frac{(bx+a)^3bcg^4}{(dx+c)^3}-\frac{(bx+a)^3adg^4}{(dx+c)^3}}\right.$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, al
gorithm="giac")
```

output
```
-1/36*(6*(2*B*b*i*n - 3*(b*x + a)*B*d*i*n/(d*x + c))*log((b*x + a)/(d*x +
c))/((b*x + a)^3*b*c*g^4/(d*x + c)^3 - (b*x + a)^3*a*d*g^4/(d*x + c)^3) +
(4*B*b*i*n - 9*(b*x + a)*B*d*i*n/(d*x + c) + 12*B*b*i*log(e) - 18*(b*x + a
)*B*d*i*log(e)/(d*x + c) + 12*A*b*i - 18*(b*x + a)*A*d*i/(d*x + c))/((b*x
+ a)^3*b*c*g^4/(d*x + c)^3 - (b*x + a)^3*a*d*g^4/(d*x + c)^3))*(b*c/(b*c -
 a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 26.50 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.07

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{\frac{6Aa^2d^2i-12Ab^2c^2i+5Ba^2d^2in-4Bb^2c^2in+6Aabcdi+5Babcdin}{6(ad-bc)}+\frac{x\left(6Aabd^2i-6Ab^2cdi-Bb^2cdin+5Babd^2in\right)}{2(ad-bc)}+}{6a^3b^2g^4+18a^2b^3g^4x+18ab^4g^4x^2+6b^5g^4x^3}$$
$$-\frac{\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(\frac{Bci}{3b}+\frac{Badi}{6b^2}+\frac{Bdix}{2b}\right)}{a^3g^4+3a^2bg^4x+3ab^2g^4x^2+b^3g^4x^3}$$
$$-\frac{Bd^3in\,\mathrm{atanh}\left(\frac{6b^4c^2g^4-6a^2b^2d^2g^4}{6b^2g^4(ad-bc)^2}-\frac{2bdx}{ad-bc}\right)}{3b^2g^4(ad-bc)^2}$$

input
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^4
,x)
```

output
```
- ((6*A*a^2*d^2*i - 12*A*b^2*c^2*i + 5*B*a^2*d^2*i*n - 4*B*b^2*c^2*i*n + 6
*A*a*b*c*d*i + 5*B*a*b*c*d*i*n)/(6*(a*d - b*c)) + (x*(6*A*a*b*d^2*i - 6*A*
b^2*c*d*i - B*b^2*c*d*i*n + 5*B*a*b*d^2*i*n))/(2*(a*d - b*c)) + (B*b^2*d^2
*i*n*x^2)/(a*d - b*c))/(6*a^3*b^2*g^4 + 6*b^5*g^4*x^3 + 18*a^2*b^3*g^4*x +
 18*a*b^4*g^4*x^2) - (log(e*((a + b*x)/(c + d*x))^n)*((B*c*i)/(3*b) + (B*a
*d*i)/(6*b^2) + (B*d*i*x)/(2*b)))/(a^3*g^4 + b^3*g^4*x^3 + 3*a*b^2*g^4*x^2
 + 3*a^2*b*g^4*x) - (B*d^3*i*n*atanh((6*b^4*c^2*g^4 - 6*a^2*b^2*d^2*g^4)/(
6*b^2*g^4*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(3*b^2*g^4*(a*d - b*c)^
2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 937, normalized size of antiderivative = 5.18

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x)
```

output

```
(i*(12*log(a + b*x)*a**4*b**2*c*d**2*n - 6*log(a + b*x)*a**3*b**3*c**2*d*n
 + 36*log(a + b*x)*a**3*b**3*c*d**2*n*x - 18*log(a + b*x)*a**2*b**4*c**2*d
*n*x + 36*log(a + b*x)*a**2*b**4*c*d**2*n*x**2 - 18*log(a + b*x)*a*b**5*c*
*2*d*n*x**2 + 12*log(a + b*x)*a*b**5*c*d**2*n*x**3 - 6*log(a + b*x)*b**6*c
**2*d*n*x**3 - 12*log(c + d*x)*a**4*b**2*c*d**2*n + 6*log(c + d*x)*a**3*b*
*3*c**2*d*n - 36*log(c + d*x)*a**3*b**3*c*d**2*n*x + 18*log(c + d*x)*a**2*
b**4*c**2*d*n*x - 36*log(c + d*x)*a**2*b**4*c*d**2*n*x**2 + 18*log(c + d*x
)*a*b**5*c**2*d*n*x**2 - 12*log(c + d*x)*a*b**5*c*d**2*n*x**3 + 6*log(c +
d*x)*b**6*c**2*d*n*x**3 - 12*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*b**2*
c*d**2 + 24*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**3*c**2*d + 18*log((
(a + b*x)**n*e)/(c + d*x)**n)*a**3*b**3*d**3*x**2 - 12*log(((a + b*x)**n*e
)/(c + d*x)**n)*a**2*b**4*c**3 - 36*log(((a + b*x)**n*e)/(c + d*x)**n)*a**
2*b**4*c*d**2*x**2 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**4*d**3*x
**3 + 18*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**5*c**2*d*x**2 - 12*log(((
a + b*x)**n*e)/(c + d*x)**n)*a*b**5*c*d**2*x**3 + 6*log(((a + b*x)**n*e)/(
c + d*x)**n)*b**6*c**2*d*x**3 - 6*a**6*d**3 - 3*a**5*b*d**3*n - 18*a**5*b*
d**3*x + 18*a**4*b**2*c**2*d - 2*a**4*b**2*c*d**2*n + 36*a**4*b**2*c*d**2*
x - 9*a**4*b**2*d**3*n*x - 12*a**3*b**3*c**3 + 9*a**3*b**3*c**2*d*n - 18*a
**3*b**3*c**2*d*x + 12*a**3*b**3*c*d**2*n*x - 4*a**2*b**4*c**3*n - 3*a**2*
b**4*c**2*d*n*x + 2*a**2*b**4*d**3*n*x**3 - 2*a*b**5*c*d**2*n*x**3))/(3...
```

3.116 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$

Optimal result . . . 1391
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Rubi [A] (verified) . . . 1392
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Optimal result

Integrand size = 41, antiderivative size = 281

$$\begin{aligned}\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx &= -\frac{Bd^2in(c+dx)^2}{4(bc-ad)^3g^5(a+bx)^2}\\ &+\frac{2bBdin(c+dx)^3}{9(bc-ad)^3g^5(a+bx)^3}\\ &-\frac{b^2Bin(c+dx)^4}{16(bc-ad)^3g^5(a+bx)^4}\\ &-\frac{d^2i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^5(a+bx)^2}\\ &+\frac{2bdi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^3g^5(a+bx)^3}\\ &-\frac{b^2i(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^3g^5(a+bx)^4}\end{aligned}$$

output

```
-1/4*B*d^2*i*n*(d*x+c)^2/(-a*d+b*c)^3/g^5/(b*x+a)^2+2/9*b*B*d*i*n*(d*x+c)^
3/(-a*d+b*c)^3/g^5/(b*x+a)^3-1/16*b^2*B*i*n*(d*x+c)^4/(-a*d+b*c)^3/g^5/(b*
x+a)^4-1/2*d^2*i*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^
5/(b*x+a)^2+2/3*b*d*i*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)
^3/g^5/(b*x+a)^3-1/4*b^2*i*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d
+b*c)^3/g^5/(b*x+a)^4
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.78

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{i\left(\frac{36Abc}{(a+bx)^4}-\frac{36aAd}{(a+bx)^4}+\frac{9bBcn}{(a+bx)^4}-\frac{9aBdn}{(a+bx)^4}+\frac{48Ad}{(a+bx)^3}+\frac{4Bdn}{(a+bx)^3}-\frac{6Bd^2n}{(bc-ad)(a+bx)^2}+\frac{12Bd^3n}{(bc-ad)^2(a+bx)}+\frac{12Bd^4n\log(a-}{(bc-ad)^3}\right.}{144b^2g^5}$$

input

```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*
g*x)^5,x]
```

output

```
-1/144*(i*((36*A*b*c)/(a + b*x)^4 - (36*a*A*d)/(a + b*x)^4 + (9*b*B*c*n)/(
a + b*x)^4 - (9*a*B*d*n)/(a + b*x)^4 + (48*A*d)/(a + b*x)^3 + (4*B*d*n)/(a
 + b*x)^3 - (6*B*d^2*n)/((b*c - a*d)*(a + b*x)^2) + (12*B*d^3*n)/((b*c - a
*d)^2*(a + b*x)) + (12*B*d^4*n*Log[a + b*x])/(b*c - a*d)^3 + (12*B*(3*b*c
 + a*d + 4*b*d*x)*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x)^4 - (12*B*d^4*n
*Log[c + d*x])/(b*c - a*d)^3))/(b^2*g^5)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {2961, 2772, 27, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^5}\,dx$$

$$\downarrow \text{2961}$$

$$\frac{i\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^5}\,d\frac{a+bx}{c+dx}}{g^5(bc-ad)^3}$$

$$\downarrow 2772$$

$$\frac{i\left(-Bn\int -\frac{(c+dx)^5\left(3b^2-\frac{8d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{12(a+bx)^5}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}\right.}{g^5(bc-ad)^3}$$

$$\downarrow 27$$

$$\frac{i\left(\frac{1}{12}Bn\int \frac{(c+dx)^5\left(3b^2-\frac{8d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}\right.}{g^5(bc-ad)^3}$$

$$\downarrow 1140$$

$$\frac{i\left(\frac{1}{12}Bn\int \left(\frac{3b^2(c+dx)^5}{(a+bx)^5}-\frac{8bd(c+dx)^4}{(a+bx)^4}+\frac{6d^2(c+dx)^3}{(a+bx)^3}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}\right.}{g^5(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{i\left(-\frac{b^2(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{2bd(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{1}{12}Bn\left(-\frac{3b^2(c\right.}{g^5(bc-ad)^3}$$

input

```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^5
,x]
```

output

```
(i*((B*n*((-3*d^2*(c + d*x)^2)/(a + b*x)^2 + (8*b*d*(c + d*x)^3)/(3*(a + b
*x)^3) - (3*b^2*(c + d*x)^4)/(4*(a + b*x)^4)))/12 - (d^2*(c + d*x)^2*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) + (2*b*d*(c + d*x)^3*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^3) - (b^2*(c + d*x)^4*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*(a + b*x)^4)))/((b*c - a*d)^3*g^
5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 985 vs. 2(269) = 538.

Time = 29.72 (sec) , antiderivative size = 986, normalized size of antiderivative = 3.51

method	result
parallelrisch	$\frac{48Bx^3\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^7bcd^4in-144Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^7bc^3d^2in+48Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^6b^2c^4din-9Bx^4a^2b^6c^5in^2-36$

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x,method=_RE
TURNVERBOSE)
```

output
```
1/144*(48*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c*d^4*i*n-144*B*x*ln(e*((b
*x+a)/(d*x+c))^n)*a^7*b*c^3*d^2*i*n+48*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b
^2*c^4*d*i*n-9*B*x^4*a^2*b^6*c^5*i*n^2-36*A*x^4*a^2*b^6*c^5*i*n-36*B*x^3*a
^3*b^5*c^5*i*n^2-144*A*x^3*a^3*b^5*c^5*i*n+36*B*x^2*a^8*c*d^4*i*n^2-54*B*x
^2*a^4*b^4*c^5*i*n^2+72*A*x^2*a^8*c*d^4*i*n-216*A*x^2*a^4*b^4*c^5*i*n+72*B
*x*a^8*c^2*d^3*i*n^2-36*B*x*a^5*b^3*c^5*i*n^2+144*A*x*a^8*c^2*d^3*i*n-144*
A*x*a^5*b^3*c^5*i*n+72*B*ln(e*((b*x+a)/(d*x+c))^n)*a^8*c^3*d^2*i*n+36*B*ln
(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^5*i*n+128*B*x^3*a^4*b^4*c^4*d*i*n^2+48*A
*x^3*a^7*b*c*d^4*i*n-288*A*x^3*a^5*b^3*c^3*d^2*i*n+384*A*x^3*a^4*b^4*c^4*d
*i*n+72*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^8*c*d^4*i*n+48*B*x^2*a^7*b*c^2*d
^3*i*n^2-222*B*x^2*a^6*b^2*c^3*d^2*i*n^2+192*B*x^2*a^5*b^3*c^4*d*i*n^2-432
*A*x^2*a^6*b^2*c^3*d^2*i*n+576*A*x^2*a^5*b^3*c^4*d*i*n+144*B*x*ln(e*((b*x+
a)/(d*x+c))^n)*a^8*c^2*d^3*i*n-168*B*x*a^7*b*c^3*d^2*i*n^2+132*B*x*a^6*b^2
*c^4*d*i*n^2-432*A*x*a^7*b*c^3*d^2*i*n+432*A*x*a^6*b^2*c^4*d*i*n-96*B*ln(e
*((b*x+a)/(d*x+c))^n)*a^7*b*c^4*d*i*n+12*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a
^6*b^2*c*d^4*i*n+13*B*x^4*a^6*b^2*c*d^4*i*n^2-36*B*x^4*a^4*b^4*c^3*d^2*i*n
^2+32*B*x^4*a^3*b^5*c^4*d*i*n^2+12*A*x^4*a^6*b^2*c*d^4*i*n-72*A*x^4*a^4*b^
4*c^3*d^2*i*n+96*A*x^4*a^3*b^5*c^4*d*i*n+40*B*x^3*a^7*b*c*d^4*i*n^2+12*B*x
^3*a^6*b^2*c^2*d^3*i*n^2-144*B*x^3*a^5*b^3*c^3*d^2*i*n^2)/g^5/(b*x+a)^4/n/
(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a^6/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(269) = 538$.

Time $= 0.11$ (sec) , antiderivative size $= 773$, normalized size of antiderivative $= 2.75$

$$\int \frac{(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag + bgx)^5}\,dx =$$

$$-\frac{12\left(Bb^4cd^3 - Bab^3d^4\right)inx^3 - 6\left(Bb^4c^2d^2 - 8\,Bab^3cd^3 + 7\,Ba^2b^2d^4\right)inx^2 + \left(9\,Bb^4c^4 - 32\,Bab^3c^3d + 36\right.}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, al
gorithm="fricas")
```

output

```
-1/144*(12*(B*b^4*c*d^3 - B*a*b^3*d^4)*i*n*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*
b^3*c*d^3 + 7*B*a^2*b^2*d^4)*i*n*x^2 + (9*B*b^4*c^4 - 32*B*a*b^3*c^3*d + 3
6*B*a^2*b^2*c^2*d^2 - 13*B*a^4*d^4)*i*n + 12*(3*A*b^4*c^4 - 8*A*a*b^3*c^3*
d + 6*A*a^2*b^2*c^2*d^2 - A*a^4*d^4)*i + 4*((B*b^4*c^3*d - 6*B*a*b^3*c^2*d
^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*b*d^4)*i*n + 12*(A*b^4*c^3*d - 3*A*a*b^
3*c^2*d^2 + 3*A*a^2*b^2*c*d^3 - A*a^3*b*d^4)*i)*x + 12*(4*(B*b^4*c^3*d - 3
*B*a*b^3*c^2*d^2 + 3*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*i*x + (3*B*b^4*c^4 - 8
*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - B*a^4*d^4)*i)*log(e) + 12*(B*b^4*d^
4*i*n*x^4 + 4*B*a*b^3*d^4*i*n*x^3 + 6*B*a^2*b^2*d^4*i*n*x^2 + 4*(B*b^4*c^3
*d - 3*B*a*b^3*c^2*d^2 + 3*B*a^2*b^2*c*d^3)*i*n*x + (3*B*b^4*c^4 - 8*B*a*b
^3*c^3*d + 6*B*a^2*b^2*c^2*d^2)*i*n)*log((b*x + a)/(d*x + c)))/((b^9*c^3 -
 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3
*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 -
 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3
 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 -
 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*b^2*d^3)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = \text{Timed out}$$

input

```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**5,x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1398 vs. $2(269) = 538$.

Time $= 0.10$ (sec) , antiderivative size $= 1398$, normalized size of antiderivative $= 4.98$

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, al
gorithm="maxima")
```

output
```
1/48*B*c*i*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^
2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*
c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3
*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4
*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a
^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 -
 a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a
^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3
*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c
^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)
) - 1/144*B*d*i*n*((7*a*b^3*c^3 - 33*a^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 13*a
^4*d^3 + 12*(4*b^4*c*d^2 - a*b^3*d^3)*x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c*d^
2 + 7*a^2*b^2*d^3)*x^2 + 4*(4*b^4*c^3 - 21*a*b^3*c^2*d + 57*a^2*b^2*c*d^2
- 13*a^3*b*d^3)*x)/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d
^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^5*d
^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*b^4
*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b
^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*b^2
*d^3)*g^5) + 12*(4*b*c*d^3 - a*d^4)*log(b*x + a)/((b^6*c^4 - 4*a*b^5*c^3*d
 + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5) - 12*(4*b*c*...
```

Giac [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.40

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{1}{144}\left(\frac{12\left(3\,Bb^2in-\frac{8\,(bx+a)Bbdin}{dx+c}+\frac{6\,(bx+a)^2Bd^2in}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^4b^2c^2g^5}{(dx+c)^4}-\frac{2\,(bx+a)^4abcdg^5}{(dx+c)^4}+\frac{(bx+a)^4a^2d^2g^5}{(dx+c)^4}}+\frac{9\,Bb^2in-\frac{32\,(bx+a)Bbdin}{dx+c}+\frac{36\,(bx+a)^2Bd^2i}{(dx+c)^2}}{}\right.$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, al
gorithm="giac")
```

output

```
-1/144*(12*(3*B*b^2*i*n - 8*(b*x + a)*B*b*d*i*n/(d*x + c) + 6*(b*x + a)^2*
B*d^2*i*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^4*b^2*c^2*g^5/(
d*x + c)^4 - 2*(b*x + a)^4*a*b*c*d*g^5/(d*x + c)^4 + (b*x + a)^4*a^2*d^2*g
^5/(d*x + c)^4) + (9*B*b^2*i*n - 32*(b*x + a)*B*b*d*i*n/(d*x + c) + 36*(b*
x + a)^2*B*d^2*i*n/(d*x + c)^2 + 36*B*b^2*i*log(e) - 96*(b*x + a)*B*b*d*i*
log(e)/(d*x + c) + 72*(b*x + a)^2*B*d^2*i*log(e)/(d*x + c)^2 + 36*A*b^2*i
- 96*(b*x + a)*A*b*d*i/(d*x + c) + 72*(b*x + a)^2*A*d^2*i/(d*x + c)^2)/((b
*x + a)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*x + a)^4*a*b*c*d*g^5/(d*x + c)^4
+ (b*x + a)^4*a^2*d^2*g^5/(d*x + c)^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*
d)^2)
```

Mupad [B] (verification not implemented)

Time = 26.91 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.17

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$$

$$= \frac{B\,d^4\,i\,n\,\operatorname{atanh}\left(\frac{12\,a^3\,b^2\,d^3\,g^5-12\,a^2\,b^3\,c\,d^2\,g^5-12\,a\,b^4\,c^2\,d\,g^5+12\,b^5\,c^3\,g^5}{12\,b^2\,g^5\,(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{6\,b^2\,g^5\,(a\,d-b\,c)^3}$$

$$- \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,c\,i}{4\,b}+\frac{B\,a\,d\,i}{12\,b^2}+\frac{B\,d\,i\,x}{3\,b}\right)}{a^4\,g^5+4\,a^3\,b\,g^5\,x+6\,a^2\,b^2\,g^5\,x^2+4\,a\,b^3\,g^5\,x^3+b^4\,g^5\,x^4}$$

$$- \frac{\frac{12\,A\,a^3\,d^3\,i+36\,A\,b^3\,c^3\,i+13\,B\,a^3\,d^3\,i\,n+9\,B\,b^3\,c^3\,i\,n-60\,A\,a\,b^2\,c^2\,d\,i+12\,A\,a^2\,b\,c\,d^2\,i-23\,B\,a\,b^2\,c^2\,d\,i\,n+13\,B\,a^2\,b\,c\,d^2\,i\,n}{12\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{x\,(12\,A\,a^2\,b\,d^3}{}}{12\,a^4\,b^2\,g^5+48\,a^3\,b^3\,g^5\,x+72\,a^2\,b^4\,g^5\,x^2+48\,a\,b^5\,g^5\,x^3+12\,b^6\,g^5\,x^4}$$

input

```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^5
,x)
```

output

```
(B*d^4*i*n*atanh((12*b^5*c^3*g^5 + 12*a^3*b^2*d^3*g^5 - 12*a*b^4*c^2*d*g^5
 - 12*a^2*b^3*c*d^2*g^5)/(12*b^2*g^5*(a*d - b*c)^3) + (2*b*d*x*(a^2*d^2 +
b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(6*b^2*g^5*(a*d - b*c)^3) - (log(e*(
(a + b*x)/(c + d*x))^n)*((B*c*i)/(4*b) + (B*a*d*i)/(12*b^2) + (B*d*i*x)/(3
*b)))/(a^4*g^5 + b^4*g^5*x^4 + 4*a*b^3*g^5*x^3 + 6*a^2*b^2*g^5*x^2 + 4*a^3
*b*g^5*x) - ((12*A*a^3*d^3*i + 36*A*b^3*c^3*i + 13*B*a^3*d^3*i*n + 9*B*b^3
*c^3*i*n - 60*A*a*b^2*c^2*d*i + 12*A*a^2*b*c*d^2*i - 23*B*a*b^2*c^2*d*i*n
+ 13*B*a^2*b*c*d^2*i*n)/(12*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(12*A*a^
2*b*d^3*i + 12*A*b^3*c^2*d*i - 24*A*a*b^2*c*d^2*i + 13*B*a^2*b*d^3*i*n + B
*b^3*c^2*d*i*n - 5*B*a*b^2*c*d^2*i*n))/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
 - (d*x^2*(B*b^3*c*d*i*n - 7*B*a*b^2*d^2*i*n))/(2*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d)) + (B*b^3*d^3*i*n*x^3)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(12*a^4*b^
2*g^5 + 12*b^6*g^5*x^4 + 48*a^3*b^3*g^5*x + 48*a*b^5*g^5*x^3 + 72*a^2*b^4*
g^5*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 994, normalized size of antiderivative = 3.54

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x)
```

output

```
(i*(12*log(a + b*x)*a**5*b*d**4*n + 48*log(a + b*x)*a**4*b**2*d**4*n*x + 7
2*log(a + b*x)*a**3*b**3*d**4*n*x**2 + 48*log(a + b*x)*a**2*b**4*d**4*n*x*
*3 + 12*log(a + b*x)*a*b**5*d**4*n*x**4 - 12*log(c + d*x)*a**5*b*d**4*n -
48*log(c + d*x)*a**4*b**2*d**4*n*x - 72*log(c + d*x)*a**3*b**3*d**4*n*x**2
 - 48*log(c + d*x)*a**2*b**4*d**4*n*x**3 - 12*log(c + d*x)*a*b**5*d**4*n*x
**4 - 12*log(((a + b*x)**n*e)/(c + d*x)**n)*a**5*b*d**4 - 48*log(((a + b*x
)**n*e)/(c + d*x)**n)*a**4*b**2*d**4*x + 72*log(((a + b*x)**n*e)/(c + d*x)
**n)*a**3*b**3*c**2*d**2 + 144*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**
3*c*d**3*x - 96*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**4*c**3*d - 144*
log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**4*c**2*d**2*x + 36*log(((a + b*
x)**n*e)/(c + d*x)**n)*a*b**5*c**4 + 48*log(((a + b*x)**n*e)/(c + d*x)**n)
*a*b**5*c**3*d*x - 12*a**6*d**4 - 10*a**5*b*d**4*n - 48*a**5*b*d**4*x + 72
*a**4*b**2*c**2*d**2 - 3*a**4*b**2*c*d**3*n + 144*a**4*b**2*c*d**3*x - 40*
a**4*b**2*d**4*n*x - 96*a**3*b**3*c**3*d + 36*a**3*b**3*c**2*d**2*n - 144*
a**3*b**3*c**2*d**2*x + 60*a**3*b**3*c*d**3*n*x - 24*a**3*b**3*d**4*n*x**2
 + 36*a**2*b**4*c**4 - 32*a**2*b**4*c**3*d*n + 48*a**2*b**4*c**3*d*x - 24*
a**2*b**4*c**2*d**2*n*x + 30*a**2*b**4*c*d**3*n*x**2 + 9*a*b**5*c**4*n + 4
*a*b**5*c**3*d*n*x - 6*a*b**5*c**2*d**2*n*x**2 + 3*a*b**5*d**4*n*x**4 - 3*
b**6*c*d**3*n*x**4))/(144*a*b**2*g**5*(a**7*d**3 - 3*a**6*b*c*d**2 + 4*a**
6*b*d**3*x + 3*a**5*b**2*c**2*d - 12*a**5*b**2*c*d**2*x + 6*a**5*b**2*d...
```

3.117 $\int (ag+bgx)^3(ci+dix)^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\, dx$

Optimal result 1401
Mathematica [A] (verified) 1402
Rubi [A] (verified) 1403
Maple [B] (verified) 1405
Fricas [B] (verification not implemented) 1406
Sympy [F(-1)] 1407
Maxima [B] (verification not implemented) 1408
Giac [B] (verification not implemented) 1409
Mupad [B] (verification not implemented) 1410
Reduce [B] (verification not implemented) 1411

Optimal result

Integrand size = 43, antiderivative size = 442

$$
\begin{aligned}
&\int (ag+bgx)^3(ci+dix)^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\, dx \\
&= \frac{B(bc-ad)^5 g^3 i^2 n x}{60 b^2 d^3} + \frac{B(bc-ad)^4 g^3 i^2 n (c+dx)^2}{120 b d^4} - \frac{19 B(bc-ad)^3 g^3 i^2 n (c+dx)^3}{180 d^4} \\
&\quad + \frac{13 b B(bc-ad)^2 g^3 i^2 n (c+dx)^4}{120 d^4} - \frac{b^2 B(bc-ad) g^3 i^2 n (c+dx)^5}{30 d^4} \\
&\quad - \frac{(bc-ad)^3 g^3 i^2 (c+dx)^3 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3 d^4} \\
&\quad + \frac{3 b (bc-ad)^2 g^3 i^2 (c+dx)^4 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4 d^4} \\
&\quad - \frac{3 b^2 (bc-ad) g^3 i^2 (c+dx)^5 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5 d^4} \\
&\quad + \frac{b^3 g^3 i^2 (c+dx)^6 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6 d^4} \\
&\quad + \frac{B(bc-ad)^6 g^3 i^2 n \log\left(\frac{a+bx}{c+dx}\right)}{60 b^3 d^4} + \frac{B(bc-ad)^6 g^3 i^2 n \log(c+dx)}{60 b^3 d^4}
\end{aligned}
$$

output
```
1/60*B*(-a*d+b*c)^5*g^3*i^2*n*x/b^2/d^3+1/120*B*(-a*d+b*c)^4*g^3*i^2*n*(d*
x+c)^2/b/d^4-19/180*B*(-a*d+b*c)^3*g^3*i^2*n*(d*x+c)^3/d^4+13/120*b*B*(-a*
d+b*c)^2*g^3*i^2*n*(d*x+c)^4/d^4-1/30*b^2*B*(-a*d+b*c)*g^3*i^2*n*(d*x+c)^5
/d^4-1/3*(-a*d+b*c)^3*g^3*i^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^
4+3/4*b*(-a*d+b*c)^2*g^3*i^2*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4
-3/5*b^2*(-a*d+b*c)*g^3*i^2*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4+
1/6*b^3*g^3*i^2*(d*x+c)^6*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4+1/60*B*(-a*d
+b*c)^6*g^3*i^2*n*ln((b*x+a)/(d*x+c))/b^3/d^4+1/60*B*(-a*d+b*c)^6*g^3*i^2*
n*ln(d*x+c)/b^3/d^4
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.00

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{g^3i^2\left(90d^4(bc-ad)^2(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+144d^5(bc-ad)(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]),x]
```

output
```
(g^3*i^2*(90*d^4*(b*c - a*d)^2*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) + 144*d^5*(b*c - a*d)*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]) + 60*d^6*(a + b*x)^6*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 15*B*
(b*c - a*d)^3*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2
+ 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^2*n
*(12*b*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c -
a*d)*(a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]) - B*
(b*c - a*d)*n*(60*b*d*(b*c - a*d)^4*x + 30*d^2*(-(b*c) + a*d)^3*(a + b*x)^
2 + 20*d^3*(b*c - a*d)^2*(a + b*x)^3 + 15*d^4*(-(b*c) + a*d)*(a + b*x)^4 +
 12*d^5*(a + b*x)^5 - 60*(b*c - a*d)^5*Log[c + d*x])))/(360*b^3*d^4)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2961, 2782, 27, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\downarrow 2961$$

$$g^3i^2(bc-ad)^6\int\frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}$$

$$\downarrow 2782$$

$$g^3i^2(bc-ad)^6\left(-Bn\int-\frac{(c+dx)\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{15d^2(a+bx)^2b}{(c+dx)^2}-\frac{20d^3(a+bx)^3}{(c+dx)^3}\right)}{60d^4(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}+\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\right.$$

$$\downarrow 27$$

$$g^3i^2(bc-ad)^6\left(\frac{Bn\int\frac{(c+dx)\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{15d^2(a+bx)^2b}{(c+dx)^2}-\frac{20d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{60d^4}+\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3b^2\left(B\log\left(e\left(\frac{a}{c}\right.\right.\right.\right.}{5d^4\left(b-\frac{d}{}\right.}$$

$$\downarrow 2123$$

$$g^3i^2(bc-ad)^6\left(\frac{Bn\int\left(-\frac{10db^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6}+\frac{26db}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{19d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3b}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2b^2}+\frac{c+dx}{(a+bx)b^3}+\frac{d}{\left(b-\frac{d(a+b}{c+d}\right.\right.\right.\right.}{60d^4}\right.$$

$$\downarrow 2009$$

$$g^3 i^2 (bc -$$

$$ad)^6 \left(\frac{b^3 \left(B \log\left(e \left(\frac{a+bx}{c+dx} \right)^n \right) + A \right)}{6d^4 \left(b - \frac{d(a+bx)}{c+dx} \right)^6} - \frac{3b^2 \left(B \log\left(e \left(\frac{a+bx}{c+dx} \right)^n \right) + A \right)}{5d^4 \left(b - \frac{d(a+bx)}{c+dx} \right)^5} + \frac{3b \left(B \log\left(e \left(\frac{a+bx}{c+dx} \right)^n \right) + A \right)}{4d^4 \left(b - \frac{d(a+bx)}{c+dx} \right)^4} - \frac{B \log\left(e \left(\right.\right.\right.}{3d^4 \left(b -}$$

input

```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
,x]
```

output

```
(b*c - a*d)^6*g^3*i^2*((b^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*d^4
*(b - (d*(a + b*x))/(c + d*x))^6) - (3*b^2*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]))/(5*d^4*(b - (d*(a + b*x))/(c + d*x))^5) + (3*b*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(4*d^4*(b - (d*(a + b*x))/(c + d*x))^4) - (A + B*Log
[e*((a + b*x)/(c + d*x))^n])/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^3) + (B*
n*((-2*b^2)/(b - (d*(a + b*x))/(c + d*x))^5 + (13*b)/(2*(b - (d*(a + b*x))
/(c + d*x))^4) - 19/(3*(b - (d*(a + b*x))/(c + d*x))^3) + 1/(2*b*(b - (d*(
a + b*x))/(c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a +
 b*x)/(c + d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(60*d^4))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2782

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1760 vs. $2(420) = 840$.

Time = 26.82 (sec) , antiderivative size = 1761, normalized size of antiderivative = 3.98

method	result	size
parallelrisch	Expression too large to display	1761

input

```
int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_
RETURNVERBOSE)
```

output

```
1/360*(540*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^5*g^3*i^2*n+720*B*x^3
*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*c*d^5*g^3*i^2*n+360*B*x^3*ln(e*((b*x+a)
/(d*x+c))^n)*a*b^5*c^2*d^4*g^3*i^2*n+360*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a
^3*b^3*c*d^5*g^3*i^2*n+540*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*c^2*d^4
*g^3*i^2*n+360*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^3*c^2*d^4*g^3*i^2*n+60*
B*x^6*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^6*g^3*i^2*n+216*B*x^5*ln(e*((b*x+a)/
(d*x+c))^n)*a*b^5*d^6*g^3*i^2*n+144*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c*
d^5*g^3*i^2*n+270*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*d^6*g^3*i^2*n+36
*B*x*a^4*b^2*c*d^5*g^3*i^2*n^2+30*B*x*a^3*b^3*c^2*d^4*g^3*i^2*n^2-90*B*x*a
^2*b^4*c^3*d^3*g^3*i^2*n^2+36*B*x*a*b^5*c^4*d^2*g^3*i^2*n^2+360*A*x*a^3*b^
3*c^2*d^4*g^3*i^2*n+120*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^3*c^3*d^3*g^3*i^
2*n-90*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*c^4*d^2*g^3*i^2*n+36*B*ln(e*((b
*x+a)/(d*x+c))^n)*a*b^5*c^5*d*g^3*i^2*n-36*B*ln(b*x+a)*a^5*b*c*d^5*g^3*i^2
*n^2+90*B*ln(b*x+a)*a^4*b^2*c^2*d^4*g^3*i^2*n^2-33*B*a^5*b*c*d^5*g^3*i^2*n
^2-168*B*a^4*b^2*c^2*d^4*g^3*i^2*n^2+150*B*a^3*b^3*c^3*d^3*g^3*i^2*n^2+72*
B*a^2*b^4*c^4*d^2*g^3*i^2*n^2-33*B*a*b^5*c^5*d*g^3*i^2*n^2-720*A*a^4*b^2*c
^2*d^4*g^3*i^2*n-900*A*a^3*b^3*c^3*d^3*g^3*i^2*n-36*B*ln(b*x+a)*a*b^5*c^5*
d*g^3*i^2*n^2+360*A*x^2*a^3*b^3*c*d^5*g^3*i^2*n+540*A*x^2*a^2*b^4*c^2*d^4*
g^3*i^2*n-120*B*ln(b*x+a)*a^3*b^3*c^3*d^3*g^3*i^2*n^2+90*B*ln(b*x+a)*a^2*b
^4*c^4*d^2*g^3*i^2*n^2+90*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^2*d^4*g...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(420) = 840$.

Time $= 0.29$ (sec) , antiderivative size $= 1074$, normalized size of antiderivative $= 2.43$

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="fricas")
```

output

```
1/360*(60*A*b^6*d^6*g^3*i^2*x^6 + 6*(15*B*a^4*b^2*c^2*d^4 - 6*B*a^5*b*c*d^
5 + B*a^6*d^6)*g^3*i^2*n*log(b*x + a) + 6*(B*b^6*c^6 - 6*B*a*b^5*c^5*d + 1
5*B*a^2*b^4*c^4*d^2 - 20*B*a^3*b^3*c^3*d^3)*g^3*i^2*n*log(d*x + c) - 12*((
B*b^6*c*d^5 - B*a*b^5*d^6)*g^3*i^2*n - 6*(2*A*b^6*c*d^5 + 3*A*a*b^5*d^6)*g
^3*i^2)*x^5 - 3*((7*B*b^6*c^2*d^4 + 6*B*a*b^5*c*d^5 - 13*B*a^2*b^4*d^6)*g^
3*i^2*n - 30*(A*b^6*c^2*d^4 + 6*A*a*b^5*c*d^5 + 3*A*a^2*b^4*d^6)*g^3*i^2)*
x^4 - 2*((B*b^6*c^3*d^3 + 39*B*a*b^5*c^2*d^4 - 21*B*a^2*b^4*c*d^5 - 19*B*a
^3*b^3*d^6)*g^3*i^2*n - 60*(3*A*a*b^5*c^2*d^4 + 6*A*a^2*b^4*c*d^5 + A*a^3*
b^3*d^6)*g^3*i^2)*x^3 + 3*((B*b^6*c^4*d^2 - 6*B*a*b^5*c^3*d^3 - 30*B*a^2*b
^4*c^2*d^4 + 34*B*a^3*b^3*c*d^5 + B*a^4*b^2*d^6)*g^3*i^2*n + 60*(3*A*a^2*b
^4*c^2*d^4 + 2*A*a^3*b^3*c*d^5)*g^3*i^2)*x^2 + 6*(60*A*a^3*b^3*c^2*d^4*g^3
*i^2 - (B*b^6*c^5*d - 6*B*a*b^5*c^4*d^2 + 15*B*a^2*b^4*c^3*d^3 - 5*B*a^3*b
^3*c^2*d^4 - 6*B*a^4*b^2*c*d^5 + B*a^5*b*d^6)*g^3*i^2*n)*x + 6*(10*B*b^6*d
^6*g^3*i^2*x^6 + 60*B*a^3*b^3*c^2*d^4*g^3*i^2*x + 12*(2*B*b^6*c*d^5 + 3*B*
a*b^5*d^6)*g^3*i^2*x^5 + 15*(B*b^6*c^2*d^4 + 6*B*a*b^5*c*d^5 + 3*B*a^2*b^4
*d^6)*g^3*i^2*x^4 + 20*(3*B*a*b^5*c^2*d^4 + 6*B*a^2*b^4*c*d^5 + B*a^3*b^3*
d^6)*g^3*i^2*x^3 + 30*(3*B*a^2*b^4*c^2*d^4 + 2*B*a^3*b^3*c*d^5)*g^3*i^2*x^
2)*log(e) + 6*(10*B*b^6*d^6*g^3*i^2*n*x^6 + 60*B*a^3*b^3*c^2*d^4*g^3*i^2*n
*x + 12*(2*B*b^6*c*d^5 + 3*B*a*b^5*d^6)*g^3*i^2*n*x^5 + 15*(B*b^6*c^2*d^4
+ 6*B*a*b^5*c*d^5 + 3*B*a^2*b^4*d^6)*g^3*i^2*n*x^4 + 20*(3*B*a*b^5*c^2*...
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

input

```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x
)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1978 vs. $2(420) = 840$.

Time = 0.10 (sec) , antiderivative size = 1978, normalized size of antiderivative = 4.48

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="maxima")
```

output
```
1/6*B*b^3*d^2*g^3*i^2*x^6*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/6*A*b
^3*d^2*g^3*i^2*x^6 + 2/5*B*b^3*c*d*g^3*i^2*x^5*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n) + 3/5*B*a*b^2*d^2*g^3*i^2*x^5*log(e*(b*x/(d*x + c) + a/(d*x +
c))^n) + 2/5*A*b^3*c*d*g^3*i^2*x^5 + 3/5*A*a*b^2*d^2*g^3*i^2*x^5 + 1/4*B*b
^3*c^2*g^3*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*B*a*b^2*c*
d*g^3*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/4*B*a^2*b*d^2*g^3
*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b^3*c^2*g^3*i^2*x^
4 + 3/2*A*a*b^2*c*d*g^3*i^2*x^4 + 3/4*A*a^2*b*d^2*g^3*i^2*x^4 + B*a*b^2*c^
2*g^3*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*B*a^2*b*c*d*g^3*i
^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*B*a^3*d^2*g^3*i^2*x^3*
log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*b^2*c^2*g^3*i^2*x^3 + 2*A*a^2
*b*c*d*g^3*i^2*x^3 + 1/3*A*a^3*d^2*g^3*i^2*x^3 + 3/2*B*a^2*b*c^2*g^3*i^2*x
^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + B*a^3*c*d*g^3*i^2*x^2*log(e*(b
*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*a^2*b*c^2*g^3*i^2*x^2 + A*a^3*c*d*g
^3*i^2*x^2 - 1/360*B*b^3*d^2*g^3*i^2*n*(60*a^6*log(b*x + a)/b^6 - 60*c^6*l
og(d*x + c)/d^6 + (12*(b^5*c*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*
b^3*d^5)*x^4 + 20*(b^5*c^3*d^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*
d^5)*x^2 + 60*(b^5*c^5 - a^5*d^5)*x)/(b^5*d^5)) + 1/30*B*b^3*c*d*g^3*i^2*n
*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^
3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5133 vs. $2(420) = 840$.

Time = 1.63 (sec) , antiderivative size = 5133, normalized size of antiderivative = 11.61

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="giac")
```

output
```
-1/360*(6*(B*b^10*c^7*g^3*i^2*n - 7*B*a*b^9*c^6*d*g^3*i^2*n - 6*(b*x + a)*
B*b^9*c^7*d*g^3*i^2*n/(d*x + c) + 21*B*a^2*b^8*c^5*d^2*g^3*i^2*n + 42*(b*x
 + a)*B*a*b^8*c^6*d^2*g^3*i^2*n/(d*x + c) + 15*(b*x + a)^2*B*b^8*c^7*d^2*g
^3*i^2*n/(d*x + c)^2 - 35*B*a^3*b^7*c^4*d^3*g^3*i^2*n - 126*(b*x + a)*B*a^
2*b^7*c^5*d^3*g^3*i^2*n/(d*x + c) - 105*(b*x + a)^2*B*a*b^7*c^6*d^3*g^3*i^
2*n/(d*x + c)^2 - 20*(b*x + a)^3*B*b^7*c^7*d^3*g^3*i^2*n/(d*x + c)^3 + 35*
B*a^4*b^6*c^3*d^4*g^3*i^2*n + 210*(b*x + a)*B*a^3*b^6*c^4*d^4*g^3*i^2*n/(d
*x + c) + 315*(b*x + a)^2*B*a^2*b^6*c^5*d^4*g^3*i^2*n/(d*x + c)^2 + 140*(b
*x + a)^3*B*a*b^6*c^6*d^4*g^3*i^2*n/(d*x + c)^3 - 21*B*a^5*b^5*c^2*d^5*g^3
*i^2*n - 210*(b*x + a)*B*a^4*b^5*c^3*d^5*g^3*i^2*n/(d*x + c) - 525*(b*x +
a)^2*B*a^3*b^5*c^4*d^5*g^3*i^2*n/(d*x + c)^2 - 420*(b*x + a)^3*B*a^2*b^5*c
^5*d^5*g^3*i^2*n/(d*x + c)^3 + 7*B*a^6*b^4*c*d^6*g^3*i^2*n + 126*(b*x + a)
*B*a^5*b^4*c^2*d^6*g^3*i^2*n/(d*x + c) + 525*(b*x + a)^2*B*a^4*b^4*c^3*d^6
*g^3*i^2*n/(d*x + c)^2 + 700*(b*x + a)^3*B*a^3*b^4*c^4*d^6*g^3*i^2*n/(d*x
+ c)^3 - B*a^7*b^3*d^7*g^3*i^2*n - 42*(b*x + a)*B*a^6*b^3*c*d^7*g^3*i^2*n/
(d*x + c) - 315*(b*x + a)^2*B*a^5*b^3*c^2*d^7*g^3*i^2*n/(d*x + c)^2 - 700*
(b*x + a)^3*B*a^4*b^3*c^3*d^7*g^3*i^2*n/(d*x + c)^3 + 6*(b*x + a)*B*a^7*b^
2*d^8*g^3*i^2*n/(d*x + c) + 105*(b*x + a)^2*B*a^6*b^2*c*d^8*g^3*i^2*n/(d*x
 + c)^2 + 420*(b*x + a)^3*B*a^5*b^2*c^2*d^8*g^3*i^2*n/(d*x + c)^3 - 15*(b*
x + a)^2*B*a^7*b*d^9*g^3*i^2*n/(d*x + c)^2 - 140*(b*x + a)^3*B*a^6*b*c*...
```

Mupad [B] (verification not implemented)

Time = 27.56 (sec) , antiderivative size = 2555, normalized size of antiderivative = 5.78

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input

```
int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))
,x)
```

output

```
x^2*((a*c*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d*n - B*b*c*n))/6 -
 (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (b*
g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2*n - 2*B*b^2*c^2*n + 60*
A*a*b*c*d - B*a*b*c*d*n))/5 + A*a*b^2*c*d*g^3*i^2))/(2*b*d) - ((60*a*d + 6
0*b*c)*((g^3*i^2*(16*A*a^3*d^3 + 4*A*b^3*c^3 + 3*B*a^3*d^3*n - B*b^3*c^3*n
 + 48*A*a*b^2*c^2*d + 72*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d
^2*n))/(4*d) + ((60*a*d + 60*b*c)*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c +
 B*a*d*n - B*b*c*n))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d +
 60*b*c))/(60*b*d) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2
*n - 2*B*b^2*c^2*n + 60*A*a*b*c*d - B*a*b*c*d*n))/5 + A*a*b^2*c*d*g^3*i^2)
)/(60*b*d) - (a*c*((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d*n - B*b*c*n
))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60))/(b*d)))/(120*b*d) + (a*g^3
*i^2*(3*A*a^3*d^3 + 12*A*b^3*c^3 + B*a^3*d^3*n - 3*B*b^3*c^3*n + 54*A*a*b^
2*c^2*d + 36*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d*n + 5*B*a^2*b*c*d^2*n))/(6*b*
d)) + x^3*((g^3*i^2*(16*A*a^3*d^3 + 4*A*b^3*c^3 + 3*B*a^3*d^3*n - B*b^3*c^
3*n + 48*A*a*b^2*c^2*d + 72*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d*n + 3*B*a^2*b*
c*d^2*n))/(12*d) + ((60*a*d + 60*b*c)*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b
*c + B*a*d*n - B*b*c*n))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a
*d + 60*b*c))/(60*b*d) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2
*d^2*n - 2*B*b^2*c^2*n + 60*A*a*b*c*d - B*a*b*c*d*n))/5 + A*a*b^2*c*d*g...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1093, normalized size of antiderivative = 2.47

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \text{Too large to display}$$

input

```
int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output

```
(g**3*( - 6*log(c + d*x)*a**6*d**6*n + 36*log(c + d*x)*a**5*b*c*d**5*n - 9
0*log(c + d*x)*a**4*b**2*c**2*d**4*n + 120*log(c + d*x)*a**3*b**3*c**3*d**
3*n - 90*log(c + d*x)*a**2*b**4*c**4*d**2*n + 36*log(c + d*x)*a*b**5*c**5*
d*n - 6*log(c + d*x)*b**6*c**6*n - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*
*6*d**6 + 36*log(((a + b*x)**n*e)/(c + d*x)**n)*a**5*b*c*d**5 - 90*log(((a
 + b*x)**n*e)/(c + d*x)**n)*a**4*b**2*c**2*d**4 - 360*log(((a + b*x)**n*e)
/(c + d*x)**n)*a**3*b**3*c**2*d**4*x - 360*log(((a + b*x)**n*e)/(c + d*x)*
*n)*a**3*b**3*c*d**5*x**2 - 120*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*
*3*d**6*x**3 - 540*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**4*c**2*d**4*
x**2 - 720*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**4*c*d**5*x**3 - 270*
log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**4*d**6*x**4 - 360*log(((a + b*x
)**n*e)/(c + d*x)**n)*a*b**5*c**2*d**4*x**3 - 540*log(((a + b*x)**n*e)/(c
+ d*x)**n)*a*b**5*c*d**5*x**4 - 216*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b
**5*d**6*x**5 - 90*log(((a + b*x)**n*e)/(c + d*x)**n)*b**6*c**2*d**4*x**4
- 144*log(((a + b*x)**n*e)/(c + d*x)**n)*b**6*c*d**5*x**5 - 60*log(((a + b
*x)**n*e)/(c + d*x)**n)*b**6*d**6*x**6 + 6*a**5*b*d**6*n*x - 360*a**4*b**2
*c**2*d**4*x - 36*a**4*b**2*c*d**5*n*x - 360*a**4*b**2*c*d**5*x**2 - 3*a**
4*b**2*d**6*n*x**2 - 120*a**4*b**2*d**6*x**3 - 30*a**3*b**3*c**2*d**4*n*x
- 540*a**3*b**3*c**2*d**4*x**2 - 102*a**3*b**3*c*d**5*n*x**2 - 720*a**3*b*
*3*c*d**5*x**3 - 38*a**3*b**3*d**6*n*x**3 - 270*a**3*b**3*d**6*x**4 + 9...
```

3.118 $\int (ag+bgx)^2(ci+dix)^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result 1412
Mathematica [A] (verified) 1413
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Optimal result

Integrand size = 43, antiderivative size = 352

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= -\frac{B(bc-ad)^4 g^2 i^2 n x}{30b^2d^2} - \frac{B(bc-ad)^3 g^2 i^2 n (c+dx)^2}{60bd^3} + \frac{B(bc-ad)^2 g^2 i^2 n (c+dx)^3}{10d^3} \\
&\quad - \frac{bB(bc-ad) g^2 i^2 n (c+dx)^4}{20d^3} + \frac{(bc-ad)^2 g^2 i^2 (c+dx)^3 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^3} \\
&\quad - \frac{b(bc-ad) g^2 i^2 (c+dx)^4 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^3} \\
&\quad + \frac{b^2 g^2 i^2 (c+dx)^5 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^3} \\
&\quad - \frac{B(bc-ad)^5 g^2 i^2 n \log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3} - \frac{B(bc-ad)^5 g^2 i^2 n \log(c+dx)}{30b^3d^3}
\end{aligned}$$

output

```
-1/30*B*(-a*d+b*c)^4*g^2*i^2*n*x/b^2/d^2-1/60*B*(-a*d+b*c)^3*g^2*i^2*n*(d*
x+c)^2/b/d^3+1/10*B*(-a*d+b*c)^2*g^2*i^2*n*(d*x+c)^3/d^3-1/20*b*B*(-a*d+b*
c)*g^2*i^2*n*(d*x+c)^4/d^3+1/3*(-a*d+b*c)^2*g^2*i^2*(d*x+c)^3*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/d^3-1/2*b*(-a*d+b*c)*g^2*i^2*(d*x+c)^4*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))/d^3+1/5*b^2*g^2*i^2*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))/d^3-1/30*B*(-a*d+b*c)^5*g^2*i^2*n*ln((b*x+a)/(d*x+c))/b^3/d^3-1/30*B*
(-a*d+b*c)^5*g^2*i^2*n*ln(d*x+c)/b^3/d^3
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.06

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{g^2i^2\left(20d^3(bc-ad)^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+30d^4(bc-ad)(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]),x]
```

output
```
(g^2*i^2*(20*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) + 30*d^4*(b*c - a*d)*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]) + 12*d^5*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 10*B*(
b*c - a*d)^3*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Lo
g[c + d*x]) - 5*B*(b*c - a*d)^2*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) +
 a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + B*
(b*c - a*d)*n*(12*b*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 +
4*d^3*(b*c - a*d)*(a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c
 + d*x])))/(60*b^3*d^3)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2961, 2782, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$
$$\downarrow \text{2961}$$
$$g^2i^2(bc-ad)^5\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\downarrow 2782$$

$$g^2i^2(bc-ad)^5\left(-Bn\int\frac{(c+dx)\left(b^2-\frac{5d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{30d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}+\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{b\left(B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.\right.$$

$$\downarrow 27$$

$$g^2i^2(bc-ad)^5\left(-\frac{Bn\int\frac{(c+dx)\left(b^2-\frac{5d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{30d^3}+\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right.$$

$$\downarrow 1195$$

$$g^2i^2(bc-ad)^5\left(-\frac{Bn\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{9d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{6bd}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a+bx}{c+dx}}{30d^3}+\right.$$

$$\downarrow 2009$$

$$g^2i^2(bc-ad)^5\left(\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\right.\right.$$

input
```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
,x]
```

output
```
(b*c - a*d)^5*g^2*i^2*((b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*d^3
*(b - (d*(a + b*x))/(c + d*x))^5) - (b*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]))/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^4) + (A + B*Log[e*((a + b*x)/(c
 + d*x))^n])/(3*d^3*(b - (d*(a + b*x))/(c + d*x))^3) - (B*n*((3*b)/(2*(b -
 (d*(a + b*x))/(c + d*x))^4) - 3/(b - (d*(a + b*x))/(c + d*x))^3 + 1/(2*b*(
b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) +
Log[(a + b*x)/(c + d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(30*
d^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1273 vs. 2(334) = 668.

Time = 11.69 (sec) , antiderivative size = 1274, normalized size of antiderivative = 3.62

method	result	size
parallelrisch	Expression too large to display	1274

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_
RETURNVERBOSE)
```

output
```
1/60*(12*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^5*g^2*i^2*n+30*B*x^4*ln(e*(
(b*x+a)/(d*x+c))^n)*a*b^4*d^5*g^2*i^2*n+30*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)
*b^5*c*d^4*g^2*i^2*n+20*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*d^5*g^2*i^
2*n+20*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^2*d^3*g^2*i^2*n+80*A*x^3*a*b^
4*c*d^4*g^2*i^2*n+15*B*x^2*a^2*b^3*c*d^4*g^2*i^2*n^2-15*B*x^2*a*b^4*c^2*d^
3*g^2*i^2*n^2+60*A*x^2*a^2*b^3*c*d^4*g^2*i^2*n+B*x^2*a^3*b^2*d^5*g^2*i^2*n
^2-B*x^2*b^5*c^3*d^2*g^2*i^2*n^2-2*B*x*a^4*b*d^5*g^2*i^2*n^2+2*B*x*b^5*c^4
*d*g^2*i^2*n^2+80*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c*d^4*g^2*i^2*n+10
*B*x*a^3*b^2*c*d^4*g^2*i^2*n^2-10*B*x*a*b^4*c^3*d^2*g^2*i^2*n^2+60*A*x*a^2
*b^3*c^2*d^3*g^2*i^2*n+2*B*a^5*d^5*g^2*i^2*n^2-2*B*b^5*c^5*g^2*i^2*n^2-25*
B*a^3*b^2*c^2*d^3*g^2*i^2*n^2+25*B*a^2*b^3*c^3*d^2*g^2*i^2*n^2+9*B*a*b^4*c
^4*d*g^2*i^2*n^2-120*A*a^3*b^2*c^2*d^3*g^2*i^2*n-120*A*a^2*b^3*c^3*d^2*g^2
*i^2*n+60*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c*d^4*g^2*i^2*n+60*B*x^2
*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^2*d^3*g^2*i^2*n+60*B*x*ln(e*((b*x+a)/(d
*x+c))^n)*a^2*b^3*c^2*d^3*g^2*i^2*n+20*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3
*c^3*d^2*g^2*i^2*n-10*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^4*d*g^2*i^2*n+10
*B*ln(b*x+a)*a*b^4*c^4*d*g^2*i^2*n^2-10*B*ln(b*x+a)*a^4*b*c*d^4*g^2*i^2*n^
2+20*B*ln(b*x+a)*a^3*b^2*c^2*d^3*g^2*i^2*n^2-20*B*ln(b*x+a)*a^2*b^3*c^3*d^
2*g^2*i^2*n^2+60*A*x^2*a*b^4*c^2*d^3*g^2*i^2*n+3*B*x^4*a*b^4*d^5*g^2*i^2*n
^2-3*B*x^4*b^5*c*d^4*g^2*i^2*n^2+30*A*x^4*a*b^4*d^5*g^2*i^2*n+30*A*x^4*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(334) = 668$.

Time $= 0.18$ (sec) , antiderivative size $= 774$, normalized size of antiderivative $= 2.20$

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{12\,Ab^5d^5g^2i^2x^5+2\left(10\,Ba^3b^2c^2d^3-5\,Ba^4bcd^4+Ba^5d^5\right)g^2i^2n\log\left(bx+a\right)-2\left(Bb^5c^5-5\,Bab^4c^4d+10\right.}{}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="fricas")
```

output
```
1/60*(12*A*b^5*d^5*g^2*i^2*x^5 + 2*(10*B*a^3*b^2*c^2*d^3 - 5*B*a^4*b*c*d^4
 + B*a^5*d^5)*g^2*i^2*n*log(b*x + a) - 2*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10
*B*a^2*b^3*c^3*d^2)*g^2*i^2*n*log(d*x + c) - 3*((B*b^5*c*d^4 - B*a*b^4*d^5
)*g^2*i^2*n - 10*(A*b^5*c*d^4 + A*a*b^4*d^5)*g^2*i^2)*x^4 - 2*(3*(B*b^5*c^
2*d^3 - B*a^2*b^3*d^5)*g^2*i^2*n - 10*(A*b^5*c^2*d^3 + 4*A*a*b^4*c*d^4 + A
*a^2*b^3*d^5)*g^2*i^2)*x^3 - ((B*b^5*c^3*d^2 + 15*B*a*b^4*c^2*d^3 - 15*B*a
^2*b^3*c*d^4 - B*a^3*b^2*d^5)*g^2*i^2*n - 60*(A*a*b^4*c^2*d^3 + A*a^2*b^3*
c*d^4)*g^2*i^2)*x^2 + 2*(30*A*a^2*b^3*c^2*d^3*g^2*i^2 + (B*b^5*c^4*d - 5*B
*a*b^4*c^3*d^2 + 5*B*a^3*b^2*c*d^4 - B*a^4*b*d^5)*g^2*i^2*n)*x + 2*(6*B*b^
5*d^5*g^2*i^2*x^5 + 30*B*a^2*b^3*c^2*d^3*g^2*i^2*x + 15*(B*b^5*c*d^4 + B*a
*b^4*d^5)*g^2*i^2*x^4 + 10*(B*b^5*c^2*d^3 + 4*B*a*b^4*c*d^4 + B*a^2*b^3*d^
5)*g^2*i^2*x^3 + 30*(B*a*b^4*c^2*d^3 + B*a^2*b^3*c*d^4)*g^2*i^2*x^2)*log(e
) + 2*(6*B*b^5*d^5*g^2*i^2*n*x^5 + 30*B*a^2*b^3*c^2*d^3*g^2*i^2*n*x + 15*(
B*b^5*c*d^4 + B*a*b^4*d^5)*g^2*i^2*n*x^4 + 10*(B*b^5*c^2*d^3 + 4*B*a*b^4*c
*d^4 + B*a^2*b^3*d^5)*g^2*i^2*n*x^3 + 30*(B*a*b^4*c^2*d^3 + B*a^2*b^3*c*d^
4)*g^2*i^2*n*x^2)*log((b*x + a)/(d*x + c)))/(b^3*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x
)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1336 vs. $2(334) = 668$.

Time = 0.07 (sec) , antiderivative size = 1336, normalized size of antiderivative = 3.80

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="maxima")
```

output

```
1/5*B*b^2*d^2*g^2*i^2*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*b
^2*d^2*g^2*i^2*x^5 + 1/2*B*b^2*c*d*g^2*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n) + 1/2*B*a*b*d^2*g^2*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n) + 1/2*A*b^2*c*d*g^2*i^2*x^4 + 1/2*A*a*b*d^2*g^2*i^2*x^4 + 1/3*B*b^2*c
^2*g^2*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 4/3*B*a*b*c*d*g^2*
i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*B*a^2*d^2*g^2*i^2*x^3
*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*b^2*c^2*g^2*i^2*x^3 + 4/3*
A*a*b*c*d*g^2*i^2*x^3 + 1/3*A*a^2*d^2*g^2*i^2*x^3 + B*a*b*c^2*g^2*i^2*x^2*
log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + B*a^2*c*d*g^2*i^2*x^2*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n) + A*a*b*c^2*g^2*i^2*x^2 + A*a^2*c*d*g^2*i^2*x^
2 + 1/60*B*b^2*d^2*g^2*i^2*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c
)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3
 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) -
1/12*B*b^2*c*d*g^2*i^2*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4
+ (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*
c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/12*B*a*b*d^2*g^2*i^2*n*(6*a^4*log(b*x + a
)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c
^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/6*B*b^2*c^
2*g^2*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d -
 a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 2/3*B*a*b*c*d*g^2...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3352 vs. $2(334) = 668$.

Time $= 1.13$ (sec) , antiderivative size $= 3352$, normalized size of antiderivative $= 9.52$

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="giac")
```

output

```
1/60*(2*(B*b^8*c^6*g^2*i^2*n - 6*B*a*b^7*c^5*d*g^2*i^2*n - 5*(b*x + a)*B*b
^7*c^6*d*g^2*i^2*n/(d*x + c) + 15*B*a^2*b^6*c^4*d^2*g^2*i^2*n + 30*(b*x +
a)*B*a*b^6*c^5*d^2*g^2*i^2*n/(d*x + c) + 10*(b*x + a)^2*B*b^6*c^6*d^2*g^2*
i^2*n/(d*x + c)^2 - 20*B*a^3*b^5*c^3*d^3*g^2*i^2*n - 75*(b*x + a)*B*a^2*b^
5*c^4*d^3*g^2*i^2*n/(d*x + c) - 60*(b*x + a)^2*B*a*b^5*c^5*d^3*g^2*i^2*n/(
d*x + c)^2 + 15*B*a^4*b^4*c^2*d^4*g^2*i^2*n + 100*(b*x + a)*B*a^3*b^4*c^3*
d^4*g^2*i^2*n/(d*x + c) + 150*(b*x + a)^2*B*a^2*b^4*c^4*d^4*g^2*i^2*n/(d*x
 + c)^2 - 6*B*a^5*b^3*c*d^5*g^2*i^2*n - 75*(b*x + a)*B*a^4*b^3*c^2*d^5*g^2
*i^2*n/(d*x + c) - 200*(b*x + a)^2*B*a^3*b^3*c^3*d^5*g^2*i^2*n/(d*x + c)^2
 + B*a^6*b^2*d^6*g^2*i^2*n + 30*(b*x + a)*B*a^5*b^2*c*d^6*g^2*i^2*n/(d*x +
 c) + 150*(b*x + a)^2*B*a^4*b^2*c^2*d^6*g^2*i^2*n/(d*x + c)^2 - 5*(b*x + a
)*B*a^6*b*d^7*g^2*i^2*n/(d*x + c) - 60*(b*x + a)^2*B*a^5*b*c*d^7*g^2*i^2*n
/(d*x + c)^2 + 10*(b*x + a)^2*B*a^6*d^8*g^2*i^2*n/(d*x + c)^2)*log((b*x +
a)/(d*x + c))/(b^5*d^3 - 5*(b*x + a)*b^4*d^4/(d*x + c) + 10*(b*x + a)^2*b^
3*d^5/(d*x + c)^2 - 10*(b*x + a)^3*b^2*d^6/(d*x + c)^3 + 5*(b*x + a)^4*b*d
^7/(d*x + c)^4 - (b*x + a)^5*d^8/(d*x + c)^5) + (2*(b*x + a)*B*b^9*c^6*d*g
^2*i^2*n/(d*x + c) - 12*(b*x + a)*B*a*b^8*c^5*d^2*g^2*i^2*n/(d*x + c) - 9*
(b*x + a)^2*B*b^8*c^6*d^2*g^2*i^2*n/(d*x + c)^2 + 30*(b*x + a)*B*a^2*b^7*c
^4*d^3*g^2*i^2*n/(d*x + c) + 54*(b*x + a)^2*B*a*b^7*c^5*d^3*g^2*i^2*n/(d*x
 + c)^2 + 9*(b*x + a)^3*B*b^7*c^6*d^3*g^2*i^2*n/(d*x + c)^3 - 40*(b*x +...
```

Mupad [B] (verification not implemented)

Time = 26.55 (sec) , antiderivative size = 1328, normalized size of antiderivative = 3.77

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input

```
int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))
,x)
```

output

```
x^2*(((30*a*d + 30*b*c)*((((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d*n - B
*b*c*n))/5 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))/30)*(30*a*d + 30*b*c))/(30*
b*d) - (g^2*i^2*(6*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 1
8*A*a*b*c*d))/2 + A*a*b*c*d*g^2*i^2))/(60*b*d) - (a*c*((b*d*g^2*i^2*(15*A*
a*d + 15*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))
/30))/(2*b*d) + (g^2*i^2*(3*A*a^3*d^3 + 3*A*b^3*c^3 + B*a^3*d^3*n - B*b^3*
c^3*n + 27*A*a*b^2*c^2*d + 27*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d*n + 3*B*a^2*
b*c*d^2*n))/(6*b*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*g^2*i^2*x^3*(a^2
*d^2 + b^2*c^2 + 4*a*b*c*d))/3 + B*a^2*c^2*g^2*i^2*x + (B*b^2*d^2*g^2*i^2*
x^5)/5 + B*a*c*g^2*i^2*x^2*(a*d + b*c) + (B*b*d*g^2*i^2*x^4*(a*d + b*c))/2
) - x^3*((((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*
b*d*g^2*i^2*(30*a*d + 30*b*c))/30)*(30*a*d + 30*b*c))/(90*b*d) - (g^2*i^2*
(6*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 18*A*a*b*c*d))/6
+ (A*a*b*c*d*g^2*i^2)/3) + x*((a*c*((((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c +
B*a*d*n - B*b*c*n))/5 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))/30)*(30*a*d + 30
*b*c))/(30*b*d) - (g^2*i^2*(6*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2*n - B*b^
2*c^2*n + 18*A*a*b*c*d))/2 + A*a*b*c*d*g^2*i^2))/(b*d) - ((30*a*d + 30*b*c
)*(((30*a*d + 30*b*c)*((((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d*n - B*b
*c*n))/5 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))/30)*(30*a*d + 30*b*c))/(30*b*
d) - (g^2*i^2*(6*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + ...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 796, normalized size of antiderivative = 2.26

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \text{Too large to display}$$

input

```
int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output

```
(g**2*( - 2*log(c + d*x)*a**5*d**5*n + 10*log(c + d*x)*a**4*b*c*d**4*n - 2
0*log(c + d*x)*a**3*b**2*c**2*d**3*n + 20*log(c + d*x)*a**2*b**3*c**3*d**2
*n - 10*log(c + d*x)*a*b**4*c**4*d*n + 2*log(c + d*x)*b**5*c**5*n - 2*log(
((a + b*x)**n*e)/(c + d*x)**n)*a**5*d**5 + 10*log(((a + b*x)**n*e)/(c + d*
x)**n)*a**4*b*c*d**4 - 20*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**2*c**
2*d**3 - 60*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*c**2*d**3*x - 60*
log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*c*d**4*x**2 - 20*log(((a + b*
x)**n*e)/(c + d*x)**n)*a**2*b**3*d**5*x**3 - 60*log(((a + b*x)**n*e)/(c +
d*x)**n)*a*b**4*c**2*d**3*x**2 - 80*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b
**4*c*d**4*x**3 - 30*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**4*d**5*x**4 -
 20*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*c**2*d**3*x**3 - 30*log(((a +
b*x)**n*e)/(c + d*x)**n)*b**5*c*d**4*x**4 - 12*log(((a + b*x)**n*e)/(c + d
*x)**n)*b**5*d**5*x**5 + 2*a**4*b*d**5*n*x - 60*a**3*b**2*c**2*d**3*x - 10
*a**3*b**2*c*d**4*n*x - 60*a**3*b**2*c*d**4*x**2 - a**3*b**2*d**5*n*x**2 -
 20*a**3*b**2*d**5*x**3 - 60*a**2*b**3*c**2*d**3*x**2 - 15*a**2*b**3*c*d**
4*n*x**2 - 80*a**2*b**3*c*d**4*x**3 - 6*a**2*b**3*d**5*n*x**3 - 30*a**2*b*
*3*d**5*x**4 + 10*a*b**4*c**3*d**2*n*x + 15*a*b**4*c**2*d**3*n*x**2 - 20*a
*b**4*c**2*d**3*x**3 - 30*a*b**4*c*d**4*x**4 - 3*a*b**4*d**5*n*x**4 - 12*a
*b**4*d**5*x**5 - 2*b**5*c**4*d*n*x + b**5*c**3*d**2*n*x**2 + 6*b**5*c**2*
d**3*n*x**3 + 3*b**5*c*d**4*n*x**4))/(60*b**2*d**3)
```

3.119 $\int (ag+bgx)(ci+dix)^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result 1423
Mathematica [A] (verified) 1424
Rubi [A] (verified) 1424
Maple [B] (verified) 1426
Fricas [B] (verification not implemented) 1427
Sympy [F(-1)] 1428
Maxima [B] (verification not implemented) 1428
Giac [B] (verification not implemented) 1429
Mupad [B] (verification not implemented) 1431
Reduce [B] (verification not implemented) 1432

Optimal result

Integrand size = 41, antiderivative size = 250

$$\begin{aligned}
&\int (ag+bgx)(ci+dix)^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= \frac{B(bc-ad)^3 gi^2 nx}{12b^2 d} + \frac{B(bc-ad)^2 gi^2 n(c+dx)^2}{24bd^2} - \frac{B(bc-ad)gi^2 n(c+dx)^3}{12d^2} \\
&\quad - \frac{(bc-ad)gi^2(c+dx)^3\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^2} \\
&\quad + \frac{bgi^2(c+dx)^4\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2} \\
&\quad + \frac{B(bc-ad)^4 gi^2 n\log\left(\frac{a+bx}{c+dx}\right)}{12b^3 d^2} + \frac{B(bc-ad)^4 gi^2 n\log(c+dx)}{12b^3 d^2}
\end{aligned}$$

output

```
1/12*B*(-a*d+b*c)^3*g*i^2*n*x/b^2/d+1/24*B*(-a*d+b*c)^2*g*i^2*n*(d*x+c)^2/
b/d^2-1/12*B*(-a*d+b*c)*g*i^2*n*(d*x+c)^3/d^2-1/3*(-a*d+b*c)*g*i^2*(d*x+c)
^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^2+1/4*b*g*i^2*(d*x+c)^4*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/d^2+1/12*B*(-a*d+b*c)^4*g*i^2*n*ln((b*x+a)/(d*x+c))/b^3
/d^2+1/12*B*(-a*d+b*c)^4*g*i^2*n*ln(d*x+c)/b^3/d^2
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.90

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{gi^2\left(\frac{4B(bc-ad)^2n\left(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx)\right)}{b^3}-\frac{B(bc-ad)n\left(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc\right.}{b^3}}{24d^{\cdot}}$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]),x]
```

output
```
(g*i^2*((4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b
*c - a*d)^2*Log[a + b*x]))/b^3 - (B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x +
 3*b^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a
 + b*x]))/b^3 - 8*(b*c - a*d)*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]) + 6*b*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(24*d^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {2961, 2782, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$
$$\downarrow 2961$$
$$gi^2(bc-ad)^4\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$
$$\downarrow 2782$$

$$gi^2(bc-ad)^4\left(-Bn\int -\frac{(c+dx)\left(b-\frac{4d(a+bx)}{c+dx}\right)}{12d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)$$

$$\downarrow 27$$

$$gi^2(bc-ad)^4\left(\frac{Bn\int\frac{(c+dx)\left(b-\frac{4d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{12d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)$$

$$\downarrow 86$$

$$gi^2(bc-ad)^4\left(\frac{Bn\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a+bx}{c+dx}}{12d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2\left(b-\frac{d(a+}{c+}\right.\right.$$

$$\downarrow 2009$$

$$gi^2(bc-ad)^4\left(-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^3}+\frac{1}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}\right.}{12d^2}\right.$$

input

```
Int[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x
]
```

output

```
(b*c - a*d)^4*g*i^2*((b*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d^2*(b
 - (d*(a + b*x))/(c + d*x))^4) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*
d^2*(b - (d*(a + b*x))/(c + d*x))^3) + (B*n*(-(b - (d*(a + b*x))/(c + d*x)
)^(-3) + 1/(2*b*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x
))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^3 - Log[b - (d*(a + b*x))/(c +
 d*x)]/b^3))/(12*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 836 vs. $2(236) = 472$.

Time = 4.97 (sec) , antiderivative size = 837, normalized size of antiderivative = 3.35

method	result
parallelrisch	$\frac{8Bx\,a^2b^2cd^3gi^2n^2-4Bxab^3c^2d^2gi^2n^2+24Axab^3c^2d^2gi^2n-8B\ln(bx+a)a^3bcd^3gi^2n^2-2Bxa^3bd^4gi^2n^2+24Bx^2\ln\left(e\left(\frac{b}{d}\right.\right.$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RE
TURNVERBOSE)
```

output
```
1/24*(8*B*x*a^2*b^2*c*d^3*g*i^2*n^2-4*B*x*a*b^3*c^2*d^2*g*i^2*n^2+24*A*x*a
*b^3*c^2*d^2*g*i^2*n-8*B*ln(b*x+a)*a^3*b*c*d^3*g*i^2*n^2-2*B*x*a^3*b*d^4*g
*i^2*n^2+24*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c*d^3*g*i^2*n+24*B*x*ln(
e*((b*x+a)/(d*x+c))^n)*a*b^3*c^2*d^2*g*i^2*n-2*B*ln(e*((b*x+a)/(d*x+c))^n)
*b^4*c^4*g*i^2*n+6*A*x^4*b^4*d^4*g*i^2*n+2*B*ln(b*x+a)*b^4*c^4*g*i^2*n^2+2
*B*ln(b*x+a)*a^4*d^4*g*i^2*n^2+12*B*ln(b*x+a)*a^2*b^2*c^2*d^2*g*i^2*n^2-8*
B*ln(b*x+a)*a*b^3*c^3*d*g*i^2*n^2+8*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c^3*
d*g*i^2*n+8*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*d^4*g*i^2*n+16*B*x^3*ln(
e*((b*x+a)/(d*x+c))^n)*b^4*c*d^3*g*i^2*n+12*B*x^2*ln(e*((b*x+a)/(d*x+c))^n
)*b^4*c^2*d^2*g*i^2*n+4*B*x^2*a*b^3*c*d^3*g*i^2*n^2+24*A*x^2*a*b^3*c*d^3*g
*i^2*n+2*B*b^4*c^4*g*i^2*n^2+2*B*a^4*d^4*g*i^2*n^2-7*B*a^3*b*c*d^3*g*i^2*n
^2-8*B*a^2*b^2*c^2*d^2*g*i^2*n^2+11*B*a*b^3*c^3*d*g*i^2*n^2-48*A*a^2*b^2*c
^2*d^2*g*i^2*n-36*A*a*b^3*c^3*d*g*i^2*n-2*B*x*b^4*c^3*d*g*i^2*n^2+6*B*x^4*
ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^4*g*i^2*n+2*B*x^3*a*b^3*d^4*g*i^2*n^2-2*B*
x^3*b^4*c*d^3*g*i^2*n^2+8*A*x^3*a*b^3*d^4*g*i^2*n+16*A*x^3*b^4*c*d^3*g*i^2
*n+B*x^2*a^2*b^2*d^4*g*i^2*n^2-5*B*x^2*b^4*c^2*d^2*g*i^2*n^2+12*A*x^2*b^4*
c^2*d^2*g*i^2*n)/b^3/d^2/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(236) = 472$.

Time $= 0.12$ (sec) , antiderivative size $= 530$, normalized size of antiderivative $= 2.12$

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{6\,Ab^4d^4gi^2x^4+2\left(6\,Ba^2b^2c^2d^2-4\,Ba^3bcd^3+Ba^4d^4\right)gi^2n\log\left(bx+a\right)+2\left(Bb^4c^4-4\,Bab^3c^3d\right)gi^2n\log\left(\right.}{}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="fricas")
```

output
```
1/24*(6*A*b^4*d^4*g*i^2*x^4 + 2*(6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3 + B
*a^4*d^4)*g*i^2*n*log(b*x + a) + 2*(B*b^4*c^4 - 4*B*a*b^3*c^3*d)*g*i^2*n*l
og(d*x + c) - 2*((B*b^4*c*d^3 - B*a*b^3*d^4)*g*i^2*n - 4*(2*A*b^4*c*d^3 +
A*a*b^3*d^4)*g*i^2)*x^3 - ((5*B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 - B*a^2*b^2*
d^4)*g*i^2*n - 12*(A*b^4*c^2*d^2 + 2*A*a*b^3*c*d^3)*g*i^2)*x^2 + 2*(12*A*a
*b^3*c^2*d^2*g*i^2 - (B*b^4*c^3*d + 2*B*a*b^3*c^2*d^2 - 4*B*a^2*b^2*c*d^3
+ B*a^3*b*d^4)*g*i^2*n)*x + 2*(3*B*b^4*d^4*g*i^2*x^4 + 12*B*a*b^3*c^2*d^2*
g*i^2*x + 4*(2*B*b^4*c*d^3 + B*a*b^3*d^4)*g*i^2*x^3 + 6*(B*b^4*c^2*d^2 + 2
*B*a*b^3*c*d^3)*g*i^2*x^2)*log(e) + 2*(3*B*b^4*d^4*g*i^2*n*x^4 + 12*B*a*b^
3*c^2*d^2*g*i^2*n*x + 4*(2*B*b^4*c*d^3 + B*a*b^3*d^4)*g*i^2*n*x^3 + 6*(B*b
^4*c^2*d^2 + 2*B*a*b^3*c*d^3)*g*i^2*n*x^2)*log((b*x + a)/(d*x + c)))/(b^3*
d^2)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(236) = 472$.

Time = 0.07 (sec) , antiderivative size = 740, normalized size of antiderivative = 2.96

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="maxima")
```

output

```
1/4*B*b*d^2*g*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b*d^2
*g*i^2*x^4 + 2/3*B*b*c*d*g*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ 1/3*B*a*d^2*g*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2/3*A*b*c
*d*g*i^2*x^3 + 1/3*A*a*d^2*g*i^2*x^3 + 1/2*B*b*c^2*g*i^2*x^2*log(e*(b*x/(d
*x + c) + a/(d*x + c))^n) + B*a*c*d*g*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n) + 1/2*A*b*c^2*g*i^2*x^2 + A*a*c*d*g*i^2*x^2 - 1/24*B*b*d^2*g*i^
2*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b
^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^
3*d^3)) + 1/3*B*b*c*d*g*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 1/
6*B*a*d^2*g*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 1/2*B*b*c^2*g*
i^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))
- B*a*c*d*g*i^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*
d)*x/(b*d)) + B*a*c^2*g*i^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a*
c^2*g*i^2*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*c^2*g*i^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1997 vs. $2(236) = 472$.

Time $= 0.85$ (sec) , antiderivative size $= 1997$, normalized size of antiderivative $= 7.99$

$$\int (ag + bgx)(ci + dix)^2 \left(A + B \log \left(e \left(\frac{a + bx}{c + dx} \right)^n \right) \right) \, dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="giac")
```

output

```
-1/24*(2*(B*b^6*c^5*g*i^2*n - 5*B*a*b^5*c^4*d*g*i^2*n - 4*(b*x + a)*B*b^5*
c^5*d*g*i^2*n/(d*x + c) + 10*B*a^2*b^4*c^3*d^2*g*i^2*n + 20*(b*x + a)*B*a*
b^4*c^4*d^2*g*i^2*n/(d*x + c) - 10*B*a^3*b^3*c^2*d^3*g*i^2*n - 40*(b*x + a
)*B*a^2*b^3*c^3*d^3*g*i^2*n/(d*x + c) + 5*B*a^4*b^2*c*d^4*g*i^2*n + 40*(b*
x + a)*B*a^3*b^2*c^2*d^4*g*i^2*n/(d*x + c) - B*a^5*b*d^5*g*i^2*n - 20*(b*x
 + a)*B*a^4*b*c*d^5*g*i^2*n/(d*x + c) + 4*(b*x + a)*B*a^5*d^6*g*i^2*n/(d*x
 + c))*log((b*x + a)/(d*x + c))/(b^4*d^2 - 4*(b*x + a)*b^3*d^3/(d*x + c) +
 6*(b*x + a)^2*b^2*d^4/(d*x + c)^2 - 4*(b*x + a)^3*b*d^5/(d*x + c)^3 + (b*
x + a)^4*d^6/(d*x + c)^4) - (B*b^8*c^5*g*i^2*n - 5*B*a*b^7*c^4*d*g*i^2*n -
 6*(b*x + a)*B*b^7*c^5*d*g*i^2*n/(d*x + c) + 10*B*a^2*b^6*c^3*d^2*g*i^2*n
+ 30*(b*x + a)*B*a*b^6*c^4*d^2*g*i^2*n/(d*x + c) + 7*(b*x + a)^2*B*b^6*c^5
*d^2*g*i^2*n/(d*x + c)^2 - 10*B*a^3*b^5*c^2*d^3*g*i^2*n - 60*(b*x + a)*B*a
^2*b^5*c^3*d^3*g*i^2*n/(d*x + c) - 35*(b*x + a)^2*B*a*b^5*c^4*d^3*g*i^2*n/
(d*x + c)^2 - 2*(b*x + a)^3*B*b^5*c^5*d^3*g*i^2*n/(d*x + c)^3 + 5*B*a^4*b^
4*c*d^4*g*i^2*n + 60*(b*x + a)*B*a^3*b^4*c^2*d^4*g*i^2*n/(d*x + c) + 70*(b
*x + a)^2*B*a^2*b^4*c^3*d^4*g*i^2*n/(d*x + c)^2 + 10*(b*x + a)^3*B*a*b^4*c
^4*d^4*g*i^2*n/(d*x + c)^3 - B*a^5*b^3*d^5*g*i^2*n - 30*(b*x + a)*B*a^4*b^
3*c*d^5*g*i^2*n/(d*x + c) - 70*(b*x + a)^2*B*a^3*b^3*c^2*d^5*g*i^2*n/(d*x
+ c)^2 - 20*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g*i^2*n/(d*x + c)^3 + 6*(b*x + a
)*B*a^5*b^2*d^6*g*i^2*n/(d*x + c) + 35*(b*x + a)^2*B*a^4*b^2*c*d^6*g*i^...
```

Mupad [B] (verification not implemented)

Time = 26.09 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.64

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$

$$= \ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(Bac^2gi^2x+\frac{Bcgi^2x^2(2ad+bc)}{2}+\frac{Bdgi^2x^3(ad+2bc)}{3}+\frac{Bbd^2gi^2x^4}{4}\right)$$

$$+x^3\left(\frac{dgi^2(8Aad+12Abc+Badn-Bbcn)}{12}-\frac{Adgi^2(12ad+12bc)}{36}\right)$$

$$+x\left(\frac{(12ad+12bc)\left(\frac{\left(\frac{dgi^2(8Aad+12Abc+Badn-Bbcn)}{4}-\frac{Adgi^2(12ad+12bc)}{12}\right)(12ad+12bc)}{12bd}-\frac{gi^2(3Aa^2d^2+9Ab^2c^2+B}{12bd}\right.\right.$$

$$\left.\left.-\frac{ac\left(\frac{dgi^2(8Aad+12Abc+Badn-Bbcn)}{4}-\frac{Adgi^2(12ad+12bc)}{12}\right)}{bd}+\frac{cgi^2(6Aa^2d^2+2Ab^2c^2+2Ba^2d^2n-Bb^2c^2n+12Aabcd-Babcdn)}{2bd}\right)\right.$$

$$-x^2\left(\frac{\left(\frac{dgi^2(8Aad+12Abc+Badn-Bbcn)}{4}-\frac{Adgi^2(12ad+12bc)}{12}\right)(12ad+12bc)}{24bd}\right.$$

$$\left.-\frac{gi^2(3Aa^2d^2+9Ab^2c^2+Ba^2d^2n-2Bb^2c^2n+18Aabcd+Babcdn)}{6b}+\frac{Aacdgi^2}{2}\right)+\frac{\ln(c+dx)\,(Bbc^4gi^2n-4Bac^3dgi^2n)}{12d^2}$$

$$+\frac{\ln(a+bx)\,(Bgna^4d^2i^2-4Bgna^3bcdi^2+6Bgna^2b^2c^2i^2)}{12b^3}+\frac{Abd^2gi^2x^4}{4}$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x
)
```

output

```
log(e*((a + b*x)/(c + d*x))^n)*(B*a*c^2*g*i^2*x + (B*c*g*i^2*x^2*(2*a*d +
b*c))/2 + (B*d*g*i^2*x^3*(a*d + 2*b*c))/3 + (B*b*d^2*g*i^2*x^4)/4) + x^3*(
(d*g*i^2*(8*A*a*d + 12*A*b*c + B*a*d*n - B*b*c*n))/12 - (A*d*g*i^2*(12*a*d
 + 12*b*c))/36) + x*(((12*a*d + 12*b*c)*((((d*g*i^2*(8*A*a*d + 12*A*b*c +
B*a*d*n - B*b*c*n))/4 - (A*d*g*i^2*(12*a*d + 12*b*c))/12)*(12*a*d + 12*b*c
))/(12*b*d) - (g*i^2*(3*A*a^2*d^2 + 9*A*b^2*c^2 + B*a^2*d^2*n - 2*B*b^2*c^
2*n + 18*A*a*b*c*d + B*a*b*c*d*n))/(3*b) + A*a*c*d*g*i^2))/(12*b*d) - (a*c
*((d*g*i^2*(8*A*a*d + 12*A*b*c + B*a*d*n - B*b*c*n))/4 - (A*d*g*i^2*(12*a*
d + 12*b*c))/12))/(b*d) + (c*g*i^2*(6*A*a^2*d^2 + 2*A*b^2*c^2 + 2*B*a^2*d^
2*n - B*b^2*c^2*n + 12*A*a*b*c*d - B*a*b*c*d*n))/(2*b*d)) - x^2*((((d*g*i^
2*(8*A*a*d + 12*A*b*c + B*a*d*n - B*b*c*n))/4 - (A*d*g*i^2*(12*a*d + 12*b*
c))/12)*(12*a*d + 12*b*c))/(24*b*d) - (g*i^2*(3*A*a^2*d^2 + 9*A*b^2*c^2 +
B*a^2*d^2*n - 2*B*b^2*c^2*n + 18*A*a*b*c*d + B*a*b*c*d*n))/(6*b) + (A*a*c*
d*g*i^2)/2) + (log(c + d*x)*(B*b*c^4*g*i^2*n - 4*B*a*c^3*d*g*i^2*n))/(12*d
^2) + (log(a + b*x)*(B*a^4*d^2*g*i^2*n + 6*B*a^2*b^2*c^2*g*i^2*n - 4*B*a^3
*b*c*d*g*i^2*n))/(12*b^3) + (A*b*d^2*g*i^2*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.31

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$=\frac{g\left(-24\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)ab^3c^2d^2x+8\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a^3bcd^3-12\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a^2b^2c^2d^2-12\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b$$

input

```
int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output

```
(g*( - 2*log(c + d*x)*a**4*d**4*n + 8*log(c + d*x)*a**3*b*c*d**3*n - 12*lo
g(c + d*x)*a**2*b**2*c**2*d**2*n + 8*log(c + d*x)*a*b**3*c**3*d*n - 2*log(
c + d*x)*b**4*c**4*n - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*d**4 + 8*
log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*c*d**3 - 12*log(((a + b*x)**n*e)
/(c + d*x)**n)*a**2*b**2*c**2*d**2 - 24*log(((a + b*x)**n*e)/(c + d*x)**n)
*a*b**3*c**2*d**2*x - 24*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c*d**3*
x**2 - 8*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*d**4*x**3 - 12*log(((a
+ b*x)**n*e)/(c + d*x)**n)*b**4*c**2*d**2*x**2 - 16*log(((a + b*x)**n*e)/(
c + d*x)**n)*b**4*c*d**3*x**3 - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*
d**4*x**4 + 2*a**3*b*d**4*n*x - 24*a**2*b**2*c**2*d**2*x - 8*a**2*b**2*c*d
**3*n*x - 24*a**2*b**2*c*d**3*x**2 - a**2*b**2*d**4*n*x**2 - 8*a**2*b**2*d
**4*x**3 + 4*a*b**3*c**2*d**2*n*x - 12*a*b**3*c**2*d**2*x**2 - 4*a*b**3*c*
d**3*n*x**2 - 16*a*b**3*c*d**3*x**3 - 2*a*b**3*d**4*n*x**3 - 6*a*b**3*d**4
*x**4 + 2*b**4*c**3*d*n*x + 5*b**4*c**2*d**2*n*x**2 + 2*b**4*c*d**3*n*x**3
))/(24*b**2*d**2)
```

3.120 $\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result 1434
Mathematica [A] (verified) 1434
Rubi [A] (verified) 1435
Maple [B] (verified) 1436
Fricas [B] (verification not implemented) 1437
Sympy [B] (verification not implemented) 1438
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Giac [B] (verification not implemented) 1440
Mupad [B] (verification not implemented) 1441
Reduce [B] (verification not implemented) 1442

Optimal result

Integrand size = 33, antiderivative size = 124

$$\begin{aligned}&\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\ &= -\frac{B(bc-ad)^2i^2nx}{3b^2} - \frac{B(bc-ad)i^2n(c+dx)^2}{6bd}\\ &\quad - \frac{B(bc-ad)^3i^2n\log(a+bx)}{3b^3d} + \frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}\end{aligned}$$

output

```
-1/3*B*(-a*d+b*c)^2*i^2*n*x/b^2-1/6*B*(-a*d+b*c)*i^2*n*(d*x+c)^2/b/d-1/3*B
*(-a*d+b*c)^3*i^2*n*ln(b*x+a)/b^3/d+1/3*i^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(
d*x+c))^n))/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.81

$$\begin{aligned}&\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\ &= \frac{i^2\left(-\frac{B(bc-ad)n\left(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx)\right)}{2b^3} + (c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{3d}\end{aligned}$$

input
```
Integrate[(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(i^2*(-1/2*(B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*
c - a*d)^2*Log[a + b*x]))/b^3 + (c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])))/(3*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)^2 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right) dx$$

$$\downarrow 2947$$

$$\frac{i^2(c+dx)^3\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d} - \frac{Bn(bc-ad)\int \frac{i^3(c+dx)^2}{a+bx}dx}{3di}$$

$$\downarrow 27$$

$$\frac{i^2(c+dx)^3\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d} - \frac{Bi^2n(bc-ad)\int \frac{(c+dx)^2}{a+bx}dx}{3d}$$

$$\downarrow 49$$

$$\frac{i^2(c+dx)^3\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d} - \frac{Bi^2n(bc-ad)\int \left(\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(bc-ad)}{b^2}+\frac{d(c+dx)}{b}\right)dx}{3d}$$

$$\downarrow 2009$$

$$\frac{i^2(c+dx)^3\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d} - \frac{Bi^2n(bc-ad)\left(\frac{(bc-ad)^2\log(a+bx)}{b^3}+\frac{dx(bc-ad)}{b^2}+\frac{(c+dx)^2}{2b}\right)}{3d}$$

input
```
Int[(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
-1/3*(B*(b*c - a*d)*i^2*n*((d*(b*c - a*d)*x)/b^2 + (c + d*x)^2/(2*b) + ((b
*c - a*d)^2*Log[a + b*x])/b^3))/d + (i^2*(c + d*x)^3*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(116) = 232$.

Time = 2.01 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.73

method	result
parallelrisch	$\frac{6Bx^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3cd^2i^2n-4Bxb^3c^2di^2n^2+6Axb^3c^2di^2n+6Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3c^2di^2n+6Bxab^2cd^2i^2n^2-6B\ln(b$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/6*(6*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d^2*i^2*n-4*B*x*b^3*c^2*d*i^2
*n^2+6*A*x*b^3*c^2*d*i^2*n+6*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^2*d*i^2*n
+6*B*x*a*b^2*c*d^2*i^2*n^2-6*B*ln(b*x+a)*a^2*b*c*d^2*i^2*n^2+6*B*ln(b*x+a)
*a*b^2*c^2*d*i^2*n^2+2*B*a^3*d^3*i^2*n^2+4*B*b^3*c^3*i^2*n^2-6*A*b^3*c^3*i
^2*n+2*A*x^3*b^3*d^3*i^2*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^3*i^2*n+2*B
*ln(b*x+a)*a^3*d^3*i^2*n^2-2*B*ln(b*x+a)*b^3*c^3*i^2*n^2-5*B*a^2*b*c*d^2*i
^2*n^2-B*a*b^2*c^2*d*i^2*n^2-12*A*a*b^2*c^2*d*i^2*n+2*B*x^3*ln(e*((b*x+a)/
(d*x+c))^n)*b^3*d^3*i^2*n+B*x^2*a*b^2*d^3*i^2*n^2-B*x^2*b^3*c*d^2*i^2*n^2+
6*A*x^2*b^3*c*d^2*i^2*n-2*B*x*a^2*b*d^3*i^2*n^2)/b^3/d/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(116) = 232$.

Time $= 0.10$ (sec) , antiderivative size $= 297$, normalized size of antiderivative $= 2.40$

$$\int (ci + dix)^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$= \frac{2\,Ab^3d^3i^2x^3 - 2\,Bb^3c^3i^2n\log(dx+c) + 2\,(3\,Bab^2c^2d - 3\,Ba^2bcd^2 + Ba^3d^3)i^2n\log(bx+a) + (6\,Ab^3cd^2i}{}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
1/6*(2*A*b^3*d^3*i^2*x^3 - 2*B*b^3*c^3*i^2*n*log(d*x + c) + 2*(3*B*a*b^2*c
^2*d - 3*B*a^2*b*c*d^2 + B*a^3*d^3)*i^2*n*log(b*x + a) + (6*A*b^3*c*d^2*i^
2 - (B*b^3*c*d^2 - B*a*b^2*d^3)*i^2*n)*x^2 + 2*(3*A*b^3*c^2*d*i^2 - (2*B*b
^3*c^2*d - 3*B*a*b^2*c*d^2 + B*a^2*b*d^3)*i^2*n)*x + 2*(B*b^3*d^3*i^2*x^3
+ 3*B*b^3*c*d^2*i^2*x^2 + 3*B*b^3*c^2*d*i^2*x)*log(e) + 2*(B*b^3*d^3*i^2*n
*x^3 + 3*B*b^3*c*d^2*i^2*n*x^2 + 3*B*b^3*c^2*d*i^2*n*x)*log((b*x + a)/(d*x
 + c)))/(b^3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(107) = 214$.

Time = 127.48 (sec) , antiderivative size = 656, normalized size of antiderivative = 5.29

$$\int (ci + dix)^2 \left(A + B \log \left(e \left(\frac{a + bx}{c + dx} \right)^n \right) \right) dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Piecewise((c**2*i**2*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (A*c
**2*i**2*x + A*c*d*i**2*x**2 + A*d**2*i**2*x**3/3 + B*c**3*i**2*log(e*(a/(
c + d*x))**n)/(3*d) + B*c**2*i**2*n*x/3 + B*c**2*i**2*x*log(e*(a/(c + d*x)
)**n) + B*c*d*i**2*n*x**2/3 + B*c*d*i**2*x**2*log(e*(a/(c + d*x))**n) + B*
d**2*i**2*n*x**3/9 + B*d**2*i**2*x**3*log(e*(a/(c + d*x))**n)/3, Eq(b, 0))
, (c**2*i**2*(A*x + B*a*log(e*(a/c + b*x/c)**n)/b - B*n*x + B*x*log(e*(a/c
 + b*x/c)**n)), Eq(d, 0)), (A*c**2*i**2*x + A*c*d*i**2*x**2 + A*d**2*i**2*
x**3/3 + B*a**3*d**2*i**2*n*log(c/d + x)/(3*b**3) + B*a**3*d**2*i**2*log(e
*(a/(c + d*x) + b*x/(c + d*x))**n)/(3*b**3) - B*a**2*c*d*i**2*n*log(c/d + 
x)/b**2 - B*a**2*c*d*i**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/b**2 - B
*a**2*d**2*i**2*n*x/(3*b**2) + B*a*c**2*i**2*n*log(c/d + x)/b + B*a*c**2*i
**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/b + B*a*c*d*i**2*n*x/b + B*a*d
**2*i**2*n*x**2/(6*b) - B*c**3*i**2*n*log(c/d + x)/(3*d) - 2*B*c**2*i**2*n
*x/3 + B*c**2*i**2*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) - B*c*d*i**2*
n*x**2/6 + B*c*d*i**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*d**
2*i**2*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(116) = 232$.

Time = 0.04 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.49

$$\begin{aligned}
&\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx \\
&= \frac{1}{3}Bd^2i^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}Ad^2i^2x^3 \\
&\quad + Bcdi^2x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Acdi^2x^2 \\
&\quad + \frac{1}{6}Bd^2i^2n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right) \\
&\quad - Bcdi^2n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) \\
&\quad + Bc^2i^2n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right) \\
&\quad + Bc^2i^2x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Ac^2i^2x
\end{aligned}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
1/3*B*d^2*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*d^2*i^2*x
^3 + B*c*d*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d*i^2*x^2
+ 1/6*B*d^2*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*c*d*i^2*n*(a
^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*c^2*
i^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c^2*i^2*x*log(e*(b*x/(d*x
+ c) + a/(d*x + c))^n) + A*c^2*i^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 990 vs. $2(116) = 232$.

Time = 0.40 (sec) , antiderivative size = 990, normalized size of antiderivative = 7.98

$$\int (ci + dix)^2 \left(A + B \log \left(e \left(\frac{a + bx}{c + dx} \right)^n \right) \right) dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
1/6*(2*(B*b^4*c^4*i^2*n - 4*B*a*b^3*c^3*d*i^2*n + 6*B*a^2*b^2*c^2*d^2*i^2*
n - 4*B*a^3*b*c*d^3*i^2*n + B*a^4*d^4*i^2*n)*log((b*x + a)/(d*x + c))/(b^3
*d - 3*(b*x + a)*b^2*d^2/(d*x + c) + 3*(b*x + a)^2*b*d^3/(d*x + c)^2 - (b*
x + a)^3*d^4/(d*x + c)^3) - (3*B*b^6*c^4*i^2*n - 12*B*a*b^5*c^3*d*i^2*n -
5*(b*x + a)*B*b^5*c^4*d*i^2*n/(d*x + c) + 18*B*a^2*b^4*c^2*d^2*i^2*n + 20*
(b*x + a)*B*a*b^4*c^3*d^2*i^2*n/(d*x + c) + 2*(b*x + a)^2*B*b^4*c^4*d^2*i^
2*n/(d*x + c)^2 - 12*B*a^3*b^3*c*d^3*i^2*n - 30*(b*x + a)*B*a^2*b^3*c^2*d^
3*i^2*n/(d*x + c) - 8*(b*x + a)^2*B*a*b^3*c^3*d^3*i^2*n/(d*x + c)^2 + 3*B*
a^4*b^2*d^4*i^2*n + 20*(b*x + a)*B*a^3*b^2*c*d^4*i^2*n/(d*x + c) + 12*(b*x
 + a)^2*B*a^2*b^2*c^2*d^4*i^2*n/(d*x + c)^2 - 5*(b*x + a)*B*a^4*b*d^5*i^2*
n/(d*x + c) - 8*(b*x + a)^2*B*a^3*b*c*d^5*i^2*n/(d*x + c)^2 + 2*(b*x + a)^
2*B*a^4*d^6*i^2*n/(d*x + c)^2 - 2*B*b^6*c^4*i^2*log(e) + 8*B*a*b^5*c^3*d*i
^2*log(e) - 12*B*a^2*b^4*c^2*d^2*i^2*log(e) + 8*B*a^3*b^3*c*d^3*i^2*log(e)
 - 2*B*a^4*b^2*d^4*i^2*log(e) - 2*A*b^6*c^4*i^2 + 8*A*a*b^5*c^3*d*i^2 - 12
*A*a^2*b^4*c^2*d^2*i^2 + 8*A*a^3*b^3*c*d^3*i^2 - 2*A*a^4*b^2*d^4*i^2)/(b^5
*d - 3*(b*x + a)*b^4*d^2/(d*x + c) + 3*(b*x + a)^2*b^3*d^3/(d*x + c)^2 - (
b*x + a)^3*b^2*d^4/(d*x + c)^3) + 2*(B*b^4*c^4*i^2*n - 4*B*a*b^3*c^3*d*i^2
*n + 6*B*a^2*b^2*c^2*d^2*i^2*n - 4*B*a^3*b*c*d^3*i^2*n + B*a^4*d^4*i^2*n)*
log(b - (b*x + a)*d/(d*x + c))/(b^3*d) - 2*(B*b^4*c^4*i^2*n - 4*B*a*b^3*c^
3*d*i^2*n + 6*B*a^2*b^2*c^2*d^2*i^2*n - 4*B*a^3*b*c*d^3*i^2*n + B*a^4*d...
```

Mupad [B] (verification not implemented)

Time = 25.77 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.44

$$\begin{aligned}
&\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx \\
&= \ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(Bc^2i^2x+Bcdi^2x^2+\frac{Bd^2i^2x^3}{3}\right) \\
&\quad - x\left(\frac{(3ad+3bc)\left(\frac{di^2(3Aad+9Abc+Badn-Bbcn)}{3b}-\frac{Adi^2(3ad+3bc)}{3b}\right)}{3bd}\right. \\
&\qquad\qquad \left. -\frac{ci^2(3Aad+3Abc+Badn-Bbcn)}{b}+\frac{Aacdi^2}{b}\right) \\
&\quad + x^2\left(\frac{di^2(3Aad+9Abc+Badn-Bbcn)}{6b}-\frac{Adi^2(3ad+3bc)}{6b}\right) \\
&\quad + \frac{\ln(a+bx)\left(Bna^3d^2i^2-3Bna^2bcdi^2+3Bnab^2c^2i^2\right)}{3b^3} \\
&\quad + \frac{Ad^2i^2x^3}{3}-\frac{Bc^3i^2n\ln(c+dx)}{3d}
\end{aligned}$$

input
```
int((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)*((B*d^2*i^2*x^3)/3 + B*c^2*i^2*x + B*c*d*i^
2*x^2) - x*(((3*a*d + 3*b*c)*((d*i^2*(3*A*a*d + 9*A*b*c + B*a*d*n - B*b*c*
n))/(3*b) - (A*d*i^2*(3*a*d + 3*b*c))/(3*b)))/(3*b*d) - (c*i^2*(3*A*a*d +
3*A*b*c + B*a*d*n - B*b*c*n))/b + (A*a*c*d*i^2)/b) + x^2*((d*i^2*(3*A*a*d
+ 9*A*b*c + B*a*d*n - B*b*c*n))/(6*b) - (A*d*i^2*(3*a*d + 3*b*c))/(6*b)) +
 (log(a + b*x)*(B*a^3*d^2*i^2*n + 3*B*a*b^2*c^2*i^2*n - 3*B*a^2*b*c*d*i^2*
n))/(3*b^3) + (A*d^2*i^2*x^3)/3 - (B*c^3*i^2*n*log(c + d*x))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.30

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{-2\log(bx+a)\,a^3d^3n+6\log(bx+a)\,a^2bc\,d^2n-6\log(bx+a)\,ab^2c^2dn+2\log(bx+a)\,b^3c^3n-2\log\left(\frac{(bx+}{(dx}\right.}$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
( - 2*log(a + b*x)*a**3*d**3*n + 6*log(a + b*x)*a**2*b*c*d**2*n - 6*log(a
+ b*x)*a*b**2*c**2*d*n + 2*log(a + b*x)*b**3*c**3*n - 2*log(((a + b*x)**n*
e)/(c + d*x)**n)*b**3*c**3 - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c**
2*d*x - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*d**2*x**2 - 2*log(((a
+ b*x)**n*e)/(c + d*x)**n)*b**3*d**3*x**3 + 2*a**2*b*d**3*n*x - 6*a*b**2*c
**2*d*x - 6*a*b**2*c*d**2*n*x - 6*a*b**2*c*d**2*x**2 - a*b**2*d**3*n*x**2
- 2*a*b**2*d**3*x**3 + 4*b**3*c**2*d*n*x + b**3*c*d**2*n*x**2)/(6*b**2*d)
```

3.121 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$

Optimal result . . . 1443
Mathematica [A] (verified) . . . 1444
Rubi [A] (verified) . . . 1444
Maple [F] . . . 1449
Fricas [F] . . . 1450
Sympy [F] . . . 1450
Maxima [B] (verification not implemented) . . . 1451
Giac [F] . . . 1452
Mupad [F(-1)] . . . 1452
Reduce [F] . . . 1452

Optimal result

Integrand size = 43, antiderivative size = 289

$$\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx \\
&= -\frac{Bd(bc-ad)i^2nx}{2b^2g} + \frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g} \\
&\quad + \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg} \\
&\quad - \frac{B(bc-ad)^2i^2n\log\left(\frac{a+bx}{c+dx}\right)}{2b^3g} - \frac{3B(bc-ad)^2i^2n\log(c+dx)}{2b^3g} \\
&\quad - \frac{(bc-ad)^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad + \frac{B(bc-ad)^2i^2n\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}
\end{aligned}$$

output
```
-1/2*B*d*(-a*d+b*c)*i^2*n*x/b^2/g+d*(-a*d+b*c)*i^2*(b*x+a)*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))/b^3/g+1/2*i^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b
/g-1/2*B*(-a*d+b*c)^2*i^2*n*ln((b*x+a)/(d*x+c))/b^3/g-3/2*B*(-a*d+b*c)^2*i
^2*n*ln(d*x+c)/b^3/g-(-a*d+b*c)^2*i^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1
-b*(d*x+c)/d/(b*x+a))/b^3/g+B*(-a*d+b*c)^2*i^2*n*polylog(2,b*(d*x+c)/d/(b*
x+a))/b^3/g
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.91

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i^2\Big(2Abd(bc-ad)x - B(bc-ad)n(bdx+(bc-ad)\log(a+bx)) + 2Bd(bc-ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g +
b*g*x),x]
```

output
```
(i^2*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a +
 b*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + b^2*
(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*(b*c - a*d)^2*Log[g
*(a + b*x)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*L
og[c + d*x] + B*(b*c - a*d)^2*n*(-(Log[g*(a + b*x)]*(Log[g*(a + b*x)] - 2*
Log[(b*(c + d*x))/(b*c - a*d)])) + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*
d)])))/(2*b^3*g)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2961, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{ag+bgx}\,dx$$

$$\downarrow 2961$$

$$\frac{i^2(bc-ad)^2\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{g}$$

$$\downarrow 2789$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\downarrow 2756$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int \frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\downarrow 54$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int \left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{}{}}{b}\right)}{g}$$

$$\downarrow 2009$$

$$i^2(bc-ad)^2\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}\right)}{g}$$

$$\downarrow 2789$$

$$i^2(bc-ad)^2\left(\frac{\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\right.\right.\right.}{b}\right.}{b}\right.}{g}$$

$$\downarrow 2751$$

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right.}{g}$$

$$\downarrow 16$$

$$i^2(bc-ad)^2\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}+d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\right.}{g}$$

$$\downarrow 2779$$

$$i^2(bc-ad)^2\left(\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(}{}\right.\right.}{b}\right.}{g}$$

$$\downarrow 2838$$

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{\frac{Bn\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right.}{b}}{g}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
,x]
```

output
```
((b*c - a*d)^2*i^2*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(2*d*(b - (
d*(a + b*x))/(c + d*x))^2) - (B*n*(1/(b*(b - (d*(a + b*x))/(c + d*x))) + L
og[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(2*d)
))/b + ((d*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b*(c + d*x
)*(b - (d*(a + b*x))/(c + d*x))) + (B*n*Log[b - (d*(a + b*x))/(c + d*x)])/
(b*d)))/b + (-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x
))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b)/b
)/b))/g
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(dix+ci)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{bgx+ag}dx$$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

output
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, al
gorithm="fricas")
```

output
```
integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B
*c*d*i^2*x + B*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n))/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i^2\left(\int \frac{Ac^2}{a+bx}\,dx + \int \frac{Ad^2x^2}{a+bx}\,dx + \int \frac{Bc^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx + \int \frac{2Acdx}{a+bx}\,dx + \int \frac{Bd^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx + \int$$
$$g$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g),x)
```

output
```
i**2*(Integral(A*c**2/(a + b*x), x) + Integral(A*d**2*x**2/(a + b*x), x) +
 Integral(B*c**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x) + I
ntegral(2*A*c*d*x/(a + b*x), x) + Integral(B*d**2*x**2*log(e*(a/(c + d*x)
+ b*x/(c + d*x))**n)/(a + b*x), x) + Integral(2*B*c*d*x*log(e*(a/(c + d*x)
 + b*x/(c + d*x))**n)/(a + b*x), x))/g
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(280) = 560$.

Time $= 0.38$ (sec) , antiderivative size $= 580$, normalized size of antiderivative $= 2.01$

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$= 2\,Acdi^2\left(\frac{x}{bg}-\frac{a\log(bx+a)}{b^2g}\right)+\frac{1}{2}\,Ad^2i^2\left(\frac{2\,a^2\log(bx+a)}{b^3g}+\frac{bx^2-2\,ax}{b^2g}\right)$$
$$+\frac{Ac^2i^2\log(bgx+ag)}{bg}-\frac{(3\,bc^2i^2n-2\,acdi^2n)B\log(dx+c)}{2\,b^2g}$$
$$+\frac{(b^2c^2i^2n-2\,abcdi^2n+a^2d^2i^2n)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^3g}$$
$$+\frac{Bb^2d^2i^2x^2\log(e)-(b^2c^2i^2n-2\,abcdi^2n+a^2d^2i^2n)B\log(bx+a)^2-((i^2n-4\,i^2\log(e))b^2cd-(i^2n-}{}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, al
gorithm="maxima")
```

output
```
2*A*c*d*i^2*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + 1/2*A*d^2*i^2*(2*a^2*log(
b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A*c^2*i^2*log(b*g*x + a*g)/(
b*g) - 1/2*(3*b*c^2*i^2*n - 2*a*c*d*i^2*n)*B*log(d*x + c)/(b^2*g) + (b^2*c
^2*i^2*n - 2*a*b*c*d*i^2*n + a^2*d^2*i^2*n)*(log(b*x + a)*log((b*d*x + a*d
)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^3*g) + 1/2*(B
*b^2*d^2*i^2*x^2*log(e) - (b^2*c^2*i^2*n - 2*a*b*c*d*i^2*n + a^2*d^2*i^2*n
)*B*log(b*x + a)^2 - ((i^2*n - 4*i^2*log(e))*b^2*c*d - (i^2*n - 2*i^2*log(
e))*a*b*d^2)*B*x + (2*b^2*c^2*i^2*log(e) + 4*(i^2*n - i^2*log(e))*a*b*c*d
- (3*i^2*n - 2*i^2*log(e))*a^2*d^2)*B*log(b*x + a) + (B*b^2*d^2*i^2*x^2 +
2*(2*b^2*c*d*i^2 - a*b*d^2*i^2)*B*x + 2*(b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^2
*d^2*i^2)*B*log(b*x + a))*log((b*x + a)^n) - (B*b^2*d^2*i^2*x^2 + 2*(2*b^2
*c*d*i^2 - a*b*d^2*i^2)*B*x + 2*(b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2
)*B*log(b*x + a))*log((d*x + c)^n))/(b^3*g)
```

Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, al
gorithm="giac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x +
a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$= \frac{-2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{bx+a}dx\right)b^4c^2 - 2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x^2}{bx+a}dx\right)b^4d^2 - 4\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x}{bx+a}dx\right)b^4cd - 2\log(bx+a)c}{2b^3g}$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

output

```
( - 2*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a + b*x),x)*b**4*c**2 - 2*in
t((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(a + b*x),x)*b**4*d**2 - 4*int
((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a + b*x),x)*b**4*c*d - 2*log(a +
b*x)*a**3*d**2 + 4*log(a + b*x)*a**2*b*c*d - 2*log(a + b*x)*a*b**2*c**2 +
2*a**2*b*d**2*x - 4*a*b**2*c*d*x - a*b**2*d**2*x**2)/(2*b**3*g)
```

3.122 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$

Optimal result . . . 1454
Mathematica [A] (verified) . . . 1455
Rubi [A] (verified) . . . 1455
Maple [F] . . . 1457
Fricas [F] . . . 1457
Sympy [F(-1)] . . . 1458
Maxima [B] (verification not implemented) . . . 1458
Giac [F] . . . 1459
Mupad [F(-1)] . . . 1460
Reduce [F] . . . 1460

Optimal result

Integrand size = 43, antiderivative size = 259

$$\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx \\
&= -\frac{B(bc-ad)i^2n(c+dx)}{b^2g^2(a+bx)} + \frac{d^2i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^2} \\
&\quad - \frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^2(a+bx)} - \frac{Bd(bc-ad)i^2n\log(c+dx)}{b^3g^2} \\
&\quad - \frac{2d(bc-ad)i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&\quad + \frac{2Bd(bc-ad)i^2n\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}
\end{aligned}$$

output

```
-B*(-a*d+b*c)*i^2*n*(d*x+c)/b^2/g^2/(b*x+a)+d^2*i^2*(b*x+a)*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))/b^3/g^2-(-a*d+b*c)*i^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))/b^2/g^2/(b*x+a)-B*d*(-a*d+b*c)*i^2*n*ln(d*x+c)/b^3/g^2-2*d*(-a*d+b*
c)*i^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^2+2
*B*d*(-a*d+b*c)*i^2*n*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.90

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$$
$$= \frac{i^2\left(Abd^2x - \frac{B(bc-ad)^2n}{a+bx} + Bd(-bc+ad)n\log(a+bx) + Bd^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - \frac{(bc-ad)^2\left(A+B\log\left(e\left(\right.\right.\right.\right.}{a+bx}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^2,x]
```

output
```
(i^2*(A*b*d^2*x - (B*(b*c - a*d)^2*n)/(a + b*x) + B*d*(-(b*c) + a*d)*n*Log[a + b*x] + B*d^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - ((b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + 2*d*(b*c - a*d)*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + B*d*(-(b*c) + a*d)*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(b^3*g^2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^2}\,dx$$
$$\downarrow \text{2961}$$
$$\frac{i^2(bc-ad)\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g^2}$$

$$\downarrow 2793$$

$$\frac{i^2(bc-ad)\int\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d^2}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d}{b^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{g^2}$$

$$\downarrow 2009$$

$$\frac{i^2(bc-ad)\left(\frac{d^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2d\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2(a+bx)}+\frac{2}{}}{g^2}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
^2,x]
```

output
```
((b*c - a*d)*i^2*(-((B*n*(c + d*x))/(b^2*(a + b*x))) - ((c + d*x)*(A + B*L
og[e*((a + b*x)/(c + d*x))^n]))/(b^2*(a + b*x)) + (d^2*(a + b*x)*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/(b^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)
)) + (B*d*n*Log[b - (d*(a + b*x))/(c + d*x)])/b^3 - (2*d*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b^3 + (2*B*d*n
*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b^3))/g^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(dix+ci)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(bgx+ag)^2}dx$$

input

```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

output

```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^2}\,dx$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x,
algorithm="fricas")
```

output

```
integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B
*c*d*i^2*x + B*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n))/(b^2*g^2*x^2 + 2*a
*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2,x
)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. $2(258) = 516$.

Time = 0.58 (sec) , antiderivative size = 1190, normalized size of antiderivative = 4.59

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x,
algorithm="maxima")
```

output

```
-B*c^2*i^2*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^
2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - A*(a^2/(b^4*g^2*x + a*b^3*g^2
) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^2))*d^2*i^2 + 2*A*c*d*i^2*(a/(b^
3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - B*c^2*i^2*log(e*(b*x/(d*x
 + c) + a/(d*x + c))^n)/(b^2*g^2*x + a*b*g^2) - A*c^2*i^2/(b^2*g^2*x + a*b
*g^2) - (b^2*c^2*d*i^2*n + a*b*c*d^2*i^2*n - a^2*d^3*i^2*n)*B*log(d*x + c)
/(b^4*c*g^2 - a*b^3*d*g^2) + ((b^3*c*d^2*i^2*log(e) - a*b^2*d^3*i^2*log(e)
)*B*x^2 + (a*b^2*c*d^2*i^2*log(e) - a^2*b*d^3*i^2*log(e))*B*x - ((b^3*c^2*
d*i^2*n - 2*a*b^2*c*d^2*i^2*n + a^2*b*d^3*i^2*n)*B*x + (a*b^2*c^2*d*i^2*n
- 2*a^2*b*c*d^2*i^2*n + a^3*d^3*i^2*n)*B)*log(b*x + a)^2 + (2*(i^2*n + i^2
*log(e))*a*b^2*c^2*d - 3*(i^2*n + i^2*log(e))*a^2*b*c*d^2 + (i^2*n + i^2*l
og(e))*a^3*d^3)*B + ((2*b^3*c^2*d*i^2*log(e) + (3*i^2*n - 4*i^2*log(e))*a*
b^2*c*d^2 - 2*(i^2*n - i^2*log(e))*a^2*b*d^3)*B*x + (2*a*b^2*c^2*d*i^2*log
(e) + (3*i^2*n - 4*i^2*log(e))*a^2*b*c*d^2 - 2*(i^2*n - i^2*log(e))*a^3*d^
3)*B)*log(b*x + a) + ((b^3*c*d^2*i^2 - a*b^2*d^3*i^2)*B*x^2 + (a*b^2*c*d^2
*i^2 - a^2*b*d^3*i^2)*B*x + (2*a*b^2*c^2*d*i^2 - 3*a^2*b*c*d^2*i^2 + a^3*d
^3*i^2)*B + 2*((b^3*c^2*d*i^2 - 2*a*b^2*c*d^2*i^2 + a^2*b*d^3*i^2)*B*x + (
a*b^2*c^2*d*i^2 - 2*a^2*b*c*d^2*i^2 + a^3*d^3*i^2)*B)*log(b*x + a))*log((b
*x + a)^n) - ((b^3*c*d^2*i^2 - a*b^2*d^3*i^2)*B*x^2 + (a*b^2*c*d^2*i^2 - a
^2*b*d^3*i^2)*B*x + (2*a*b^2*c^2*d*i^2 - 3*a^2*b*c*d^2*i^2 + a^3*d^3*i^...
```

Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^2}\,dx$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x,
algorithm="giac")
```

output

```
integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x +
a*g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(a\,g+b\,g\,x)^2}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^2,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^2, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

output

```
( - int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(a**2 + 2*a*b*x + b**2*x
**2),x)*a**3*b**4*d**3 + int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(a*
*2 + 2*a*b*x + b**2*x**2),x)*a**2*b**5*c*d**2 - int((log(((a + b*x)**n*e)/
(c + d*x)**n)*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**5*d**3*x + int
((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)
*a*b**6*c*d**2*x - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**2 + 2*
a*b*x + b**2*x**2),x)*a**3*b**4*c*d**2 + 2*int((log(((a + b*x)**n*e)/(c +
d*x)**n)*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**5*c**2*d - 2*int((log(
((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**
5*c*d**2*x + 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**2 + 2*a*b*x
+ b**2*x**2),x)*a*b**6*c**2*d*x + 2*log(a + b*x)*a**5*d**3 - 4*log(a + b*x
)*a**4*b*c*d**2 + 2*log(a + b*x)*a**4*b*d**3*x + 2*log(a + b*x)*a**3*b**2*
c**2*d - 4*log(a + b*x)*a**3*b**2*c*d**2*x + 2*log(a + b*x)*a**2*b**3*c**2
*d*x - log(a + b*x)*a*b**4*c**3*n - log(a + b*x)*b**5*c**3*n*x + log(c + d
*x)*a*b**4*c**3*n + log(c + d*x)*b**5*c**3*n*x - log(((a + b*x)**n*e)/(c +
 d*x)**n)*a*b**4*c**2*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*c**3*x
 - 2*a**4*b*d**3*x + 4*a**3*b**2*c*d**2*x - a**3*b**2*d**3*x**2 - 3*a**2*b
**3*c**2*d*x + a**2*b**3*c*d**2*x**2 + a*b**4*c**3*x - a*b**4*c**2*d*n*x +
 b**5*c**3*n*x)/(a*b**3*g**2*(a**2*d - a*b*c + a*b*d*x - b**2*c*x))
```

3.123 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$

Optimal result . . . 1462
Mathematica [A] (verified) . . . 1463
Rubi [A] (verified) . . . 1463
Maple [F] . . . 1466
Fricas [F] . . . 1467
Sympy [F] . . . 1467
Maxima [F] . . . 1468
Giac [F] . . . 1468
Mupad [F(-1)] . . . 1469
Reduce [F] . . . 1469

Optimal result

Integrand size = 43, antiderivative size = 242

$$\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx \\
&= -\frac{Bdi^2n(c+dx)}{b^2g^3(a+bx)} - \frac{Bi^2n(c+dx)^2}{4bg^3(a+bx)^2} - \frac{di^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^3(a+bx)} \\
&\quad - \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2} \\
&\quad - \frac{d^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} + \frac{Bd^2i^2n\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}
\end{aligned}$$

output

```
-B*d*i^2*n*(d*x+c)/b^2/g^3/(b*x+a)-1/4*B*i^2*n*(d*x+c)^2/b/g^3/(b*x+a)^2-d
*i^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^3/(b*x+a)-1/2*i^2*(d*x+
c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^3/(b*x+a)^2-d^2*i^2*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^3+B*d^2*i^2*n*polylog(2
,b*(d*x+c)/d/(b*x+a))/b^3/g^3
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.07

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$$

$$= \frac{i^2\left(-\frac{B(bc-ad)^2n}{(a+bx)^2}+\frac{6Bd(-bc+ad)n}{a+bx}-6Bd^2n\log(a+bx)-\frac{2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}+\frac{8d(-bc+ad)\left(A+B\log\left(e\left(\right.\right.\right.\right.}{a+bx}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^3,x]
```

output
```
(i^2*(-((B*(b*c - a*d)^2*n)/(a + b*x)^2) + (6*B*d*(-(b*c) + a*d)*n)/(a + b*x) - 6*B*d^2*n*Log[a + b*x] - (2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 + (8*d*(-(b*c) + a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + 4*d^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*B*d^2*n*Log[c + d*x] - 2*B*d^2*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(4*b^3*g^3)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {2961, 2780, 2741, 2780, 2741, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{i^2\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\,d\frac{a+bx}{c+dx}}{g^3}$$

$$\downarrow 2780$$

$$\frac{i^2\left(\frac{\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^3}$$

$$\downarrow 2741$$

$$\frac{i^2\left(\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}}{b}\right)}{g^3}$$

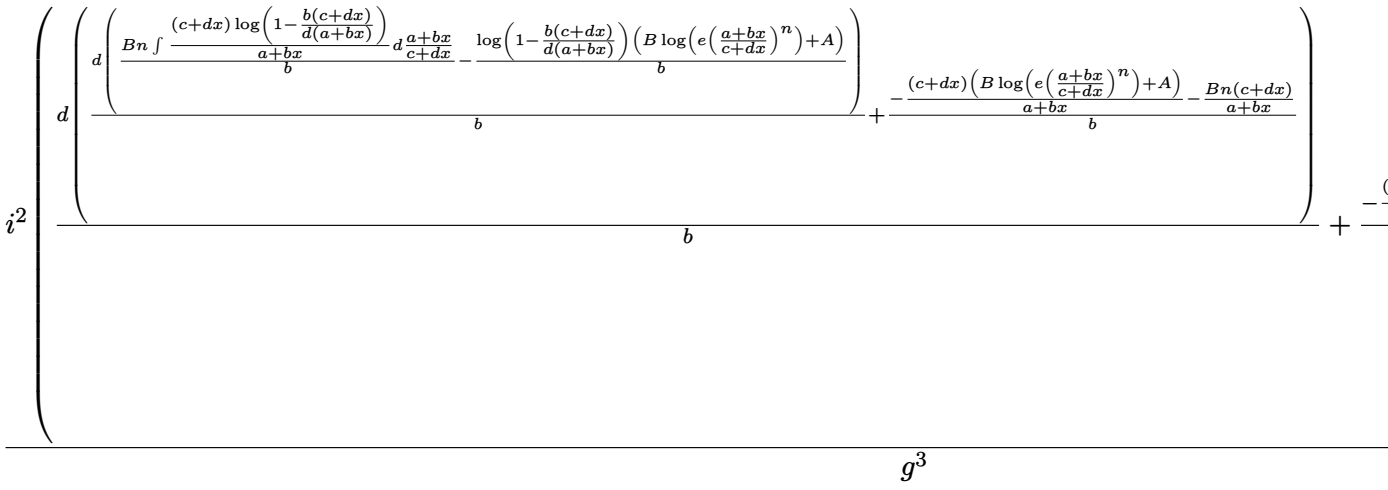
$$\downarrow 2780$$

$$\frac{i^2\left(\frac{d\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}}{b}\right)}{g^3}$$

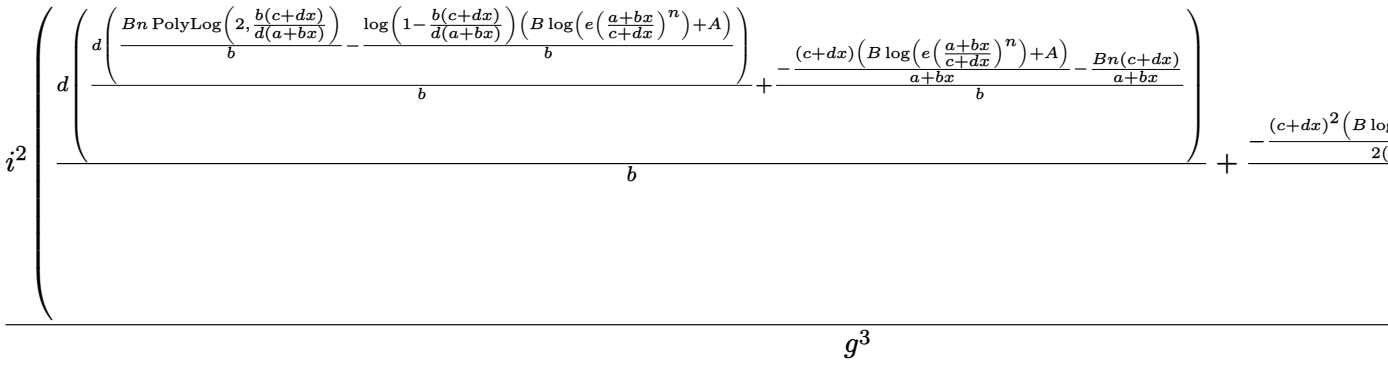
$$\downarrow 2741$$

$$\frac{i^2\left(\frac{d\left(\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}}{b}\right)}{g^3}$$

$$\downarrow 2779$$



↓ 2838



input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
^3,x]
```

output
```
(i^2*((-1/4*(B*n*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]))/(2*(a + b*x)^2))/b + (d*((-((B*n*(c + d*x))/(a + b*
x)) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x))/b + (d
*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a +
b*x))])/b) + (B*n*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b))/b))/b))/g^3
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(dix+ci)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(bgx+ag)^3}dx$$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
```

output
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,
algorithm="fricas")
```

output
```
integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B
*c*d*i^2*x + B*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n))/(b^3*g^3*x^3 + 3*a
*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$$
$$= \frac{i^2\left(\int \frac{Ac^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{Ad^2x^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{Bc^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{2Acdx}{a^3+3a^2bx+3ab^2x^2+}\right.}{g^3}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3,x
)
```

output
```
i**2*(Integral(A*c**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x)
+ Integral(A*d**2*x**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x)
 + Integral(B*c**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*
b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*c*d*x/(a**3 + 3*a**2*b
*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(B*d**2*x**2*log(e*(a/(c + d
*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),
x) + Integral(2*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**3 + 3*
a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x))/g**3
```

Maxima [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,
algorithm="maxima")
```

output
```
-1/2*B*c*d*i^2*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c - a*b^
4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3
) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2
)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^
2*d^2)*g^3)) + 1/4*B*c^2*i^2*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)
*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*
d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x
 + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) + 1/2*A*d^2*i^2*((4*a*b*x
 + 3*a^2)/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^
3*g^3)) + 1/2*B*d^2*i^2*(((4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*l
og(b*x + a))*log((b*x + a)^n) - (4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x +
a^2)*log(b*x + a))*log((d*x + c)^n))/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^
3*g^3) + 2*integrate(1/2*(2*b^3*d*x^3*log(e) + 2*b^3*c*x^2*log(e) - 3*a^2*
b*c*n + 3*a^3*d*n - 4*(a*b^2*c*n - a^2*b*d*n)*x - 2*(a^2*b*c*n - a^3*d*n +
 (b^3*c*n - a*b^2*d*n)*x^2 + 2*(a*b^2*c*n - a^2*b*d*n)*x)*log(b*x + a))/(b
^6*d*g^3*x^4 + a^3*b^3*c*g^3 + (b^6*c*g^3 + 3*a*b^5*d*g^3)*x^3 + 3*(a*b^5*
c*g^3 + a^2*b^4*d*g^3)*x^2 + (3*a^2*b^4*c*g^3 + a^3*b^3*d*g^3)*x), x)) - (
2*b*x + a)*B*c*d*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^3*x^2 +
 2*a*b^3*g^3*x + a^2*b^2*g^3) - (2*b*x + a)*A*c*d*i^2/(b^4*g^3*x^2 + 2*a*b
^3*g^3*x + a^2*b^2*g^3) - 1/2*B*c^2*i^2*log(e*(b*x/(d*x + c) + a/(d*x +...
```

Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,
algorithm="giac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x +
a*g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(a\,g+b\,g\,x)^3}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^3,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^3, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
```

output

```
( - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(a**3 + 3*a**2*b*x + 3
*a*b**2*x**2 + b**3*x**3),x)*a**6*b**4*d**4 + 8*int((log(((a + b*x)**n*e)/
(c + d*x)**n)*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**
5*b**5*c*d**3 - 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(a**3 + 3*
a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**5*b**5*d**4*x - 4*int((log(((a
 + b*x)**n*e)/(c + d*x)**n)*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**
3*x**3),x)*a**4*b**6*c**2*d**2 + 16*int((log(((a + b*x)**n*e)/(c + d*x)**n
)*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**4*b**6*c*d**
3*x - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(a**3 + 3*a**2*b*x +
 3*a*b**2*x**2 + b**3*x**3),x)*a**4*b**6*d**4*x**2 - 8*int((log(((a + b*x)
**n*e)/(c + d*x)**n)*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)
,x)*a**3*b**7*c**2*d**2*x + 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2
)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**3*b**7*c*d**3*x**2
 - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(a**3 + 3*a**2*b*x + 3*
a*b**2*x**2 + b**3*x**3),x)*a**2*b**8*c**2*d**2*x**2 - 4*log(a + b*x)*a**7
*d**4 + 8*log(a + b*x)*a**6*b*c*d**3 - 8*log(a + b*x)*a**6*b*d**4*x - 4*lo
g(a + b*x)*a**5*b**2*c**2*d**2 + 16*log(a + b*x)*a**5*b**2*c*d**3*x - 4*lo
g(a + b*x)*a**5*b**2*d**4*x**2 - 8*log(a + b*x)*a**4*b**3*c**2*d**2*x + 8*
log(a + b*x)*a**4*b**3*c*d**3*x**2 - 4*log(a + b*x)*a**3*b**4*c**2*d**2*x*
*2 + 2*log(a + b*x)*a**2*b**5*c**4*n + 4*log(a + b*x)*a*b**6*c**4*n*x +...
```

3.124 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$

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Optimal result

Integrand size = 43, antiderivative size = 93

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = -\frac{Bi^2n(c+dx)^3}{9(bc-ad)g^4(a+bx)^3} - \frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)g^4(a+bx)^3}$$

output

```
-1/9*B*i^2*n*(d*x+c)^3/(-a*d+b*c)/g^4/(b*x+a)^3-1/3*i^2*(d*x+c)^3*(A+B*ln(
e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/g^4/(b*x+a)^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 329 vs. 2(93) = 186.

Time = 0.40 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.54

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = -\frac{i^2\left(3Ab^3c^3-3a^3Ad^3+b^3Bc^3n-a^3Bd^3n+9Ab^3c^2dx-9a^2Abd^3x+3b^3Bc^2dnx-3a^2bBd^3nx+9Ab^3\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^4,x]
```

output
```
-1/9*(i^2*(3*A*b^3*c^3 - 3*a^3*A*d^3 + b^3*B*c^3*n - a^3*B*d^3*n + 9*A*b^3*c^2*d*x - 9*a^2*A*b*d^3*x + 3*b^3*B*c^2*d*n*x - 3*a^2*b*B*d^3*n*x + 9*A*b^3*c*d^2*x^2 - 9*a*A*b^2*d^3*x^2 + 3*b^3*B*c*d^2*n*x^2 - 3*a*b^2*B*d^3*n*x^2 + 3*B*d^3*n*(a + b*x)^3*Log[a + b*x] + 3*B*(b*c - a*d)*(a^2*d^2 + a*b*d*(c + 3*d*x) + b^2*(c^2 + 3*c*d*x + 3*d^2*x^2))*Log[e*((a + b*x)/(c + d*x))^n] - 3*a^3*B*d^3*n*Log[c + d*x] - 9*a^2*b*B*d^3*n*x*Log[c + d*x] - 9*a*b^2*B*d^3*n*x^2*Log[c + d*x] - 3*b^3*B*d^3*n*x^3*Log[c + d*x]))/(b^3*(b*c - a*d)*g^4*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2961, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^4}\,dx$$

$$\downarrow 2961$$

$$\frac{i^2\int \frac{(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{i^2\left(-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}-\frac{Bn(c+dx)^3}{9(a+bx)^3}\right)}{g^4(bc-ad)}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^4,x]
```

output
```
(i^2*(-1/9*(B*n*(c + d*x)^3)/(a + b*x)^3 - ((c + d*x)^3*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(3*(a + b*x)^3)))/((b*c - a*d)*g^4)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(89) = 178$.

Time = 11.94 (sec) , antiderivative size = 376, normalized size of antiderivative = 4.04

method	result
parallelrisch	$-\frac{B a^3 b^2 d^4 i^2 n^2 - B b^5 c^3 d i^2 n^2 + 3A a^3 b^2 d^4 i^2 n - 9A x^2 b^5 c d^3 i^2 n + 9A x^2 a b^4 d^4 i^2 n - 3B x^2 b^5 c d^3 i^2 n^2 + 3B x^2 a b^4 d^4 i^2 n^2 - 3B x$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,method=_
RETURNVERBOSE)
```

output

```
-1/9*(B*a^3*b^2*d^4*i^2*n^2-B*b^5*c^3*d*i^2*n^2+3*A*a^3*b^2*d^4*i^2*n-9*A*
x^2*b^5*c*d^3*i^2*n+9*A*x^2*a*b^4*d^4*i^2*n-3*B*x^2*b^5*c*d^3*i^2*n^2+3*B*
x^2*a*b^4*d^4*i^2*n^2-3*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^4*i^2*n-9*B*
x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d^3*i^2*n-3*A*b^5*c^3*d*i^2*n-9*B*x*ln
(e*((b*x+a)/(d*x+c))^n)*b^5*c^2*d^2*i^2*n-3*B*x*b^5*c^2*d^2*i^2*n^2+9*A*x*
a^2*b^3*d^4*i^2*n-9*A*x*b^5*c^2*d^2*i^2*n-3*B*ln(e*((b*x+a)/(d*x+c))^n)*b^
5*c^3*d*i^2*n+3*B*x*a^2*b^3*d^4*i^2*n^2)/g^4/(b*x+a)^3/b^5/d/n/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(89) = 178$.

Time $= 0.09$ (sec) , antiderivative size $= 409$, normalized size of antiderivative $= 4.40$

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{(Bb^3c^3-Ba^3d^3)i^2n+3\left(Ab^3c^3-Aa^3d^3\right)i^2+3\left((Bb^3cd^2-Bab^2d^3)i^2n+3\left(Ab^3cd^2-Aab^2d^3\right)i^2\right)x^2+}{}$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,
algorithm="fricas")
```

output

```
-1/9*((B*b^3*c^3 - B*a^3*d^3)*i^2*n + 3*(A*b^3*c^3 - A*a^3*d^3)*i^2 + 3*((
B*b^3*c*d^2 - B*a*b^2*d^3)*i^2*n + 3*(A*b^3*c*d^2 - A*a*b^2*d^3)*i^2)*x^2
+ 3*((B*b^3*c^2*d - B*a^2*b*d^3)*i^2*n + 3*(A*b^3*c^2*d - A*a^2*b*d^3)*i^2
)*x + 3*(3*(B*b^3*c*d^2 - B*a*b^2*d^3)*i^2*x^2 + 3*(B*b^3*c^2*d - B*a^2*b*
d^3)*i^2*x + (B*b^3*c^3 - B*a^3*d^3)*i^2)*log(e) + 3*(B*b^3*d^3*i^2*n*x^3
+ 3*B*b^3*c*d^2*i^2*n*x^2 + 3*B*b^3*c^2*d*i^2*n*x + B*b^3*c^3*i^2*n)*log((
b*x + a)/(d*x + c)))/((b^7*c - a*b^6*d)*g^4*x^3 + 3*(a*b^6*c - a^2*b^5*d)*
g^4*x^2 + 3*(a^2*b^5*c - a^3*b^4*d)*g^4*x + (a^3*b^4*c - a^4*b^3*d)*g^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4,x
)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1544 vs. $2(89) = 178$.

Time = 0.10 (sec) , antiderivative size = 1544, normalized size of antiderivative = 16.60

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,
algorithm="maxima")
```

output

```
-1/18*B*d^2*i^2*n*((11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*c^
2 - 3*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a^
3*b*d^2)*x)/((b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c^2
 - 2*a^2*b^6*c*d + a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d +
 a^4*b^4*d^2)*g^4*x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6*
(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2
*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 +
 a^2*d^3)*log(d*x + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^
3*d^3)*g^4)) - 1/18*B*c^2*i^2*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d +
11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*
d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^
2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*
c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a
^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^
2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/18*B*c*d*i^2*n*((5*a*b^2*c^2
 - 22*a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2
 - 16*a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^
4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c
^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a
^5*b^2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b...
```

Giac [A] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.16

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{1}{9}\left(\frac{3\,(dx+c)^3Bi^2n\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^3g^4}+\frac{(Bi^2n+3\,Bi^2\log(e)+3\,Ai^2)(dx+c)^3}{(bx+a)^3g^4}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right.$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,
algorithm="giac")
```

output

```
-1/9*(3*(d*x + c)^3*B*i^2*n*log((b*x + a)/(d*x + c))/((b*x + a)^3*g^4) + (
B*i^2*n + 3*B*i^2*log(e) + 3*A*i^2)*(d*x + c)^3/((b*x + a)^3*g^4))*(b*c/(b
*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 26.83 (sec) , antiderivative size = 421, normalized size of antiderivative = 4.53

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{x\left(3\,A\,a\,b\,d^2\,i^2+3\,A\,b^2\,c\,d\,i^2+B\,a\,b\,d^2\,i^2\,n+B\,b^2\,c\,d\,i^2\,n\right)+x^2\left(3\,A\,b^2\,d^2\,i^2+B\,b^2\,d^2\,i^2\,n\right)+A\,a^2\,d^2\,i^2}{3\,a^3\,b^3\,g^4+9\,a^2\,b^4\,g^4\,x+9\,a\,b^5\,g^4\,x^2+3\,b^6\,g^4}$$

$$-\frac{\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(a\left(\frac{B\,a\,d^2\,i^2}{3\,b^3}+\frac{B\,c\,d\,i^2}{3\,b^2}\right)+x\left(b\left(\frac{B\,a\,d^2\,i^2}{3\,b^3}+\frac{B\,c\,d\,i^2}{3\,b^2}\right)+\frac{2\,B\,a\,d^2\,i^2}{3\,b^2}+\frac{2\,B\,c\,d\,i^2}{3\,b}\right)+\frac{B\,c^2\,i^2}{3\,b}+\frac{B\,d^2\,i^2\,x^2}{b}\right.}{a^3\,g^4+3\,a^2\,b\,g^4\,x+3\,a\,b^2\,g^4\,x^2+b^3\,g^4\,x^3}$$

$$-\frac{B\,d^3\,i^2\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{3\,b^3\,g^4\,(a\,d-b\,c)}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^4,x)
```

output
```
- (x*(3*A*a*b*d^2*i^2 + 3*A*b^2*c*d*i^2 + B*a*b*d^2*i^2*n + B*b^2*c*d*i^2*
n) + x^2*(3*A*b^2*d^2*i^2 + B*b^2*d^2*i^2*n) + A*a^2*d^2*i^2 + A*b^2*c^2*i
^2 + (B*a^2*d^2*i^2*n)/3 + (B*b^2*c^2*i^2*n)/3 + A*a*b*c*d*i^2 + (B*a*b*c*
d*i^2*n)/3)/(3*a^3*b^3*g^4 + 3*b^6*g^4*x^3 + 9*a^2*b^4*g^4*x + 9*a*b^5*g^4
*x^2) - (log(e*((a + b*x)/(c + d*x))^n)*(a*((B*a*d^2*i^2)/(3*b^3) + (B*c*d
*i^2)/(3*b^2)) + x*(b*((B*a*d^2*i^2)/(3*b^3) + (B*c*d*i^2)/(3*b^2)) + (2*B
*a*d^2*i^2)/(3*b^2) + (2*B*c*d*i^2)/(3*b)) + (B*c^2*i^2)/(3*b) + (B*d^2*i^
2*x^2)/b))/(a^3*g^4 + b^3*g^4*x^3 + 3*a*b^2*g^4*x^2 + 3*a^2*b*g^4*x) - (B*
d^3*i^2*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(3*b^3*g^4*(a*d -
 b*c))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 560, normalized size of antiderivative = 6.02

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$$

$$=\frac{3a^4cd^2-3a^2b^2c^3-3\log(bx+a)\,a^3bcd^2n-3\log(bx+a)\,b^4cd^2nx^3+3\log(dx+c)\,b^4cd^2nx^3+9\log\left(\frac{}{}\right.}{}$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x)
```

output
```
( - 3*log(a + b*x)*a**3*b*c*d**2*n - 9*log(a + b*x)*a**2*b**2*c*d**2*n*x -
 9*log(a + b*x)*a*b**3*c*d**2*n*x**2 - 3*log(a + b*x)*b**4*c*d**2*n*x**3 +
 3*log(c + d*x)*a**3*b*c*d**2*n + 9*log(c + d*x)*a**2*b**2*c*d**2*n*x + 9*
log(c + d*x)*a*b**3*c*d**2*n*x**2 + 3*log(c + d*x)*b**4*c*d**2*n*x**3 + 3*
log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*c*d**2 + 9*log(((a + b*x)**n*e)/
(c + d*x)**n)*a**2*b**2*c*d**2*x - 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a*
b**3*c**3 - 9*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c**2*d*x - 3*log((
(a + b*x)**n*e)/(c + d*x)**n)*a*b**3*d**3*x**3 + 3*log(((a + b*x)**n*e)/(c
 + d*x)**n)*b**4*c*d**2*x**3 + 3*a**4*c*d**2 + a**3*b*c*d**2*n + 9*a**3*b*
c*d**2*x - 3*a**2*b**2*c**3 - 9*a**2*b**2*c**2*d*x + 3*a**2*b**2*c*d**2*n*
x - 3*a**2*b**2*d**3*x**3 - a*b**3*c**3*n - 3*a*b**3*c**2*d*n*x + 3*a*b**3
*c*d**2*x**3 - a*b**3*d**3*n*x**3 + b**4*c*d**2*n*x**3)/(9*a*b**2*g**4*(a*
*4*d - a**3*b*c + 3*a**3*b*d*x - 3*a**2*b**2*c*x + 3*a**2*b**2*d*x**2 - 3*
a*b**3*c*x**2 + a*b**3*d*x**3 - b**4*c*x**3))
```

3.125 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$

Optimal result 1479
Mathematica [B] (verified) 1480
Rubi [A] (verified) 1481
Maple [B] (verified) 1483
Fricas [B] (verification not implemented) 1484
Sympy [F(-1)] 1484
Maxima [B] (verification not implemented) 1485
Giac [A] (verification not implemented) 1486
Mupad [B] (verification not implemented) 1486
Reduce [B] (verification not implemented) 1487

Optimal result

Integrand size = 43, antiderivative size = 189

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = \frac{Bdi^2n(c+dx)^3}{9(bc-ad)^2g^5(a+bx)^3} - \frac{bBi^2n(c+dx)^4}{16(bc-ad)^2g^5(a+bx)^4} + \frac{di^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^2g^5(a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^2g^5(a+bx)^4}$$

output

```
1/9*B*d*i^2*n*(d*x+c)^3/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/16*b*B*i^2*n*(d*x+c)^
4/(-a*d+b*c)^2/g^5/(b*x+a)^4+1/3*d*i^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/4*b*i^2*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(
d*x+c))^n))/(-a*d+b*c)^2/g^5/(b*x+a)^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 474 vs. $2(189) = 378$.

Time $= 0.46$ (sec) , antiderivative size $= 474$, normalized size of antiderivative $= 2.51$

$$\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx \\
&= -\frac{(bc-ad)^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^3g^5(a+bx)^4} \\
&\quad - \frac{2d(bc-ad)i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3g^5(a+bx)^3} - \frac{d^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^3g^5(a+bx)^2} \\
&\quad - \frac{Bd^2i^2n\left(\frac{1}{(a+bx)^2} - \frac{2d}{(bc-ad)(a+bx)} - \frac{2d^2\log(a+bx)}{(bc-ad)^2} + \frac{2d^2\log(c+dx)}{(bc-ad)^2}\right)}{4b^3g^5} \\
&\quad - \frac{Bdi^2n\left(\frac{2(bc-ad)}{(a+bx)^3} - \frac{3d}{(a+bx)^2} + \frac{6d^2}{(bc-ad)(a+bx)} + \frac{6d^3\log(a+bx)}{(bc-ad)^2} - \frac{6d^3\log(c+dx)}{(bc-ad)^2}\right)}{9b^3g^5} \\
&\quad - \frac{Bi^2n\left(\frac{3(bc-ad)^2}{(a+bx)^4} - \frac{4d(bc-ad)}{(a+bx)^3} + \frac{6d^2}{(a+bx)^2} - \frac{12d^3}{(bc-ad)(a+bx)} - \frac{12d^4\log(a+bx)}{(bc-ad)^2} + \frac{12d^4\log(c+dx)}{(bc-ad)^2}\right)}{48b^3g^5}
\end{aligned}$$

input

```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g +
b*g*x)^5,x]
```

output

```
-1/4*((b*c - a*d)^2*i^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^3*g^5*(
a + b*x)^4) - (2*d*(b*c - a*d)*i^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
/(3*b^3*g^5*(a + b*x)^3) - (d^2*i^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
)/(2*b^3*g^5*(a + b*x)^2) - (B*d^2*i^2*n*((a + b*x)^(-2) - (2*d)/((b*c - a
*d)*(a + b*x)) - (2*d^2*Log[a + b*x])/(b*c - a*d)^2 + (2*d^2*Log[c + d*x])
/(b*c - a*d)^2))/(4*b^3*g^5) - (B*d*i^2*n*((2*(b*c - a*d))/(a + b*x)^3 - (
3*d)/(a + b*x)^2 + (6*d^2)/((b*c - a*d)*(a + b*x)) + (6*d^3*Log[a + b*x])/
(b*c - a*d)^2 - (6*d^3*Log[c + d*x])/(b*c - a*d)^2))/(9*b^3*g^5) - (B*i^2*
n*((3*(b*c - a*d)^2)/(a + b*x)^4 - (4*d*(b*c - a*d))/(a + b*x)^3 + (6*d^2)
/(a + b*x)^2 - (12*d^3)/((b*c - a*d)*(a + b*x)) - (12*d^4*Log[a + b*x])/(b
*c - a*d)^2 + (12*d^4*Log[c + d*x])/(b*c - a*d)^2))/(48*b^3*g^5)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2961, 2772, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^5}\,dx$$

$$\downarrow 2961$$

$$\frac{i^2\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^2}$$

$$\downarrow 2772$$

$$\frac{i^2\left(-Bn\int -\frac{(c+dx)^5\left(3b-\frac{4d(a+bx)}{c+dx}\right)}{12(a+bx)^5}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}\right)}{g^5(bc-ad)^2}$$

$$\downarrow 27$$

$$\frac{i^2\left(\frac{1}{12}Bn\int \frac{(c+dx)^5\left(3b-\frac{4d(a+bx)}{c+dx}\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}\right)}{g^5(bc-ad)^2}$$

$$\downarrow 53$$

$$\frac{i^2\left(\frac{1}{12}Bn\int \left(\frac{3b(c+dx)^5}{(a+bx)^5}-\frac{4d(c+dx)^4}{(a+bx)^4}\right)d\frac{a+bx}{c+dx}-\frac{b(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}\right)}{g^5(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{i^2\left(-\frac{b(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{1}{12}Bn\left(\frac{4d(c+dx)^3}{3(a+bx)^3}-\frac{3b(c+dx)^4}{4(a+bx)^4}\right)\right)}{g^5(bc-ad)^2}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
^5,x]
```

output
```
(i^2*((B*n*((4*d*(c + d*x)^3)/(3*(a + b*x)^3) - (3*b*(c + d*x)^4)/(4*(a +
b*x)^4)))/12 + (d*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(
a + b*x)^3) - (b*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*(a
 + b*x)^4)))/((b*c - a*d)^2*g^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. 2(181) = 362.

Time = 29.83 (sec) , antiderivative size = 855, normalized size of antiderivative = 4.52

method	result
parallelrisch	$\frac{48Ax^3a^7cd^4i^2n+144Ax^3a^3b^4c^5i^2n+48Bx^2a^7c^2d^3i^2n^2+54Bx^2a^4b^3c^5i^2n^2+144Ax^2a^7c^2d^3i^2n+216Ax^2a^4b^3c^5i^2n+48B$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x,method=_
RETURNVERBOSE)
```

output
```
1/144*(48*A*x^3*a^7*c*d^4*i^2*n+144*A*x^3*a^3*b^4*c^5*i^2*n+48*B*x^2*a^7*c
^2*d^3*i^2*n^2+54*B*x^2*a^4*b^3*c^5*i^2*n^2+144*A*x^2*a^7*c^2*d^3*i^2*n+21
6*A*x^2*a^4*b^3*c^5*i^2*n+48*B*x*a^7*c^3*d^2*i^2*n^2+36*B*x*a^5*b^2*c^5*i^
2*n^2+144*A*x*a^7*c^3*d^2*i^2*n+144*A*x*a^5*b^2*c^5*i^2*n+48*B*ln(e*((b*x+
a)/(d*x+c))^n)*a^7*c^4*d*i^2*n-36*B*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^5*i^
2*n+12*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c*d^4*i^2*n-72*B*x^2*ln(e*((b
*x+a)/(d*x+c))^n)*a^6*b*c^3*d^2*i^2*n-96*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^6
*b*c^4*d*i^2*n+9*B*x^4*a^2*b^5*c^5*i^2*n^2+36*A*x^4*a^2*b^5*c^5*i^2*n+16*B
*x^3*a^7*c*d^4*i^2*n^2+36*B*x^3*a^3*b^4*c^5*i^2*n^2+7*B*x^4*a^6*b*c*d^4*i^
2*n^2-16*B*x^4*a^3*b^4*c^4*d*i^2*n^2+12*A*x^4*a^6*b*c*d^4*i^2*n-48*A*x^4*a
^3*b^4*c^4*d*i^2*n+48*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^7*c*d^4*i^2*n+12*B
*x^3*a^6*b*c^2*d^3*i^2*n^2-64*B*x^3*a^4*b^3*c^4*d*i^2*n^2-192*A*x^3*a^4*b^
3*c^4*d*i^2*n+144*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^7*c^2*d^3*i^2*n-6*B*x^
2*a^6*b*c^3*d^2*i^2*n^2-96*B*x^2*a^5*b^2*c^4*d*i^2*n^2-72*A*x^2*a^6*b*c^3*
d^2*i^2*n-288*A*x^2*a^5*b^2*c^4*d*i^2*n+144*B*x*ln(e*((b*x+a)/(d*x+c))^n)*
a^7*c^3*d^2*i^2*n-84*B*x*a^6*b*c^4*d*i^2*n^2-288*A*x*a^6*b*c^4*d*i^2*n)/g^
5/(b*x+a)^4/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)/a^6/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(181) = 362$.

Time = 0.11 (sec) , antiderivative size = 710, normalized size of antiderivative = 3.76

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{12\left(Bb^4cd^3-Bab^3d^4\right)i^2nx^3-\left(9\,Bb^4c^4-16\,Bab^3c^3d+7\,Ba^4d^4\right)i^2n-12\left(3\,Ab^4c^4-4\,Aab^3c^3d+Aa^4d^4\right.}{}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x,
algorithm="fricas")
```

output
```
1/144*(12*(B*b^4*c*d^3 - B*a*b^3*d^4)*i^2*n*x^3 - (9*B*b^4*c^4 - 16*B*a*b^
3*c^3*d + 7*B*a^4*d^4)*i^2*n - 12*(3*A*b^4*c^4 - 4*A*a*b^3*c^3*d + A*a^4*d
^4)*i^2 - 6*((B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4)*i^2*n + 1
2*(A*b^4*c^2*d^2 - 2*A*a*b^3*c*d^3 + A*a^2*b^2*d^4)*i^2)*x^2 - 4*((5*B*b^4
*c^3*d - 12*B*a*b^3*c^2*d^2 + 7*B*a^3*b*d^4)*i^2*n + 12*(2*A*b^4*c^3*d - 3
*A*a*b^3*c^2*d^2 + A*a^3*b*d^4)*i^2)*x - 12*(6*(B*b^4*c^2*d^2 - 2*B*a*b^3*
c*d^3 + B*a^2*b^2*d^4)*i^2*x^2 + 4*(2*B*b^4*c^3*d - 3*B*a*b^3*c^2*d^2 + B*
a^3*b*d^4)*i^2*x + (3*B*b^4*c^4 - 4*B*a*b^3*c^3*d + B*a^4*d^4)*i^2)*log(e)
 + 12*(B*b^4*d^4*i^2*n*x^4 + 4*B*a*b^3*d^4*i^2*n*x^3 - 6*(B*b^4*c^2*d^2 -
2*B*a*b^3*c*d^3)*i^2*n*x^2 - 4*(2*B*b^4*c^3*d - 3*B*a*b^3*c^2*d^2)*i^2*n*x
 - (3*B*b^4*c^4 - 4*B*a*b^3*c^3*d)*i^2*n)*log((b*x + a)/(d*x + c)))/((b^9*
c^2 - 2*a*b^8*c*d + a^2*b^7*d^2)*g^5*x^4 + 4*(a*b^8*c^2 - 2*a^2*b^7*c*d +
a^3*b^6*d^2)*g^5*x^3 + 6*(a^2*b^7*c^2 - 2*a^3*b^6*c*d + a^4*b^5*d^2)*g^5*x
^2 + 4*(a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*g^5*x + (a^4*b^5*c^2 -
2*a^5*b^4*c*d + a^6*b^3*d^2)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**5,x
)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2247 vs. 2(181) = 362.

Time = 0.14 (sec) , antiderivative size = 2247, normalized size of antiderivative = 11.89

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x,
algorithm="maxima")
```

output
```
1/48*B*c^2*i^2*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*
c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*
b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 -
 a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 -
 a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2
 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d
^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2
 - a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2
*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b
^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*
g^5)) - 1/144*B*d^2*i^2*n*((13*a^2*b^3*c^3 - 75*a^3*b^2*c^2*d + 33*a^4*b*c
*d^2 - 7*a^5*d^3 - 12*(6*b^5*c^2*d - 4*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3 + 6*
(6*b^5*c^3 - 46*a*b^4*c^2*d + 29*a^2*b^3*c*d^2 - 7*a^3*b^2*d^3)*x^2 + 4*(1
0*a*b^4*c^3 - 63*a^2*b^3*c^2*d + 33*a^3*b^2*c*d^2 - 7*a^4*b*d^3)*x)/((b^10
*c^3 - 3*a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*g^5*x^4 + 4*(a*b^9*c
^3 - 3*a^2*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*g^5*x^3 + 6*(a^2*b^8
*c^3 - 3*a^3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*g^5*x^2 + 4*(a^3*b
^7*c^3 - 3*a^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*g^5*x + (a^4*b^6
*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3)*g^5) - 12*(6*b^2*c
^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*log(b*x + a)/((b^7*c^4 - 4*a*b^6*c^3*d ...
```

Giac [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.32

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{1}{144}\left(\frac{12\left(3\,Bbi^2n-\frac{4\,(bx+a)Bdi^2n}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^4bcg^5}{(dx+c)^4}-\frac{(bx+a)^4adg^5}{(dx+c)^4}}+\frac{9\,Bbi^2n-\frac{16\,(bx+a)Bdi^2n}{dx+c}+36\,Bbi^2\log(e)-\frac{48\,(bx+a)Bd}{dx+c}}{\frac{(bx+a)^4bcg^5}{(dx+c)^4}-\frac{(bx+a)^4adg^5}{(dx+c)^4}}\right.$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x,
algorithm="giac")
```

output
```
-1/144*(12*(3*B*b*i^2*n - 4*(b*x + a)*B*d*i^2*n/(d*x + c))*log((b*x + a)/(
d*x + c))/((b*x + a)^4*b*c*g^5/(d*x + c)^4 - (b*x + a)^4*a*d*g^5/(d*x + c)
^4) + (9*B*b*i^2*n - 16*(b*x + a)*B*d*i^2*n/(d*x + c) + 36*B*b*i^2*log(e)
- 48*(b*x + a)*B*d*i^2*log(e)/(d*x + c) + 36*A*b*i^2 - 48*(b*x + a)*A*d*i^
2/(d*x + c))/((b*x + a)^4*b*c*g^5/(d*x + c)^4 - (b*x + a)^4*a*d*g^5/(d*x +
 c)^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 27.08 (sec) , antiderivative size = 652, normalized size of antiderivative = 3.45

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{\frac{12\,Aa^3d^3i^2-36\,Ab^3c^3i^2+7\,Ba^3d^3i^2n-9\,Bb^3c^3i^2n+12\,Aab^2c^2di^2+12\,Aa^2bcd^2i^2+7\,Bab^2c^2di^2n+7\,Ba^2bcd^2i^2n}{12\,(ad-bc)}+\frac{x\,(12\,Aa}{12\,a^4b^3g^5+48\,a^3b}$$

$$-\frac{\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(a\left(\frac{Bad^2i^2}{12b^3}+\frac{Bcdi^2}{6b^2}\right)+x\left(b\left(\frac{Bad^2i^2}{12b^3}+\frac{Bcdi^2}{6b^2}\right)+\frac{Bad^2i^2}{4b^2}+\frac{Bcdi^2}{2b}\right)+\frac{Bc^2i^2}{4b}+\frac{Bd^2i^2x^2}{2b}\right)}{a^4g^5+4a^3bg^5x+6a^2b^2g^5x^2+4ab^3g^5x^3+b^4g^5x^4}$$

$$-\frac{Bd^4i^2n\,\operatorname{atanh}\left(\frac{12b^5c^2g^5-12a^2b^3d^2g^5}{12b^3g^5(ad-bc)^2}-\frac{2bdx}{ad-bc}\right)}{6b^3g^5(ad-bc)^2}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^5,x)
```

output

```
- ((12*A*a^3*d^3*i^2 - 36*A*b^3*c^3*i^2 + 7*B*a^3*d^3*i^2*n - 9*B*b^3*c^3*
i^2*n + 12*A*a*b^2*c^2*d*i^2 + 12*A*a^2*b*c*d^2*i^2 + 7*B*a*b^2*c^2*d*i^2*
n + 7*B*a^2*b*c*d^2*i^2*n)/(12*(a*d - b*c)) + (x*(12*A*a^2*b*d^3*i^2 - 24*
A*b^3*c^2*d*i^2 + 12*A*a*b^2*c*d^2*i^2 + 7*B*a^2*b*d^3*i^2*n - 5*B*b^3*c^2
*d*i^2*n + 7*B*a*b^2*c*d^2*i^2*n))/(3*(a*d - b*c)) + (x^2*(12*A*a*b^2*d^3*
i^2 - 12*A*b^3*c*d^2*i^2 + 7*B*a*b^2*d^3*i^2*n - B*b^3*c*d^2*i^2*n))/(2*(a
*d - b*c)) + (B*b^3*d^3*i^2*n*x^3)/(a*d - b*c))/(12*a^4*b^3*g^5 + 12*b^7*g
^5*x^4 + 48*a^3*b^4*g^5*x + 48*a*b^6*g^5*x^3 + 72*a^2*b^5*g^5*x^2) - (log(
e*((a + b*x)/(c + d*x))^n)*(a*((B*a*d^2*i^2)/(12*b^3) + (B*c*d*i^2)/(6*b^2
)) + x*(b*((B*a*d^2*i^2)/(12*b^3) + (B*c*d*i^2)/(6*b^2)) + (B*a*d^2*i^2)/(
4*b^2) + (B*c*d*i^2)/(2*b)) + (B*c^2*i^2)/(4*b) + (B*d^2*i^2*x^2)/(2*b)))/
(a^4*g^5 + b^4*g^5*x^4 + 4*a*b^3*g^5*x^3 + 6*a^2*b^2*g^5*x^2 + 4*a^3*b*g^5
*x) - (B*d^4*i^2*n*atanh((12*b^5*c^2*g^5 - 12*a^2*b^3*d^2*g^5)/(12*b^3*g^5
*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(6*b^3*g^5*(a*d - b*c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1333, normalized size of antiderivative = 7.05

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x)
```

output

```
( - 24*log(a + b*x)*a**5*b**2*c*d**3*n + 12*log(a + b*x)*a**4*b**3*c**2*d*
*2*n - 96*log(a + b*x)*a**4*b**3*c*d**3*n*x + 48*log(a + b*x)*a**3*b**4*c*
*2*d**2*n*x - 144*log(a + b*x)*a**3*b**4*c*d**3*n*x**2 + 72*log(a + b*x)*a
**2*b**5*c**2*d**2*n*x**2 - 96*log(a + b*x)*a**2*b**5*c*d**3*n*x**3 + 48*l
og(a + b*x)*a*b**6*c**2*d**2*n*x**3 - 24*log(a + b*x)*a*b**6*c*d**3*n*x**4
 + 12*log(a + b*x)*b**7*c**2*d**2*n*x**4 + 24*log(c + d*x)*a**5*b**2*c*d**
3*n - 12*log(c + d*x)*a**4*b**3*c**2*d**2*n + 96*log(c + d*x)*a**4*b**3*c*
d**3*n*x - 48*log(c + d*x)*a**3*b**4*c**2*d**2*n*x + 144*log(c + d*x)*a**3
*b**4*c*d**3*n*x**2 - 72*log(c + d*x)*a**2*b**5*c**2*d**2*n*x**2 + 96*log(
c + d*x)*a**2*b**5*c*d**3*n*x**3 - 48*log(c + d*x)*a*b**6*c**2*d**2*n*x**3
 + 24*log(c + d*x)*a*b**6*c*d**3*n*x**4 - 12*log(c + d*x)*b**7*c**2*d**2*n
*x**4 + 24*log(((a + b*x)**n*e)/(c + d*x)**n)*a**5*b**2*c*d**3 - 12*log(((
a + b*x)**n*e)/(c + d*x)**n)*a**4*b**3*c**2*d**2 + 96*log(((a + b*x)**n*e)
/(c + d*x)**n)*a**4*b**3*c*d**3*x - 48*log(((a + b*x)**n*e)/(c + d*x)**n)*
a**3*b**4*c**3*d - 192*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**4*c**2*d
**2*x - 48*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**4*d**4*x**3 + 36*log
(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**5*c**4 + 96*log(((a + b*x)**n*e)/(
c + d*x)**n)*a**2*b**5*c**3*d*x + 96*log(((a + b*x)**n*e)/(c + d*x)**n)*a*
*2*b**5*c*d**3*x**3 - 12*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**5*d**4
*x**4 - 48*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**6*c**2*d**2*x**3 + 2...
```

3.126 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx$

Optimal result 1489
Mathematica [A] (verified) 1490
Rubi [A] (verified) 1490
Maple [B] (verified) 1492
Fricas [B] (verification not implemented) 1493
Sympy [F(-1)] 1494
Maxima [B] (verification not implemented) 1495
Giac [A] (verification not implemented) 1496
Mupad [B] (verification not implemented) 1496
Reduce [B] (verification not implemented) 1497

Optimal result

Integrand size = 43, antiderivative size = 293

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx &= -\frac{Bd^2i^2n(c+dx)^3}{9(bc-ad)^3g^6(a+bx)^3}\\ &+\frac{bBdi^2n(c+dx)^4}{8(bc-ad)^3g^6(a+bx)^4}\\ &-\frac{b^2Bi^2n(c+dx)^5}{25(bc-ad)^3g^6(a+bx)^5}\\ &-\frac{d^2i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^3g^6(a+bx)^3}\\ &+\frac{bdi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^6(a+bx)^4}\\ &-\frac{b^2i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5(bc-ad)^3g^6(a+bx)^5}\end{aligned}$$

output

```
-1/9*B*d^2*i^2*n*(d*x+c)^3/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/8*b*B*d*i^2*n*(d*x
+c)^4/(-a*d+b*c)^3/g^6/(b*x+a)^4-1/25*b^2*B*i^2*n*(d*x+c)^5/(-a*d+b*c)^3/g
^6/(b*x+a)^5-1/3*d^2*i^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b
*c)^3/g^6/(b*x+a)^3+1/2*b*d*i^2*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/
(-a*d+b*c)^3/g^6/(b*x+a)^4-1/5*b^2*i^2*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))/(-a*d+b*c)^3/g^6/(b*x+a)^5
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.22

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx$$

$$= \frac{i^2\left(-\frac{360Ab^2c^2}{(a+bx)^5}+\frac{720aAbcd}{(a+bx)^5}-\frac{360a^2Ad^2}{(a+bx)^5}-\frac{72b^2Bc^2n}{(a+bx)^5}+\frac{144abBcdn}{(a+bx)^5}-\frac{72a^2Bd^2n}{(a+bx)^5}-\frac{900Abcd}{(a+bx)^4}+\frac{900aAd^2}{(a+bx)^4}-\frac{135bBcdn}{(a+bx)^4}+\frac{135}{(a-}\right.}{}$$

input

```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^6,x]
```

output

```
(i^2*((-360*A*b^2*c^2)/(a + b*x)^5 + (720*a*A*b*c*d)/(a + b*x)^5 - (360*a^2*A*d^2)/(a + b*x)^5 - (72*b^2*B*c^2*n)/(a + b*x)^5 + (144*a*b*B*c*d*n)/(a + b*x)^5 - (72*a^2*B*d^2*n)/(a + b*x)^5 - (900*A*b*c*d)/(a + b*x)^4 + (900*a*A*d^2)/(a + b*x)^4 - (135*b*B*c*d*n)/(a + b*x)^4 + (135*a*B*d^2*n)/(a + b*x)^4 - (600*A*d^2)/(a + b*x)^3 - (20*B*d^2*n)/(a + b*x)^3 + (30*B*d^3*n)/((b*c - a*d)*(a + b*x)^2) - (60*B*d^4*n)/((b*c - a*d)^2*(a + b*x)) - (60*B*d^5*n*Log[a + b*x])/(b*c - a*d)^3 - (60*B*(a^2*d^2 + a*b*d*(3*c + 5*d*x) + b^2*(6*c^2 + 15*c*d*x + 10*d^2*x^2))*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x)^5 + (60*B*d^5*n*Log[c + d*x])/(b*c - a*d)^3))/(1800*b^3*g^6)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2961, 2772, 27, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^6}\,dx$$

$$\downarrow 2961$$

$$\frac{i^2\int\frac{(c+dx)^6\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^6}d\frac{a+bx}{c+dx}}{g^6(bc-ad)^3}$$

$$\downarrow 2772$$

$$\frac{i^2\left(-Bn\int-\frac{(c+dx)^6\left(6b^2-\frac{15d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{30(a+bx)^6}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{3(a+bx)^3}\right.}{g^6(bc-ad)^3}$$

$$\downarrow 27$$

$$\frac{i^2\left(\frac{1}{30}Bn\int\frac{(c+dx)^6\left(6b^2-\frac{15d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^6}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right.}{3(a+bx)^3}\right.}{g^6(bc-ad)^3}$$

$$\downarrow 1140$$

$$\frac{i^2\left(\frac{1}{30}Bn\int\left(\frac{6b^2(c+dx)^6}{(a+bx)^6}-\frac{15bd(c+dx)^5}{(a+bx)^5}+\frac{10d^2(c+dx)^4}{(a+bx)^4}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right.}{3(a+bx)^3}\right.}{g^6(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{i^2\left(-\frac{b^2(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{bd(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^4}+\frac{1}{30}Bn\left(-\frac{6b^2(}{5(a}\right.\right.}{g^6(bc-ad)^3}$$

input

```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
^6,x]
```

output

```
(i^2*((B*n*((-10*d^2*(c + d*x)^3)/(3*(a + b*x)^3) + (15*b*d*(c + d*x)^4)/(
4*(a + b*x)^4) - (6*b^2*(c + d*x)^5)/(5*(a + b*x)^5)))/30 - (d^2*(c + d*x)
^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^3) + (b*d*(c + d*x
)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^4) - (b^2*(c + d*
x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*(a + b*x)^5)))/((b*c - a*d
)^3*g^6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. $2(281) = 562$.

Time = 61.25 (sec) , antiderivative size = 1049, normalized size of antiderivative = 3.58

method	result	size
parallelrisch	Expression too large to display	1049

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^6,x,method=_
RETURNVERBOSE)
```

output

```
-1/1800*(1800*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^2*d^4*i^2*n-1800*B*x
*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*c^2*d^4*i^2*n+2400*B*x*ln(e*((b*x+a)/(d
*x+c))^n)*a*b^8*c^3*d^3*i^2*n-300*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*d^
6*i^2*n-600*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*d^6*i^2*n-300*B*x^3*a*
b^8*c*d^5*i^2*n^2-600*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^3*d^3*i^2*n-60
0*B*x^2*a^2*b^7*c*d^5*i^2*n^2-60*B*x^4*b^9*c*d^5*i^2*n^2+270*B*x^3*a^2*b^7
*d^6*i^2*n^2+30*B*x^3*b^9*c^2*d^4*i^2*n^2+470*B*x^2*a^3*b^6*d^6*i^2*n^2-20
*B*x^2*b^9*c^3*d^3*i^2*n^2+600*A*x^2*a^3*b^6*d^6*i^2*n-600*A*x^2*b^9*c^3*d
^3*i^2*n+235*B*x*a^4*b^5*d^6*i^2*n^2-135*B*x*b^9*c^4*d^2*i^2*n^2+300*A*x*a
^4*b^5*d^6*i^2*n-900*A*x*b^9*c^4*d^2*i^2*n-360*B*ln(e*((b*x+a)/(d*x+c))^n)
*b^9*c^5*d*i^2*n-1800*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*c*d^5*i^2*n+
150*B*x^2*a*b^8*c^2*d^4*i^2*n^2-1800*A*x^2*a^2*b^7*c*d^5*i^2*n+1800*A*x^2*
a*b^8*c^2*d^4*i^2*n-900*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^4*d^2*i^2*n-60
0*B*x*a^2*b^7*c^2*d^4*i^2*n^2+500*B*x*a*b^8*c^3*d^3*i^2*n^2-1800*A*x*a^2*b
^7*c^2*d^4*i^2*n+2400*A*x*a*b^8*c^3*d^3*i^2*n-600*B*ln(e*((b*x+a)/(d*x+c))
^n)*a^2*b^7*c^3*d^3*i^2*n+900*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^4*d^2*i^
2*n+47*B*a^5*b^4*d^6*i^2*n^2-72*B*b^9*c^5*d*i^2*n^2+60*A*a^5*b^4*d^6*i^2*n
-360*A*b^9*c^5*d*i^2*n-200*B*a^2*b^7*c^3*d^3*i^2*n^2+225*B*a*b^8*c^4*d^2*i
^2*n^2-600*A*a^2*b^7*c^3*d^3*i^2*n+900*A*a*b^8*c^4*d^2*i^2*n-60*B*x^5*ln(e
*((b*x+a)/(d*x+c))^n)*b^9*d^6*i^2*n+60*B*x^4*a*b^8*d^6*i^2*n^2)/g^6/(b*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1087 vs. $2(281) = 562$.

Time $= 0.13$ (sec) , antiderivative size $= 1087$, normalized size of antiderivative $= 3.71$

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^6,x,
algorithm="fricas")
```

output

```
-1/1800*(60*(B*b^5*c*d^4 - B*a*b^4*d^5)*i^2*n*x^4 - 30*(B*b^5*c^2*d^3 - 10
*B*a*b^4*c*d^4 + 9*B*a^2*b^3*d^5)*i^2*n*x^3 + (72*B*b^5*c^5 - 225*B*a*b^4*
c^4*d + 200*B*a^2*b^3*c^3*d^2 - 47*B*a^5*d^5)*i^2*n + 60*(6*A*b^5*c^5 - 15
*A*a*b^4*c^4*d + 10*A*a^2*b^3*c^3*d^2 - A*a^5*d^5)*i^2 + 10*((2*B*b^5*c^3*
d^2 - 15*B*a*b^4*c^2*d^3 + 60*B*a^2*b^3*c*d^4 - 47*B*a^3*b^2*d^5)*i^2*n +
60*(A*b^5*c^3*d^2 - 3*A*a*b^4*c^2*d^3 + 3*A*a^2*b^3*c*d^4 - A*a^3*b^2*d^5)
*i^2)*x^2 + 5*((27*B*b^5*c^4*d - 100*B*a*b^4*c^3*d^2 + 120*B*a^2*b^3*c^2*d
^3 - 47*B*a^4*b*d^5)*i^2*n + 60*(3*A*b^5*c^4*d - 8*A*a*b^4*c^3*d^2 + 6*A*a
^2*b^3*c^2*d^3 - A*a^4*b*d^5)*i^2)*x + 60*(10*(B*b^5*c^3*d^2 - 3*B*a*b^4*c
^2*d^3 + 3*B*a^2*b^3*c*d^4 - B*a^3*b^2*d^5)*i^2*x^2 + 5*(3*B*b^5*c^4*d - 8
*B*a*b^4*c^3*d^2 + 6*B*a^2*b^3*c^2*d^3 - B*a^4*b*d^5)*i^2*x + (6*B*b^5*c^5
 - 15*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - B*a^5*d^5)*i^2)*log(e) + 60*(
B*b^5*d^5*i^2*n*x^5 + 5*B*a*b^4*d^5*i^2*n*x^4 + 10*B*a^2*b^3*d^5*i^2*n*x^3
 + 10*(B*b^5*c^3*d^2 - 3*B*a*b^4*c^2*d^3 + 3*B*a^2*b^3*c*d^4)*i^2*n*x^2 +
5*(3*B*b^5*c^4*d - 8*B*a*b^4*c^3*d^2 + 6*B*a^2*b^3*c^2*d^3)*i^2*n*x + (6*B
*b^5*c^5 - 15*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2)*i^2*n)*log((b*x + a)/(
d*x + c)))/((b^11*c^3 - 3*a*b^10*c^2*d + 3*a^2*b^9*c*d^2 - a^3*b^8*d^3)*g^
6*x^5 + 5*(a*b^10*c^3 - 3*a^2*b^9*c^2*d + 3*a^3*b^8*c*d^2 - a^4*b^7*d^3)*g
^6*x^4 + 10*(a^2*b^9*c^3 - 3*a^3*b^8*c^2*d + 3*a^4*b^7*c*d^2 - a^5*b^6*d^3
)*g^6*x^3 + 10*(a^3*b^8*c^3 - 3*a^4*b^7*c^2*d + 3*a^5*b^6*c*d^2 - a^6*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx = \text{Timed out}$$

input

```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**6,x
)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3058 vs. $2(281) = 562$.

Time $= 0.18$ (sec) , antiderivative size $= 3058$, normalized size of antiderivative $= 10.44$

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^6,x,
algorithm="maxima")
```

output

```
-1/300*B*c^2*i^2*n*((60*b^4*d^4*x^4 + 12*b^4*c^4 - 63*a*b^3*c^3*d + 137*a^
2*b^2*c^2*d^2 - 163*a^3*b*c*d^3 + 137*a^4*d^4 - 30*(b^4*c*d^3 - 9*a*b^3*d^
4)*x^3 + 10*(2*b^4*c^2*d^2 - 13*a*b^3*c*d^3 + 47*a^2*b^2*d^4)*x^2 - 5*(3*b
^4*c^3*d - 17*a*b^3*c^2*d^2 + 43*a^2*b^2*c*d^3 - 77*a^3*b*d^4)*x)/((b^10*c
^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*g^
6*x^5 + 5*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d
^3 + a^5*b^5*d^4)*g^6*x^4 + 10*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*
c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*g^6*x^3 + 10*(a^3*b^7*c^4 - 4*a^4
*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*g^6*x^2 +
5*(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a
^8*b^2*d^4)*g^6*x + (a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4
*a^8*b^2*c*d^3 + a^9*b*d^4)*g^6) + 60*d^5*log(b*x + a)/((b^6*c^5 - 5*a*b^5
*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b
*d^5)*g^6) - 60*d^5*log(d*x + c)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^
3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6)) - 1/1800*B
*d^2*i^2*n*((47*a^2*b^4*c^4 - 278*a^3*b^3*c^3*d + 822*a^4*b^2*c^2*d^2 - 27
8*a^5*b*c*d^3 + 47*a^6*d^4 + 60*(10*b^6*c^2*d^2 - 5*a*b^5*c*d^3 + a^2*b^4*
d^4)*x^4 - 30*(10*b^6*c^3*d - 95*a*b^5*c^2*d^2 + 46*a^2*b^4*c*d^3 - 9*a^3*
b^3*d^4)*x^3 + 10*(20*b^6*c^4 - 140*a*b^5*c^3*d + 537*a^2*b^4*c^2*d^2 - 24
8*a^3*b^3*c*d^3 + 47*a^4*b^2*d^4)*x^2 + 5*(35*a*b^5*c^4 - 218*a^2*b^4*c...
```

Giac [A] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.43

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx =$$

$$-\frac{1}{1800}\left(\frac{60\left(6\,Bb^2i^2n-\frac{15\,(bx+a)Bbdi^2n}{dx+c}+\frac{10\,(bx+a)^2Bd^2i^2n}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^5b^2c^2g^6}{(dx+c)^5}-\frac{2\,(bx+a)^5abcdg^6}{(dx+c)^5}+\frac{(bx+a)^5a^2d^2g^6}{(dx+c)^5}}+\frac{72\,Bb^2i^2n-\frac{225\,(bx+a)Bbdi^2n}{dx+c}+\frac{200}{}\right.$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^6,x,
algorithm="giac")
```

output
```
-1/1800*(60*(6*B*b^2*i^2*n - 15*(b*x + a)*B*b*d*i^2*n/(d*x + c) + 10*(b*x
+ a)^2*B*d^2*i^2*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^5*b^2*
c^2*g^6/(d*x + c)^5 - 2*(b*x + a)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*x + a)^5*
a^2*d^2*g^6/(d*x + c)^5) + (72*B*b^2*i^2*n - 225*(b*x + a)*B*b*d*i^2*n/(d*
x + c) + 200*(b*x + a)^2*B*d^2*i^2*n/(d*x + c)^2 + 360*B*b^2*i^2*log(e) -
900*(b*x + a)*B*b*d*i^2*log(e)/(d*x + c) + 600*(b*x + a)^2*B*d^2*i^2*log(e
)/(d*x + c)^2 + 360*A*b^2*i^2 - 900*(b*x + a)*A*b*d*i^2/(d*x + c) + 600*(b
*x + a)^2*A*d^2*i^2/(d*x + c)^2)/((b*x + a)^5*b^2*c^2*g^6/(d*x + c)^5 - 2*
(b*x + a)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*x + a)^5*a^2*d^2*g^6/(d*x + c)^5)
)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 27.94 (sec) , antiderivative size = 954, normalized size of antiderivative = 3.26

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx$$

$$=\frac{B\,d^5\,i^2\,n\,\operatorname{atanh}\left(\frac{30\,a^3\,b^3\,d^3\,g^6-30\,a^2\,b^4\,c\,d^2\,g^6-30\,a\,b^5\,c^2\,d\,g^6+30\,b^6\,c^3\,g^6}{30\,b^3\,g^6\,(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{15\,b^3\,g^6\,(a\,d-b\,c)^3}$$

$$-\frac{\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(a\left(\frac{B\,a\,d^2\,i^2}{30\,b^3}+\frac{B\,c\,d\,i^2}{10\,b^2}\right)+x\left(b\left(\frac{B\,a\,d^2\,i^2}{30\,b^3}+\frac{B\,c\,d\,i^2}{10\,b^2}\right)+\frac{2\,B\,a\,d^2\,i^2}{15\,b^2}+\frac{2\,B\,c\,d\,i^2}{5\,b}\right)+\frac{B\,c^2\,i^2}{5\,b}+\frac{B\,d^2\,i^2}{3\,b}\right.}{a^5\,g^6+5\,a^4\,b\,g^6\,x+10\,a^3\,b^2\,g^6\,x^2+10\,a^2\,b^3\,g^6\,x^3+5\,a\,b^4\,g^6\,x^4+b^5\,g^6\,x^5}$$

$$-\frac{60\,A\,a^4\,d^4\,i^2+360\,A\,b^4\,c^4\,i^2+47\,B\,a^4\,d^4\,i^2\,n+72\,B\,b^4\,c^4\,i^2\,n+60\,A\,a^2\,b^2\,c^2\,d^2\,i^2-540\,A\,a\,b^3\,c^3\,d\,i^2+60\,A\,a^3\,b\,c\,d^3\,i^2-153\,B\,a\,b^3\,c^3\,d\,i^2\,n+47}{60\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^6,x)
```

output
```
(B*d^5*i^2*n*atanh((30*b^6*c^3*g^6 + 30*a^3*b^3*d^3*g^6 - 30*a*b^5*c^2*d*g
^6 - 30*a^2*b^4*c*d^2*g^6)/(30*b^3*g^6*(a*d - b*c)^3) + (2*b*d*x*(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(15*b^3*g^6*(a*d - b*c)^3) - (log(
e*((a + b*x)/(c + d*x))^n)*(a*((B*a*d^2*i^2)/(30*b^3) + (B*c*d*i^2)/(10*b^
2)) + x*(b*((B*a*d^2*i^2)/(30*b^3) + (B*c*d*i^2)/(10*b^2)) + (2*B*a*d^2*i^
2)/(15*b^2) + (2*B*c*d*i^2)/(5*b)) + (B*c^2*i^2)/(5*b) + (B*d^2*i^2*x^2)/(
3*b)))/(a^5*g^6 + b^5*g^6*x^5 + 5*a*b^4*g^6*x^4 + 10*a^3*b^2*g^6*x^2 + 10*
a^2*b^3*g^6*x^3 + 5*a^4*b*g^6*x) - ((60*A*a^4*d^4*i^2 + 360*A*b^4*c^4*i^2
+ 47*B*a^4*d^4*i^2*n + 72*B*b^4*c^4*i^2*n + 60*A*a^2*b^2*c^2*d^2*i^2 - 540
*A*a*b^3*c^3*d*i^2 + 60*A*a^3*b*c*d^3*i^2 - 153*B*a*b^3*c^3*d*i^2*n + 47*B
*a^3*b*c*d^3*i^2*n + 47*B*a^2*b^2*c^2*d^2*i^2*n)/(60*(a^2*d^2 + b^2*c^2 -
2*a*b*c*d)) + (x^2*(60*A*a^2*b^2*d^4*i^2 + 60*A*b^4*c^2*d^2*i^2 + 47*B*a^2
*b^2*d^4*i^2*n + 2*B*b^4*c^2*d^2*i^2*n - 120*A*a*b^3*c*d^3*i^2 - 13*B*a*b^
3*c*d^3*i^2*n))/(6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(60*A*a^3*b*d^4*i
^2 + 180*A*b^4*c^3*d*i^2 - 300*A*a*b^3*c^2*d^2*i^2 + 60*A*a^2*b^2*c*d^3*i^
2 + 47*B*a^3*b*d^4*i^2*n + 27*B*b^4*c^3*d*i^2*n - 73*B*a*b^3*c^2*d^2*i^2*n
 + 47*B*a^2*b^2*c*d^3*i^2*n))/(12*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d*x^
3*(9*B*a*b^3*d^3*i^2*n - B*b^4*c*d^2*i^2*n))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b
*c*d)) + (B*b^4*d^4*i^2*n*x^4)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(30*a^5*b^
3*g^6 + 30*b^8*g^6*x^5 + 150*a^4*b^4*g^6*x + 150*a*b^7*g^6*x^4 + 300*a^...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1378, normalized size of antiderivative = 4.70

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^6,x)
```

output

```
( - 60*log(a + b*x)*a**6*b*d**5*n - 300*log(a + b*x)*a**5*b**2*d**5*n*x -
600*log(a + b*x)*a**4*b**3*d**5*n*x**2 - 600*log(a + b*x)*a**3*b**4*d**5*n
*x**3 - 300*log(a + b*x)*a**2*b**5*d**5*n*x**4 - 60*log(a + b*x)*a*b**6*d*
*5*n*x**5 + 60*log(c + d*x)*a**6*b*d**5*n + 300*log(c + d*x)*a**5*b**2*d**
5*n*x + 600*log(c + d*x)*a**4*b**3*d**5*n*x**2 + 600*log(c + d*x)*a**3*b**
4*d**5*n*x**3 + 300*log(c + d*x)*a**2*b**5*d**5*n*x**4 + 60*log(c + d*x)*a
*b**6*d**5*n*x**5 + 60*log(((a + b*x)**n*e)/(c + d*x)**n)*a**6*b*d**5 + 30
0*log(((a + b*x)**n*e)/(c + d*x)**n)*a**5*b**2*d**5*x + 600*log(((a + b*x)
**n*e)/(c + d*x)**n)*a**4*b**3*d**5*x**2 - 600*log(((a + b*x)**n*e)/(c + d
*x)**n)*a**3*b**4*c**3*d**2 - 1800*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3
*b**4*c**2*d**3*x - 1800*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**4*c*d*
*4*x**2 + 900*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**5*c**4*d + 2400*l
og(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**5*c**3*d**2*x + 1800*log(((a + b
*x)**n*e)/(c + d*x)**n)*a**2*b**5*c**2*d**3*x**2 - 360*log(((a + b*x)**n*e
)/(c + d*x)**n)*a*b**6*c**5 - 900*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**
6*c**4*d*x - 600*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**6*c**3*d**2*x**2
+ 60*a**7*d**5 + 35*a**6*b*d**5*n + 300*a**6*b*d**5*x + 12*a**5*b**2*c*d**
4*n + 175*a**5*b**2*d**5*n*x + 600*a**5*b**2*d**5*x**2 - 600*a**4*b**3*c**
3*d**2 - 1800*a**4*b**3*c**2*d**3*x + 60*a**4*b**3*c*d**4*n*x - 1800*a**4*
b**3*c*d**4*x**2 + 350*a**4*b**3*d**5*n*x**2 + 900*a**3*b**4*c**4*d - 2...
```

3.127 $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result 1499
Mathematica [A] (verified) 1500
Rubi [A] (verified) 1501
Maple [B] (verified) 1503
Fricas [B] (verification not implemented) 1504
Sympy [F(-1)] 1505
Maxima [B] (verification not implemented) 1506
Giac [B] (verification not implemented) 1507
Mupad [B] (verification not implemented) 1508
Reduce [B] (verification not implemented) 1509

Optimal result

Integrand size = 43, antiderivative size = 477

$$
\begin{aligned}
&\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx \\
&= \frac{B(bc-ad)^6g^3i^3nx}{140b^3d^3}+\frac{B(bc-ad)^5g^3i^3n(c+dx)^2}{280b^2d^4}+\frac{B(bc-ad)^4g^3i^3n(c+dx)^3}{420bd^4} \\
&\quad -\frac{17B(bc-ad)^3g^3i^3n(c+dx)^4}{280d^4}+\frac{bB(bc-ad)^2g^3i^3n(c+dx)^5}{14d^4} \\
&\quad -\frac{b^2B(bc-ad)g^3i^3n(c+dx)^6}{42d^4}-\frac{(bc-ad)^3g^3i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^4} \\
&\quad +\frac{3b(bc-ad)^2g^3i^3(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^4} \\
&\quad -\frac{b^2(bc-ad)g^3i^3(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^4} \\
&\quad +\frac{b^3g^3i^3(c+dx)^7\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{7d^4} \\
&\quad +\frac{B(bc-ad)^7g^3i^3n\log\left(\frac{a+bx}{c+dx}\right)}{140b^4d^4}+\frac{B(bc-ad)^7g^3i^3n\log(c+dx)}{140b^4d^4}
\end{aligned}
$$

output

```
1/140*B*(-a*d+b*c)^6*g^3*i^3*n*x/b^3/d^3+1/280*B*(-a*d+b*c)^5*g^3*i^3*n*(d
*x+c)^2/b^2/d^4+1/420*B*(-a*d+b*c)^4*g^3*i^3*n*(d*x+c)^3/b/d^4-17/280*B*(-
a*d+b*c)^3*g^3*i^3*n*(d*x+c)^4/d^4+1/14*b*B*(-a*d+b*c)^2*g^3*i^3*n*(d*x+c)
^5/d^4-1/42*b^2*B*(-a*d+b*c)*g^3*i^3*n*(d*x+c)^6/d^4-1/4*(-a*d+b*c)^3*g^3*
i^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4+3/5*b*(-a*d+b*c)^2*g^3*i
^3*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4-1/2*b^2*(-a*d+b*c)*g^3*i^
3*(d*x+c)^6*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4+1/7*b^3*g^3*i^3*(d*x+c)^7*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4+1/140*B*(-a*d+b*c)^7*g^3*i^3*n*ln((b*x
+a)/(d*x+c))/b^4/d^4+1/140*B*(-a*d+b*c)^7*g^3*i^3*n*ln(d*x+c)/b^4/d^4
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.32

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{g^3i^3\left(210d^4(bc-ad)^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+504d^5(bc-ad)^2(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{}\right.\right.\right.}{}$$

input

```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]),x]
```

output

```
(g^3*i^3*(210*d^4*(b*c - a*d)^3*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]) + 504*d^5*(b*c - a*d)^2*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]) + 420*d^6*(b*c - a*d)*(a + b*x)^6*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]) + 120*d^7*(a + b*x)^7*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 35
*B*(b*c - a*d)^4*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)
^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 42*B*(b*c - a*d)^
3*n*(12*b*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c
 - a*d)*(a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]) -
 7*B*(b*c - a*d)^2*n*(60*b*d*(b*c - a*d)^4*x + 30*d^2*(-(b*c) + a*d)^3*(a
+ b*x)^2 + 20*d^3*(b*c - a*d)^2*(a + b*x)^3 + 15*d^4*(-(b*c) + a*d)*(a + b
*x)^4 + 12*d^5*(a + b*x)^5 - 60*(b*c - a*d)^5*Log[c + d*x]) + 2*B*(b*c - a
*d)*n*(60*b*d*(b*c - a*d)^5*x - 30*d^2*(b*c - a*d)^4*(a + b*x)^2 + 20*d^3*
(b*c - a*d)^3*(a + b*x)^3 - 15*d^4*(b*c - a*d)^2*(a + b*x)^4 + 12*d^5*(b*c
 - a*d)*(a + b*x)^5 - 10*d^6*(a + b*x)^6 - 60*(b*c - a*d)^6*Log[c + d*x]))
)/(840*b^4*d^4)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2961, 2782, 27, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2961}$$

$$g^3i^3(bc-ad)^7\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^8}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2782}$$

$$g^3i^3(bc-ad)^7\left(-Bn\int -\frac{(c+dx)\left(b^3-\frac{7d(a+bx)b^2}{c+dx}+\frac{21d^2(a+bx)^2b}{(c+dx)^2}-\frac{35d^3(a+bx)^3}{(c+dx)^3}\right)}{140d^4(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}+\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{7d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}\right.$$

$$\Big\downarrow \text{27}$$

$$g^3i^3(bc-ad)^7\left(\frac{Bn\int \frac{(c+dx)\left(b^3-\frac{7d(a+bx)b^2}{c+dx}+\frac{21d^2(a+bx)^2b}{(c+dx)^2}-\frac{35d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}}{140d^4}+\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{7d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{b^2\left(B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{2d^4\left(b-\frac{d(}{}\right.}\right.$$

$$\Big\downarrow \text{2123}$$

$$g^3i^3(bc-ad)^7\left(\frac{Bn\int\left(-\frac{20db^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^7}+\frac{50db}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{34d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4b}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3b^2}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2b^3}+\frac{}{(a}\right.}{140d^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$g^3 i^3 (bc -$$

$$ad)^7 \left(\frac{b^3 \left(B \log\left(e \left(\frac{a+bx}{c+dx} \right)^n \right) + A \right)}{7d^4 \left(b - \frac{d(a+bx)}{c+dx} \right)^7} - \frac{b^2 \left(B \log\left(e \left(\frac{a+bx}{c+dx} \right)^n \right) + A \right)}{2d^4 \left(b - \frac{d(a+bx)}{c+dx} \right)^6} + \frac{3b \left(B \log\left(e \left(\frac{a+bx}{c+dx} \right)^n \right) + A \right)}{5d^4 \left(b - \frac{d(a+bx)}{c+dx} \right)^5} - \frac{B \log\left(e \left(\frac{a}{c-} \right.\right.}{4d^4 \left(b - \right.}$$

input

```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
,x]
```

output

```
(b*c - a*d)^7*g^3*i^3*((b^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(7*d^4
*(b - (d*(a + b*x))/(c + d*x))^7) - (b^2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^6) + (3*b*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(5*d^4*(b - (d*(a + b*x))/(c + d*x))^5) - (A + B*Log[e
*((a + b*x)/(c + d*x))^n])/(4*d^4*(b - (d*(a + b*x))/(c + d*x))^4) + (B*n*
((-10*b^2)/(3*(b - (d*(a + b*x))/(c + d*x))^6) + (10*b)/(b - (d*(a + b*x))
/(c + d*x))^5 - 17/(2*(b - (d*(a + b*x))/(c + d*x))^4) + 1/(3*b*(b - (d*(a
 + b*x))/(c + d*x))^3) + 1/(2*b^2*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^
3*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^4 - Log[b -
(d*(a + b*x))/(c + d*x)]/b^4))/(140*d^4))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2782

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2178 vs. $2(453) = 906$.

Time = 54.63 (sec) , antiderivative size = 2179, normalized size of antiderivative = 4.57

method	result	size
parallelrisch	Expression too large to display	2179

input

```
int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_
RETURNVERBOSE)
```

output

```
1/840*(120*B*x^7*ln(e*((b*x+a)/(d*x+c))^n)*b^7*d^7*g^3*i^3*n-6*B*a^7*d^7*g
^3*i^3*n^2+6*B*b^7*c^7*g^3*i^3*n^2+840*A*x*a^3*b^4*c^3*d^4*g^3*i^3*n+210*B
*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^4*c^4*d^3*g^3*i^3*n-126*B*ln(e*((b*x+a)/(
d*x+c))^n)*a^2*b^5*c^5*d^2*g^3*i^3*n+1260*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*
a^2*b^5*c^3*d^4*g^3*i^3*n+840*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^4*c^3*d^
4*g^3*i^3*n+1512*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c*d^6*g^3*i^3*n+189
0*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*c*d^6*g^3*i^3*n+1890*B*x^4*ln(e*
((b*x+a)/(d*x+c))^n)*a*b^6*c^2*d^5*g^3*i^3*n+840*B*x^3*ln(e*((b*x+a)/(d*x+
c))^n)*a^3*b^4*c*d^6*g^3*i^3*n+2520*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^
5*c^2*d^5*g^3*i^3*n+840*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c^3*d^4*g^3*
i^3*n+1260*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^4*c^2*d^5*g^3*i^3*n+21*B*
x^2*a^4*b^3*c*d^6*g^3*i^3*n^2-210*B*ln(b*x+a)*a^3*b^4*c^4*d^3*g^3*i^3*n^2+
126*B*ln(b*x+a)*a^2*b^5*c^5*d^2*g^3*i^3*n^2-42*B*ln(b*x+a)*a*b^6*c^6*d*g^3
*i^3*n^2+420*B*x^6*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*d^7*g^3*i^3*n-42*B*x*a^
5*b^2*c*d^6*g^3*i^3*n^2+126*B*x*a^4*b^3*c^2*d^5*g^3*i^3*n^2-126*B*x*a^2*b^
5*c^4*d^3*g^3*i^3*n^2+42*B*x*a*b^6*c^5*d^2*g^3*i^3*n^2+252*B*x^2*a^3*b^4*c
^2*d^5*g^3*i^3*n^2-252*B*x^2*a^2*b^5*c^3*d^4*g^3*i^3*n^2-21*B*x^2*a*b^6*c^
4*d^3*g^3*i^3*n^2+1260*A*x^2*a^3*b^4*c^2*d^5*g^3*i^3*n+42*B*ln(e*((b*x+a)/
(d*x+c))^n)*a*b^6*c^6*d*g^3*i^3*n+42*B*ln(b*x+a)*a^6*b*c*d^6*g^3*i^3*n^2-1
26*B*ln(b*x+a)*a^5*b^2*c^2*d^5*g^3*i^3*n^2+210*B*ln(b*x+a)*a^4*b^3*c^3*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1336 vs. $2(453) = 906$.

Time $= 0.59$ (sec) , antiderivative size $= 1336$, normalized size of antiderivative $= 2.80$

$$\int (ag + bgx)^3 (ci + dix)^3 \left(A + B \log \left(e \left(\frac{a + bx}{c + dx} \right)^n \right) \right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="fricas")
```

output

```
1/840*(120*A*b^7*d^7*g^3*i^3*x^7 + 6*(35*B*a^4*b^3*c^3*d^4 - 21*B*a^5*b^2*
c^2*d^5 + 7*B*a^6*b*c*d^6 - B*a^7*d^7)*g^3*i^3*n*log(b*x + a) + 6*(B*b^7*c
^7 - 7*B*a*b^6*c^6*d + 21*B*a^2*b^5*c^5*d^2 - 35*B*a^3*b^4*c^4*d^3)*g^3*i^
3*n*log(d*x + c) - 20*((B*b^7*c*d^6 - B*a*b^6*d^7)*g^3*i^3*n - 21*(A*b^7*c
*d^6 + A*a*b^6*d^7)*g^3*i^3)*x^6 - 12*(5*(B*b^7*c^2*d^5 - B*a^2*b^5*d^7)*g
^3*i^3*n - 42*(A*b^7*c^2*d^5 + 3*A*a*b^6*c*d^6 + A*a^2*b^5*d^7)*g^3*i^3)*x
^5 - 3*((17*B*b^7*c^3*d^4 + 49*B*a*b^6*c^2*d^5 - 49*B*a^2*b^5*c*d^6 - 17*B
*a^3*b^4*d^7)*g^3*i^3*n - 70*(A*b^7*c^3*d^4 + 9*A*a*b^6*c^2*d^5 + 9*A*a^2*
b^5*c*d^6 + A*a^3*b^4*d^7)*g^3*i^3)*x^4 - 2*((B*b^7*c^4*d^3 + 98*B*a*b^6*c
^3*d^4 - 98*B*a^3*b^4*c*d^6 - B*a^4*b^3*d^7)*g^3*i^3*n - 420*(A*a*b^6*c^3*
d^4 + 3*A*a^2*b^5*c^2*d^5 + A*a^3*b^4*c*d^6)*g^3*i^3)*x^3 + 3*((B*b^7*c^5*
d^2 - 7*B*a*b^6*c^4*d^3 - 84*B*a^2*b^5*c^3*d^4 + 84*B*a^3*b^4*c^2*d^5 + 7*
B*a^4*b^3*c*d^6 - B*a^5*b^2*d^7)*g^3*i^3*n + 420*(A*a^2*b^5*c^3*d^4 + A*a^
3*b^4*c^2*d^5)*g^3*i^3)*x^2 + 6*(140*A*a^3*b^4*c^3*d^4*g^3*i^3 - (B*b^7*c^
6*d - 7*B*a*b^6*c^5*d^2 + 21*B*a^2*b^5*c^4*d^3 - 21*B*a^4*b^3*c^2*d^5 + 7*
B*a^5*b^2*c*d^6 - B*a^6*b*d^7)*g^3*i^3*n)*x + 6*(20*B*b^7*d^7*g^3*i^3*x^7
+ 140*B*a^3*b^4*c^3*d^4*g^3*i^3*x + 70*(B*b^7*c*d^6 + B*a*b^6*d^7)*g^3*i^3
*x^6 + 84*(B*b^7*c^2*d^5 + 3*B*a*b^6*c*d^6 + B*a^2*b^5*d^7)*g^3*i^3*x^5 +
35*(B*b^7*c^3*d^4 + 9*B*a*b^6*c^2*d^5 + 9*B*a^2*b^5*c*d^6 + B*a^3*b^4*d^7)
*g^3*i^3*x^4 + 140*(B*a*b^6*c^3*d^4 + 3*B*a^2*b^5*c^2*d^5 + B*a^3*b^4*c...
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

input

```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x
)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2901 vs. $2(453) = 906$.

Time $= 0.13$ (sec) , antiderivative size $= 2901$, normalized size of antiderivative $= 6.08$

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="maxima")
```

output

```
1/7*B*b^3*d^3*g^3*i^3*x^7*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/7*A*b
^3*d^3*g^3*i^3*x^7 + 1/2*B*b^3*c*d^2*g^3*i^3*x^6*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n) + 1/2*B*a*b^2*d^3*g^3*i^3*x^6*log(e*(b*x/(d*x + c) + a/(d*x
+ c))^n) + 1/2*A*b^3*c*d^2*g^3*i^3*x^6 + 1/2*A*a*b^2*d^3*g^3*i^3*x^6 + 3/5
*B*b^3*c^2*d*g^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 9/5*B*a*
b^2*c*d^2*g^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/5*B*a^2*b
*d^3*g^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/5*A*b^3*c^2*d*
g^3*i^3*x^5 + 9/5*A*a*b^2*c*d^2*g^3*i^3*x^5 + 3/5*A*a^2*b*d^3*g^3*i^3*x^5
+ 1/4*B*b^3*c^3*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 9/4*B
*a*b^2*c^2*d*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 9/4*B*a^
2*b*c*d^2*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*B*a^3*d
^3*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b^3*c^3*g^3*
i^3*x^4 + 9/4*A*a*b^2*c^2*d*g^3*i^3*x^4 + 9/4*A*a^2*b*c*d^2*g^3*i^3*x^4 +
1/4*A*a^3*d^3*g^3*i^3*x^4 + B*a*b^2*c^3*g^3*i^3*x^3*log(e*(b*x/(d*x + c) +
 a/(d*x + c))^n) + 3*B*a^2*b*c^2*d*g^3*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n) + B*a^3*c*d^2*g^3*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n) + A*a*b^2*c^3*g^3*i^3*x^3 + 3*A*a^2*b*c^2*d*g^3*i^3*x^3 + A*a^3*c*d^2*g
^3*i^3*x^3 + 3/2*B*a^2*b*c^3*g^3*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n) + 3/2*B*a^3*c^2*d*g^3*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
 + 3/2*A*a^2*b*c^3*g^3*i^3*x^2 + 3/2*A*a^3*c^2*d*g^3*i^3*x^2 + 1/420*B*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5934 vs. $2(453) = 906$.

Time = 2.35 (sec) , antiderivative size = 5934, normalized size of antiderivative = 12.44

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="giac")
```

output
```
-1/840*(6*(B*b^11*c^8*g^3*i^3*n - 8*B*a*b^10*c^7*d*g^3*i^3*n - 7*(b*x + a)
*B*b^10*c^8*d*g^3*i^3*n/(d*x + c) + 28*B*a^2*b^9*c^6*d^2*g^3*i^3*n + 56*(b
*x + a)*B*a*b^9*c^7*d^2*g^3*i^3*n/(d*x + c) + 21*(b*x + a)^2*B*b^9*c^8*d^2
*g^3*i^3*n/(d*x + c)^2 - 56*B*a^3*b^8*c^5*d^3*g^3*i^3*n - 196*(b*x + a)*B*
a^2*b^8*c^6*d^3*g^3*i^3*n/(d*x + c) - 168*(b*x + a)^2*B*a*b^8*c^7*d^3*g^3*
i^3*n/(d*x + c)^2 - 35*(b*x + a)^3*B*b^8*c^8*d^3*g^3*i^3*n/(d*x + c)^3 + 7
0*B*a^4*b^7*c^4*d^4*g^3*i^3*n + 392*(b*x + a)*B*a^3*b^7*c^5*d^4*g^3*i^3*n/
(d*x + c) + 588*(b*x + a)^2*B*a^2*b^7*c^6*d^4*g^3*i^3*n/(d*x + c)^2 + 280*
(b*x + a)^3*B*a*b^7*c^7*d^4*g^3*i^3*n/(d*x + c)^3 - 56*B*a^5*b^6*c^3*d^5*g
^3*i^3*n - 490*(b*x + a)*B*a^4*b^6*c^4*d^5*g^3*i^3*n/(d*x + c) - 1176*(b*x
 + a)^2*B*a^3*b^6*c^5*d^5*g^3*i^3*n/(d*x + c)^2 - 980*(b*x + a)^3*B*a^2*b^
6*c^6*d^5*g^3*i^3*n/(d*x + c)^3 + 28*B*a^6*b^5*c^2*d^6*g^3*i^3*n + 392*(b*
x + a)*B*a^5*b^5*c^3*d^6*g^3*i^3*n/(d*x + c) + 1470*(b*x + a)^2*B*a^4*b^5*
c^4*d^6*g^3*i^3*n/(d*x + c)^2 + 1960*(b*x + a)^3*B*a^3*b^5*c^5*d^6*g^3*i^3
*n/(d*x + c)^3 - 8*B*a^7*b^4*c*d^7*g^3*i^3*n - 196*(b*x + a)*B*a^6*b^4*c^2
*d^7*g^3*i^3*n/(d*x + c) - 1176*(b*x + a)^2*B*a^5*b^4*c^3*d^7*g^3*i^3*n/(d
*x + c)^2 - 2450*(b*x + a)^3*B*a^4*b^4*c^4*d^7*g^3*i^3*n/(d*x + c)^3 + B*a
^8*b^3*d^8*g^3*i^3*n + 56*(b*x + a)*B*a^7*b^3*c*d^8*g^3*i^3*n/(d*x + c) +
588*(b*x + a)^2*B*a^6*b^3*c^2*d^8*g^3*i^3*n/(d*x + c)^2 + 1960*(b*x + a)^3
*B*a^5*b^3*c^3*d^8*g^3*i^3*n/(d*x + c)^3 - 7*(b*x + a)*B*a^8*b^2*d^9*g^...
```

Mupad [B] (verification not implemented)

Time = 28.09 (sec) , antiderivative size = 4476, normalized size of antiderivative = 9.38

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input

```
int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))
,x)
```

output

```
x^4*((g^3*i^3*(20*A*a^3*d^3 + 20*A*b^3*c^3 + 3*B*a^3*d^3*n - 3*B*b^3*c^3*n
 + 120*A*a*b^2*c^2*d + 120*A*a^2*b*c*d^2 - 6*B*a*b^2*c^2*d*n + 6*B*a^2*b*c
*d^2*n))/20 + ((140*a*d + 140*b*c)*((((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*
c + B*a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140)*(
140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2
+ B*a^2*d^2*n - B*b^2*c^2*n + 32*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^3*i^3))/(
560*b*d) - (a*c*((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d*n - B*b*c*n
))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140))/(4*b*d)) + x^3*((g^3*
i^3*(4*A*a^4*d^4 + 4*A*b^4*c^4 + B*a^4*d^4*n - B*b^4*c^4*n + 144*A*a^2*b^2
*c^2*d^2 + 64*A*a*b^3*c^3*d + 64*A*a^3*b*c*d^3 - 8*B*a*b^3*c^3*d*n + 8*B*a
^3*b*c*d^3*n))/(12*b*d) - ((140*a*d + 140*b*c)*((g^3*i^3*(20*A*a^3*d^3 + 2
0*A*b^3*c^3 + 3*B*a^3*d^3*n - 3*B*b^3*c^3*n + 120*A*a*b^2*c^2*d + 120*A*a^
2*b*c*d^2 - 6*B*a*b^2*c^2*d*n + 6*B*a^2*b*c*d^2*n))/5 + ((140*a*d + 140*b*
c)*((((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d*n - B*b*c*n))/7 - (A*b
^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) -
(b*d*g^3*i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 32
*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^3*i^3))/(140*b*d) - (a*c*((b^2*d^2*g^3*i^
3*(28*A*a*d + 28*A*b*c + B*a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(140*a
*d + 140*b*c))/140))/(b*d)))/(420*b*d) + (a*c*((((b^2*d^2*g^3*i^3*(28*A*a*
d + 28*A*b*c + B*a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 14...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1419, normalized size of antiderivative = 2.97

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \text{Too large to display}$$

input

```
int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output

```
(g**3*i*(6*log(a + b*x)*a**7*d**7*n - 42*log(a + b*x)*a**6*b*c*d**6*n + 12
6*log(a + b*x)*a**5*b**2*c**2*d**5*n - 210*log(a + b*x)*a**4*b**3*c**3*d**
4*n + 210*log(a + b*x)*a**3*b**4*c**4*d**3*n - 126*log(a + b*x)*a**2*b**5*
c**5*d**2*n + 42*log(a + b*x)*a*b**6*c**6*d*n - 6*log(a + b*x)*b**7*c**7*n
 - 210*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**4*c**4*d**3 - 840*log(((
a + b*x)**n*e)/(c + d*x)**n)*a**3*b**4*c**3*d**4*x - 1260*log(((a + b*x)**
n*e)/(c + d*x)**n)*a**3*b**4*c**2*d**5*x**2 - 840*log(((a + b*x)**n*e)/(c
+ d*x)**n)*a**3*b**4*c*d**6*x**3 - 210*log(((a + b*x)**n*e)/(c + d*x)**n)*
a**3*b**4*d**7*x**4 + 126*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**5*c**
5*d**2 - 1260*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**5*c**3*d**4*x**2
- 2520*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**5*c**2*d**5*x**3 - 1890*
log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**5*c*d**6*x**4 - 504*log(((a + b
*x)**n*e)/(c + d*x)**n)*a**2*b**5*d**7*x**5 - 42*log(((a + b*x)**n*e)/(c +
 d*x)**n)*a*b**6*c**6*d - 840*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**6*c*
*3*d**4*x**3 - 1890*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**6*c**2*d**5*x*
*4 - 1512*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**6*c*d**6*x**5 - 420*log(
((a + b*x)**n*e)/(c + d*x)**n)*a*b**6*d**7*x**6 + 6*log(((a + b*x)**n*e)/(
c + d*x)**n)*b**7*c**7 - 210*log(((a + b*x)**n*e)/(c + d*x)**n)*b**7*c**3*
d**4*x**4 - 504*log(((a + b*x)**n*e)/(c + d*x)**n)*b**7*c**2*d**5*x**5 - 4
20*log(((a + b*x)**n*e)/(c + d*x)**n)*b**7*c*d**6*x**6 - 120*log(((a + ...
```

3.128 $\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result . . . 1510
Mathematica [A] (verified) . . . 1511
Rubi [A] (verified) . . . 1512
Maple [B] (verified) . . . 1514
Fricas [B] (verification not implemented) . . . 1515
Sympy [F(-1)] . . . 1516
Maxima [B] (verification not implemented) . . . 1517
Giac [B] (verification not implemented) . . . 1518
Mupad [B] (verification not implemented) . . . 1519
Reduce [B] (verification not implemented) . . . 1520

Optimal result

Integrand size = 43, antiderivative size = 387

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx \\
&= -\frac{B(bc-ad)^5g^2i^3nx}{60b^3d^2} - \frac{B(bc-ad)^4g^2i^3n(c+dx)^2}{120b^2d^3} \\
&\quad - \frac{B(bc-ad)^3g^2i^3n(c+dx)^3}{180bd^3} + \frac{7B(bc-ad)^2g^2i^3n(c+dx)^4}{120d^3} \\
&\quad - \frac{bB(bc-ad)g^2i^3n(c+dx)^5}{30d^3} + \frac{(bc-ad)^2g^2i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^3} \\
&\quad - \frac{2b(bc-ad)g^2i^3(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^3} \\
&\quad + \frac{b^2g^2i^3(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^3} \\
&\quad - \frac{B(bc-ad)^6g^2i^3n\log\left(\frac{a+bx}{c+dx}\right)}{60b^4d^3} - \frac{B(bc-ad)^6g^2i^3n\log(c+dx)}{60b^4d^3}
\end{aligned}$$

output
```
-1/60*B*(-a*d+b*c)^5*g^2*i^3*n*x/b^3/d^2-1/120*B*(-a*d+b*c)^4*g^2*i^3*n*(d
*x+c)^2/b^2/d^3-1/180*B*(-a*d+b*c)^3*g^2*i^3*n*(d*x+c)^3/b/d^3+7/120*B*(-a
*d+b*c)^2*g^2*i^3*n*(d*x+c)^4/d^3-1/30*b*B*(-a*d+b*c)*g^2*i^3*n*(d*x+c)^5/
d^3+1/4*(-a*d+b*c)^2*g^2*i^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3
-2/5*b*(-a*d+b*c)*g^2*i^3*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3+1/
6*b^2*g^2*i^3*(d*x+c)^6*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3-1/60*B*(-a*d+b
*c)^6*g^2*i^3*n*ln((b*x+a)/(d*x+c))/b^4/d^3-1/60*B*(-a*d+b*c)^6*g^2*i^3*n*
ln(d*x+c)/b^4/d^3
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.14

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{g^2i^3\left(-15B(bc-ad)^3n(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]),x]
```

output
```
(g^2*i^3*(-15*B*(b*c - a*d)^3*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)
*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]) + 12*B*(b
*c - a*d)^2*n*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2*(c + d*x)^2 +
4*b^3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c - a*d)^4*Log[a
 + b*x]) - B*(b*c - a*d)*n*(60*b*d*(b*c - a*d)^4*x + 30*b^2*(b*c - a*d)^3*
(c + d*x)^2 + 20*b^3*(b*c - a*d)^2*(c + d*x)^3 + 15*b^4*(b*c - a*d)*(c + d
*x)^4 + 12*b^5*(c + d*x)^5 + 60*(b*c - a*d)^5*Log[a + b*x]) + 90*b^4*(b*c
- a*d)^2*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 144*b^5*(b*c
 - a*d)*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 60*b^6*(c + d
*x)^6*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(360*b^4*d^3)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2961, 2782, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\,dx$$

$$\downarrow 2961$$

$$g^2i^3(bc-ad)^6\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}$$

$$\downarrow 2782$$

$$g^2i^3(bc-ad)^6\left(-Bn\int\frac{(c+dx)\left(b^2-\frac{6d(a+bx)b}{c+dx}+\frac{15d^2(a+bx)^2}{(c+dx)^2}\right)}{60d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}+\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{2b\left(B\log\left(e\left(\frac{a}{c}\right.\right.\right.\right.$$

$$\downarrow 27$$

$$g^2i^3(bc-ad)^6\left(-\frac{Bn\int\frac{(c+dx)\left(b^2-\frac{6d(a+bx)b}{c+dx}+\frac{15d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{60d^3}+\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{2b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right.}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}$$

$$\downarrow 1195$$

$$g^2i^3(bc-ad)^6\left(-\frac{Bn\int\left(\frac{d}{b^4\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{14d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{10bd}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6}+\frac{b}{}\right.}{60d^3}\right.$$

$$\downarrow 2009$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(\frac{b^2 \left(B \log\left(e \left(\frac{a+bx}{c+dx} \right)^n \right) + A \right)}{6d^3 \left(b - \frac{d(a+bx)}{c+dx} \right)^6} - \frac{2b \left(B \log\left(e \left(\frac{a+bx}{c+dx} \right)^n \right) + A \right)}{5d^3 \left(b - \frac{d(a+bx)}{c+dx} \right)^5} + \frac{B \log\left(e \left(\frac{a+bx}{c+dx} \right)^n \right) + A}{4d^3 \left(b - \frac{d(a+bx)}{c+dx} \right)^4} - \frac{Bn \left(\frac{\log\left(\frac{a+bx}{c+dx} \right)}{b^4} -$$

input
```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
,x]
```

output
```
(b*c - a*d)^6*g^2*i^3*((b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*d^3
*(b - (d*(a + b*x))/(c + d*x))^6) - (2*b*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(5*d^3*(b - (d*(a + b*x))/(c + d*x))^5) + (A + B*Log[e*((a + b*x)/(
c + d*x))^n])/(4*d^3*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*((2*b)/(b - (
d*(a + b*x))/(c + d*x))^5 - 7/(2*(b - (d*(a + b*x))/(c + d*x))^4) + 1/(3*b
*(b - (d*(a + b*x))/(c + d*x))^3) + 1/(2*b^2*(b - (d*(a + b*x))/(c + d*x))
^2) + 1/(b^3*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^4
 - Log[b - (d*(a + b*x))/(c + d*x)]/b^4))/(60*d^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2782

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1720 vs. $2(367) = 734$.

Time = 26.81 (sec) , antiderivative size = 1721, normalized size of antiderivative = 4.45

method	result	size
parallelrisch	Expression too large to display	1721

input

```
int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_
RETURNVERBOSE)
```

output

```
1/360*(540*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*c^2*d^4*g^2*i^3*n+360*B
*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^3*d^3*g^2*i^3*n+360*B*x*ln(e*((b*x+
a)/(d*x+c))^n)*a^2*b^4*c^3*d^3*g^2*i^3*n+540*B*x^4*ln(e*((b*x+a)/(d*x+c))^
n)*a*b^5*c*d^5*g^2*i^3*n+360*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*c*d^5
*g^2*i^3*n+720*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^2*d^4*g^2*i^3*n-90*
B*ln(b*x+a)*a^2*b^4*c^4*d^2*g^2*i^3*n^2+360*A*x^2*a*b^5*c^3*d^3*g^2*i^3*n+
78*B*x^3*a^2*b^4*c*d^5*g^2*i^3*n^2-42*B*x^3*a*b^5*c^2*d^4*g^2*i^3*n^2+360*
A*x^3*a^2*b^4*c*d^5*g^2*i^3*n+720*A*x^3*a*b^5*c^2*d^4*g^2*i^3*n+270*A*x^4*
b^6*c^2*d^4*g^2*i^3*n+60*A*x^6*b^6*d^6*g^2*i^3*n+6*B*ln(e*((b*x+a)/(d*x+c)
)^n)*b^6*c^6*g^2*i^3*n-6*B*ln(b*x+a)*b^6*c^6*g^2*i^3*n^2-6*B*ln(b*x+a)*a^6
*d^6*g^2*i^3*n^2-36*B*x*a*b^5*c^4*d^2*g^2*i^3*n^2+360*A*x*a^2*b^4*c^3*d^3*
g^2*i^3*n+90*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*c^4*d^2*g^2*i^3*n-36*B*ln
(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^5*d*g^2*i^3*n+36*B*ln(b*x+a)*a*b^5*c^5*d*g
^2*i^3*n^2+36*B*ln(b*x+a)*a^5*b*c*d^5*g^2*i^3*n^2-90*B*ln(b*x+a)*a^4*b^2*c
^2*d^4*g^2*i^3*n^2+120*B*ln(b*x+a)*a^3*b^3*c^3*d^3*g^2*i^3*n^2+18*B*x^2*a^
3*b^3*c*d^5*g^2*i^3*n^2+90*B*x^2*a^2*b^4*c^2*d^4*g^2*i^3*n^2-102*B*x^2*a*b
^5*c^3*d^3*g^2*i^3*n^2+540*A*x^2*a^2*b^4*c^2*d^4*g^2*i^3*n+216*B*x^5*ln(e*
((b*x+a)/(d*x+c))^n)*b^6*c*d^5*g^2*i^3*n+90*B*x^4*ln(e*((b*x+a)/(d*x+c))^n
)*a^2*b^4*d^6*g^2*i^3*n+270*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^2*d^4*g^
2*i^3*n+18*B*x^4*a*b^5*c*d^5*g^2*i^3*n^2+540*A*x^4*a*b^5*c*d^5*g^2*i^3*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1075 vs. $2(367) = 734$.

Time $= 0.37$ (sec) , antiderivative size $= 1075$, normalized size of antiderivative $= 2.78$

$$\int (ag + bgx)^2 (ci + dix)^3 \left(A + B \log \left(e \left(\frac{a + bx}{c + dx} \right)^n \right) \right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="fricas")
```

output

```
1/360*(60*A*b^6*d^6*g^2*i^3*x^6 + 6*(20*B*a^3*b^3*c^3*d^3 - 15*B*a^4*b^2*c
^2*d^4 + 6*B*a^5*b*c*d^5 - B*a^6*d^6)*g^2*i^3*n*log(b*x + a) - 6*(B*b^6*c^
6 - 6*B*a*b^5*c^5*d + 15*B*a^2*b^4*c^4*d^2)*g^2*i^3*n*log(d*x + c) - 12*((
B*b^6*c*d^5 - B*a*b^5*d^6)*g^2*i^3*n - 6*(3*A*b^6*c*d^5 + 2*A*a*b^5*d^6)*g
^2*i^3)*x^5 - 3*((13*B*b^6*c^2*d^4 - 6*B*a*b^5*c*d^5 - 7*B*a^2*b^4*d^6)*g^
2*i^3*n - 30*(3*A*b^6*c^2*d^4 + 6*A*a*b^5*c*d^5 + A*a^2*b^4*d^6)*g^2*i^3)*
x^4 - 2*((19*B*b^6*c^3*d^3 + 21*B*a*b^5*c^2*d^4 - 39*B*a^2*b^4*c*d^5 - B*a
^3*b^3*d^6)*g^2*i^3*n - 60*(A*b^6*c^3*d^3 + 6*A*a*b^5*c^2*d^4 + 3*A*a^2*b^
4*c*d^5)*g^2*i^3)*x^3 - 3*((B*b^6*c^4*d^2 + 34*B*a*b^5*c^3*d^3 - 30*B*a^2*
b^4*c^2*d^4 - 6*B*a^3*b^3*c*d^5 + B*a^4*b^2*d^6)*g^2*i^3*n - 60*(2*A*a*b^5
*c^3*d^3 + 3*A*a^2*b^4*c^2*d^4)*g^2*i^3)*x^2 + 6*(60*A*a^2*b^4*c^3*d^3*g^2
*i^3 + (B*b^6*c^5*d - 6*B*a*b^5*c^4*d^2 - 5*B*a^2*b^4*c^3*d^3 + 15*B*a^3*b
^3*c^2*d^4 - 6*B*a^4*b^2*c*d^5 + B*a^5*b*d^6)*g^2*i^3*n)*x + 6*(10*B*b^6*d
^6*g^2*i^3*x^6 + 60*B*a^2*b^4*c^3*d^3*g^2*i^3*x + 12*(3*B*b^6*c*d^5 + 2*B*
a*b^5*d^6)*g^2*i^3*x^5 + 15*(3*B*b^6*c^2*d^4 + 6*B*a*b^5*c*d^5 + B*a^2*b^4
*d^6)*g^2*i^3*x^4 + 20*(B*b^6*c^3*d^3 + 6*B*a*b^5*c^2*d^4 + 3*B*a^2*b^4*c*
d^5)*g^2*i^3*x^3 + 30*(2*B*a*b^5*c^3*d^3 + 3*B*a^2*b^4*c^2*d^4)*g^2*i^3*x^
2)*log(e) + 6*(10*B*b^6*d^6*g^2*i^3*n*x^6 + 60*B*a^2*b^4*c^3*d^3*g^2*i^3*n
*x + 12*(3*B*b^6*c*d^5 + 2*B*a*b^5*d^6)*g^2*i^3*n*x^5 + 15*(3*B*b^6*c^2*d^
4 + 6*B*a*b^5*c*d^5 + B*a^2*b^4*d^6)*g^2*i^3*n*x^4 + 20*(B*b^6*c^3*d^3 ...
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

input

```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x
)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1978 vs. 2(367) = 734.

Time = 0.09 (sec) , antiderivative size = 1978, normalized size of antiderivative = 5.11

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="maxima")
```

output

```
1/6*B*b^2*d^3*g^2*i^3*x^6*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/6*A*b
^2*d^3*g^2*i^3*x^6 + 3/5*B*b^2*c*d^2*g^2*i^3*x^5*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n) + 2/5*B*a*b*d^3*g^2*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x +
c))^n) + 3/5*A*b^2*c*d^2*g^2*i^3*x^5 + 2/5*A*a*b*d^3*g^2*i^3*x^5 + 3/4*B*b
^2*c^2*d*g^2*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*B*a*b*c*
d^2*g^2*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*B*a^2*d^3*g^2
*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/4*A*b^2*c^2*d*g^2*i^3*
x^4 + 3/2*A*a*b*c*d^2*g^2*i^3*x^4 + 1/4*A*a^2*d^3*g^2*i^3*x^4 + 1/3*B*b^2*
c^3*g^2*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*B*a*b*c^2*d*g^2
*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + B*a^2*c*d^2*g^2*i^3*x^3*
log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*b^2*c^3*g^2*i^3*x^3 + 2*A*a
*b*c^2*d*g^2*i^3*x^3 + A*a^2*c*d^2*g^2*i^3*x^3 + B*a*b*c^3*g^2*i^3*x^2*log
(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*B*a^2*c^2*d*g^2*i^3*x^2*log(e*(b
*x/(d*x + c) + a/(d*x + c))^n) + A*a*b*c^3*g^2*i^3*x^2 + 3/2*A*a^2*c^2*d*g
^2*i^3*x^2 - 1/360*B*b^2*d^3*g^2*i^3*n*(60*a^6*log(b*x + a)/b^6 - 60*c^6*l
og(d*x + c)/d^6 + (12*(b^5*c*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*
b^3*d^5)*x^4 + 20*(b^5*c^3*d^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*
d^5)*x^2 + 60*(b^5*c^5 - a^5*d^5)*x)/(b^5*d^5)) + 1/20*B*b^2*c*d^2*g^2*i^3
*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*
b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4300 vs. $2(367) = 734$.

Time = 1.64 (sec) , antiderivative size = 4300, normalized size of antiderivative = 11.11

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="giac")
```

output
```
1/360*(6*(B*b^9*c^7*g^2*i^3*n - 7*B*a*b^8*c^6*d*g^2*i^3*n - 6*(b*x + a)*B*
b^8*c^7*d*g^2*i^3*n/(d*x + c) + 21*B*a^2*b^7*c^5*d^2*g^2*i^3*n + 42*(b*x +
 a)*B*a*b^7*c^6*d^2*g^2*i^3*n/(d*x + c) + 15*(b*x + a)^2*B*b^7*c^7*d^2*g^2
*i^3*n/(d*x + c)^2 - 35*B*a^3*b^6*c^4*d^3*g^2*i^3*n - 126*(b*x + a)*B*a^2*
b^6*c^5*d^3*g^2*i^3*n/(d*x + c) - 105*(b*x + a)^2*B*a*b^6*c^6*d^3*g^2*i^3*
n/(d*x + c)^2 + 35*B*a^4*b^5*c^3*d^4*g^2*i^3*n + 210*(b*x + a)*B*a^3*b^5*c
^4*d^4*g^2*i^3*n/(d*x + c) + 315*(b*x + a)^2*B*a^2*b^5*c^5*d^4*g^2*i^3*n/(
d*x + c)^2 - 21*B*a^5*b^4*c^2*d^5*g^2*i^3*n - 210*(b*x + a)*B*a^4*b^4*c^3*
d^5*g^2*i^3*n/(d*x + c) - 525*(b*x + a)^2*B*a^3*b^4*c^4*d^5*g^2*i^3*n/(d*x
 + c)^2 + 7*B*a^6*b^3*c*d^6*g^2*i^3*n + 126*(b*x + a)*B*a^5*b^3*c^2*d^6*g^
2*i^3*n/(d*x + c) + 525*(b*x + a)^2*B*a^4*b^3*c^3*d^6*g^2*i^3*n/(d*x + c)^
2 - B*a^7*b^2*d^7*g^2*i^3*n - 42*(b*x + a)*B*a^6*b^2*c*d^7*g^2*i^3*n/(d*x
 + c) - 315*(b*x + a)^2*B*a^5*b^2*c^2*d^7*g^2*i^3*n/(d*x + c)^2 + 6*(b*x +
 a)*B*a^7*b*d^8*g^2*i^3*n/(d*x + c) + 105*(b*x + a)^2*B*a^6*b*c*d^8*g^2*i^3
*n/(d*x + c)^2 - 15*(b*x + a)^2*B*a^7*d^9*g^2*i^3*n/(d*x + c)^2)*log((b*x
 + a)/(d*x + c))/(b^6*d^3 - 6*(b*x + a)*b^5*d^4/(d*x + c) + 15*(b*x + a)^2*
b^4*d^5/(d*x + c)^2 - 20*(b*x + a)^3*b^3*d^6/(d*x + c)^3 + 15*(b*x + a)^4*
b^2*d^7/(d*x + c)^4 - 6*(b*x + a)^5*b*d^8/(d*x + c)^5 + (b*x + a)^6*d^9/(d
*x + c)^6) - (2*B*b^12*c^7*g^2*i^3*n - 14*B*a*b^11*c^6*d*g^2*i^3*n - 18*(b
*x + a)*B*b^11*c^7*d*g^2*i^3*n/(d*x + c) + 42*B*a^2*b^10*c^5*d^2*g^2*i^...
```

Mupad [B] (verification not implemented)

Time = 27.72 (sec) , antiderivative size = 2547, normalized size of antiderivative = 6.58

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \text{Too large to display}$$

input

```
int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))
,x)
```

output

```
x^2*((a*c*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d*n - B*b*c*n))/6 -
 (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (d*
g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2*n - 3*B*b^2*c^2*n + 60*
A*a*b*c*d + B*a*b*c*d*n))/5 + A*a*b*c*d^2*g^2*i^3))/(2*b*d) - ((60*a*d + 6
0*b*c)*((g^2*i^3*(4*A*a^3*d^3 + 16*A*b^3*c^3 + B*a^3*d^3*n - 3*B*b^3*c^3*n
 + 72*A*a*b^2*c^2*d + 48*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d*n + 5*B*a^2*b*c*d
^2*n))/(4*b) + ((60*a*d + 60*b*c)*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c +
 B*a*d*n - B*b*c*n))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d +
 60*b*c))/(60*b*d) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2
*n - 3*B*b^2*c^2*n + 60*A*a*b*c*d + B*a*b*c*d*n))/5 + A*a*b*c*d^2*g^2*i^3)
)/(60*b*d) - (a*c*((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d*n - B*b*c*n
))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60))/(b*d)))/(120*b*d) + (c*g^2
*i^3*(12*A*a^3*d^3 + 3*A*b^3*c^3 + 3*B*a^3*d^3*n - B*b^3*c^3*n + 36*A*a*b^
2*c^2*d + 54*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n))/(6*b*
d)) + x^3*((g^2*i^3*(4*A*a^3*d^3 + 16*A*b^3*c^3 + B*a^3*d^3*n - 3*B*b^3*c^
3*n + 72*A*a*b^2*c^2*d + 48*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d*n + 5*B*a^2*b*
c*d^2*n))/(12*b) + ((60*a*d + 60*b*c)*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b
*c + B*a*d*n - B*b*c*n))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a
*d + 60*b*c))/(60*b*d) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2
*d^2*n - 3*B*b^2*c^2*n + 60*A*a*b*c*d + B*a*b*c*d*n))/5 + A*a*b*c*d^2*g...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1097, normalized size of antiderivative = 2.83

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \text{Too large to display}$$

input

```
int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output

```
(g**2*i*(6*log(a + b*x)*a**6*d**6*n - 36*log(a + b*x)*a**5*b*c*d**5*n + 90
*log(a + b*x)*a**4*b**2*c**2*d**4*n - 120*log(a + b*x)*a**3*b**3*c**3*d**3
*n + 90*log(a + b*x)*a**2*b**4*c**4*d**2*n - 36*log(a + b*x)*a*b**5*c**5*d
*n + 6*log(a + b*x)*b**6*c**6*n - 90*log(((a + b*x)**n*e)/(c + d*x)**n)*a*
*2*b**4*c**4*d**2 - 360*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**4*c**3*
d**3*x - 540*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**4*c**2*d**4*x**2 -
 360*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**4*c*d**5*x**3 - 90*log(((a
 + b*x)**n*e)/(c + d*x)**n)*a**2*b**4*d**6*x**4 + 36*log(((a + b*x)**n*e)/
(c + d*x)**n)*a*b**5*c**5*d - 360*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**
5*c**3*d**3*x**2 - 720*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**5*c**2*d**4
*x**3 - 540*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**5*c*d**5*x**4 - 144*lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a*b**5*d**6*x**5 - 6*log(((a + b*x)**n*e)
/(c + d*x)**n)*b**6*c**6 - 120*log(((a + b*x)**n*e)/(c + d*x)**n)*b**6*c**
3*d**3*x**3 - 270*log(((a + b*x)**n*e)/(c + d*x)**n)*b**6*c**2*d**4*x**4 -
 216*log(((a + b*x)**n*e)/(c + d*x)**n)*b**6*c*d**5*x**5 - 60*log(((a + b*
x)**n*e)/(c + d*x)**n)*b**6*d**6*x**6 - 6*a**5*b*d**6*n*x + 36*a**4*b**2*c
*d**5*n*x + 3*a**4*b**2*d**6*n*x**2 - 360*a**3*b**3*c**3*d**3*x - 90*a**3*
b**3*c**2*d**4*n*x - 540*a**3*b**3*c**2*d**4*x**2 - 18*a**3*b**3*c*d**5*n*
x**2 - 360*a**3*b**3*c*d**5*x**3 - 2*a**3*b**3*d**6*n*x**3 - 90*a**3*b**3*
d**6*x**4 + 30*a**2*b**4*c**3*d**3*n*x - 360*a**2*b**4*c**3*d**3*x**2 -...
```

3.129 $\int (ag+bgx)(ci+dix)^3 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result 1521
Mathematica [A] (verified) 1522
Rubi [A] (verified) 1522
Maple [B] (verified) 1524
Fricas [B] (verification not implemented) 1525
Sympy [F(-1)] 1526
Maxima [B] (verification not implemented) 1526
Giac [B] (verification not implemented) 1527
Mupad [B] (verification not implemented) 1528
Reduce [B] (verification not implemented) 1529

Optimal result

Integrand size = 41, antiderivative size = 283

$$\begin{aligned}
&\int (ag+bgx)(ci+dix)^3 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= \frac{B(bc-ad)^4 gi^3 nx}{20b^3 d} + \frac{B(bc-ad)^3 gi^3 n(c+dx)^2}{40b^2 d^2} + \frac{B(bc-ad)^2 gi^3 n(c+dx)^3}{60bd^2} \\
&\quad - \frac{B(bc-ad)gi^3 n(c+dx)^4}{20d^2} - \frac{(bc-ad)gi^3(c+dx)^4\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2} \\
&\quad + \frac{bgi^3(c+dx)^5\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^2} \\
&\quad + \frac{B(bc-ad)^5 gi^3 n\log\left(\frac{a+bx}{c+dx}\right)}{20b^4 d^2} + \frac{B(bc-ad)^5 gi^3 n\log(c+dx)}{20b^4 d^2}
\end{aligned}$$

output

```
1/20*B*(-a*d+b*c)^4*g*i^3*n*x/b^3/d+1/40*B*(-a*d+b*c)^3*g*i^3*n*(d*x+c)^2/
b^2/d^2+1/60*B*(-a*d+b*c)^2*g*i^3*n*(d*x+c)^3/b/d^2-1/20*B*(-a*d+b*c)*g*i^
3*n*(d*x+c)^4/d^2-1/4*(-a*d+b*c)*g*i^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))/d^2+1/5*b*g*i^3*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^2+1/20*
B*(-a*d+b*c)^5*g*i^3*n*ln((b*x+a)/(d*x+c))/b^4/d^2+1/20*B*(-a*d+b*c)^5*g*i
^3*n*ln(d*x+c)/b^4/d^2
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.95

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{gi^3\left(\frac{5B(bc-ad)^2n(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx))}{b^4}-\frac{2B(bc-ad)n(12bd(bc-ad)^3x+6b^2(bc-ad)^2}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]),x]
```

output
```
(g*i^3*((5*B*(b*c - a*d)^2*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c
 + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]))/b^4 - (2*B*
(b*c - a*d)*n*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2*(c + d*x)^2 +
4*b^3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c - a*d)^4*Log[a
 + b*x]))/b^4 - 30*(b*c - a*d)*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) + 24*b*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(120*d
^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {2961, 2782, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$
$$\downarrow 2961$$
$$gi^3(bc-ad)^5\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$
$$\downarrow 2782$$

$$gi^3(bc-ad)^5\left(-Bn\int -\frac{(c+dx)\left(b-\frac{5d(a+bx)}{c+dx}\right)}{20d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}\right)$$

$$\downarrow 27$$

$$gi^3(bc-ad)^5\left(\frac{Bn\int\frac{(c+dx)\left(b-\frac{5d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{20d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}\right)$$

$$\downarrow 86$$

$$gi^3(bc-ad)^5\left(\frac{Bn\int\left(\frac{d}{b^4\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{4d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{c+dx}{b^4(a+bx)}\right)d\frac{a+bx}{c+dx}}{20d^2}-\right.$$

$$\downarrow 2009$$

$$gi^3(bc-ad)^5\left(-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^4}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^4}+\frac{1}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\right.}{}\right.$$

input

```
Int[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x
]
```

output

```
(b*c - a*d)^5*g*i^3*((b*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*d^2*(b
- (d*(a + b*x))/(c + d*x))^5) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(4*
d^2*(b - (d*(a + b*x))/(c + d*x))^4) + (B*n*(-(b - (d*(a + b*x))/(c + d*x)
)^(-4) + 1/(3*b*(b - (d*(a + b*x))/(c + d*x))^3) + 1/(2*b^2*(b - (d*(a + b
*x))/(c + d*x))^2) + 1/(b^3*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)
/(c + d*x)]/b^4 - Log[b - (d*(a + b*x))/(c + d*x)]/b^4))/(20*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1118 vs. $2(267) = 534$.

Time = 11.69 (sec) , antiderivative size = 1119, normalized size of antiderivative = 3.95

method	result	size
parallelrisch	Expression too large to display	1119

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RE
TURNVERBOSE)
```

output
```
1/120*(15*B*x^2*a^2*b^3*c*d^4*g*i^3*n^2+15*B*x^2*a*b^4*c^2*d^3*g*i^3*n^2+1
80*A*x^2*a*b^4*c^2*d^3*g*i^3*n-30*B*x*a^3*b^2*c*d^4*g*i^3*n^2+60*B*x*a^2*b
^3*c^2*d^3*g*i^3*n^2-30*B*x*a*b^4*c^3*d^2*g*i^3*n^2+120*A*x*a*b^4*c^3*d^2*
g*i^3*n+30*B*ln(b*x+a)*a^4*b*c*d^4*g*i^3*n^2-60*B*ln(b*x+a)*a^3*b^2*c^2*d^
3*g*i^3*n^2+60*B*ln(b*x+a)*a^2*b^3*c^3*d^2*g*i^3*n^2-30*B*ln(b*x+a)*a*b^4*
c^4*d*g*i^3*n^2+30*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^4*d*g*i^3*n+180*B*x
^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^2*d^3*g*i^3*n+120*B*x*ln(e*((b*x+a)/(
d*x+c))^n)*a*b^4*c^3*d^2*g*i^3*n+120*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4
*c*d^4*g*i^3*n+24*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^5*g*i^3*n+30*B*x^4
*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*d^5*g*i^3*n+90*B*x^4*ln(e*((b*x+a)/(d*x+c
))^n)*b^5*c*d^4*g*i^3*n+120*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^2*d^3*g*
i^3*n+20*B*x^3*a*b^4*c*d^4*g*i^3*n^2+120*A*x^3*a*b^4*c*d^4*g*i^3*n+60*B*x^
2*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^3*d^2*g*i^3*n+6*B*b^5*c^5*g*i^3*n^2+6*B*
x^4*a*b^4*d^5*g*i^3*n^2-6*B*x^4*b^5*c*d^4*g*i^3*n^2+30*A*x^4*a*b^4*d^5*g*i
^3*n+90*A*x^4*b^5*c*d^4*g*i^3*n+2*B*x^3*a^2*b^3*d^5*g*i^3*n^2-22*B*x^3*b^5
*c^2*d^3*g*i^3*n^2+120*A*x^3*b^5*c^2*d^3*g*i^3*n-3*B*x^2*a^3*b^2*d^5*g*i^3
*n^2-27*B*x^2*b^5*c^3*d^2*g*i^3*n^2+60*A*x^2*b^5*c^3*d^2*g*i^3*n+6*B*x*a^4
*b*d^5*g*i^3*n^2-6*B*x*b^5*c^4*d*g*i^3*n^2-6*B*ln(e*((b*x+a)/(d*x+c))^n)*b
^5*c^5*g*i^3*n+24*A*x^5*b^5*d^5*g*i^3*n+6*B*ln(b*x+a)*b^5*c^5*g*i^3*n^2-6*
B*ln(b*x+a)*a^5*d^5*g*i^3*n^2-6*B*a^5*d^5*g*i^3*n^2+27*B*a^4*b*c*d^4*g*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(267) = 534$.

Time $= 0.16$ (sec) , antiderivative size $= 721$, normalized size of antiderivative $= 2.55$

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{24\,Ab^5d^5gi^3x^5+6\,(10\,Ba^2b^3c^3d^2-10\,Ba^3b^2c^2d^3+5\,Ba^4bcd^4-Ba^5d^5)gi^3n\log(bx+a)+6\,(Bb^5c^5-5}{}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="fricas")
```

output

```
1/120*(24*A*b^5*d^5*g*i^3*x^5 + 6*(10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2
*d^3 + 5*B*a^4*b*c*d^4 - B*a^5*d^5)*g*i^3*n*log(b*x + a) + 6*(B*b^5*c^5 -
5*B*a*b^4*c^4*d)*g*i^3*n*log(d*x + c) - 6*((B*b^5*c*d^4 - B*a*b^4*d^5)*g*i
^3*n - 5*(3*A*b^5*c*d^4 + A*a*b^4*d^5)*g*i^3)*x^4 - 2*((11*B*b^5*c^2*d^3 -
 10*B*a*b^4*c*d^4 - B*a^2*b^3*d^5)*g*i^3*n - 60*(A*b^5*c^2*d^3 + A*a*b^4*c
*d^4)*g*i^3)*x^3 - 3*((9*B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 - 5*B*a^2*b^3*c
*d^4 + B*a^3*b^2*d^5)*g*i^3*n - 20*(A*b^5*c^3*d^2 + 3*A*a*b^4*c^2*d^3)*g*i
^3)*x^2 + 6*(20*A*a*b^4*c^3*d^2*g*i^3 - (B*b^5*c^4*d + 5*B*a*b^4*c^3*d^2 -
 10*B*a^2*b^3*c^2*d^3 + 5*B*a^3*b^2*c*d^4 - B*a^4*b*d^5)*g*i^3*n)*x + 6*(4
*B*b^5*d^5*g*i^3*x^5 + 20*B*a*b^4*c^3*d^2*g*i^3*x + 5*(3*B*b^5*c*d^4 + B*a
*b^4*d^5)*g*i^3*x^4 + 20*(B*b^5*c^2*d^3 + B*a*b^4*c*d^4)*g*i^3*x^3 + 10*(B
*b^5*c^3*d^2 + 3*B*a*b^4*c^2*d^3)*g*i^3*x^2)*log(e) + 6*(4*B*b^5*d^5*g*i^3
*n*x^5 + 20*B*a*b^4*c^3*d^2*g*i^3*n*x + 5*(3*B*b^5*c*d^4 + B*a*b^4*d^5)*g*
i^3*n*x^4 + 20*(B*b^5*c^2*d^3 + B*a*b^4*c*d^4)*g*i^3*n*x^3 + 10*(B*b^5*c^3
*d^2 + 3*B*a*b^4*c^2*d^3)*g*i^3*n*x^2)*log((b*x + a)/(d*x + c)))/(b^4*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1118 vs. $2(267) = 534$.

Time = 0.07 (sec) , antiderivative size = 1118, normalized size of antiderivative = 3.95

$$\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="maxima")
```

output

```
1/5*B*b*d^3*g*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*b*d^3
*g*i^3*x^5 + 3/4*B*b*c*d^2*g*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
) + 1/4*B*a*d^3*g*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/4*A*b
*c*d^2*g*i^3*x^4 + 1/4*A*a*d^3*g*i^3*x^4 + B*b*c^2*d*g*i^3*x^3*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n) + B*a*c*d^2*g*i^3*x^3*log(e*(b*x/(d*x + c) + a
/(d*x + c))^n) + A*b*c^2*d*g*i^3*x^3 + A*a*c*d^2*g*i^3*x^3 + 1/2*B*b*c^3*g
*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*B*a*c^2*d*g*i^3*x^2*
log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*b*c^3*g*i^3*x^2 + 3/2*A*a*c
^2*d*g*i^3*x^2 + 1/60*B*b*d^3*g*i^3*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*lo
g(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2
*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4
*d^4)) - 1/8*B*b*c*d^2*g*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c
)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6
*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/24*B*a*d^3*g*i^3*n*(6*a^4*log(b*x +
 a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3
*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*b*c^
2*d*g*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d -
 a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 1/2*B*a*c*d^2*g*i^3*
n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*
x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 1/2*B*b*c^3*g*i^3*n*(a^2*lo...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2584 vs. $2(267) = 534$.

Time $= 1.02$ (sec) , antiderivative size $= 2584$, normalized size of antiderivative $= 9.13$

$$\int (ag + bgx)(ci + dix)^3 \left(A + B \log\left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right) dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="giac")
```

output

```
-1/120*(6*(B*b^7*c^6*g*i^3*n - 6*B*a*b^6*c^5*d*g*i^3*n - 5*(b*x + a)*B*b^6
*c^6*d*g*i^3*n/(d*x + c) + 15*B*a^2*b^5*c^4*d^2*g*i^3*n + 30*(b*x + a)*B*a
*b^5*c^5*d^2*g*i^3*n/(d*x + c) - 20*B*a^3*b^4*c^3*d^3*g*i^3*n - 75*(b*x +
a)*B*a^2*b^4*c^4*d^3*g*i^3*n/(d*x + c) + 15*B*a^4*b^3*c^2*d^4*g*i^3*n + 10
0*(b*x + a)*B*a^3*b^3*c^3*d^4*g*i^3*n/(d*x + c) - 6*B*a^5*b^2*c*d^5*g*i^3*
n - 75*(b*x + a)*B*a^4*b^2*c^2*d^5*g*i^3*n/(d*x + c) + B*a^6*b*d^6*g*i^3*n
 + 30*(b*x + a)*B*a^5*b*c*d^6*g*i^3*n/(d*x + c) - 5*(b*x + a)*B*a^6*d^7*g*
i^3*n/(d*x + c))*log((b*x + a)/(d*x + c))/(b^5*d^2 - 5*(b*x + a)*b^4*d^3/(
d*x + c) + 10*(b*x + a)^2*b^3*d^4/(d*x + c)^2 - 10*(b*x + a)^3*b^2*d^5/(d*
x + c)^3 + 5*(b*x + a)^4*b*d^6/(d*x + c)^4 - (b*x + a)^5*d^7/(d*x + c)^5)
- (5*B*b^10*c^6*g*i^3*n - 30*B*a*b^9*c^5*d*g*i^3*n - 31*(b*x + a)*B*b^9*c^
6*d*g*i^3*n/(d*x + c) + 75*B*a^2*b^8*c^4*d^2*g*i^3*n + 186*(b*x + a)*B*a*b
^8*c^5*d^2*g*i^3*n/(d*x + c) + 47*(b*x + a)^2*B*b^8*c^6*d^2*g*i^3*n/(d*x +
 c)^2 - 100*B*a^3*b^7*c^3*d^3*g*i^3*n - 465*(b*x + a)*B*a^2*b^7*c^4*d^3*g*
i^3*n/(d*x + c) - 282*(b*x + a)^2*B*a*b^7*c^5*d^3*g*i^3*n/(d*x + c)^2 - 27
*(b*x + a)^3*B*b^7*c^6*d^3*g*i^3*n/(d*x + c)^3 + 75*B*a^4*b^6*c^2*d^4*g*i^
3*n + 620*(b*x + a)*B*a^3*b^6*c^3*d^4*g*i^3*n/(d*x + c) + 705*(b*x + a)^2*
B*a^2*b^6*c^4*d^4*g*i^3*n/(d*x + c)^2 + 162*(b*x + a)^3*B*a*b^6*c^5*d^4*g*
i^3*n/(d*x + c)^3 + 6*(b*x + a)^4*B*b^6*c^6*d^4*g*i^3*n/(d*x + c)^4 - 30*B
*a^5*b^5*c*d^5*g*i^3*n - 465*(b*x + a)*B*a^4*b^5*c^2*d^5*g*i^3*n/(d*x +...
```

Mupad [B] (verification not implemented)

Time = 27.04 (sec) , antiderivative size = 1234, normalized size of antiderivative = 4.36

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input

```
int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x
)
```

output

```
x*((a*c*(((20*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d*n - B
*b*c*n))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(20*b*d) - (d*g*i^3*(4*A
*a^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2*n - 3*B*b^2*c^2*n + 32*A*a*b*c*d + 2*B
*a*b*c*d*n))/(4*b) + A*a*c*d^2*g*i^3))/(b*d) - ((20*a*d + 20*b*c)*(((20*a*
d + 20*b*c)*((20*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d*n
 - B*b*c*n))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(20*b*d) - (d*g*i^3*
(4*A*a^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2*n - 3*B*b^2*c^2*n + 32*A*a*b*c*d +
 2*B*a*b*c*d*n))/(4*b) + A*a*c*d^2*g*i^3))/(20*b*d) - (a*c*((d^2*g*i^3*(10
*A*a*d + 20*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c)
)/20))/(b*d) + (c*g*i^3*(4*A*a^2*d^2 + 4*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c
^2*n + 12*A*a*b*c*d))/b))/(20*b*d) + (c^2*g*i^3*(12*A*a^2*d^2 + 2*A*b^2*c^
2 + 3*B*a^2*d^2*n - B*b^2*c^2*n + 16*A*a*b*c*d - 2*B*a*b*c*d*n))/(2*b*d))
- x^3*(((20*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d*n - B*b
*c*n))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(60*b*d) - (d*g*i^3*(4*A*a
^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2*n - 3*B*b^2*c^2*n + 32*A*a*b*c*d + 2*B*a
*b*c*d*n))/(12*b) + (A*a*c*d^2*g*i^3)/3) + x^2*(((20*a*d + 20*b*c)*(((20*a
*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d*n - B*b*c*n))/5 - (A
*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(20*b*d) - (d*g*i^3*(4*A*a^2*d^2 + 24*A
*b^2*c^2 + B*a^2*d^2*n - 3*B*b^2*c^2*n + 32*A*a*b*c*d + 2*B*a*b*c*d*n))/(4
*b) + A*a*c*d^2*g*i^3))/(40*b*d) - (a*c*((d^2*g*i^3*(10*A*a*d + 20*A*b*...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 743, normalized size of antiderivative = 2.63

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input

```
int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output

```
(g*i*(6*log(a + b*x)*a**5*d**5*n - 30*log(a + b*x)*a**4*b*c*d**4*n + 60*lo
g(a + b*x)*a**3*b**2*c**2*d**3*n - 60*log(a + b*x)*a**2*b**3*c**3*d**2*n +
 30*log(a + b*x)*a*b**4*c**4*d*n - 6*log(a + b*x)*b**5*c**5*n - 30*log(((a
 + b*x)**n*e)/(c + d*x)**n)*a*b**4*c**4*d - 120*log(((a + b*x)**n*e)/(c +
d*x)**n)*a*b**4*c**3*d**2*x - 180*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**
4*c**2*d**3*x**2 - 120*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**4*c*d**4*x*
*3 - 30*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**4*d**5*x**4 + 6*log(((a +
b*x)**n*e)/(c + d*x)**n)*b**5*c**5 - 60*log(((a + b*x)**n*e)/(c + d*x)**n)
*b**5*c**3*d**2*x**2 - 120*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*c**2*d*
*3*x**3 - 90*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*c*d**4*x**4 - 24*log(
((a + b*x)**n*e)/(c + d*x)**n)*b**5*d**5*x**5 - 6*a**4*b*d**5*n*x + 30*a**
3*b**2*c*d**4*n*x + 3*a**3*b**2*d**5*n*x**2 - 120*a**2*b**3*c**3*d**2*x -
60*a**2*b**3*c**2*d**3*n*x - 180*a**2*b**3*c**2*d**3*x**2 - 15*a**2*b**3*c
*d**4*n*x**2 - 120*a**2*b**3*c*d**4*x**3 - 2*a**2*b**3*d**5*n*x**3 - 30*a*
*2*b**3*d**5*x**4 + 30*a*b**4*c**3*d**2*n*x - 60*a*b**4*c**3*d**2*x**2 - 1
5*a*b**4*c**2*d**3*n*x**2 - 120*a*b**4*c**2*d**3*x**3 - 20*a*b**4*c*d**4*n
*x**3 - 90*a*b**4*c*d**4*x**4 - 6*a*b**4*d**5*n*x**4 - 24*a*b**4*d**5*x**5
 + 6*b**5*c**4*d*n*x + 27*b**5*c**3*d**2*n*x**2 + 22*b**5*c**2*d**3*n*x**3
 + 6*b**5*c*d**4*n*x**4))/(120*b**3*d**2)
```

3.130 $\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result . . . 1531
Mathematica [A] (verified) . . . 1531
Rubi [A] (verified) . . . 1532
Maple [B] (verified) . . . 1533
Fricas [B] (verification not implemented) . . . 1534
Sympy [F(-1)] . . . 1535
Maxima [B] (verification not implemented) . . . 1535
Giac [B] (verification not implemented) . . . 1537
Mupad [B] (verification not implemented) . . . 1538
Reduce [B] (verification not implemented) . . . 1539

Optimal result

Integrand size = 33, antiderivative size = 156

$$\begin{aligned}&\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\ &= -\frac{B(bc-ad)^3 i^3 nx}{4b^3} - \frac{B(bc-ad)^2 i^3 n(c+dx)^2}{8b^2 d} - \frac{B(bc-ad) i^3 n(c+dx)^3}{12bd}\\ &\quad - \frac{B(bc-ad)^4 i^3 n\log(a+bx)}{4b^4 d} + \frac{i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d}\end{aligned}$$

output

```
-1/4*B*(-a*d+b*c)^3*i^3*n*x/b^3-1/8*B*(-a*d+b*c)^2*i^3*n*(d*x+c)^2/b^2/d-1
/12*B*(-a*d+b*c)*i^3*n*(d*x+c)^3/b/d-1/4*B*(-a*d+b*c)^4*i^3*n*ln(b*x+a)/b^
4/d+1/4*i^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\begin{aligned}&\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\ &= \frac{i^3\left(-\frac{B(bc-ad)n\left(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx)\right)}{6b^4} + (c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right.}{4d}\end{aligned}$$

input
```
Integrate[(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(i^3*(-1/6*(B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c
+ d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]))/b^4 + (c + d
*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(4*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)^3 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right) dx$$

$$\downarrow 2947$$

$$\frac{i^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d} - \frac{Bn(bc-ad)\int \frac{i^4(c+dx)^3}{a+bx}dx}{4di}$$

$$\downarrow 27$$

$$\frac{i^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d} - \frac{Bi^3n(bc-ad)\int \frac{(c+dx)^3}{a+bx}dx}{4d}$$

$$\downarrow 49$$

$$\frac{i^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d} - \frac{Bi^3n(bc-ad)\int \left(\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)^2}{b^3}+\frac{d(c+dx)(bc-ad)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx}{4d}$$

$$\downarrow 2009$$

$$\frac{i^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d} - \frac{Bi^3n(bc-ad)\left(\frac{(bc-ad)^3\log(a+bx)}{b^4}+\frac{dx(bc-ad)^2}{b^3}+\frac{(c+dx)^2(bc-ad)}{2b^2}+\frac{(c+dx)^3}{3b}\right)}{4d}$$

input
```
Int[(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
-1/4*(B*(b*c - a*d)*i^3*n*((d*(b*c - a*d)^2*x)/b^3 + ((b*c - a*d)*(c + d*x
)^2)/(2*b^2) + (c + d*x)^3/(3*b) + ((b*c - a*d)^3*Log[a + b*x])/b^4))/d +
(i^3*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(146) = 292$.

Time = 4.89 (sec) , antiderivative size = 652, normalized size of antiderivative = 4.18

method	result
parallelrisch	$\frac{24Bx^3\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4cd^3i^3n+6Bx^4\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4d^4i^3n+2Bx^3ab^3d^4i^3n^2-2Bx^3b^4cd^3i^3n^2+24Ax^3b^4cd^3i^3n-3B$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/24*(24*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^3*i^3*n+6*B*x^4*ln(e*((b*
x+a)/(d*x+c))^n)*b^4*d^4*i^3*n+2*B*x^3*a*b^3*d^4*i^3*n^2-2*B*x^3*b^4*c*d^3
*i^3*n^2+24*A*x^3*b^4*c*d^3*i^3*n-3*B*x^2*a^2*b^2*d^4*i^3*n^2-9*B*x^2*b^4*
c^2*d^2*i^3*n^2+36*A*x^2*b^4*c^2*d^2*i^3*n+6*B*x*a^3*b*d^4*i^3*n^2-18*B*x*
b^4*c^3*d*i^3*n^2+24*A*x*b^4*c^3*d*i^3*n-36*B*ln(b*x+a)*a^2*b^2*c^2*d^2*i^
3*n^2+24*B*ln(b*x+a)*a*b^3*c^3*d*i^3*n^2-6*B*a^4*d^4*i^3*n^2+18*B*b^4*c^4*
i^3*n^2-24*A*b^4*c^4*i^3*n+21*B*a^3*b*c*d^3*i^3*n^2-24*B*a^2*b^2*c^2*d^2*i
^3*n^2-9*B*a*b^3*c^3*d*i^3*n^2-60*A*a*b^3*c^3*d*i^3*n+6*A*x^4*b^4*d^4*i^3*
n+6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^4*i^3*n-6*B*ln(b*x+a)*a^4*d^4*i^3*n^
2-6*B*ln(b*x+a)*b^4*c^4*i^3*n^2+36*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2
*d^2*i^3*n+12*B*x^2*a*b^3*c*d^3*i^3*n^2+24*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b
^4*c^3*d*i^3*n-24*B*x*a^2*b^2*c*d^3*i^3*n^2+36*B*x*a*b^3*c^2*d^2*i^3*n^2+2
4*B*ln(b*x+a)*a^3*b*c*d^3*i^3*n^2)/b^4/d/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(146) = 292$.

Time $= 0.11$ (sec) , antiderivative size $= 429$, normalized size of antiderivative $= 2.75$

$$\int (ci + dix)^3 \left(A + B \log\left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right) dx$$
$$= \frac{6\,Ab^4d^4i^3x^4 - 6\,Bb^4c^4i^3n\log(dx + c) + 6\,(4\,Bab^3c^3d - 6\,Ba^2b^2c^2d^2 + 4\,Ba^3bcd^3 - Ba^4d^4)i^3n\log(bx +$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
1/24*(6*A*b^4*d^4*i^3*x^4 - 6*B*b^4*c^4*i^3*n*log(d*x + c) + 6*(4*B*a*b^3*
c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3*b*c*d^3 - B*a^4*d^4)*i^3*n*log(b*x +
 a) + 2*(12*A*b^4*c*d^3*i^3 - (B*b^4*c*d^3 - B*a*b^3*d^4)*i^3*n)*x^3 + 3*(
12*A*b^4*c^2*d^2*i^3 - (3*B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)
*i^3*n)*x^2 + 6*(4*A*b^4*c^3*d*i^3 - (3*B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 +
4*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*i^3*n)*x + 6*(B*b^4*d^4*i^3*x^4 + 4*B*b^4
*c*d^3*i^3*x^3 + 6*B*b^4*c^2*d^2*i^3*x^2 + 4*B*b^4*c^3*d*i^3*x)*log(e) + 6
*(B*b^4*d^4*i^3*n*x^4 + 4*B*b^4*c*d^3*i^3*n*x^3 + 6*B*b^4*c^2*d^2*i^3*n*x^
2 + 4*B*b^4*c^3*d*i^3*n*x)*log((b*x + a)/(d*x + c)))/(b^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int (ci + dix)^3 \left(A + B \log\left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right) dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(146) = 292$.

Time = 0.06 (sec) , antiderivative size = 479, normalized size of antiderivative = 3.07

$$\begin{aligned}
&\int (ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx \\
&= \frac{1}{4}Bd^3i^3x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{4}Ad^3i^3x^4 \\
&\quad + Bcd^2i^3x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Acd^2i^3x^3 \\
&\quad + \frac{3}{2}Bc^2di^3x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{3}{2}Ac^2di^3x^2 \\
&\quad - \frac{1}{24}Bd^3i^3n\left(\frac{6\,a^4\log(bx+a)}{b^4}-\frac{6\,c^4\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c^3\right. \\
&\quad + \frac{1}{2}Bcd^2i^3n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right) \\
&\quad - \frac{3}{2}Bc^2di^3n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) \\
&\quad + Bc^3i^3n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right) \\
&\quad + Bc^3i^3x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Ac^3i^3x
\end{aligned}$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output

```
1/4*B*d^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*d^3*i^3*x
^4 + B*c*d^2*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d^2*i^3*
x^3 + 3/2*B*c^2*d*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*c
^2*d*i^3*x^2 - 1/24*B*d^3*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x +
c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 +
6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*c*d^2*i^3*n*(2*a^3*log(b*x + a
)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a
^2*d^2)*x)/(b^2*d^2)) - 3/2*B*c^2*d*i^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(
d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*c^3*i^3*n*(a*log(b*x + a)/b - c*lo
g(d*x + c)/d) + B*c^3*i^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c^3
*i^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(146) = 292$.

Time = 0.60 (sec) , antiderivative size = 1402, normalized size of antiderivative = 8.99

$$\int (ci + dix)^3 \left(A + B \log \left(e \left(\frac{a + bx}{c + dx} \right)^n \right) \right) dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
1/24*(6*(B*b^5*c^5*i^3*n - 5*B*a*b^4*c^4*d*i^3*n + 10*B*a^2*b^3*c^3*d^2*i^
3*n - 10*B*a^3*b^2*c^2*d^3*i^3*n + 5*B*a^4*b*c*d^4*i^3*n - B*a^5*d^5*i^3*n
)*log((b*x + a)/(d*x + c))/(b^4*d - 4*(b*x + a)*b^3*d^2/(d*x + c) + 6*(b*x
 + a)^2*b^2*d^3/(d*x + c)^2 - 4*(b*x + a)^3*b*d^4/(d*x + c)^3 + (b*x + a)^
4*d^5/(d*x + c)^4) - (11*B*b^8*c^5*i^3*n - 55*B*a*b^7*c^4*d*i^3*n - 26*(b*
x + a)*B*b^7*c^5*d*i^3*n/(d*x + c) + 110*B*a^2*b^6*c^3*d^2*i^3*n + 130*(b*
x + a)*B*a*b^6*c^4*d^2*i^3*n/(d*x + c) + 21*(b*x + a)^2*B*b^6*c^5*d^2*i^3*
n/(d*x + c)^2 - 110*B*a^3*b^5*c^2*d^3*i^3*n - 260*(b*x + a)*B*a^2*b^5*c^3*
d^3*i^3*n/(d*x + c) - 105*(b*x + a)^2*B*a*b^5*c^4*d^3*i^3*n/(d*x + c)^2 -
6*(b*x + a)^3*B*b^5*c^5*d^3*i^3*n/(d*x + c)^3 + 55*B*a^4*b^4*c*d^4*i^3*n +
 260*(b*x + a)*B*a^3*b^4*c^2*d^4*i^3*n/(d*x + c) + 210*(b*x + a)^2*B*a^2*b
^4*c^3*d^4*i^3*n/(d*x + c)^2 + 30*(b*x + a)^3*B*a*b^4*c^4*d^4*i^3*n/(d*x +
 c)^3 - 11*B*a^5*b^3*d^5*i^3*n - 130*(b*x + a)*B*a^4*b^3*c*d^5*i^3*n/(d*x
+ c) - 210*(b*x + a)^2*B*a^3*b^3*c^2*d^5*i^3*n/(d*x + c)^2 - 60*(b*x + a)^
3*B*a^2*b^3*c^3*d^5*i^3*n/(d*x + c)^3 + 26*(b*x + a)*B*a^5*b^2*d^6*i^3*n/(
d*x + c) + 105*(b*x + a)^2*B*a^4*b^2*c*d^6*i^3*n/(d*x + c)^2 + 60*(b*x + a
)^3*B*a^3*b^2*c^2*d^6*i^3*n/(d*x + c)^3 - 21*(b*x + a)^2*B*a^5*b*d^7*i^3*n
/(d*x + c)^2 - 30*(b*x + a)^3*B*a^4*b*c*d^7*i^3*n/(d*x + c)^3 + 6*(b*x + a
)^3*B*a^5*d^8*i^3*n/(d*x + c)^3 - 6*B*b^8*c^5*i^3*log(e) + 30*B*a*b^7*c^4*
d*i^3*log(e) - 60*B*a^2*b^6*c^3*d^2*i^3*log(e) + 60*B*a^3*b^5*c^2*d^3*i...
```

Mupad [B] (verification not implemented)

Time = 26.27 (sec) , antiderivative size = 588, normalized size of antiderivative = 3.77

$$
\begin{aligned}
&\int (ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx \\
&= x^3\left(\frac{d^2i^3(4Aad+16Abc+Badn-Bbcn)}{12b}-\frac{Ad^2i^3(4ad+4bc)}{12b}\right) \\
&\quad -x^2\left(\frac{\left(\frac{d^2i^3(4Aad+16Abc+Badn-Bbcn)}{4b}-\frac{Ad^2i^3(4ad+4bc)}{4b}\right)(4ad+4bc)}{8bd}\right. \\
&\qquad \left.-\frac{cdi^3(4Aad+6Abc+Badn-Bbcn)}{2b}+\frac{Aacd^2i^3}{2b}\right) \\
&\quad +\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(Bc^3i^3x+\frac{3Bc^2di^3x^2}{2}+Bcd^2i^3x^3+\frac{Bd^3i^3x^4}{4}\right) \\
&\quad +x\left(\frac{(4ad+4bc)\left(\frac{\left(\frac{d^2i^3(4Aad+16Abc+Badn-Bbcn)}{4b}-\frac{Ad^2i^3(4ad+4bc)}{4b}\right)(4ad+4bc)}{4bd}-\frac{cdi^3(4Aad+6Abc+Badn-Bbc}{b}\right.}{4bd}\right. \\
&\qquad \left.\left.+\frac{c^2i^3(12Aad+8Abc+3Badn-3Bbcn)}{2b}-\frac{ac\left(\frac{d^2i^3(4Aad+16Abc+Badn-Bbcn)}{4b}-\frac{Ad^2i^3(4ad+4bc)}{4b}\right)}{bd}\right) \\
&\quad -\frac{\ln(a+bx)\left(Bna^4d^3i^3-4Bna^3bcd^2i^3+6Bna^2b^2c^2di^3-4Bnab^3c^3i^3\right)}{4b^4} \\
&\quad +\frac{Ad^3i^3x^4}{4}-\frac{Bc^4i^3n\ln(c+dx)}{4d}
\end{aligned}
$$

input

```
int((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output

```
x^3*((d^2*i^3*(4*A*a*d + 16*A*b*c + B*a*d*n - B*b*c*n))/(12*b) - (A*d^2*i^
3*(4*a*d + 4*b*c))/(12*b)) - x^2*((((d^2*i^3*(4*A*a*d + 16*A*b*c + B*a*d*n
 - B*b*c*n))/(4*b) - (A*d^2*i^3*(4*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(
8*b*d) - (c*d*i^3*(4*A*a*d + 6*A*b*c + B*a*d*n - B*b*c*n))/(2*b) + (A*a*c*
d^2*i^3)/(2*b)) + log(e*((a + b*x)/(c + d*x))^n)*((B*d^3*i^3*x^4)/4 + B*c^
3*i^3*x + (3*B*c^2*d*i^3*x^2)/2 + B*c*d^2*i^3*x^3) + x*(((4*a*d + 4*b*c)*(
(((d^2*i^3*(4*A*a*d + 16*A*b*c + B*a*d*n - B*b*c*n))/(4*b) - (A*d^2*i^3*(4
*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(4*b*d) - (c*d*i^3*(4*A*a*d + 6*A*b
*c + B*a*d*n - B*b*c*n))/b + (A*a*c*d^2*i^3)/b))/(4*b*d) + (c^2*i^3*(12*A*
a*d + 8*A*b*c + 3*B*a*d*n - 3*B*b*c*n))/(2*b) - (a*c*((d^2*i^3*(4*A*a*d +
16*A*b*c + B*a*d*n - B*b*c*n))/(4*b) - (A*d^2*i^3*(4*a*d + 4*b*c))/(4*b)))
/(b*d)) - (log(a + b*x)*(B*a^4*d^3*i^3*n - 4*B*a*b^3*c^3*i^3*n - 4*B*a^3*b
*c*d^2*i^3*n + 6*B*a^2*b^2*c^2*d*i^3*n))/(4*b^4) + (A*d^3*i^3*x^4)/4 - (B*
c^4*i^3*n*log(c + d*x))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 507, normalized size of antiderivative = 3.25

$$\int (ci+dix)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$

$$= \frac{i\left(-24\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) a^3 bc\, d^3 + 36\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) a^2 b^2 c^2 d^2 - 24\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) a\, b^3 c^3 d - 24\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) b^4$$

input

```
int((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output

```
(i*(6*log(c + d*x)*a**4*d**4*n - 24*log(c + d*x)*a**3*b*c*d**3*n + 36*log(
c + d*x)*a**2*b**2*c**2*d**2*n - 24*log(c + d*x)*a*b**3*c**3*d*n + 6*log(c
 + d*x)*b**4*c**4*n + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*d**4 - 24*
log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*c*d**3 + 36*log(((a + b*x)**n*e)
/(c + d*x)**n)*a**2*b**2*c**2*d**2 - 24*log(((a + b*x)**n*e)/(c + d*x)**n)
*a*b**3*c**3*d - 24*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*c**3*d*x - 36*
log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*c**2*d**2*x**2 - 24*log(((a + b*x)
**n*e)/(c + d*x)**n)*b**4*c*d**3*x**3 - 6*log(((a + b*x)**n*e)/(c + d*x)**
n)*b**4*d**4*x**4 - 6*a**3*b*d**4*n*x + 24*a**2*b**2*c*d**3*n*x + 3*a**2*b
**2*d**4*n*x**2 - 24*a*b**3*c**3*d*x - 36*a*b**3*c**2*d**2*n*x - 36*a*b**3
*c**2*d**2*x**2 - 12*a*b**3*c*d**3*n*x**2 - 24*a*b**3*c*d**3*x**3 - 2*a*b*
*3*d**4*n*x**3 - 6*a*b**3*d**4*x**4 + 18*b**4*c**3*d*n*x + 9*b**4*c**2*d**
2*n*x**2 + 2*b**4*c*d**3*n*x**3))/(24*b**3*d)
```

3.131 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$

Optimal result . . . 1541
Mathematica [A] (verified) . . . 1542
Rubi [A] (verified) . . . 1543
Maple [F] . . . 1549
Fricas [F] . . . 1549
Sympy [F] . . . 1550
Maxima [B] (verification not implemented) . . . 1550
Giac [F] . . . 1551
Mupad [F(-1)] . . . 1552
Reduce [F] . . . 1552

Optimal result

Integrand size = 43, antiderivative size = 373

$$
\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx \\
&= -\frac{5Bd(bc-ad)^2i^3nx}{6b^3g} - \frac{B(bc-ad)i^3n(c+dx)^2}{6b^2g} \\
&\quad + \frac{d(bc-ad)^2i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4g} \\
&\quad + \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g} \\
&\quad + \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bg} \\
&\quad - \frac{5B(bc-ad)^3i^3n\log\left(\frac{a+bx}{c+dx}\right)}{6b^4g} - \frac{11B(bc-ad)^3i^3n\log(c+dx)}{6b^4g} \\
&\quad - \frac{(bc-ad)^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} \\
&\quad + \frac{B(bc-ad)^3i^3n\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}
\end{aligned}
$$

output

```
-5/6*B*d*(-a*d+b*c)^2*i^3*n*x/b^3/g-1/6*B*(-a*d+b*c)*i^3*n*(d*x+c)^2/b^2/g
+d*(-a*d+b*c)^2*i^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/g+1/2*(-a*
d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g+1/3*i^3*(d*x+c)
^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g-5/6*B*(-a*d+b*c)^3*i^3*n*ln((b*x+a)
/(d*x+c))/b^4/g-11/6*B*(-a*d+b*c)^3*i^3*n*ln(d*x+c)/b^4/g-(-a*d+b*c)^3*i^3
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g+B*(-a*d+b
*c)^3*i^3*n*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.99

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i^3\Big(6Abd(bc-ad)^2x-3B(bc-ad)^2n(bdx+(bc-ad)\log(a+bx))-B(bc-ad)n(2bd(bc-ad)x+b^2($$

input

```
Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g +
b*g*x),x]
```

output

```
(i^3*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*c - a*d)*L
og[a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*
(b*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*
x)/(c + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) +
 6*(b*c - a*d)^3*Log[g*(a + b*x)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) -
 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[g*(a + b*x)]*
(Log[g*(a + b*x)] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)])))/(6*b^4*g)
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.28, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.326$, Rules used = {2961, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{ag+bgx}\,dx$$

$$\downarrow 2961$$

$$\frac{i^3(bc-ad)^3\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{g}$$

$$\downarrow 2789$$

$$\frac{i^3(bc-ad)^3\left(\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\downarrow 2756$$

$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int \frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\downarrow 54$$

$$i^3(bc-ad)^3\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a+bx}{c+dx}}{3d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B}{(a+bx)\left(b}}{g}$$

$$\downarrow 2009$$

$$i^3(bc-ad)^3\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^3}+\frac{1}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{}{2b\left(}\right.}{3d}\right)}{b}}{g}$$

$$\downarrow 2789$$

$$i^3(bc-ad)^3\left(\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\right.}{}\right.}{}\right)}{g}$$

$$\downarrow 2756$$

$$i^3(bc-ad)^3\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right.}{g}$$

$$\downarrow 54$$

$$i^3(bc-ad)^3\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+b}{c+d}}{b}\right)}{b}$$

$$\downarrow 2009$$

$$i^3(bc-ad)^3\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}\right)}{b}+\frac{}{g}$$

$$\downarrow 2789$$

$$i^3(bc-ad)^3\left(\frac{\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d}{}\right.}{b^2}\right.}{2}\right.}{b}\right.}{g}$$

$$\downarrow 2751$$

$$i^3(bc-ad)^3\left(\frac{\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right.}{b}\right.}{}$$

$$\downarrow 16$$

$$i^3(bc-ad)^3\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\right.}{b}\right.}{}$$

$$\downarrow 2779$$

$$i^3(bc-ad)^3\left(\frac{\frac{\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(\cdots}{\cdots}\right)}{bd}\cdots\right)}{b}}{b}\cdots}{b}\cdots\right)$$

$\downarrow$ 2838

$$i^3(bc-ad)^3\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{\frac{\frac{Bn\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)(\cdots}{b}}{b}\cdots}{b}\cdots\right)$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
,x]
```

output
```
((b*c - a*d)^3*i^3*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*d*(b - (
d*(a + b*x))/(c + d*x))^3) - (B*n*(1/(2*b*(b - (d*(a + b*x))/(c + d*x))^2)
 + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^3 -
Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(3*d)))/b + ((d*((A + B*Log[e*((a +
 b*x)/(c + d*x))^n])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*(1/(b*(b
 - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(
a + b*x))/(c + d*x)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*Log[e*((a +
b*x)/(c + d*x))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*n*Lo
g[b - (d*(a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*((a + b*x)/(c
 + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b
*(c + d*x))/(d*(a + b*x))])/b)/b)/b)/b))/g
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(dix+ci)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{bgx+ag}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, al
gorithm="fricas")
```

output
```
integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3
+ (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log(e*
((b*x + a)/(d*x + c))^n))/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$

$$= \frac{i^3\left(\int \frac{Ac^3}{a+bx}\,dx+\int \frac{Ad^3x^3}{a+bx}\,dx+\int \frac{Bc^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx+\int \frac{3Acd^2x^2}{a+bx}\,dx+\int \frac{3Ac^2dx}{a+bx}\,dx+\int \frac{Bd^3x^3\log\left(e\left(\frac{a}{c+dx}\right.\right.\right.}{a+bx}}{g}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g),x)
```

output
```
i**3*(Integral(A*c**3/(a + b*x), x) + Integral(A*d**3*x**3/(a + b*x), x) +
 Integral(B*c**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x) + I
ntegral(3*A*c*d**2*x**2/(a + b*x), x) + Integral(3*A*c**2*d*x/(a + b*x), x
) + Integral(B*d**3*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x)
, x) + Integral(3*B*c*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a
 + b*x), x) + Integral(3*B*c**2*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n
)/(a + b*x), x))/g
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(360) = 720$.

Time $= 0.38$ (sec) , antiderivative size $= 935$, normalized size of antiderivative $= 2.51$

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, al
gorithm="maxima")
```

output

```
3*A*c^2*d*i^3*(x/(b*g) - a*log(b*x + a)/(b^2*g)) - 1/6*A*d^3*i^3*(6*a^3*lo
g(b*x + a)/(b^4*g) - (2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/(b^3*g)) + 3/2*A*c*
d^2*i^3*(2*a^2*log(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A*c^3*i^3
*log(b*g*x + a*g)/(b*g) - 1/6*(11*b^2*c^3*i^3*n - 15*a*b*c^2*d*i^3*n + 6*a
^2*c*d^2*i^3*n)*B*log(d*x + c)/(b^3*g) + (b^3*c^3*i^3*n - 3*a*b^2*c^2*d*i^
3*n + 3*a^2*b*c*d^2*i^3*n - a^3*d^3*i^3*n)*(log(b*x + a)*log((b*d*x + a*d)
/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^4*g) + 1/6*(2*
B*b^3*d^3*i^3*x^3*log(e) - ((i^3*n - 9*i^3*log(e))*b^3*c*d^2 - (i^3*n - 3*
i^3*log(e))*a*b^2*d^3)*B*x^2 - 3*(b^3*c^3*i^3*n - 3*a*b^2*c^2*d*i^3*n + 3*
a^2*b*c*d^2*i^3*n - a^3*d^3*i^3*n)*B*log(b*x + a)^2 - ((7*i^3*n - 18*i^3*l
og(e))*b^3*c^2*d - 6*(2*i^3*n - 3*i^3*log(e))*a*b^2*c*d^2 + (5*i^3*n - 6*i
^3*log(e))*a^2*b*d^3)*B*x + (6*b^3*c^3*i^3*log(e) + 18*(i^3*n - i^3*log(e)
)*a*b^2*c^2*d - 9*(3*i^3*n - 2*i^3*log(e))*a^2*b*c*d^2 + (11*i^3*n - 6*i^3
*log(e))*a^3*d^3)*B*log(b*x + a) + (2*B*b^3*d^3*i^3*x^3 + 3*(3*b^3*c*d^2*i
^3 - a*b^2*d^3*i^3)*B*x^2 + 6*(3*b^3*c^2*d*i^3 - 3*a*b^2*c*d^2*i^3 + a^2*b
*d^3*i^3)*B*x + 6*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*d^2*i^3 - a
^3*d^3*i^3)*B*log(b*x + a))*log((b*x + a)^n) - (2*B*b^3*d^3*i^3*x^3 + 3*(3
*b^3*c*d^2*i^3 - a*b^2*d^3*i^3)*B*x^2 + 6*(3*b^3*c^2*d*i^3 - 3*a*b^2*c*d^2
*i^3 + a^2*b*d^3*i^3)*B*x + 6*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c
*d^2*i^3 - a^3*d^3*i^3)*B*log(b*x + a))*log((d*x + c)^n))/(b^4*g)
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{bgx+ag}\,dx$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, al
gorithm="giac")
```

output

```
integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x +
a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i\left(-6\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{bx+a}dx\right)b^5c^3 - 6\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x^3}{bx+a}dx\right)b^5d^3 - 18\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x^2}{bx+a}dx\right)b^5c\,d^2 - 18\left(\int \frac{\log}{}\right.\right.\right.}{}$$

input
```
int((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

output
```
(i*( - 6*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a + b*x),x)*b**5*c**3 - 6
*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(a + b*x),x)*b**5*d**3 - 18
*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(a + b*x),x)*b**5*c*d**2 -
18*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a + b*x),x)*b**5*c**2*d + 6
*log(a + b*x)*a**4*d**3 - 18*log(a + b*x)*a**3*b*c*d**2 + 18*log(a + b*x)*
a**2*b**2*c**2*d - 6*log(a + b*x)*a*b**3*c**3 - 6*a**3*b*d**3*x + 18*a**2*
b**2*c*d**2*x + 3*a**2*b**2*d**3*x**2 - 18*a*b**3*c**2*d*x - 9*a*b**3*c*d*
*2*x**2 - 2*a*b**3*d**3*x**3))/(6*b**4*g)
```

3.132 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$

Optimal result . . . 1553
Mathematica [A] (verified) . . . 1554
Rubi [A] (verified) . . . 1555
Maple [F] . . . 1556
Fricas [F] . . . 1557
Sympy [F(-1)] . . . 1557
Maxima [B] (verification not implemented) . . . 1558
Giac [F] . . . 1559
Mupad [F(-1)] . . . 1559
Reduce [F] . . . 1559

Optimal result

Integrand size = 43, antiderivative size = 390

$$
\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx \\
&= -\frac{Bd^2(bc-ad)i^3nx}{2b^3g^2} - \frac{B(bc-ad)^2i^3n(c+dx)}{b^3g^2(a+bx)} \\
&\quad + \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4g^2} \\
&\quad - \frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^2(a+bx)} \\
&\quad + \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^2} \\
&\quad - \frac{Bd(bc-ad)^2i^3n\log\left(\frac{a+bx}{c+dx}\right)}{2b^4g^2} - \frac{5Bd(bc-ad)^2i^3n\log(c+dx)}{2b^4g^2} \\
&\quad - \frac{3d(bc-ad)^2i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} \\
&\quad + \frac{3Bd(bc-ad)^2i^3n\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}
\end{aligned}
$$

output
```
-1/2*B*d^2*(-a*d+b*c)*i^3*n*x/b^3/g^2-B*(-a*d+b*c)^2*i^3*n*(d*x+c)/b^3/g^2
/(b*x+a)+2*d^2*(-a*d+b*c)*i^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/
g^2-(-a*d+b*c)^2*i^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/g^2/(b*x+
a)+1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^2-1/2*B*d*(-a
*d+b*c)^2*i^3*n*ln((b*x+a)/(d*x+c))/b^4/g^2-5/2*B*d*(-a*d+b*c)^2*i^3*n*ln(
d*x+c)/b^4/g^2-3*d*(-a*d+b*c)^2*i^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b
*(d*x+c)/d/(b*x+a))/b^4/g^2+3*B*d*(-a*d+b*c)^2*i^3*n*polylog(2,b*(d*x+c)/d
/(b*x+a))/b^4/g^2
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.01

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$$
$$= \frac{i^3\left(2Abd^2(3bc-2ad)x - bBd^2(bc-ad)nx - \frac{2B(bc-ad)^3n}{a+bx} - a^2Bd^3n\log(a+bx) - 2Bd(bc-ad)^2n\log(a\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g +
b*g*x)^2,x]
```

output
```
(i^3*(2*A*b*d^2*(3*b*c - 2*a*d)*x - b*B*d^2*(b*c - a*d)*n*x - (2*B*(b*c -
a*d)^3*n)/(a + b*x) - a^2*B*d^3*n*Log[a + b*x] - 2*B*d*(b*c - a*d)^2*n*Log
[a + b*x] + 2*B*d^2*(3*b*c - 2*a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^
n] + b^2*d^3*x^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (2*(b*c - a*d)^3
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + 6*d*(b*c - a*d)^2*Log
[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + b^2*B*c^2*d*n*Log[c + d
*x] + 2*B*d*(b*c - a*d)^2*n*Log[c + d*x] - 2*B*d*(-(b*c) + a*d)*(-3*b*c +
2*a*d)*n*Log[c + d*x] - 3*B*d*(b*c - a*d)^2*n*(Log[a + b*x]*(Log[a + b*x]
- 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) +
 a*d)])))/(2*b^4*g^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^2}\,dx$$

$$\downarrow 2961$$

$$\frac{i^3(bc-ad)^2\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{g^2}$$

$$\downarrow 2793$$

$$\frac{i^3(bc-ad)^2\int\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d^2}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d^2}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d}{b^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(c+dx)^2\left(A+B\log}{b^3(a+b}\right.}{g^2}$$

$$\downarrow 2009$$

$$\frac{i^3(bc-ad)^2\left(\frac{2d^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{3d\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3(a+bx)}+\right.}{}$$

input

```
Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
^2,x]
```

output
```
((b*c - a*d)^2*i^3*(-((B*n*(c + d*x))/(b^3*(a + b*x))) - (B*d*n)/(2*b^3*(b
 - (d*(a + b*x))/(c + d*x))) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(b^3*(a + b*x)) + (d*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*b^
2*(b - (d*(a + b*x))/(c + d*x))^2) + (2*d^2*(a + b*x)*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(b^4*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (B*d*n
*Log[(a + b*x)/(c + d*x)])/(2*b^4) + (5*B*d*n*Log[b - (d*(a + b*x))/(c + d
*x)])/(2*b^4) - (3*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c
 + d*x))/(d*(a + b*x))])/b^4 + (3*B*d*n*PolyLog[2, (b*(c + d*x))/(d*(a + b*
x))])/b^4))/g^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(dix+ci)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(bgx+ag)^2}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x,
algorithm="fricas")
```

output
```
integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3
+ (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log(e*
((b*x + a)/(d*x + c))^n))/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2,x
)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1785 vs. $2(381) = 762$.

Time $= 0.38$ (sec) , antiderivative size $= 1785$, normalized size of antiderivative $= 4.58$

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x,
algorithm="maxima")
```

output

```
-B*c^3*i^3*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^
2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - 3*A*(a^2/(b^4*g^2*x + a*b^3*g
^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^2))*c*d^2*i^3 + 1/2*(2*a^3/(b^
5*g^2*x + a*b^4*g^2) + 6*a^2*log(b*x + a)/(b^4*g^2) + (b*x^2 - 4*a*x)/(b^3
*g^2))*A*d^3*i^3 + 3*A*c^2*d*i^3*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)
/(b^2*g^2)) - B*c^3*i^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^2*g^2*x
+ a*b*g^2) - A*c^3*i^3/(b^2*g^2*x + a*b*g^2) - 1/2*(5*b^3*c^3*d*i^3*n - 3*
a*b^2*c^2*d^2*i^3*n - 2*a^2*b*c*d^3*i^3*n + 2*a^3*d^4*i^3*n)*B*log(d*x + c
)/(b^5*c*g^2 - a*b^4*d*g^2) + 1/2*((b^4*c*d^3*i^3*log(e) - a*b^3*d^4*i^3*l
og(e))*B*x^3 - ((i^3*n - 6*i^3*log(e))*b^4*c^2*d^2 - (2*i^3*n - 9*i^3*log(
e))*a*b^3*c*d^3 + (i^3*n - 3*i^3*log(e))*a^2*b^2*d^4)*B*x^2 - ((i^3*n - 6*
i^3*log(e))*a*b^3*c^2*d^2 - 2*(i^3*n - 5*i^3*log(e))*a^2*b^2*c*d^3 + (i^3*
n - 4*i^3*log(e))*a^3*b*d^4)*B*x - 3*((b^4*c^3*d*i^3*n - 3*a*b^3*c^2*d^2*i
^3*n + 3*a^2*b^2*c*d^3*i^3*n - a^3*b*d^4*i^3*n)*B*x + (a*b^3*c^3*d*i^3*n -
 3*a^2*b^2*c^2*d^2*i^3*n + 3*a^3*b*c*d^3*i^3*n - a^4*d^4*i^3*n)*B)*log(b*x
 + a)^2 + 2*(3*(i^3*n + i^3*log(e))*a*b^3*c^3*d - 6*(i^3*n + i^3*log(e))*a
^2*b^2*c^2*d^2 + 4*(i^3*n + i^3*log(e))*a^3*b*c*d^3 - (i^3*n + i^3*log(e))
*a^4*d^4)*B + ((6*b^4*c^3*d*i^3*log(e) + 6*(2*i^3*n - 3*i^3*log(e))*a*b^3*
c^2*d^2 - (17*i^3*n - 18*i^3*log(e))*a^2*b^2*c*d^3 + (7*i^3*n - 6*i^3*log(
e))*a^3*b*d^4)*B*x + (6*a*b^3*c^3*d*i^3*log(e) + 6*(2*i^3*n - 3*i^3*log...
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x,
algorithm="giac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x +
a*g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(a\,g+b\,g\,x)^2}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^2,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^2, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

input
```
int((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

output

```
(i*( - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(a**2 + 2*a*b*x + b
**2*x**2),x)*a**3*b**5*d**4 + 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x*
*3)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**6*c*d**3 - 2*int((log(((a + b*
x)**n*e)/(c + d*x)**n)*x**3)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**6*d**
4*x + 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(a**2 + 2*a*b*x + b*
*2*x**2),x)*a*b**7*c*d**3*x - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x*
*2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**3*b**5*c*d**3 + 6*int((log(((a + b*
x)**n*e)/(c + d*x)**n)*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**6*c**
2*d**2 - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(a**2 + 2*a*b*x +
 b**2*x**2),x)*a**2*b**6*c*d**3*x + 6*int((log(((a + b*x)**n*e)/(c + d*x)*
*n)*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a*b**7*c**2*d**2*x - 6*int((log(
((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**3*b**
5*c**2*d**2 + 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**2 + 2*a*b*x
 + b**2*x**2),x)*a**2*b**6*c**3*d - 6*int((log(((a + b*x)**n*e)/(c + d*x)*
*n)*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**6*c**2*d**2*x + 6*int((log(
((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a*b**7*c
**3*d*x - 6*log(a + b*x)*a**6*d**4 + 18*log(a + b*x)*a**5*b*c*d**3 - 6*log
(a + b*x)*a**5*b*d**4*x - 18*log(a + b*x)*a**4*b**2*c**2*d**2 + 18*log(a +
 b*x)*a**4*b**2*c*d**3*x + 6*log(a + b*x)*a**3*b**3*c**3*d - 18*log(a + b*
x)*a**3*b**3*c**2*d**2*x + 6*log(a + b*x)*a**2*b**4*c**3*d*x - 2*log(a ...
```

3.133 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$

Optimal result . . . 1561
Mathematica [A] (verified) . . . 1562
Rubi [A] (verified) . . . 1563
Maple [F] . . . 1564
Fricas [F] . . . 1565
Sympy [F] . . . 1565
Maxima [B] (verification not implemented) . . . 1566
Giac [F] . . . 1567
Mupad [F(-1)] . . . 1567
Reduce [F] . . . 1567

Optimal result

Integrand size = 43, antiderivative size = 361

$$
\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx \\
&= -\frac{2Bd(bc-ad)i^3n(c+dx)}{b^3g^3(a+bx)} - \frac{B(bc-ad)i^3n(c+dx)^2}{4b^2g^3(a+bx)^2} \\
&\quad + \frac{d^3i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4g^3} \\
&\quad - \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^3(a+bx)} \\
&\quad - \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^3(a+bx)^2} - \frac{Bd^2(bc-ad)i^3n\log(c+dx)}{b^4g^3} \\
&\quad - \frac{3d^2(bc-ad)i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} \\
&\quad + \frac{3Bd^2(bc-ad)i^3n\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}
\end{aligned}
$$

output

```
-2*B*d*(-a*d+b*c)*i^3*n*(d*x+c)/b^3/g^3/(b*x+a)-1/4*B*(-a*d+b*c)*i^3*n*(d*
x+c)^2/b^2/g^3/(b*x+a)^2+d^3*i^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b
^4/g^3-2*d*(-a*d+b*c)*i^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/g^3/
(b*x+a)-1/2*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g
^3/(b*x+a)^2-B*d^2*(-a*d+b*c)*i^3*n*ln(d*x+c)/b^4/g^3-3*d^2*(-a*d+b*c)*i^3
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^3+3*B*d^2
*(-a*d+b*c)*i^3*n*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^3
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.92

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$$

$$= \frac{i^3\left(4Abd^3x - \frac{B(bc-ad)^3n}{(a+bx)^2} - \frac{10Bd(bc-ad)^2n}{a+bx} + 10Bd^2(-bc+ad)n\log(a+bx) + 4Bd^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{}$$

input

```
Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + 
b*g*x)^3,x]
```

output

```
(i^3*(4*A*b*d^3*x - (B*(b*c - a*d)^3*n)/(a + b*x)^2 - (10*B*d*(b*c - a*d)^
2*n)/(a + b*x) + 10*B*d^2*(-(b*c) + a*d)*n*Log[a + b*x] + 4*B*d^3*(a + b*x
)*Log[e*((a + b*x)/(c + d*x))^n] - (2*(b*c - a*d)^3*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]))/(a + b*x)^2 - (12*d*(b*c - a*d)^2*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]))/(a + b*x) + 12*d^2*(b*c - a*d)*Log[a + b*x]*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]) + 6*B*d^2*(b*c - a*d)*n*Log[c + d*x] + 6*B*d^2*
(-(b*c) + a*d)*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - 
a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(4*b^4*g^3)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{i^3(bc-ad)\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g^3}$$

$$\downarrow 2793$$

$$\frac{i^3(bc-ad)\int \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d^3}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d^2}{b^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d}{b^3(a+bx)^2}+\frac{(c+dx)^3\left(A}{}\right.}{g^3}$$

$$\downarrow 2009$$

$$\frac{i^3(bc-ad)\left(\frac{d^3(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{3d^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4}-\frac{2d(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3(a+bx)}-\right.}{g^3}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
^3,x]
```

output
```
((b*c - a*d)*i^3*((-2*B*d*n*(c + d*x))/(b^3*(a + b*x)) - (B*n*(c + d*x)^2)
/(4*b^2*(a + b*x)^2) - (2*d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]))/(b^3*(a + b*x)) - ((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
/(2*b^2*(a + b*x)^2) + (d^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]))/(b^4*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*d^2*n*Log[b - (d*(a
 + b*x))/(c + d*x)])/b^4 - (3*d^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*L
og[1 - (b*(c + d*x))/(d*(a + b*x))])/b^4 + (3*B*d^2*n*PolyLog[2, (b*(c + d
*x))/(d*(a + b*x))])/b^4))/g^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(dix+ci)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(bgx+ag)^3}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,
algorithm="fricas")
```

output
```
integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3
+ (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log(e*
((b*x + a)/(d*x + c))^n))/(b^3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x +
 a^3*g^3), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$$
$$= \frac{i^3\left(\int \frac{Ac^3}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{Ad^3x^3}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{Bc^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{3Acd^2x^2}{a^3+3a^2bx+3ab^2x^2+}\right.}{}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3,x
)
```

output
```
i**3*(Integral(A*c**3/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x)
+ Integral(A*d**3*x**3/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x)
 + Integral(B*c**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*
b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(3*A*c*d**2*x**2/(a**3 + 3*
a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(3*A*c**2*d*x/(a**3 + 
3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(B*d**3*x**3*log(e*(
a/(c + d*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3
*x**3), x) + Integral(3*B*c*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(3*B*c**
2*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**
2*x**2 + b**3*x**3), x))/g**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2746 vs. $2(356) = 712$.

Time $= 0.43$ (sec) , antiderivative size $= 2746$, normalized size of antiderivative $= 7.61$

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,
algorithm="maxima")
```

output

```
-3/4*B*c^2*d*i^3*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c - a*
b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g
^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d
^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*
b^2*d^2)*g^3)) + 1/4*B*c^3*i^3*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*
d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) +
2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d
*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A*d^3*i^3*((6*a^2
*b*x + 5*a^3)/(b^6*g^3*x^2 + 2*a*b^5*g^3*x + a^2*b^4*g^3) - 2*x/(b^3*g^3)
+ 6*a*log(b*x + a)/(b^4*g^3)) + 3/2*A*c*d^2*i^3*((4*a*b*x + 3*a^2)/(b^5*g^
3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3)) - 3/2*(2*
b*x + a)*B*c^2*d*i^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^3*x^2 +
 2*a*b^3*g^3*x + a^2*b^2*g^3) - 3/2*(2*b*x + a)*A*c^2*d*i^3/(b^4*g^3*x^2 +
 2*a*b^3*g^3*x + a^2*b^2*g^3) - 1/2*B*c^3*i^3*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A*c^3*i^3/(b^3*
g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*(2*b^3*c^3*d^2*i^3*n + 8*a*b^2*
c^2*d^3*i^3*n - 13*a^2*b*c*d^4*i^3*n + 5*a^3*d^5*i^3*n)*B*log(d*x + c)/(b^
6*c^2*g^3 - 2*a*b^5*c*d*g^3 + a^2*b^4*d^2*g^3) + 1/4*(4*(b^5*c^2*d^3*i^3*l
og(e) - 2*a*b^4*c*d^4*i^3*log(e) + a^2*b^3*d^5*i^3*log(e))*B*x^3 + 8*(a*b^
4*c^2*d^3*i^3*log(e) - 2*a^2*b^3*c*d^4*i^3*log(e) + a^3*b^2*d^5*i^3*log...
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,
algorithm="giac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x +
a*g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(a\,g+b\,g\,x)^3}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^3,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^3, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \text{too large to display}$$

input
```
int((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
```

output

```
(i*( - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(a**3 + 3*a**2*b*x
+ 3*a*b**2*x**2 + b**3*x**3),x)*a**6*b**5*d**5 + 8*int((log(((a + b*x)**n*
e)/(c + d*x)**n)*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*
a**5*b**6*c*d**4 - 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(a**3 +
 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**5*b**6*d**5*x - 4*int((log(
((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 +
b**3*x**3),x)*a**4*b**7*c**2*d**3 + 16*int((log(((a + b*x)**n*e)/(c + d*x)
**n)*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**4*b**7*c*
d**4*x - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(a**3 + 3*a**2*b*
x + 3*a*b**2*x**2 + b**3*x**3),x)*a**4*b**7*d**5*x**2 - 8*int((log(((a + b
*x)**n*e)/(c + d*x)**n)*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x*
*3),x)*a**3*b**8*c**2*d**3*x + 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x
**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**3*b**8*c*d**4*x
**2 - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(a**3 + 3*a**2*b*x +
 3*a*b**2*x**2 + b**3*x**3),x)*a**2*b**9*c**2*d**3*x**2 - 12*int((log(((a
+ b*x)**n*e)/(c + d*x)**n)*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3
*x**3),x)*a**6*b**5*c*d**4 + 24*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x*
*2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**5*b**6*c**2*d**3
 - 24*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(a**3 + 3*a**2*b*x + 3
*a*b**2*x**2 + b**3*x**3),x)*a**5*b**6*c*d**4*x - 12*int((log(((a + b*x...
```

3.134 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$

Optimal result . . . 1569
Mathematica [A] (verified) . . . 1570
Rubi [A] (verified) . . . 1570
Maple [F] . . . 1575
Fricas [F] . . . 1576
Sympy [F] . . . 1576
Maxima [F] . . . 1577
Giac [F] . . . 1578
Mupad [F(-1)] . . . 1579
Reduce [F] . . . 1579

Optimal result

Integrand size = 43, antiderivative size = 326

$$\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx \\
&= -\frac{Bd^2i^3n(c+dx)}{b^3g^4(a+bx)} - \frac{Bdi^3n(c+dx)^2}{4b^2g^4(a+bx)^2} - \frac{Bi^3n(c+dx)^3}{9bg^4(a+bx)^3} \\
&\quad - \frac{d^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^4(a+bx)} \\
&\quad - \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^4(a+bx)^2} - \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bg^4(a+bx)^3} \\
&\quad - \frac{d^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} + \frac{Bd^3i^3n\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4}
\end{aligned}$$

output

```
-B*d^2*i^3*n*(d*x+c)/b^3/g^4/(b*x+a)-1/4*B*d*i^3*n*(d*x+c)^2/b^2/g^4/(b*x+
a)^2-1/9*B*i^3*n*(d*x+c)^3/b/g^4/(b*x+a)^3-d^2*i^3*(d*x+c)*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))/b^3/g^4/(b*x+a)-1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))/b^2/g^4/(b*x+a)^2-1/3*i^3*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))/b/g^4/(b*x+a)^3-d^3*i^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c
)/d/(b*x+a))/b^4/g^4+B*d^3*i^3*n*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^4
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$$
$$= \frac{i^3\left(-\frac{4B(bc-ad)^3n}{(a+bx)^3}-\frac{21Bd(bc-ad)^2n}{(a+bx)^2}+\frac{66Bd^2(-bc+ad)n}{a+bx}-66Bd^3n\log(a+bx)-\frac{12(bc-ad)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}-\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^4,x]
```

output
```
(i^3*((-4*B*(b*c - a*d)^3*n)/(a + b*x)^3 - (21*B*d*(b*c - a*d)^2*n)/(a + b*x)^2 + (66*B*d^2*(-(b*c) + a*d)*n)/(a + b*x) - 66*B*d^3*n*Log[a + b*x] - (12*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^3 - (54*d*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 + (108*d^2*(-(b*c) + a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + 36*d^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 66*B*d^3*n*Log[c + d*x] - 18*B*d^3*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(36*b^4*g^4)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.209$, Rules used = {2961, 2780, 2741, 2780, 2741, 2780, 2741, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^4}\,dx$$
$$\downarrow \text{2961}$$

$$\frac{i^3 \int \frac{(c+dx)^4 \left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)} d\frac{a+bx}{c+dx}}{g^4}$$

$$\downarrow 2780$$

$$\frac{i^3 \left(\frac{\int \frac{(c+dx)^4 \left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4} d\frac{a+bx}{c+dx}}{b} + \frac{d \int \frac{(c+dx)^3 \left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)} d\frac{a+bx}{c+dx}}{b} \right)}{g^4}$$

$$\downarrow 2741$$

$$\frac{i^3 \left(\frac{d \int \frac{(c+dx)^3 \left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)} d\frac{a+bx}{c+dx}}{b} + \frac{-\frac{(c+dx)^3 \left(B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3} - \frac{Bn(c+dx)^3}{9(a+bx)^3}}{b} \right)}{g^4}$$

$$\downarrow 2780$$

$$\frac{i^3 \left(\frac{d \left(\frac{\int \frac{(c+dx)^3 \left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3} d\frac{a+bx}{c+dx}}{b} + \frac{d \int \frac{(c+dx)^2 \left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)} d\frac{a+bx}{c+dx}}{b} \right)}{b} + \frac{-\frac{(c+dx)^3 \left(B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3} - \frac{Bn(c+dx)^3}{9(a+bx)^3}}{b} \right)}{g^4}$$

$$\downarrow 2741$$

$$\frac{i^3 \left(\frac{d \left(\frac{d \int \frac{(c+dx)^2 \left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)} d\frac{a+bx}{c+dx}}{b} + \frac{-\frac{(c+dx)^2 \left(B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2} - \frac{Bn(c+dx)^2}{4(a+bx)^2}}{b} \right)}{b} + \frac{-\frac{(c+dx)^3 \left(B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3} - \frac{Bn(c+}{9(a+b}}{b} \right)}{g^4}$$

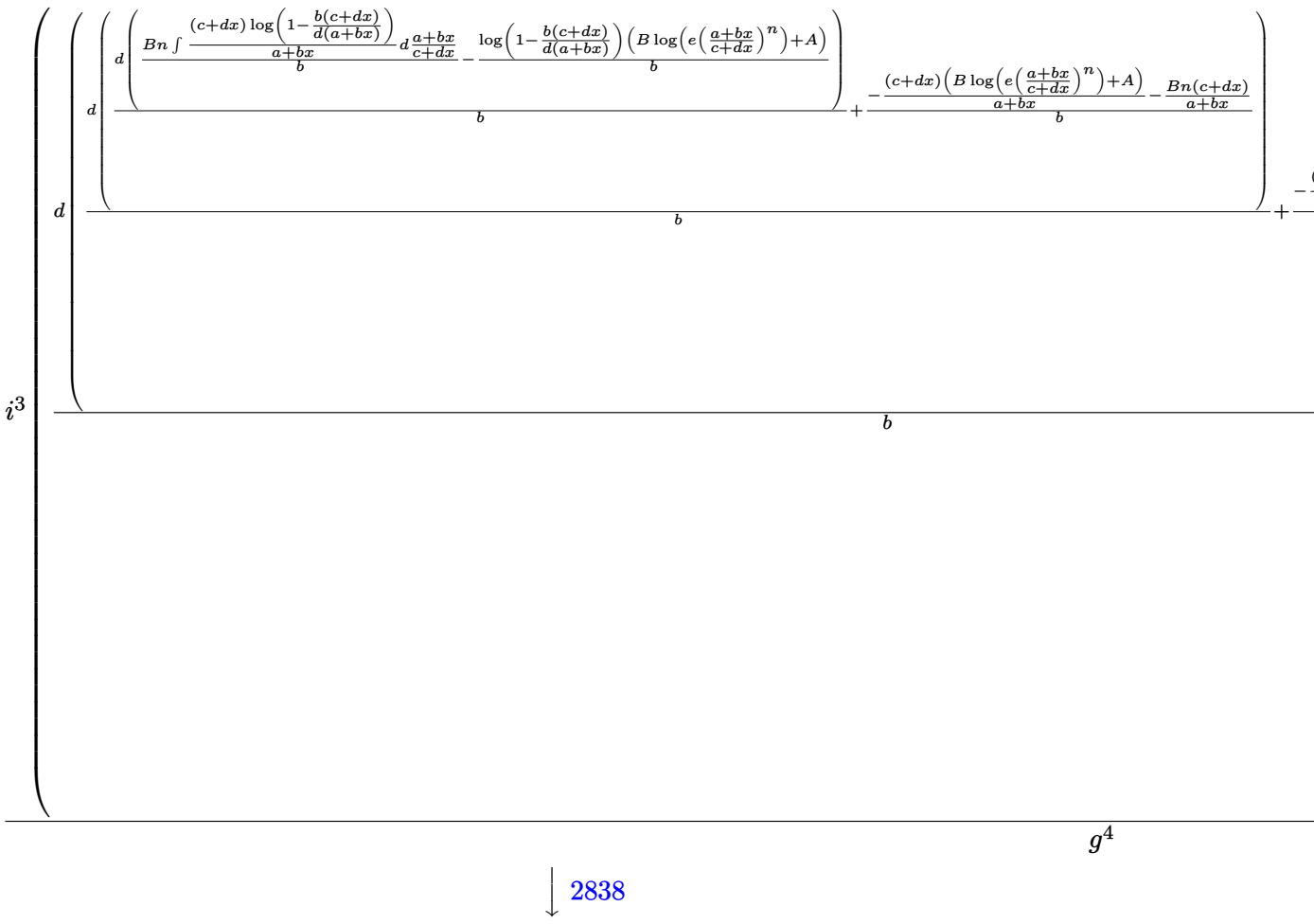
$$\downarrow 2780$$

$$i^3\left(\frac{d\left(\frac{d\left(\frac{\int\frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}}{b}\right)}{b}\right)}{g^4}$$

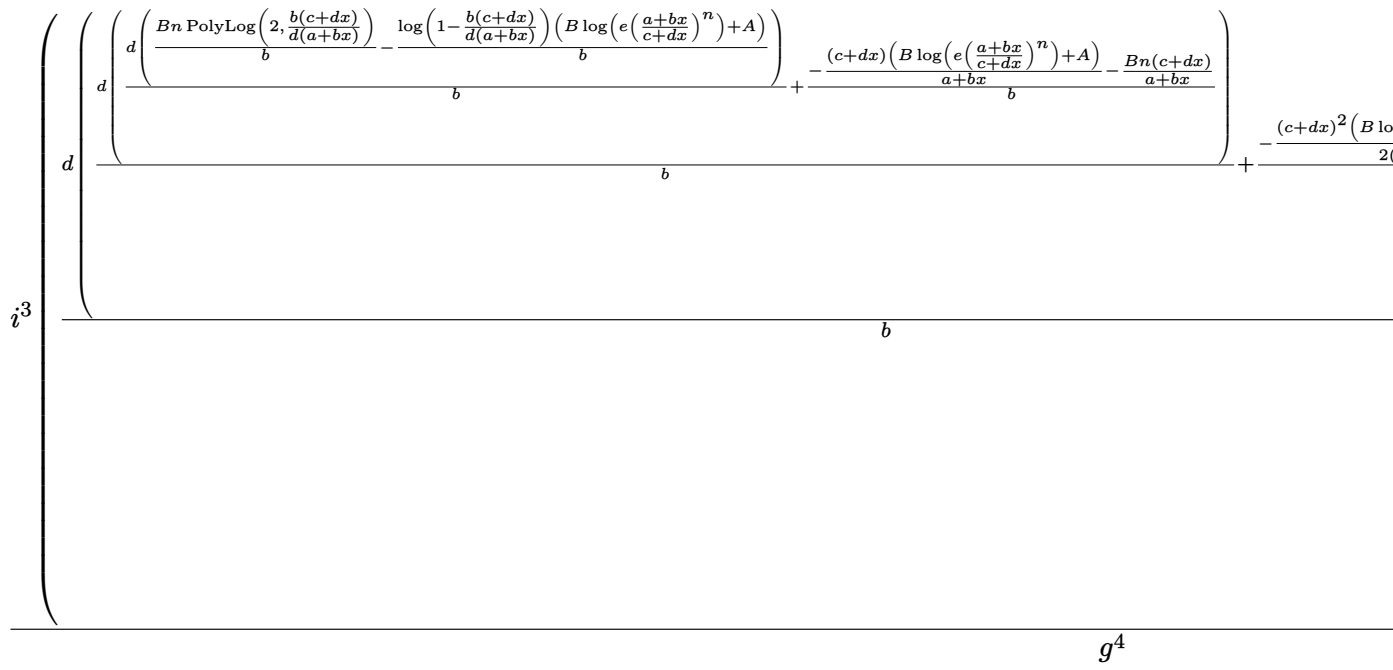
$$\downarrow 2741$$

$$i^3\left(\frac{d\left(\frac{d\left(\frac{d\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}}{b}\right)}{b}\right)}{g^4}$$

$$\downarrow 2779$$



$\downarrow$ 2838



input
```
Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
^4,x]
```

output
```
(i^3*((-1/9*(B*n*(c + d*x)^3)/(a + b*x)^3 - ((c + d*x)^3*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]))/(3*(a + b*x)^3))/b + (d*((-1/4*(B*n*(c + d*x)^2)/(a
  + b*x)^2 - ((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b
 *x)^2))/b + (d*((-((B*n*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[e*((
 a + b*x)/(c + d*x))^n]))/(a + b*x))/b + (d*(-(((A + B*Log[e*((a + b*x)/(c
 + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*
 (c + d*x))/(d*(a + b*x))])/b))/b))/b))/b))/g^4
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(dix+ci)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(bgx+ag)^4}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^4}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,
algorithm="fricas")
```

output
```
integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3
+ (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log(e*
((b*x + a)/(d*x + c))^n))/(b^4*g^4*x^4 + 4*a*b^3*g^4*x^3 + 6*a^2*b^2*g^4*x
^2 + 4*a^3*b*g^4*x + a^4*g^4), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$$
$$= \frac{i^3\left(\int \frac{Ac^3}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \int \frac{Ad^3x^3}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \int \frac{Bc^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx\right.}{}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4,x
)
```

output

```
i**3*(Integral(A*c**3/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**
3 + b**4*x**4), x) + Integral(A*d**3*x**3/(a**4 + 4*a**3*b*x + 6*a**2*b**2
*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(B*c**3*log(e*(a/(c + d*x
) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x*
*3 + b**4*x**4), x) + Integral(3*A*c*d**2*x**2/(a**4 + 4*a**3*b*x + 6*a**2
*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(3*A*c**2*d*x/(a**4
+ 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integra
l(B*d**3*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x +
 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(3*B*c*d**2*x
**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2*b*
*2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(3*B*c**2*d*x*log(e*(a/
(c + d*x) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a
*b**3*x**3 + b**4*x**4), x))/g**4
```

Maxima [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^4}\,dx$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,
algorithm="maxima")
```

output

```
-1/6*B*c*d^2*i^3*n*((11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*c
^2 - 3*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a
^3*b*d^2)*x)/((b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c^2
 - 2*a^2*b^6*c*d + a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d +
 a^4*b^4*d^2)*g^4*x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6
*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^
2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 +
 a^2*d^3)*log(d*x + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b
^3*d^3)*g^4)) - 1/18*B*c^3*i^3*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d +
 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4
*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a
^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2
*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*
a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c
^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/12*B*c^2*d*i^3*n*((5*a*b^2*c
^2 - 22*a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c
^2 - 16*a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)
*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^
5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d
+ a^5*b^2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*...
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^4}\,dx$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,
algorithm="giac")
```

output

```
integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x +
a*g)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(a\,g+b\,g\,x)^4}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^4,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^4, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \text{too large to display}$$

input
```
int((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x)
```

output

```
(i*( - 36*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(a**4 + 4*a**3*b*x
 + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4),x)*a**8*b**5*d**6 + 108*i
nt((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b
**2*x**2 + 4*a*b**3*x**3 + b**4*x**4),x)*a**7*b**6*c*d**5 - 108*int((log((
(a + b*x)**n*e)/(c + d*x)**n)*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2
+ 4*a*b**3*x**3 + b**4*x**4),x)*a**7*b**6*d**6*x - 108*int((log(((a + b*x)
**n*e)/(c + d*x)**n)*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**
3*x**3 + b**4*x**4),x)*a**6*b**7*c**2*d**4 + 324*int((log(((a + b*x)**n*e)
/(c + d*x)**n)*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3
 + b**4*x**4),x)*a**6*b**7*c*d**5*x - 108*int((log(((a + b*x)**n*e)/(c + d
*x)**n)*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4
*x**4),x)*a**6*b**7*d**6*x**2 + 36*int((log(((a + b*x)**n*e)/(c + d*x)**n)
*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4),
x)*a**5*b**8*c**3*d**3 - 324*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)
/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4),x)*a**
5*b**8*c**2*d**4*x + 324*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(a*
*4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4),x)*a**5*b*
*8*c*d**5*x**2 - 36*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(a**4 +
4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4),x)*a**5*b**8*d*
*6*x**3 + 108*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(a**4 + 4*a...
```

3.135 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$

Optimal result . . . 1581
Mathematica [A] (verified) . . . 1582
Rubi [A] (verified) . . . 1582
Maple [F] . . . 1585
Fricas [F] . . . 1585
Sympy [F] . . . 1586
Maxima [B] (verification not implemented) . . . 1586
Giac [F(-1)] . . . 1587
Mupad [F(-1)] . . . 1588
Reduce [F] . . . 1588

Optimal result

Integrand size = 43, antiderivative size = 269

$$\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx \\
&= \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3di} \\
&\quad - \frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^2i} \\
&\quad + \frac{(bc-ad)^2g^3(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^3i} \\
&\quad + \frac{(bc-ad)^3g^3\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{6d^4i} \\
&\quad + \frac{B(bc-ad)^3g^3n\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}
\end{aligned}$$

output

```
1/3*g^3*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i-1/6*(-a*d+b*c)*g^3*(
b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/d^2/i+1/6*(-a*d+b*c)^2*g^
3*(b*x+a)*(6*A+5*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))/d^3/i+1/6*(-a*d+b*c)^3
*g^3*(6*A+11*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d
^4/i+B*(-a*d+b*c)^3*g^3*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.38

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$=\frac{g^3\Big(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3d^2(-bc+ad)(a+bx)^2\left(A+B\log\left(e\left(\frac{}{}\right.\right.\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x),x]
```

output
```
(g^3*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) - 6*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[i*(c + d*x)] + 3*B*(b*c - a*d)^3*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[i*(c + d*x)])*Log[i*(c + d*x)] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(6*d^4*i)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {2961, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{ci+dix}\,dx$$
$$\downarrow 2961$$

$$\frac{g^3(bc-ad)^3\int\frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{i}$$

$$\downarrow 2784$$

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int\frac{(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{i}$$

$$\downarrow 2784$$

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}}{3d}\right)}{i}$$

$$\downarrow 2784$$

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+5Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{6A+11}{}}{2d}}{3d}\right)}{i}$$

$$\downarrow 2754$$

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+5Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{6Bn\int\frac{(c}{}}{}}{3d}\right)}{i}$$

$$\Big\downarrow \text{2838}$$

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+5Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\log\left(1-\right.}{}}{}}{3d}\right.}{i}$$

input
```
Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)
,x]
```

output
```
((b*c - a*d)^3*g^3*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(
3*d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a + b*x)^2*(3*A + B*
n + 3*B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x
))/(c + d*x))^2) - (((a + b*x)*(6*A + 5*B*n + 6*B*Log[e*((a + b*x)/(c + d*
x))^n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((6*A + 11*B*n +
 6*B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])
/d) - (6*B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d))/(3*d)))
/i
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{dix+ci}dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, al
gorithm="fricas")
```

output
```
integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
+ (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log(e*
((b*x + a)/(d*x + c))^n))/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$= \frac{g^3\left(\int \frac{Aa^3}{c+dx}\,dx+\int \frac{Ab^3x^3}{c+dx}\,dx+\int \frac{Ba^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx+\int \frac{3Aab^2x^2}{c+dx}\,dx+\int \frac{3Aa^2bx}{c+dx}\,dx+\int \frac{Bb^3x^3\log\left(e\left(\frac{a}{c+dx}\right.\right.\right.}{i}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i),x)
```

output
```
g**3*(Integral(A*a**3/(c + d*x), x) + Integral(A*b**3*x**3/(c + d*x), x) +
 Integral(B*a**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x) + I
ntegral(3*A*a*b**2*x**2/(c + d*x), x) + Integral(3*A*a**2*b*x/(c + d*x), x
) + Integral(B*b**3*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x)
, x) + Integral(3*B*a*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c
 + d*x), x) + Integral(3*B*a**2*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n
)/(c + d*x), x))/i
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. $2(260) = 520$.

Time $= 0.36$ (sec) , antiderivative size $= 1003$, normalized size of antiderivative $= 3.73$

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, al
gorithm="maxima")
```

output

```
3*A*a^2*b*g^3*(x/(d*i) - c*log(d*x + c)/(d^2*i)) - 1/6*A*b^3*g^3*(6*c^3*lo
g(d*x + c)/(d^4*i) - (2*d^2*x^3 - 3*c*d*x^2 + 6*c^2*x)/(d^3*i)) + 3/2*A*a*
b^2*g^3*(2*c^2*log(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A*a^3*g^3
*log(d*i*x + c*i)/(d*i) - (b^3*c^3*g^3*n - 3*a*b^2*c^2*d*g^3*n + 3*a^2*b*c
*d^2*g^3*n - a^3*d^3*g^3*n)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) +
1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^4*i) + 1/6*(6*a^3*d^3*g^3*log
(e) - (11*g^3*n + 6*g^3*log(e))*b^3*c^3 + 9*(3*g^3*n + 2*g^3*log(e))*a*b^2
*c^2*d - 18*(g^3*n + g^3*log(e))*a^2*b*c*d^2)*B*log(d*x + c)/(d^4*i) + 1/6
*(2*B*b^3*d^3*g^3*x^3*log(e) - ((g^3*n + 3*g^3*log(e))*b^3*c*d^2 - (g^3*n
+ 9*g^3*log(e))*a*b^2*d^3)*B*x^2 + 6*(b^3*c^3*g^3*n - 3*a*b^2*c^2*d*g^3*n
+ 3*a^2*b*c*d^2*g^3*n - a^3*d^3*g^3*n)*B*log(b*x + a)*log(d*x + c) - 3*(b^
3*c^3*g^3*n - 3*a*b^2*c^2*d*g^3*n + 3*a^2*b*c*d^2*g^3*n - a^3*d^3*g^3*n)*B
*log(d*x + c)^2 + ((5*g^3*n + 6*g^3*log(e))*b^3*c^2*d - 6*(2*g^3*n + 3*g^3
*log(e))*a*b^2*c*d^2 + (7*g^3*n + 18*g^3*log(e))*a^2*b*d^3)*B*x + (6*a*b^2
*c^2*d*g^3*n - 15*a^2*b*c*d^2*g^3*n + 11*a^3*d^3*g^3*n)*B*log(b*x + a) + (
2*B*b^3*d^3*g^3*x^3 - 3*(b^3*c*d^2*g^3 - 3*a*b^2*d^3*g^3)*B*x^2 + 6*(b^3*c
^2*d*g^3 - 3*a*b^2*c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B*x - 6*(b^3*c^3*g^3 - 3*a
*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*B*log(d*x + c))*log((b*x
 + a)^n) - (2*B*b^3*d^3*g^3*x^3 - 3*(b^3*c*d^2*g^3 - 3*a*b^2*d^3*g^3)*B*x^
2 + 6*(b^3*c^2*d*g^3 - 3*a*b^2*c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B*x - 6*(b^...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \text{Timed out}$$

input

```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, al
gorithm="giac")
```

output

```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \int \frac{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$= \frac{g^3 i\left(-6\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{dx+c}dx\right)a^3 b\,d^4 - 6\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x^3}{dx+c}dx\right)b^4 d^4 - 18\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x^2}{dx+c}dx\right)a\,b^3 d^4 - 18\Big($$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

output
```
(g**3*i*( - 6*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(c + d*x),x)*a**3*b*d
**4 - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(c + d*x),x)*b**4*d*
*4 - 18*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c + d*x),x)*a*b**3*
d**4 - 18*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c + d*x),x)*a**2*b**
2*d**4 - 6*log(c + d*x)*a**4*d**3 + 18*log(c + d*x)*a**3*b*c*d**2 - 18*log
(c + d*x)*a**2*b**2*c**2*d + 6*log(c + d*x)*a*b**3*c**3 - 18*a**3*b*d**3*x
 + 18*a**2*b**2*c*d**2*x - 9*a**2*b**2*d**3*x**2 - 6*a*b**3*c**2*d*x + 3*a
*b**3*c*d**2*x**2 - 2*a*b**3*d**3*x**3))/(6*d**4)
```

3.136 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$

Optimal result . . . 1589
Mathematica [A] (verified) . . . 1590
Rubi [A] (verified) . . . 1590
Maple [F] . . . 1593
Fricas [F] . . . 1593
Sympy [F] . . . 1593
Maxima [B] (verification not implemented) . . . 1594
Giac [B] (verification not implemented) . . . 1595
Mupad [F(-1)] . . . 1596
Reduce [F] . . . 1597

Optimal result

Integrand size = 43, antiderivative size = 211

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx \\
&= \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di} \\
&\quad - \frac{(bc-ad)g^2(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i} \\
&\quad - \frac{(bc-ad)^2g^2\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2d^3i} \\
&\quad - \frac{B(bc-ad)^2g^2n\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}
\end{aligned}$$

output

```
1/2*g^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i-1/2*(-a*d+b*c)*g^2*(
b*x+a)*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))/d^2/i-1/2*(-a*d+b*c)^2*g^2*
(2*A+3*B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^3/i-B
*(-a*d+b*c)^2*g^2*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.26

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$=\frac{g^2\Big(-2Abd(bc-ad)x+2Bd(-bc+ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x),x]
```

output
```
(g^2*(-2*A*b*d*(b*c - a*d)*x + 2*B*d*(-(b*c) + a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*B*(b*c - a*d)^2*n*Log[c + d*x] - B*(b*c - a*d)*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[i*(c + d*x)] - B*(b*c - a*d)^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[i*(c + d*x)])*Log[i*(c + d*x)] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(2*d^3*i)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2961, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{ci+dix}\,dx$$
$$\downarrow \text{2961}$$
$$\frac{g^2(bc-ad)^2\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{i}$$

$$\downarrow 2784$$

$$\frac{g^2(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{i}$$

$$\downarrow 2784$$

$$\frac{g^2(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}}{2d}\right)}{i}$$

$$\downarrow 2754$$

$$\frac{g^2(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2Bn\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a}{b(c-}\right.}{2d}\right)}{i}$$

$$\downarrow 2838$$

$$\frac{g^2(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+3Bn\right)}{d}}{2d}\right)}{i}$$

input

```
Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)
,x]
```

output
```
((b*c - a*d)^2*g^2*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(
2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(2*A + B*n
+ 2*B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c
+ d*x))) - (-(((2*A + 3*B*n + 2*B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 -
(d*(a + b*x))/(b*(c + d*x))])/d) - (2*B*n*PolyLog[2, (d*(a + b*x))/(b*(c +
 d*x))])/d)/d)/(2*d)))/i
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{dix+ci}dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, al
gorithm="fricas")
```

output
```
integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B
*a*b*g^2*x + B*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n))/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$= \frac{g^2\left(\int \frac{Aa^2}{c+dx}\,dx + \int \frac{Ab^2x^2}{c+dx}\,dx + \int \frac{Ba^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx + \int \frac{2Aabx}{c+dx}\,dx + \int \frac{Bb^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx + \int}{i}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i),x)
```

output

```
g**2*(Integral(A*a**2/(c + d*x), x) + Integral(A*b**2*x**2/(c + d*x), x) +
 Integral(B*a**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x) + I
ntegral(2*A*a*b*x/(c + d*x), x) + Integral(B*b**2*x**2*log(e*(a/(c + d*x)
+ b*x/(c + d*x))**n)/(c + d*x), x) + Integral(2*B*a*b*x*log(e*(a/(c + d*x)
 + b*x/(c + d*x))**n)/(c + d*x), x))/i
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(204) = 408$.

Time $= 0.35$ (sec) , antiderivative size $= 627$, normalized size of antiderivative $= 2.97$

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = 2\,Aabg^2\left(\frac{x}{di}-\frac{c\log(dx+c)}{d^2i}\right)$$
$$+\frac{1}{2}\,Ab^2g^2\left(\frac{2\,c^2\log(dx+c)}{d^3i}+\frac{dx^2-2\,cx}{d^2i}\right)+\frac{Aa^2g^2\log(dix+ci)}{di}$$
$$+\frac{(b^2c^2g^2n-2\,abcdg^2n+a^2d^2g^2n)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^3i}$$
$$+\frac{(2\,a^2d^2g^2\log(e)+(3\,g^2n+2\,g^2\log(e))b^2c^2-4\,(g^2n+g^2\log(e))abcd)B\log(dx+c)}{2\,d^3i}$$
$$+\frac{Bb^2d^2g^2x^2\log(e)-2\,(b^2c^2g^2n-2\,abcdg^2n+a^2d^2g^2n)B\log(bx+a)\log(dx+c)+(b^2c^2g^2n-2\,abcd}{}$$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, al
gorithm="maxima")
```

output

```
2*A*a*b*g^2*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + 1/2*A*b^2*g^2*(2*c^2*log(
d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A*a^2*g^2*log(d*i*x + c*i)/(
d*i) + (b^2*c^2*g^2*n - 2*a*b*c*d*g^2*n + a^2*d^2*g^2*n)*(log(b*x + a)*log
((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^
3*i) + 1/2*(2*a^2*d^2*g^2*log(e) + (3*g^2*n + 2*g^2*log(e))*b^2*c^2 - 4*(g
^2*n + g^2*log(e))*a*b*c*d)*B*log(d*x + c)/(d^3*i) + 1/2*(B*b^2*d^2*g^2*x^
2*log(e) - 2*(b^2*c^2*g^2*n - 2*a*b*c*d*g^2*n + a^2*d^2*g^2*n)*B*log(b*x +
 a)*log(d*x + c) + (b^2*c^2*g^2*n - 2*a*b*c*d*g^2*n + a^2*d^2*g^2*n)*B*log
(d*x + c)^2 - ((g^2*n + 2*g^2*log(e))*b^2*c*d - (g^2*n + 4*g^2*log(e))*a*b
*d^2)*B*x - (2*a*b*c*d*g^2*n - 3*a^2*d^2*g^2*n)*B*log(b*x + a) + (B*b^2*d^
2*g^2*x^2 - 2*(b^2*c*d*g^2 - 2*a*b*d^2*g^2)*B*x + 2*(b^2*c^2*g^2 - 2*a*b*c
*d*g^2 + a^2*d^2*g^2)*B*log(d*x + c))*log((b*x + a)^n) - (B*b^2*d^2*g^2*x^
2 - 2*(b^2*c*d*g^2 - 2*a*b*d^2*g^2)*B*x + 2*(b^2*c^2*g^2 - 2*a*b*c*d*g^2 +
 a^2*d^2*g^2)*B*log(d*x + c))*log((d*x + c)^n))/(d^3*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2499 vs. $2(204) = 408$.

Time = 131.85 (sec) , antiderivative size = 2499, normalized size of antiderivative = 11.84

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, al
gorithm="giac")
```

output

```
1/24*(2*(B*b^7*c^5*g^2*n - 5*B*a*b^6*c^4*d*g^2*n - 4*(b*x + a)*B*b^6*c^5*d
*g^2*n/(d*x + c) + 10*B*a^2*b^5*c^3*d^2*g^2*n + 20*(b*x + a)*B*a*b^5*c^4*d
^2*g^2*n/(d*x + c) + 6*(b*x + a)^2*B*b^5*c^5*d^2*g^2*n/(d*x + c)^2 - 10*B*
a^3*b^4*c^2*d^3*g^2*n - 40*(b*x + a)*B*a^2*b^4*c^3*d^3*g^2*n/(d*x + c) - 3
0*(b*x + a)^2*B*a*b^4*c^4*d^3*g^2*n/(d*x + c)^2 + 5*B*a^4*b^3*c*d^4*g^2*n
+ 40*(b*x + a)*B*a^3*b^3*c^2*d^4*g^2*n/(d*x + c) + 60*(b*x + a)^2*B*a^2*b^
3*c^3*d^4*g^2*n/(d*x + c)^2 - B*a^5*b^2*d^5*g^2*n - 20*(b*x + a)*B*a^4*b^2
*c*d^5*g^2*n/(d*x + c) - 60*(b*x + a)^2*B*a^3*b^2*c^2*d^5*g^2*n/(d*x + c)^
2 + 4*(b*x + a)*B*a^5*b*d^6*g^2*n/(d*x + c) + 30*(b*x + a)^2*B*a^4*b*c*d^6
*g^2*n/(d*x + c)^2 - 6*(b*x + a)^2*B*a^5*d^7*g^2*n/(d*x + c)^2)*log((b*x +
 a)/(d*x + c))/(b^4*d^3*i - 4*(b*x + a)*b^3*d^4*i/(d*x + c) + 6*(b*x + a)^
2*b^2*d^5*i/(d*x + c)^2 - 4*(b*x + a)^3*b*d^6*i/(d*x + c)^3 + (b*x + a)^4*
d^7*i/(d*x + c)^4) + (B*b^8*c^5*g^2*n - 5*B*a*b^7*c^4*d*g^2*n - 2*(b*x + a
)*B*b^7*c^5*d*g^2*n/(d*x + c) + 10*B*a^2*b^6*c^3*d^2*g^2*n + 10*(b*x + a)*
B*a*b^6*c^4*d^2*g^2*n/(d*x + c) - (b*x + a)^2*B*b^6*c^5*d^2*g^2*n/(d*x + c
)^2 - 10*B*a^3*b^5*c^2*d^3*g^2*n - 20*(b*x + a)*B*a^2*b^5*c^3*d^3*g^2*n/(d
*x + c) + 5*(b*x + a)^2*B*a*b^5*c^4*d^3*g^2*n/(d*x + c)^2 + 2*(b*x + a)^3*
B*b^5*c^5*d^3*g^2*n/(d*x + c)^3 + 5*B*a^4*b^4*c*d^4*g^2*n + 20*(b*x + a)*B
*a^3*b^4*c^2*d^4*g^2*n/(d*x + c) - 10*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^2*n/
(d*x + c)^2 - 10*(b*x + a)^3*B*a*b^4*c^4*d^4*g^2*n/(d*x + c)^3 - B*a^5*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{c\,i+d\,i\,x}\,dx$$

input

```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
,x)
```

output

```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$

$$= \frac{g^2 i\left(-2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{dx+c}dx\right)a^2 b\,d^3 - 2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x^2}{dx+c}dx\right)b^3 d^3 - 4\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x}{dx+c}dx\right)a\,b^2 d^3 - 2\log(\right.}{2d^3}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

output
```
(g**2*i*( - 2*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(c + d*x),x)*a**2*b*d
**3 - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c + d*x),x)*b**3*d*
*3 - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c + d*x),x)*a*b**2*d**3
 - 2*log(c + d*x)*a**3*d**2 + 4*log(c + d*x)*a**2*b*c*d - 2*log(c + d*x)*a
*b**2*c**2 - 4*a**2*b*d**2*x + 2*a*b**2*c*d*x - a*b**2*d**2*x**2))/(2*d**3
)
```

3.137 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$

Optimal result . . . 1598
Mathematica [A] (verified) . . . 1599
Rubi [A] (verified) . . . 1599
Maple [F] . . . 1601
Fricas [F] . . . 1602
Sympy [F] . . . 1602
Maxima [B] (verification not implemented) . . . 1603
Giac [B] (verification not implemented) . . . 1603
Mupad [F(-1)] . . . 1604
Reduce [F] . . . 1605

Optimal result

Integrand size = 41, antiderivative size = 134

$$\begin{aligned}
&\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx \\
&= \frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di} \\
&\quad + \frac{(bc-ad)g\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2 i} \\
&\quad + \frac{B(bc-ad)gn\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2 i}
\end{aligned}$$

output

```
g*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i+(-a*d+b*c)*g*(A+B*n+B*ln(e*(
(b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^2/i+B*(-a*d+b*c)*g*n*polyl
og(2,d*(b*x+a)/b/(d*x+c))/d^2/i
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.27

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$= \frac{g\left(2Abdx+2Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-2B(bc-ad)n\log(c+dx)-2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.\right.}{2d^2}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*
i*x),x]
```

output
```
(g*(2*A*b*d*x + 2*B*d*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*B*(b*c
- a*d)*n*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[c + d*x] + B*(b*c - a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] -
Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(2
*d^2*i)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {2961, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{ci+dix}\,dx$$
$$\downarrow 2961$$
$$\frac{g(bc-ad)\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i}$$
$$\downarrow 2784$$

$$\frac{g(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{i}$$

$$\downarrow 2754$$

$$\frac{g(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A+Bn\right)}{d}}{d}\right)}{i}$$

$$\downarrow 2838$$

$$\frac{g(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A+Bn\right)}{d}-\frac{Bn\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d}}{d}\right)}{i}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x),x
]
```

output
```
((b*c - a*d)*g*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c +
 d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((A + B*n + B*Log[e*((a + b*x)/(
c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (B*n*PolyLog[2, (
d*(a + b*x))/(b*(c + d*x))])/d)/d))/i
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])/(e*(q + 1))), x] - Simp[f/(e*(q + 1))  Int[(f*x)^(m - 1)*(d + e*x)^(q + 1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}{dix+ci}dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algo
rithm="fricas")
```

output
```
integral((A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log(e*((b*x + a)/(d*x + c))^
n))/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \frac{g\left(\int \frac{Aa}{c+dx}\,dx + \int \frac{Abx}{c+dx}\,dx + \int \frac{Ba\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx + \int \frac{Bbx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx\right)}{i}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i),x)
```

output
```
g*(Integral(A*a/(c + d*x), x) + Integral(A*b*x/(c + d*x), x) + Integral(B*
a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x) + Integral(B*b*x*l
og(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x))/i
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(133) = 266.

Time = 0.35 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.28

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$= Abg\left(\frac{x}{di}-\frac{c\log(dx+c)}{d^2i}\right)+\frac{Aag\log(dix+ci)}{di}$$
$$-\frac{(bcgn-adgn)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^2i}$$
$$+\frac{(adg\log(e)-(gn+g\log(e))bc)B\log(dx+c)}{d^2i}$$
$$+\frac{2\,Badgn\log(bx+a)+2\,Bbdgx\log(e)+2\,(bcgn-adgn)B\log(bx+a)\log(dx+c)-(bcgn-adgn)}{}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algo
rithm="maxima")
```

output
```
A*b*g*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + A*a*g*log(d*i*x + c*i)/(d*i) -
(b*c*g*n - a*d*g*n)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dil
og(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^2*i) + (a*d*g*log(e) - (g*n + g*log(e
))*b*c)*B*log(d*x + c)/(d^2*i) + 1/2*(2*B*a*d*g*n*log(b*x + a) + 2*B*b*d*g
*x*log(e) + 2*(b*c*g*n - a*d*g*n)*B*log(b*x + a)*log(d*x + c) - (b*c*g*n -
 a*d*g*n)*B*log(d*x + c)^2 + 2*(B*b*d*g*x - (b*c*g - a*d*g)*B*log(d*x + c)
)*log((b*x + a)^n) - 2*(B*b*d*g*x - (b*c*g - a*d*g)*B*log(d*x + c))*log((d
*x + c)^n))/(d^2*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. 2(133) = 266.

Time = 80.68 (sec) , antiderivative size = 1242, normalized size of antiderivative = 9.27

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algo
rithm="giac")
```

output
```
-1/6*((B*b^5*c^4*g*n - 4*B*a*b^4*c^3*d*g*n - 3*(b*x + a)*B*b^4*c^4*d*g*n/(
d*x + c) + 6*B*a^2*b^3*c^2*d^2*g*n + 12*(b*x + a)*B*a*b^3*c^3*d^2*g*n/(d*x
 + c) - 4*B*a^3*b^2*c*d^3*g*n - 18*(b*x + a)*B*a^2*b^2*c^2*d^3*g*n/(d*x + 
c) + B*a^4*b*d^4*g*n + 12*(b*x + a)*B*a^3*b*c*d^4*g*n/(d*x + c) - 3*(b*x + 
 a)*B*a^4*d^5*g*n/(d*x + c))*log((b*x + a)/(d*x + c))/(b^3*d^2*i - 3*(b*x
 + a)*b^2*d^3*i/(d*x + c) + 3*(b*x + a)^2*b*d^4*i/(d*x + c)^2 - (b*x + a)^3
*d^5*i/(d*x + c)^3) + ((b*x + a)*B*b^5*c^4*d*g*n/(d*x + c) - 4*(b*x + a)*B
*a*b^4*c^3*d^2*g*n/(d*x + c) - (b*x + a)^2*B*b^4*c^4*d^2*g*n/(d*x + c)^2 +
 6*(b*x + a)*B*a^2*b^3*c^2*d^3*g*n/(d*x + c) + 4*(b*x + a)^2*B*a*b^3*c^3*d
^3*g*n/(d*x + c)^2 - 4*(b*x + a)*B*a^3*b^2*c*d^4*g*n/(d*x + c) - 6*(b*x + 
a)^2*B*a^2*b^2*c^2*d^4*g*n/(d*x + c)^2 + (b*x + a)*B*a^4*b*d^5*g*n/(d*x + 
c) + 4*(b*x + a)^2*B*a^3*b*c*d^5*g*n/(d*x + c)^2 - (b*x + a)^2*B*a^4*d^6*g
*n/(d*x + c)^2 + B*b^6*c^4*g*log(e) - 4*B*a*b^5*c^3*d*g*log(e) - 3*(b*x + 
a)*B*b^5*c^4*d*g*log(e)/(d*x + c) + 6*B*a^2*b^4*c^2*d^2*g*log(e) + 12*(b*x
 + a)*B*a*b^4*c^3*d^2*g*log(e)/(d*x + c) - 4*B*a^3*b^3*c*d^3*g*log(e) - 18
*(b*x + a)*B*a^2*b^3*c^2*d^3*g*log(e)/(d*x + c) + B*a^4*b^2*d^4*g*log(e) + 
 12*(b*x + a)*B*a^3*b^2*c*d^4*g*log(e)/(d*x + c) - 3*(b*x + a)*B*a^4*b*d^5
*g*log(e)/(d*x + c) + A*b^6*c^4*g - 4*A*a*b^5*c^3*d*g - 3*(b*x + a)*A*b^5*
c^4*d*g/(d*x + c) + 6*A*a^2*b^4*c^2*d^2*g + 12*(b*x + a)*A*a*b^4*c^3*d^2*g
/(d*x + c) - 4*A*a^3*b^3*c*d^3*g - 18*(b*x + a)*A*a^2*b^3*c^2*d^3*g/(d*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x),x
)
```

output
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x),
x)
```

Reduce [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$= \frac{gi\left(-\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{dx+c}dx\right) ab\,d^2 - \left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x}{dx+c}dx\right) b^2d^2 - \log(dx+c)\,a^2d + \log(dx+c)\,abc - abdx\right)}{d^2}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

output
```
(g*i*( - int(log(((a + b*x)**n*e)/(c + d*x)**n)/(c + d*x),x)*a*b*d**2 - in
t((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c + d*x),x)*b**2*d**2 - log(c +
d*x)*a**2*d + log(c + d*x)*a*b*c - a*b*d*x))/d**2
```

3.138 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx$

Optimal result . . . 1606
Mathematica [A] (verified) . . . 1606
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Optimal result

Integrand size = 33, antiderivative size = 85

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{di} - \frac{Bn\,\mathrm{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{di}$$

output
```
-(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d/i-B*n*polylog(
2,1-(-a*d+b*c)/b/(d*x+c))/d/i
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.19

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx = \frac{\log(i(c+dx))\left(2A-2Bn\log\left(\frac{d(a+bx)}{-bc+ad}\right)+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log(i(c+dx))\right)-2Bn\,\mathrm{PolyLog}\left(2,\right.}{2di}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x),x]
```

output

```
(Log[i*(c + d*x)]*(2*A - 2*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*B*Log
[e*((a + b*x)/(c + d*x))^n] + B*n*Log[i*(c + d*x)]) - 2*B*n*PolyLog[2, (b*
(c + d*x))/(b*c - a*d)])/(2*d*i)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2943, 2858, 27, 25, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{ci + dix}\, dx$$

$$\downarrow 2943$$

$$\frac{Bn(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx}{di} - \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di}$$

$$\downarrow 2858$$

$$\frac{Bn(bc-ad)\int \frac{d\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)\left(\left(a-\frac{bc}{d}\right)d+b(c+dx)\right)}d(c+dx)}{d^2 i} - \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di}$$

$$\downarrow 27$$

$$\frac{Bn(bc-ad)\int -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{di} - \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di}$$

$$\downarrow 25$$

$$-\frac{Bn(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{di} - \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di}$$

$$\downarrow 2778$$

$$\frac{Bn(bc-ad)\int \frac{(c+dx)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bc-ad-b(c+dx)}d\frac{1}{c+dx}}{di} - \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di}$$

$$\downarrow 2005$$

$$\frac{Bn(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{\frac{bc-ad}{c+dx}-b}d\frac{1}{c+dx}}{di}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di}$$

$$\downarrow 2752$$

$$-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di}-\frac{Bn\,\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{di}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x),x]
```

output
```
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/
(d*i)) - (B*n*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(d*i)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2943

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c + d*x
))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Simp[B*n*((b*c - a*d)
/g)   Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[d*f - c
*g, 0]
```

Maple [F]

$$\int \frac{A + B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{dix + ci} dx$$

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

output

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci + dix} dx = \int \frac{B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{dix + ci} dx$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algorithm="frica
s")
```

output

```
integral((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci + dix}\,dx = \frac{\int \frac{A}{c+dx}\,dx + \int \frac{B \log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{i}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i),x)
```

output
```
(Integral(A/(c + d*x), x) + Integral(B*log(e*(a/(c + d*x) + b*x/(c + d*x))
**n)/(c + d*x), x))/i
```

Maxima [F]

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci + dix}\,dx = \int \frac{B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{dix + ci}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algorithm="maxim
a")
```

output
```
-1/2*B*((2*n*log(b*x + a)*log(d*x + c) - n*log(d*x + c)^2 - 2*log(d*x + c)
*log((b*x + a)^n) + 2*log(d*x + c)*log((d*x + c)^n))/(d*i) - 2*integrate((
n*log(b*x + a) + log(e))/(d*i*x + c*i), x)) + A*log(d*i*x + c*i)/(d*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(84) = 168$.

Time $= 53.92$ (sec) , antiderivative size $= 566$, normalized size of antiderivative $= 6.66$

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci + dix}\,dx$$

$$= \frac{1}{2}\left(\frac{\left(Bb^3c^3n - 3\,Bab^2c^2dn + 3\,Ba^2bcd^2n - Ba^3d^3n\right)\log\left(\frac{bx+a}{dx+c}\right)}{b^2di - \frac{2\,(bx+a)bd^2i}{dx+c} + \frac{(bx+a)^2d^3i}{(dx+c)^2}} - \frac{Bb^4c^3n - 3\,Bab^3c^2dn - \frac{(bx+a)Bb^3c^3dn}{dx+c}}{}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algorithm="giac"
)
```

output
```
1/2*((B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*l
og((b*x + a)/(d*x + c))/(b^2*d*i - 2*(b*x + a)*b*d^2*i/(d*x + c) + (b*x +
a)^2*d^3*i/(d*x + c)^2) - (B*b^4*c^3*n - 3*B*a*b^3*c^2*d*n - (b*x + a)*B*b
^3*c^3*d*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*n + 3*(b*x + a)*B*a*b^2*c^2*d^2*n
/(d*x + c) - B*a^3*b*d^3*n - 3*(b*x + a)*B*a^2*b*c*d^3*n/(d*x + c) + (b*x
+ a)*B*a^3*d^4*n/(d*x + c) - B*b^4*c^3*log(e) + 3*B*a*b^3*c^2*d*log(e) - 3
*B*a^2*b^2*c*d^2*log(e) + B*a^3*b*d^3*log(e) - A*b^4*c^3 + 3*A*a*b^3*c^2*d
 - 3*A*a^2*b^2*c*d^2 + A*a^3*b*d^3)/(b^3*d*i - 2*(b*x + a)*b^2*d^2*i/(d*x
+ c) + (b*x + a)^2*b*d^3*i/(d*x + c)^2) + (B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n
 + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^2*d
*i) - (B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*
log((b*x + a)/(d*x + c))/(b^2*d*i))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2
)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci + dix}\,dx = \int \frac{A + B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{c\,i + d\,i\,x}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*i + d*i*x),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*i + d*i*x), x)
```

Reduce [F]

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci + dix}\, dx = -\frac{i\left(\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{dx+c} dx\right) bd + \log(dx+c)\, a\right)}{d}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

output

```
( - i*(int(log(((a + b*x)**n*e)/(c + d*x)**n)/(c + d*x),x)*b*d + log(c + d
*x)*a))/d
```

3.139 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx$

Optimal result . . . 1613
Mathematica [C] (verified) . . . 1613
Rubi [A] (verified) . . . 1614
Maple [A] (verified) . . . 1615
Fricas [A] (verification not implemented) . . . 1616
Sympy [F] . . . 1616
Maxima [B] (verification not implemented) . . . 1616
Giac [A] (verification not implemented) . . . 1617
Mupad [B] (verification not implemented) . . . 1618
Reduce [B] (verification not implemented) . . . 1618

Optimal result

Integrand size = 43, antiderivative size = 50

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx = \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2B(bc-ad)gin}$$

output
```
1/2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/B/(-a*d+b*c)/g/i/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 219, normalized size of antiderivative = 4.38

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx$$
$$= \frac{2A\log(a+bx) - Bn\log^2(a+bx) + 2B\log(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - 2A\log(c+dx) + 2Bn\log\left(\frac{d(a+bx)}{-bc+ad}\right.}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i
*x)),x]
```

output
```
(2*A*Log[a + b*x] - B*n*Log[a + b*x]^2 + 2*B*Log[a + b*x]*Log[e*((a + b*x)
/(c + d*x))^n] - 2*A*Log[c + d*x] + 2*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)
]*Log[c + d*x] - 2*B*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x] - B*n*Log
[c + d*x]^2 + 2*B*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 2*B*n*Po
lyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*B*n*PolyLog[2, (b*(c + d*x))/(b
*c - a*d)])/(2*(b*c - a*d)*g*i)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2961, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{(ag+bgx)(ci+dix)}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx} d\frac{a+bx}{c+dx}}{gi(bc-ad)}$$

$$\downarrow 2738$$

$$\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2Bgin(bc-ad)}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i*x)),x
]
```

output
```
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(2*B*(b*c - a*d)*g*i*n)
```

Defintions of rubi rules used

rule 2738

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.68

method	result	size
parallelrisch	$-\frac{B a^2 c^2 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2+2A\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^2c^2}{2ig\,a^2c^2n(da-bc)}$	84
default	$\frac{A\left(\frac{\ln(dx+c)}{da-bc}-\frac{\ln(bx+a)}{da-bc}\right)}{gi}-\frac{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}{2gin(da-bc)}$	88
parts	$\frac{A\left(\frac{\ln(dx+c)}{da-bc}-\frac{\ln(bx+a)}{da-bc}\right)}{gi}-\frac{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}{2gin(da-bc)}$	88

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i),x,method=_RETURNVERBOSE)
```

output

```
-1/2*(B*a^2*c^2*ln(e*((b*x+a)/(d*x+c))^n)^2+2*A*ln(e*((b*x+a)/(d*x+c))^n)*a^2*c^2)/i/g/a^2/c^2/n/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.48

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)(ci + dix)}\,dx = \frac{Bn \log\left(\frac{bx+a}{dx+c}\right)^2 + 2\,B \log(e) \log\left(\frac{bx+a}{dx+c}\right) + 2\,A \log\left(\frac{bx+a}{dx+c}\right)}{2\,(bc - ad)gi}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i),x, algo
rithm="fricas")
```

output
```
1/2*(B*n*log((b*x + a)/(d*x + c))^2 + 2*B*log(e)*log((b*x + a)/(d*x + c))
+ 2*A*log((b*x + a)/(d*x + c)))/((b*c - a*d)*g*i)
```

Sympy [F]

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)(ci + dix)}\,dx = \frac{\int \frac{A}{ac+adx+bcx+bdx^2}\,dx + \int \frac{B \log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{ac+adx+bcx+bdx^2}\,dx}{gi}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)/(d*i*x+c*i),x)
```

output
```
(Integral(A/(a*c + a*d*x + b*c*x + b*d*x**2), x) + Integral(B*log(e*(a/(c
+ d*x) + b*x/(c + d*x))**n)/(a*c + a*d*x + b*c*x + b*d*x**2), x))/(g*i)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(48) = 96$.

Time = 0.04 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.50

$$\begin{aligned}
&\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx \\
&= B\left(\frac{\log(bx+a)}{(bc-ad)gi} - \frac{\log(dx+c)}{(bc-ad)gi}\right)\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) \\
&\quad - \frac{\left(\log(bx+a)^2 - 2\log(bx+a)\log(dx+c) + \log(dx+c)^2\right)Bn}{2(bcgi-adgi)} \\
&\quad + A\left(\frac{\log(bx+a)}{(bc-ad)gi} - \frac{\log(dx+c)}{(bc-ad)gi}\right)
\end{aligned}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i),x, algo
rithm="maxima")
```

output
```
B*(log(b*x + a)/((b*c - a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))*log(e*
(b*x/(d*x + c) + a/(d*x + c))^n) - 1/2*(log(b*x + a)^2 - 2*log(b*x + a)*lo
g(d*x + c) + log(d*x + c)^2)*B*n/(b*c*g*i - a*d*g*i) + A*(log(b*x + a)/((b
*c - a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.84

$$\begin{aligned}
&\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx \\
&= \frac{\left(Bn\log\left(\frac{bx+a}{dx+c}\right)^2 + 2B\log(e)\log\left(\frac{bx+a}{dx+c}\right) + 2A\log\left(\frac{bx+a}{dx+c}\right)\right)\left(\frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2}\right)}{2gi}
\end{aligned}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i),x, algo
rithm="giac")
```

output
```
1/2*(B*n*log((b*x + a)/(d*x + c))^2 + 2*B*log(e)*log((b*x + a)/(d*x + c))
+ 2*A*log((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(g
*i)
```

Mupad [B] (verification not implemented)

Time = 26.80 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.52

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx = -\frac{B\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^2 - An\,\mathrm{atan}\left(\frac{bc\,2\mathrm{i}+bdx\,2\mathrm{i}}{ad-bc}+1\mathrm{i}\right)\,4\mathrm{i}}{2gin\,(ad-bc)}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)*(c*i + d*i*x)),x
)
```

output
```
-(B*log(e*((a + b*x)/(c + d*x))^n)^2 - A*n*atan((b*c*2i + b*d*x*2i)/(a*d -
 b*c) + 1i)*4i)/(2*g*i*n*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx = \frac{i\left(2\log(bx+a)\,an - 2\log(dx+c)\,an + \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 b\right)}{2gn\,(ad-bc)}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i),x)
```

output
```
(i*(2*log(a + b*x)*a*n - 2*log(c + d*x)*a*n + log(((a + b*x)**n*e)/(c + d*
x)**n)**2*b))/(2*g*n*(a*d - b*c))
```

3.140 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx$

Optimal result 1619
Mathematica [C] (verified) 1620
Rubi [A] (verified) 1620
Maple [A] (verified) 1622
Fricas [A] (verification not implemented) 1623
Sympy [F(-1)] 1623
Maxima [B] (verification not implemented) 1624
Giac [F] 1624
Mupad [B] (verification not implemented) 1625
Reduce [B] (verification not implemented) 1625

Optimal result

Integrand size = 43, antiderivative size = 181

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{bBn(c+dx)}{(bc-ad)^2g^2i(a+bx)} - \frac{b(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i} + \frac{Bdn\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^2g^2i}$$

output

```
-b*B*n*(d*x+c)/(-a*d+b*c)^2/g^2/i/(b*x+a)-b*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))/(-a*d+b*c)^2/g^2/i/(b*x+a)-d*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(
(b*x+a)/(d*x+c))/(-a*d+b*c)^2/g^2/i+1/2*B*d*n*ln((b*x+a)/(d*x+c))^2/(-a*d+
b*c)^2/g^2/i
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.68

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx =$$

$$-\frac{2(bc-ad)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + 2d(a+bx)\log(a+bx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) - 2d(a+bx)\left(A$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d
*i*x)),x]
```

output
```
-1/2*(2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d*(a + b*x)
*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*d*(a + b*x)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 2*B*n*(b*c - a*d + d*(a +
 b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*d*n*(a + b*x)*(Log[a +
b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(
a + b*x))/(-(b*c) + a*d)]) + B*d*n*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c)
 + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)]))/((b*c - a*d)^2*g^2*i*(a + b*x))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2961, 2772, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{(ag+bgx)^2(ci+dix)}\,dx$$

$$\downarrow \text{2961}$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i(bc-ad)^2}$$

$$\downarrow 2772$$

$$\frac{-Bn\int -\frac{(c+dx)^2\left(b+\frac{d(a+bx)\log\left(\frac{a+bx}{c+dx}\right)}{c+dx}\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{b(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right.}{g^2i(bc-ad)^2}$$

$$\downarrow 25$$

$$\frac{Bn\int \frac{(c+dx)^2\left(b+\frac{d(a+bx)\log\left(\frac{a+bx}{c+dx}\right)}{c+dx}\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{b(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2i(bc-ad)^2}$$

$$\downarrow 2010$$

$$\frac{Bn\int \left(\frac{b(c+dx)^2}{(a+bx)^2}+\frac{d\log\left(\frac{a+bx}{c+dx}\right)(c+dx)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right.}{g^2i(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{-\frac{b(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+Bn\left(\frac{1}{2}d\log^2\left(\frac{a+bx}{c+dx}\right)-\frac{b(c+dx)}{a+bx}\right)}{g^2i(bc-ad)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d*i*x))
,x]
```

output
```
(-((b*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)) - d*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] + B*n*(-((b*(
c + d*x))/(a + b*x)) + (d*Log[(a + b*x)/(c + d*x)]^2)/2))/((b*c - a*d)^2*g
^2*i)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [A] (verified)

Time = 5.07 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.49

method	result
parallelrisch	$-\frac{2Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{4}d^{3}n+2B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{4}c\,d^{2}n-2Ba\,b^{3}d^{3}n^{2}+2B\,b^{4}c\,d^{2}n^{2}-2Aa\,b^{3}d^{3}n+2A\,b^{4}c\,d^{2}n+Bx\ln\left(e\left(\frac{bx+}{dx+}\right.\right.}{2i\,g^{2}(bx+a)n(a^{2}d^{2}-2acdb+c^{2}b^{2})}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i),x,method=_RE
TURNVERBOSE)
```

output
```
-1/2*(2*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^3*n+2*B*ln(e*((b*x+a)/(d*x+c))
^n)*b^4*c*d^2*n-2*B*a*b^3*d^3*n^2+2*B*b^4*c*d^2*n^2-2*A*a*b^3*d^3*n+2*A*b^
4*c*d^2*n+B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*d^3+2*A*x*ln(e*((b*x+a)/(d*x
+c))^n)*b^4*d^3+B*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^3*d^3+2*A*ln(e*((b*x+a)/
(d*x+c))^n)*a*b^3*d^3)/i/g^2/(b*x+a)/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/d^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.07

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx =$$
$$-\frac{2\,Abc - 2\,Aad + (Bbdnx + Badn)\log\left(\frac{bx+a}{dx+c}\right)^2 + 2\,(Bbc - Bad)n + 2\left(Bbc - Bad + (Bbdx + Bad)\lo}{2\left((b^3c^2 - 2\,ab^2cd + a^2bd^2)g^2ix + (ab^2c^2 - 2\,a^2bcd}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i),x, al
gorithm="fricas")
```

output
```
-1/2*(2*A*b*c - 2*A*a*d + (B*b*d*n*x + B*a*d*n)*log((b*x + a)/(d*x + c))^2
 + 2*(B*b*c - B*a*d)*n + 2*(B*b*c - B*a*d + (B*b*d*x + B*a*d)*log((b*x + a
)/(d*x + c)))*log(e) + 2*(B*b*c*n + A*a*d + (B*b*d*n + A*b*d)*x)*log((b*x
+ a)/(d*x + c)))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^2*i*x + (a*b^2*c^2
 - 2*a^2*b*c*d + a^3*d^2)*g^2*i)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2/(d*i*x+c*i),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(179) = 358$.

Time $= 0.06$ (sec) , antiderivative size $= 427$, normalized size of antiderivative $= 2.36$

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx =$$
$$-B\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)\log\left($$
$$+\frac{((bdx+ad)\log(bx+a)^2+(bdx+ad)\log(dx+c)^2-2\,bc+2\,ad-2\,(bdx+ad)\log(bx+a)+2\,(bdx}{2\,(ab^2c^2g^2i-2\,a^2bcdg^2i+a^3d^2g^2i+(b^3c^2g^2i-2\,ab^2cdg^2i+a^}$$
$$-A\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i),x, al
gorithm="maxima")
```

output
```
-B*(1/((b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/(
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*g^2*i))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*((b*d*
x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d - 2
*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))
*log(d*x + c))*B*n/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*i + a^3*d^2*g^2*i +
(b^3*c^2*g^2*i - 2*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*x) - A*(1/((b^2*c -
a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g
^2*i))
```

Giac [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{(bgx+ag)^2(dix+ci)}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i),x, al
gorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/((b*g*x + a*g)^2*(d*i*x +
 c*i)), x)
```

Mupad [B] (verification not implemented)

Time = 27.18 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.32

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx = \frac{A}{g^2\,i\,(a\,d-b\,c)\,(a+b\,x)} + \frac{B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{g^2\,i\,(a\,d-b\,c)\,(a+b\,x)} + \frac{B\,n}{g^2\,i\,(a\,d-b\,c)\,(a+b\,x)} - \frac{B\,d\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{2\,g^2\,i\,n\,(a\,d-b\,c)^2} + \frac{A\,d\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g^2\,i\,(a\,d-b\,c)^2} + \frac{B\,d\,n\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g^2\,i\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^2*(c*i + d*i*x))
,x)
```

output
```
A/(g^2*i*(a*d - b*c)*(a + b*x)) + (B*log(e*((a + b*x)/(c + d*x))^n))/(g^2*
i*(a*d - b*c)*(a + b*x)) + (A*d*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b
*c))*2i)/(g^2*i*(a*d - b*c)^2) + (B*n)/(g^2*i*(a*d - b*c)*(a + b*x)) + (B*
d*n*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g^2*i*(a*d - b*c)^
2) - (B*d*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g^2*i*n*(a*d - b*c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.85

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx$$

$$= \frac{i\left(2\log(bx+a)\,a^3dn + 2\log(bx+a)\,a^2bdnx + 2\log(bx+a)\,ab^2cn^2 + 2\log(bx+a)\,b^3cn^2x - 2\log(dx+$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i),x)
```

output
```
(i*(2*log(a + b*x)*a**3*d*n + 2*log(a + b*x)*a**2*b*d*n*x + 2*log(a + b*x)
*a*b**2*c*n**2 + 2*log(a + b*x)*b**3*c*n**2*x - 2*log(c + d*x)*a**3*d*n -
2*log(c + d*x)*a**2*b*d*n*x - 2*log(c + d*x)*a*b**2*c*n**2 - 2*log(c + d*x
)*b**3*c*n**2*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b*d + log(((a
 + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*d*x + 2*log(((a + b*x)**n*e)/(c + d*
x)**n)*a*b**2*d*n*x - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*n*x + 2*
a**2*b*d*n*x - 2*a*b**2*c*n*x + 2*a*b**2*d*n**2*x - 2*b**3*c*n**2*x))/(2*a
*g**2*n*(a**3*d**2 - 2*a**2*b*c*d + a**2*b*d**2*x + a*b**2*c**2 - 2*a*b**2
*c*d*x + b**3*c**2*x))
```

3.141 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx$

Optimal result 1627
Mathematica [C] (verified) 1628
Rubi [A] (verified) 1628
Maple [B] (verified) 1630
Fricas [A] (verification not implemented) 1631
Sympy [F(-1)] 1632
Maxima [B] (verification not implemented) 1632
Giac [A] (verification not implemented) 1633
Mupad [B] (verification not implemented) 1634
Reduce [B] (verification not implemented) 1635

Optimal result

Integrand size = 43, antiderivative size = 266

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx &= -\frac{Bn(c+dx)^2\left(b-\frac{4d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3g^3i(a+bx)^2}\\ &+\frac{2bd(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^3i(a+bx)}\\ &-\frac{b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2}\\ &+\frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i}\\ &-\frac{Bd^2n\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^3g^3i}\end{aligned}$$

output

```
-1/4*B*n*(d*x+c)^2*(b-4*d*(b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^3/i/(b*x+a)^2+
2*b*d*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^3/i/(b*x+a)-1
/2*b^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^3/i/(b*x+a
)^2+d^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c)^3/g
^3/i-1/2*B*d^2*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^3/i
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.63

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx$$
$$= \frac{-2(bc-ad)^2\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + 4d(bc-ad)(a+bx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + 4d^2(a+bx)^2\log$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d
*i*x)),x]
```

output
```
(-2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d*(b*c - a*d)
*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d^2*(a + b*x)^2*Log[
a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 4*d^2*(a + b*x)^2*(A + B
*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 4*B*d*n*(a + b*x)*(b*c - a
*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*n*((b*c - a*
d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d
^2*(a + b*x)^2*Log[c + d*x]) - 2*B*d^2*n*(a + b*x)^2*(Log[a + b*x]*(Log[a
+ b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-
(b*c) + a*d)]) + 2*B*d^2*n*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
)/(4*(b*c - a*d)^3*g^3*i*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2961, 2772, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{(ag+bgx)^3(ci+dix)}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3i(bc-ad)^3}$$

$$\downarrow 2772$$

$$\frac{-Bn\int -\frac{(c+dx)^3\left(b^2-\frac{4d(a+bx)b}{c+dx}-\frac{2d^2(a+bx)^2\log\left(\frac{a+bx}{c+dx}\right)}{(c+dx)^2}\right)}{2(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e}{g^3i(bc-ad)^3}$$

$$\downarrow 27$$

$$\frac{\frac{1}{2}Bn\int \frac{(c+dx)^3\left(b^2-\frac{4d(a+bx)b}{c+dx}-\frac{2d^2(a+bx)^2\log\left(\frac{a+bx}{c+dx}\right)}{(c+dx)^2}\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(}{g^3i(bc-ad)^3}$$

$$\downarrow 2010$$

$$\frac{\frac{1}{2}Bn\int \left(\frac{b(c+dx)^3\left(b-\frac{4d(a+bx)}{c+dx}\right)}{(a+bx)^3}-\frac{2d^2(c+dx)\log\left(\frac{a+bx}{c+dx}\right)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log}{g^3i(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{2bd(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}+\frac{1}{2}Bn}{g^3i(bc-ad)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d*i*x))
,x]
```

output
```
((2*b*d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - (b^2
*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) + d^2
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] + (B*n*(-
1/2*((c + d*x)^2*(b - (4*d*(a + b*x))/(c + d*x))^2)/(a + b*x)^2 - d^2*Log[
(a + b*x)/(c + d*x)]^2))/2)/((b*c - a*d)^3*g^3*i)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(260) = 520$.

Time = 13.37 (sec) , antiderivative size = 633, normalized size of antiderivative = 2.38

method	result
parallelrisch	$-\frac{2Bx^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2a^4b^2c^2d^2+7Bx^2a^4b^2c^2d^2n^2-8Bx^2a^3b^3c^3dn^2+4Ax^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^4b^2c^2d^2+6Ax^2a^4b^2c^2d^2n$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i),x,method=_RE
TURNVERBOSE)
```

output
```
-1/4*(2*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b^2*c^2*d^2+7*B*x^2*a^4*b^2*
c^2*d^2*n^2-8*B*x^2*a^3*b^3*c^3*d*n^2+4*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^
4*b^2*c^2*d^2+6*A*x^2*a^4*b^2*c^2*d^2*n-8*A*x^2*a^3*b^3*c^3*d*n+4*B*x*ln(e
*((b*x+a)/(d*x+c))^n)^2*a^5*b*c^2*d^2+8*B*x*a^5*b*c^2*d^2*n^2+2*B*ln(e*((b
*x+a)/(d*x+c))^n)^2*a^6*c^2*d^2+4*A*ln(e*((b*x+a)/(d*x+c))^n)*a^6*c^2*d^2-
10*B*x*a^4*b^2*c^3*d*n^2+8*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b*c^2*d^2+8*A
*x*a^5*b*c^2*d^2*n-12*A*x*a^4*b^2*c^3*d*n+8*B*ln(e*((b*x+a)/(d*x+c))^n)*a^
5*b*c^3*d*n+6*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^2*d^2*n+8*B*x*ln(e
*((b*x+a)/(d*x+c))^n)*a^5*b*c^2*d^2*n+4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*
b^2*c^3*d*n+B*x^2*a^2*b^4*c^4*n^2+2*A*x^2*a^2*b^4*c^4*n+2*B*x*a^3*b^3*c^4*
n^2+4*A*x*a^3*b^3*c^4*n-2*B*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^4*n)/i/g^3
/(b*x+a)^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a^4/c^2/n
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.82

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx =$$
$$-\frac{2\,Ab^2c^2 - 8\,Aabcd + 6\,Aa^2d^2 - 2\,(Bb^2d^2nx^2 + 2\,Babd^2nx + Ba^2d^2n)\log\left(\frac{bx+a}{dx+c}\right)^2 + (Bb^2c^2 - 8\,Babcd$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i),x, al
gorithm="fricas")
```

output
```
-1/4*(2*A*b^2*c^2 - 8*A*a*b*c*d + 6*A*a^2*d^2 - 2*(B*b^2*d^2*n*x^2 + 2*B*a
*b*d^2*n*x + B*a^2*d^2*n)*log((b*x + a)/(d*x + c))^2 + (B*b^2*c^2 - 8*B*a*
b*c*d + 7*B*a^2*d^2)*n - 2*(2*A*b^2*c*d - 2*A*a*b*d^2 + 3*(B*b^2*c*d - B*a
*b*d^2)*n)*x + 2*(B*b^2*c^2 - 4*B*a*b*c*d + 3*B*a^2*d^2 - 2*(B*b^2*c*d - B
*a*b*d^2)*x - 2*(B*b^2*d^2*x^2 + 2*B*a*b*d^2*x + B*a^2*d^2)*log((b*x + a)/
(d*x + c)))*log(e) - 2*(2*A*a^2*d^2 + (3*B*b^2*d^2*n + 2*A*b^2*d^2)*x^2 -
(B*b^2*c^2 - 4*B*a*b*c*d)*n + 2*(2*A*a*b*d^2 + (B*b^2*c*d + 2*B*a*b*d^2)*n
)*x)*log((b*x + a)/(d*x + c)))/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2
 - a^3*b^2*d^3)*g^3*i*x^2 + 2*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d
^2 - a^4*b*d^3)*g^3*i*x + (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 -
 a^5*d^3)*g^3*i)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^3(ci + dix)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3/(d*i*x+c*i),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 888 vs. $2(260) = 520$.

Time = 0.08 (sec) , antiderivative size = 888, normalized size of antiderivative = 3.34

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^3(ci + dix)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i),x, al
gorithm="maxima")
```

output

```
1/2*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3*
i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a^2*b^2*c^2 -
 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n) - 1/4*(b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b
*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)
*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x +
 a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b
^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c))*B*n/(a^2*b
^3*c^3*g^3*i - 3*a^3*b^2*c^2*d*g^3*i + 3*a^4*b*c*d^2*g^3*i - a^5*d^3*g^3*i
 + (b^5*c^3*g^3*i - 3*a*b^4*c^2*d*g^3*i + 3*a^2*b^3*c*d^2*g^3*i - a^3*b^2*
d^3*g^3*i)*x^2 + 2*(a*b^4*c^3*g^3*i - 3*a^2*b^3*c^2*d*g^3*i + 3*a^3*b^2*c*
d^2*g^3*i - a^4*b*d^3*g^3*i)*x) + 1/2*A*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2
 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a
^3*b*d^2)*g^3*i*x + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*l
og(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) -
2*d^2*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^
3*i))
```

Giac [A] (verification not implemented)

Time = 101.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.39

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx =$$
$$-\frac{1}{4}\left(\frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2}\right)^2\left(\frac{2\,(dx+c)^2 Bn\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^2 g^3 i} + \frac{(Bn + 2\,B\log(e) + 2\,A)(dx+c)^2}{(bx+a)^2 g^3 i}\right)$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i),x, al
gorithm="giac")
```

output

```
-1/4*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2*(2*(d*x + c)^2*B*n*log((b*x
 + a)/(d*x + c))/((b*x + a)^2*g^3*i) + (B*n + 2*B*log(e) + 2*A)*(d*x + c)^
2/((b*x + a)^2*g^3*i))
```

Mupad [B] (verification not implemented)

Time = 27.50 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.15

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx$$
$$= \frac{B\,d^2\,\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(\frac{g^3\,i\,n\,(a\,d-b\,c)\,(2\,a\,d-b\,c)}{2\,d^2}+\frac{a\,g^3\,i\,n\,(a\,d-b\,c)}{2\,d}+\frac{b\,g^3\,i\,n\,x\,(a\,d-b\,c)}{d}\right)}{g^3\,i\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)\,(i\,a^2\,g^3+2\,i\,a\,b\,g^3\,x+i\,b^2\,g^3\,x^2)}$$
$$- \frac{B\,d^2\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}{2\,g^3\,i\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}$$
$$- \frac{\frac{6\,A\,a\,d-2\,A\,b\,c+7\,B\,a\,d\,n-B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{d\,x\,(2\,A\,b+3\,B\,b\,n)}{a\,d-b\,c}}{x^2\,(2\,b^3\,c\,g^3\,i-2\,a\,b^2\,d\,g^3\,i)+x\,(4\,a\,b^2\,c\,g^3\,i-4\,a^2\,b\,d\,g^3\,i)-2\,a^3\,d\,g^3\,i+2\,a^2\,b\,c\,g^3\,i}$$
$$+ \frac{d^2\,\mathrm{atan}\left(\frac{d^2\left(A+\frac{3\,B\,n}{2}\right)(2\,i\,a^3\,d^3\,g^3-2\,i\,a^2\,b\,c\,d^2\,g^3-2\,i\,a\,b^2\,c^2\,d\,g^3+2\,i\,b^3\,c^3\,g^3)\,1\mathrm{i}}{g^3\,i\,(2\,A\,d^2+3\,B\,d^2\,n)\,(a\,d-b\,c)^3}+\frac{b\,d^3\,x\left(A+\frac{3\,B\,n}{2}\right)(i\,a^2\,d^2\,g^3-2\,i\,a\,b\,c\,d\,g^3+i\,b^2\,c^2\,g}{g^3\,i\,(2\,A\,d^2+3\,B\,d^2\,n)\,(a\,d-b\,c)^3}\right.}{g^3\,i\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^3*(c*i + d*i*x))
,x)
```

output
```
(d^2*atan((d^2*(A + (3*B*n)/2)*(2*a^3*d^3*g^3*i + 2*b^3*c^3*g^3*i - 2*a*b^
2*c^2*d*g^3*i - 2*a^2*b*c*d^2*g^3*i)*1i)/(g^3*i*(2*A*d^2 + 3*B*d^2*n)*(a*d
 - b*c)^3) + (b*d^3*x*(A + (3*B*n)/2)*(a^2*d^2*g^3*i + b^2*c^2*g^3*i - 2*a
*b*c*d*g^3*i)*4i)/(g^3*i*(2*A*d^2 + 3*B*d^2*n)*(a*d - b*c)^3))*(A + (3*B*n
)/2)*2i)/(g^3*i*(a*d - b*c)^3) - ((6*A*a*d - 2*A*b*c + 7*B*a*d*n - B*b*c*n
)/(2*(a*d - b*c)) + (d*x*(2*A*b + 3*B*b*n))/(a*d - b*c))/(x^2*(2*b^3*c*g^3
*i - 2*a*b^2*d*g^3*i) + x*(4*a*b^2*c*g^3*i - 4*a^2*b*d*g^3*i) - 2*a^3*d*g^
3*i + 2*a^2*b*c*g^3*i) - (B*d^2*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g^3*i
*n*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B*d^2*log(e*((a + b*x)/
(c + d*x))^n)*((g^3*i*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2) + (a*g^3*i*n*(a
*d - b*c))/(2*d) + (b*g^3*i*n*x*(a*d - b*c))/d))/(g^3*i*n*(a*d - b*c)*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*g^3*i + b^2*g^3*i*x^2 + 2*a*b*g^3*i*x))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 747, normalized size of antiderivative = 2.81

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^3(ci + dix)}\, dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i),x)
```

output

```
(i*(4*log(a + b*x)*a**4*d**2*n + 6*log(a + b*x)*a**3*b*d**2*n**2 + 8*log(a
 + b*x)*a**3*b*d**2*n*x + 12*log(a + b*x)*a**2*b**2*d**2*n**2*x + 4*log(a
+ b*x)*a**2*b**2*d**2*n*x**2 + 6*log(a + b*x)*a*b**3*d**2*n**2*x**2 - 4*lo
g(c + d*x)*a**4*d**2*n - 6*log(c + d*x)*a**3*b*d**2*n**2 - 8*log(c + d*x)*
a**3*b*d**2*n*x - 12*log(c + d*x)*a**2*b**2*d**2*n**2*x - 4*log(c + d*x)*a
**2*b**2*d**2*n*x**2 - 6*log(c + d*x)*a*b**3*d**2*n**2*x**2 + 2*log(((a +
b*x)**n*e)/(c + d*x)**n)**2*a**3*b*d**2 + 4*log(((a + b*x)**n*e)/(c + d*x)
**n)**2*a**2*b**2*d**2*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3*
d**2*x**2 - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*d**2*n + 8*log(((a
 + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*c*d*n - 4*log(((a + b*x)**n*e)/(c +
d*x)**n)*a**2*b**2*d**2*n*x - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*
c**2*n + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c*d*n*x - 4*a**4*d**2
*n + 6*a**3*b*c*d*n - 4*a**3*b*d**2*n**2 - 2*a**2*b**2*c**2*n + 5*a**2*b**
2*c*d*n**2 + 2*a**2*b**2*d**2*n*x**2 - a*b**3*c**2*n**2 - 2*a*b**3*c*d*n*x
**2 + 3*a*b**3*d**2*n**2*x**2 - 3*b**4*c*d*n**2*x**2))/(4*a*g**3*n*(a**5*d
**3 - 3*a**4*b*c*d**2 + 2*a**4*b*d**3*x + 3*a**3*b**2*c**2*d - 6*a**3*b**2
*c*d**2*x + a**3*b**2*d**3*x**2 - a**2*b**3*c**3 + 6*a**2*b**3*c**2*d*x -
3*a**2*b**3*c*d**2*x**2 - 2*a*b**4*c**3*x + 3*a*b**4*c**2*d*x**2 - b**5*c*
*3*x**2))
```

3.142 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx$

Optimal result . . . 1636
Mathematica [C] (verified) . . . 1637
Rubi [A] (verified) . . . 1638
Maple [B] (verified) . . . 1640
Fricas [B] (verification not implemented) . . . 1641
Sympy [F(-1)] . . . 1642
Maxima [B] (verification not implemented) . . . 1643
Giac [A] (verification not implemented) . . . 1644
Mupad [B] (verification not implemented) . . . 1644
Reduce [B] (verification not implemented) . . . 1645

Optimal result

Integrand size = 43, antiderivative size = 389

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx &= -\frac{3bBd^2n(c+dx)}{(bc-ad)^4g^4i(a+bx)}+\frac{3b^2Bdn(c+dx)^2}{4(bc-ad)^4g^4i(a+bx)^2}\\ &\quad-\frac{b^3Bn(c+dx)^3}{9(bc-ad)^4g^4i(a+bx)^3}\\ &\quad-\frac{3bd^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^4i(a+bx)}\\ &\quad+\frac{3b^2d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2}\\ &\quad-\frac{b^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3}\\ &\quad-\frac{d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i}\\ &\quad+\frac{Bd^3n\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^4i}\end{aligned}$$

output

```
-3*b*B*d^2*n*(d*x+c)/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/4*b^2*B*d*n*(d*x+c)^2/(-
a*d+b*c)^4/g^4/i/(b*x+a)^2-1/9*b^3*B*n*(d*x+c)^3/(-a*d+b*c)^4/g^4/i/(b*x+a
)^3-3*b*d^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^4/i/(b*
x+a)+3/2*b^2*d*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^4/
i/(b*x+a)^2-1/3*b^3*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4
/g^4/i/(b*x+a)^3-d^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(
-a*d+b*c)^4/g^4/i+1/2*B*d^3*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^4/i
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.71 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.33

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx$$

$$= \frac{-\frac{12A(bc-ad)^3}{(a+bx)^3} - \frac{4B(bc-ad)^3n}{(a+bx)^3} + \frac{18Ad(bc-ad)^2}{(a+bx)^2} + \frac{15Bd(bc-ad)^2n}{(a+bx)^2} + \frac{36Ad^2(-bc+ad)}{a+bx} + \frac{66Bd^2(-bc+ad)n}{a+bx} - 36Ad^3\log(a+$$

input

```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d
*i*x)),x]
```

output

```
((-12*A*(b*c - a*d)^3)/(a + b*x)^3 - (4*B*(b*c - a*d)^3*n)/(a + b*x)^3 + (
18*A*d*(b*c - a*d)^2)/(a + b*x)^2 + (15*B*d*(b*c - a*d)^2*n)/(a + b*x)^2 +
 (36*A*d^2*(-(b*c) + a*d))/(a + b*x) + (66*B*d^2*(-(b*c) + a*d)*n)/(a + b*
x) - 36*A*d^3*Log[a + b*x] - 66*B*d^3*n*Log[a + b*x] + 18*B*d^3*n*Log[a +
b*x]^2 - (12*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x)^3 +
 (18*B*d*(b*c - a*d)^2*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x)^2 + (36*B
*d^2*(-(b*c) + a*d)*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x) - 36*B*d^3*L
og[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n] + 36*A*d^3*Log[c + d*x] + 66*B*
d^3*n*Log[c + d*x] - 36*B*d^3*n*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c +
d*x] + 36*B*d^3*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x] + 18*B*d^3*n*L
og[c + d*x]^2 - 36*B*d^3*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 3
6*B*d^3*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 36*B*d^3*n*PolyLog[2,
 (b*(c + d*x))/(b*c - a*d)])/(36*(b*c - a*d)^4*g^4*i)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2961, 2772, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{(ag+bgx)^4(ci+dix)}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4}\,d\frac{a+bx}{c+dx}}{g^4 i(bc-ad)^4}$$

$$\downarrow 2772$$

$$\frac{-Bn\int -\frac{(c+dx)^4\left(2b^3-\frac{9d(a+bx)b^2}{c+dx}+\frac{18d^2(a+bx)^2b}{(c+dx)^2}+\frac{6d^3(a+bx)^3\log\left(\frac{a+bx}{c+dx}\right)}{(c+dx)^3}\right)}{6(a+bx)^4}\,d\frac{a+bx}{c+dx} - \frac{b^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3} + \frac{3b^2d(c+dx)^2}{}}{g^4 i(bc-ad)^4}$$

$$\downarrow 27$$

$$\frac{\frac{1}{6}Bn\int \frac{(c+dx)^4\left(2b^3-\frac{9d(a+bx)b^2}{c+dx}+\frac{18d^2(a+bx)^2b}{(c+dx)^2}+\frac{6d^3(a+bx)^3\log\left(\frac{a+bx}{c+dx}\right)}{(c+dx)^3}\right)}{(a+bx)^4}\,d\frac{a+bx}{c+dx} - \frac{b^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3} + \frac{3b^2d(c+dx)^2}{}}{g^4 i(bc-ad)^4}$$

$$\downarrow 2010$$

$$\frac{\frac{1}{6}Bn\int \left(\frac{b\left(2b^2-\frac{9d(a+bx)b}{c+dx}+\frac{18d^2(a+bx)^2}{(c+dx)^2}\right)(c+dx)^4}{(a+bx)^4}+\frac{6d^3\log\left(\frac{a+bx}{c+dx}\right)(c+dx)}{a+bx}\right)d\frac{a+bx}{c+dx} - \frac{b^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3} + \frac{3b^2d(c+}{}}{g^4 i(bc-ad)}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3} + \frac{3b^2d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2} - \left(d^3\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right) - }{g^4 i(bc-ad)^4}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d*i*x))
,x]
```

output
```
((-3*b*d^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (
3*b^2*d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2
) - (b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^
3) - d^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] +
 (B*n*((-18*b*d^2*(c + d*x))/(a + b*x) + (9*b^2*d*(c + d*x)^2)/(2*(a + b*x
)^2) - (2*b^3*(c + d*x)^3)/(3*(a + b*x)^3) + 3*d^3*Log[(a + b*x)/(c + d*x)
]^2))/6)/((b*c - a*d)^4*g^4*i)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1089 vs. $2(379) = 758$.

Time = 27.76 (sec) , antiderivative size = 1090, normalized size of antiderivative = 2.80

method	result	size
parallelrisch	Expression too large to display	1090

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i),x,method=_RE
TURNVERBOSE)
```

output

```
-1/36*(18*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^8*c^2*d^3+36*A*ln(e*((b*x+a)/(d*
x+c))^n)*a^8*c^2*d^3+108*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^2*d^3+1
62*A*x^2*a^6*b^2*c^2*d^3*n-288*A*x^2*a^5*b^3*c^3*d^2*n+162*A*x^2*a^4*b^4*c
^4*d*n+54*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^7*b*c^2*d^3+108*B*x*a^7*b*c^2*
d^3*n^2-162*B*x*a^6*b^2*c^3*d^2*n^2+66*B*x*a^5*b^3*c^4*d*n^2+108*A*x*ln(e*
((b*x+a)/(d*x+c))^n)*a^7*b*c^2*d^3+108*A*x*a^7*b*c^2*d^3*n-216*A*x*a^6*b^2
*c^3*d^2*n+144*A*x*a^5*b^3*c^4*d*n+108*B*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c
^3*d^2*n-54*B*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^4*d*n+18*B*x^3*ln(e*((b*
x+a)/(d*x+c))^n)^2*a^5*b^3*c^2*d^3+85*B*x^3*a^5*b^3*c^2*d^3*n^2-108*B*x^3*
a^4*b^4*c^3*d^2*n^2+27*B*x^3*a^3*b^5*c^4*d*n^2+36*A*x^3*ln(e*((b*x+a)/(d*x
+c))^n)*a^5*b^3*c^2*d^3+66*A*x^3*a^5*b^3*c^2*d^3*n-108*A*x^3*a^4*b^4*c^3*d
^2*n+54*A*x^3*a^3*b^5*c^4*d*n+54*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^2
*c^2*d^3+189*B*x^2*a^6*b^2*c^2*d^3*n^2-258*B*x^2*a^5*b^3*c^3*d^2*n^2+81*B*
x^2*a^4*b^4*c^4*d*n^2+66*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^3*c^2*d^3*n
+162*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^2*d^3*n+36*B*x^2*ln(e*((b*x
+a)/(d*x+c))^n)*a^5*b^3*c^3*d^2*n+108*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*
c^2*d^3*n+108*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^3*d^2*n-18*B*x*ln(e*
((b*x+a)/(d*x+c))^n)*a^5*b^3*c^4*d*n-4*B*x^3*a^2*b^6*c^5*n^2-12*A*x^3*a^2*
b^6*c^5*n-12*B*x^2*a^3*b^5*c^5*n^2-36*A*x^2*a^3*b^5*c^5*n-12*B*x*a^4*b^4*c
^5*n^2-36*A*x*a^4*b^4*c^5*n+12*B*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^3*c^5*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(379) = 758$.

Time = 0.10 (sec) , antiderivative size = 859, normalized size of antiderivative = 2.21

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i),x, al
gorithm="fricas")
```

output

```
-1/36*(12*A*b^3*c^3 - 54*A*a*b^2*c^2*d + 108*A*a^2*b*c*d^2 - 66*A*a^3*d^3
+ 6*(6*A*b^3*c*d^2 - 6*A*a*b^2*d^3 + 11*(B*b^3*c*d^2 - B*a*b^2*d^3)*n)*x^2
 + 18*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + B*a^3*d
^3*n)*log((b*x + a)/(d*x + c))^2 + (4*B*b^3*c^3 - 27*B*a*b^2*c^2*d + 108*B
*a^2*b*c*d^2 - 85*B*a^3*d^3)*n - 3*(6*A*b^3*c^2*d - 36*A*a*b^2*c*d^2 + 30*
A*a^2*b*d^3 + (5*B*b^3*c^2*d - 54*B*a*b^2*c*d^2 + 49*B*a^2*b*d^3)*n)*x + 6
*(2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2 - 11*B*a^3*d^3 + 6*(B*b
^3*c*d^2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b
*d^3)*x + 6*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x^2 + 3*B*a^2*b*d^3*x + B*a^3*d
^3)*log((b*x + a)/(d*x + c)))*log(e) + 6*(6*A*a^3*d^3 + (11*B*b^3*d^3*n +
6*A*b^3*d^3)*x^3 + 3*(6*A*a*b^2*d^3 + (2*B*b^3*c*d^2 + 9*B*a*b^2*d^3)*n)*x
^2 + (2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2)*n + 3*(6*A*a^2*b*d
^3 - (B*b^3*c^2*d - 6*B*a*b^2*c*d^2 - 6*B*a^2*b*d^3)*n)*x)*log((b*x + a)/(
d*x + c)))/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3
 + a^4*b^3*d^4)*g^4*i*x^3 + 3*(a*b^6*c^4 - 4*a^2*b^5*c^3*d + 6*a^3*b^4*c^2
*d^2 - 4*a^4*b^3*c*d^3 + a^5*b^2*d^4)*g^4*i*x^2 + 3*(a^2*b^5*c^4 - 4*a^3*b
^4*c^3*d + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^3 + a^6*b*d^4)*g^4*i*x + (a^3
*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*
g^4*i)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx = \text{Timed out}$$

input

```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4/(d*i*x+c*i),x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1472 vs. $2(379) = 758$.

Time $= 0.13$ (sec) , antiderivative size $= 1472$, normalized size of antiderivative $= 3.78$

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^4(ci + dix)}\, dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i),x, al
gorithm="maxima")
```

output

```
-1/6*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*
g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3
)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d
^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*g^
4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i))*log(e*(b*x/(d*
x + c) + a/(d*x + c))^n) - 1/36*(4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*
d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*
b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*(b^3*d^3*x^3 +
3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d
 - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 +
3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x
^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^
2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))*B*n/(a^3*b^4*c^4*g^4*i -
4*a^4*b^3*c^3*d*g^4*i + 6*a^5*b^2*c^2*d^2*g^4*i - 4*a^6*b*c*d^3*g^4*i + a^
7*d^4*g^4*i + (b^7*c^4*g^4*i - 4*a*b^6*c^3*d*g^4*i + 6*a^2*b^5*c^2*d^2*g^4
*i - 4*a^3*b^4*c*d^3*g^4*i + a^4*b^3*d^4*g^4*i)*x^3 + 3*(a*b^6*c^4*g^4*i -
 4*a^2*b^5*c^3*d*g^4*i + 6*a^3*b^4*c^2*d^2*g^4*i - 4*a^4*b^3*c*d^3*g^4*...
```

Giac [A] (verification not implemented)

Time = 138.57 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.60

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx =$$
$$-\frac{1}{36}\left(\frac{6\left(2\,Bbn-\frac{3\,(bx+a)Bdn}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^3bcg^4i}{(dx+c)^3}-\frac{(bx+a)^3adg^4i}{(dx+c)^3}}+\frac{4\,Bbn-\frac{9\,(bx+a)Bdn}{dx+c}+12\,Bb\log(e)-\frac{18\,(bx+a)Bd\log(e)}{dx+c}+12\,A}{\frac{(bx+a)^3bcg^4i}{(dx+c)^3}-\frac{(bx+a)^3adg^4i}{(dx+c)^3}}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i),x, al
gorithm="giac")
```

output
```
-1/36*(6*(2*B*b*n - 3*(b*x + a)*B*d*n/(d*x + c))*log((b*x + a)/(d*x + c))/
((b*x + a)^3*b*c*g^4*i/(d*x + c)^3 - (b*x + a)^3*a*d*g^4*i/(d*x + c)^3) +
(4*B*b*n - 9*(b*x + a)*B*d*n/(d*x + c) + 12*B*b*log(e) - 18*(b*x + a)*B*d*
log(e)/(d*x + c) + 12*A*b - 18*(b*x + a)*A*d/(d*x + c))/((b*x + a)^3*b*c*g
^4*i/(d*x + c)^3 - (b*x + a)^3*a*d*g^4*i/(d*x + c)^3))*(b*c/(b*c - a*d)^2
- a*d/(b*c - a*d)^2)^2
```

Mupad [B] (verification not implemented)

Time = 28.44 (sec) , antiderivative size = 986, normalized size of antiderivative = 2.53

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^4*(c*i + d*i*x))
,x)
```

output

```
((66*A*a^2*d^2 + 12*A*b^2*c^2 + 85*B*a^2*d^2*n + 4*B*b^2*c^2*n - 42*A*a*b*
c*d - 23*B*a*b*c*d*n)/(6*(a*d - b*c)) + (x*(30*A*a*b*d^2 - 6*A*b^2*c*d + 4
9*B*a*b*d^2*n - 5*B*b^2*c*d*n))/(2*(a*d - b*c)) + (d*x^2*(6*A*b^2*d + 11*B
*b^2*d*n))/(a*d - b*c))/(x*(18*a^4*b*d^2*g^4*i + 18*a^2*b^3*c^2*g^4*i - 36
*a^3*b^2*c*d*g^4*i) + x^2*(18*a*b^4*c^2*g^4*i + 18*a^3*b^2*d^2*g^4*i - 36*
a^2*b^3*c*d*g^4*i) + x^3*(6*b^5*c^2*g^4*i + 6*a^2*b^3*d^2*g^4*i - 12*a*b^4
*c*d*g^4*i) + 6*a^5*d^2*g^4*i + 6*a^3*b^2*c^2*g^4*i - 12*a^4*b*c*d*g^4*i)
+ (d^3*atan((d^3*((a^4*d^4*g^4*i - b^4*c^4*g^4*i + 2*a*b^3*c^3*d*g^4*i - 2
*a^3*b*c*d^3*g^4*i)/(a^3*d^3*g^4*i - b^3*c^3*g^4*i + 3*a*b^2*c^2*d*g^4*i -
 3*a^2*b*c*d^2*g^4*i) + 2*b*d*x)*(A + (11*B*n)/6)*(a^3*d^3*g^4*i - b^3*c^3
*g^4*i + 3*a*b^2*c^2*d*g^4*i - 3*a^2*b*c*d^2*g^4*i)*6i)/(g^4*i*(6*A*d^3 +
11*B*d^3*n)*(a*d - b*c)^4))*(A + (11*B*n)/6)*2i)/(g^4*i*(a*d - b*c)^4) - (
B*d^3*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g^4*i*n*(a*d - b*c)*(a^3*d^3 -
b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B*d^3*log(e*((a + b*x)/(c + d
*x))^n)*(x*(b*((g^4*i*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2) + (a*g^4*i*n*(a
*d - b*c))/(3*d)) + (2*a*b*g^4*i*n*(a*d - b*c))/(3*d) + (b*g^4*i*n*(a*d -
b*c)*(3*a*d - b*c))/(3*d^2)) + a*((g^4*i*n*(a*d - b*c)*(3*a*d - b*c))/(6*d
^2) + (a*g^4*i*n*(a*d - b*c))/(3*d)) + (g^4*i*n*(a*d - b*c)*(3*a^2*d^2 + b
^2*c^2 - 3*a*b*c*d))/(3*d^3) + (b^2*g^4*i*n*x^2*(a*d - b*c))/d))/(g^4*i*n*
(a*d - b*c)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^3*g^...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1414, normalized size of antiderivative = 3.63

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^4(ci + dix)}\, dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i),x)
```

output

```
(i*(36*log(a + b*x)*a**5*d**3*n + 54*log(a + b*x)*a**4*b*d**3*n**2 + 108*l
og(a + b*x)*a**4*b*d**3*n*x + 12*log(a + b*x)*a**3*b**2*c*d**2*n**2 + 162*
log(a + b*x)*a**3*b**2*d**3*n**2*x + 108*log(a + b*x)*a**3*b**2*d**3*n*x**
2 + 36*log(a + b*x)*a**2*b**3*c*d**2*n**2*x + 162*log(a + b*x)*a**2*b**3*d
**3*n**2*x**2 + 36*log(a + b*x)*a**2*b**3*d**3*n*x**3 + 36*log(a + b*x)*a*
b**4*c*d**2*n**2*x**2 + 54*log(a + b*x)*a*b**4*d**3*n**2*x**3 + 12*log(a +
 b*x)*b**5*c*d**2*n**2*x**3 - 36*log(c + d*x)*a**5*d**3*n - 54*log(c + d*x
)*a**4*b*d**3*n**2 - 108*log(c + d*x)*a**4*b*d**3*n*x - 12*log(c + d*x)*a*
*3*b**2*c*d**2*n**2 - 162*log(c + d*x)*a**3*b**2*d**3*n**2*x - 108*log(c +
 d*x)*a**3*b**2*d**3*n*x**2 - 36*log(c + d*x)*a**2*b**3*c*d**2*n**2*x - 16
2*log(c + d*x)*a**2*b**3*d**3*n**2*x**2 - 36*log(c + d*x)*a**2*b**3*d**3*n
*x**3 - 36*log(c + d*x)*a*b**4*c*d**2*n**2*x**2 - 54*log(c + d*x)*a*b**4*d
**3*n**2*x**3 - 12*log(c + d*x)*b**5*c*d**2*n**2*x**3 + 18*log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*a**4*b*d**3 + 54*log(((a + b*x)**n*e)/(c + d*x)**n)
**2*a**3*b**2*d**3*x + 54*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**3*
d**3*x**2 + 18*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**4*d**3*x**3 - 54
*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*b*d**3*n + 96*log(((a + b*x)**n*e
)/(c + d*x)**n)*a**3*b**2*c*d**2*n - 54*log(((a + b*x)**n*e)/(c + d*x)**n)
*a**3*b**2*d**3*n*x - 54*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*c**2
*d*n + 72*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*c*d**2*n*x + 12*...
```

3.143 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$

Optimal result . . . 1647
Mathematica [A] (verified) . . . 1648
Rubi [A] (verified) . . . 1649
Maple [F] . . . 1651
Fricas [F] . . . 1652
Sympy [F(-1)] . . . 1652
Maxima [B] (verification not implemented) . . . 1652
Giac [F(-1)] . . . 1653
Mupad [F(-1)] . . . 1654
Reduce [F] . . . 1654

Optimal result

Integrand size = 43, antiderivative size = 359

$$\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx \\
&= \frac{3B(bc-ad)^2g^3n(a+bx)}{d^3i^2(c+dx)} - \frac{(bc-ad)^2g^3(6A+5Bn)(a+bx)}{2d^3i^2(c+dx)} \\
&\quad - \frac{3B(bc-ad)^2g^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^2(c+dx)} + \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^2(c+dx)} \\
&\quad - \frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^2(c+dx)} \\
&\quad - \frac{b(bc-ad)^2g^3\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2d^4i^2} \\
&\quad - \frac{3bB(bc-ad)^2g^3n\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}
\end{aligned}$$

output
```
3*B*(-a*d+b*c)^2*g^3*n*(b*x+a)/d^3/i^2/(d*x+c)-1/2*(-a*d+b*c)^2*g^3*(5*B*n
+6*A)*(b*x+a)/d^3/i^2/(d*x+c)-3*B*(-a*d+b*c)^2*g^3*(b*x+a)*ln(e*((b*x+a)/(
d*x+c))^n)/d^3/i^2/(d*x+c)+1/2*g^3*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n
))/d/i^2/(d*x+c)-1/2*(-a*d+b*c)*g^3*(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(
d*x+c))^n))/d^2/i^2/(d*x+c)-1/2*b*(-a*d+b*c)^2*g^3*(6*A+5*B*n+6*B*ln(e*((b
*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^4/i^2-3*b*B*(-a*d+b*c)^2*g^3
*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^2
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.04

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$$
$$= \frac{g^3\Bigg(-2Ab^2d(2bc-3ad)x-2bBd(2bc-3ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+b^3d^2x^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i +
d*i*x)^2,x]
```

output
```
(g^3*(-2*A*b^2*d*(2*b*c - 3*a*d)*x - 2*b*B*d*(2*b*c - 3*a*d)*(a + b*x)*Log
[e*((a + b*x)/(c + d*x))^n] + b^3*d^2*x^2*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]) + (2*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x
) + 2*b*B*(2*b*c - 3*a*d)*(b*c - a*d)*n*Log[c + d*x] + 6*b*(b*c - a*d)^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*B*(b*c - a*d)^2*n*(
(b*c - a*d)/(c + d*x) + b*Log[a + b*x] - b*Log[c + d*x]) + b*B*n*(-(a^2*d^
2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 3*b*B*(b*
c - a*d)^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
 d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(2*d^4*i^2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2961, 2784, 2784, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ci+dix)^2}\,dx$$

$$\downarrow 2961$$

$$\frac{g^3(bc-ad)^2\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{i^2}$$

$$\downarrow 2784$$

$$\frac{g^3(bc-ad)^2\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int \frac{(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{i^2}$$

$$\downarrow 2784$$

$$\frac{g^3(bc-ad)^2\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int \frac{(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{d}}{2d}\right)}{i^2}$$

$$\downarrow 2793$$

$$\frac{g^3(bc-ad)^2\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\left(-\frac{6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d}-\frac{b\left(6A+5Bn+6\right.}{d\left(\frac{d(}{}\right.}\right.}{2d}\right)}{i^2}$$

$$\downarrow 2009$$

$$\frac{g^3(bc-ad)^2\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+5Bn\right.}{d^2}}{2}\right)}{i^2}$$

input
```
Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)
^2,x]
```

output
```
((b*c - a*d)^2*g^3*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(
2*d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)^2*(3*A + B*
n + 3*B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)^2*(b - (d*(a + b*x))
/(c + d*x))) - ((6*B*n*(a + b*x))/(d*(c + d*x)) - ((6*A + 5*B*n)*(a + b*x)
)/(d*(c + d*x)) - (6*B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d*(c + d
*x)) - (b*(6*A + 5*B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a
 + b*x))/(b*(c + d*x))])/d^2 - (6*b*B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d
*x))])/d^2)/d)/(2*d)))/i^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(dix+ci)^2}dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x,
algorithm="fricas")
```

output
```
integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
+ (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log(e*
((b*x + a)/(d*x + c))^n))/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**2,x
)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1892 vs. $2(350) = 700$.

Time $= 0.37$ (sec) , antiderivative size $= 1892$, normalized size of antiderivative $= 5.27$

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x,
algorithm="maxima")
```

output

```
B*a^3*g^3*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2
) - b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) + 1/2*(2*c^3/(d^5*i^2*x + c*d^4*
i^2) + 6*c^2*log(d*x + c)/(d^4*i^2) + (d*x^2 - 4*c*x)/(d^3*i^2))*A*b^3*g^3
 - 3*A*a*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2) - x/(d^2*i^2) + 2*c*log(d*x + c)
/(d^3*i^2))*g^3 + 3*A*a^2*b*g^3*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*x + c)/
(d^2*i^2)) - B*a^3*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^2*i^2*x +
 c*d*i^2) - A*a^3*g^3/(d^2*i^2*x + c*d*i^2) - 1/2*(6*a^3*b*d^3*g^3*log(e)
- (7*g^3*n + 6*g^3*log(e))*b^4*c^3 + (17*g^3*n + 18*g^3*log(e))*a*b^3*c^2*
d - 6*(2*g^3*n + 3*g^3*log(e))*a^2*b^2*c*d^2)*B*log(d*x + c)/(b*c*d^4*i^2
- a*d^5*i^2) + 1/2*((b^4*c*d^3*g^3*log(e) - a*b^3*d^4*g^3*log(e))*B*x^3 -
((g^3*n + 3*g^3*log(e))*b^4*c^2*d^2 - (2*g^3*n + 9*g^3*log(e))*a*b^3*c*d^3
 + (g^3*n + 6*g^3*log(e))*a^2*b^2*d^4)*B*x^2 - ((g^3*n + 4*g^3*log(e))*b^4
*c^3*d - 2*(g^3*n + 5*g^3*log(e))*a*b^3*c^2*d^2 + (g^3*n + 6*g^3*log(e))*a
^2*b^2*c*d^3)*B*x - 6*((b^4*c^3*d*g^3*n - 3*a*b^3*c^2*d^2*g^3*n + 3*a^2*b^
2*c*d^3*g^3*n - a^3*b*d^4*g^3*n)*B*x + (b^4*c^4*g^3*n - 3*a*b^3*c^3*d*g^3*
n + 3*a^2*b^2*c^2*d^2*g^3*n - a^3*b*c*d^3*g^3*n)*B)*log(b*x + a)*log(d*x +
 c) + 3*((b^4*c^3*d*g^3*n - 3*a*b^3*c^2*d^2*g^3*n + 3*a^2*b^2*c*d^3*g^3*n
- a^3*b*d^4*g^3*n)*B*x + (b^4*c^4*g^3*n - 3*a*b^3*c^3*d*g^3*n + 3*a^2*b^2*
c^2*d^2*g^3*n - a^3*b*c*d^3*g^3*n)*B)*log(d*x + c)^2 - 2*((g^3*n - g^3*log
(e))*b^4*c^4 - 4*(g^3*n - g^3*log(e))*a*b^3*c^3*d + 6*(g^3*n - g^3*log(...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Timed out}$$

input

```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x,
algorithm="giac")
```

output

```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(c\,i+d\,i\,x)^2}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^2,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^2, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

output

```
(g**3*( - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(c**2 + 2*c*d*x
+ d**2*x**2),x)*a*b**4*c**2*d**5 - 2*int((log(((a + b*x)**n*e)/(c + d*x)**
n)*x**3)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**4*c*d**6*x + 2*int((log(((a
+ b*x)**n*e)/(c + d*x)**n)*x**3)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**5*c**3
*d**4 + 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(c**2 + 2*c*d*x +
d**2*x**2),x)*b**5*c**2*d**5*x - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)
*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b**3*c**2*d**5 - 6*int((log(((
a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b*
*3*c*d**6*x + 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c**2 + 2*c*
d*x + d**2*x**2),x)*a*b**4*c**3*d**4 + 6*int((log(((a + b*x)**n*e)/(c + d*
x)**n)*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**4*c**2*d**5*x - 6*int((l
og(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**3*
b**2*c**2*d**5 - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c**2 + 2*c*
d*x + d**2*x**2),x)*a**3*b**2*c*d**6*x + 6*int((log(((a + b*x)**n*e)/(c +
d*x)**n)*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b**3*c**3*d**4 + 6*int((l
og(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*
b**3*c**2*d**5*x + 2*log(a + b*x)*a**4*b*c*d**4*n + 2*log(a + b*x)*a**4*b*
d**5*n*x - 6*log(c + d*x)*a**4*b*c**2*d**3 - 2*log(c + d*x)*a**4*b*c*d**4*
n - 6*log(c + d*x)*a**4*b*c*d**4*x - 2*log(c + d*x)*a**4*b*d**5*n*x + 18*l
og(c + d*x)*a**3*b**2*c**3*d**2 + 18*log(c + d*x)*a**3*b**2*c**2*d**3*x...
```

3.144 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$

Optimal result . . . 1656
Mathematica [A] (verified) . . . 1657
Rubi [A] (verified) . . . 1657
Maple [F] . . . 1659
Fricas [F] . . . 1660
Sympy [F(-1)] . . . 1660
Maxima [B] (verification not implemented) . . . 1660
Giac [F(-1)] . . . 1661
Mupad [F(-1)] . . . 1662
Reduce [F] . . . 1662

Optimal result

Integrand size = 43, antiderivative size = 275

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx \\
&= -\frac{2B(bc-ad)g^2n(a+bx)}{d^2i^2(c+dx)} + \frac{(bc-ad)g^2(2A+Bn)(a+bx)}{d^2i^2(c+dx)} \\
&\quad + \frac{2B(bc-ad)g^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^2(c+dx)} + \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di^2(c+dx)} \\
&\quad + \frac{b(bc-ad)g^2\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^2} \\
&\quad + \frac{2bB(bc-ad)g^2n\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}
\end{aligned}$$

output

```
-2*B*(-a*d+b*c)*g^2*n*(b*x+a)/d^2/i^2/(d*x+c)+(-a*d+b*c)*g^2*(B*n+2*A)*(b*
x+a)/d^2/i^2/(d*x+c)+2*B*(-a*d+b*c)*g^2*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/
d^2/i^2/(d*x+c)+g^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i^2/(d*x+c
)+b*(-a*d+b*c)*g^2*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b
/(d*x+c))/d^3/i^2+2*b*B*(-a*d+b*c)*g^2*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^
3/i^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.92

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$$
$$= \frac{g^2\left(Ab^2dx+\frac{B(bc-ad)^2n}{c+dx}+bB(bc-ad)n\log(a+bx)+bBd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-\frac{(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.\right.}{c+dx}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i +
d*i*x)^2,x]
```

output
```
(g^2*(A*b^2*d*x + (B*(b*c - a*d)^2*n)/(c + d*x) + b*B*(b*c - a*d)*n*Log[a
+ b*x] + b*B*d*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - ((b*c - a*d)^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) - 2*b*B*(b*c - a*d)*n*Log
[c + d*x] - 2*b*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c +
 d*x] + b*B*(b*c - a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c +
d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(d^3*i^2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {2961, 2784, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ci+dix)^2}\,dx$$
$$\downarrow \text{2961}$$
$$\frac{g^2(bc-ad)\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i^2}$$

$$\downarrow 2784$$

$$\frac{g^2(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{d}\right)}{i^2}$$

$$\downarrow 2793$$

$$\frac{g^2(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\left(-\frac{2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d}-\frac{b\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{d}\right)}{i^2}$$

$$\downarrow 2009$$

$$\frac{g^2(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d^2}-\frac{(a+bx)(2A+Bn)}{d(c+dx)}-\frac{2bBn\,\mathrm{PolyLog}\left(2,\frac{d(a-}{b(c+}\right.}{d}\right)}{i^2}$$

input
```
Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)
^2,x]
```

output
```
((b*c - a*d)*g^2*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*
(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))) - ((2*B*n*(a + b*x))/(d*(c + d*
x)) - ((2*A + B*n)*(a + b*x))/(d*(c + d*x)) - (2*B*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n])/(d*(c + d*x)) - (b*(2*A + B*n + 2*B*Log[e*((a + b*x)/(c
 + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 - (2*b*B*n*PolyLog[
2, (d*(a + b*x))/(b*(c + d*x))])/d^2)/d))/i^2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2784

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2793

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(dix+ci)^2}dx$$

input

```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

output

```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x,
algorithm="fricas")
```

output
```
integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B
*a*b*g^2*x + B*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n))/(d^2*i^2*x^2 + 2*c
*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**2,x
)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1273 vs. $2(274) = 548$.

Time $= 0.35$ (sec) , antiderivative size $= 1273$, normalized size of antiderivative $= 4.63$

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x,
algorithm="maxima")
```

output

```
B*a^2*g^2*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2
) - b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) - A*b^2*(c^2/(d^4*i^2*x + c*d^3*
i^2) - x/(d^2*i^2) + 2*c*log(d*x + c)/(d^3*i^2))*g^2 + 2*A*a*b*g^2*(c/(d^3
*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2)) - B*a^2*g^2*log(e*(b*x/(d*x
+ c) + a/(d*x + c))^n)/(d^2*i^2*x + c*d*i^2) - A*a^2*g^2/(d^2*i^2*x + c*d*
i^2) - (2*a^2*b*d^2*g^2*log(e) + 2*(g^2*n + g^2*log(e))*b^3*c^2 - (3*g^2*n
 + 4*g^2*log(e))*a*b^2*c*d)*B*log(d*x + c)/(b*c*d^3*i^2 - a*d^4*i^2) + ((b
^3*c*d^2*g^2*log(e) - a*b^2*d^3*g^2*log(e))*B*x^2 + (b^3*c^2*d*g^2*log(e)
- a*b^2*c*d^2*g^2*log(e))*B*x + 2*((b^3*c^2*d*g^2*n - 2*a*b^2*c*d^2*g^2*n
+ a^2*b*d^3*g^2*n)*B*x + (b^3*c^3*g^2*n - 2*a*b^2*c^2*d*g^2*n + a^2*b*c*d^
2*g^2*n)*B)*log(b*x + a)*log(d*x + c) - ((b^3*c^2*d*g^2*n - 2*a*b^2*c*d^2*
g^2*n + a^2*b*d^3*g^2*n)*B*x + (b^3*c^3*g^2*n - 2*a*b^2*c^2*d*g^2*n + a^2*
b*c*d^2*g^2*n)*B)*log(d*x + c)^2 + ((g^2*n - g^2*log(e))*b^3*c^3 - 3*(g^2*
n - g^2*log(e))*a*b^2*c^2*d + 2*(g^2*n - g^2*log(e))*a^2*b*c*d^2)*B + ((b^
3*c^2*d*g^2*n - a*b^2*c*d^2*g^2*n - a^2*b*d^3*g^2*n)*B*x + (b^3*c^3*g^2*n
- a*b^2*c^2*d*g^2*n - a^2*b*c*d^2*g^2*n)*B)*log(b*x + a) + ((b^3*c*d^2*g^2
 - a*b^2*d^3*g^2)*B*x^2 + (b^3*c^2*d*g^2 - a*b^2*c*d^2*g^2)*B*x - (b^3*c^3
*g^2 - 3*a*b^2*c^2*d*g^2 + 2*a^2*b*c*d^2*g^2)*B - 2*((b^3*c^2*d*g^2 - 2*a*
b^2*c*d^2*g^2 + a^2*b*d^3*g^2)*B*x + (b^3*c^3*g^2 - 2*a*b^2*c^2*d*g^2 + a^
2*b*c*d^2*g^2)*B)*log(d*x + c))*log((b*x + a)^n) - ((b^3*c*d^2*g^2 - a*...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Timed out}$$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x,
algorithm="giac")
```

output

```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(c\,i+d\,i\,x)^2}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^2,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^2, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

output

```
(g**2*( - int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c**2 + 2*c*d*x +
d**2*x**2),x)*a*b**3*c**2*d**4 - int((log(((a + b*x)**n*e)/(c + d*x)**n)*x
**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**3*c*d**5*x + int((log(((a + b*x)
**n*e)/(c + d*x)**n)*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**4*c**3*d**3
+ int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c**2 + 2*c*d*x + d**2*x**
2),x)*b**4*c**2*d**4*x - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c**
2 + 2*c*d*x + d**2*x**2),x)*a**2*b**2*c**2*d**4 - 2*int((log(((a + b*x)**n
*e)/(c + d*x)**n)*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b**2*c*d**5*x +
2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c**2 + 2*c*d*x + d**2*x**2),
x)*a*b**3*c**3*d**3 + 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c**2 +
 2*c*d*x + d**2*x**2),x)*a*b**3*c**2*d**4*x + log(a + b*x)*a**3*b*c*d**3*n
 + log(a + b*x)*a**3*b*d**4*n*x - 2*log(c + d*x)*a**3*b*c**2*d**2 - log(c
+ d*x)*a**3*b*c*d**3*n - 2*log(c + d*x)*a**3*b*c*d**3*x - log(c + d*x)*a**
3*b*d**4*n*x + 4*log(c + d*x)*a**2*b**2*c**3*d + 4*log(c + d*x)*a**2*b**2*
c**2*d**2*x - 2*log(c + d*x)*a*b**3*c**4 - 2*log(c + d*x)*a*b**3*c**3*d*x
- log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*d**4*x + log(((a + b*x)**n*e)/
(c + d*x)**n)*a**2*b**2*c*d**3*x - a**4*d**4*x + 3*a**3*b*c*d**3*x + a**3*
b*d**4*n*x - 4*a**2*b**2*c**2*d**2*x - a**2*b**2*c*d**3*n*x - a**2*b**2*c*
d**3*x**2 + 2*a*b**3*c**3*d*x + a*b**3*c**2*d**2*x**2))/(c*d**3*(a*c*d + a
*d**2*x - b*c**2 - b*c*d*x))
```

3.145 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$

Optimal result . . . 1664
Mathematica [A] (verified) . . . 1665
Rubi [A] (verified) . . . 1665
Maple [F] . . . 1667
Fricas [F] . . . 1667
Sympy [F(-1)] . . . 1667
Maxima [F] . . . 1668
Giac [B] (verification not implemented) . . . 1668
Mupad [F(-1)] . . . 1669
Reduce [F] . . . 1670

Optimal result

Integrand size = 41, antiderivative size = 168

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = -\frac{Ag(a+bx)}{di^2(c+dx)} + \frac{Bgn(a+bx)}{di^2(c+dx)} - \frac{Bg(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{di^2(c+dx)} - \frac{bg\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i^2} - \frac{bBgn\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}$$

output

```
-A*g*(b*x+a)/d/i^2/(d*x+c)+B*g*n*(b*x+a)/d/i^2/(d*x+c)-B*g*(b*x+a)*ln(e*((
b*x+a)/(d*x+c))^n)/d/i^2/(d*x+c)-b*g*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-
a*d+b*c)/b/(d*x+c))/d^2/i^2-b*B*g*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^2/i^2
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.09

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$$
$$= \frac{g\left(\frac{2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{c+dx} + 2b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx) - 2Bn\left(\frac{bc-ad}{c+dx} + b\log(a+bx) - b\l\right.}{2d^2i^2}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*
i*x)^2,x]
```

output
```
(g*((2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) + 2*b
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*B*n*((b*c - a*d)/
(c + d*x) + b*Log[a + b*x] - b*Log[c + d*x]) - b*B*n*((2*Log[(d*(a + b*x))
/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))
/(b*c - a*d)])))/(2*d^2*i^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {2961, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ci+dix)^2}\,dx$$
$$\downarrow 2961$$
$$\frac{g\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{i^2}$$
$$\downarrow 2793$$

$$\frac{g\int\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d}-\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{i^2}$$

$$\downarrow \text{2009}$$

$$\frac{g\left(-\frac{b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^2}-\frac{A(a+bx)}{d(c+dx)}-\frac{bBn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2}-\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d(c+dx)}+\frac{Bn(a+bx)}{d(c+dx)}\right)}{i^2}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)^2
,x]
```

output
```
(g*(-((A*(a + b*x))/(d*(c + d*x))) + (B*n*(a + b*x))/(d*(c + d*x)) - (B*(a
 + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d*(c + d*x)) - (b*(A + B*Log[e*((
a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 - (b*B*n
*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^2))/i^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(dix+ci)^2}dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}dx = \int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^2}dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, al
gorithm="fricas")
```

output
```
integral((A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log(e*((b*x + a)/(d*x + c))^
n))/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, al
gorithm="maxima")
```

output
```
B*a*g*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2) -
b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) - 1/2*B*b*g*((2*(d*n*x + c*n)*log(b*
x + a)*log(d*x + c) - (d*n*x + c*n)*log(d*x + c)^2 - 2*((d*x + c)*log(d*x
+ c) + c)*log((b*x + a)^n) + 2*((d*x + c)*log(d*x + c) + c)*log((d*x + c)^
n))/(d^3*i^2*x + c*d^2*i^2) - 2*integrate((b*d^2*x^2*log(e) + a*d^2*x*log(
e) - b*c^2*n + a*c*d*n + (b*d^2*n*x^2 + a*c*d*n + (b*c*d*n + a*d^2*n)*x)*l
og(b*x + a))/(b*d^4*i^2*x^3 + a*c^2*d^2*i^2 + (2*b*c*d^3*i^2 + a*d^4*i^2)*
x^2 + (b*c^2*d^2*i^2 + 2*a*c*d^3*i^2)*x), x)) + A*b*g*(c/(d^3*i^2*x + c*d^
2*i^2) + log(d*x + c)/(d^2*i^2)) - B*a*g*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n)/(d^2*i^2*x + c*d*i^2) - A*a*g/(d^2*i^2*x + c*d*i^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(167) = 334$.

Time = 142.58 (sec) , antiderivative size = 906, normalized size of antiderivative = 5.39

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, al
gorithm="giac")
```

output

```
-1/2*((B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - 2*(b*x + a)*B*b^3*c^3*d*g*n/(
d*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 6*(b*x + a)*B*a*b^2*c^2*d^2*g*n/(d*x +
c) - B*a^3*b*d^3*g*n - 6*(b*x + a)*B*a^2*b*c*d^3*g*n/(d*x + c) + 2*(b*x +
a)*B*a^3*d^4*g*n/(d*x + c))*log((b*x + a)/(d*x + c))/(b^2*d^2*i^2 - 2*(b*x
 + a)*b*d^3*i^2/(d*x + c) + (b*x + a)^2*d^4*i^2/(d*x + c)^2) + (B*b^4*c^3*
g*n - 3*B*a*b^3*c^2*d*g*n - (b*x + a)*B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*
b^2*c*d^2*g*n + 3*(b*x + a)*B*a*b^2*c^2*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*
n - 3*(b*x + a)*B*a^2*b*c*d^3*g*n/(d*x + c) + (b*x + a)*B*a^3*d^4*g*n/(d*x
 + c) + B*b^4*c^3*g*log(e) - 3*B*a*b^3*c^2*d*g*log(e) - 2*(b*x + a)*B*b^3*
c^3*d*g*log(e)/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*log(e) + 6*(b*x + a)*B*a*b^
2*c^2*d^2*g*log(e)/(d*x + c) - B*a^3*b*d^3*g*log(e) - 6*(b*x + a)*B*a^2*b*
c*d^3*g*log(e)/(d*x + c) + 2*(b*x + a)*B*a^3*d^4*g*log(e)/(d*x + c) + A*b^
4*c^3*g - 3*A*a*b^3*c^2*d*g - 2*(b*x + a)*A*b^3*c^3*d*g/(d*x + c) + 3*A*a^
2*b^2*c*d^2*g + 6*(b*x + a)*A*a*b^2*c^2*d^2*g/(d*x + c) - A*a^3*b*d^3*g -
6*(b*x + a)*A*a^2*b*c*d^3*g/(d*x + c) + 2*(b*x + a)*A*a^3*d^4*g/(d*x + c))
/(b^2*d^2*i^2 - 2*(b*x + a)*b*d^3*i^2/(d*x + c) + (b*x + a)^2*d^4*i^2/(d*x
 + c)^2) + (B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*
a^3*d^3*g*n)*log(-b + (b*x + a)*d/(d*x + c))/(b*d^2*i^2) - (B*b^3*c^3*g*n
- 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log((b*x + a)
/(d*x + c))/(b*d^2*i^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(c\,i+d\,i\,x)^2}\,dx$$

input

```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^2
,x)
```

output

```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^2
, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$$

$$= \frac{g\left(-\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x}{d^2x^2+2cdx+c^2}dx\right) a\,b^2c^2d^3 - \left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x}{d^2x^2+2cdx+c^2}dx\right) a\,b^2c\,d^4x + \left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x}{d^2x^2+2cdx+c^2}dx\right) b^3c^3d^2 + \left(\int \frac{1}{}\right.\right.$$

input

```
int((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

output

```
(g*( - int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c**2 + 2*c*d*x + d**2*x
**2),x)*a*b**2*c**2*d**3 - int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c**
2 + 2*c*d*x + d**2*x**2),x)*a*b**2*c*d**4*x + int((log(((a + b*x)**n*e)/(c
 + d*x)**n)*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**3*c**3*d**2 + int((log((
(a + b*x)**n*e)/(c + d*x)**n)*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**3*c**2
*d**3*x + log(a + b*x)*a**2*b*c*d**2*n + log(a + b*x)*a**2*b*d**3*n*x - lo
g(c + d*x)*a**2*b*c**2*d - log(c + d*x)*a**2*b*c*d**2*n - log(c + d*x)*a**
2*b*c*d**2*x - log(c + d*x)*a**2*b*d**3*n*x + log(c + d*x)*a*b**2*c**3 + l
og(c + d*x)*a*b**2*c**2*d*x - log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*
*3*x + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*d**2*x - a**3*d**3*x +
2*a**2*b*c*d**2*x + a**2*b*d**3*n*x - a*b**2*c**2*d*x - a*b**2*c*d**2*n*x)
)/(c*d**2*(a*c*d + a*d**2*x - b*c**2 - b*c*d*x))
```

3.146 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx$

Optimal result 1671
Mathematica [A] (verified) 1671
Rubi [A] (verified) 1672
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Optimal result

Integrand size = 33, antiderivative size = 102

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx = \frac{A(a+bx)}{(bc-ad)i^2(c+dx)} - \frac{Bn(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)i^2(c+dx)}$$

output
```
A*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)-B*n*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)+B*(b*x
+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/i^2/(d*x+c)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{di(ci+dix)} + \frac{B(bc-ad)n\left(\frac{1}{(bc-ad)(c+dx)} + \frac{b\log(a+bx)}{(bc-ad)^2} - \frac{b\log(c+dx)}{(bc-ad)^2}\right)}{di^2}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x)^2,x]
```

output
```
-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(d*i*(c*i + d*i*x))) + (B*(b*c -
a*d)*n*(1/((b*c - a*d)*(c + d*x)) + (b*Log[a + b*x])/(b*c - a*d)^2 - (b*Lo
g[c + d*x])/(b*c - a*d)^2))/(d*i^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2951, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{(ci+dix)^2}\,dx$$

$$\downarrow 2951$$

$$\frac{\int \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2009$$

$$\frac{\frac{A(a+bx)}{c+dx} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} - \frac{Bn(a+bx)}{c+dx}}{i^2(bc-ad)}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x)^2,x]
```

output
```
((A*(a + b*x))/(c + d*x) - (B*n*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[e*
((a + b*x)/(c + d*x))^n])/(c + d*x))/((b*c - a*d)*i^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2951

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{A}{i^2(dx+c)d}-\frac{B\left(\frac{(bx+a)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{dx+c}-\frac{n(bx+a)}{dx+c}\right)}{i^2(da-bc)}$	81
parts	$-\frac{A}{i^2(dx+c)d}-\frac{B\left(\frac{(bx+a)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{dx+c}-\frac{n(bx+a)}{dx+c}\right)}{i^2(da-bc)}$	81
parallelrisch	$-\frac{-Bab\,d^3n^2+B\,b^2c\,d^2n^2+Aab\,d^3n-A\,b^2c\,d^2n+Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^2d^3n+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)ab\,d^3n}{i^2(dx+c)b\,d^3n(da-bc)}$	129

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/i^2*A/(d*x+c)/d-1/i^2*B/(a*d-b*c)*((b*x+a)/(d*x+c)*ln(e*((b*x+a)/(d*x+c
))^n)-n*(b*x+a)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx$$
$$= -\frac{Abc - Aad - (Bbc - Bad)n + (Bbc - Bad)\log(e) - (Bbdnx + Badn)\log\left(\frac{bx+a}{dx+c}\right)}{(bcd^2 - ad^3)i^2x + (bc^2d - acd^2)i^2}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, algorithm="fri
cas")
```

output
```
-(A*b*c - A*a*d - (B*b*c - B*a*d)*n + (B*b*c - B*a*d)*log(e) - (B*b*d*n*x
+ B*a*d*n)*log((b*x + a)/(d*x + c)))/((b*c*d^2 - a*d^3)*i^2*x + (b*c^2*d -
 a*c*d^2)*i^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(82) = 164.

Time = 39.03 (sec) , antiderivative size = 444, normalized size of antiderivative = 4.35

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx$$
$$= \begin{cases} -\frac{A}{cdi^2+d^2i^2x} - \frac{B\log\left(e\left(\frac{bc}{cd+d^2x}+\frac{bx}{c+dx}\right)^n\right)}{cdi^2+d^2i^2x} \\ \frac{Ax+\frac{Ba\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b} - Bnx + Bx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{c^2i^2} \\ -\frac{Aad}{acd^2i^2+ad^3i^2x-bc^2di^2-bcd^2i^2x} + \frac{Abc}{acd^2i^2+ad^3i^2x-bc^2di^2-bcd^2i^2x} + \frac{Badn}{acd^2i^2+ad^3i^2x-bc^2di^2-bcd^2i^2x} - \frac{Bad\log\left(e\left(\frac{a}{c+dx}+\frac{b}{c+}\right.\right.}{acd^2i^2+ad^3i^2x-bc^2di^2-} \end{cases}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**2,x)
```

output
```
Piecewise((-A/(c*d*i**2 + d**2*i**2*x) - B*log(e*(b*c/(c*d + d**2*x) + b*x
/(c + d*x))**n)/(c*d*i**2 + d**2*i**2*x), Eq(a, b*c/d)), ((A*x + B*a*log(e
*(a/c + b*x/c)**n)/b - B*n*x + B*x*log(e*(a/c + b*x/c)**n))/(c**2*i**2), E
q(d, 0)), (-A*a*d/(a*c*d**2*i**2 + a*d**3*i**2*x - b*c**2*d*i**2 - b*c*d**
2*i**2*x) + A*b*c/(a*c*d**2*i**2 + a*d**3*i**2*x - b*c**2*d*i**2 - b*c*d**
2*i**2*x) + B*a*d*n/(a*c*d**2*i**2 + a*d**3*i**2*x - b*c**2*d*i**2 - b*c*d
**2*i**2*x) - B*a*d*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a*c*d**2*i**2
 + a*d**3*i**2*x - b*c**2*d*i**2 - b*c*d**2*i**2*x) - B*b*c*n/(a*c*d**2*i*
*2 + a*d**3*i**2*x - b*c**2*d*i**2 - b*c*d**2*i**2*x) - B*b*d*x*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n)/(a*c*d**2*i**2 + a*d**3*i**2*x - b*c**2*d*i*
*2 - b*c*d**2*i**2*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.33

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx = Bn\left(\frac{1}{d^2i^2x + cdi^2} + \frac{b\log(bx+a)}{(bcd-ad^2)i^2} - \frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right) - \frac{B\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{d^2i^2x + cdi^2} - \frac{A}{d^2i^2x + cdi^2}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, algorithm="max
ima")
```

output
```
B*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2) - b*lo
g(d*x + c)/((b*c*d - a*d^2)*i^2)) - B*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)/(d^2*i^2*x + c*d*i^2) - A/(d^2*i^2*x + c*d*i^2)
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx = \left(\frac{(bx+a)Bn\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)i^2} - \frac{(Bn - B\log(e) - A)(bx+a)}{(dx+c)i^2}\right)\left(\frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2}\right)$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, algorithm="gia
c")
```

output
```
((b*x + a)*B*n*log((b*x + a)/(d*x + c))/((d*x + c)*i^2) - (B*n - B*log(e)
- A)*(b*x + a)/((d*x + c)*i^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 25.80 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx = -\frac{A-B\,n}{x\,d^2\,i^2+c\,d\,i^2} - \frac{B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{d\,\left(c\,i^2+d\,i^2\,x\right)} + \frac{B\,b\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{d\,i^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*i + d*i*x)^2,x)
```

output
```
(B*b*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(d*i^2*(a*d - b*c))
- (B*log(e*((a + b*x)/(c + d*x))^n))/(d*(c*i^2 + d*i^2*x)) - (A - B*n)/(d^
2*i^2*x + c*d*i^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.51

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx$$
$$= \frac{\log(bx+a)\,abcn+\log(bx+a)\,abdnx-\log(dx+c)\,abcn-\log(dx+c)\,abdnx-\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)abdx+\mathrm{lo}}{c\left(a\,d^2x-bcdx+acd-b\,c^2\right)}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

output

```
(log(a + b*x)*a*b*c*n + log(a + b*x)*a*b*d*n*x - log(c + d*x)*a*b*c*n - lo
g(c + d*x)*a*b*d*n*x - log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x + log(((
a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x - a**2*d*x + a*b*c*x + a*b*d*n*x - b
**2*c*n*x)/(c*(a*c*d + a*d**2*x - b*c**2 - b*c*d*x))
```

3.147 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx$

Optimal result 1678
Mathematica [C] (verified) 1678
Rubi [A] (verified) 1679
Maple [A] (verified) 1681
Fricas [A] (verification not implemented) 1681
Sympy [F(-1)] 1682
Maxima [B] (verification not implemented) 1682
Giac [A] (verification not implemented) 1683
Mupad [B] (verification not implemented) 1684
Reduce [B] (verification not implemented) 1684

Optimal result

Integrand size = 43, antiderivative size = 166

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx = -\frac{Ad(a+bx)}{(bc-ad)^2gi^2(c+dx)} + \frac{Bdn(a+bx)}{(bc-ad)^2gi^2(c+dx)} - \frac{Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^2gi^2(c+dx)} + \frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2B(bc-ad)^2gi^2n}$$

output

```
-A*d*(b*x+a)/(-a*d+b*c)^2/g/i^2/(d*x+c)+B*d*n*(b*x+a)/(-a*d+b*c)^2/g/i^2/(
d*x+c)-B*d*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^2/g/i^2/(d*x+c)+1/
2*b*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/B/(-a*d+b*c)^2/g/i^2/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.83

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2b(c+dx)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2b(c+dx)\left(A+\right.}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i
*x)^2),x]
```

output
```
(2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*b*(c + d*x)*Log[
a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b*(c + d*x)*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*B*n*(b*c - a*d + b*(c + d*x)
*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]) - b*B*n*(c + d*x)*(Log[a + b*x]*
(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b
*x))/(-(b*c) + a*d)]) + b*B*n*(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*
d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
))/(2*(b*c - a*d)^2*g*i^2*(c + d*x))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.72, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {2961, 2788, 2009, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{(ag+bgx)(ci+dix)^2}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx} d\frac{a+bx}{c+dx}}{gi^2(bc-ad)^2}$$

$$\downarrow 2788$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx} d\frac{a+bx}{c+dx} - d\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) d\frac{a+bx}{c+dx}}{gi^2(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx} d\frac{a+bx}{c+dx} - d\left(\frac{A(a+bx)}{c+dx} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} - \frac{Bn(a+bx)}{c+dx}\right)}{gi^2(bc-ad)^2}$$

$$\downarrow 2738$$

$$\frac{\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2Bn}-d\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}-\frac{Bn(a+bx)}{c+dx}\right)}{gi^2(bc-ad)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i*x)^2)
,x]
```

output
```
((b*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*B*n) - d*((A*(a + b*x))/(
c + d*x) - (B*n*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[e*((a + b*x)/(c +
d*x))^n])/(c + d*x)))/((b*c - a*d)^2*g*i^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.62

method	result
parallelrisch	$\frac{Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2b^3d^4+2Ax\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3d^4+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2b^3cd^3+2A\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3cd^3+2Ba\,b^2d^4n^2-2B}{2i^2g(dx+c)(a^2d^2-2acdb+c^2b^2)b^2d^3}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^2,x,method=_RE
TURNVERBOSE)
```

output
```
1/2*(B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*d^4+2*A*x*ln(e*((b*x+a)/(d*x+c))^
n)*b^3*d^4+B*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*c*d^3+2*A*ln(e*((b*x+a)/(d*x+
c))^n)*b^3*c*d^3+2*B*a*b^2*d^4*n^2-2*B*b^3*c*d^3*n^2-2*A*a*b^2*d^4*n+2*A*b
^3*c*d^3*n-2*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^4*n-2*B*ln(e*((b*x+a)/(d*
x+c))^n)*a*b^2*d^4*n)/i^2/g/(d*x+c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/d^3/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.18

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$=\frac{2\,Abc-2\,Aad+(Bbdnx+Bbcn)\log\left(\frac{bx+a}{dx+c}\right)^2-2\,(Bbc-Bad)n+2\left(Bbc-Bad+(Bbdx+Bbc)\log\right.}{2\left((b^2c^2d-2\,abcd^2+a^2d^3)gi^2x+(b^2c^3-2\,abc^2d+\right.}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, al
gorithm="fricas")
```

output
```
1/2*(2*A*b*c - 2*A*a*d + (B*b*d*n*x + B*b*c*n)*log((b*x + a)/(d*x + c))^2
- 2*(B*b*c - B*a*d)*n + 2*(B*b*c - B*a*d + (B*b*d*x + B*b*c)*log((b*x + a)
/(d*x + c)))*log(e) - 2*(B*a*d*n - A*b*c + (B*b*d*n - A*b*d)*x)*log((b*x +
 a)/(d*x + c)))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g*i^2*x + (b^2*c^3 -
2*a*b*c^2*d + a^2*c*d^2)*g*i^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(164) = 328.

Time = 0.06 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.55

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= B\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)\log\left(e\left($$
$$-\frac{((bdx+bc)\log(bx+a)^2+(bdx+bc)\log(dx+c)^2+2\,bc-2\,ad+2\,(bdx+bc)\log(bx+a)-2\,(bdx}{2\,(b^2c^3gi^2-2\,abc^2dgi^2+a^2cd^2gi^2+(b^2c^2dgi^2-2\,abcd^2gi^2+a^2}$$
$$+A\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, al
gorithm="maxima")
```

output

```
B*(1/((b*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*g*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/2*((b*d*x
 + b*c)*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*
(b*d*x + b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*
log(d*x + c))*B*n/(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 + a^2*c*d^2*g*i^2 + (
b^2*c^2*d*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x) + A*(1/((b*c*d - a
*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*
i^2))
```

Giac [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.25

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{1}{2}\left(\frac{Bbn\log\left(\frac{bx+a}{dx+c}\right)^2}{bcgi^2 - adgi^2} - \frac{2\,(bx+a)Bdn\log\left(\frac{bx+a}{dx+c}\right)}{(bcgi^2 - adgi^2)(dx+c)} + \frac{2\,(Bb\log(e) + Ab)\log\left(\frac{bx+a}{dx+c}\right)}{bcgi^2 - adgi^2} + \frac{2\,(Bdn - Bd\log(e)}{(bcgi^2 - adgi^2}\right.$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, al
gorithm="giac")
```

output

```
1/2*(B*b*n*log((b*x + a)/(d*x + c))^2/(b*c*g*i^2 - a*d*g*i^2) - 2*(b*x + a
)*B*d*n*log((b*x + a)/(d*x + c))/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)) + 2*(
B*b*log(e) + A*b)*log((b*x + a)/(d*x + c))/(b*c*g*i^2 - a*d*g*i^2) + 2*(B*
d*n - B*d*log(e) - A*d)*(b*x + a)/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)))*(b*
c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 25.83 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.45

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx = \frac{B\,n}{g\,i^2\,(a\,d-b\,c)\,(c+d\,x)} - \frac{A}{g\,i^2\,(a\,d-b\,c)\,(c+d\,x)} - \frac{B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{g\,i^2\,(a\,d-b\,c)\,(c+d\,x)} + \frac{B\,b\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{2\,g\,i^2\,n\,(a\,d-b\,c)^2} - \frac{A\,b\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g\,i^2\,(a\,d-b\,c)^2} + \frac{B\,b\,n\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g\,i^2\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)*(c*i + d*i*x)^2)
,x)
```

output
```
(B*n)/(g*i^2*(a*d - b*c)*(c + d*x)) - A/(g*i^2*(a*d - b*c)*(c + d*x)) - (B
*log(e*((a + b*x)/(c + d*x))^n))/(g*i^2*(a*d - b*c)*(c + d*x)) - (A*b*atan
((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g*i^2*(a*d - b*c)^2) + (B*
b*n*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g*i^2*(a*d - b*c)^
2) + (B*b*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g*i^2*n*(a*d - b*c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.04

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{-2\log(bx+a)\,ab\,c^2n + 2\log(bx+a)\,abcd\,n^2 - 2\log(bx+a)\,abcdnx + 2\log(bx+a)\,ab\,d^2n^2x + 2\log(dx$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^2,x)
```

output

```
( - 2*log(a + b*x)*a*b*c**2*n + 2*log(a + b*x)*a*b*c*d*n**2 - 2*log(a + b*
x)*a*b*c*d*n*x + 2*log(a + b*x)*a*b*d**2*n**2*x + 2*log(c + d*x)*a*b*c**2*
n - 2*log(c + d*x)*a*b*c*d*n**2 + 2*log(c + d*x)*a*b*c*d*n*x - 2*log(c + d
*x)*a*b*d**2*n**2*x - log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**2*c**2 - lo
g(((a + b*x)**n*e)/(c + d*x)**n)**2*b**2*c*d*x - 2*log(((a + b*x)**n*e)/(c
 + d*x)**n)*a*b*d**2*n*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*d*n
*x - 2*a**2*d**2*n*x + 2*a*b*c*d*n*x + 2*a*b*d**2*n**2*x - 2*b**2*c*d*n**2
*x)/(2*c*g*n*(a**2*c*d**2 + a**2*d**3*x - 2*a*b*c**2*d - 2*a*b*c*d**2*x +
b**2*c**3 + b**2*c**2*d*x))
```

3.148 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$

Optimal result . . . 1686
Mathematica [C] (verified) . . . 1687
Rubi [A] (verified) . . . 1687
Maple [B] (verified) . . . 1689
Fricas [A] (verification not implemented) . . . 1690
Sympy [F(-1)] . . . 1690
Maxima [B] (verification not implemented) . . . 1691
Giac [A] (verification not implemented) . . . 1691
Mupad [B] (verification not implemented) . . . 1692
Reduce [B] (verification not implemented) . . . 1693

Optimal result

Integrand size = 43, antiderivative size = 273

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx &= -\frac{Bd^2n(a+bx)}{(bc-ad)^3g^2i^2(c+dx)} - \frac{b^2Bn(c+dx)}{(bc-ad)^3g^2i^2(a+bx)} \\ &\quad + \frac{d^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^2i^2(c+dx)} \\ &\quad - \frac{b^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^2i^2(a+bx)} \\ &\quad - \frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2} \\ &\quad + \frac{bBdn\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}\end{aligned}$$

output

```
-B*d^2*n*(b*x+a)/(-a*d+b*c)^3/g^2/i^2/(d*x+c)-b^2*B*n*(d*x+c)/(-a*d+b*c)^3
/g^2/i^2/(b*x+a)+d^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/
g^2/i^2/(d*x+c)-b^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g
^2/i^2/(b*x+a)-2*b*d*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(
-a*d+b*c)^3/g^2/i^2+b*B*d*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^2/i^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.25

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$$
$$= \frac{-\frac{b^2Bcn}{a+bx}+\frac{abBdn}{a+bx}+\frac{bBcdn}{c+dx}-\frac{aBd^2n}{c+dx}-\frac{b(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}+\frac{d(-bc+ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{c+dx}-2bd\log(a-$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d
*i*x)^2),x]
```

output
```
(-((b^2*B*c*n)/(a + b*x)) + (a*b*B*d*n)/(a + b*x) + (b*B*c*d*n)/(c + d*x)
- (a*B*d^2*n)/(c + d*x) - (b*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(a + b*x) + (d*(-(b*c) + a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
))/(c + d*x) - 2*b*d*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) +
 2*b*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + b*B*d*n*(Log[
a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2,
(d*(a + b*x))/(-(b*c) + a*d)]) - b*B*d*n*((2*Log[(d*(a + b*x))/(-(b*c) + a
*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)
]))/((b*c - a*d)^3*g^2*i^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^2(ci+dix)^2}\,dx$$
$$\downarrow \text{2961}$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i^2(bc-ad)^3}$$

$$\downarrow 2772$$

$$\frac{-Bn\int\left(d^2-\frac{2b(c+dx)\log\left(\frac{a+bx}{c+dx}\right)d}{a+bx}-\frac{b^2(c+dx)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}+\frac{d^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right.}{c+dx}}{g^2i^2(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}+\frac{d^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{c+dx}-2bd\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\left(}{g^2i^2(bc-ad)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d*i*x)^
2),x]
```

output
```
((d^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) - (b^2*(
c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - 2*b*d*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] - B*n*((d^2*(a + 
b*x))/(c + d*x) + (b^2*(c + d*x))/(a + b*x) - b*d*Log[(a + b*x)/(c + d*x)]
^2))/((b*c - a*d)^3*g^2*i^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a + 
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(273) = 546$.

Time = 9.76 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.46

method	result
parallelrisch	$\frac{2Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^3b^2c^4dn+Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2a^3b^2c^4d+Bx\,a^4b\,c^3d^2n^2+Bx\,a^3b^2c^4d\,n^2+2Ax\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^4b\,c^3d^2-$

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,method=_
RETURNVERBOSE)
```

output

```
(2*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^4*d*n+B*x*ln(e*((b*x+a)/(d*x+c)
)^n)^2*a^3*b^2*c^4*d+B*x*a^4*b*c^3*d^2*n^2+B*x*a^3*b^2*c^4*d*n^2+2*A*x*ln(
e*((b*x+a)/(d*x+c))^n)*a^4*b*c^3*d^2+2*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b
^2*c^4*d-A*x*a^4*b*c^3*d^2*n+A*x*a^3*b^2*c^4*d*n-B*ln(e*((b*x+a)/(d*x+c))^
n)*a^5*c^3*d^2*n+B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^5*n+2*A*ln(e*((b*x+
a)/(d*x+c))^n)*a^4*b*c^4*d-2*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b*c^3*d^2*n
-B*x*a^5*c^2*d^3*n^2-B*x*a^2*b^3*c^5*n^2+A*x*a^5*c^2*d^3*n-A*x*a^2*b^3*c^5
*n+B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b*c^4*d+B*x^2*ln(e*((b*x+a)/(d*x+c))^
n)^2*a^3*b^2*c^3*d^2-B*x^2*a^4*b*c^2*d^3*n^2+2*B*x^2*a^3*b^2*c^3*d^2*n^2-B
*x^2*a^2*b^3*c^4*d*n^2+2*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^3*d^2+A
*x^2*a^4*b*c^2*d^3*n-A*x^2*a^2*b^3*c^4*d*n+B*x*ln(e*((b*x+a)/(d*x+c))^n)^2
*a^4*b*c^3*d^2)/i^2/g^2/(d*x+c)/(b*x+a)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2
*d-b^3*c^3)/a^3/c^3/n
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.65

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx =$$

$$-\frac{Ab^2c^2 - Aa^2d^2 + (Bb^2d^2nx^2 + Babcdn + (Bb^2cd + Babd^2)nx)\log\left(\frac{bx+a}{dx+c}\right)^2 + (Bb^2c^2 - 2\,Babcd + Ba$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,
algorithm="fricas")
```

output
```
-(A*b^2*c^2 - A*a^2*d^2 + (B*b^2*d^2*n*x^2 + B*a*b*c*d*n + (B*b^2*c*d + B*
a*b*d^2)*n*x)*log((b*x + a)/(d*x + c))^2 + (B*b^2*c^2 - 2*B*a*b*c*d + B*a^
2*d^2)*n + 2*(A*b^2*c*d - A*a*b*d^2)*x + (B*b^2*c^2 - B*a^2*d^2 + 2*(B*b^2
*c*d - B*a*b*d^2)*x + 2*(B*b^2*d^2*x^2 + B*a*b*c*d + (B*b^2*c*d + B*a*b*d^
2)*x)*log((b*x + a)/(d*x + c)))*log(e) + (2*A*b^2*d^2*x^2 + 2*A*a*b*c*d +
(B*b^2*c^2 - B*a^2*d^2)*n + 2*(A*b^2*c*d + A*a*b*d^2 + (B*b^2*c*d - B*a*b*
d^2)*n)*x)*log((b*x + a)/(d*x + c)))/((b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2
*b^2*c*d^3 - a^3*b*d^4)*g^2*i^2*x^2 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c
*d^3 - a^4*d^4)*g^2*i^2*x + (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2
 - a^4*c*d^3)*g^2*i^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2/(d*i*x+c*i)**2,x
)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 862 vs. 2(273) = 546.

Time = 0.08 (sec) , antiderivative size = 862, normalized size of antiderivative = 3.16

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,
algorithm="maxima")
```

output
```
-B*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i^2
*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*g^2*i^2*x + (a*b^2*
c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d*log(b*x + a)/((b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2) - 2*b*d*log(d*x + c)/((
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2))*log(e*(b*x/(d
*x + c) + a/(d*x + c))^n) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*d^2*x^2
+ a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + a*b*c
*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)*log(d*x + c) - (b^2*d^2*x^2 + a*b
*c*d + (b^2*c*d + a*b*d^2)*x)*log(d*x + c)^2)*B*n/(a*b^3*c^4*g^2*i^2 - 3*a
^2*b^2*c^3*d*g^2*i^2 + 3*a^3*b*c^2*d^2*g^2*i^2 - a^4*c*d^3*g^2*i^2 + (b^4*
c^3*d*g^2*i^2 - 3*a*b^3*c^2*d^2*g^2*i^2 + 3*a^2*b^2*c*d^3*g^2*i^2 - a^3*b*
d^4*g^2*i^2)*x^2 + (b^4*c^4*g^2*i^2 - 2*a*b^3*c^3*d*g^2*i^2 + 2*a^3*b*c*d^
3*g^2*i^2 - a^4*d^4*g^2*i^2)*x) - A*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2
*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d
^2 + a^3*d^3)*g^2*i^2*x + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2)
 + 2*b*d*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*g^2*i^2) - 2*b*d*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*g^2*i^2))
```

Giac [A] (verification not implemented)

Time = 103.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.35

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx =$$
$$-\left(\frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2}\right)^2\left(\frac{(dx+c)Bn\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)g^2i^2} + \frac{(Bn + B\log(e) + A)(dx+c)}{(bx+a)g^2i^2}\right)$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,
algorithm="giac")
```

output
```
-(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2*((d*x + c)*B*n*log((b*x + a)/(d
*x + c))/((b*x + a)*g^2*i^2) + (B*n + B*log(e) + A)*(d*x + c)/((b*x + a)*g
^2*i^2))
```

Mupad [B] (verification not implemented)

Time = 26.37 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.58

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = \frac{B\,b\,d\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{g^2\,i^2\,n\,(a\,d-b\,c)^3} - \frac{A\,b\,c}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{A\,a\,d}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} + \frac{B\,a\,d\,n}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{B\,b\,c\,n}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{2\,A\,b\,d\,x}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{B\,a\,d\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{B\,b\,c\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{2\,B\,b\,d\,x\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{A\,b\,d\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,4\mathrm{i}}{g^2\,i^2\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^2*(c*i + d*i*x)^
2),x)
```

output

```
(B*b*d*log(e*((a + b*x)/(c + d*x))^n)^2)/(g^2*i^2*n*(a*d - b*c)^3) - (A*a*
d)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (A*b*c)/(g^2*i^2*(a*d - b
*c)^2*(a + b*x)*(c + d*x)) - (A*b*d*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d
 - b*c))*4i)/(g^2*i^2*(a*d - b*c)^3) + (B*a*d*n)/(g^2*i^2*(a*d - b*c)^2*(a
 + b*x)*(c + d*x)) - (B*b*c*n)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x))
 - (2*A*b*d*x)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (B*a*d*log(e*
((a + b*x)/(c + d*x))^n))/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (B
*b*c*log(e*((a + b*x)/(c + d*x))^n))/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c +
 d*x)) - (2*B*b*d*x*log(e*((a + b*x)/(c + d*x))^n))/(g^2*i^2*(a*d - b*c)^2
*(a + b*x)*(c + d*x))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 961, normalized size of antiderivative = 3.52

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^2(ci + dix)^2}\, dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x)
```

output

```
( - 2*log(a + b*x)*a**3*b*c*d**2*n - 2*log(a + b*x)*a**3*b*d**3*n*x - 2*lo
g(a + b*x)*a**2*b**2*c**2*d*n - 4*log(a + b*x)*a**2*b**2*c*d**2*n*x - 2*lo
g(a + b*x)*a**2*b**2*d**3*n*x**2 - 2*log(a + b*x)*a*b**3*c**2*d*n*x - 2*lo
g(a + b*x)*a*b**3*c*d**2*n*x**2 + 2*log(c + d*x)*a**3*b*c*d**2*n + 2*log(c
 + d*x)*a**3*b*d**3*n*x + 2*log(c + d*x)*a**2*b**2*c**2*d*n + 4*log(c + d*
x)*a**2*b**2*c*d**2*n*x + 2*log(c + d*x)*a**2*b**2*d**3*n*x**2 + 2*log(c +
 d*x)*a*b**3*c**2*d*n*x + 2*log(c + d*x)*a*b**3*c*d**2*n*x**2 - log(((a +
b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*c*d**2 - log(((a + b*x)**n*e)/(c + d
*x)**n)**2*a**2*b**2*d**3*x - log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3
*c**2*d - 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3*c*d**2*x - log(((
a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3*d**3*x**2 - log(((a + b*x)**n*e)/(c
 + d*x)**n)**2*b**4*c**2*d*x - log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*
c*d**2*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*d**3*n + log(((a +
 b*x)**n*e)/(c + d*x)**n)*a**2*b**2*c*d**2*n + 2*log(((a + b*x)**n*e)/(c +
 d*x)**n)*a**2*b**2*d**3*n*x - log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c
**2*d*n - log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*c**3*n - 2*log(((a + b*x
)**n*e)/(c + d*x)**n)*b**4*c**2*d*n*x + a**4*d**3*n - a**3*b*c*d**2*n - a*
*3*b*d**3*n**2 + a**2*b**2*c**2*d*n + a**2*b**2*c*d**2*n**2 - 2*a**2*b**2*
d**3*n*x**2 - a*b**3*c**3*n + a*b**3*c**2*d*n**2 + 2*a*b**3*c*d**2*n*x**2
- b**4*c**3*n**2)/(g**2*n*(a**5*c*d**4 + a**5*d**5*x - 2*a**4*b*c**2*d*...
```

3.149 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx$

Optimal result . . . 1695
Mathematica [C] (verified) . . . 1696
Rubi [A] (verified) . . . 1697
Maple [B] (verified) . . . 1699
Fricas [B] (verification not implemented) . . . 1700
Sympy [F(-1)] . . . 1701
Maxima [B] (verification not implemented) . . . 1702
Giac [A] (verification not implemented) . . . 1703
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Reduce [B] (verification not implemented) . . . 1704

Optimal result

Integrand size = 43, antiderivative size = 361

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx &= \frac{Bd^3n(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} - \frac{bBn(c+dx)^2\left(b-\frac{6d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^4g^3i^2(a+bx)^2}\\ &\quad - \frac{d^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(c+dx)}\\ &\quad + \frac{3b^2d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(a+bx)}\\ &\quad - \frac{b^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2}\\ &\quad + \frac{3bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2}\\ &\quad - \frac{3bBd^2n\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2}\end{aligned}$$

output

```
B*d^3*n*(b*x+a)/(-a*d+b*c)^4/g^3/i^2/(d*x+c)-1/4*b*B*n*(d*x+c)^2*(b-6*d*(b
*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2-d^3*(b*x+a)*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+3*b^2*d*(d*x+c)*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)-1/2*b^3*(d*x+c)^2*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2+3*b*d^2*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c)^4/g^3/i^2-3/2*b*B*d^
2*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^3/i^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.32

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx$$

$$= \frac{-\frac{bB(bc-ad)^2n}{(a+bx)^2} + \frac{8b^2Bcdn}{a+bx} - \frac{8abBd^2n}{a+bx} + \frac{2bBd(bc-ad)n}{a+bx} - \frac{4bBcd^2n}{c+dx} + \frac{4aBd^3n}{c+dx} + 6bBd^2n\log(a+bx) - \frac{2b(bc-ad)^2\left(A+B}{(a+}$$

input

```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d
*i*x)^2),x]
```

output

```
(-((b*B*(b*c - a*d)^2*n)/(a + b*x)^2) + (8*b^2*B*c*d*n)/(a + b*x) - (8*a*b
*B*d^2*n)/(a + b*x) + (2*b*B*d*(b*c - a*d)*n)/(a + b*x) - (4*b*B*c*d^2*n)/
(c + d*x) + (4*a*B*d^3*n)/(c + d*x) + 6*b*B*d^2*n*Log[a + b*x] - (2*b*(b*c
 - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 + (8*b*d*(b*
c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (4*d^2*(b*c -
 a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) + 12*b*d^2*Log[a +
 b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*b*B*d^2*n*Log[c + d*x] -
12*b*d^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 6*b*B*d^2*n
*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyL
og[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 6*b*B*d^2*n*((2*Log[(d*(a + b*x))/(
-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(
b*c - a*d)]))/(4*(b*c - a*d)^4*g^3*i^2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2961, 2772, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^3(ci+dix)^2}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3i^2(bc-ad)^4}$$

$$\downarrow 2772$$

$$\frac{-Bn\int -\frac{(c+dx)^3\left(b^3-\frac{6d(a+bx)b^2}{c+dx}-\frac{6d^2(a+bx)^2\log\left(\frac{a+bx}{c+dx}\right)b}{(c+dx)^2}+\frac{2d^3(a+bx)^3}{(c+dx)^3}\right)}{2(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx)\left(B}{}}{g^3i^2(bc-ad)^4}$$

$$\downarrow 27$$

$$\frac{\frac{1}{2}Bn\int \frac{(c+dx)^3\left(b^3-\frac{6d(a+bx)b^2}{c+dx}-\frac{6d^2(a+bx)^2\log\left(\frac{a+bx}{c+dx}\right)b}{(c+dx)^2}+\frac{2d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx)\left(B\log\right.}{a}}{g^3i^2(bc-ad)^4}$$

$$\downarrow 2010$$

$$\frac{\frac{1}{2}Bn\int\left(\frac{(c+dx)^3\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{2d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)^3}-\frac{6bd^2(c+dx)\log\left(\frac{a+bx}{c+dx}\right)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+}{}}{g^3i^2(bc-ad)^4}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^3(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{d^3(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{c+dx}+3bd^2\log\left(\frac{a+bx}{c+dx}\right)}{g^3i^2(bc-ad)^4}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d*i*x)^
2),x]
```

output
```
(-((d^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x)) + (3*
b^2*d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - (b^3*(
c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) + 3*b*d
^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] + (B*n*
((2*d^3*(a + b*x))/(c + d*x) + (6*b^2*d*(c + d*x))/(a + b*x) - (b^3*(c + d
*x)^2)/(2*(a + b*x)^2) - 3*b*d^2*Log[(a + b*x)/(c + d*x)]^2))/2)/((b*c - a
*d)^4*g^3*i^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(355) = 710$.

Time = 30.93 (sec) , antiderivative size = 977, normalized size of antiderivative = 2.71

method	result
parallelrisch	$\frac{6Bx^3\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^7d^6n+12Bx^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2ab^6d^6+6Bx^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2b^7cd^5-6Bx^2ab^6d^6n^2+6Bx^2b^7cd^5n^2$

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,method=_
RETURNVERBOSE)
```

output

```
1/4*(6*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^7*d^6*n+12*B*x^2*ln(e*((b*x+a)/(d
*x+c))^n)^2*a*b^6*d^6+6*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^7*c*d^5-6*B*x^
2*a*b^6*d^6*n^2+6*B*x^2*b^7*c*d^5*n^2+24*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a
*b^6*d^6+12*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c*d^5-12*A*x^2*a*b^6*d^6*n
+12*A*x^2*b^7*c*d^5*n+6*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^5*d^6-3*B*x*
a^2*b^5*d^6*n^2+9*B*x*b^7*c^2*d^4*n^2+12*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2
*b^5*d^6-18*A*x*a^2*b^5*d^6*n+6*A*x*b^7*c^2*d^4*n+6*B*ln(e*((b*x+a)/(d*x+c
))^n)^2*a^2*b^5*c*d^5-4*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^4*d^6*n-2*B*ln(e
*((b*x+a)/(d*x+c))^n)*b^7*c^3*d^3*n+18*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^7
*c*d^5*n+12*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^6*c*d^5-12*B*x*ln(e*((b*x+
a)/(d*x+c))^n)*a^2*b^5*d^6*n+6*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^2*d^4*n
-6*B*x*a*b^6*c*d^5*n^2+24*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c*d^5+12*A*x
*a*b^6*c*d^5*n+12*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c^2*d^4*n+24*B*x*ln(e*
((b*x+a)/(d*x+c))^n)*a*b^6*c*d^5*n+6*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^7
*d^6+12*A*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^7*d^6+4*B*a^3*b^4*d^6*n^2-B*b^7*
c^3*d^3*n^2-4*A*a^3*b^4*d^6*n-2*A*b^7*c^3*d^3*n-6*A*a^2*b^5*c*d^5*n+12*A*a
*b^6*c^2*d^4*n+12*A*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*c*d^5-15*B*a^2*b^5*c
*d^5*n^2+12*B*a*b^6*c^2*d^4*n^2)/i^2/g^3/(d*x+c)/(b*x+a)^2/(a^3*d^3-3*a^2*
b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/n/(a*d-b*c)/b^4/d^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 946 vs. $2(355) = 710$.

Time = 0.11 (sec) , antiderivative size = 946, normalized size of antiderivative = 2.62

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^3(ci + dix)^2}\, dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,
algorithm="fricas")
```

output

```
-1/4*(2*A*b^3*c^3 - 12*A*a*b^2*c^2*d + 6*A*a^2*b*c*d^2 + 4*A*a^3*d^3 - 6*(
2*A*b^3*c*d^2 - 2*A*a*b^2*d^3 + (B*b^3*c*d^2 - B*a*b^2*d^3)*n)*x^2 - 6*(B*
b^3*d^3*n*x^3 + B*a^2*b*c*d^2*n + (B*b^3*c*d^2 + 2*B*a*b^2*d^3)*n*x^2 + (2
*B*a*b^2*c*d^2 + B*a^2*b*d^3)*n*x)*log((b*x + a)/(d*x + c))^2 + (B*b^3*c^3
 - 12*B*a*b^2*c^2*d + 15*B*a^2*b*c*d^2 - 4*B*a^3*d^3)*n - 3*(2*A*b^3*c^2*d
 + 4*A*a*b^2*c*d^2 - 6*A*a^2*b*d^3 + (3*B*b^3*c^2*d - 2*B*a*b^2*c*d^2 - B*
a^2*b*d^3)*n)*x + 2*(B*b^3*c^3 - 6*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2 + 2*B*a
^3*d^3 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*c^2*d + 2*B*a*b^2*c*
d^2 - 3*B*a^2*b*d^3)*x - 6*(B*b^3*d^3*x^3 + B*a^2*b*c*d^2 + (B*b^3*c*d^2 +
 2*B*a*b^2*d^3)*x^2 + (2*B*a*b^2*c*d^2 + B*a^2*b*d^3)*x)*log((b*x + a)/(d*
x + c)))*log(e) - 2*(6*A*a^2*b*c*d^2 + 3*(B*b^3*d^3*n + 2*A*b^3*d^3)*x^3 +
 3*(3*B*b^3*c*d^2*n + 2*A*b^3*c*d^2 + 4*A*a*b^2*d^3)*x^2 - (B*b^3*c^3 - 6*
B*a*b^2*c^2*d + 2*B*a^3*d^3)*n + 3*(4*A*a*b^2*c*d^2 + 2*A*a^2*b*d^3 + (B*b
^3*c^2*d + 4*B*a*b^2*c*d^2 - 2*B*a^2*b*d^3)*n)*x)*log((b*x + a)/(d*x + c))
)/((b^6*c^4*d - 4*a*b^5*c^3*d^2 + 6*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*d^4 + a^
4*b^2*d^5)*g^3*i^2*x^3 + (b^6*c^5 - 2*a*b^5*c^4*d - 2*a^2*b^4*c^3*d^2 + 8*
a^3*b^3*c^2*d^3 - 7*a^4*b^2*c*d^4 + 2*a^5*b*d^5)*g^3*i^2*x^2 + (2*a*b^5*c^
5 - 7*a^2*b^4*c^4*d + 8*a^3*b^3*c^3*d^2 - 2*a^4*b^2*c^2*d^3 - 2*a^5*b*c*d^
4 + a^6*d^5)*g^3*i^2*x + (a^2*b^4*c^5 - 4*a^3*b^3*c^4*d + 6*a^4*b^2*c^3*d^
2 - 4*a^5*b*c^2*d^3 + a^6*c*d^4)*g^3*i^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Timed out}$$

input

```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3/(d*i*x+c*i)**2,x
)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1724 vs. $2(355) = 710$.

Time = 0.15 (sec) , antiderivative size = 1724, normalized size of antiderivative = 4.78

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,
algorithm="maxima")
```

output
```
1/2*B*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a
*b*d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*
g^3*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3
 - 2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c
^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d
 + 3*a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4
- 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) -
6*b*d^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)*g^3*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/4
*(b^3*c^3 - 12*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3 - 6*(b^3*c*d^2 - a
*b^2*d^3)*x^2 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x
^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a)^2 + 6*(b^3*d^3*x^3 + a^2*
b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*l
og(d*x + c)^2 - 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x - 6*(b^3*d^3
*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*
b*d^3)*x)*log(b*x + a) + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b
^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x - 2*(b^3*d^3*x^3 + a^2*b*c*d^2
 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x
+ a))*log(d*x + c))*B*n/(a^2*b^4*c^5*g^3*i^2 - 4*a^3*b^3*c^4*d*g^3*i^2 + 6
*a^4*b^2*c^3*d^2*g^3*i^2 - 4*a^5*b*c^2*d^3*g^3*i^2 + a^6*c*d^4*g^3*i^2 ...
```

Giac [A] (verification not implemented)

Time = 134.38 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.66

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx =$$
$$-\frac{1}{4}\left(\frac{2\left(Bbn-\frac{2\,(bx+a)Bdn}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^2bcg^3i^2}{(dx+c)^2}-\frac{(bx+a)^2adg^3i^2}{(dx+c)^2}}+\frac{Bbn-\frac{4\,(bx+a)Bdn}{dx+c}+2\,Bb\log\left(e\right)-\frac{4\,(bx+a)Bd\log(e)}{dx+c}+2\,Ab-\frac{4\,(b}{\frac{(bx+a)^2bcg^3i^2}{(dx+c)^2}-\frac{(bx+a)^2adg^3i^2}{(dx+c)^2}}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,
algorithm="giac")
```

output
```
-1/4*(2*(B*b*n - 2*(b*x + a)*B*d*n/(d*x + c))*log((b*x + a)/(d*x + c))/((b
*x + a)^2*b*c*g^3*i^2/(d*x + c)^2 - (b*x + a)^2*a*d*g^3*i^2/(d*x + c)^2) +
 (B*b*n - 4*(b*x + a)*B*d*n/(d*x + c) + 2*B*b*log(e) - 4*(b*x + a)*B*d*log
(e)/(d*x + c) + 2*A*b - 4*(b*x + a)*A*d/(d*x + c))/((b*x + a)^2*b*c*g^3*i^
2/(d*x + c)^2 - (b*x + a)^2*a*d*g^3*i^2/(d*x + c)^2))*(b*c/(b*c - a*d)^2 -
 a*d/(b*c - a*d)^2)^2
```

Mupad [B] (verification not implemented)

Time = 28.21 (sec) , antiderivative size = 1016, normalized size of antiderivative = 2.81

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^3*(c*i + d*i*x)^
2),x)
```

output

```
(3*B*b*d^2*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g^3*i^2*n*(a*d - b*c)^4) -
 ((4*A*a^2*d^2 - 2*A*b^2*c^2 - 4*B*a^2*d^2*n - B*b^2*c^2*n + 10*A*a*b*c*d
+ 11*B*a*b*c*d*n)/(2*(a*d - b*c)) + (3*x^2*(2*A*b^2*d^2 + B*b^2*d^2*n))/(a
*d - b*c) + (3*x*(6*A*a*b*d^2 + 2*A*b^2*c*d + B*a*b*d^2*n + 3*B*b^2*c*d*n)
)/(2*(a*d - b*c)))/(x*(2*a^4*d^3*g^3*i^2 + 4*a*b^3*c^3*g^3*i^2 - 6*a^2*b^2
*c^2*d*g^3*i^2) + x^2*(2*b^4*c^3*g^3*i^2 + 4*a^3*b*d^3*g^3*i^2 - 6*a^2*b^2
*c*d^2*g^3*i^2) + x^3*(2*a^2*b^2*d^3*g^3*i^2 + 2*b^4*c^2*d*g^3*i^2 - 4*a*b
^3*c*d^2*g^3*i^2) + 2*a^2*b^2*c^3*g^3*i^2 + 2*a^4*c*d^2*g^3*i^2 - 4*a^3*b*
c^2*d*g^3*i^2) - (b*d^2*atan((b*d^2*(2*A + B*n)*((a^4*d^4*g^3*i^2 - b^4*c^
4*g^3*i^2 + 2*a*b^3*c^3*d*g^3*i^2 - 2*a^3*b*c*d^3*g^3*i^2)/(a^3*d^3*g^3*i^
2 - b^3*c^3*g^3*i^2 + 3*a*b^2*c^2*d*g^3*i^2 - 3*a^2*b*c*d^2*g^3*i^2) + 2*b
*d*x)*(a^3*d^3*g^3*i^2 - b^3*c^3*g^3*i^2 + 3*a*b^2*c^2*d*g^3*i^2 - 3*a^2*b
*c*d^2*g^3*i^2)*3i)/(g^3*i^2*(6*A*b*d^2 + 3*B*b*d^2*n)*(a*d - b*c)^4))*(2*
A + B*n)*3i)/(g^3*i^2*(a*d - b*c)^4) - log(e*((a + b*x)/(c + d*x))^n)*(((B
*(2*a*d + b*c))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (3*B*b*d*x)/(2*(a^2*
d^2 + b^2*c^2 - 2*a*b*c*d)))/(x*(a^2*d*g^3*i^2 + 2*a*b*c*g^3*i^2) + x^2*(b
^2*c*g^3*i^2 + 2*a*b*d*g^3*i^2) + a^2*c*g^3*i^2 + b^2*d*g^3*i^2*x^3) + (3*
B*b*d^2*(b*g^3*i^2*n*x^2*(a*d - b*c) + (a*c*g^3*i^2*n*(a*d - b*c))/d + (g^
3*i^2*n*x*(a*d + b*c)*(a*d - b*c))/d))/(g^3*i^2*n*(a*d - b*c)^4*(x*(a^2*d*
g^3*i^2 + 2*a*b*c*g^3*i^2) + x^2*(b^2*c*g^3*i^2 + 2*a*b*d*g^3*i^2) + a^...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1921, normalized size of antiderivative = 5.32

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^3(ci + dix)^2}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x)
```

output

```
( - 24*log(a + b*x)*a**4*b*c*d**3*n - 24*log(a + b*x)*a**4*b*d**4*n*x - 12
*log(a + b*x)*a**3*b**2*c**2*d**2*n - 60*log(a + b*x)*a**3*b**2*c*d**3*n*x
 - 48*log(a + b*x)*a**3*b**2*d**4*n*x**2 - 18*log(a + b*x)*a**2*b**3*c**2*
d**2*n**2 - 24*log(a + b*x)*a**2*b**3*c**2*d**2*n*x - 18*log(a + b*x)*a**2
*b**3*c*d**3*n**2*x - 48*log(a + b*x)*a**2*b**3*c*d**3*n*x**2 - 24*log(a +
 b*x)*a**2*b**3*d**4*n*x**3 - 36*log(a + b*x)*a*b**4*c**2*d**2*n**2*x - 12
*log(a + b*x)*a*b**4*c**2*d**2*n*x**2 - 36*log(a + b*x)*a*b**4*c*d**3*n**2
*x**2 - 12*log(a + b*x)*a*b**4*c*d**3*n*x**3 - 18*log(a + b*x)*b**5*c**2*d
**2*n**2*x**2 - 18*log(a + b*x)*b**5*c*d**3*n**2*x**3 + 24*log(c + d*x)*a*
*4*b*c*d**3*n + 24*log(c + d*x)*a**4*b*d**4*n*x + 12*log(c + d*x)*a**3*b**
2*c**2*d**2*n + 60*log(c + d*x)*a**3*b**2*c*d**3*n*x + 48*log(c + d*x)*a**
3*b**2*d**4*n*x**2 + 18*log(c + d*x)*a**2*b**3*c**2*d**2*n**2 + 24*log(c +
 d*x)*a**2*b**3*c**2*d**2*n*x + 18*log(c + d*x)*a**2*b**3*c*d**3*n**2*x +
48*log(c + d*x)*a**2*b**3*c*d**3*n*x**2 + 24*log(c + d*x)*a**2*b**3*d**4*n
*x**3 + 36*log(c + d*x)*a*b**4*c**2*d**2*n**2*x + 12*log(c + d*x)*a*b**4*c
**2*d**2*n*x**2 + 36*log(c + d*x)*a*b**4*c*d**3*n**2*x**2 + 12*log(c + d*x
)*a*b**4*c*d**3*n*x**3 + 18*log(c + d*x)*b**5*c**2*d**2*n**2*x**2 + 18*log
(c + d*x)*b**5*c*d**3*n**2*x**3 - 12*log(((a + b*x)**n*e)/(c + d*x)**n)**2
*a**3*b**2*c*d**3 - 12*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*b**2*d**
4*x - 6*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**3*c**2*d**2 - 30*...
```

3.150 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx$

Optimal result 1706
Mathematica [C] (verified) 1707
Rubi [A] (verified) 1708
Maple [B] (verified) 1710
Fricas [B] (verification not implemented) 1711
Sympy [F(-1)] 1712
Maxima [B] (verification not implemented) 1712
Giac [A] (verification not implemented) 1713
Mupad [B] (verification not implemented) 1714
Reduce [B] (verification not implemented) 1715

Optimal result

Integrand size = 43, antiderivative size = 477

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx &= -\frac{Bd^4n(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2Bd^2n(c+dx)}{(bc-ad)^5g^4i^2(a+bx)} \\ &+ \frac{b^3Bdn(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4Bn(c+dx)^3}{9(bc-ad)^5g^4i^2(a+bx)^3} \\ &+ \frac{d^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(c+dx)} \\ &- \frac{6b^2d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)} \\ &+ \frac{2b^3d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} \\ &- \frac{b^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^5g^4i^2(a+bx)^3} \\ &- \frac{4bd^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2} \\ &+ \frac{2bBd^3n\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}\end{aligned}$$

output

```
-B*d^4*n*(b*x+a)/(-a*d+b*c)^5/g^4/i^2/(d*x+c)-6*b^2*B*d^2*n*(d*x+c)/(-a*d+
b*c)^5/g^4/i^2/(b*x+a)+b^3*B*d*n*(d*x+c)^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-
1/9*b^4*B*n*(d*x+c)^3/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3+d^4*(b*x+a)*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(d*x+c)-6*b^2*d^2*(d*x+c)*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)+2*b^3*d*(d*x+c)^2
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-1/3*b^4*(d
*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3-4*b
*d^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c)^5/g^4/
i^2+2*b*B*d^3*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^5/g^4/i^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.31 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.15

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx =$$

$$-\frac{\frac{bB(bc-ad)^3n}{(a+bx)^3} - \frac{6bBd(bc-ad)^2n}{(a+bx)^2} + \frac{27b^2Bcd^2n}{a+bx} - \frac{27abBd^3n}{a+bx} + \frac{12bBd^2(bc-ad)n}{a+bx} - \frac{9bBcd^3n}{c+dx} + \frac{9aBd^4n}{c+dx} + 30bBd^3n\log(a$$

input

```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d
*i*x)^2),x]
```

output

```
-1/9*((b*B*(b*c - a*d)^3*n)/(a + b*x)^3 - (6*b*B*d*(b*c - a*d)^2*n)/(a + b
*x)^2 + (27*b^2*B*c*d^2*n)/(a + b*x) - (27*a*b*B*d^3*n)/(a + b*x) + (12*b*
B*d^2*(b*c - a*d)*n)/(a + b*x) - (9*b*B*c*d^3*n)/(c + d*x) + (9*a*B*d^4*n)
/(c + d*x) + 30*b*B*d^3*n*Log[a + b*x] + (3*b*(b*c - a*d)^3*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]))/(a + b*x)^3 - (9*b*d*(b*c - a*d)^2*(A + B*Log[e*
((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 + (27*b*d^2*(b*c - a*d)*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - (9*d^3*(-(b*c) + a*d)*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]))/(c + d*x) + 36*b*d^3*Log[a + b*x]*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]) - 30*b*B*d^3*n*Log[c + d*x] - 36*b*d^3*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 18*b*B*d^3*n*(Log[a + b*x]*
(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b
*x))/(-(b*c) + a*d)]) + 18*b*B*d^3*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
 - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/
((b*c - a*d)^5*g^4*i^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{(ag+bgx)^4(ci+dix)^2}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4} d\frac{a+bx}{c+dx}}{g^4 i^2 (bc-ad)^5}$$

$$\downarrow 2772$$

$$\frac{-Bn\int\left(d^4 - \frac{4b(c+dx)\log\left(\frac{a+bx}{c+dx}\right)d^3}{a+bx} - \frac{6b^2(c+dx)^2d^2}{(a+bx)^2} + \frac{2b^3(c+dx)^3d}{(a+bx)^3} - \frac{b^4(c+dx)^4}{3(a+bx)^4}\right) d\frac{a+bx}{c+dx} - \frac{b^4(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}}$$

$$\downarrow 2009$$

$$-\frac{b^4(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{2b^3d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(a+bx)^2}-\frac{6b^2d^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}+\frac{d^4(a+bx)\left(B\log\right.}{c}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d*i*x)^
2),x]
```

output
```
((d^4*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) - (6*b^2
*d^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (2*b^3*
d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 - (b^4*(
c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^3) - 4*b*d
^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] - B*n*(
(d^4*(a + b*x))/(c + d*x) + (6*b^2*d^2*(c + d*x))/(a + b*x) - (b^3*d*(c +
d*x)^2)/(a + b*x)^2 + (b^4*(c + d*x)^3)/(9*(a + b*x)^3) - 2*b*d^3*Log[(a +
 b*x)/(c + d*x)]^2))/((b*c - a*d)^5*g^4*i^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1909 vs. 2(473) = 946.

Time = 49.06 (sec) , antiderivative size = 1910, normalized size of antiderivative = 4.00

method	result	size
parallelrisch	Expression too large to display	1910

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,method=_
RETURNVERBOSE)
```

output
```
1/9*(-54*A*x^4*a^4*b^5*c^4*d^3*n+18*A*x^4*a^3*b^6*c^5*d^2*n-3*A*x^4*a^2*b^
7*c^6*d*n+54*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^3*c^3*d^4+18*B*x^3*ln
(e*((b*x+a)/(d*x+c))^n)^2*a^5*b^4*c^4*d^3-27*B*x^3*a^7*b^2*c^2*d^5*n^2+126
*B*x^3*a^6*b^3*c^3*d^4*n^2-77*B*x^3*a^5*b^4*c^4*d^3*n^2-27*B*x^3*a^4*b^5*c
^5*d^2*n^2+6*B*x^3*a^3*b^6*c^6*d*n^2+54*B*x*a^7*b^2*c^4*d^3*n^2+108*A*x*ln
(e*((b*x+a)/(d*x+c))^n)*a^7*b^2*c^4*d^3-9*A*x*a^8*b*c^3*d^4*n+54*A*x*a^7*b
^2*c^4*d^3*n-90*A*x*a^6*b^3*c^5*d^2*n+45*A*x*a^5*b^4*c^6*d*n+108*A*x^3*ln(
e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c^3*d^4+36*A*x^3*ln(e*((b*x+a)/(d*x+c))^n)*
a^5*b^4*c^4*d^3+27*A*x^3*a^7*b^2*c^2*d^5*n+63*A*x^3*a^6*b^3*c^3*d^4*n-96*A
*x^3*a^5*b^4*c^4*d^3*n+9*A*x^3*a^3*b^6*c^6*d*n+54*B*x^2*ln(e*((b*x+a)/(d*x
+c))^n)^2*a^7*b^2*c^3*d^4+54*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^3*c^4
*d^3-27*B*x^2*a^8*b*c^2*d^5*n^2+81*B*x^2*a^7*b^2*c^3*d^4*n^2+27*B*x^2*a^6*
b^3*c^4*d^3*n^2-102*B*x^2*a^5*b^4*c^5*d^2*n^2+24*B*x^2*a^4*b^5*c^6*d*n^2+1
08*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b^2*c^3*d^4+108*A*x^2*ln(e*((b*x+a)
/(d*x+c))^n)*a^6*b^3*c^4*d^3+27*A*x^2*a^8*b*c^2*d^5*n+27*A*x^2*a^7*b^2*c^3
*d^4*n-90*A*x^2*a^5*b^4*c^5*d^2*n+45*A*x^2*a^4*b^5*c^6*d*n+18*B*x*ln(e*((b
*x+a)/(d*x+c))^n)^2*a^8*b*c^3*d^4-72*B*x*a^6*b^3*c^5*d^2*n^2+21*B*x*a^5*b^
4*c^6*d*n^2+36*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a^8*b*c^3*d^4+162*B*x^2*ln(e*
((b*x+a)/(d*x+c))^n)*a^6*b^3*c^4*d^3*n+18*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*
a^5*b^4*c^5*d^2*n-36*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^8*b*c^3*d^4*n+108*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1458 vs. $2(473) = 946$.

Time $= 0.14$ (sec) , antiderivative size $= 1458$, normalized size of antiderivative $= 3.06$

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,
algorithm="fricas")
```

output

```
-1/9*(3*A*b^4*c^4 - 18*A*a*b^3*c^3*d + 54*A*a^2*b^2*c^2*d^2 - 30*A*a^3*b*c
*d^3 - 9*A*a^4*d^4 + 6*(6*A*b^4*c*d^3 - 6*A*a*b^3*d^4 + 5*(B*b^4*c*d^3 - B
*a*b^3*d^4)*n)*x^3 + 3*(6*A*b^4*c^2*d^2 + 24*A*a*b^3*c*d^3 - 30*A*a^2*b^2*
d^4 + (11*B*b^4*c^2*d^2 + 8*B*a*b^3*c*d^3 - 19*B*a^2*b^2*d^4)*n)*x^2 + 18*
(B*b^4*d^4*n*x^4 + B*a^3*b*c*d^3*n + (B*b^4*c*d^3 + 3*B*a*b^3*d^4)*n*x^3 +
 3*(B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*n*x^2 + (3*B*a^2*b^2*c*d^3 + B*a^3*b*d^
4)*n*x)*log((b*x + a)/(d*x + c))^2 + (B*b^4*c^4 - 9*B*a*b^3*c^3*d + 54*B*a
^2*b^2*c^2*d^2 - 55*B*a^3*b*c*d^3 + 9*B*a^4*d^4)*n - (6*A*b^4*c^3*d - 54*A
*a*b^3*c^2*d^2 - 18*A*a^2*b^2*c*d^3 + 66*A*a^3*b*d^4 + (5*B*b^4*c^3*d - 81
*B*a*b^3*c^2*d^2 + 57*B*a^2*b^2*c*d^3 + 19*B*a^3*b*d^4)*n)*x + 3*(B*b^4*c^
4 - 6*B*a*b^3*c^3*d + 18*B*a^2*b^2*c^2*d^2 - 10*B*a^3*b*c*d^3 - 3*B*a^4*d^
4 + 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*x^3 + 6*(B*b^4*c^2*d^2 + 4*B*a*b^3*c*d^
3 - 5*B*a^2*b^2*d^4)*x^2 - 2*(B*b^4*c^3*d - 9*B*a*b^3*c^2*d^2 - 3*B*a^2*b^
2*c*d^3 + 11*B*a^3*b*d^4)*x + 12*(B*b^4*d^4*x^4 + B*a^3*b*c*d^3 + (B*b^4*c
*d^3 + 3*B*a*b^3*d^4)*x^3 + 3*(B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*x^2 + (3*B*a
^2*b^2*c*d^3 + B*a^3*b*d^4)*x)*log((b*x + a)/(d*x + c)))*log(e) + 3*(12*A*
a^3*b*c*d^3 + 2*(5*B*b^4*d^4*n + 6*A*b^4*d^4)*x^4 + 2*(6*A*b^4*c*d^3 + 18*
A*a*b^3*d^4 + (11*B*b^4*c*d^3 + 9*B*a*b^3*d^4)*n)*x^3 + 6*(6*A*a*b^3*c*d^3
 + 6*A*a^2*b^2*d^4 + (B*b^4*c^2*d^2 + 9*B*a*b^3*c*d^3)*n)*x^2 + (B*b^4*c^4
 - 6*B*a*b^3*c^3*d + 18*B*a^2*b^2*c^2*d^2 - 3*B*a^4*d^4)*n + 2*(18*A*a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4/(d*i*x+c*i)**2,x
)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2563 vs. $2(473) = 946$.

Time = 0.22 (sec) , antiderivative size = 2563, normalized size of antiderivative = 5.37

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,
algorithm="maxima")
```

output

```
-1/3*B*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 13*a^2*b*c*d^2 + 3*a^3
*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 8*a*b^2*c*d^2 - 11
*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b
^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c
^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*x^
3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d^3
 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^4*
c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*g^
4*i^2*x + (a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2
*d^3 + a^7*c*d^4)*g^4*i^2) + 12*b*d^3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4
*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^
4*i^2) - 12*b*d^3*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*
d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2))*log(e*(b*x/(
d*x + c) + a/(d*x + c))^n) - 1/9*(b^4*c^4 - 9*a*b^3*c^3*d + 54*a^2*b^2*c^2
*d^2 - 55*a^3*b*c*d^3 + 9*a^4*d^4 + 30*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 3*(11
*b^4*c^2*d^2 + 8*a*b^3*c*d^3 - 19*a^2*b^2*d^4)*x^2 - 18*(b^4*d^4*x^4 + a^3
*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x
^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a)^2 - 18*(b^4*d^4*x^4 + a
^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)
*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(d*x + c)^2 - (5*b^4*c^3*d -...
```

Giac [A] (verification not implemented)

Time = 173.65 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.84

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx =$$

$$-\frac{1}{18}\left(\frac{6\left(Bb^2n - \frac{3\,(bx+a)Bbdn}{dx+c} + \frac{3\,(bx+a)^2Bd^2n}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^3b^2c^2g^4i^2}{(dx+c)^3} - \frac{2\,(bx+a)^3abcdg^4i^2}{(dx+c)^3} + \frac{(bx+a)^3a^2d^2g^4i^2}{(dx+c)^3}} + \frac{2\,Bb^2n - \frac{9\,(bx+a)Bbdn}{dx+c} + \frac{18\,(bx+a)^2Bd^2n}{(dx+c)^2} + 6\,Bb}{\frac{(bx+}{(}}\right.$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,
algorithm="giac")
```

output

```
-1/18*(6*(B*b^2*n - 3*(b*x + a)*B*b*d*n/(d*x + c) + 3*(b*x + a)^2*B*d^2*n/
(d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^3*b^2*c^2*g^4*i^2/(d*x +
c)^3 - 2*(b*x + a)^3*a*b*c*d*g^4*i^2/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4
*i^2/(d*x + c)^3) + (2*B*b^2*n - 9*(b*x + a)*B*b*d*n/(d*x + c) + 18*(b*x +
 a)^2*B*d^2*n/(d*x + c)^2 + 6*B*b^2*log(e) - 18*(b*x + a)*B*b*d*log(e)/(d*
x + c) + 18*(b*x + a)^2*B*d^2*log(e)/(d*x + c)^2 + 6*A*b^2 - 18*(b*x + a)*
A*b*d/(d*x + c) + 18*(b*x + a)^2*A*d^2/(d*x + c)^2)/((b*x + a)^3*b^2*c^2*g
^4*i^2/(d*x + c)^3 - 2*(b*x + a)^3*a*b*c*d*g^4*i^2/(d*x + c)^3 + (b*x + a)
^3*a^2*d^2*g^4*i^2/(d*x + c)^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2
```

Mupad [B] (verification not implemented)

Time = 31.14 (sec) , antiderivative size = 1665, normalized size of antiderivative = 3.49

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^4 (ci + dix)^2}\, dx = \text{Too large to display}$$

input

```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^4*(c*i + d*i*x)^
2),x)
```

output

```
(2*B*b*d^3*log(e*((a + b*x)/(c + d*x))^n)^2)/(g^4*i^2*n*(a*d - b*c)^3*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d)) - log(e*((a + b*x)/(c + d*x))^n)*(((B*(3*a*d
+ b*c))/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (4*B*b*d*x)/(3*(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d)))/(x^3*(b^3*c*g^4*i^2 + 3*a*b^2*d*g^4*i^2) + x^2*(3*a*b
^2*c*g^4*i^2 + 3*a^2*b*d*g^4*i^2) + x*(a^3*d*g^4*i^2 + 3*a^2*b*c*g^4*i^2)
+ a^3*c*g^4*i^2 + b^3*d*g^4*i^2*x^4) + (4*B*b*d^3*(x*((a*d + b*c)*((a*g^4*
i^2*n*(a*d - b*c))/(2*d) + (g^4*i^2*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2))
+ (a*b*c*g^4*i^2*n*(a*d - b*c))/d) + x^2*(b*d*((a*g^4*i^2*n*(a*d - b*c))/(
2*d) + (g^4*i^2*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2)) + (b*g^4*i^2*n*(a*d
+ b*c)*(a*d - b*c))/d) + a*c*((a*g^4*i^2*n*(a*d - b*c))/(2*d) + (g^4*i^2*n
*(a*d - b*c)*(2*a*d - b*c))/(2*d^2)) + b^2*g^4*i^2*n*x^3*(a*d - b*c)))/(g^
4*i^2*n*(a*d - b*c)^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(x^3*(b^3*c*g^4*i^2
+ 3*a*b^2*d*g^4*i^2) + x^2*(3*a*b^2*c*g^4*i^2 + 3*a^2*b*d*g^4*i^2) + x*(a^
3*d*g^4*i^2 + 3*a^2*b*c*g^4*i^2) + a^3*c*g^4*i^2 + b^3*d*g^4*i^2*x^4))) -
(b*d^3*atan((b*d^3*((a^5*d^5*g^4*i^2 + b^5*c^5*g^4*i^2 - 3*a*b^4*c^4*d*g^4
*i^2 - 3*a^4*b*c*d^4*g^4*i^2 + 2*a^2*b^3*c^3*d^2*g^4*i^2 + 2*a^3*b^2*c^2*d
^3*g^4*i^2)/(a^4*d^4*g^4*i^2 + b^4*c^4*g^4*i^2 - 4*a*b^3*c^3*d*g^4*i^2 - 4
*a^3*b*c*d^3*g^4*i^2 + 6*a^2*b^2*c^2*d^2*g^4*i^2) + 2*b*d*x)*(6*A + 5*B*n)
*(a^4*d^4*g^4*i^2 + b^4*c^4*g^4*i^2 - 4*a*b^3*c^3*d*g^4*i^2 - 4*a^3*b*c*d^
3*g^4*i^2 + 6*a^2*b^2*c^2*d^2*g^4*i^2)*2i)/(g^4*i^2*(12*A*b*d^3 + 10*B*...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 3127, normalized size of antiderivative = 6.56

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^4(ci + dix)^2}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x)
```

output

```
( - 108*log(a + b*x)*a**5*b*c*d**4*n - 108*log(a + b*x)*a**5*b*d**5*n*x -
36*log(a + b*x)*a**4*b**2*c**2*d**3*n - 54*log(a + b*x)*a**4*b**2*c*d**4*n
**2 - 360*log(a + b*x)*a**4*b**2*c*d**4*n*x - 54*log(a + b*x)*a**4*b**2*d*
*5*n**2*x - 324*log(a + b*x)*a**4*b**2*d**5*n*x**2 - 66*log(a + b*x)*a**3*
b**3*c**2*d**3*n**2 - 108*log(a + b*x)*a**3*b**3*c**2*d**3*n*x - 228*log(a
 + b*x)*a**3*b**3*c*d**4*n**2*x - 432*log(a + b*x)*a**3*b**3*c*d**4*n*x**2
 - 162*log(a + b*x)*a**3*b**3*d**5*n**2*x**2 - 324*log(a + b*x)*a**3*b**3*
d**5*n*x**3 - 198*log(a + b*x)*a**2*b**4*c**2*d**3*n**2*x - 108*log(a + b*
x)*a**2*b**4*c**2*d**3*n*x**2 - 360*log(a + b*x)*a**2*b**4*c*d**4*n**2*x**
2 - 216*log(a + b*x)*a**2*b**4*c*d**4*n*x**3 - 162*log(a + b*x)*a**2*b**4*
d**5*n**2*x**3 - 108*log(a + b*x)*a**2*b**4*d**5*n*x**4 - 198*log(a + b*x)
*a*b**5*c**2*d**3*n**2*x**2 - 36*log(a + b*x)*a*b**5*c**2*d**3*n*x**3 - 25
2*log(a + b*x)*a*b**5*c*d**4*n**2*x**3 - 36*log(a + b*x)*a*b**5*c*d**4*n*x
**4 - 54*log(a + b*x)*a*b**5*d**5*n**2*x**4 - 66*log(a + b*x)*b**6*c**2*d*
*3*n**2*x**3 - 66*log(a + b*x)*b**6*c*d**4*n**2*x**4 + 108*log(c + d*x)*a*
*5*b*c*d**4*n + 108*log(c + d*x)*a**5*b*d**5*n*x + 36*log(c + d*x)*a**4*b*
*2*c**2*d**3*n + 54*log(c + d*x)*a**4*b**2*c*d**4*n**2 + 360*log(c + d*x)*
a**4*b**2*c*d**4*n*x + 54*log(c + d*x)*a**4*b**2*d**5*n**2*x + 324*log(c +
 d*x)*a**4*b**2*d**5*n*x**2 + 66*log(c + d*x)*a**3*b**3*c**2*d**3*n**2 + 1
08*log(c + d*x)*a**3*b**3*c**2*d**3*n*x + 228*log(c + d*x)*a**3*b**3*c*...
```

3.151 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$

Optimal result . . . 1717
Mathematica [A] (verified) . . . 1718
Rubi [A] (verified) . . . 1719
Maple [F] . . . 1721
Fricas [F] . . . 1721
Sympy [F] . . . 1721
Maxima [B] (verification not implemented) . . . 1722
Giac [F] . . . 1723
Mupad [F(-1)] . . . 1724
Reduce [F] . . . 1724

Optimal result

Integrand size = 43, antiderivative size = 382

$$
\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx \\
&= -\frac{3B(bc-ad)g^3n(a+bx)^2}{4d^2i^3(c+dx)^2} - \frac{3bB(bc-ad)g^3n(a+bx)}{d^3i^3(c+dx)} \\
&\quad + \frac{b(bc-ad)g^3(3A+Bn)(a+bx)}{d^3i^3(c+dx)} + \frac{3bB(bc-ad)g^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^3(c+dx)} \\
&\quad + \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di^3(c+dx)^2} \\
&\quad + \frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^3(c+dx)^2} \\
&\quad + \frac{b^2(bc-ad)g^3\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^3} \\
&\quad + \frac{3b^2B(bc-ad)g^3n\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}
\end{aligned}
$$

output

```
-3/4*B*(-a*d+b*c)*g^3*n*(b*x+a)^2/d^2/i^3/(d*x+c)^2-3*b*B*(-a*d+b*c)*g^3*n
*(b*x+a)/d^3/i^3/(d*x+c)+b*(-a*d+b*c)*g^3*(B*n+3*A)*(b*x+a)/d^3/i^3/(d*x+c
)+3*b*B*(-a*d+b*c)*g^3*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/d^3/i^3/(d*x+c)+g
^3*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i^3/(d*x+c)^2+1/2*(-a*d+b*c
)*g^3*(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/d^2/i^3/(d*x+c)^2+
b^2*(-a*d+b*c)*g^3*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b
/(d*x+c))/d^4/i^3+3*b^2*B*(-a*d+b*c)*g^3*n*polylog(2,d*(b*x+a)/b/(d*x+c))/
d^4/i^3
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.87

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{g^3\left(4Ab^3dx - \frac{B(bc-ad)^3n}{(c+dx)^2} + \frac{10bB(bc-ad)^2n}{c+dx} + 10b^2B(bc-ad)n\log(a+bx) + 4b^2Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{}$$

input

```
Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i +
d*i*x)^3,x]
```

output

```
(g^3*(4*A*b^3*d*x - (B*(b*c - a*d)^3*n)/(c + d*x)^2 + (10*b*B*(b*c - a*d)^
2*n)/(c + d*x) + 10*b^2*B*(b*c - a*d)*n*Log[a + b*x] + 4*b^2*B*d*(a + b*x)
*Log[e*((a + b*x)/(c + d*x))^n] + (2*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(c + d*x)^2 - (12*b*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(c + d*x) - 14*b^2*B*(b*c - a*d)*n*Log[c + d*x] - 12*b^2*
(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 6*b^2*B*
(b*c - a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c
+ d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(4*d^4*i^3)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {2961, 2784, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ci+dix)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{g^3(bc-ad)\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i^3}$$

$$\downarrow 2784$$

$$\frac{g^3(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int \frac{(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{d}\right)}{i^3}$$

$$\downarrow 2793$$

$$\frac{g^3(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\left(-\frac{\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)}-\frac{\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{d^2}-\frac{(a+bx)\left(3A+Bn+3\right.}{d(c}\right.}{d}\right)}{i^3}$$

$$\downarrow 2009$$

$$\frac{g^3(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{b^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{d^3}-\frac{b(a+bx)(3A+Bn)}{d^2(c+dx)}-\frac{(a+bx)^2\left(3B\log\left(e\left(\right.\right.\right.}{2d(c}}{i^3}$$

input
```
Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)
^3,x]
```

output
```
((b*c - a*d)*g^3*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*
(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))) - ((3*B*n*(a + b*x)^2)/(4*d*(c
+ d*x)^2) + (3*b*B*n*(a + b*x))/(d^2*(c + d*x)) - (b*(3*A + B*n)*(a + b*x)
)/(d^2*(c + d*x)) - (3*b*B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d^2*
(c + d*x)) - ((a + b*x)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^n])
)/(2*d*(c + d*x)^2) - (b^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^n]
)*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 - (3*b^2*B*n*PolyLog[2, (d*(a
+ b*x))/(b*(c + d*x))])/d^3)/d))/i^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(dix+ci)^3}dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,
algorithm="fricas")
```

output
```
integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
+ (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log(e*
((b*x + a)/(d*x + c))^n))/(d^3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x +
 c^3*i^3), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{g^3\left(\int \frac{Aa^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{Ab^3x^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{Ba^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3Aab^2x^2}{c^3+3c^2dx+3cd^2x^2+}\right.}{}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**3,x
)
```

output
```
g**3*(Integral(A*a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x)
+ Integral(A*b**3*x**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x)
 + Integral(B*a**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*
d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(3*A*a*b**2*x**2/(c**3 + 3*
c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(3*A*a**2*b*x/(c**3 +
3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(B*b**3*x**3*log(e*(
a/(c + d*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3), x) + Integral(3*B*a*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(3*B*a**
2*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**
2*x**2 + d**3*x**3), x))/i**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2894 vs. $2(377) = 754$.

Time $= 0.43$ (sec) , antiderivative size $= 2894$, normalized size of antiderivative $= 7.58$

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,
algorithm="maxima")
```

output

```
3/4*B*a^2*b*g^3*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 - a
*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^
3) + 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^
4)*i^3) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a
^2*d^4)*i^3)) + 1/4*B*a^3*g^3*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4
)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2
*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*
x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*A*b^3*g^3*((6*c^2*
d*x + 5*c^3)/(d^6*i^3*x^2 + 2*c*d^5*i^3*x + c^2*d^4*i^3) - 2*x/(d^3*i^3) +
 6*c*log(d*x + c)/(d^4*i^3)) + 3/2*A*a*b^2*g^3*((4*c*d*x + 3*c^2)/(d^5*i^3
*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) + 2*log(d*x + c)/(d^3*i^3)) - 3/2*(2*d
*x + c)*B*a^2*b*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^4*i^3*x^2 +
2*c*d^3*i^3*x + c^2*d^2*i^3) - 3/2*(2*d*x + c)*A*a^2*b*g^3/(d^4*i^3*x^2 +
2*c*d^3*i^3*x + c^2*d^2*i^3) - 1/2*B*a^3*g^3*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n)/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A*a^3*g^3/(d^3*i
^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) + 1/2*(6*a^3*b^2*d^3*g^3*log(e) - (7*g
^3*n + 6*g^3*log(e))*b^5*c^3 + (19*g^3*n + 18*g^3*log(e))*a*b^4*c^2*d - 2*
(7*g^3*n + 9*g^3*log(e))*a^2*b^3*c*d^2)*B*log(d*x + c)/(b^2*c^2*d^4*i^3 -
2*a*b*c*d^5*i^3 + a^2*d^6*i^3) + 1/4*(4*(b^5*c^2*d^3*g^3*log(e) - 2*a*b^4*
c*d^4*g^3*log(e) + a^2*b^3*d^5*g^3*log(e))*B*x^3 + 8*(b^5*c^3*d^2*g^3*l...
```

Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^3}\,dx$$

input

```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,
algorithm="giac")
```

output

```
integrate((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(d*i*x +
c*i)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(c\,i+d\,i\,x)^3}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^3,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^3, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x)
```

output

```
(g**3*i*(4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(c**3 + 3*c**2*d*
x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*b**4*c**4*d**6 + 8*int((log(((a + b
*x)**n*e)/(c + d*x)**n)*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x*
*3),x)*a**2*b**4*c**3*d**7*x + 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x
**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*b**4*c**2*d**
8*x**2 - 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(c**3 + 3*c**2*d*
x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**5*c**5*d**5 - 16*int((log(((a + b*x
)**n*e)/(c + d*x)**n)*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3
),x)*a*b**5*c**4*d**6*x - 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/
(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**5*c**3*d**7*x**2 +
 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3),x)*b**6*c**6*d**4 + 8*int((log(((a + b*x)**n*e)/(c
+ d*x)**n)*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**6*c
**5*d**5*x + 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(c**3 + 3*c**
2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**6*c**4*d**6*x**2 + 12*int((log(((
a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d*
*3*x**3),x)*a**3*b**3*c**4*d**6 + 24*int((log(((a + b*x)**n*e)/(c + d*x)**
n)*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**3*b**3*c**3
*d**7*x + 12*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c**3 + 3*c**2*
d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**3*b**3*c**2*d**8*x**2 - 24*int((...
```

3.152 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$

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Mathematica [A] (verified) . . . 1727
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Giac [F] . . . 1731
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Optimal result

Integrand size = 43, antiderivative size = 263

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx \\
&= \frac{Bg^2n(a+bx)^2}{4di^3(c+dx)^2} - \frac{Abg^2(a+bx)}{d^2i^3(c+dx)} + \frac{bBg^2n(a+bx)}{d^2i^3(c+dx)} \\
&\quad - \frac{bBg^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^3(c+dx)} - \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^3(c+dx)^2} \\
&\quad - \frac{b^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^3} - \frac{b^2Bg^2n\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}
\end{aligned}$$

output

```
1/4*B*g^2*n*(b*x+a)^2/d/i^3/(d*x+c)^2-A*b*g^2*(b*x+a)/d^2/i^3/(d*x+c)+b*B*
g^2*n*(b*x+a)/d^2/i^3/(d*x+c)-b*B*g^2*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/d^
2/i^3/(d*x+c)-1/2*g^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i^3/(d*x
+c)^2-b^2*g^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^3
/i^3-b^2*B*g^2*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^3
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.98

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{g^2\left(\frac{B(bc-ad)^2n}{(c+dx)^2} - \frac{6bB(bc-ad)n}{c+dx} - 6b^2Bn\log(a+bx) - \frac{2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2} + \frac{8b(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.\right.}{c+dx}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)^3,x]
```

output
```
(g^2*((B*(b*c - a*d)^2*n)/(c + d*x)^2 - (6*b*B*(b*c - a*d)*n)/(c + d*x) - 6*b^2*B*n*Log[a + b*x] - (2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x)^2 + (8*b*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) + 6*b^2*B*n*Log[c + d*x] + 4*b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*b^2*B*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4*d^3*i^3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ci+dix)^3}\,dx$$
$$\downarrow 2961$$
$$\frac{g^2\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}\,d\frac{a+bx}{c+dx}}{i^3}$$

$$\downarrow 2793$$

$$\frac{g^2\int\left(-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{d^2}-\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(c+dx)}\right)d\frac{a+bx}{c+dx}}{i^3}$$

$$\downarrow 2009$$

$$\frac{g^2\left(-\frac{b^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3}-\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2}-\frac{Ab(a+bx)}{d^2(c+dx)}-\frac{b^2Bn\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3}-\frac{bB(}{}\right.}{i^3}$$

input

```
Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)
^3,x]
```

output

```
(g^2*((B*n*(a + b*x)^2)/(4*d*(c + d*x)^2) - (A*b*(a + b*x))/(d^2*(c + d*x)
) + (b*B*n*(a + b*x))/(d^2*(c + d*x)) - (b*B*(a + b*x)*Log[e*((a + b*x)/(c
 + d*x))^n])/(d^2*(c + d*x)) - ((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]))/(2*d*(c + d*x)^2) - (b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*
Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 - (b^2*B*n*PolyLog[2, (d*(a + b*
x))/(b*(c + d*x))])/d^3))/i^3
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(dix+ci)^3}dx$$

input

```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x)
```

output

```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^3}\,dx$$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,
algorithm="fricas")
```

output

```
integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B
*a*b*g^2*x + B*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n))/(d^3*i^3*x^3 + 3*c
*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{g^2\left(\int \frac{Aa^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{Ab^2x^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{Ba^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{2Aabx}{c^3+3c^2dx+3cd^2x^2+}\right.}{i^3}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**3,x
)
```

output
```
g**2*(Integral(A*a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x)
+ Integral(A*b**2*x**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x)
 + Integral(B*a**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*
d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*a*b*x/(c**3 + 3*c**2*d
*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(B*b**2*x**2*log(e*(a/(c + d
*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(2*B*a*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**3 + 3*
c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/i**3
```

Maxima [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,
algorithm="maxima")
```

output

```
1/2*B*a*b*g^2*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 - a*d
^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3)
 + 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*i^3) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2
*d^4)*i^3)) + 1/4*B*a^2*g^2*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*
i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b
^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x
 + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) + 1/2*A*b^2*g^2*((4*c*d*x
 + 3*c^2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) + 2*log(d*x + c)/(d^3
*i^3)) - 1/2*B*b^2*g^2*((2*(d^2*n*x^2 + 2*c*d*n*x + c^2*n)*log(b*x + a)*lo
g(d*x + c) - (d^2*n*x^2 + 2*c*d*n*x + c^2*n)*log(d*x + c)^2 - (4*c*d*x + 3
*c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*log(d*x + c))*log((b*x + a)^n) + (4*c*d
*x + 3*c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*log(d*x + c))*log((d*x + c)^n))/(
d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) - 2*integrate(1/2*(2*b*d^3*x^3*
log(e) + 2*a*d^3*x^2*log(e) - 3*b*c^3*n + 3*a*c^2*d*n - 4*(b*c^2*d*n - a*c
*d^2*n)*x + 2*(b*d^3*n*x^3 + a*c^2*d*n + (2*b*c*d^2*n + a*d^3*n)*x^2 + (b*
c^2*d*n + 2*a*c*d^2*n)*x)*log(b*x + a))/(b*d^6*i^3*x^4 + a*c^3*d^3*i^3 + (
3*b*c*d^5*i^3 + a*d^6*i^3)*x^3 + 3*(b*c^2*d^4*i^3 + a*c*d^5*i^3)*x^2 + (b*
c^3*d^3*i^3 + 3*a*c^2*d^4*i^3)*x), x)) - (2*d*x + c)*B*a*b*g^2*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n)/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3)...
```

Giac [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^3}\,dx$$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,
algorithm="giac")
```

output

```
integrate((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(d*i*x +
c*i)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(c\,i+d\,i\,x)^3}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^3,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^3, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x)
```

output

```
(g**2*i*(4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c**3 + 3*c**2*d*
x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*b**3*c**4*d**5 + 8*int((log(((a + b
*x)**n*e)/(c + d*x)**n)*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x*
*3),x)*a**2*b**3*c**3*d**6*x + 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x
**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*b**3*c**2*d**
7*x**2 - 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c**3 + 3*c**2*d*
x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**4*c**5*d**4 - 16*int((log(((a + b*x
)**n*e)/(c + d*x)**n)*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3
),x)*a*b**4*c**4*d**5*x - 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/
(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**4*c**3*d**6*x**2 +
 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3),x)*b**5*c**6*d**3 + 8*int((log(((a + b*x)**n*e)/(c
+ d*x)**n)*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**5*c
**5*d**4*x + 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c**3 + 3*c**
2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**5*c**4*d**5*x**2 - 2*log(a + b*x)
*a**4*b*c**2*d**4*n - 4*log(a + b*x)*a**4*b*c*d**5*n*x - 2*log(a + b*x)*a*
*4*b*d**6*n*x**2 + 2*log(c + d*x)*a**4*b*c**2*d**4*n + 4*log(c + d*x)*a**4
*b*c*d**5*n*x + 2*log(c + d*x)*a**4*b*d**6*n*x**2 + 4*log(c + d*x)*a**3*b*
*2*c**4*d**2 + 8*log(c + d*x)*a**3*b**2*c**3*d**3*x + 4*log(c + d*x)*a**3*
b**2*c**2*d**4*x**2 - 8*log(c + d*x)*a**2*b**3*c**5*d - 16*log(c + d*x)...
```

3.153 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$

Optimal result . . . 1734
Mathematica [B] (verified) . . . 1734
Rubi [A] (verified) . . . 1735
Maple [B] (verified) . . . 1736
Fricas [B] (verification not implemented) . . . 1737
Sympy [F(-1)] . . . 1737
Maxima [B] (verification not implemented) . . . 1738
Giac [A] (verification not implemented) . . . 1739
Mupad [B] (verification not implemented) . . . 1739
Reduce [B] (verification not implemented) . . . 1740

Optimal result

Integrand size = 41, antiderivative size = 89

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = -\frac{Bgn(a+bx)^2}{4(bc-ad)i^3(c+dx)^2} + \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)i^3(c+dx)^2}$$

output

```
-1/4*B*g*n*(b*x+a)^2/(-a*d+b*c)/i^3/(d*x+c)^2+1/2*g*(b*x+a)^2*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/(-a*d+b*c)/i^3/(d*x+c)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 215 vs. 2(89) = 178.

Time = 0.18 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.42

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \frac{g\left(\frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2(c+dx)^2} - \frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d^2(c+dx)} + \frac{bBn\left(\frac{1}{c+dx}+\frac{b\log(a+bx)}{bc-ad}-\frac{b\log(c+dx)}{bc-ad}\right)}{d^2} - \frac{Bn\left(\frac{bc-ad}{(c+dx)^2}+\frac{2b}{c+dx}+\frac{2b^2}{b}\right.}{4d}\right.}{i^3}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*
i*x)^3,x]
```

output
```
(g*(((b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d^2*(c + d*x)^
2) - (b*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d^2*(c + d*x)) + (b*B*n*(
(c + d*x)^(-1) + (b*Log[a + b*x])/(b*c - a*d) - (b*Log[c + d*x])/(b*c - a*
d)))/d^2 - (B*n*((b*c - a*d)/(c + d*x)^2 + (2*b)/(c + d*x) + (2*b^2*Log[a
+ b*x])/(b*c - a*d) - (2*b^2*Log[c + d*x])/(b*c - a*d)))/(4*d^2)))/i^3
```

Rubi [A] (verified)

Time $= 0.27$ (sec) , antiderivative size $= 76$, normalized size of antiderivative $= 0.85$, number of steps used $= 3$, number of rules used $= 2$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$, Rules used $= \{2961, 2741\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ci+dix)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{g\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{c+dx}d\frac{a+bx}{c+dx}}{i^3(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{g\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(c+dx)^2}-\frac{Bn(a+bx)^2}{4(c+dx)^2}\right)}{i^3(bc-ad)}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)^3
,x]
```

output
```
(g*(-1/4*(B*n*(a + b*x)^2)/(c + d*x)^2 + ((a + b*x)^2*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(2*(c + d*x)^2)))/((b*c - a*d)*i^3)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(85) = 170$.

Time = 4.89 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.66

method	result
parallelrisch	$-\frac{4Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)ab^2d^4gn-Ba^2bd^4gn^2+Bb^3c^2d^2gn^2+2Aa^2bd^4gn-2Ab^3c^2d^2gn+2B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^2bd^4gn+2B}{4i^3(dx+c)^2bd^4n(da-bc)}$

input
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,method=_RE
TURNVERBOSE)
```

output
```
-1/4*(4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^4*g*n-B*a^2*b*d^4*g*n^2+B*b^
3*c^2*d^2*g*n^2+2*A*a^2*b*d^4*g*n-2*A*b^3*c^2*d^2*g*n+2*B*ln(e*((b*x+a)/(d
*x+c))^n)*a^2*b*d^4*g*n+2*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^4*g*n-2*B*
x*a*b^2*d^4*g*n^2+2*B*x*b^3*c*d^3*g*n^2+4*A*x*a*b^2*d^4*g*n-4*A*x*b^3*c*d^
3*g*n)/i^3/(d*x+c)^2/b/d^4/n/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(85) = 170$.

Time = 0.11 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.81

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{(Bb^2c^2-Ba^2d^2)gn-2\left(Ab^2c^2-Aa^2d^2\right)g+2\left((Bb^2cd-Babd^2)gn-2\left(Ab^2cd-Aabd^2\right)g\right)x-2\left(2\left(Bb^2\right.\right.}{4\left(\left(bcd^4-ad^5\right)i^3x^2+2\left(bc^2d^3-a\right.\right.}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, al
gorithm="fricas")
```

output
```
1/4*((B*b^2*c^2 - B*a^2*d^2)*g*n - 2*(A*b^2*c^2 - A*a^2*d^2)*g + 2*((B*b^2
*c*d - B*a*b*d^2)*g*n - 2*(A*b^2*c*d - A*a*b*d^2)*g)*x - 2*(2*(B*b^2*c*d -
 B*a*b*d^2)*g*x + (B*b^2*c^2 - B*a^2*d^2)*g)*log(e) + 2*(B*b^2*d^2*g*n*x^2
 + 2*B*a*b*d^2*g*n*x + B*a^2*d^2*g*n)*log((b*x + a)/(d*x + c)))/((b*c*d^4
- a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)
*i^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(85) = 170$.

Time $= 0.05$ (sec) , antiderivative size $= 578$, normalized size of antiderivative $= 6.49$

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{1}{4}Bbgn\left(\frac{bc^2-3acd+2(bcd-2ad^2)x}{(bcd^4-ad^5)i^3x^2+2(bc^2d^3-acd^4)i^3x+(bc^3d^2-ac^2d^3)i^3}+\frac{2(b^2c-2abd)\log(bx+a)}{(b^2c^2d^2-2abcd^3+a^2d^4)i^3}-\frac{2}{(b\right.$$
$$+\frac{1}{4}Bagn\left(\frac{2bdx+3bc-ad}{(bcd^3-ad^4)i^3x^2+2(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3}+\frac{2b^2\log(bx+a)}{(b^2c^2d-2abcd^2+a^2d^3)i^3}-\frac{}{(b\right.$$
$$-\frac{(2dx+c)Bbg\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2(d^4i^3x^2+2cd^3i^3x+c^2d^2i^3)}-\frac{(2dx+c)Abg}{2(d^4i^3x^2+2cd^3i^3x+c^2d^2i^3)}$$
$$-\frac{Bag\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2(d^3i^3x^2+2cd^2i^3x+c^2di^3)}-\frac{Aag}{2(d^3i^3x^2+2cd^2i^3x+c^2di^3)}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, al
gorithm="maxima")
```

output
```
1/4*B*b*g*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 - a*d^5)*
i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3) + 2
*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3
) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4
)*i^3)) + 1/4*B*a*g*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2
+ 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log(b
*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*(2*d*x + c)*B*b*g*log(e*(b*x
/(d*x + c) + a/(d*x + c))^n)/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) -
 1/2*(2*d*x + c)*A*b*g/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - 1/2*B
*a*g*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^3*i^3*x^2 + 2*c*d^2*i^3*x +
 c^2*d*i^3) - 1/2*A*a*g/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3)
```

Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.13

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(\frac{2\,(bx+a)^2 Bgn\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)^2 i^3}-\frac{(Bgn-2\,Bg\log(e)-2\,Ag)(bx+a)^2}{(dx+c)^2 i^3}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, al
gorithm="giac")
```

output
```
1/4*(2*(b*x + a)^2*B*g*n*log((b*x + a)/(d*x + c))/((d*x + c)^2*i^3) - (B*g
*n - 2*B*g*log(e) - 2*A*g)*(b*x + a)^2/((d*x + c)^2*i^3))*(b*c/(b*c - a*d)
^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 26.33 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.30

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$=-\frac{x\,(2\,Abdg-Bbdgn)+Aadg+Abcg-\frac{Badgn}{2}-\frac{Bbcgn}{2}}{2\,c^2 d^2 i^3+4\,cd^3 i^3 x+2\,d^4 i^3 x^2}$$
$$-\frac{\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(\frac{Bag}{2d}+\frac{Bbcg}{2d^2}+\frac{Bbgx}{d}\right)}{c^2 i^3+2\,cdi^3 x+d^2 i^3 x^2}+\frac{B\,b^2\,g\,n\,\mathrm{atan}\left(\frac{bc\,2\mathrm{i}+bdx\,2\mathrm{i}}{ad-bc}+1\mathrm{i}\right)\,1\mathrm{i}}{d^2 i^3\,(ad-bc)}$$

input
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^3
,x)
```

output
```
(B*b^2*g*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*1i)/(d^2*i^3*(a*d -
b*c)) - (log(e*((a + b*x)/(c + d*x))^n)*((B*a*g)/(2*d) + (B*b*c*g)/(2*d^2)
 + (B*b*g*x)/d))/(c^2*i^3 + d^2*i^3*x^2 + 2*c*d*i^3*x) - (x*(2*A*b*d*g - B
*b*d*g*n) + A*a*d*g + A*b*c*g - (B*a*d*g*n)/2 - (B*b*c*g*n)/2)/(2*c^2*d^2*
i^3 + 2*d^4*i^3*x^2 + 4*c*d^3*i^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 357, normalized size of antiderivative = 4.01

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$

$$= \frac{gi\Big(-2\log(bx+a)\,ab^2c^2n-4\log(bx+a)\,ab^2cdnx-2\log(bx+a)\,ab^2d^2n\,x^2+2\log(dx+c)\,ab^2c^2n+4}{}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x)
```

output
```
(g*i*( - 2*log(a + b*x)*a*b**2*c**2*n - 4*log(a + b*x)*a*b**2*c*d*n*x - 2*
log(a + b*x)*a*b**2*d**2*n*x**2 + 2*log(c + d*x)*a*b**2*c**2*n + 4*log(c +
 d*x)*a*b**2*c*d*n*x + 2*log(c + d*x)*a*b**2*d**2*n*x**2 - 2*log(((a + b*x
)**n*e)/(c + d*x)**n)*a**2*b*c*d + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*
b**2*c**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**2*x**2 - 2*log(
((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*d*x**2 - 2*a**3*c*d + 2*a**2*b*c**2
+ a**2*b*c*d*n + 2*a**2*b*d**2*x**2 - a*b**2*c**2*n - 2*a*b**2*c*d*x**2 -
a*b**2*d**2*n*x**2 + b**3*c*d*n*x**2))/(4*c*d*(a*c**2*d + 2*a*c*d**2*x + a
*d**3*x**2 - b*c**3 - 2*b*c**2*d*x - b*c*d**2*x**2))
```

3.154 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx$

Optimal result 1741
Mathematica [A] (verified) 1741
Rubi [A] (verified) 1742
Maple [A] (verified) 1744
Fricas [A] (verification not implemented) 1744
Sympy [F(-1)] 1745
Maxima [A] (verification not implemented) 1745
Giac [A] (verification not implemented) 1746
Mupad [B] (verification not implemented) 1746
Reduce [B] (verification not implemented) 1747

Optimal result

Integrand size = 33, antiderivative size = 151

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx = \frac{Bn}{4di^3(c+dx)^2}+\frac{bBn}{2d(bc-ad)i^3(c+dx)}+\frac{b^2Bn\log(a+bx)}{2d(bc-ad)^2i^3} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2di^3(c+dx)^2}-\frac{b^2Bn\log(c+dx)}{2d(bc-ad)^2i^3}$$

output

```
1/4*B*n/d/i^3/(d*x+c)^2+1/2*b*B*n/d/(-a*d+b*c)/i^3/(d*x+c)+1/2*b^2*B*n*ln(
b*x+a)/d/(-a*d+b*c)^2/i^3-1/2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i^3/(d*x+c
)^2-1/2*b^2*B*n*ln(d*x+c)/d/(-a*d+b*c)^2/i^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx = \frac{-2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn((bc-ad)(3bc-ad+2bdx)+2b^2(c+dx)^2\log(a+bx)-2b^2(c+dx)^2\log(c+dx))}{(bc-ad)^2}}{4di^3(c+dx)^2}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x)^3,x]
```

output
```
(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(3*b*c - a*
d + 2*b*d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c +
d*x]))/(b*c - a*d)^2)/(4*d*i^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{(ci+dix)^3}\,dx$$

$$\downarrow 2947$$

$$\frac{Bn(bc-ad)\int \frac{1}{i^2(a+bx)(c+dx)^3}\,dx}{2di} - \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{2di^3(c+dx)^2}$$

$$\downarrow 27$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)^3}\,dx}{2di^3} - \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{2di^3(c+dx)^2}$$

$$\downarrow 54$$

$$\frac{Bn(bc-ad)\int \left(\frac{b^3}{(bc-ad)^3(a+bx)} - \frac{db^2}{(bc-ad)^3(c+dx)} - \frac{db}{(bc-ad)^2(c+dx)^2} - \frac{d}{(bc-ad)(c+dx)^3}\right)dx}{2di^3} - \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{2di^3(c+dx)^2}$$

$$\downarrow 2009$$

$$\frac{Bn(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}+\frac{b}{(c+dx)(bc-ad)^2}+\frac{1}{2(c+dx)^2(bc-ad)}\right)}{2di^3}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2di^3(c+dx)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x)^3,x]
```

output
```
-1/2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(d*i^3*(c + d*x)^2) + (B*(b*c
- a*d)*n*(1/(2*(b*c - a*d)*(c + d*x)^2) + b/((b*c - a*d)^2*(c + d*x)) + (b
^2*Log[a + b*x])/(b*c - a*d)^3 - (b^2*Log[c + d*x])/(b*c - a*d)^3))/(2*d*i
^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

Maple [A] (verified)

Time = 4.75 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.80

method	result
parallelrisch	$-\frac{-B\,a^2b\,d^5n^2-3B\,b^3c^2d^3n^2+2A\,a^2b\,d^5n+2A\,b^3c^2d^3n+4Ba\,b^2c\,d^4n^2-4Aa\,b^2c\,d^4n-2B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3d^5n+2Bxa\,b}{4i^3(dx+c)^2n(a^2d^2-2acd}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-B*a^2*b*d^5*n^2-3*B*b^3*c^2*d^3*n^2+2*A*a^2*b*d^5*n+2*A*b^3*c^2*d^3
*n+4*B*a*b^2*c*d^4*n^2-4*A*a*b^2*c*d^4*n-2*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)
*b^3*d^5*n+2*B*x*a*b^2*d^5*n^2-2*B*x*b^3*c*d^4*n^2+2*B*ln(e*((b*x+a)/(d*x+
c))^n)*a^2*b*d^5*n-4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d^4*n-4*B*ln(e*((
b*x+a)/(d*x+c))^n)*a*b^2*c*d^4*n)/i^3/(d*x+c)^2/n/(a^2*d^2-2*a*b*c*d+b^2*c
^2)/b/d^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.76

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx =$$
$$-\frac{2\,Ab^2c^2-4\,Aabcd+2\,Aa^2d^2-2\,(Bb^2cd-Babd^2)nx-(3\,Bb^2c^2-4\,Babcd+Ba^2d^2)n+2\,(Bb^2c^2-}{4\,((b^2c^2d^3-2\,abcd^4+a^2d^5)i^3x^2+2\,(b^2c^3d^2-2\,abc^2d^3+}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, algorithm="fri
cas")
```

output
```
-1/4*(2*A*b^2*c^2 - 4*A*a*b*c*d + 2*A*a^2*d^2 - 2*(B*b^2*c*d - B*a*b*d^2)*
n*x - (3*B*b^2*c^2 - 4*B*a*b*c*d + B*a^2*d^2)*n + 2*(B*b^2*c^2 - 2*B*a*b*c
*d + B*a^2*d^2)*log(e) - 2*(B*b^2*d^2*n*x^2 + 2*B*b^2*c*d*n*x + (2*B*a*b*c
*d - B*a^2*d^2)*n)*log((b*x + a)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 +
 a^2*d^5)*i^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*i^3*x + (b
^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*i^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.72

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx$$
$$= \frac{1}{4}Bn\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)i^3x^2+2\,(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3}+\frac{2\,b^2\log(bx+a)}{(b^2c^2d-2\,abcd^2+a^2d^3)i^3}-\frac{2}{(b^2c^2d}\right.$$
$$\left.-\frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}-\frac{A}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, algorithm="max
ima")
```

output
```
1/4*B*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2
 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log(b*x + a)/((b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a
*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(
d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A/(d^3*i^3*x^2 + 2*c*d^2*i^
3*x + c^2*d*i^3)
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.37

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx$$
$$= \frac{1}{4}\left(2\left(\frac{2\,(bx+a)Bbn}{(bci^3-adi^3)(dx+c)} - \frac{(bx+a)^2 Bdn}{(bci^3-adi^3)(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right) + \frac{(Bdn - 2\,Bd\log(e) - 2\,Ad)(b$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, algorithm="gia
c")
```

output
```
1/4*(2*(2*(b*x + a)*B*b*n/((b*c*i^3 - a*d*i^3)*(d*x + c)) - (b*x + a)^2*B*
d*n/((b*c*i^3 - a*d*i^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c)) + (B*d*n -
 2*B*d*log(e) - 2*A*d)*(b*x + a)^2/((b*c*i^3 - a*d*i^3)*(d*x + c)^2) - 4*(
B*b*n - B*b*log(e) - A*b)*(b*x + a)/((b*c*i^3 - a*d*i^3)*(d*x + c)))*(b*c/
(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 26.21 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.46

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx = \frac{B\,b^2\,n\,\operatorname{atanh}\left(\frac{2\,a^2\,d^3\,i^3 - 2\,b^2\,c^2\,d\,i^3}{2\,d\,i^3\,(a\,d-b\,c)^2} + \frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{d\,i^3\,(a\,d-b\,c)^2}$$
$$- \frac{B\,\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2\,d\,\left(c^2\,i^3 + 2\,c\,d\,i^3\,x + d^2\,i^3\,x^2\right)}$$
$$- \frac{\frac{2\,A\,a\,d - 2\,A\,b\,c - B\,a\,d\,n + 3\,B\,b\,c\,n}{2\,(a\,d-b\,c)} + \frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,c^2\,d\,i^3 + 4\,c\,d^2\,i^3\,x + 2\,d^3\,i^3\,x^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*i + d*i*x)^3,x)
```

output
```
(B*b^2*n*atanh((2*a^2*d^3*i^3 - 2*b^2*c^2*d*i^3)/(2*d*i^3*(a*d - b*c)^2) +
 (2*b*d*x)/(a*d - b*c)))/(d*i^3*(a*d - b*c)^2) - (B*log(e*((a + b*x)/(c +
d*x))^n))/(2*d*(c^2*i^3 + d^2*i^3*x^2 + 2*c*d*i^3*x)) - ((2*A*a*d - 2*A*b*
c - B*a*d*n + 3*B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)/(a*d - b*c))/(2*c^2
*d*i^3 + 2*d^3*i^3*x^2 + 4*c*d^2*i^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 606, normalized size of antiderivative = 4.01

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx$$

$$= \frac{i\Big(4\log(dx+c)\,a^2bc\,d^3nx - 8\log(dx+c)\,a\,b^2c^2d^2nx - 4\log(dx+c)\,a\,b^2c\,d^3n\,x^2 - 4\log(bx+a)\,a^2bc\,d^3n$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x)
```

output
```
(i*( - 2*log(a + b*x)*a**2*b*c**2*d**2*n - 4*log(a + b*x)*a**2*b*c*d**3*n*
x - 2*log(a + b*x)*a**2*b*d**4*n*x**2 + 4*log(a + b*x)*a*b**2*c**3*d*n + 8
*log(a + b*x)*a*b**2*c**2*d**2*n*x + 4*log(a + b*x)*a*b**2*c*d**3*n*x**2 +
 2*log(c + d*x)*a**2*b*c**2*d**2*n + 4*log(c + d*x)*a**2*b*c*d**3*n*x + 2*
log(c + d*x)*a**2*b*d**4*n*x**2 - 4*log(c + d*x)*a*b**2*c**3*d*n - 8*log(c
 + d*x)*a*b**2*c**2*d**2*n*x - 4*log(c + d*x)*a*b**2*c*d**3*n*x**2 + 4*log
(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c*d**3*x + 2*log(((a + b*x)**n*e)/(
c + d*x)**n)*a**2*b*d**4*x**2 - 8*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**
2*c**2*d**2*x - 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*d**3*x**2 +
4*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c**3*d*x + 2*log(((a + b*x)**n*e
)/(c + d*x)**n)*b**3*c**2*d**2*x**2 - 2*a**3*c**2*d**2 + 4*a**2*b*c**3*d +
 a**2*b*c**2*d**2*n - 2*a*b**2*c**4 - 3*a*b**2*c**3*d*n + a*b**2*c*d**3*n*
x**2 + 2*b**3*c**4*n - b**3*c**2*d**2*n*x**2))/(4*c**2*d*(a**2*c**2*d**2 +
 2*a**2*c*d**3*x + a**2*d**4*x**2 - 2*a*b*c**3*d - 4*a*b*c**2*d**2*x - 2*a
*b*c*d**3*x**2 + b**2*c**4 + 2*b**2*c**3*d*x + b**2*c**2*d**2*x**2))
```

3.155 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$

Optimal result . . . 1748
Mathematica [C] (verified) . . . 1749
Rubi [A] (verified) . . . 1749
Maple [B] (verified) . . . 1751
Fricas [A] (verification not implemented) . . . 1752
Sympy [F(-1)] . . . 1752
Maxima [B] (verification not implemented) . . . 1753
Giac [A] (verification not implemented) . . . 1754
Mupad [B] (verification not implemented) . . . 1754
Reduce [B] (verification not implemented) . . . 1755

Optimal result

Integrand size = 43, antiderivative size = 254

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx &= -\frac{Bn\left(4b-\frac{d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3gi^3}\\ &+\frac{d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2}\\ &-\frac{2bd(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3gi^3(c+dx)}\\ &+\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}\\ &-\frac{b^2Bn\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^3gi^3}\end{aligned}$$

output

```
-1/4*B*n*(4*b-d*(b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g/i^3+1/2*d^2*(b*x+a)^2*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g/i^3/(d*x+c)^2-2*b*d*(b*x+a)*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g/i^3/(d*x+c)+b^2*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c)^3/g/i^3-1/2*b^2*B*n*ln(
(b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g/i^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.71

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \frac{2(bc-ad)^2\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + 4b(bc-ad)(c+dx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + 4b^2(c+dx)^2\log(a$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i
*x)^3),x]
```

output
```
(2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b*(b*c - a*d)*
(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b^2*(c + d*x)^2*Log[a
 + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 4*b^2*(c + d*x)^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 4*b*B*n*(c + d*x)*(b*c - a*
d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]) - B*n*((b*c - a*d
)^2 + 2*b*(b*c - a*d)*(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(
c + d*x)^2*Log[c + d*x]) - 2*b^2*B*n*(c + d*x)^2*(Log[a + b*x]*(Log[a + b*
x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c
) + a*d)]) + 2*b^2*B*n*(c + d*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] -
 Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4
*(b*c - a*d)^3*g*i^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{(ag+bgx)(ci+dix)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}d\frac{a+bx}{c+dx}}{gi^3(bc-ad)^3}$$

$$\downarrow 2772$$

$$\frac{-Bn\int\left(\frac{b^2(c+dx)\log\left(\frac{a+bx}{c+dx}\right)}{a+bx}-\frac{1}{2}d\left(4b-\frac{d(a+bx)}{c+dx}\right)\right)d\frac{a+bx}{c+dx}+b^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{d^2(a+bx)^2\left(B\right.}{2}}{gi^3(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{b^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{d^2(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(c+dx)^2}-\frac{2bd(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{c+dx}-Bn\left(\frac{1}{2}\right.}{gi^3(bc-ad)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i*x)^3)
,x]
```

output
```
((d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(c + d*x)^2)
- (2*b*d*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) + b^2
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] - B*n*((4
*b - (d*(a + b*x))/(c + d*x))^2/4 + (b^2*Log[(a + b*x)/(c + d*x)]^2)/2))/(
(b*c - a*d)^3*g*i^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(248) = 496.

Time = 13.38 (sec) , antiderivative size = 633, normalized size of antiderivative = 2.49

method	result
parallelrisch	$-\frac{2Bx^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2a^2b^2c^4d^2-8Bx^2a^3bc^3d^3n^2+7Bx^2a^2b^2c^4d^2n^2+4Ax^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^2b^2c^4d^2+8Ax^2a^3bc^3d^3n-$

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^3,x,method=_RE
TURNVERBOSE)
```

output

```
-1/4*(2*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c^4*d^2-8*B*x^2*a^3*b*c^
3*d^3*n^2+7*B*x^2*a^2*b^2*c^4*d^2*n^2+4*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^
2*b^2*c^4*d^2+8*A*x^2*a^3*b*c^3*d^3*n-6*A*x^2*a^2*b^2*c^4*d^2*n-8*B*x*ln(e
*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^5*d*n-4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*
b*c^4*d^2*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c^6+4*A*ln(e*((b*x+a)/
(d*x+c))^n)*a^2*b^2*c^6+4*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c^5*d-10
*B*x*a^3*b*c^4*d^2*n^2+8*B*x*a^2*b^2*c^5*d*n^2+8*A*x*ln(e*((b*x+a)/(d*x+c)
)^n)*a^2*b^2*c^5*d+12*A*x*a^3*b*c^4*d^2*n-8*A*x*a^2*b^2*c^5*d*n-8*B*ln(e*(
(b*x+a)/(d*x+c))^n)*a^3*b*c^5*d*n-6*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^
2*c^4*d^2*n+B*x^2*a^4*c^2*d^4*n^2-2*A*x^2*a^4*c^2*d^4*n+2*B*x*a^4*c^3*d^3*
n^2-4*A*x*a^4*c^3*d^3*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*a^4*c^4*d^2*n)/i^3/g
/(d*x+c)^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a^2/c^4/n
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.92

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \frac{6\,Ab^2c^2 - 8\,Aabcd + 2\,Aa^2d^2 + 2\,(Bb^2d^2nx^2 + 2\,Bb^2cdnx + Bb^2c^2n)\log\left(\frac{bx+a}{dx+c}\right)^2 - (7\,Bb^2c^2 - 8\,Babcd +$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, al
gorithm="fricas")
```

output
```
1/4*(6*A*b^2*c^2 - 8*A*a*b*c*d + 2*A*a^2*d^2 + 2*(B*b^2*d^2*n*x^2 + 2*B*b^
2*c*d*n*x + B*b^2*c^2*n)*log((b*x + a)/(d*x + c))^2 - (7*B*b^2*c^2 - 8*B*a
*b*c*d + B*a^2*d^2)*n + 2*(2*A*b^2*c*d - 2*A*a*b*d^2 - 3*(B*b^2*c*d - B*a*
b*d^2)*n)*x + 2*(3*B*b^2*c^2 - 4*B*a*b*c*d + B*a^2*d^2 + 2*(B*b^2*c*d - B*
a*b*d^2)*x + 2*(B*b^2*d^2*x^2 + 2*B*b^2*c*d*x + B*b^2*c^2)*log((b*x + a)/(
d*x + c)))*log(e) + 2*(2*A*b^2*c^2 - (3*B*b^2*d^2*n - 2*A*b^2*d^2)*x^2 - (
4*B*a*b*c*d - B*a^2*d^2)*n + 2*(2*A*b^2*c*d - (2*B*b^2*c*d + B*a*b*d^2)*n)
*x)*log((b*x + a)/(d*x + c)))/((b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*
d^4 - a^3*d^5)*g*i^3*x^2 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^
3 - a^3*c*d^4)*g*i^3*x + (b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*
c^2*d^3)*g*i^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)/(d*i*x+c*i)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 888 vs. $2(248) = 496$.

Time = 0.09 (sec) , antiderivative size = 888, normalized size of antiderivative = 3.50

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)(ci + dix)^3}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, al
gorithm="maxima")
```

output
```
1/2*B*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g*i^
3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b^2*c^4 - 2*a
*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n) - 1/4*(7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*log(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))*B*n/(b^3*c
^5*g*i^3 - 3*a*b^2*c^4*d*g*i^3 + 3*a^2*b*c^3*d^2*g*i^3 - a^3*c^2*d^3*g*i^3
 + (b^3*c^3*d^2*g*i^3 - 3*a*b^2*c^2*d^3*g*i^3 + 3*a^2*b*c*d^4*g*i^3 - a^3*
d^5*g*i^3)*x^2 + 2*(b^3*c^4*d*g*i^3 - 3*a*b^2*c^3*d^2*g*i^3 + 3*a^2*b*c^2*
d^3*g*i^3 - a^3*c*d^4*g*i^3)*x) + 1/2*A*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2
*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g*i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a
^2*c*d^3)*g*i^3*x + (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*l
og(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) -
2*b^2*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*
i^3))
```

Giac [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.65

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(\frac{2\,Bb^2n\log\left(\frac{bx+a}{dx+c}\right)^2}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}-2\left(\frac{4\,(bx+a)Bbdn}{(b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3)(dx+c)}-\frac{(bx+a)}{(b^2c^2gi^3-2\,abcdgi^3+\right.\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, al
gorithm="giac")
```

output
```
1/4*(2*B*b^2*n*log((b*x + a)/(d*x + c))^2/(b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3
 + a^2*d^2*g*i^3) - 2*(4*(b*x + a)*B*b*d*n/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i
^3 + a^2*d^2*g*i^3)*(d*x + c)) - (b*x + a)^2*B*d^2*n/((b^2*c^2*g*i^3 - 2*a
*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c)) + 4*(
B*b^2*log(e) + A*b^2)*log((b*x + a)/(d*x + c))/(b^2*c^2*g*i^3 - 2*a*b*c*d*
g*i^3 + a^2*d^2*g*i^3) - (B*d^2*n - 2*B*d^2*log(e) - 2*A*d^2)*(b*x + a)^2/
((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)^2) + 8*(B*b*d
*n - B*b*d*log(e) - A*b*d)*(b*x + a)/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a
^2*d^2*g*i^3)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 27.88 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.26

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$=\frac{B\,b^2\,\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(\frac{c\,g\,i^3\,n\,(a\,d-b\,c)}{2\,b}-\frac{g\,i^3\,n\,(a\,d-b\,c)\,(a\,d-2\,b\,c)}{2\,b^2}+\frac{d\,g\,i^3\,n\,x\,(a\,d-b\,c)}{b}\right)}{g\,i^3\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)\,(g\,c^2\,i^3+2\,g\,c\,d\,i^3\,x+g\,d^2\,i^3\,x^2)}$$
$$-\frac{B\,b^2\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}{2\,g\,i^3\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}$$
$$-\frac{\frac{2\,A\,a\,d-6\,A\,b\,c-B\,a\,d\,n+7\,B\,b\,c\,n}{2\,(a\,d-b\,c)}-\frac{b\,x\,(2\,A\,d-3\,B\,d\,n)}{a\,d-b\,c}}{x^2\,(2\,a\,d^3\,g\,i^3-2\,b\,c\,d^2\,g\,i^3)+x\,(4\,a\,c\,d^2\,g\,i^3-4\,b\,c^2\,d\,g\,i^3)-2\,b\,c^3\,g\,i^3+2\,a\,c^2\,d\,g\,i^3}$$
$$+\frac{b^2\,\mathrm{atan}\left(\frac{b^2\left(A-\frac{3\,B\,n}{2}\right)(2\,g\,a^3\,d^3\,i^3-2\,g\,a^2\,b\,c\,d^2\,i^3-2\,g\,a\,b^2\,c^2\,d\,i^3+2\,g\,b^3\,c^3\,i^3)\,1\mathrm{i}}{g\,i^3\,(2\,A\,b^2-3\,B\,b^2\,n)\,(a\,d-b\,c)^3}+\frac{b^3\,d\,x\left(A-\frac{3\,B\,n}{2}\right)(g\,a^2\,d^2\,i^3-2\,g\,a\,b\,c\,d\,i^3+g\,b^2\,c^2\,i}{g\,i^3\,(2\,A\,b^2-3\,B\,b^2\,n)\,(a\,d-b\,c)^3}\right.}{g\,i^3\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)*(c*i + d*i*x)^3)
,x)
```

output
```
(b^2*atan((b^2*(A - (3*B*n)/2)*(2*a^3*d^3*g*i^3 + 2*b^3*c^3*g*i^3 - 2*a*b^
2*c^2*d*g*i^3 - 2*a^2*b*c*d^2*g*i^3)*1i)/(g*i^3*(2*A*b^2 - 3*B*b^2*n)*(a*d
 - b*c)^3) + (b^3*d*x*(A - (3*B*n)/2)*(a^2*d^2*g*i^3 + b^2*c^2*g*i^3 - 2*a
*b*c*d*g*i^3)*4i)/(g*i^3*(2*A*b^2 - 3*B*b^2*n)*(a*d - b*c)^3))*(A - (3*B*n
)/2)*2i)/(g*i^3*(a*d - b*c)^3) - ((2*A*a*d - 6*A*b*c - B*a*d*n + 7*B*b*c*n
)/(2*(a*d - b*c)) - (b*x*(2*A*d - 3*B*d*n))/(a*d - b*c))/(x^2*(2*a*d^3*g*i
^3 - 2*b*c*d^2*g*i^3) + x*(4*a*c*d^2*g*i^3 - 4*b*c^2*d*g*i^3) - 2*b*c^3*g*
i^3 + 2*a*c^2*d*g*i^3) - (B*b^2*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g*i^3
*n*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B*b^2*log(e*((a + b*x)/
(c + d*x))^n)*((c*g*i^3*n*(a*d - b*c))/(2*b) - (g*i^3*n*(a*d - b*c)*(a*d -
 2*b*c))/(2*b^2) + (d*g*i^3*n*x*(a*d - b*c))/b))/(g*i^3*n*(a*d - b*c)*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d)*(c^2*g*i^3 + d^2*g*i^3*x^2 + 2*c*d*g*i^3*x))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.92

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^3,x)
```

output

```
(i*( - 4*log(a + b*x)*a*b**2*c**3*n - 8*log(a + b*x)*a*b**2*c**2*d*n*x - 4
*log(a + b*x)*a*b**2*c*d**2*n*x**2 + 6*log(a + b*x)*b**3*c**3*n**2 + 12*lo
g(a + b*x)*b**3*c**2*d*n**2*x + 6*log(a + b*x)*b**3*c*d**2*n**2*x**2 + 4*l
og(c + d*x)*a*b**2*c**3*n + 8*log(c + d*x)*a*b**2*c**2*d*n*x + 4*log(c + d
*x)*a*b**2*c*d**2*n*x**2 - 6*log(c + d*x)*b**3*c**3*n**2 - 12*log(c + d*x)
*b**3*c**2*d*n**2*x - 6*log(c + d*x)*b**3*c*d**2*n**2*x**2 - 2*log(((a + b
*x)**n*e)/(c + d*x)**n)**2*b**3*c**3 - 4*log(((a + b*x)**n*e)/(c + d*x)**n
)**2*b**3*c**2*d*x - 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*d**2*x
**2 - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c*d**2*n + 8*log(((a + b
*x)**n*e)/(c + d*x)**n)*a*b**2*c**2*d*n + 4*log(((a + b*x)**n*e)/(c + d*x)
**n)*a*b**2*c*d**2*n*x - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c**3*n
- 4*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c**2*d*n*x - 2*a**3*c*d**2*n +
 6*a**2*b*c**2*d*n + a**2*b*c*d**2*n**2 - 2*a**2*b*d**3*n*x**2 - 4*a*b**2*
c**3*n - 5*a*b**2*c**2*d*n**2 + 2*a*b**2*c*d**2*n*x**2 + 3*a*b**2*d**3*n**
2*x**2 + 4*b**3*c**3*n**2 - 3*b**3*c*d**2*n**2*x**2))/(4*c*g*n*(a**3*c**2*
d**3 + 2*a**3*c*d**4*x + a**3*d**5*x**2 - 3*a**2*b*c**3*d**2 - 6*a**2*b*c*
*2*d**3*x - 3*a**2*b*c*d**4*x**2 + 3*a*b**2*c**4*d + 6*a*b**2*c**3*d**2*x
+ 3*a*b**2*c**2*d**3*x**2 - b**3*c**5 - 2*b**3*c**4*d*x - b**3*c**3*d**2*x
**2))
```

3.156 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$

Optimal result 1757
Mathematica [C] (verified) 1758
Rubi [A] (verified) 1759
Maple [B] (verified) 1760
Fricas [B] (verification not implemented) 1761
Sympy [F(-1)] 1762
Maxima [B] (verification not implemented) 1763
Giac [F] 1764
Mupad [B] (verification not implemented) 1764
Reduce [B] (verification not implemented) 1765

Optimal result

Integrand size = 43, antiderivative size = 381

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx &= \frac{Bd^3n(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2}-\frac{3bBd^2n(a+bx)}{(bc-ad)^4g^2i^3(c+dx)}\\ &-\frac{b^3Bn(c+dx)}{(bc-ad)^4g^2i^3(a+bx)}\\ &-\frac{d^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2}\\ &+\frac{3bd^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^2i^3(c+dx)}\\ &-\frac{b^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^2i^3(a+bx)}\\ &-\frac{3b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}\\ &+\frac{3b^2Bdn\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^2i^3}\end{aligned}$$

output

```
1/4*B*d^3*n*(b*x+a)^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2-3*b*B*d^2*n*(b*x+a)/(
-a*d+b*c)^4/g^2/i^3/(d*x+c)-b^3*B*n*(d*x+c)/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-1
/2*d^3*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^2/i^3/(d*x
+c)^2+3*b*d^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^2/i^3
/(d*x+c)-b^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^2/i^3/
(b*x+a)-3*b^2*d*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+
b*c)^4/g^2/i^3+3/2*b^2*B*d*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^2/i^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.25

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$$

$$= \frac{-\frac{4b^3Bcn}{a+bx} + \frac{4ab^2Bdn}{a+bx} + \frac{Bd(bc-ad)^2n}{(c+dx)^2} + \frac{8b^2Bcdn}{c+dx} - \frac{8abBd^2n}{c+dx} + \frac{2bBd(bc-ad)n}{c+dx} + 6b^2Bdn\log(a+bx) - \frac{4b^2(bc-ad)\left(A+B\right.}{a}$$

input

```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d
*i*x)^3),x]
```

output

```
((-4*b^3*B*c*n)/(a + b*x) + (4*a*b^2*B*d*n)/(a + b*x) + (B*d*(b*c - a*d)^2
*n)/(c + d*x)^2 + (8*b^2*B*c*d*n)/(c + d*x) - (8*a*b*B*d^2*n)/(c + d*x) +
(2*b*B*d*(b*c - a*d)*n)/(c + d*x) + 6*b^2*B*d*n*Log[a + b*x] - (4*b^2*(b*c
 - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - (2*d*(b*c - a*
d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x)^2 - (8*b*d*(b*c - a
*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) - 12*b^2*d*Log[a + b
*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*b^2*B*d*n*Log[c + d*x] + 12
*b^2*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 6*b^2*B*d*n*(
Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog
[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 6*b^2*B*d*n*((2*Log[(d*(a + b*x))/(-(
b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)]))/(4*(b*c - a*d)^4*g^2*i^3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^2(ci+dix)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i^3(bc-ad)^4}$$

$$\downarrow 2772$$

$$\frac{-Bn\int\left(-\frac{(c+dx)^2b^3}{(a+bx)^2}-\frac{3d(c+dx)\log\left(\frac{a+bx}{c+dx}\right)b^2}{a+bx}+3d^2b-\frac{d^3(a+bx)}{2(c+dx)}\right)d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-3b^2d\log\left(\frac{a}{c}\right.}{g^2i^3(bc-ad)^4}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-3b^2d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{d^3(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(c+dx)^2}+\frac{3bd^2}{}}{g^2i^3(bc-ad)^4}$$

input

```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d*i*x)^
3),x]
```

output

```
(-1/2*(d^3*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x)^2
 + (3*b*d^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) -
(b^3*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - 3*b^2*d
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] - B*n*(-1
/4*(d^3*(a + b*x)^2)/(c + d*x)^2 + (3*b*d^2*(a + b*x))/(c + d*x) + (b^3*(c
 + d*x))/(a + b*x) - (3*b^2*d*Log[(a + b*x)/(c + d*x)]^2)/2))/((b*c - a*d)
^4*g^2*i^3)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(375) = 750$.

Time = 30.64 (sec) , antiderivative size = 977, normalized size of antiderivative = 2.56

method	result
parallelrisch	$-\frac{6Bx^3\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2b^6d^7+12Ax^3\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^6d^7-12Axab^5cd^6n-12B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^2b^4cd^6n-Ba^3b^3d^7n^2+4$

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,method=_
RETURNVERBOSE)
```

output

```
-1/4*(6*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^6*d^7+12*A*x^3*ln(e*((b*x+a)/(
d*x+c))^n)*b^6*d^7-12*A*x*a*b^5*c*d^6*n-12*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2
*b^4*c*d^6*n-B*a^3*b^3*d^7*n^2+4*B*b^6*c^3*d^4*n^2+2*A*a^3*b^3*d^7*n+4*A*b
^6*c^3*d^4*n-18*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^7*n+12*B*x*ln(e*((
b*x+a)/(d*x+c))^n)^2*a*b^5*c*d^6-6*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*d
^7*n+12*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^2*d^5*n-6*B*x*a*b^5*c*d^6*n^2+
24*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^6+12*B*a^2*b^4*c*d^6*n^2-15*B*a
*b^5*c^2*d^5*n^2-12*A*a^2*b^4*c*d^6*n+6*A*a*b^5*c^2*d^5*n-24*B*x*ln(e*((b*
x+a)/(d*x+c))^n)*a*b^5*c*d^6*n-6*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^7*n
+6*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^5*d^7+12*B*x^2*ln(e*((b*x+a)/(d*x
+c))^n)^2*b^6*c*d^6+6*B*x^2*a*b^5*d^7*n^2-6*B*x^2*b^6*c*d^6*n^2+12*A*x^2*l
n(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^7+24*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^6*
c*d^6-12*A*x^2*a*b^5*d^7*n+12*A*x^2*b^6*c*d^6*n+6*B*x*ln(e*((b*x+a)/(d*x+c
))^n)^2*b^6*c^2*d^5+9*B*x*a^2*b^4*d^7*n^2-3*B*x*b^6*c^2*d^5*n^2+12*A*x*ln(
e*((b*x+a)/(d*x+c))^n)*b^6*c^2*d^5-6*A*x*a^2*b^4*d^7*n+18*A*x*b^6*c^2*d^5*
n+6*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^5*c^2*d^5+2*B*ln(e*((b*x+a)/(d*x+c))
^n)*a^3*b^3*d^7*n+4*B*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^3*d^4*n+12*A*ln(e*((
b*x+a)/(d*x+c))^n)*a*b^5*c^2*d^5)/i^3/g^2/(d*x+c)^2/(b*x+a)/(a^3*d^3-3*a^2
*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/n/(a*d-b*c)/b^3/d^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(375) = 750$.

Time = 0.12 (sec) , antiderivative size = 949, normalized size of antiderivative = 2.49

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^2(ci + dix)^3}\, dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,
algorithm="fricas")
```

output

```
-1/4*(4*A*b^3*c^3 + 6*A*a*b^2*c^2*d - 12*A*a^2*b*c*d^2 + 2*A*a^3*d^3 + 6*(
2*A*b^3*c*d^2 - 2*A*a*b^2*d^3 - (B*b^3*c*d^2 - B*a*b^2*d^3)*n)*x^2 + 6*(B*
b^3*d^3*n*x^3 + B*a*b^2*c^2*d*n + (2*B*b^3*c*d^2 + B*a*b^2*d^3)*n*x^2 + (B
*b^3*c^2*d + 2*B*a*b^2*c*d^2)*n*x)*log((b*x + a)/(d*x + c))^2 + (4*B*b^3*c
^3 - 15*B*a*b^2*c^2*d + 12*B*a^2*b*c*d^2 - B*a^3*d^3)*n + 3*(6*A*b^3*c^2*d
 - 4*A*a*b^2*c*d^2 - 2*A*a^2*b*d^3 - (B*b^3*c^2*d + 2*B*a*b^2*c*d^2 - 3*B*
a^2*b*d^3)*n)*x + 2*(2*B*b^3*c^3 + 3*B*a*b^2*c^2*d - 6*B*a^2*b*c*d^2 + B*a
^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 + 3*(3*B*b^3*c^2*d - 2*B*a*b^2*
c*d^2 - B*a^2*b*d^3)*x + 6*(B*b^3*d^3*x^3 + B*a*b^2*c^2*d + (2*B*b^3*c*d^2
 + B*a*b^2*d^3)*x^2 + (B*b^3*c^2*d + 2*B*a*b^2*c*d^2)*x)*log((b*x + a)/(d*
x + c)))*log(e) + 2*(6*A*a*b^2*c^2*d - 3*(B*b^3*d^3*n - 2*A*b^3*d^3)*x^3 -
 3*(3*B*a*b^2*d^3*n - 4*A*b^3*c*d^2 - 2*A*a*b^2*d^3)*x^2 + (2*B*b^3*c^3 -
6*B*a^2*b*c*d^2 + B*a^3*d^3)*n + 3*(2*A*b^3*c^2*d + 4*A*a*b^2*c*d^2 + (2*B
*b^3*c^2*d - 4*B*a*b^2*c*d^2 - B*a^2*b*d^3)*n)*x)*log((b*x + a)/(d*x + c))
)/((b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 +
a^4*b*d^6)*g^2*i^3*x^3 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^
3 - 2*a^3*b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*g^2*i^3*x^2 + (b^5*c^6 -
2*a*b^4*c^5*d - 2*a^2*b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 +
2*a^5*c*d^5)*g^2*i^3*x + (a*b^4*c^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2
- 4*a^4*b*c^3*d^3 + a^5*c^2*d^4)*g^2*i^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Timed out}$$

input

```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2/(d*i*x+c*i)**3,x
)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1724 vs. $2(375) = 750$.

Time $= 0.14$ (sec) , antiderivative size $= 1724$, normalized size of antiderivative $= 4.52$

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,
algorithm="maxima")
```

output

```
-1/2*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d +
a*b*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)
*g^2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*
c*d^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2
+ 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*d
+ 3*a^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c^4
 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3) -
 6*b^2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^
3*b*c*d^3 + a^4*d^4)*g^2*i^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/
4*(4*b^3*c^3 - 15*a*b^2*c^2*d + 12*a^2*b*c*d^2 - a^3*d^3 - 6*(b^3*c*d^2 -
a*b^2*d^3)*x^2 - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*
x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a)^2 - 6*(b^3*d^3*x^3 + a*b
^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*
log(d*x + c)^2 - 3*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3*d^
3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2
*c*d^2)*x)*log(b*x + a) + 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*
b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x + 2*(b^3*d^3*x^3 + a*b^2*c^2*
d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x
 + a))*log(d*x + c))*B*n/(a*b^4*c^6*g^2*i^3 - 4*a^2*b^3*c^5*d*g^2*i^3 + 6*
a^3*b^2*c^4*d^2*g^2*i^3 - 4*a^4*b*c^3*d^3*g^2*i^3 + a^5*c^2*d^4*g^2*i^3...
```

Giac [F]

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \int \frac{B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{(bgx+ag)^2(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,
algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/((b*g*x + a*g)^2*(d*i*x +
 c*i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 28.64 (sec) , antiderivative size = 1018, normalized size of antiderivative = 2.67

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^2*(c*i + d*i*x)^
3),x)
```

output

```
((4*A*b^2*c^2 - 2*A*a^2*d^2 + B*a^2*d^2*n + 4*B*b^2*c^2*n + 10*A*a*b*c*d -
 11*B*a*b*c*d*n)/(2*(a*d - b*c)) + (3*x^2*(2*A*b^2*d^2 - B*b^2*d^2*n))/(a*
d - b*c) + (3*x*(2*A*a*b*d^2 + 6*A*b^2*c*d - 3*B*a*b*d^2*n - B*b^2*c*d*n))
/(2*(a*d - b*c)))/(x*(2*b^3*c^4*g^2*i^3 + 4*a^3*c*d^3*g^2*i^3 - 6*a^2*b*c^
2*d^2*g^2*i^3) + x^2*(2*a^3*d^4*g^2*i^3 + 4*b^3*c^3*d*g^2*i^3 - 6*a*b^2*c^
2*d^2*g^2*i^3) + x^3*(2*b^3*c^2*d^2*g^2*i^3 + 2*a^2*b*d^4*g^2*i^3 - 4*a*b^
2*c*d^3*g^2*i^3) + 2*a^3*c^2*d^2*g^2*i^3 + 2*a*b^2*c^4*g^2*i^3 - 4*a^2*b*c
^3*d*g^2*i^3) - log(e*((a + b*x)/(c + d*x))^n)*(((B*(a*d + 2*b*c))/(2*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d)) + (3*B*b*d*x)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c
*d)))/(x*(b*c^2*g^2*i^3 + 2*a*c*d*g^2*i^3) + x^2*(a*d^2*g^2*i^3 + 2*b*c*d*
g^2*i^3) + a*c^2*g^2*i^3 + b*d^2*g^2*i^3*x^3) - (3*B*b^2*d*(d*g^2*i^3*n*x^
2*(a*d - b*c) + (a*c*g^2*i^3*n*(a*d - b*c))/b + (g^2*i^3*n*x*(a*d + b*c)*(
a*d - b*c))/b))/(g^2*i^3*n*(a*d - b*c)^4*(x*(b*c^2*g^2*i^3 + 2*a*c*d*g^2*i
^3) + x^2*(a*d^2*g^2*i^3 + 2*b*c*d*g^2*i^3) + a*c^2*g^2*i^3 + b*d^2*g^2*i^
3*x^3))) + (b^2*d*atan((b^2*d*(2*A - B*n)*((a^4*d^4*g^2*i^3 - b^4*c^4*g^2*
i^3 + 2*a*b^3*c^3*d*g^2*i^3 - 2*a^3*b*c*d^3*g^2*i^3)/(a^3*d^3*g^2*i^3 - b^
3*c^3*g^2*i^3 + 3*a*b^2*c^2*d*g^2*i^3 - 3*a^2*b*c*d^2*g^2*i^3) + 2*b*d*x)*
(a^3*d^3*g^2*i^3 - b^3*c^3*g^2*i^3 + 3*a*b^2*c^2*d*g^2*i^3 - 3*a^2*b*c*d^2
*g^2*i^3)*3i)/(g^2*i^3*(6*A*b^2*d - 3*B*b^2*d*n)*(a*d - b*c)^4))*(2*A - B*
n)*3i)/(g^2*i^3*(a*d - b*c)^4) - (3*B*b^2*d*log(e*((a + b*x)/(c + d*x))...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1928, normalized size of antiderivative = 5.06

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x)
```

output

```
(i*( - 12*log(a + b*x)*a**3*b**2*c**2*d**2*n - 24*log(a + b*x)*a**3*b**2*c
*d**3*n*x - 12*log(a + b*x)*a**3*b**2*d**4*n*x**2 - 24*log(a + b*x)*a**2*b
**3*c**3*d*n + 18*log(a + b*x)*a**2*b**3*c**2*d**2*n**2 - 60*log(a + b*x)*
a**2*b**3*c**2*d**2*n*x + 36*log(a + b*x)*a**2*b**3*c*d**3*n**2*x - 48*log
(a + b*x)*a**2*b**3*c*d**3*n*x**2 + 18*log(a + b*x)*a**2*b**3*d**4*n**2*x*
*2 - 12*log(a + b*x)*a**2*b**3*d**4*n*x**3 - 24*log(a + b*x)*a*b**4*c**3*d
*n*x + 18*log(a + b*x)*a*b**4*c**2*d**2*n**2*x - 48*log(a + b*x)*a*b**4*c*
*2*d**2*n*x**2 + 36*log(a + b*x)*a*b**4*c*d**3*n**2*x**2 - 24*log(a + b*x)
*a*b**4*c*d**3*n*x**3 + 18*log(a + b*x)*a*b**4*d**4*n**2*x**3 + 12*log(c +
 d*x)*a**3*b**2*c**2*d**2*n + 24*log(c + d*x)*a**3*b**2*c*d**3*n*x + 12*lo
g(c + d*x)*a**3*b**2*d**4*n*x**2 + 24*log(c + d*x)*a**2*b**3*c**3*d*n - 18
*log(c + d*x)*a**2*b**3*c**2*d**2*n**2 + 60*log(c + d*x)*a**2*b**3*c**2*d*
*2*n*x - 36*log(c + d*x)*a**2*b**3*c*d**3*n**2*x + 48*log(c + d*x)*a**2*b*
*3*c*d**3*n*x**2 - 18*log(c + d*x)*a**2*b**3*d**4*n**2*x**2 + 12*log(c + d
*x)*a**2*b**3*d**4*n*x**3 + 24*log(c + d*x)*a*b**4*c**3*d*n*x - 18*log(c +
 d*x)*a*b**4*c**2*d**2*n**2*x + 48*log(c + d*x)*a*b**4*c**2*d**2*n*x**2 -
36*log(c + d*x)*a*b**4*c*d**3*n**2*x**2 + 24*log(c + d*x)*a*b**4*c*d**3*n*
x**3 - 18*log(c + d*x)*a*b**4*d**4*n**2*x**3 - 6*log(((a + b*x)**n*e)/(c +
 d*x)**n)**2*a**2*b**3*c**2*d**2 - 12*log(((a + b*x)**n*e)/(c + d*x)**n)**
2*a**2*b**3*c*d**3*x - 6*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**...
```

3.157 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx$

Optimal result . . . 1767
Mathematica [C] (verified) . . . 1768
Rubi [A] (verified) . . . 1769
Maple [B] (verified) . . . 1771
Fricas [B] (verification not implemented) . . . 1772
Sympy [F(-1)] . . . 1773
Maxima [B] (verification not implemented) . . . 1773
Giac [F] . . . 1774
Mupad [B] (verification not implemented) . . . 1775
Reduce [B] (verification not implemented) . . . 1776

Optimal result

Integrand size = 43, antiderivative size = 483

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx &= -\frac{Bd^4n(a+bx)^2}{4(bc-ad)^5g^3i^3(c+dx)^2}+\frac{4bBd^3n(a+bx)}{(bc-ad)^5g^3i^3(c+dx)}\\ &+\frac{4b^3Bdn(c+dx)}{(bc-ad)^5g^3i^3(a+bx)}-\frac{b^4Bn(c+dx)^2}{4(bc-ad)^5g^3i^3(a+bx)^2}\\ &+\frac{d^4(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2}\\ &-\frac{4bd^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^3i^3(c+dx)}\\ &+\frac{4b^3d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^3i^3(a+bx)}\\ &-\frac{b^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2}\\ &+\frac{6b^2d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}\\ &-\frac{3b^2Bd^2n\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}\end{aligned}$$

output

```
-1/4*B*d^4*n*(b*x+a)^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2+4*b*B*d^3*n*(b*x+a)/
(-a*d+b*c)^5/g^3/i^3/(d*x+c)+4*b^3*B*d*n*(d*x+c)/(-a*d+b*c)^5/g^3/i^3/(b*x
+a)-1/4*b^4*B*n*(d*x+c)^2/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+1/2*d^4*(b*x+a)^2
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2-4*b*d^3*(b
*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(d*x+c)+4*b^3*d
*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/2*
b^4*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(b*x+a)
^2+6*b^2*d^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c
)^5/g^3/i^3-3*b^2*B*d^2*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^5/g^3/i^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.20 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.16

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx =$$

$$-\frac{\frac{b^2B(bc-ad)^2n}{(a+bx)^2} - \frac{12b^3Bcdn}{a+bx} + \frac{12ab^2Bd^2n}{a+bx} - \frac{2b^2Bd(bc-ad)n}{a+bx} + \frac{Bd^2(bc-ad)^2n}{(c+dx)^2} + \frac{12b^2Bcd^2n}{c+dx} - \frac{12abBd^3n}{c+dx} + \frac{2bBd^2(bc-ad)n}{c+dx}}{}$$

input

```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d
*i*x)^3),x]
```

output

```
-1/4*((b^2*B*(b*c - a*d)^2*n)/(a + b*x)^2 - (12*b^3*B*c*d*n)/(a + b*x) + (
12*a*b^2*B*d^2*n)/(a + b*x) - (2*b^2*B*d*(b*c - a*d)*n)/(a + b*x) + (B*d^2
*(b*c - a*d)^2*n)/(c + d*x)^2 + (12*b^2*B*c*d^2*n)/(c + d*x) - (12*a*b*B*d
^3*n)/(c + d*x) + (2*b*B*d^2*(b*c - a*d)*n)/(c + d*x) + (2*b^2*(b*c - a*d)
^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 - (12*b^2*d*(b*c -
a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - (2*d^2*(b*c - a*d
)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x)^2 - (12*b*d^2*(b*c -
 a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) - 24*b^2*d^2*Log[a
 + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 24*b^2*d^2*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n])*Log[c + d*x] + 12*b^2*B*d^2*n*(Log[a + b*x]*(Log[
a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)]) - 12*b^2*B*d^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] -
Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b
*c - a*d)^5*g^3*i^3)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{(ag+bgx)^3(ci+dix)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3} d\frac{a+bx}{c+dx}}{g^3 i^3 (bc-ad)^5}$$

$$\downarrow 2772$$

$$-Bn\int \left(\frac{6b^2(c+dx)\log\left(\frac{a+bx}{c+dx}\right)d^2}{a+bx} + \frac{1}{2}\left(-\frac{(c+dx)^3 b^4}{(a+bx)^3} + \frac{8d(c+dx)^2 b^3}{(a+bx)^2} - 8d^3 b + \frac{d^4(a+bx)}{c+dx}\right)\right) d\frac{a+bx}{c+dx} - \frac{b^4(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{2(a+bx)^2}$$

$$\downarrow 2009$$

$$-\frac{b^4(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{4b^3d(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}+6b^2d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d*i*x)^
3),x]
```

output
```
((d^4*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(c + d*x)^2)
- (4*b*d^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) + (
4*b^3*d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - (b^4
*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) + 6*b
^2*d^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] - B
*n*((d^4*(a + b*x)^2)/(4*(c + d*x)^2) - (4*b*d^3*(a + b*x))/(c + d*x) - (4
*b^3*d*(c + d*x))/(a + b*x) + (b^4*(c + d*x)^2)/(4*(a + b*x)^2) + 3*b^2*d^
2*Log[(a + b*x)/(c + d*x)]^2))/((b*c - a*d)^5*g^3*i^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1449 vs. $2(475) = 950$.

Time = 58.84 (sec) , antiderivative size = 1450, normalized size of antiderivative = 3.00

method	result	size
parallelrisch	Expression too large to display	1450

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,method=_
RETURNVERBOSE)
```

output
```
-1/4*(-B*a^4*b^4*d^8*n^2-B*b^8*c^4*d^4*n^2+2*A*a^4*b^4*d^8*n-2*A*b^8*c^4*d
^4*n+36*A*x^2*b^8*c^2*d^6*n+12*B*x*a^3*b^5*d^8*n^2+12*B*x*b^8*c^3*d^5*n^2-
8*A*x*a^3*b^5*d^8*n+8*A*x*b^8*c^3*d^5*n+12*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a
^2*b^6*c^2*d^6+2*B*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^4*d^8*n-2*B*ln(e*((b*x+
a)/(d*x+c))^n)*b^8*c^4*d^4*n+24*A*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^6*c^2*d^
6+12*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*b^8*d^8+24*A*x^4*ln(e*((b*x+a)/(d*x
+c))^n)*b^8*d^8-24*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*d^8*n+24*B*x^3*ln
(e*((b*x+a)/(d*x+c))^n)*b^8*c*d^7*n+48*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a
*b^7*c*d^7-36*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^6*d^8*n+36*B*x^2*ln(e*
((b*x+a)/(d*x+c))^n)*b^8*c^2*d^6*n-24*B*x^2*a*b^7*c*d^7*n^2+96*A*x^2*ln(e*
((b*x+a)/(d*x+c))^n)*a*b^7*c*d^7+24*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^
6*c*d^7+24*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^7*c^2*d^6-8*B*x*ln(e*((b*x+
a)/(d*x+c))^n)*a^3*b^5*d^8*n+8*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^8*c^3*d^5*n
-12*B*x*a^2*b^6*c*d^7*n^2-12*B*x*a*b^7*c^2*d^6*n^2+48*A*x*ln(e*((b*x+a)/(d
*x+c))^n)*a^2*b^6*c*d^7+48*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*c^2*d^6-48*
A*x*a^2*b^6*c*d^7*n+48*A*x*a*b^7*c^2*d^6*n-16*B*ln(e*((b*x+a)/(d*x+c))^n)*
a^3*b^5*c*d^7*n+16*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*c^3*d^5*n+16*B*a^3*b^
5*c*d^7*n^2-30*B*a^2*b^6*c^2*d^6*n^2+16*B*a*b^7*c^3*d^5*n^2-16*A*a^3*b^5*c
*d^7*n+16*A*a*b^7*c^3*d^5*n+24*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^7*d^8
+24*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^8*c*d^7+48*A*x^3*ln(e*((b*x+a)/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. $2(475) = 950$.

Time $= 0.14$ (sec) , antiderivative size $= 1416$, normalized size of antiderivative $= 2.93$

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,
algorithm="fricas")
```

output

```
-1/4*(2*A*b^4*c^4 - 16*A*a*b^3*c^3*d + 16*A*a^3*b*c*d^3 - 2*A*a^4*d^4 - 24
*(A*b^4*c*d^3 - A*a*b^3*d^4)*x^3 - 12*(3*A*b^4*c^2*d^2 - 3*A*a^2*b^2*d^4 +
 (B*b^4*c^2*d^2 - 2*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*n)*x^2 - 12*(B*b^4*d^4*
n*x^4 + B*a^2*b^2*c^2*d^2*n + 2*(B*b^4*c*d^3 + B*a*b^3*d^4)*n*x^3 + (B*b^4
*c^2*d^2 + 4*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*n*x^2 + 2*(B*a*b^3*c^2*d^2 + B
*a^2*b^2*c*d^3)*n*x)*log((b*x + a)/(d*x + c))^2 + (B*b^4*c^4 - 16*B*a*b^3*
c^3*d + 30*B*a^2*b^2*c^2*d^2 - 16*B*a^3*b*c*d^3 + B*a^4*d^4)*n - 4*(2*A*b^
4*c^3*d + 12*A*a*b^3*c^2*d^2 - 12*A*a^2*b^2*c*d^3 - 2*A*a^3*b*d^4 + 3*(B*b
^4*c^3*d - B*a*b^3*c^2*d^2 - B*a^2*b^2*c*d^3 + B*a^3*b*d^4)*n)*x + 2*(B*b^
4*c^4 - 8*B*a*b^3*c^3*d + 8*B*a^3*b*c*d^3 - B*a^4*d^4 - 12*(B*b^4*c*d^3 -
B*a*b^3*d^4)*x^3 - 18*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*x^2 - 4*(B*b^4*c^3*d
 + 6*B*a*b^3*c^2*d^2 - 6*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*x - 12*(B*b^4*d^4*
x^4 + B*a^2*b^2*c^2*d^2 + 2*(B*b^4*c*d^3 + B*a*b^3*d^4)*x^3 + (B*b^4*c^2*d
^2 + 4*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*x^2 + 2*(B*a*b^3*c^2*d^2 + B*a^2*b^2
*c*d^3)*x)*log((b*x + a)/(d*x + c)))*log(e) - 2*(12*A*b^4*d^4*x^4 + 12*A*a
^2*b^2*c^2*d^2 + 12*(2*A*b^4*c*d^3 + 2*A*a*b^3*d^4 + (B*b^4*c*d^3 - B*a*b^
3*d^4)*n)*x^3 + 6*(2*A*b^4*c^2*d^2 + 8*A*a*b^3*c*d^3 + 2*A*a^2*b^2*d^4 + 3
*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*n)*x^2 - (B*b^4*c^4 - 8*B*a*b^3*c^3*d + 8
*B*a^3*b*c*d^3 - B*a^4*d^4)*n + 4*(6*A*a*b^3*c^2*d^2 + 6*A*a^2*b^2*c*d^3 +
 (B*b^4*c^3*d + 6*B*a*b^3*c^2*d^2 - 6*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*n)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^3(ci + dix)^3}\, dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3/(d*i*x+c*i)**3,x
)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2383 vs. $2(475) = 950$.

Time = 0.16 (sec) , antiderivative size = 2383, normalized size of antiderivative = 4.93

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^3(ci + dix)^3}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,
algorithm="maxima")
```

output

```
1/2*B*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3
 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b*d
^3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d
^5 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4
*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3 +
(b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^
6*d^6)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 +
2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^6
- 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g^3
*i^3) + 12*b^2*d^2*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3
*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d^2
*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*
c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3))*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n) - 1/4*(b^4*c^4 - 16*a*b^3*c^3*d + 30*a^2*b^2*c^2*d^2 - 16*a^3*b*
c*d^3 + a^4*d^4 - 12*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 12*
(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*
d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)
*x)*log(b*x + a)^2 - 24*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*
b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*
c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a)*log(d*x + c) + 12*(b^4*d^4*x^4...
```

Giac [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{(bgx+ag)^3(dix+ci)^3}\,dx$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,
algorithm="giac")
```

output

```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/((b*g*x + a*g)^3*(d*i*x +
 c*i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 28.74 (sec) , antiderivative size = 1341, normalized size of antiderivative = 2.78

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^3*(c*i + d*i*x)^
3),x)
```

output

```
((2*x*(2*A*a^2*b*d^3 + 2*A*b^3*c^2*d + 14*A*a*b^2*c*d^2 - 3*B*a^2*b*d^3*n
+ 3*B*b^3*c^2*d*n))/(a*d - b*c) - (2*A*a^3*d^3 + 2*A*b^3*c^3 - B*a^3*d^3*n
 + B*b^3*c^3*n - 14*A*a*b^2*c^2*d - 14*A*a^2*b*c*d^2 - 15*B*a*b^2*c^2*d*n
+ 15*B*a^2*b*c*d^2*n)/(2*(a*d - b*c)) + (6*x^2*(3*A*a*b^2*d^3 + 3*A*b^3*c*
d^2 - B*a*b^2*d^3*n + B*b^3*c*d^2*n))/(a*d - b*c) + (12*A*b^3*d^3*x^3)/(a*
d - b*c))/(x^4*(2*a^3*b^2*d^5*g^3*i^3 - 2*b^5*c^3*d^2*g^3*i^3 + 6*a*b^4*c^
2*d^3*g^3*i^3 - 6*a^2*b^3*c*d^4*g^3*i^3) - x*(4*a*b^4*c^5*g^3*i^3 - 4*a^5*
c*d^4*g^3*i^3 - 8*a^2*b^3*c^4*d*g^3*i^3 + 8*a^4*b*c^2*d^3*g^3*i^3) + x^3*(
4*a^4*b*d^5*g^3*i^3 - 4*b^5*c^4*d*g^3*i^3 + 8*a*b^4*c^3*d^2*g^3*i^3 - 8*a^
3*b^2*c*d^4*g^3*i^3) + x^2*(2*a^5*d^5*g^3*i^3 - 2*b^5*c^5*g^3*i^3 - 2*a*b^
4*c^4*d*g^3*i^3 + 2*a^4*b*c*d^4*g^3*i^3 + 16*a^2*b^3*c^3*d^2*g^3*i^3 - 16*
a^3*b^2*c^2*d^3*g^3*i^3) - 2*a^2*b^3*c^5*g^3*i^3 + 2*a^5*c^2*d^3*g^3*i^3 +
 6*a^3*b^2*c^4*d*g^3*i^3 - 6*a^4*b*c^3*d^2*g^3*i^3) + (log(e*((a + b*x)/(c
 + d*x))^n)*(x*((3*B*b*d*(a*d + b*c)^2)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2
- (B*b*d)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d) + (6*B*a*b^2*c*d^2)/(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d)^2) - (B*(a*d + b*c))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d
)) + (6*B*b^3*d^3*x^3)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2 + (9*B*b^2*d^2*x^
2*(a*d + b*c))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2 + (3*B*a*b*c*d*(a*d + b*c
))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2))/(x*(2*a*b*c^2*g^3*i^3 + 2*a^2*c*d*g
^3*i^3) + x^3*(2*a*b*d^2*g^3*i^3 + 2*b^2*c*d*g^3*i^3) + x^2*(a^2*d^2*g^...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2958, normalized size of antiderivative = 6.12

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x)
```

output

```
(i*( - 24*log(a + b*x)*a**4*b**2*c**2*d**3*n - 48*log(a + b*x)*a**4*b**2*c
*d**4*n*x - 24*log(a + b*x)*a**4*b**2*d**5*n*x**2 - 24*log(a + b*x)*a**3*b
**3*c**3*d**2*n + 12*log(a + b*x)*a**3*b**3*c**2*d**3*n**2 - 96*log(a + b*
x)*a**3*b**3*c**2*d**3*n*x + 24*log(a + b*x)*a**3*b**3*c*d**4*n**2*x - 120
*log(a + b*x)*a**3*b**3*c*d**4*n*x**2 + 12*log(a + b*x)*a**3*b**3*d**5*n**
2*x**2 - 48*log(a + b*x)*a**3*b**3*d**5*n*x**3 - 12*log(a + b*x)*a**2*b**4
*c**3*d**2*n**2 - 48*log(a + b*x)*a**2*b**4*c**3*d**2*n*x - 120*log(a + b*
x)*a**2*b**4*c**2*d**3*n*x**2 + 36*log(a + b*x)*a**2*b**4*c*d**4*n**2*x**2
 - 96*log(a + b*x)*a**2*b**4*c*d**4*n*x**3 + 24*log(a + b*x)*a**2*b**4*d**
5*n**2*x**3 - 24*log(a + b*x)*a**2*b**4*d**5*n*x**4 - 24*log(a + b*x)*a*b*
*5*c**3*d**2*n**2*x - 24*log(a + b*x)*a*b**5*c**3*d**2*n*x**2 - 36*log(a +
 b*x)*a*b**5*c**2*d**3*n**2*x**2 - 48*log(a + b*x)*a*b**5*c**2*d**3*n*x**3
 - 24*log(a + b*x)*a*b**5*c*d**4*n*x**4 + 12*log(a + b*x)*a*b**5*d**5*n**2
*x**4 - 12*log(a + b*x)*b**6*c**3*d**2*n**2*x**2 - 24*log(a + b*x)*b**6*c*
*2*d**3*n**2*x**3 - 12*log(a + b*x)*b**6*c*d**4*n**2*x**4 + 24*log(c + d*x
)*a**4*b**2*c**2*d**3*n + 48*log(c + d*x)*a**4*b**2*c*d**4*n*x + 24*log(c
+ d*x)*a**4*b**2*d**5*n*x**2 + 24*log(c + d*x)*a**3*b**3*c**3*d**2*n - 12*
log(c + d*x)*a**3*b**3*c**2*d**3*n**2 + 96*log(c + d*x)*a**3*b**3*c**2*d**
3*n*x - 24*log(c + d*x)*a**3*b**3*c*d**4*n**2*x + 120*log(c + d*x)*a**3*b*
*3*c*d**4*n*x**2 - 12*log(c + d*x)*a**3*b**3*d**5*n**2*x**2 + 48*log(c ...
```

3.158 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx$

Optimal result 1778
Mathematica [C] (verified) 1779
Rubi [A] (verified) 1780
Maple [B] (verified) 1782
Fricas [B] (verification not implemented) 1783
Sympy [F(-1)] 1784
Maxima [B] (verification not implemented) 1785
Giac [F] 1786
Mupad [B] (verification not implemented) 1786
Reduce [B] (verification not implemented) 1787

Optimal result

Integrand size = 43, antiderivative size = 587

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = &\frac{Bd^5n(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2}-\frac{5bBd^4n(a+bx)}{(bc-ad)^6g^4i^3(c+dx)}\\ &-\frac{10b^3Bd^2n(c+dx)}{(bc-ad)^6g^4i^3(a+bx)}+\frac{5b^4Bdn(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2}\\ &-\frac{b^5Bn(c+dx)^3}{9(bc-ad)^6g^4i^3(a+bx)^3}\\ &-\frac{d^5(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2}\\ &+\frac{5bd^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(c+dx)}\\ &-\frac{10b^3d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(a+bx)}\\ &+\frac{5b^4d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2}\\ &-\frac{b^5(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3}\\ &-\frac{10b^2d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}\\ &+\frac{5b^2Bd^3n\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}\end{aligned}$$

output

```
1/4*B*d^5*n*(b*x+a)^2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2-5*b*B*d^4*n*(b*x+a)/(
-a*d+b*c)^6/g^4/i^3/(d*x+c)-10*b^3*B*d^2*n*(d*x+c)/(-a*d+b*c)^6/g^4/i^3/(b
*x+a)+5/4*b^4*B*d*n*(d*x+c)^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-1/9*b^5*B*n*(
d*x+c)^3/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^3-1/2*d^5*(b*x+a)^2*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2+5*b*d^4*(b*x+a)*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(d*x+c)-10*b^3*d^2*(d*x+c)*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/2*b^4*d*(d*x+c)^
2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-1/3*b^5*(
d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^3-10
*b^2*d^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c)^6/
g^4/i^3+5*b^2*B*d^3*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^6/g^4/i^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.63 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.14

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx =$$

$$-\frac{\frac{4b^2B(bc-ad)^3n}{(a+bx)^3} - \frac{33b^2Bd(bc-ad)^2n}{(a+bx)^2} + \frac{216b^3Bcd^2n}{a+bx} - \frac{216ab^2Bd^3n}{a+bx} + \frac{66b^2Bd^2(bc-ad)n}{a+bx} - \frac{9Bd^3(bc-ad)^2n}{(c+dx)^2} - \frac{144b^2Bcd^3n}{c+dx} +$$

input

```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d
*i*x)^3),x]
```

output

```
-1/36*((4*b^2*B*(b*c - a*d)^3*n)/(a + b*x)^3 - (33*b^2*B*d*(b*c - a*d)^2*n
)/(a + b*x)^2 + (216*b^3*B*c*d^2*n)/(a + b*x) - (216*a*b^2*B*d^3*n)/(a + b
*x) + (66*b^2*B*d^2*(b*c - a*d)*n)/(a + b*x) - (9*B*d^3*(b*c - a*d)^2*n)/(
c + d*x)^2 - (144*b^2*B*c*d^3*n)/(c + d*x) + (144*a*b*B*d^4*n)/(c + d*x) -
 (18*b*B*d^3*(b*c - a*d)*n)/(c + d*x) + 120*b^2*B*d^3*n*Log[a + b*x] + (12
*b^2*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^3 - (
54*b^2*d*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2
 + (216*b^2*d^2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b
*x) + (18*d^3*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d
*x)^2 + (144*b*d^3*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c
+ d*x) + 360*b^2*d^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) -
 120*b^2*B*d^3*n*Log[c + d*x] - 360*b^2*d^3*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])*Log[c + d*x] - 180*b^2*B*d^3*n*(Log[a + b*x]*(Log[a + b*x] - 2*Lo
g[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]
) + 180*b^2*B*d^3*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*
Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^6*g^
4*i^3)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.69, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2961, 2772, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{(ag+bgx)^4(ci+dix)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4}\,d\frac{a+bx}{c+dx}}{g^4i^3(bc-ad)^6}$$

$$\downarrow 2772$$

$$-Bn\int -\frac{(c+dx)^4\left(2b^5-\frac{15d(a+bx)b^4}{c+dx}+\frac{60d^2(a+bx)^2b^3}{(c+dx)^2}+\frac{60d^3(a+bx)^3\log\left(\frac{a+bx}{c+dx}\right)b^2}{(c+dx)^3}-\frac{30d^4(a+bx)^4b}{(c+dx)^4}+\frac{3d^5(a+bx)^5}{(c+dx)^5}\right)}{6(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b^5(c+dx)^3\left(B\log\left(\right.\right.}{3(a+}$$

$$\downarrow 27$$

$$\frac{1}{6}Bn\int \frac{(c+dx)^4\left(2b^5-\frac{15d(a+bx)b^4}{c+dx}+\frac{60d^2(a+bx)^2b^3}{(c+dx)^2}+\frac{60d^3(a+bx)^3\log\left(\frac{a+bx}{c+dx}\right)b^2}{(c+dx)^3}-\frac{30d^4(a+bx)^4b}{(c+dx)^4}+\frac{3d^5(a+bx)^5}{(c+dx)^5}\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b^5(c+dx)^3\left(B\log\left(e\right.\right.}{3(a+bx)}$$

$$\downarrow 2010$$

$$\frac{1}{6}Bn\int \left(\frac{\left(2b^5-\frac{15d(a+bx)b^4}{c+dx}+\frac{60d^2(a+bx)^2b^3}{(c+dx)^2}-\frac{30d^4(a+bx)^4b}{(c+dx)^4}+\frac{3d^5(a+bx)^5}{(c+dx)^5}\right)(c+dx)^4}{(a+bx)^4}+\frac{60b^2d^3\log\left(\frac{a+bx}{c+dx}\right)(c+dx)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b^5(c+dx)^3\left(\right.}{}$$

$$\downarrow 2009$$

$$-\frac{b^5(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{5b^4d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{10b^3d^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-10b^2d^3\log\left(\right.$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d*i*x)^
3),x]
```

output
```
(-1/2*(d^5*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x)^2
 + (5*b*d^4*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) -
(10*b^3*d^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) +
(5*b^4*d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^
2) - (b^5*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)
^3) - 10*b^2*d^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c +
 d*x)] + (B*n*((3*d^5*(a + b*x)^2)/(2*(c + d*x)^2) - (30*b*d^4*(a + b*x))/
(c + d*x) - (60*b^3*d^2*(c + d*x))/(a + b*x) + (15*b^4*d*(c + d*x)^2)/(2*(
a + b*x)^2) - (2*b^5*(c + d*x)^3)/(3*(a + b*x)^3) + 30*b^2*d^3*Log[(a + b*
x)/(c + d*x)]^2))/6)/((b*c - a*d)^6*g^4*i^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2148 vs. 2(575) = 1150.

Time = 211.34 (sec) , antiderivative size = 2149, normalized size of antiderivative = 3.66

method	result	size
parallelrisch	Expression too large to display	2149

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,method=_
RETURNVERBOSE)
```

output

```
-1/36*(-720*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^7*c*d^8*n-540*B*x^2*ln(e*(
(b*x+a)/(d*x+c))^n)*a^3*b^7*d^9*n+120*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^10
*c^3*d^6*n-780*B*x^2*a^2*b^8*c*d^8*n^2+420*B*x^2*a*b^9*c^2*d^7*n^2+2160*A*
x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*c*d^8+1080*A*x^2*ln(e*((b*x+a)/(d*x+
c))^n)*a*b^9*c^2*d^7-720*A*x^2*a^2*b^8*c*d^8*n+1260*A*x^2*a*b^9*c^2*d^7*n+
360*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^7*c*d^8+540*B*x*ln(e*((b*x+a)/(d
*x+c))^n)^2*a^2*b^8*c^2*d^7-90*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^6*d^9*n
-30*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c^4*d^5*n-260*B*x*a^3*b^7*c*d^8*n^2
-390*B*x*a^2*b^8*c^2*d^7*n^2+540*B*x*a*b^9*c^3*d^6*n^2+720*A*x*ln(e*((b*x+
a)/(d*x+c))^n)*a^3*b^7*c*d^8+1080*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*c^
2*d^7-960*A*x*a^3*b^7*c*d^8*n+720*A*x*a^2*b^8*c^2*d^7*n+360*A*x*a*b^9*c^3*
d^6*n-180*B*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^6*c*d^8*n+360*B*ln(e*((b*x+a)/
(d*x+c))^n)*a^2*b^8*c^3*d^6*n-90*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c^4*d^5
*n+120*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^10*d^9*n+540*B*x^4*ln(e*((b*x+a)/
(d*x+c))^n)^2*a*b^9*d^9+360*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*b^10*c*d^8-1
20*B*x^4*a*b^9*d^9*n^2+120*B*x^4*b^10*c*d^8*n^2+1080*A*x^4*ln(e*((b*x+a)/(
d*x+c))^n)*a*b^9*d^9+720*A*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c*d^8-360*A*
x^4*a*b^9*d^9*n+360*A*x^4*b^10*c*d^8*n+540*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)
^2*a^2*b^8*d^9+180*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^10*c^2*d^7-120*B*x^
3*a^2*b^8*d^9*n^2+360*B*x^3*b^10*c^2*d^7*n^2+1080*A*x^3*ln(e*((b*x+a)/(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2181 vs. $2(575) = 1150$.

Time $= 0.18$ (sec) , antiderivative size $= 2181$, normalized size of antiderivative $= 3.72$

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,
algorithm="fricas")
```

output

```
-1/36*(12*A*b^5*c^5 - 90*A*a*b^4*c^4*d + 360*A*a^2*b^3*c^3*d^2 - 120*A*a^3
*b^2*c^2*d^3 - 180*A*a^4*b*c*d^4 + 18*A*a^5*d^5 + 120*(3*A*b^5*c*d^4 - 3*A
*a*b^4*d^5 + (B*b^5*c*d^4 - B*a*b^4*d^5)*n)*x^4 + 60*(9*A*b^5*c^2*d^3 + 6*
A*a*b^4*c*d^4 - 15*A*a^2*b^3*d^5 + 2*(3*B*b^5*c^2*d^3 - 2*B*a*b^4*c*d^4 -
B*a^2*b^3*d^5)*n)*x^3 + 20*(6*A*b^5*c^3*d^2 + 63*A*a*b^4*c^2*d^3 - 36*A*a^
2*b^3*c*d^4 - 33*A*a^3*b^2*d^5 + (11*B*b^5*c^3*d^2 + 21*B*a*b^4*c^2*d^3 -
39*B*a^2*b^3*c*d^4 + 7*B*a^3*b^2*d^5)*n)*x^2 + 180*(B*b^5*d^5*n*x^5 + B*a^
3*b^2*c^2*d^3*n + (2*B*b^5*c*d^4 + 3*B*a*b^4*d^5)*n*x^4 + (B*b^5*c^2*d^3 +
 6*B*a*b^4*c*d^4 + 3*B*a^2*b^3*d^5)*n*x^3 + (3*B*a*b^4*c^2*d^3 + 6*B*a^2*b
^3*c*d^4 + B*a^3*b^2*d^5)*n*x^2 + (3*B*a^2*b^3*c^2*d^3 + 2*B*a^3*b^2*c*d^4
)*n*x)*log((b*x + a)/(d*x + c))^2 + (4*B*b^5*c^5 - 45*B*a*b^4*c^4*d + 360*
B*a^2*b^3*c^3*d^2 - 490*B*a^3*b^2*c^2*d^3 + 180*B*a^4*b*c*d^4 - 9*B*a^5*d^
5)*n - 5*(6*A*b^5*c^4*d - 72*A*a*b^4*c^3*d^2 - 144*A*a^2*b^3*c^2*d^3 + 192
*A*a^3*b^2*c*d^4 + 18*A*a^4*b*d^5 + (5*B*b^5*c^4*d - 108*B*a*b^4*c^3*d^2 +
 78*B*a^2*b^3*c^2*d^3 + 52*B*a^3*b^2*c*d^4 - 27*B*a^4*b*d^5)*n)*x + 6*(2*B
*b^5*c^5 - 15*B*a*b^4*c^4*d + 60*B*a^2*b^3*c^3*d^2 - 20*B*a^3*b^2*c^2*d^3
- 30*B*a^4*b*c*d^4 + 3*B*a^5*d^5 + 60*(B*b^5*c*d^4 - B*a*b^4*d^5)*x^4 + 30
*(3*B*b^5*c^2*d^3 + 2*B*a*b^4*c*d^4 - 5*B*a^2*b^3*d^5)*x^3 + 10*(2*B*b^5*c
^3*d^2 + 21*B*a*b^4*c^2*d^3 - 12*B*a^2*b^3*c*d^4 - 11*B*a^3*b^2*d^5)*x^2 -
 5*(B*b^5*c^4*d - 12*B*a*b^4*c^3*d^2 - 24*B*a^2*b^3*c^2*d^3 + 32*B*a^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^4(ci + dix)^3}\, dx = \text{Timed out}$$

input

```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4/(d*i*x+c*i)**3,x
)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3819 vs. $2(575) = 1150$.

Time = 0.33 (sec) , antiderivative size = 3819, normalized size of antiderivative = 6.51

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,
algorithm="maxima")
```

output

```
-1/6*B*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2*b^2*c^2*d^2
+ 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 10*(2*
b^4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3*d - 11*a*b
^3*c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/((b^8*c^5*d^2 - 5*a*b^7*c^
4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^
3*d^7)*g^4*i^3*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 +
10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7
)*g^4*i^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c
^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*
d^7)*g^4*i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a
^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a
^8*d^7)*g^4*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d
^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^
6)*g^4*i^3*x + (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^
6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i^3) + 60*b^2*d^3*log(b
*x + a)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^
3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3) - 60*b^2*d^3*lo
g(d*x + c)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3
*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3))*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n) - 1/36*(4*b^5*c^5 - 45*a*b^4*c^4*d + 360*a^...
```

Giac [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{(bgx+ag)^4(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,
algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/((b*g*x + a*g)^4*(d*i*x +
 c*i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 31.38 (sec) , antiderivative size = 2400, normalized size of antiderivative = 4.09

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^4*(c*i + d*i*x)^
3),x)
```

output

```
log(e*((a + b*x)/(c + d*x))^n)*((x*((5*B*(2*a*b*d^2 + b^2*c*d)*(a*d + b*c)
)/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) - (5*B*b*d)/(6*(a^2*d^2 + b^2*c^2
- 2*a*b*c*d)) + (5*B*a*b^2*c*d^2)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + x^2
*((5*B*b*d*(2*a*b*d^2 + b^2*c*d))/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) +
(5*B*b^2*d^2*(a*d + b*c))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) - (B*(3*a*d +
 2*b*c))/(6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (5*B*a*c*(2*a*b*d^2 + b^2*c
*d))/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + (5*B*b^3*d^3*x^3)/(a^2*d^2 +
b^2*c^2 - 2*a*b*c*d)^2)/(x*(2*a^3*c*d*g^4*i^3 + 3*a^2*b*c^2*g^4*i^3) + x^2
*(a^3*d^2*g^4*i^3 + 3*a*b^2*c^2*g^4*i^3 + 6*a^2*b*c*d*g^4*i^3) + x^3*(b^3*
c^2*g^4*i^3 + 3*a^2*b*d^2*g^4*i^3 + 6*a*b^2*c*d*g^4*i^3) + x^4*(2*b^3*c*d*
g^4*i^3 + 3*a*b^2*d^2*g^4*i^3) + a^3*c^2*g^4*i^3 + b^3*d^2*g^4*i^3*x^5) +
(10*B*b^2*d^3*(x^2*((g^4*i^3*n*(a*d + b*c)^2*(a*d - b*c))/d + 2*a*b*c*g^4*
i^3*n*(a*d - b*c)) + b^2*d*g^4*i^3*n*x^4*(a*d - b*c) + (a^2*c^2*g^4*i^3*n*
(a*d - b*c))/d + 2*b*g^4*i^3*n*x^3*(a*d + b*c)*(a*d - b*c) + (2*a*c*g^4*i^
3*n*x*(a*d + b*c)*(a*d - b*c))/d))/(g^4*i^3*n*(a*d - b*c)^6*(x*(2*a^3*c*d*
g^4*i^3 + 3*a^2*b*c^2*g^4*i^3) + x^2*(a^3*d^2*g^4*i^3 + 3*a*b^2*c^2*g^4*i^
3 + 6*a^2*b*c*d*g^4*i^3) + x^3*(b^3*c^2*g^4*i^3 + 3*a^2*b*d^2*g^4*i^3 + 6*
a*b^2*c*d*g^4*i^3) + x^4*(2*b^3*c*d*g^4*i^3 + 3*a*b^2*d^2*g^4*i^3) + a^3*c
^2*g^4*i^3 + b^3*d^2*g^4*i^3*x^5))) + ((12*A*b^4*c^4 - 18*A*a^4*d^4 + 9*B*
a^4*d^4*n + 4*B*b^4*c^4*n + 282*A*a^2*b^2*c^2*d^2 - 78*A*a*b^3*c^3*d + ...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 4291, normalized size of antiderivative = 7.31

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^4(ci + dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x)
```

output

```
(i*( - 1080*log(a + b*x)*a**5*b**2*c**2*d**4*n - 2160*log(a + b*x)*a**5*b*
*2*c*d**5*n*x - 1080*log(a + b*x)*a**5*b**2*d**6*n*x**2 - 720*log(a + b*x)
*a**4*b**3*c**3*d**3*n - 4680*log(a + b*x)*a**4*b**3*c**2*d**4*n*x - 7200*
log(a + b*x)*a**4*b**3*c*d**5*n*x**2 - 3240*log(a + b*x)*a**4*b**3*d**6*n*
x**3 - 600*log(a + b*x)*a**3*b**4*c**3*d**3*n**2 - 2160*log(a + b*x)*a**3*
b**4*c**3*d**3*n*x - 1200*log(a + b*x)*a**3*b**4*c**2*d**4*n**2*x - 7560*l
og(a + b*x)*a**3*b**4*c**2*d**4*n*x**2 - 600*log(a + b*x)*a**3*b**4*c*d**5
*n**2*x**2 - 8640*log(a + b*x)*a**3*b**4*c*d**5*n*x**3 - 3240*log(a + b*x)
*a**3*b**4*d**6*n*x**4 - 1800*log(a + b*x)*a**2*b**5*c**3*d**3*n**2*x - 21
60*log(a + b*x)*a**2*b**5*c**3*d**3*n*x**2 - 3600*log(a + b*x)*a**2*b**5*c
**2*d**4*n**2*x**2 - 5400*log(a + b*x)*a**2*b**5*c**2*d**4*n*x**3 - 1800*l
og(a + b*x)*a**2*b**5*c*d**5*n**2*x**3 - 4320*log(a + b*x)*a**2*b**5*c*d**
5*n*x**4 - 1080*log(a + b*x)*a**2*b**5*d**6*n*x**5 - 1800*log(a + b*x)*a*b
**6*c**3*d**3*n**2*x**2 - 720*log(a + b*x)*a*b**6*c**3*d**3*n*x**3 - 3600*
log(a + b*x)*a*b**6*c**2*d**4*n**2*x**3 - 1440*log(a + b*x)*a*b**6*c**2*d*
*4*n*x**4 - 1800*log(a + b*x)*a*b**6*c*d**5*n**2*x**4 - 720*log(a + b*x)*a
*b**6*c*d**5*n*x**5 - 600*log(a + b*x)*b**7*c**3*d**3*n**2*x**3 - 1200*log
(a + b*x)*b**7*c**2*d**4*n**2*x**4 - 600*log(a + b*x)*b**7*c*d**5*n**2*x**
5 + 1080*log(c + d*x)*a**5*b**2*c**2*d**4*n + 2160*log(c + d*x)*a**5*b**2*
c*d**5*n*x + 1080*log(c + d*x)*a**5*b**2*d**6*n*x**2 + 720*log(c + d*x)...
```

3.159 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result . . . 1789
Mathematica [A] (verified) . . . 1790
Rubi [A] (verified) . . . 1791
Maple [F] . . . 1797
Fricas [F] . . . 1797
Sympy [F(-1)] . . . 1798
Maxima [B] (verification not implemented) . . . 1798
Giac [F(-1)] . . . 1799
Mupad [F(-1)] . . . 1800
Reduce [F] . . . 1800

Optimal result

Integrand size = 43, antiderivative size = 584

$$
\begin{aligned}
&\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{3B^2(bc-ad)^4g^3in^2x}{10bd^3} - \frac{3B^2(bc-ad)^3g^3in^2(c+dx)^2}{20d^4} \\
&\quad + \frac{bB^2(bc-ad)^2g^3in^2(c+dx)^3}{30d^4} - \frac{B(bc-ad)^2g^3in(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^2d} \\
&\quad - \frac{B(bc-ad)g^3in(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^2} \\
&\quad + \frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2} \\
&\quad + \frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} \\
&\quad + \frac{B(bc-ad)^3g^3in(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^2d^2} \\
&\quad - \frac{B(bc-ad)^4g^3in(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^2d^3} \\
&\quad - \frac{B(bc-ad)^5g^3in\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{60b^2d^4} \\
&\quad - \frac{B^2(bc-ad)^5g^3in^2\log(c+dx)}{10b^2d^4} - \frac{B^2(bc-ad)^5g^3in^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{10b^2d^4}
\end{aligned}
$$

output

```
3/10*B^2*(-a*d+b*c)^4*g^3*i*n^2*x/b/d^3-3/20*B^2*(-a*d+b*c)^3*g^3*i*n^2*(d
*x+c)^2/d^4+1/30*b*B^2*(-a*d+b*c)^2*g^3*i*n^2*(d*x+c)^3/d^4-1/30*B*(-a*d+b
*c)^2*g^3*i*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d-1/10*B*(-a*d
+b*c)*g^3*i*n*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2+1/20*(-a*d+b*c
)*g^3*i*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2+1/5*g^3*i*(b*x+a)^
4*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/60*B*(-a*d+b*c)^3*g^3*i*n*
(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d^2-1/60*B*(-a*d+b*c
)^4*g^3*i*n*(b*x+a)*(6*A+5*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d^3-1/60
*B*(-a*d+b*c)^5*g^3*i*n*(6*A+11*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*
d+b*c)/b/(d*x+c))/b^2/d^4-1/10*B^2*(-a*d+b*c)^5*g^3*i*n^2*ln(d*x+c)/b^2/d^
4-1/10*B^2*(-a*d+b*c)^5*g^3*i*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^4
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 949, normalized size of antiderivative = 1.62

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

input

```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^2,x]
```

output

```
(g^3*i*(5*(b*c - a*d)*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
 + 4*d*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (5*B*(b*c -
a*d)^2*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((
a + b*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[e*
((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(b*c - a*d
)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*
n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n*((2*Log[(d*(
a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(
c + d*x))/(b*c - a*d)])))/(3*d^4) + (B*(b*c - a*d)*n*(24*A*b*d*(b*c - a*d)
^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 12*
d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 8*d
^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*d^4*
(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 24*B*(b*c - a*d)^4*n*
Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log
[c + d*x] + 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2
*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*
d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c
 + d*x]) + 12*B*(b*c - a*d)^3*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) +...
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.256$, Rules used = {2961, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$

$$\downarrow 2961$$

$$g^3i(bc-ad)^5\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$g^3 i(bc-ad)^5\left(-\frac{2Bn\int\frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{\int\frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{(a+bx)^4\left(B\log\left(e\left(\frac{}{}\right.\right.\right.}{5b(c+dx)^4\left(b-\right.}$$

$$\downarrow 2773$$

$$g^3 i(bc-ad)^5\left(-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int\frac{(a+bx)^3}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}\right)}{5b}+\frac{\int\frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+b}{c+d}}{5b}$$

$$\downarrow 49$$

$$g^3 i(bc-ad)^5\left(-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int\left(\frac{b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{1}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)d\frac{a}{c}}{4b}\right)}{5b}$$

$$\downarrow 2009$$

$$g^3 i(bc-ad)^5\left(\frac{\int\frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{b^3}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3b^2}{2d^4\left(b-\frac{d(a+b}{c+d}\right.\right.\right.}{5b}$$

$$\downarrow 2781$$

$$g^3 i(bc-$$

$$ad)^5\left(\frac{\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int\frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}}{5b}-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\right.$$

$$\downarrow 2784$$

$$g^3 i(bc-$$

$$ad)^5\left(\frac{\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int\frac{(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{2b}}{5b}-\frac{2Bn}{}$$

$$\downarrow 2784$$

$$g^3 i(bc-$$

$$ad)^5\left(\frac{\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(6A+5Bn+\right.}{(c+dx)\left(b-\right.}}{3d}\right)}{2b}}{5b}\right.$$

$$\downarrow 2784$$

$$g^3 i(bc-$$

$$ad)^5 \left(\frac{(a+bx)^4 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn \left(\frac{(a+bx)^3 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{(a+bx)^2 \left(3B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{\frac{(a+bx)\left(6B \log\left(e\left(\frac{a}{c}\right.\right.\right.}{d(c+dx)\left(b-\right.}}{3} \right)}{2b} \right)}{5b}$$

$$\downarrow 2754$$

$$g^3 i(bc-$$

$$ad)^5 \left(\frac{(a+bx)^4 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn \left(\frac{(a+bx)^3 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{(a+bx)^2 \left(3B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{\frac{(a+bx)\left(6B \log\left(e\left(\frac{a}{c}\right.\right.\right.}{d(c+dx)\left(b-\right.}}{3} \right)}{2b} \right)}{5b}$$

$$\downarrow 2838$$

$$g^3 i(bc-ad)^5 \left(-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{b^3}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3b^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{3b}{d^4\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^4}\right)}{4b}\right)}{5b}\right)$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
,x]
```

output
```
(b*c - a*d)^5*g^3*i*(((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
)/(5*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*n*(((a + b*x)^4
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b*(c + d*x)^4*(b - (d*(a + b*x
))/(c + d*x))^4) - (B*n*(b^3/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^3) - (3*
b^2)/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (3*b)/(d^4*(b - (d*(a + b*x
))/(c + d*x))) + Log[b - (d*(a + b*x))/(c + d*x)]/d^4))/(4*b)))/(5*b) + ((
(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*b*(c + d*x)^4*(b
- (d*(a + b*x))/(c + d*x))^4) - (B*n*(((a + b*x)^3*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a
+ b*x)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)^
2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(6*A + 5*B*n + 6*B*Log[e*
((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (
-(((6*A + 11*B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x
))/(b*(c + d*x))])/d) - (6*B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)
/d)/(2*d))/(3*d)))/(2*b))/(5*b))
```

Defintions of rubi rules used

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2783

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)^3 (dix+ci)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (bgx+ag)^3(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="fricas")
```

output
```
integral(A^2*b^3*d*g^3*i*x^4 + A^2*a^3*c*g^3*i + (A^2*b^3*c + 3*A^2*a*b^2*
d)*g^3*i*x^3 + 3*(A^2*a*b^2*c + A^2*a^2*b*d)*g^3*i*x^2 + (3*A^2*a^2*b*c +
A^2*a^3*d)*g^3*i*x + (B^2*b^3*d*g^3*i*x^4 + B^2*a^3*c*g^3*i + (B^2*b^3*c +
 3*B^2*a*b^2*d)*g^3*i*x^3 + 3*(B^2*a*b^2*c + B^2*a^2*b*d)*g^3*i*x^2 + (3*B
^2*a^2*b*c + B^2*a^3*d)*g^3*i*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B
*b^3*d*g^3*i*x^4 + A*B*a^3*c*g^3*i + (A*B*b^3*c + 3*A*B*a*b^2*d)*g^3*i*x^3
 + 3*(A*B*a*b^2*c + A*B*a^2*b*d)*g^3*i*x^2 + (3*A*B*a^2*b*c + A*B*a^3*d)*g
^3*i*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x
)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3764 vs. $2(559) = 1118$.

Time = 0.64 (sec) , antiderivative size = 3764, normalized size of antiderivative = 6.45

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="maxima")
```

output

```
2/5*A*B*b^3*d*g^3*i*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A^2*b
^3*d*g^3*i*x^5 + 1/2*A*B*b^3*c*g^3*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n) + 3/2*A*B*a*b^2*d*g^3*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
 1/4*A^2*b^3*c*g^3*i*x^4 + 3/4*A^2*a*b^2*d*g^3*i*x^4 + 2*A*B*a*b^2*c*g^3*i
*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*B*a^2*b*d*g^3*i*x^3*log(
e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*b^2*c*g^3*i*x^3 + A^2*a^2*b*d*g
^3*i*x^3 + 3*A*B*a^2*b*c*g^3*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ A*B*a^3*d*g^3*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A^2*a^2
*b*c*g^3*i*x^2 + 1/2*A^2*a^3*d*g^3*i*x^2 + 1/30*A*B*b^3*d*g^3*i*n*(12*a^5*
log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^
4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12
*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/12*A*B*b^3*c*g^3*i*n*(6*a^4*log(b*x
 + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b
^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/4*A*B*
a*b^2*d*g^3*i*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3
*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3
*d^3)*x)/(b^3*d^3)) + A*B*a*b^2*c*g^3*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*
log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^
2*d^2)) + A*B*a^2*b*d*g^3*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) -...
```

Giac [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input

```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="giac")
```

output

```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^3\,(c\,i+d\,i\,x)\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2
,x)
```

output
```
int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2
, x)
```

Reduce [F]

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
(g**3*i*( - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*
c*x + b*d*x**2),x)*a**5*b**2*d**6*n + 30*int((log(((a + b*x)**n*e)/(c + d*
x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**4*b**3*c*d**5*n - 60*int(
(log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)
*a**3*b**4*c**2*d**4*n + 60*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*
c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**5*c**3*d**3*n - 30*int((log(((a +
 b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**6*c*
*4*d**2*n + 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*
c*x + b*d*x**2),x)*b**7*c**5*d*n - 6*log(a + b*x)*a**6*d**5*n + 30*log(a +
 b*x)*a**5*b*c*d**4*n - 5*log(a + b*x)*a**5*b*d**5*n**2 - 60*log(a + b*x)*
a**4*b**2*c**2*d**3*n + 25*log(a + b*x)*a**4*b**2*c*d**4*n**2 + 60*log(a +
 b*x)*a**3*b**3*c**3*d**2*n - 50*log(a + b*x)*a**3*b**3*c**2*d**3*n**2 - 3
0*log(a + b*x)*a**2*b**4*c**4*d*n + 50*log(a + b*x)*a**2*b**4*c**3*d**2*n*
*2 + 6*log(a + b*x)*a*b**5*c**5*n - 25*log(a + b*x)*a*b**5*c**4*d*n**2 + 5
*log(a + b*x)*b**6*c**5*n**2 + 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**
4*b**2*c*d**4 + 18*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*b**3*c**2*d*
*3 + 60*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*b**3*c*d**4*x + 30*log(
((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*b**3*d**5*x**2 - 12*log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*a**2*b**4*c**3*d**2 + 90*log(((a + b*x)**n*e)/(c +
d*x)**n)**2*a**2*b**4*c*d**4*x**2 + 60*log(((a + b*x)**n*e)/(c + d*x)**...
```

3.160 $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result . . . 1802
Mathematica [A] (verified) . . . 1803
Rubi [A] (verified) . . . 1804
Maple [F] . . . 1810
Fricas [F] . . . 1810
Sympy [F] . . . 1811
Maxima [B] (verification not implemented) . . . 1812
Giac [F(-1)] . . . 1813
Mupad [F(-1)] . . . 1814
Reduce [F] . . . 1814

Optimal result

Integrand size = 43, antiderivative size = 487

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= -\frac{B^2(bc-ad)^3g^2in^2x}{3bd^2} + \frac{B^2(bc-ad)^2g^2in^2(c+dx)^2}{12d^3} \\
&\quad - \frac{B(bc-ad)^2g^2in(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2d} \\
&\quad - \frac{B(bc-ad)g^2in(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^2} \\
&\quad + \frac{(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{12b^2} \\
&\quad + \frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} \\
&\quad + \frac{B(bc-ad)^3g^2in(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2d^2} \\
&\quad + \frac{B(bc-ad)^4g^2in\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{12b^2d^3} \\
&\quad + \frac{B^2(bc-ad)^4g^2in^2\log(c+dx)}{6b^2d^3} + \frac{B^2(bc-ad)^4g^2in^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{6b^2d^3}
\end{aligned}$$

output

```
-1/3*B^2*(-a*d+b*c)^3*g^2*i*n^2*x/b/d^2+1/12*B^2*(-a*d+b*c)^2*g^2*i*n^2*(d
*x+c)^2/d^3-1/12*B*(-a*d+b*c)^2*g^2*i*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))/b^2/d-1/6*B*(-a*d+b*c)*g^2*i*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))/b^2+1/12*(-a*d+b*c)*g^2*i*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
^2/b^2+1/4*g^2*i*(b*x+a)^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/1
2*B*(-a*d+b*c)^3*g^2*i*n*(b*x+a)*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))/b
^2/d^2+1/12*B*(-a*d+b*c)^4*g^2*i*n*(2*A+3*B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n
))*ln((-a*d+b*c)/b/(d*x+c))/b^2/d^3+1/6*B^2*(-a*d+b*c)^4*g^2*i*n^2*ln(d*x+
c)/b^2/d^3+1/6*B^2*(-a*d+b*c)^4*g^2*i*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b
^2/d^3
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.47

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$

$$= \frac{g^2 i\Big(4(bc-ad)(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+3d(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{4B(bc-ad)^2n\left(}{}$$

input

```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^2,x]
```

output

```
(g^2*i*(4*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
 + 3*d*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (4*B*(b*c -
a*d)^2*n*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a +
b*x)/(c + d*x))^n] - d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
) - 2*B*(b*c - a*d)^2*n*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[e*((a +
b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(b*d*x + (-(b*c) + a*d)
*Log[c + d*x]) + B*(b*c - a*d)^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] -
 Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d
^3 - (B*(b*c - a*d)*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a +
b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b
*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(
b*c - a*d)^2*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n
*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*Po
lyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(12*b^2)
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2961, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$

$$\downarrow 2961$$

$$g^2i(bc-ad)^4\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$g^2 i(bc-ad)^4\left(-\frac{Bn\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}+\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a}{c}\right.\right.\right.}{4b(c+dx)^3\left(b-\right.}\right.$$

$$\downarrow 2773$$

$$g^2 i(bc-ad)^4\left(-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\frac{(a+bx)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}\right)}{2b}+\frac{\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}$$

$$\downarrow 49$$

$$g^2 i(bc-ad)^4\left(-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\left(\frac{b^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{1}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}}{3b}\right)}{2b}+\frac{\int\frac{(a+bx}{(}$$

$$\downarrow 2009$$

$$g^2 i(bc-ad)^4\left(\frac{\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{b^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right.\right.}\right.}{3b}\right.}{2b}$$

$$\downarrow 2781$$

$$g^2 i(bc-$$

$$ad)^4\left(\frac{\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}}{4b}-Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\right.$$

$$\downarrow 2784$$

$$g^2 i(bc-$$

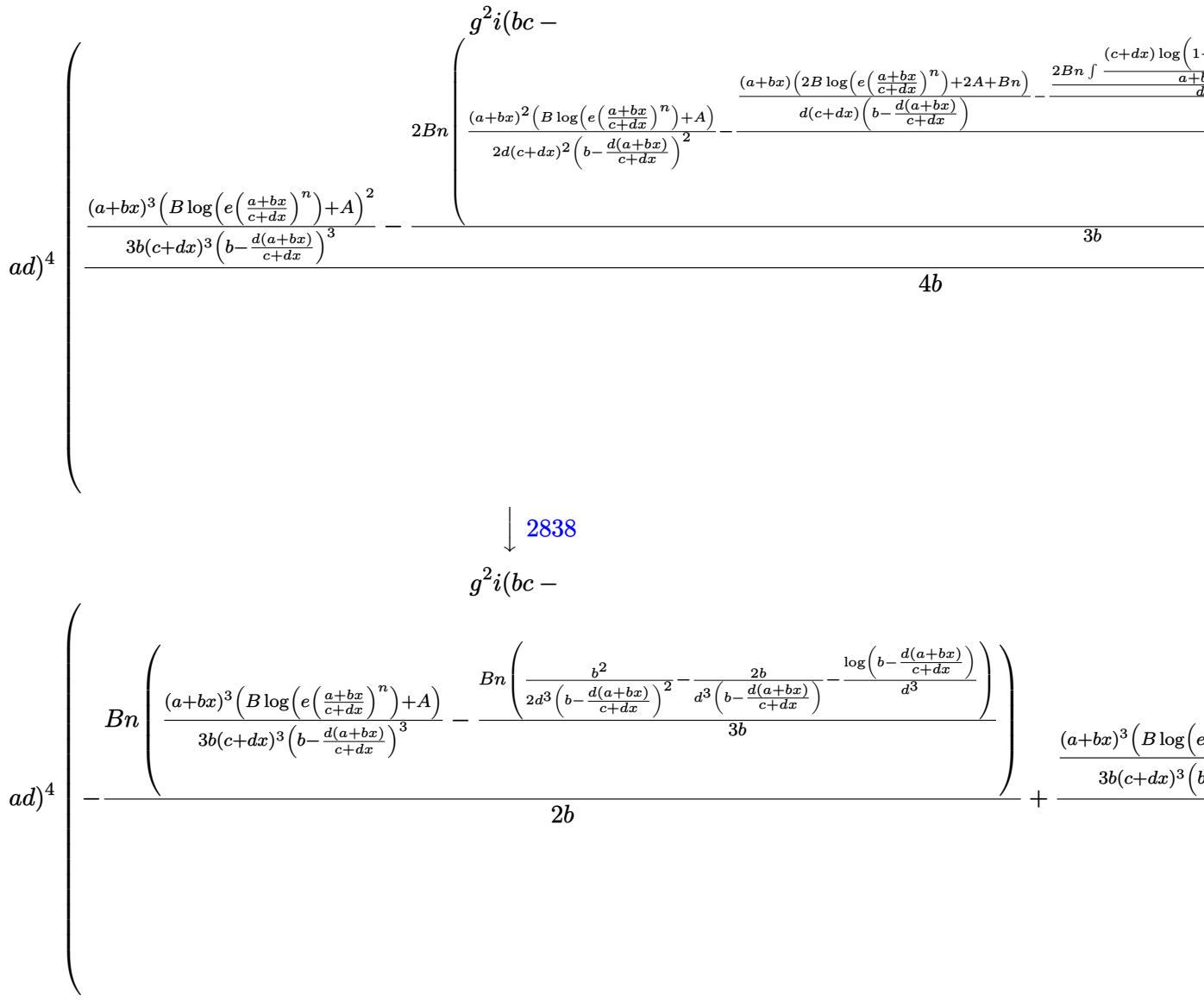
$$ad)^4\left(\frac{\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{3b}}{4b}-Bn\left(\right.$$

$$\downarrow 2784$$

$$g^2 i(bc-$$

$$ad)^4\left(\frac{\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{2A+3Bn+2B\log\left(e\right.}{b-\frac{d(a+b}{c+d}}}{2d}\right)}{3b}}{4b}\right.$$

$$\downarrow 2754$$



input
```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
,x]
```

output
```
(b*c - a*d)^4*g^2*i*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
)/(4*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*(((a + b*x)^3*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b*(c + d*x)^3*(b - (d*(a + b*x))
/(c + d*x))^3) - (B*n*(b^2/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^2) - (2*b)
/(d^3*(b - (d*(a + b*x))/(c + d*x))) - Log[b - (d*(a + b*x))/(c + d*x)]/d^
3))/(3*b)))/(2*b) + (((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
)/(3*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*n*(((a + b*x)^2
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x
))/(c + d*x))^2) - (((a + b*x)*(2*A + B*n + 2*B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((2*A + 3*B*n + 2*
B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d)
 - (2*B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d)))/(3*b))/(4
*b))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2783

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)^2 (dix+ci) \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (bgx+ag)^2(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="fricas")
```

output
```
integral(A^2*b^2*d*g^2*i*x^3 + A^2*a^2*c*g^2*i + (A^2*b^2*c + 2*A^2*a*b*d)
*g^2*i*x^2 + (2*A^2*a*b*c + A^2*a^2*d)*g^2*i*x + (B^2*b^2*d*g^2*i*x^3 + B^
2*a^2*c*g^2*i + (B^2*b^2*c + 2*B^2*a*b*d)*g^2*i*x^2 + (2*B^2*a*b*c + B^2*a
^2*d)*g^2*i*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^2*d*g^2*i*x^3 +
 A*B*a^2*c*g^2*i + (A*B*b^2*c + 2*A*B*a*b*d)*g^2*i*x^2 + (2*A*B*a*b*c + A*
B*a^2*d)*g^2*i*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F]

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= g^2 i\left(\int A^2a^2c\,dx+\int A^2a^2dx\,dx+\int A^2b^2cx^2\,dx+\int A^2b^2dx^3\,dx\right.$$
$$+\int B^2a^2c\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+\int 2ABa^2c\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx+\int 2A^2abcx\,dx+\int 2A^2abdx^2\,dx$$
$$+\int B^2a^2dx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+\int B^2b^2cx^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+\int B^2b^2dx^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+\int 2ABa^2dx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx$$
$$+\int 2ABb^2cx^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx$$
$$+\int 2ABb^2dx^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx$$
$$+\int 2B^2abcx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+\int 2B^2abdx^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+\int 4ABabcx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx$$
$$\left.+\int 4ABabdx^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx\right)$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x
)
```

output

```
g**2*i*(Integral(A**2*a**2*c, x) + Integral(A**2*a**2*d*x, x) + Integral(A
**2*b**2*c*x**2, x) + Integral(A**2*b**2*d*x**3, x) + Integral(B**2*a**2*c
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*a**2*c*lo
g(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Integral(2*A**2*a*b*c*x, x) +
Integral(2*A**2*a*b*d*x**2, x) + Integral(B**2*a**2*d*x*log(e*(a/(c + d*x)
 + b*x/(c + d*x))**n)**2, x) + Integral(B**2*b**2*c*x**2*log(e*(a/(c + d*x
) + b*x/(c + d*x))**n)**2, x) + Integral(B**2*b**2*d*x**3*log(e*(a/(c + d*
x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*a**2*d*x*log(e*(a/(c + d*x
) + b*x/(c + d*x))**n), x) + Integral(2*A*B*b**2*c*x**2*log(e*(a/(c + d*x)
 + b*x/(c + d*x))**n), x) + Integral(2*A*B*b**2*d*x**3*log(e*(a/(c + d*x)
+ b*x/(c + d*x))**n), x) + Integral(2*B**2*a*b*c*x*log(e*(a/(c + d*x) + b*
x/(c + d*x))**n)**2, x) + Integral(2*B**2*a*b*d*x**2*log(e*(a/(c + d*x) +
b*x/(c + d*x))**n)**2, x) + Integral(4*A*B*a*b*c*x*log(e*(a/(c + d*x) + b*
x/(c + d*x))**n), x) + Integral(4*A*B*a*b*d*x**2*log(e*(a/(c + d*x) + b*x/
(c + d*x))**n), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2691 vs. $2(466) = 932$.

Time $= 0.60$ (sec) , antiderivative size $= 2691$, normalized size of antiderivative $= 5.53$

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="maxima")
```

output

```
1/2*A*B*b^2*d*g^2*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*b
^2*d*g^2*i*x^4 + 2/3*A*B*b^2*c*g^2*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n) + 4/3*A*B*a*b*d*g^2*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1
/3*A^2*b^2*c*g^2*i*x^3 + 2/3*A^2*a*b*d*g^2*i*x^3 + 2*A*B*a*b*c*g^2*i*x^2*l
og(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*B*a^2*d*g^2*i*x^2*log(e*(b*x/(d*
x + c) + a/(d*x + c))^n) + A^2*a*b*c*g^2*i*x^2 + 1/2*A^2*a^2*d*g^2*i*x^2 -
 1/12*A*B*b^2*d*g^2*i*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 +
 (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c
^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/3*A*B*b^2*c*g^2*i*n*(2*a^3*log(b*x + a)/b^
3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d
^2)*x)/(b^2*d^2)) + 2/3*A*B*a*b*d*g^2*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*
log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^
2*d^2)) - 2*A*B*a*b*c*g^2*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2
 + (b*c - a*d)*x/(b*d)) - A*B*a^2*d*g^2*i*n*(a^2*log(b*x + a)/b^2 - c^2*lo
g(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a^2*c*g^2*i*n*(a*log(b*x + a
)/b - c*log(d*x + c)/d) + 2*A*B*a^2*c*g^2*i*x*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n) + A^2*a^2*c*g^2*i*x - 1/12*(2*a^3*c*d^3*g^2*i*n^2 + (g^2*i*n^2
+ 2*g^2*i*n*log(e))*b^3*c^4 - 2*(g^2*i*n^2 + 4*g^2*i*n*log(e))*a*b^2*c^3*d
 - (g^2*i*n^2 - 12*g^2*i*n*log(e))*a^2*b*c^2*d^2)*B^2*log(d*x + c)/(b*d^3)
 - 1/6*(b^4*c^4*g^2*i*n^2 - 4*a*b^3*c^3*d*g^2*i*n^2 + 6*a^2*b^2*c^2*d^2...
```

Giac [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input

```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="giac")
```

output

```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2
,x)
```

output
```
int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2
, x)
```

Reduce [F]

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
(g**2*i*( - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*
c*x + b*d*x**2),x)*a**4*b**2*d**5*n + 8*int((log(((a + b*x)**n*e)/(c + d*x
)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**3*c*d**4*n - 12*int((
log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*
a**2*b**4*c**2*d**3*n + 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c
+ a*d*x + b*c*x + b*d*x**2),x)*a*b**5*c**3*d**2*n - 2*int((log(((a + b*x)*
*n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**6*c**4*d*n -
 2*log(a + b*x)*a**5*d**4*n + 8*log(a + b*x)*a**4*b*c*d**3*n - log(a + b*x
)*a**4*b*d**4*n**2 - 12*log(a + b*x)*a**3*b**2*c**2*d**2*n + 4*log(a + b*x
)*a**3*b**2*c*d**3*n**2 + 8*log(a + b*x)*a**2*b**3*c**3*d*n - 6*log(a + b*
x)*a**2*b**3*c**2*d**2*n**2 - 2*log(a + b*x)*a*b**4*c**4*n + 4*log(a + b*x
)*a*b**4*c**3*d*n**2 - log(a + b*x)*b**5*c**4*n**2 + log(((a + b*x)**n*e)/
(c + d*x)**n)**2*a**3*b**2*c*d**3 + 3*log(((a + b*x)**n*e)/(c + d*x)**n)**
2*a**2*b**3*c**2*d**2 + 12*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**3
*c*d**3*x + 6*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**3*d**4*x**2 -
log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**4*c**3*d + 12*log(((a + b*x)**n
*e)/(c + d*x)**n)**2*a*b**4*c*d**3*x**2 + 8*log(((a + b*x)**n*e)/(c + d*x)
**n)**2*a*b**4*d**4*x**3 + 4*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**5*c*
d**3*x**3 + 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**5*d**4*x**4 + 12*lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**2*c**2*d**2 + 2*log(((a + b*x)...
```

3.161 $\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result . . . 1816
Mathematica [B] (verified) . . . 1817
Rubi [A] (verified) . . . 1818
Maple [F] . . . 1823
Fricas [F] . . . 1823
Sympy [F] . . . 1824
Maxima [B] (verification not implemented) . . . 1825
Giac [F] . . . 1826
Mupad [F(-1)] . . . 1826
Reduce [F] . . . 1827

Optimal result

Integrand size = 41, antiderivative size = 372

$$\begin{aligned}
&\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^2 gin^2 x}{3bd} - \frac{B(bc-ad)^2 gin(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2 d} \\
&\quad - \frac{B(bc-ad)gin(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2} \\
&\quad + \frac{(bc-ad)gi(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b^2} \\
&\quad + \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} \\
&\quad - \frac{B(bc-ad)^3 gin\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3b^2 d^2} \\
&\quad - \frac{B^2(bc-ad)^3 gin^2\log(c+dx)}{3b^2 d^2} - \frac{B^2(bc-ad)^3 gin^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^2 d^2}
\end{aligned}$$

output

```
1/3*B^2*(-a*d+b*c)^2*g*i*n^2*x/b/d-1/3*B*(-a*d+b*c)^2*g*i*n*(b*x+a)*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))/b^2/d-1/3*B*(-a*d+b*c)*g*i*n*(b*x+a)^2*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))/b^2+1/6*(-a*d+b*c)*g*i*(b*x+a)^2*(A+B*ln(e*((b*x+a)
/(d*x+c))^n))^2/b^2+1/3*g*i*(b*x+a)^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))^2/b-1/3*B*(-a*d+b*c)^3*g*i*n*(A+B*n+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-
a*d+b*c)/b/(d*x+c))/b^2/d^2-1/3*B^2*(-a*d+b*c)^3*g*i*n^2*ln(d*x+c)/b^2/d^2
-1/3*B^2*(-a*d+b*c)^3*g*i*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 937 vs. $2(372) = 744$.

Time $= 0.84$ (sec) , antiderivative size $= 937$, normalized size of antiderivative $= 2.52$

$$\int (ag + bgx)(ci + dix)\left(A + B\log\left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input

```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])^2,x]
```

output

```
(g*i*(-6*A*b^2*B*c*d*(b*c - a*d)*n*x + 6*a*A*b*B*d^2*(-(b*c) + a*d)*n*x +
4*A*b*B*d*(b*c - a*d)*(b*c + a*d)*n*x - 6*b*B^2*c*d*(b*c - a*d)*n*(a + b*x
)*Log[e*((a + b*x)/(c + d*x))^n] + 6*a*B^2*d^2*(-(b*c) + a*d)*n*(a + b*x)*
Log[e*((a + b*x)/(c + d*x))^n] + 4*B^2*d*(b*c - a*d)*(b*c + a*d)*n*(a + b*
x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*b^2*B*d^2*(b*c - a*d)*n*x^2*(A + B*L
og[e*((a + b*x)/(c + d*x))^n]) + 6*a^2*b*B*c*d^2*n*Log[a + b*x]*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]) - 2*a^3*B*d^3*n*Log[a + b*x]*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]) + 6*a*b^2*c*d^2*x*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2 + 3*b^2*d^2*(b*c + a*d)*x^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2 + 2*b^3*d^3*x^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 6*b*B^2*c*(b
*c - a*d)^2*n^2*Log[c + d*x] + 6*a*B^2*d*(b*c - a*d)^2*n^2*Log[c + d*x] -
4*B^2*(b*c - a*d)^2*(b*c + a*d)*n^2*Log[c + d*x] + 2*b^3*B*c^3*n*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 6*a*b^2*B*c^2*d*n*(A + B*Log[
e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 2*B^2*(b*c - a*d)*n^2*(a^2*d^2*
Log[a + b*x] - b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 3*a^2*b*B^2*
c*d^2*n^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)])
- 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + a^3*B^2*d^3*n^2*(Log[a + b
*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)]) - b^3*B^2*c^3*n^2*((2*Log[(d*(a + b*x))/(-(b*c)
+ a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c ...
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.220$, Rules used = {2961, 2783, 2773, 49, 2009, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$

$$\downarrow 2961$$

$$gi(bc-ad)^3\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$gi(bc-ad)^3\left(-\frac{2Bn\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{3b(c+dx)^2\left(b-\frac{d}{}\right.}$$

$$\downarrow 2773$$

$$gi(bc-ad)^3\left(-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{\frac{a+bx}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2b}\right)}{3b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}\right.$$

$$\downarrow 49$$

$$gi(bc-ad)^3\left(-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{b}{d\left(\frac{d(a+bx)}{c+dx}-b\right)^2}+\frac{1}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{2b}\right)}{3b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}$$

$$\downarrow 2009$$

$$gi(bc-ad)^3\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^2}\right.\right.}{2b}\right.$$

$$\downarrow 2781$$

$$gi(bc-ad)^3\left(\frac{\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{3b}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn}{}\right.}{3b}$$

$$\downarrow 2784$$

$$gi(bc-ad)^3\left(\frac{\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{b}}{3b}-\frac{2Bn\left(\frac{(a+bx)^2\left(\right.\right.}{2b(c+}$$

$$\downarrow 2754$$

$$gi(bc-ad)^3\left(\frac{\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\right.}{d}}{b}\right)}{3b}$$

$$\downarrow 2838$$

$$gi(bc-ad)^3\left(-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^2}\right)}{2b}\right)}{3b}+\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right.$$

input

```
Int[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x
]
```

output

```
(b*c - a*d)^3*g*i*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/
(3*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*n*(((a + b*x)^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*b*(c + d*x)^2*(b - (d*(a + b*x))
/(c + d*x))^2) - (B*n*(b/(d^2*(b - (d*(a + b*x))/(c + d*x))) + Log[b - (d*
(a + b*x))/(c + d*x)]/d^2))/(2*b)))/(3*b) + (((a + b*x)^2*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])^2)/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2
) - (B*n*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*
(b - (d*(a + b*x))/(c + d*x))) - (-(((A + B*n + B*Log[e*((a + b*x)/(c + d*
x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (B*n*PolyLog[2, (d*(a +
 b*x))/(b*(c + d*x))])/d)/d))/b)/(3*b))
```

Defintions of rubi rules used

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2783

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)\,(dix+ci)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}dx$$

input

```
int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}dx$$
$$=\int (bgx+ag)(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}dx$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, al
gorithm="fricas")
```

output

```
integral(A^2*b*d*g*i*x^2 + A^2*a*c*g*i + (A^2*b*c + A^2*a*d)*g*i*x + (B^2*
b*d*g*i*x^2 + B^2*a*c*g*i + (B^2*b*c + B^2*a*d)*g*i*x)*log(e*((b*x + a)/(d
*x + c))^n)^2 + 2*(A*B*b*d*g*i*x^2 + A*B*a*c*g*i + (A*B*b*c + A*B*a*d)*g*i
*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F]

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= gi\left(\int A^2ac\,dx + \int A^2adx\,dx + \int A^2bcx\,dx + \int A^2bdx^2\,dx \right.$$
$$+ \int B^2ac\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+ \int 2ABac\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right) dx$$
$$+ \int B^2adx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+ \int B^2bcx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+ \int B^2bdx^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+ \int 2ABadx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right) dx$$
$$+ \int 2ABbcx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right) dx$$
$$\left. + \int 2ABbdx^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right) dx\right)$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
g*i*(Integral(A**2*a*c, x) + Integral(A**2*a*d*x, x) + Integral(A**2*b*c*x
, x) + Integral(A**2*b*d*x**2, x) + Integral(B**2*a*c*log(e*(a/(c + d*x) +
 b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*a*c*log(e*(a/(c + d*x) + b*x/(
c + d*x))**n), x) + Integral(B**2*a*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x)
)**n)**2, x) + Integral(B**2*b*c*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)
**2, x) + Integral(B**2*b*d*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**
2, x) + Integral(2*A*B*a*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) +
 Integral(2*A*B*b*c*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Integr
al(2*A*B*b*d*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1542 vs. $2(355) = 710$.

Time = 0.57 (sec) , antiderivative size = 1542, normalized size of antiderivative = 4.15

$$\int (ag + bgx)(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, al
gorithm="maxima")
```

output

```
2/3*A*B*b*d*g*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*b*d*g
*i*x^3 + A*B*b*c*g*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*B*a*d*
g*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*b*c*g*i*x^2 + 1/2
*A^2*a*d*g*i*x^2 + 1/3*A*B*b*d*g*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d
*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2
)) - A*B*b*c*g*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a
*d)*x/(b*d)) - A*B*a*d*g*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2
+ (b*c - a*d)*x/(b*d)) + 2*A*B*a*c*g*i*n*(a*log(b*x + a)/b - c*log(d*x + c
)/d) + 2*A*B*a*c*g*i*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*c*g*
i*x - 1/3*(a^2*c*d^2*g*i*n^2 - b^2*c^3*g*i*n*log(e) - (g*i*n^2 - 3*g*i*n*l
og(e))*a*b*c^2*d)*B^2*log(d*x + c)/(b*d^2) + 1/3*(b^3*c^3*g*i*n^2 - 3*a*b^
2*c^2*d*g*i*n^2 + 3*a^2*b*c*d^2*g*i*n^2 - a^3*d^3*g*i*n^2)*(log(b*x + a)*l
og((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2
/(b^2*d^2) + 1/6*(2*B^2*b^3*d^3*g*i*x^3*log(e)^2 - ((2*g*i*n*log(e) - 3*g*
i*log(e)^2)*b^3*c*d^2 - (2*g*i*n*log(e) + 3*g*i*log(e)^2)*a*b^2*d^3)*B^2*x
^2 - (3*a^2*b*c*d^2*g*i*n^2 - a^3*d^3*g*i*n^2)*B^2*log(b*x + a)^2 - 2*(b^3
*c^3*g*i*n^2 - 3*a*b^2*c^2*d*g*i*n^2)*B^2*log(b*x + a)*log(d*x + c) + (b^3
*c^3*g*i*n^2 - 3*a*b^2*c^2*d*g*i*n^2)*B^2*log(d*x + c)^2 + 2*((g*i*n^2 - g
*i*n*log(e))*b^3*c^2*d - (2*g*i*n^2 - 3*g*i*log(e)^2)*a*b^2*c*d^2 + (g*i*n
^2 + g*i*n*log(e))*a^2*b*d^3)*B^2*x - 2*(a*b^2*c^2*d*g*i*n^2 + a^3*d^3*...
```

Giac [F]

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (bgx+ag)(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, al
gorithm="giac")
```

output
```
integrate((b*g*x + a*g)*(d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) +
A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)\,(c\,i+d\,i\,x)\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x
)
```

output
```
int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,
x)
```

Reduce [F]

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input

```
int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
(g*i*( - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x
 + b*d*x**2),x)*a**3*b**2*d**4*n + 6*int((log(((a + b*x)**n*e)/(c + d*x)**
n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**3*c*d**3*n - 6*int((log(
((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b*
*4*c**2*d**2*n + 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x
 + b*c*x + b*d*x**2),x)*b**5*c**3*d*n - 2*log(a + b*x)*a**4*d**3*n + 6*log
(a + b*x)*a**3*b*c*d**2*n - 6*log(a + b*x)*a**2*b**2*c**2*d*n + 2*log(a +
b*x)*a*b**3*c**3*n + log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*c*d**
2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3*c**2*d + 6*log(((a + b*x)
**n*e)/(c + d*x)**n)**2*a*b**3*c*d**2*x + 3*log(((a + b*x)**n*e)/(c + d*x)
**n)**2*a*b**3*d**3*x**2 + 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*c*
d**2*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*d**3*x**3 + 6*log
(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*c**2*d + 2*log(((a + b*x)**n*e)/
(c + d*x)**n)*a**2*b**2*c*d**2*n + 12*log(((a + b*x)**n*e)/(c + d*x)**n)*a
**2*b**2*c*d**2*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*d**3*n*
x + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*d**3*x**2 - 2*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a*b**3*c**3 - 2*log(((a + b*x)**n*e)/(c + d*x)**
n)*a*b**3*c**2*d*n + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c*d**2*x*
*2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*d**3*n*x**2 + 4*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a*b**3*d**3*x**3 - 2*log(((a + b*x)**n*e)/(c ...
```

3.162 $\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result . . . 1828
Mathematica [A] (verified) . . . 1829
Rubi [A] (verified) . . . 1829
Maple [F] . . . 1833
Fricas [F] . . . 1833
Sympy [F] . . . 1834
Maxima [B] (verification not implemented) . . . 1834
Giac [F] . . . 1835
Mupad [F(-1)] . . . 1836
Reduce [F] . . . 1836

Optimal result

Integrand size = 33, antiderivative size = 220

$$\begin{aligned}
&\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= -\frac{B(bc-ad)in(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2} \\
&\quad + \frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d} + \frac{B^2(bc-ad)^2in^2\log(c+dx)}{b^2d} \\
&\quad + \frac{B(bc-ad)^2in\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d} \\
&\quad - \frac{B^2(bc-ad)^2in^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d}
\end{aligned}$$

output

```
-B*(-a*d+b*c)*i*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2+1/2*i*(d*x+c
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d+B^2*(-a*d+b*c)^2*i*n^2*ln(d*x+c)/b
^2/d+B*(-a*d+b*c)^2*i*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(
b*x+a))/b^2/d-B^2*(-a*d+b*c)^2*i*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.98

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{i\left((c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{B(bc-ad)n\left(B(bc-ad)n\log^2(a+bx)-2\left(Abdx+Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+B(-bc+ad\right.\right.}{}\right.}{2d}$$

input
```
Integrate[(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(i*((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d)*
n*(B*(b*c - a*d)*n*Log[a + b*x]^2 - 2*(A*b*d*x + B*d*(a + b*x)*Log[e*((a +
 b*x)/(c + d*x))^n] + B*(-(b*c) + a*d)*n*Log[c + d*x]) - 2*(b*c - a*d)*Log
[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(b*(c + d*x))/(b
*c - a*d)]) + 2*B*(-(b*c) + a*d)*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)
]))/b^2))/(2*d)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2951, 2756, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2\,dx$$

$$\Big\downarrow \text{2951}$$

$$i(bc-ad)^2\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2756}$$

$$i(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{d}\right)$$

$\downarrow$ 2789

$$i(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{d}\right)$$

$\downarrow$ 2751

$$i(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right.}}{b}\right.}{d}\right.$$

$\downarrow$ 16

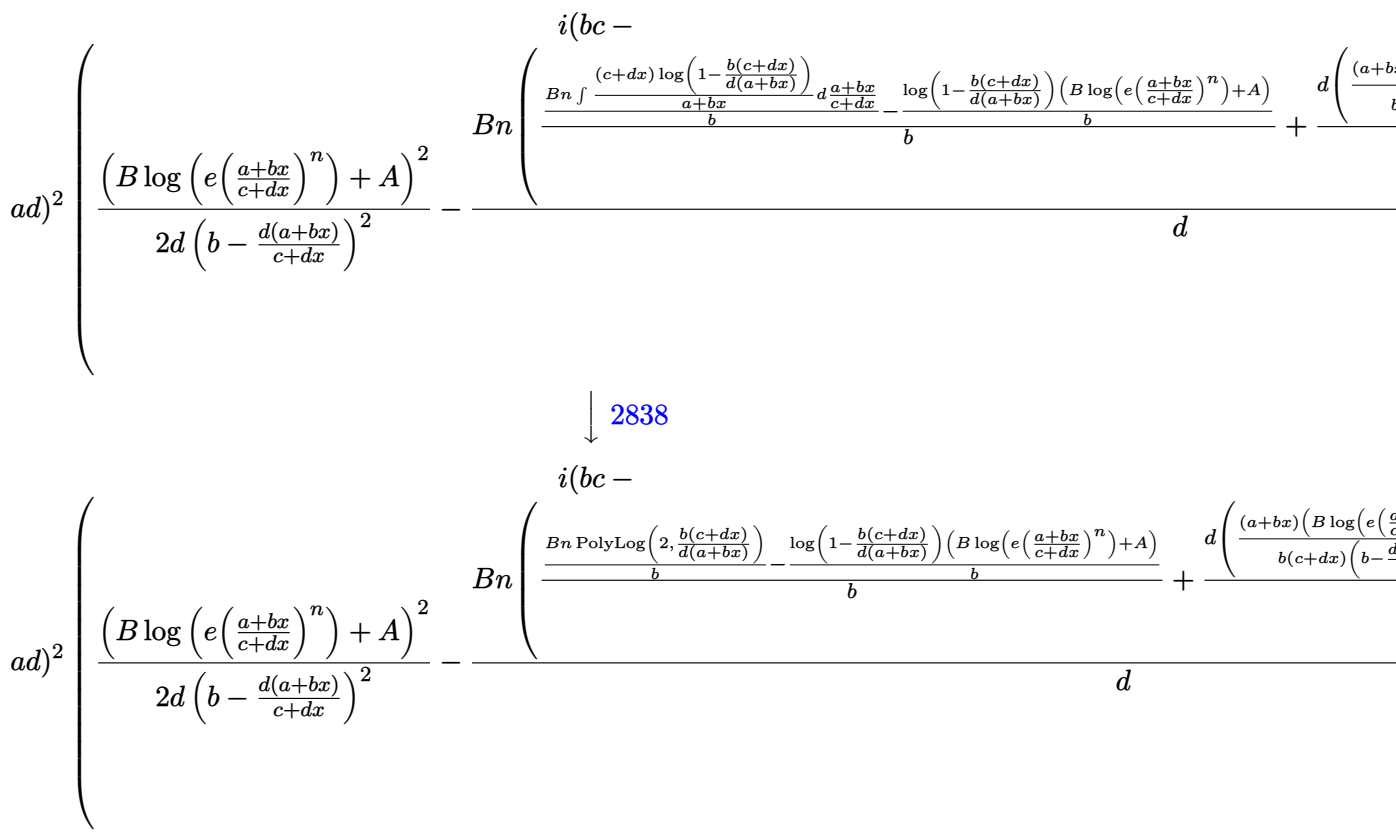
$$i(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(}{c}\right.}{bd}\right.}{b}\right.}{d}\right.$$

$\downarrow$ 2779

$$i(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+b\cdots}{b\cdots}\right.}{\cdots}\right.}{d}\right.$$

$\downarrow$ 2838

$$i(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\frac{Bn\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a\cdots}{c\cdots}\right.\right.\right.}{b(c+dx)\left(b-\frac{d\cdots}{\cdots}\right.}\right.}{\cdots}\right.}{d}\right.$$



input
```
Int[(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^2*i*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(2*d*(b - (d*(a
+ b*x))/(c + d*x))^2) - (B*n*((d*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*n*Log[b - (d*(
a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*(c + d*x)
)/(d*(a + b*x))])/b)/b))/d)
```

Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2951

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [F]

$$\int (dix + ci)\left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input

```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (dix + ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2 dx$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fri
cas")
```

output

```
integral(A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log(e*((b*x + a)/(d*x
 + c))^n)^2 + 2*(A*B*d*i*x + A*B*c*i)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F]

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= i\left(\int A^2 c\,dx + \int A^2 dx\,dx + \int B^2 c\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx\right.$$
$$+ \int 2ABc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right) dx$$
$$+ \int B^2 dx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$\left.+ \int 2ABdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right) dx\right)$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
i*(Integral(A**2*c, x) + Integral(A**2*d*x, x) + Integral(B**2*c*log(e*(a/
(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*c*log(e*(a/(c + d*x
) + b*x/(c + d*x))**n), x) + Integral(B**2*d*x*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)**2, x) + Integral(2*A*B*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x
))**n), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(217) = 434$.

Time = 0.54 (sec) , antiderivative size = 825, normalized size of antiderivative = 3.75

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="max
ima")
```

output

```
A*B*d*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*d*i*x^2 - A*B
*d*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))
 + 2*A*B*c*i*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c*i*x*log(e*(
b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c*i*x - (a*c*d*i*n^2 - (i*n^2 - i*n*
log(e))*b*c^2)*B^2*log(d*x + c)/(b*d) - (b^2*c^2*i*n^2 - 2*a*b*c*d*i*n^2 +
 a^2*d^2*i*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-
(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d) + 1/2*(2*B^2*b^2*c^2*i*n^2*log(b*x
 + a)*log(d*x + c) - B^2*b^2*c^2*i*n^2*log(d*x + c)^2 + B^2*b^2*d^2*i*x^2*
log(e)^2 - (2*a*b*c*d*i*n^2 - a^2*d^2*i*n^2)*B^2*log(b*x + a)^2 + 2*(a*b*d
^2*i*n*log(e) - (i*n*log(e) - i*log(e)^2)*b^2*c*d)*B^2*x - 2*((i*n^2 - 2*i
*n*log(e))*a*b*c*d - (i*n^2 - i*n*log(e))*a^2*d^2)*B^2*log(b*x + a) + (B^2
*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*i*x^
2 + 2*B^2*b^2*c*d*i*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*i*x^2*log(e) -
B^2*b^2*c^2*i*n*log(d*x + c) + (a*b*d^2*i*n - (i*n - 2*i*log(e))*b^2*c*d)*
B^2*x + (2*a*b*c*d*i*n - a^2*d^2*i*n)*B^2*log(b*x + a))*log((b*x + a)^n) -
 2*(B^2*b^2*d^2*i*x^2*log(e) - B^2*b^2*c^2*i*n*log(d*x + c) + (a*b*d^2*i*n
 - (i*n - 2*i*log(e))*b^2*c*d)*B^2*x + (2*a*b*c*d*i*n - a^2*d^2*i*n)*B^2*l
og(b*x + a) + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x)*log((b*x + a)^n))*lo
g((d*x + c)^n))/(b^2*d)
```

Giac [F]

$$\int (ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (dix + ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2 dx$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gia
c")
```

output

```
integrate((d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (c\,i+d\,i\,x)\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [F]

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(i*( - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x +
 b*d*x**2),x)*a**2*b**2*d**3*n + 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)
*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**3*c*d**2*n - 2*int((log(((a +
 b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**4*c**2
*d*n - 2*log(c + d*x)*a**3*d**2*n + 4*log(c + d*x)*a**2*b*c*d*n + 2*log(c
 + d*x)*a**2*b*d**2*n**2 - 2*log(c + d*x)*a*b**2*c**2*n - 4*log(c + d*x)*a*
b**2*c*d*n**2 + 2*log(c + d*x)*b**3*c**2*n**2 + log(((a + b*x)**n*e)/(c +
d*x)**n)**2*a*b**2*c*d + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*d*
x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d**2*x**2 - 2*log(((a + b*x
)**n*e)/(c + d*x)**n)*a**3*d**2 + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**
2*b*c*d + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d**2*n - 2*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a*b**2*c*d*n + 4*log(((a + b*x)**n*e)/(c + d*x)*
*n)*a*b**2*c*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**2*n*x +
2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**2*x**2 - 2*log(((a + b*x)**
n*e)/(c + d*x)**n)*b**3*c*d*n*x + 2*a**2*b*c*d*x + 2*a**2*b*d**2*n*x + a**
2*b*d**2*x**2 - 2*a*b**2*c*d*n*x))/(2*b*d)
```

3.163 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

Optimal result . . . 1837
Mathematica [B] (verified) . . . 1838
Rubi [A] (verified) . . . 1839
Maple [F] . . . 1844
Fricas [F] . . . 1844
Sympy [F] . . . 1844
Maxima [F] . . . 1845
Giac [F] . . . 1846
Mupad [F(-1)] . . . 1846
Reduce [F] . . . 1846

Optimal result

Integrand size = 43, antiderivative size = 306

$$
\begin{aligned}
&\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx \\
&= \frac{di(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2g} \\
&\quad + \frac{2B(bc-ad)in\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^2g} \\
&\quad - \frac{(bc-ad)i\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g} \\
&\quad + \frac{2B^2(bc-ad)in^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2g} \\
&\quad + \frac{2B(bc-ad)in\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g} \\
&\quad + \frac{2B^2(bc-ad)in^2\,\mathrm{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}
\end{aligned}
$$

output

```
d*i*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2/g+2*B*(-a*d+b*c)*i*n*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^2/g-(-a*d+b*c)*i*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^2/g+2*B^2*(-a
*d+b*c)*i*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/g+2*B*(-a*d+b*c)*i*n*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/g+2*B^2*(-a
*d+b*c)*i*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^2/g
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1372 vs. $2(306) = 612$.

Time $= 1.11$ (sec) , antiderivative size $= 1372$, normalized size of antiderivative $= 4.48$

$$\int \frac{(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag + bgx}\,dx = \text{Too large to display}$$

input

```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
b*g*x),x]
```

output

```
(i*(3*b*d*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c +
 d*x)])^2 + 3*(b*c - a*d)*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^
n] - B*n*Log[(a + b*x)/(c + d*x)])^2 - 3*B*n*(A + B*Log[e*((a + b*x)/(c +
d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(a*d*Log[a/b + x]^2 - 2*a*d*Log[a
/b + x]*(1 + Log[a + b*x]) + 2*(-(b*c) + a*d + Log[c/d + x]*(b*c + a*d*Log
[a + b*x] - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)]) + (-(b*d*x) + a*d*Log[a
 + b*x])*Log[(a + b*x)/(c + d*x)]) - 2*a*d*PolyLog[2, (b*(c + d*x))/(b*c -
 a*d)]) + 3*b*B*c*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b
*x)/(c + d*x)])*(Log[a/b + x]^2 - 2*Log[a + b*x]*(Log[a/b + x] - Log[c/d +
 x] - Log[(a + b*x)/(c + d*x)]) - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c
) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - B^2*n^2*(a*d*Log[a/b
 + x]^3 - 3*d*(2*b*x - 2*(a + b*x)*Log[a/b + x] + (a + b*x)*Log[a/b + x]^2
) - 3*b*(2*d*x - 2*(c + d*x)*Log[c/d + x] + (c + d*x)*Log[c/d + x]^2) - 3*
d*(b*x - a*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c
+ d*x)])^2 + 6*(a*d + 2*b*d*x - b*d*x*Log[c/d + x] - b*c*Log[c + d*x] + Lo
g[a/b + x]*(-(d*(a + b*x)) + d*(a + b*x)*Log[c/d + x] + (b*c - a*d)*Log[(b
*(c + d*x))/(b*c - a*d)]) + (b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) +
 a*d)]) - 3*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*(-2*b
*c + 2*a*d - 2*d*(a + b*x)*Log[a/b + x] + a*d*Log[a/b + x]^2 + 2*Log[c/d +
 x]*(b*(c + d*x) - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)]) - 2*a*d*PolyL...
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.186$, Rules used = {2961, 2789, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{ag+bgx}\,dx$$

$$\downarrow 2961$$

$$\frac{i(bc-ad)\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g}$$

$$\downarrow 2789$$

$$\frac{i(bc-ad)\left(\frac{d\int\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\downarrow 2755$$

$$\frac{i(bc-ad)\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2Bn\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\downarrow 2754$$

$$\frac{i(bc-ad)\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2Bn\left(\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1\right.}{b}\right.}{b}\right.}{b}\right.}{g}$$

$$\downarrow 2779$$

$$i(bc-ad)\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2Bn\left(\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}\right)}{b}\right)}{b}+\frac{2}{-}\right)$$

$$g$$

$$\downarrow 2821$$

$$i(bc-ad)\left(\frac{\frac{2Bn\left(\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int\frac{(c+dx)\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{b}}{b}\right)$$

$$g$$

$$\downarrow 2838$$

$$i(bc-ad)\left(\frac{\frac{2Bn\left(\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int\frac{(c+dx)\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{b}}{b}\right)$$

$$g$$

$$\downarrow 7143$$

$$i(bc-ad)\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2Bn\left(-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}-\frac{Bn\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d}\right)}{b}\right)}{b}+\frac{2Bn\left(\mathrm{PolyLo}\right.}{g}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x)
,x]
```

output
```
((b*c - a*d)*i*((d*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(
b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (2*B*n*(-(((A + B*Log[e*((a +
 b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (B*n*PolyL
og[2, (d*(a + b*x))/(b*(c + d*x))])/d))/b))/b + (-(((A + B*Log[e*((a + b*x
)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (2*B*n*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))]
 + B*n*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/b)/b))/g
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2779

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))  Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d  Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d  Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)  Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(dix+ci)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{bgx+ag}dx$$

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

output
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, al
gorithm="fricas")
```

output
```
integral((A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log(e*((b*x + a)/(d*
x + c))^n)^2 + 2*(A*B*d*i*x + A*B*c*i)*log(e*((b*x + a)/(d*x + c))^n))/(b*
g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}dx$$
$$= \frac{i\left(\int \frac{A^2c}{a+bx}\,dx + \int \frac{A^2dx}{a+bx}\,dx + \int \frac{B^2c\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a+bx}\,dx + \int \frac{2ABc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx + \int \frac{B^2dx\log\left(e\left(\frac{a}{c+dx}\right.\right.\right.}{a+bx}$$
$$g$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g),x)
```

output
```
i*(Integral(A**2*c/(a + b*x), x) + Integral(A**2*d*x/(a + b*x), x) + Integ
ral(B**2*c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a + b*x), x) + Inte
gral(2*A*B*c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x) + Integ
ral(B**2*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a + b*x), x) + In
tegral(2*A*B*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, al
gorithm="maxima")
```

output
```
A^2*d*i*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + A^2*c*i*log(b*g*x + a*g)/(b*g
) + (B^2*b*d*i*x + (b*c*i - a*d*i)*B^2*log(b*x + a))*log((d*x + c)^n)^2/(b
^2*g) - integrate(-(B^2*b^2*c^2*i*log(e)^2 + 2*A*B*b^2*c^2*i*log(e) + (B^2
*b^2*d^2*i*log(e)^2 + 2*A*B*b^2*d^2*i*log(e))*x^2 + (B^2*b^2*d^2*i*x^2 + 2
*B^2*b^2*c*d*i*x + B^2*b^2*c^2*i)*log((b*x + a)^n)^2 + 2*(B^2*b^2*c*d*i*lo
g(e)^2 + 2*A*B*b^2*c*d*i*log(e))*x + 2*(B^2*b^2*c^2*i*log(e) + A*B*b^2*c^2
*i + (B^2*b^2*d^2*i*log(e) + A*B*b^2*d^2*i)*x^2 + 2*(B^2*b^2*c*d*i*log(e)
+ A*B*b^2*c*d*i)*x)*log((b*x + a)^n) - 2*(B^2*b^2*c^2*i*log(e) + A*B*b^2*c
^2*i + ((i*n + i*log(e))*B^2*b^2*d^2 + A*B*b^2*d^2*i)*x^2 + (2*A*B*b^2*c*d
*i + (a*b*d^2*i*n + 2*b^2*c*d*i*log(e))*B^2)*x + ((b^2*c*d*i*n - a*b*d^2*i
*n)*B^2*x + (a*b*c*d*i*n - a^2*d^2*i*n)*B^2)*log(b*x + a) + (B^2*b^2*d^2*i
*x^2 + 2*B^2*b^2*c*d*i*x + B^2*b^2*c^2*i)*log((b*x + a)^n))*log((d*x + c)^
n))/(b^3*d*g*x^2 + a*b^2*c*g + (b^3*c*g + a*b^2*d*g)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, al
gorithm="giac")
```

output
```
integrate((d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x +
a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)
,x)
```

output
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)
, x)
```

Reduce [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{i\left(\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{bx+a}dx\right) b^4c + 2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{bx+a}dx\right) a\,b^3c + \left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 x}{bx+a}dx\right) b^4d + 2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x}{bx+a}\right.\right.}{b^2g}$$

input
```
int((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

output
```
(i*(int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(a + b*x),x)*b**4*c + 2*int(
log(((a + b*x)**n*e)/(c + d*x)**n)/(a + b*x),x)*a*b**3*c + int((log(((a +
b*x)**n*e)/(c + d*x)**n)**2*x)/(a + b*x),x)*b**4*d + 2*int((log(((a + b*x)
**n*e)/(c + d*x)**n)*x)/(a + b*x),x)*a*b**3*d - log(a + b*x)*a**3*d + log(
a + b*x)*a**2*b*c + a**2*b*d*x))/(b**2*g)
```

3.164 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result . . . 1848
Mathematica [B] (verified) . . . 1849
Rubi [A] (verified) . . . 1850
Maple [F] . . . 1853
Fricas [F] . . . 1853
Sympy [F(-1)] . . . 1854
Maxima [F] . . . 1854
Giac [F] . . . 1855
Mupad [F(-1)] . . . 1856
Reduce [F] . . . 1856

Optimal result

Integrand size = 43, antiderivative size = 261

$$\begin{aligned}
&\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx \\
&= -\frac{2B^2in^2(c+dx)}{bg^2(a+bx)} - \frac{2Bin(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)} \\
&\quad - \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} \\
&\quad + \frac{2Bdin\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} \\
&\quad + \frac{2B^2din^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}
\end{aligned}$$

output

```
-2*B^2*i*n^2*(d*x+c)/b/g^2/(b*x+a)-2*B*i*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x
+c))^n))/b/g^2/(b*x+a)-i*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b/g^2/(
b*x+a)-d*i*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^2
/g^2+2*B*d*i*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+
a))/b^2/g^2+2*B^2*d*i*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^2/g^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1564 vs. $2(261) = 522$.

Time = 2.00 (sec) , antiderivative size = 1564, normalized size of antiderivative = 5.99

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

input

```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
b*g*x)^2,x]
```

output

```
(i*((-3*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b
*x)/(c + d*x)])^2)/(a + b*x) + 3*d*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 + (6*b*B*c*n*(-A - B*Log[e*(
(a + b*x)/(c + d*x))^n] + B*n*Log[(a + b*x)/(c + d*x)])*(-(d*(a + b*x)*Log
[c/d + x]) + d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) + a*d)] + (b*c - a*d)*(
1 + Log[(a + b*x)/(c + d*x)])))/((b*c - a*d)*(a + b*x)) + (3*b*B^2*c*n^2*(
-2*b*c + 2*a*d - 2*d*(a + b*x)*Log[a + b*x] - 2*(b*c - a*d)*Log[(a + b*x)/
(c + d*x)] - 2*d*(a + b*x)*Log[a + b*x]*Log[(a + b*x)/(c + d*x)] - (b*c -
a*d)*Log[(a + b*x)/(c + d*x)]^2 + 2*d*(a + b*x)*Log[c + d*x] - 2*d*(a + b*
x)*Log[(a + b*x)/(c + d*x)]*Log[(b*c - a*d)/(b*c + b*d*x)] + d*(a + b*x)*(
Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog
[2, (d*(a + b*x))/(-(b*c) + a*d)]) + d*(a + b*x)*(Log[(b*c - a*d)/(b*c + b
*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c + b*d*x
)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/((b*c - a*d)*(a + b*x)) +
 3*B*d*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*
x)])*(Log[a/b + x]^2 - 2*Log[a/b + x]*Log[a + b*x] - 2*Log[c/d + x]*Log[(d
*(a + b*x))/(-(b*c) + a*d)] + 2*Log[a + b*x]*((a*d)/(b*c - a*d) + Log[c/d
+ x] + Log[(a + b*x)/(c + d*x)]) + 2*a*((a + b*x)^(-1) + Log[(a + b*x)/(c
+ d*x)]/(a + b*x) + (d*Log[c + d*x])/(-(b*c) + a*d)) - 2*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)]) + (B^2*d*n^2*((b*c - a*d)*(a + b*x)*Log[a/b + x]^...
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {2961, 2780, 2742, 2741, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^2}\,dx$$

$$\downarrow 2961$$

$$\frac{i\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g^2}$$

$$\downarrow 2780$$

$$\frac{i\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^2}$$

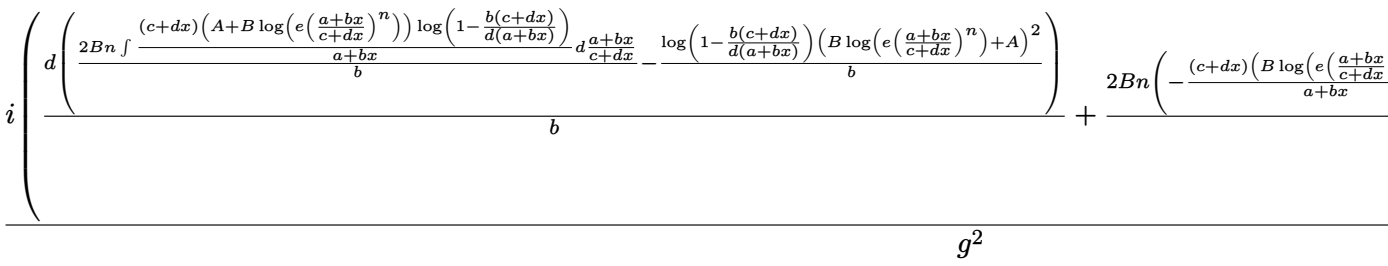
$$\downarrow 2742$$

$$\frac{i\left(\frac{2Bn\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^2}$$

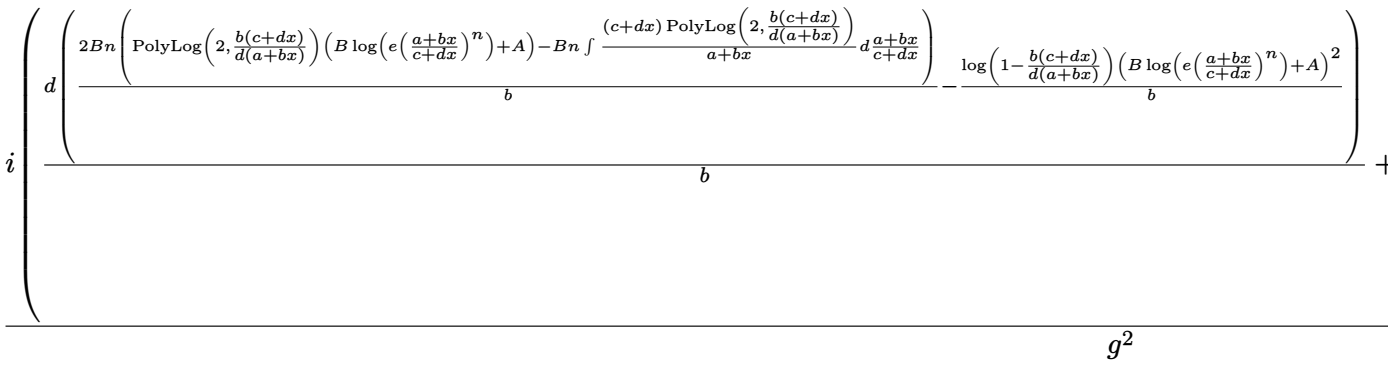
$$\downarrow 2741$$

$$\frac{i\left(\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{2Bn\left(-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{b}\right)}{g^2}$$

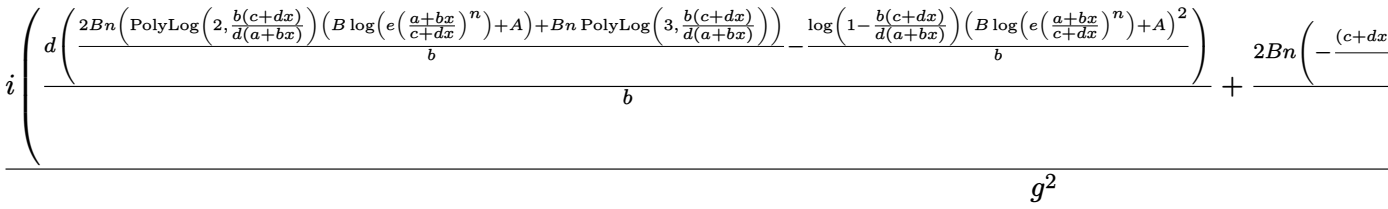
$$\downarrow 2779$$



↓ 2821



↓ 7143



input
```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x)
^2,x]
```

output
```
(i*((-(((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)) + 2
*B*n*(-((B*n*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/(a + b*x)))/b + (d*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (2*B*n*((A + B*Log[e*((a +
b*x)/(c + d*x))^n])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + B*n*PolyLog[
3, (b*(c + d*x))/(d*(a + b*x))]))/b))/b))/g^2
```

Defintions of rubi rules used

rule 2741

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2779

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2821

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(dix+ci)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(bgx+ag)^2}dx$$

input

```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

output

```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}dx$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x,
algorithm="fricas")
```

output

```
integral((A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log(e*((b*x + a)/(d*
x + c))^n)^2 + 2*(A*B*d*i*x + A*B*c*i)*log(e*((b*x + a)/(d*x + c))^n))/(b^
2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2,x
)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x,
algorithm="maxima")
```

output

```
-2*A*B*c*i*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^
2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) + A^2*d*i*(a/(b^3*g^2*x + a*b^2
*g^2) + log(b*x + a)/(b^2*g^2)) - 2*A*B*c*i*log(e*(b*x/(d*x + c) + a/(d*x
+ c))^n)/(b^2*g^2*x + a*b*g^2) - A^2*c*i/(b^2*g^2*x + a*b*g^2) - ((b*c*i -
 a*d*i)*B^2 - (B^2*b*d*i*x + B^2*a*d*i)*log(b*x + a))*log((d*x + c)^n)^2/(
b^3*g^2*x + a*b^2*g^2) - integrate(-(B^2*b^2*c^2*i*log(e)^2 + (B^2*b^2*d^2
*i*log(e)^2 + 2*A*B*b^2*d^2*i*log(e))*x^2 + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2
*c*d*i*x + B^2*b^2*c^2*i)*log((b*x + a)^n)^2 + 2*(B^2*b^2*c*d*i*log(e)^2 +
 A*B*b^2*c*d*i*log(e))*x + 2*(B^2*b^2*c^2*i*log(e) + (B^2*b^2*d^2*i*log(e)
 + A*B*b^2*d^2*i)*x^2 + (2*B^2*b^2*c*d*i*log(e) + A*B*b^2*c*d*i)*x)*log((b
*x + a)^n) + 2*((a*b*c*d*i*n - a^2*d^2*i*n - b^2*c^2*i*log(e))*B^2 - (B^2*
b^2*d^2*i*log(e) + A*B*b^2*d^2*i)*x^2 - (A*B*b^2*c*d*i + (a*b*d^2*i*n - (i
*n - 2*i*log(e))*b^2*c*d)*B^2)*x - (B^2*b^2*d^2*i*n*x^2 + 2*B^2*a*b*d^2*i*
n*x + B^2*a^2*d^2*i*n)*log(b*x + a) - (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i
*x + B^2*b^2*c^2*i)*log((b*x + a)^n))*log((d*x + c)^n))/(b^4*d*g^2*x^3 + a
^2*b^2*c*g^2 + (b^4*c*g^2 + 2*a*b^3*d*g^2)*x^2 + (2*a*b^3*c*g^2 + a^2*b^2*
d*g^2)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x,
algorithm="giac")
```

output

```
integrate((d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x +
a*g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(a\,g+b\,g\,x)^2}\,dx$$

input
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)
^2,x)
```

output
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)
^2, x)
```

Reduce [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

input
```
int((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

output

```
(i*(int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a**2 + 2*a*b*x + b**2*x
**2),x)*a**3*b**4*d**2 - int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a*
*2 + 2*a*b*x + b**2*x**2),x)*a**2*b**5*c*d + int((log(((a + b*x)**n*e)/(c
 + d*x)**n)**2*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**5*d**2*x - int((l
og(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a*
b**6*c*d*x + 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**2 + 2*a*b*x
 + b**2*x**2),x)*a**4*b**3*d**2 - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)
*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**3*b**4*c*d + 2*int((log(((a + b*x)*
*n*e)/(c + d*x)**n)*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**3*b**4*d**2*x -
2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**2 + 2*a*b*x + b**2*x**2),
x)*a**2*b**5*c*d*x + log(a + b*x)*a**5*d**2 - log(a + b*x)*a**4*b*c*d + lo
g(a + b*x)*a**4*b*d**2*x - log(a + b*x)*a**3*b**2*c*d*x + 2*log(a + b*x)*a
**2*b**3*c**2*n + 2*log(a + b*x)*a*b**4*c**2*n**2 + 2*log(a + b*x)*a*b**4*
c**2*n*x + 2*log(a + b*x)*b**5*c**2*n**2*x - 2*log(c + d*x)*a**2*b**3*c**2
*n - 2*log(c + d*x)*a*b**4*c**2*n**2 - 2*log(c + d*x)*a*b**4*c**2*n*x - 2*
log(c + d*x)*b**5*c**2*n**2*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b*
*4*c**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**4*c*d*x + 2*log(((a +
 b*x)**n*e)/(c + d*x)**n)*a**2*b**3*c*d*x - 2*log(((a + b*x)**n*e)/(c + d*
x)**n)*a*b**4*c**2*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**4*c*d*n*x
 - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**5*c**2*n*x - a**4*b*d**2*x +...
```

3.165 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result . . . 1858
Mathematica [C] (verified) . . . 1859
Rubi [A] (verified) . . . 1860
Maple [B] (verified) . . . 1861
Fricas [B] (verification not implemented) . . . 1862
Sympy [F] . . . 1863
Maxima [B] (verification not implemented) . . . 1864
Giac [A] (verification not implemented) . . . 1865
Mupad [B] (verification not implemented) . . . 1865
Reduce [B] (verification not implemented) . . . 1866

Optimal result

Integrand size = 43, antiderivative size = 151

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{B^2in^2(c+dx)^2}{4(bc-ad)g^3(a+bx)^2} - \frac{Bin(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)g^3(a+bx)^2} - \frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)g^3(a+bx)^2}$$

output

```
-1/4*B^2*i*n^2*(d*x+c)^2/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*B*i*n*(d*x+c)^2*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*i*(d*x+c)^2*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/g^3/(b*x+a)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.94 (sec) , antiderivative size = 801, normalized size of antiderivative = 5.30

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{i\Big(2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-4d(-bc+ad)(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+4Bdn(a+$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x)^3,x]
```

output
```
-1/4*(i*(2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 4*B*d*n*(a + b*x)*(2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*d*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 2*B*n*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*d*n*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*n*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + B*n*(2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 4*B*d*n*(a + b*x)*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*n*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 2*B*d^2*n*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*n*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b^2*(b*c - a*d)*g^3*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{i\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3(bc-ad)}$$

$$\downarrow 2742$$

$$\frac{i\left(Bn\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx} - \frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}\right)}{g^3(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{i\left(Bn\left(-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2} - \frac{Bn(c+dx)^2}{4(a+bx)^2}\right) - \frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}\right)}{g^3(bc-ad)}$$

input

```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x)
^3,x]
```

output

```
(i*(-1/2*((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)^
2 + B*n*(-1/4*(B*n*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2))))/((b*c - a*d)*g^3)
```

Defintions of rubi rules used

rule 2741

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(145) = 290.

Time = 4.81 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.51

method	result
parallelrisch	$-\frac{2AB\,a^2b^2d^3in^2-2AB\,b^4c^2din^2+2B^2xab^3d^3in^3-2B^2xb^4cd^2in^3-2B^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2b^4c^2din-2B^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b$

input

```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,method=_
RETURNVERBOSE)
```

output

```
-1/4*(2*A*B*a^2*b^2*d^3*i*n^2-2*A*B*b^4*c^2*d*i*n^2+2*B^2*x*a*b^3*d^3*i*n^
3-2*B^2*x*b^4*c*d^2*i*n^3-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*c^2*d*i*n-
2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d*i*n^2-8*A*B*x*ln(e*((b*x+a)/(d*x
+c))^n)*b^4*c*d^2*i*n-4*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^3*i*n-4*B^
2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*c*d^2*i*n-4*B^2*x*ln(e*((b*x+a)/(d*x+c
))^n)*b^4*c*d^2*i*n^2+4*A*B*x*a*b^3*d^3*i*n^2-4*A*B*x*b^4*c*d^2*i*n^2-4*A*
B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d*i*n+B^2*a^2*b^2*d^3*i*n^3-B^2*b^4*c^
2*d*i*n^3+2*A^2*a^2*b^2*d^3*i*n-2*A^2*b^4*c^2*d*i*n+4*A^2*x*a*b^3*d^3*i*n-
4*A^2*x*b^4*c*d^2*i*n-2*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*d^3*i*n-2*
B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^3*i*n^2)/g^3/(b*x+a)^2/b^4/d/n/(a*
d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(145) = 290$.

Time $= 0.10$ (sec) , antiderivative size $= 600$, normalized size of antiderivative $= 3.97$

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{(B^2b^2c^2-B^2a^2d^2)in^2+2\left(ABb^2c^2-ABa^2d^2\right)in+2\left(2\left(B^2b^2cd-B^2abd^2\right)ix+\left(B^2b^2c^2-B^2a^2d^2\right)i\right)$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,
algorithm="fricas")
```

output

```
-1/4*((B^2*b^2*c^2 - B^2*a^2*d^2)*i*n^2 + 2*(A*B*b^2*c^2 - A*B*a^2*d^2)*i*
n + 2*(2*(B^2*b^2*c*d - B^2*a*b*d^2)*i*x + (B^2*b^2*c^2 - B^2*a^2*d^2)*i)*
log(e)^2 + 2*(B^2*b^2*d^2*i*n^2*x^2 + 2*B^2*b^2*c*d*i*n^2*x + B^2*b^2*c^2*
i*n^2)*log((b*x + a)/(d*x + c))^2 + 2*(A^2*b^2*c^2 - A^2*a^2*d^2)*i + 2*((
B^2*b^2*c*d - B^2*a*b*d^2)*i*n^2 + 2*(A*B*b^2*c*d - A*B*a*b*d^2)*i*n + 2*(
A^2*b^2*c*d - A^2*a*b*d^2)*i)*x + 2*((B^2*b^2*c^2 - B^2*a^2*d^2)*i*n + 2*(
A*B*b^2*c^2 - A*B*a^2*d^2)*i + 2*((B^2*b^2*c*d - B^2*a*b*d^2)*i*n + 2*(A*B
*b^2*c*d - A*B*a*b*d^2)*i)*x + 2*(B^2*b^2*d^2*i*n*x^2 + 2*B^2*b^2*c*d*i*n*
x + B^2*b^2*c^2*i*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*(B^2*b^2*c^2*i*n
^2 + 2*A*B*b^2*c^2*i*n + (B^2*b^2*d^2*i*n^2 + 2*A*B*b^2*d^2*i*n)*x^2 + 2*(
B^2*b^2*c*d*i*n^2 + 2*A*B*b^2*c*d*i*n)*x)*log((b*x + a)/(d*x + c)))/((b^5*
c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^
2*d)*g^3)
```

Sympy [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$

$$= \frac{i\left(\int \frac{A^2c}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{A^2dx}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{B^2c\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{2ABc\log\left(e\left(\frac{a}{c+dx}+\right.\right.\right.}{a^3+3a^2bx+3ab^2x}}{g^3}$$

input

```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3,x
)
```

output

```
i*(Integral(A**2*c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + I
ntegral(A**2*d*x/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Int
egral(B**2*c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a**3 + 3*a**2*b*x
 + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*c*log(e*(a/(c + d*x) +
b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + I
ntegral(B**2*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a**3 + 3*a**2
*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*d*x*log(e*(a/(c + d
*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),
x))/g**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2017 vs. $2(145) = 290$.

Time $= 0.14$ (sec) , antiderivative size $= 2017$, normalized size of antiderivative $= 13.36$

$$\int \frac{(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^3}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,
algorithm="maxima")
```

output

```
-1/2*A*B*d*i*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c - a*b^4*
d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3)
+ 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*
g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*
d^2)*g^3)) + 1/2*A*B*c*i*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3
*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*
log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c
)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*(2*b*x + a)*B^2*d*i*log
(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b
^2*g^3) + 1/4*(2*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2
*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x
+ a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*
c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
) - (b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*
d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)
^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log
(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 +
2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c))*n^2/(a^2*b^3*c^2*g^3 -
2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b
^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^...
```

Giac [A] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.29

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx =$$
$$-\frac{1}{4}\left(\frac{2\,(dx+c)^2B^2in^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)^2g^3}+\frac{2\,(B^2in^2+2\,B^2in\log(e)+2\,ABin)(dx+c)^2\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^2g^3}+\frac{(B^2in$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,
algorithm="giac")
```

output
```
-1/4*(2*(d*x + c)^2*B^2*i*n^2*log((b*x + a)/(d*x + c))^2/((b*x + a)^2*g^3)
 + 2*(B^2*i*n^2 + 2*B^2*i*n*log(e) + 2*A*B*i*n)*(d*x + c)^2*log((b*x + a)/
(d*x + c))/((b*x + a)^2*g^3) + (B^2*i*n^2 + 2*B^2*i*n*log(e) + 2*B^2*i*log
(e)^2 + 2*A*B*i*n + 4*A*B*i*log(e) + 2*A^2*i)*(d*x + c)^2/((b*x + a)^2*g^3
))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 27.52 (sec) , antiderivative size = 561, normalized size of antiderivative = 3.72

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= -\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^2\left(\frac{\frac{B^2ci}{2b}+\frac{B^2dix}{b}+\frac{B^2adi}{2b^2}}{a^2g^3+2abg^3x+b^2g^3x^2}-\frac{B^2d^2i}{2b^2g^3\,(ad-bc)}\right)$$
$$-\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(\frac{ABadi+ABbci-B^2adin+B^2bcin+2ABbdix}{a^2b^2g^3+2ab^3g^3x+b^4g^3x^2}\right.$$
$$\left.+\frac{B^2d^2i\left(\frac{ab^2g^3n(ad-bc)}{2d}+\frac{b^3g^3nx(ad-bc)}{d}+\frac{b^2g^3n(ad-bc)(2ad-bc)}{2d^2}\right)}{b^2g^3\,(ad-bc)\,(a^2b^2g^3+2ab^3g^3x+b^4g^3x^2)}\right)$$
$$-\frac{x\,(2bdiA^2+2bdiABn+bdiB^2n^2)+A^2adi+A^2bci+\frac{B^2adin^2}{2}+\frac{B^2bcin^2}{2}+ABadin+AB}{2a^2b^2g^3+4ab^3g^3x+2b^4g^3x^2}$$
$$-\frac{Bd^2in\,\mathrm{atan}\left(\frac{B\,d^2\,i\,n\,(2A+Bn)\left(\frac{cb^3g^3+adb^2g^3}{b^2g^3}+2bdx\right)1\mathrm{i}}{(ad-bc)\,(iB^2d^2n^2+2AiBd^2n)}\right)(2A+Bn)\,1\mathrm{i}}{b^2g^3\,(ad-bc)}$$

input
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)
^3,x)
```

output
```
- log(e*((a + b*x)/(c + d*x))^n)^2*(((B^2*c*i)/(2*b) + (B^2*d*i*x)/b + (B^
2*a*d*i)/(2*b^2))/(a^2*g^3 + b^2*g^3*x^2 + 2*a*b*g^3*x) - (B^2*d^2*i)/(2*b
^2*g^3*(a*d - b*c))) - log(e*((a + b*x)/(c + d*x))^n)*((A*B*a*d*i + A*B*b*
c*i - B^2*a*d*i*n + B^2*b*c*i*n + 2*A*B*b*d*i*x)/(a^2*b^2*g^3 + b^4*g^3*x^
2 + 2*a*b^3*g^3*x) + (B^2*d^2*i*((a*b^2*g^3*n*(a*d - b*c))/(2*d) + (b^3*g^
3*n*x*(a*d - b*c))/d + (b^2*g^3*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2)))/(b^
2*g^3*(a*d - b*c)*(a^2*b^2*g^3 + b^4*g^3*x^2 + 2*a*b^3*g^3*x))) - (x*(2*A^
2*b*d*i + B^2*b*d*i*n^2 + 2*A*B*b*d*i*n) + A^2*a*d*i + A^2*b*c*i + (B^2*a*
d*i*n^2)/2 + (B^2*b*c*i*n^2)/2 + A*B*a*d*i*n + A*B*b*c*i*n)/(2*a^2*b^2*g^3
 + 2*b^4*g^3*x^2 + 4*a*b^3*g^3*x) - (B*d^2*i*n*atan((B*d^2*i*n*(2*A + B*n)
*((b^3*c*g^3 + a*b^2*d*g^3)/(b^2*g^3) + 2*b*d*x)*1i)/((a*d - b*c)*(B^2*d^2
*i*n^2 + 2*A*B*d^2*i*n)))*(2*A + B*n)*1i)/(b^2*g^3*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 750, normalized size of antiderivative = 4.97

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
int((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

output

```
(i*(4*log(a + b*x)*a**3*b*c*d*n + 2*log(a + b*x)*a**2*b**2*c*d*n**2 + 8*lo
g(a + b*x)*a**2*b**2*c*d*n*x + 4*log(a + b*x)*a*b**3*c*d*n**2*x + 4*log(a
+ b*x)*a*b**3*c*d*n*x**2 + 2*log(a + b*x)*b**4*c*d*n**2*x**2 - 4*log(c + d
*x)*a**3*b*c*d*n - 2*log(c + d*x)*a**2*b**2*c*d*n**2 - 8*log(c + d*x)*a**2
*b**2*c*d*n*x - 4*log(c + d*x)*a*b**3*c*d*n**2*x - 4*log(c + d*x)*a*b**3*c
*d*n*x**2 - 2*log(c + d*x)*b**4*c*d*n**2*x**2 + 2*log(((a + b*x)**n*e)/(c
+ d*x)**n)**2*a*b**3*c**2 + 4*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3
*c*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3*d**2*x**2 - 4*log(
((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*c*d + 4*log(((a + b*x)**n*e)/(c + d*
x)**n)*a**2*b**2*c**2 - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*c*d
*n + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*d**2*x**2 + 2*log(((a
+ b*x)**n*e)/(c + d*x)**n)*a*b**3*c**2*n - 4*log(((a + b*x)**n*e)/(c + d*x
)**n)*a*b**3*c*d*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*d**2*n
*x**2 - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*c*d*n*x**2 - 2*a**4*c*d
+ 2*a**3*b*c**2 - 2*a**3*b*c*d*n + 2*a**3*b*d**2*x**2 + 2*a**2*b**2*c**2*n
 - a**2*b**2*c*d*n**2 - 2*a**2*b**2*c*d*x**2 + 2*a**2*b**2*d**2*n*x**2 + a
*b**3*c**2*n**2 - 2*a*b**3*c*d*n*x**2 + a*b**3*d**2*n**2*x**2 - b**4*c*d*n
**2*x**2))/(4*a*b*g**3*(a**3*d - a**2*b*c + 2*a**2*b*d*x - 2*a*b**2*c*x +
a*b**2*d*x**2 - b**3*c*x**2))
```

3.166 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result . . . 1868
Mathematica [C] (verified) . . . 1869
Rubi [A] (verified) . . . 1870
Maple [B] (verified) . . . 1872
Fricas [B] (verification not implemented) . . . 1873
Sympy [F] . . . 1874
Maxima [B] (verification not implemented) . . . 1875
Giac [A] (verification not implemented) . . . 1876
Mupad [B] (verification not implemented) . . . 1877
Reduce [B] (verification not implemented) . . . 1878

Optimal result

Integrand size = 43, antiderivative size = 307

$$\begin{aligned}\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx &= \frac{B^2din^2(c+dx)^2}{4(bc-ad)^2g^4(a+bx)^2}\\ &\quad-\frac{2bB^2in^2(c+dx)^3}{27(bc-ad)^2g^4(a+bx)^3}\\ &\quad+\frac{Bdin(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^4(a+bx)^2}\\ &\quad-\frac{2bBin(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^2g^4(a+bx)^3}\\ &\quad+\frac{di(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2g^4(a+bx)^2}\\ &\quad-\frac{bi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^2g^4(a+bx)^3}\end{aligned}$$

output

```
1/4*B^2*d*i*n^2*(d*x+c)^2/(-a*d+b*c)^2/g^4/(b*x+a)^2-2/27*b*B^2*i*n^2*(d*x
+c)^3/(-a*d+b*c)^2/g^4/(b*x+a)^3+1/2*B*d*i*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))/(-a*d+b*c)^2/g^4/(b*x+a)^2-2/9*b*B*i*n*(d*x+c)^3*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/g^4/(b*x+a)^3+1/2*d*i*(d*x+c)^2*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^4/(b*x+a)^2-1/3*b*i*(d*x+c)^3*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^4/(b*x+a)^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.14 (sec) , antiderivative size = 1082, normalized size of antiderivative = 3.52

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input

```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
b*g*x)^4,x]
```

output

```
-1/108*(i*(36*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 54*
d*(b*c - a*d)^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 2*B*n
*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3*n - 18*A*d*(b*c - a*d)^2*(a + b*x
) - 15*B*d*(b*c - a*d)^2*n*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 +
66*B*d^2*(b*c - a*d)*n*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] + 6
6*B*d^3*n*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*n*(a + b*x)^3*Log[a + b*x]^2
 + 12*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n] - 18*B*d*(b*c - a*d)^
2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^2*(b*c - a*d)*(a + b*x
)^2*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log
[e*((a + b*x)/(c + d*x))^n] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3
*n*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*n*(a + b*x)^3*Log[(d*(a + b*x))/(-(
b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[e*((a + b*x)/(c + d*x
))^n]*Log[c + d*x] - 18*B*d^3*n*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*n*(a
 + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*n*(a + b*
x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*n*(a + b*x)^3*Pol
yLog[2, (b*(c + d*x))/(b*c - a*d)]) + 27*B*d*n*(a + b*x)*(2*(b*c - a*d)^2*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d^2*(a + b*x)^2*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])*Log[c + d*x] - 4*B*d*n*(a + b*x)*(b*c - a*d + d*(a + ...
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.78, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^4}\,dx$$

$$\downarrow \text{2961}$$

$$\frac{i\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)^2}$$

$$\downarrow \text{2795}$$

$$\frac{i\int\left(\frac{b(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}-\frac{d(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}\right)d\frac{a+bx}{c+dx}}{g^4(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{i\left(-\frac{b(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}-\frac{2bBn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}+\frac{Bdn(c+dx)^2\left(\right.}{g^4(bc-ad)^2}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x)
^4,x]
```

output
```
(i*((B^2*d*n^2*(c + d*x)^2)/(4*(a + b*x)^2) - (2*b*B^2*n^2*(c + d*x)^3)/(2
7*(a + b*x)^3) + (B*d*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
)/(2*(a + b*x)^2) - (2*b*B*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(9*(a + b*x)^3) + (d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^2)/(2*(a + b*x)^2) - (b*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2)/(3*(a + b*x)^3)))/((b*c - a*d)^2*g^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(295) = 590$.

Time = 10.21 (sec) , antiderivative size = 1099, normalized size of antiderivative = 3.58

method	result	size
parallelrisch	Expression too large to display	1099

input

```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,method=_
RETURNVERBOSE)
```

output

```
-1/108*(18*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^2*d^2*i*n^2+8*B^2*b^6*c^3
*d*i*n^3+18*A^2*a^3*b^3*d^4*i*n+36*A^2*b^6*c^3*d*i*n-54*B^2*x*a*b^5*c*d^3*
i*n^3+90*A*B*x*a^2*b^4*d^4*i*n^2+18*A*B*x*b^6*c^2*d^2*i*n^2-54*B^2*ln(e*((
b*x+a)/(d*x+c))^n)^2*a*b^5*c^2*d^2*i*n-54*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*
b^5*c^2*d^2*i*n^2-108*A^2*x*a*b^5*c*d^3*i*n+72*A*B*ln(e*((b*x+a)/(d*x+c))^
n)*b^6*c^3*d*i*n-27*B^2*a*b^5*c^2*d^2*i*n^3+30*A*B*a^3*b^3*d^4*i*n^2+24*A*
B*b^6*c^3*d*i*n^2-54*A^2*a*b^5*c^2*d^2*i*n-36*A*B*x^3*ln(e*((b*x+a)/(d*x+c
))^n)*b^6*d^4*i*n-54*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^5*d^4*i*n-54*
B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^4*i*n^2-36*B^2*x^2*ln(e*((b*x+a)
/(d*x+c))^n)*b^6*c*d^3*i*n^2+36*A*B*x^2*a*b^5*d^4*i*n^2-36*A*B*x^2*b^6*c*d
^3*i*n^2+54*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^6*c^2*d^2*i*n-54*A*B*a*b^5
*c^2*d^2*i*n^2+19*B^2*a^3*b^3*d^4*i*n^3-108*A*B*x^2*ln(e*((b*x+a)/(d*x+c))
^n)*a*b^5*d^4*i*n-108*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^5*c*d^3*i*n-10
8*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^3*i*n^2+108*A*B*x*ln(e*((b*x+a
)/(d*x+c))^n)*b^6*c^2*d^2*i*n-108*A*B*x*a*b^5*c*d^3*i*n^2-108*A*B*ln(e*((b
*x+a)/(d*x+c))^n)*a*b^5*c^2*d^2*i*n-18*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2
*b^6*d^4*i*n-30*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^4*i*n^2+30*B^2*x^2
*a*b^5*d^4*i*n^3-30*B^2*x^2*b^6*c*d^3*i*n^3+57*B^2*x*a^2*b^4*d^4*i*n^3-3*B
^2*x*b^6*c^2*d^2*i*n^3+36*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^6*c^3*d*i*n+24
*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^3*d*i*n^2+54*A^2*x*a^2*b^4*d^4*i*n...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1167 vs. $2(295) = 590$.

Time $= 0.12$ (sec) , antiderivative size $= 1167$, normalized size of antiderivative $= 3.80$

$$\int \frac{(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^4}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,
algorithm="fricas")
```

output

```
-1/108*((8*B^2*b^3*c^3 - 27*B^2*a*b^2*c^2*d + 19*B^2*a^3*d^3)*i*n^2 + 6*(4
*A*B*b^3*c^3 - 9*A*B*a*b^2*c^2*d + 5*A*B*a^3*d^3)*i*n - 6*(5*(B^2*b^3*c*d^
2 - B^2*a*b^2*d^3)*i*n^2 + 6*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*i*n)*x^2 + 18
*(3*(B^2*b^3*c^2*d - 2*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*i*x + (2*B^2*b^3*c
^3 - 3*B^2*a*b^2*c^2*d + B^2*a^3*d^3)*i)*log(e)^2 - 18*(B^2*b^3*d^3*i*n^2*
x^3 + 3*B^2*a*b^2*d^3*i*n^2*x^2 - 3*(B^2*b^3*c^2*d - 2*B^2*a*b^2*c*d^2)*i*
n^2*x - (2*B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d)*i*n^2)*log((b*x + a)/(d*x + c)
)^2 + 18*(2*A^2*b^3*c^3 - 3*A^2*a*b^2*c^2*d + A^2*a^3*d^3)*i - 3*((B^2*b^3
*c^2*d + 18*B^2*a*b^2*c*d^2 - 19*B^2*a^2*b*d^3)*i*n^2 - 6*(A*B*b^3*c^2*d -
 6*A*B*a*b^2*c*d^2 + 5*A*B*a^2*b*d^3)*i*n - 18*(A^2*b^3*c^2*d - 2*A^2*a*b^
2*c*d^2 + A^2*a^2*b*d^3)*i)*x - 6*(6*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*i*n*x
^2 - (4*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 5*B^2*a^3*d^3)*i*n - 6*(2*A*B*b^
3*c^3 - 3*A*B*a*b^2*c^2*d + A*B*a^3*d^3)*i - 3*((B^2*b^3*c^2*d - 6*B^2*a*b
^2*c*d^2 + 5*B^2*a^2*b*d^3)*i*n + 6*(A*B*b^3*c^2*d - 2*A*B*a*b^2*c*d^2 + A
*B*a^2*b*d^3)*i)*x + 6*(B^2*b^3*d^3*i*n*x^3 + 3*B^2*a*b^2*d^3*i*n*x^2 - 3*
(B^2*b^3*c^2*d - 2*B^2*a*b^2*c*d^2)*i*n*x - (2*B^2*b^3*c^3 - 3*B^2*a*b^2*c
^2*d)*i*n)*log((b*x + a)/(d*x + c)))*log(e) + 6*((4*B^2*b^3*c^3 - 9*B^2*a*
b^2*c^2*d)*i*n^2 - (5*B^2*b^3*d^3*i*n^2 + 6*A*B*b^3*d^3*i*n)*x^3 + 6*(2*A*
B*b^3*c^3 - 3*A*B*a*b^2*c^2*d)*i*n - 3*(6*A*B*a*b^2*d^3*i*n + (2*B^2*b^3*c
*d^2 + 3*B^2*a*b^2*d^3)*i*n^2)*x^2 + 3*((B^2*b^3*c^2*d - 6*B^2*a*b^2*c*...
```

Sympy [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$$

$$= \frac{i\left(\int \frac{A^2c}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \int \frac{A^2dx}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \int \frac{B^2c\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx -}{g}$$

input

```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4,x
)
```

output

```
i*(Integral(A**2*c/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 +
 b**4*x**4), x) + Integral(A**2*d*x/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2
+ 4*a*b**3*x**3 + b**4*x**4), x) + Integral(B**2*c*log(e*(a/(c + d*x) + b*
x/(c + d*x))**n)**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3
+ b**4*x**4), x) + Integral(2*A*B*c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n
)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) +
 Integral(B**2*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a**4 + 4*a*
*3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(2*A*
B*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2*
b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x))/g**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3312 vs. $2(295) = 590$.

Time $= 0.23$ (sec) , antiderivative size $= 3312$, normalized size of antiderivative $= 10.79$

$$\int \frac{(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^4}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,
algorithm="maxima")
```

output

```
-1/9*A*B*c*i*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b
^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*
(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3
*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*
g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^
3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c
*d^2 - a^3*b*d^3)*g^4)) - 1/18*A*B*d*i*n*((5*a*b^2*c^2 - 22*a^2*b*c*d + 5*
a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*a*b^2*c*d + 5*
a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^
2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d
+ a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4) -
6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c
*d^2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x + c)/((b^5*c^3 -
3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4)) - 1/6*(3*b*x + a)*B^2
*d*i*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(b^5*g^4*x^3 + 3*a*b^4*g^4*x
^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/54*(6*n*((6*b^2*d^2*x^2 + 2*b^2*c^
2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^
5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)
*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*
c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - ...
```

Giac [A] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.63

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{1}{108}\left(\frac{18\left(2\,B^2bin^2-\frac{3\,(bx+a)B^2din^2}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)^2}{\frac{(bx+a)^3bcg^4}{(dx+c)^3}-\frac{(bx+a)^3adg^4}{(dx+c)^3}}+\frac{6\left(4\,B^2bin^2-\frac{9\,(bx+a)B^2din^2}{dx+c}+12\,B^2bin\log(e)-\frac{18}{\frac{(bx+a)^3bcg}{(dx+c)^3}}\right.\right.$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,
algorithm="giac")
```

output

```
-1/108*(18*(2*B^2*b*i*n^2 - 3*(b*x + a)*B^2*d*i*n^2/(d*x + c))*log((b*x +
a)/(d*x + c))^2/((b*x + a)^3*b*c*g^4/(d*x + c)^3 - (b*x + a)^3*a*d*g^4/(d*
x + c)^3) + 6*(4*B^2*b*i*n^2 - 9*(b*x + a)*B^2*d*i*n^2/(d*x + c) + 12*B^2*
b*i*n*log(e) - 18*(b*x + a)*B^2*d*i*n*log(e)/(d*x + c) + 12*A*B*b*i*n - 18
*(b*x + a)*A*B*d*i*n/(d*x + c))*log((b*x + a)/(d*x + c))/((b*x + a)^3*b*c*
g^4/(d*x + c)^3 - (b*x + a)^3*a*d*g^4/(d*x + c)^3) + (8*B^2*b*i*n^2 - 27*(
b*x + a)*B^2*d*i*n^2/(d*x + c) + 24*B^2*b*i*n*log(e) - 54*(b*x + a)*B^2*d*
i*n*log(e)/(d*x + c) + 36*B^2*b*i*log(e)^2 - 54*(b*x + a)*B^2*d*i*log(e)^2
/(d*x + c) + 24*A*B*b*i*n - 54*(b*x + a)*A*B*d*i*n/(d*x + c) + 72*A*B*b*i*
log(e) - 108*(b*x + a)*A*B*d*i*log(e)/(d*x + c) + 36*A^2*b*i - 54*(b*x + a
)*A^2*d*i/(d*x + c))/((b*x + a)^3*b*c*g^4/(d*x + c)^3 - (b*x + a)^3*a*d*g^
4/(d*x + c)^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 28.89 (sec) , antiderivative size = 993, normalized size of antiderivative = 3.23

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input

```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)
^4,x)
```

output

```
- ((18*A^2*a^2*d^2*i - 36*A^2*b^2*c^2*i + 19*B^2*a^2*d^2*i*n^2 - 8*B^2*b^2
*c^2*i*n^2 + 30*A*B*a^2*d^2*i*n - 24*A*B*b^2*c^2*i*n + 18*A^2*a*b*c*d*i +
19*B^2*a*b*c*d*i*n^2 + 30*A*B*a*b*c*d*i*n)/(6*(a*d - b*c)) + (x*(18*A^2*a*
b*d^2*i - 18*A^2*b^2*c*d*i + 19*B^2*a*b*d^2*i*n^2 + B^2*b^2*c*d*i*n^2 + 30
*A*B*a*b*d^2*i*n - 6*A*B*b^2*c*d*i*n))/(2*(a*d - b*c)) + (x^2*(5*B^2*b^2*d
^2*i*n^2 + 6*A*B*b^2*d^2*i*n))/(a*d - b*c))/(18*a^3*b^2*g^4 + 18*b^5*g^4*x
^3 + 54*a^2*b^3*g^4*x + 54*a*b^4*g^4*x^2) - log(e*((a + b*x)/(c + d*x))^n)
^2*(((B^2*c*i)/(3*b) + (B^2*d*i*x)/(2*b) + (B^2*a*d*i)/(6*b^2))/(a^3*g^4 +
 b^3*g^4*x^3 + 3*a*b^2*g^4*x^2 + 3*a^2*b*g^4*x) - (B^2*d^3*i)/(6*b^2*g^4*(
a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - log(e*((a + b*x)/(c + d*x))^n)*((A*B*a*
d*i + 2*A*B*b*c*i - B^2*a*d*i*n + B^2*b*c*i*n + 3*A*B*b*d*i*x)/(3*a^3*b^2*
g^4 + 3*b^5*g^4*x^3 + 9*a^2*b^3*g^4*x + 9*a*b^4*g^4*x^2) + (B^2*d^3*i*(x*(
b*((a*b^2*g^4*n*(a*d - b*c))/d + (b^2*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*
d^2)) + (2*a*b^3*g^4*n*(a*d - b*c))/d + (b^3*g^4*n*(a*d - b*c)*(3*a*d - b*
c))/d^2) + a*((a*b^2*g^4*n*(a*d - b*c))/d + (b^2*g^4*n*(a*d - b*c)*(3*a*d
- b*c))/(2*d^2)) + (3*b^4*g^4*n*x^2*(a*d - b*c))/d + (b^2*g^4*n*(a*d - b*c
)*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/d^3))/(3*b^2*g^4*(a^2*d^2 + b^2*c^2 -
 2*a*b*c*d)*(3*a^3*b^2*g^4 + 3*b^5*g^4*x^3 + 9*a^2*b^3*g^4*x + 9*a*b^4*g^4
*x^2))) - (B*d^3*i*n*atan((B*d^3*i*n*(6*A + 5*B*n)*(2*b*d*x - (b^4*c^2*g^4
 - a^2*b^2*d^2*g^4)/(b^2*g^4*(a*d - b*c)))*1i)/((a*d - b*c)*(5*B^2*d^3*...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1624, normalized size of antiderivative = 5.29

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x)
```

output

```
(i*(36*log(a + b*x)*a**5*b*d**3*n + 18*log(a + b*x)*a**4*b**2*d**3*n**2 +
108*log(a + b*x)*a**4*b**2*d**3*n*x + 12*log(a + b*x)*a**3*b**3*c*d**2*n**
2 + 54*log(a + b*x)*a**3*b**3*d**3*n**2*x + 108*log(a + b*x)*a**3*b**3*d**
3*n*x**2 + 36*log(a + b*x)*a**2*b**4*c*d**2*n**2*x + 54*log(a + b*x)*a**2*
b**4*d**3*n**2*x**2 + 36*log(a + b*x)*a**2*b**4*d**3*n*x**3 + 36*log(a + b
*x)*a*b**5*c*d**2*n**2*x**2 + 18*log(a + b*x)*a*b**5*d**3*n**2*x**3 + 12*l
og(a + b*x)*b**6*c*d**2*n**2*x**3 - 36*log(c + d*x)*a**5*b*d**3*n - 18*log
(c + d*x)*a**4*b**2*d**3*n**2 - 108*log(c + d*x)*a**4*b**2*d**3*n*x - 12*l
og(c + d*x)*a**3*b**3*c*d**2*n**2 - 54*log(c + d*x)*a**3*b**3*d**3*n**2*x
- 108*log(c + d*x)*a**3*b**3*d**3*n*x**2 - 36*log(c + d*x)*a**2*b**4*c*d**
2*n**2*x - 54*log(c + d*x)*a**2*b**4*d**3*n**2*x**2 - 36*log(c + d*x)*a**2
*b**4*d**3*n*x**3 - 36*log(c + d*x)*a*b**5*c*d**2*n**2*x**2 - 18*log(c + d
*x)*a*b**5*d**3*n**2*x**3 - 12*log(c + d*x)*b**6*c*d**2*n**2*x**3 + 54*log
(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**4*c**2*d + 108*log(((a + b*x)**
n*e)/(c + d*x)**n)**2*a**2*b**4*c*d**2*x + 54*log(((a + b*x)**n*e)/(c + d*
x)**n)**2*a**2*b**4*d**3*x**2 - 36*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a
*b**5*c**3 - 54*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**5*c**2*d*x + 18
*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**5*d**3*x**3 - 36*log(((a + b*x
)**n*e)/(c + d*x)**n)*a**5*b*d**3 - 18*log(((a + b*x)**n*e)/(c + d*x)**n)*
a**4*b**2*d**3*n - 108*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*b**2*d**...
```

3.167 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result . . . 1880
Mathematica [C] (verified) . . . 1881
Rubi [A] (verified) . . . 1882
Maple [B] (verified) . . . 1884
Fricas [B] (verification not implemented) . . . 1885
Sympy [F(-1)] . . . 1886
Maxima [B] (verification not implemented) . . . 1887
Giac [A] (verification not implemented) . . . 1888
Mupad [B] (verification not implemented) . . . 1889
Reduce [B] (verification not implemented) . . . 1890

Optimal result

Integrand size = 43, antiderivative size = 475

$$\begin{aligned}\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx &= -\frac{B^2d^2in^2(c+dx)^2}{4(bc-ad)^3g^5(a+bx)^2}\\ &+\frac{4bB^2din^2(c+dx)^3}{27(bc-ad)^3g^5(a+bx)^3}\\ &-\frac{b^2B^2in^2(c+dx)^4}{32(bc-ad)^3g^5(a+bx)^4}\\ &-\frac{Bd^2in(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^5(a+bx)^2}\\ &+\frac{4bBdin(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^5(a+bx)^3}\\ &-\frac{b^2Bin(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^3g^5(a+bx)^4}\\ &-\frac{d^2i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3g^5(a+bx)^2}\\ &+\frac{2bdi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^3g^5(a+bx)^3}\\ &-\frac{b^2i(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4(bc-ad)^3g^5(a+bx)^4}\end{aligned}$$

output

```
-1/4*B^2*d^2*i*n^2*(d*x+c)^2/(-a*d+b*c)^3/g^5/(b*x+a)^2+4/27*b*B^2*d*i*n^2
*(d*x+c)^3/(-a*d+b*c)^3/g^5/(b*x+a)^3-1/32*b^2*B^2*i*n^2*(d*x+c)^4/(-a*d+b
*c)^3/g^5/(b*x+a)^4-1/2*B*d^2*i*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)/(-a*d+b*c)^3/g^5/(b*x+a)^2+4/9*b*B*d*i*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))/(-a*d+b*c)^3/g^5/(b*x+a)^3-1/8*b^2*B*i*n*(d*x+c)^4*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^5/(b*x+a)^4-1/2*d^2*i*(d*x+c)^2*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^5/(b*x+a)^2+2/3*b*d*i*(d*x+c)^3*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^5/(b*x+a)^3-1/4*b^2*i*(d*
x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^5/(b*x+a)^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.31 (sec) , antiderivative size = 1319, normalized size of antiderivative = 2.78

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
b*g*x)^5,x]
```

output

```
-1/864*(i*(216*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - 28
8*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 +
16*B*d*n*(a + b*x)*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3*n - 18*A*d*(b*c
 - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*n*(a + b*x) + 36*A*d^2*(b*c - a
*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*n*(a + b*x)^2 + 36*A*d^3*(a + b*x)^
3*Log[a + b*x] + 66*B*d^3*n*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*n*(a + b*x
)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n] - 1
8*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^2*(b
*c - a*d)*(a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^3*(a + b*x)^
3*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n] - 36*A*d^3*(a + b*x)^3*Log[c
 + d*x] - 66*B*d^3*n*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*n*(a + b*x)^3*Log
[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[e*(
(a + b*x)/(c + d*x))^n]*Log[c + d*x] - 18*B*d^3*n*(a + b*x)^3*Log[c + d*x]
^2 + 36*B*d^3*n*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] +
36*B*d^3*n*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3
*n*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 3*B*n*(36*A*(b*c -
 a*d)^4 + 9*B*(b*c - a*d)^4*n + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d
*(-(b*c) + a*d)^3*n*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*
d^2*(b*c - a*d)^2*n*(a + b*x)^2 + 144*A*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 3
00*B*d^3*(-(b*c) + a*d)*n*(a + b*x)^3 - 144*A*d^4*(a + b*x)^4*Log[a + b...
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^5}\,dx$$

$$\downarrow 2961$$

$$\frac{i\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^3}$$

$$\downarrow 2795$$

$$\frac{i\int\left(\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^5}{(a+bx)^5}-\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^4}{(a+bx)^4}+\frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^3}{(a+bx)^3}\right)d\frac{a+bx}{c+dx}}{g^5(bc-ad)^3}$$

$$\downarrow \text{2009}$$

$$\frac{i\left(-\frac{b^2(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4(a+bx)^4}-\frac{b^2Bn(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{8(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}-\frac{Bd^2n(c+dx}{}\right.}{}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x)
^5,x]
```

output
```
(i*(-1/4*(B^2*d^2*n^2*(c + d*x)^2)/(a + b*x)^2 + (4*b*B^2*d*n^2*(c + d*x)^
3)/(27*(a + b*x)^3) - (b^2*B^2*n^2*(c + d*x)^4)/(32*(a + b*x)^4) - (B*d^2*
n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) + (4
*b*B*d*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(a + b*x)^
3) - (b^2*B*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(8*(a +
b*x)^4) - (d^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a
 + b*x)^2) + (2*b*d*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/
(3*(a + b*x)^3) - (b^2*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2)/(4*(a + b*x)^4)))/((b*c - a*d)^3*g^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2219 vs. $2(457) = 914$.

Time = 29.84 (sec) , antiderivative size = 2220, normalized size of antiderivative = 4.67

method	result	size
parallelrisch	Expression too large to display	2220

input

```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,method=_
RETURNVERBOSE)
```

output

```
1/864*(72*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^2*c*d^4*i*n+156*B^2*x^
4*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c*d^4*i*n^2+156*A*B*x^4*a^6*b^2*c*d^4*
i*n^2+864*A^2*x*a^8*c^2*d^3*i*n-864*A^2*x*a^5*b^3*c^5*i*n+144*A*B*x^4*ln(e
*((b*x+a)/(d*x+c))^n)*a^6*b^2*c*d^4*i*n+576*A*B*x^3*ln(e*((b*x+a)/(d*x+c))
^n)*a^7*b*c*d^4*i*n-1728*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^3*d^2*i*n
+576*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^4*d*i*n+115*B^2*x^4*a^6*b^2
*c*d^4*i*n^3-216*B^2*x^4*a^4*b^4*c^3*d^2*i*n^3+128*B^2*x^4*a^3*b^5*c^4*d*i
*n^3-108*A*B*x^4*a^2*b^6*c^5*i*n^2+304*B^2*x^3*a^7*b*c*d^4*i*n^3-27*B^2*x^
4*a^2*b^6*c^5*i*n^3-108*B^2*x^3*a^3*b^5*c^5*i*n^3-216*A^2*x^4*a^2*b^6*c^5*
i*n+1536*A*B*x^3*a^4*b^4*c^4*d*i*n^2+576*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)
*a^7*b*c^2*d^3*i*n^2-72*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^3*d^2*
i*n^2+864*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^8*c*d^4*i*n+576*A*B*x^2*a^7*
b*c^2*d^3*i*n^2-2664*A*B*x^2*a^6*b^2*c^3*d^2*i*n^2+2304*A*B*x^2*a^5*b^3*c^
4*d*i*n^2+216*B^2*x^2*a^8*c*d^4*i*n^3-162*B^2*x^2*a^4*b^4*c^5*i*n^3-864*A^
2*x^3*a^3*b^5*c^5*i*n+432*B^2*x*a^8*c^2*d^3*i*n^3-108*B^2*x*a^5*b^3*c^5*i*
n^3+432*A^2*x^2*a^8*c*d^4*i*n-1296*A^2*x^2*a^4*b^4*c^5*i*n+432*B^2*ln(e*((
b*x+a)/(d*x+c))^n)^2*a^8*c^3*d^2*i*n+216*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a
^6*b^2*c^5*i*n+432*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^8*c^3*d^2*i*n^2+108*B^2
*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^5*i*n^2+432*A*B*ln(e*((b*x+a)/(d*x+c)
)^n)*a^6*b^2*c^5*i*n-864*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^7*b*c^3*d^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1868 vs. $2(457) = 914$.

Time $= 0.15$ (sec) , antiderivative size $= 1868$, normalized size of antiderivative $= 3.93$

$$\int \frac{(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,
algorithm="fricas")
```

output

```
-1/864*((27*B^2*b^4*c^4 - 128*B^2*a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 -
115*B^2*a^4*d^4)*i*n^2 + 12*(13*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)*i*n^2 + 12
*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*i*n)*x^3 + 12*(9*A*B*b^4*c^4 - 32*A*B*a*b
^3*c^3*d + 36*A*B*a^2*b^2*c^2*d^2 - 13*A*B*a^4*d^4)*i*n - 6*((B^2*b^4*c^2*
d^2 - 80*B^2*a*b^3*c*d^3 + 79*B^2*a^2*b^2*d^4)*i*n^2 + 12*(A*B*b^4*c^2*d^2
 - 8*A*B*a*b^3*c*d^3 + 7*A*B*a^2*b^2*d^4)*i*n)*x^2 + 72*(4*(B^2*b^4*c^3*d
- 3*B^2*a*b^3*c^2*d^2 + 3*B^2*a^2*b^2*c*d^3 - B^2*a^3*b*d^4)*i*x + (3*B^2*
b^4*c^4 - 8*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - B^2*a^4*d^4)*i)*log(
e)^2 + 72*(B^2*b^4*d^4*i*n^2*x^4 + 4*B^2*a*b^3*d^4*i*n^2*x^3 + 6*B^2*a^2*b
^2*d^4*i*n^2*x^2 + 4*(B^2*b^4*c^3*d - 3*B^2*a*b^3*c^2*d^2 + 3*B^2*a^2*b^2*
c*d^3)*i*n^2*x + (3*B^2*b^4*c^4 - 8*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^
2)*i*n^2)*log((b*x + a)/(d*x + c))^2 + 72*(3*A^2*b^4*c^4 - 8*A^2*a*b^3*c^3
*d + 6*A^2*a^2*b^2*c^2*d^2 - A^2*a^4*d^4)*i - 4*((5*B^2*b^4*c^3*d - 12*B^2
*a*b^3*c^2*d^2 - 108*B^2*a^2*b^2*c*d^3 + 115*B^2*a^3*b*d^4)*i*n^2 - 12*(A*
B*b^4*c^3*d - 6*A*B*a*b^3*c^2*d^2 + 18*A*B*a^2*b^2*c*d^3 - 13*A*B*a^3*b*d^
4)*i*n - 72*(A^2*b^4*c^3*d - 3*A^2*a*b^3*c^2*d^2 + 3*A^2*a^2*b^2*c*d^3 - A
^2*a^3*b*d^4)*i)*x + 12*(12*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)*i*n*x^3 - 6*(B
^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + 7*B^2*a^2*b^2*d^4)*i*n*x^2 + (9*B^2*b
^4*c^4 - 32*B^2*a*b^3*c^3*d + 36*B^2*a^2*b^2*c^2*d^2 - 13*B^2*a^4*d^4)*i*n
 + 12*(3*A*B*b^4*c^4 - 8*A*B*a*b^3*c^3*d + 6*A*B*a^2*b^2*c^2*d^2 - A*B*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Timed out}$$

input

```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**5,x
)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4838 vs. $2(457) = 914$.

Time $= 0.36$ (sec) , antiderivative size $= 4838$, normalized size of antiderivative $= 10.19$

$$\int \frac{(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,
algorithm="maxima")
```

output

```
1/24*A*B*c*i*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*
d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^
2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a
^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a
^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 -
 a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2
 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 -
 a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b
^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5
*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^
5)) - 1/72*A*B*d*i*n*((7*a*b^3*c^3 - 33*a^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 1
3*a^4*d^3 + 12*(4*b^4*c*d^2 - a*b^3*d^3)*x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c
*d^2 + 7*a^2*b^2*d^3)*x^2 + 4*(4*b^4*c^3 - 21*a*b^3*c^2*d + 57*a^2*b^2*c*d
^2 - 13*a^3*b*d^3)*x)/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^
6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^
5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*
b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^
6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*
b^2*d^3)*g^5) + 12*(4*b*c*d^3 - a*d^4)*log(b*x + a)/((b^6*c^4 - 4*a*b^5*c^
3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5) - 12*(4*b...
```

Giac [A] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 871, normalized size of antiderivative = 1.83

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,
algorithm="giac")
```

output

```
-1/864*(72*(3*B^2*b^2*i*n^2 - 8*(b*x + a)*B^2*b*d*i*n^2/(d*x + c) + 6*(b*x
 + a)^2*B^2*d^2*i*n^2/(d*x + c)^2)*log((b*x + a)/(d*x + c))^2/((b*x + a)^4
*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*x + a)^4*a*b*c*d*g^5/(d*x + c)^4 + (b*x +
a)^4*a^2*d^2*g^5/(d*x + c)^4) + 12*(9*B^2*b^2*i*n^2 - 32*(b*x + a)*B^2*b*d
*i*n^2/(d*x + c) + 36*(b*x + a)^2*B^2*d^2*i*n^2/(d*x + c)^2 + 36*B^2*b^2*i
*n*log(e) - 96*(b*x + a)*B^2*b*d*i*n*log(e)/(d*x + c) + 72*(b*x + a)^2*B^2
*d^2*i*n*log(e)/(d*x + c)^2 + 36*A*B*b^2*i*n - 96*(b*x + a)*A*B*b*d*i*n/(d
*x + c) + 72*(b*x + a)^2*A*B*d^2*i*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))
/((b*x + a)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*x + a)^4*a*b*c*d*g^5/(d*x + c
)^4 + (b*x + a)^4*a^2*d^2*g^5/(d*x + c)^4) + (27*B^2*b^2*i*n^2 - 128*(b*x
+ a)*B^2*b*d*i*n^2/(d*x + c) + 216*(b*x + a)^2*B^2*d^2*i*n^2/(d*x + c)^2 +
 108*B^2*b^2*i*n*log(e) - 384*(b*x + a)*B^2*b*d*i*n*log(e)/(d*x + c) + 432
*(b*x + a)^2*B^2*d^2*i*n*log(e)/(d*x + c)^2 + 216*B^2*b^2*i*log(e)^2 - 576
*(b*x + a)*B^2*b*d*i*log(e)^2/(d*x + c) + 432*(b*x + a)^2*B^2*d^2*i*log(e)
^2/(d*x + c)^2 + 108*A*B*b^2*i*n - 384*(b*x + a)*A*B*b*d*i*n/(d*x + c) + 4
32*(b*x + a)^2*A*B*d^2*i*n/(d*x + c)^2 + 432*A*B*b^2*i*log(e) - 1152*(b*x
+ a)*A*B*b*d*i*log(e)/(d*x + c) + 864*(b*x + a)^2*A*B*d^2*i*log(e)/(d*x +
c)^2 + 216*A^2*b^2*i - 576*(b*x + a)*A^2*b*d*i/(d*x + c) + 432*(b*x + a)^2
*A^2*d^2*i/(d*x + c)^2)/((b*x + a)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*x + a)
^4*a*b*c*d*g^5/(d*x + c)^4 + (b*x + a)^4*a^2*d^2*g^5/(d*x + c)^4))*(b*c...
```

Mupad [B] (verification not implemented)

Time = 31.16 (sec) , antiderivative size = 1794, normalized size of antiderivative = 3.78

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)
^5,x)
```

output

```
((72*A^2*a^3*d^3*i + 216*A^2*b^3*c^3*i + 115*B^2*a^3*d^3*i*n^2 + 27*B^2*b^
3*c^3*i*n^2 + 156*A*B*a^3*d^3*i*n + 108*A*B*b^3*c^3*i*n - 360*A^2*a*b^2*c^
2*d*i + 72*A^2*a^2*b*c*d^2*i - 101*B^2*a*b^2*c^2*d*i*n^2 + 115*B^2*a^2*b*c
*d^2*i*n^2 - 276*A*B*a*b^2*c^2*d*i*n + 156*A*B*a^2*b*c*d^2*i*n)/(12*(a*d -
 b*c)) + (x^2*(79*B^2*a*b^2*d^3*i*n^2 - B^2*b^3*c*d^2*i*n^2 + 84*A*B*a*b^2
*d^3*i*n - 12*A*B*b^3*c*d^2*i*n))/(2*(a*d - b*c)) + (x*(72*A^2*a^2*b*d^3*i
 + 72*A^2*b^3*c^2*d*i + 115*B^2*a^2*b*d^3*i*n^2 - 5*B^2*b^3*c^2*d*i*n^2 -
144*A^2*a*b^2*c*d^2*i + 7*B^2*a*b^2*c*d^2*i*n^2 + 156*A*B*a^2*b*d^3*i*n +
12*A*B*b^3*c^2*d*i*n - 60*A*B*a*b^2*c*d^2*i*n))/(3*(a*d - b*c)) + (d*x^3*(
13*B^2*b^3*d^2*i*n^2 + 12*A*B*b^3*d^2*i*n))/(a*d - b*c))/(x*(288*a^3*b^4*c
*g^5 - 288*a^4*b^3*d*g^5) - x^3*(288*a^2*b^5*d*g^5 - 288*a*b^6*c*g^5) + x^
4*(72*b^7*c*g^5 - 72*a*b^6*d*g^5) + x^2*(432*a^2*b^5*c*g^5 - 432*a^3*b^4*d
*g^5) + 72*a^4*b^3*c*g^5 - 72*a^5*b^2*d*g^5) - log(e*((a + b*x)/(c + d*x))
^n)^2*(((B^2*c*i)/(4*b) + (B^2*d*i*x)/(3*b) + (B^2*a*d*i)/(12*b^2))/(a^4*g
^5 + b^4*g^5*x^4 + 4*a*b^3*g^5*x^3 + 6*a^2*b^2*g^5*x^2 + 4*a^3*b*g^5*x) -
(B^2*d^4*i)/(12*b^2*g^5*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
))) - log(e*((a + b*x)/(c + d*x))^n)*((A*B*a*d*i + 3*A*B*b*c*i - B^2*a*d*i
*n + B^2*b*c*i*n + 4*A*B*b*d*i*x)/(6*a^4*b^2*g^5 + 6*b^6*g^5*x^4 + 24*a^3*
b^3*g^5*x + 24*a*b^5*g^5*x^3 + 36*a^2*b^4*g^5*x^2) + (B^2*d^4*i*(x^2*(b*(b
*((3*a*b^2*g^5*n*(a*d - b*c))/(2*d) + (b^2*g^5*n*(a*d - b*c)*(4*a*d - b...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 2426, normalized size of antiderivative = 5.11

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x)
```

output

```
(i*(144*log(a + b*x)*a**6*b*d**4*n + 120*log(a + b*x)*a**5*b**2*d**4*n**2
+ 576*log(a + b*x)*a**5*b**2*d**4*n*x + 36*log(a + b*x)*a**4*b**3*c*d**3*n
**2 + 480*log(a + b*x)*a**4*b**3*d**4*n**2*x + 864*log(a + b*x)*a**4*b**3*
d**4*n*x**2 + 144*log(a + b*x)*a**3*b**4*c*d**3*n**2*x + 720*log(a + b*x)*
a**3*b**4*d**4*n**2*x**2 + 576*log(a + b*x)*a**3*b**4*d**4*n*x**3 + 216*lo
g(a + b*x)*a**2*b**5*c*d**3*n**2*x**2 + 480*log(a + b*x)*a**2*b**5*d**4*n*
*2*x**3 + 144*log(a + b*x)*a**2*b**5*d**4*n*x**4 + 144*log(a + b*x)*a*b**6
*c*d**3*n**2*x**3 + 120*log(a + b*x)*a*b**6*d**4*n**2*x**4 + 36*log(a + b*
x)*b**7*c*d**3*n**2*x**4 - 144*log(c + d*x)*a**6*b*d**4*n - 120*log(c + d*
x)*a**5*b**2*d**4*n**2 - 576*log(c + d*x)*a**5*b**2*d**4*n*x - 36*log(c +
d*x)*a**4*b**3*c*d**3*n**2 - 480*log(c + d*x)*a**4*b**3*d**4*n**2*x - 864*
log(c + d*x)*a**4*b**3*d**4*n*x**2 - 144*log(c + d*x)*a**3*b**4*c*d**3*n**
2*x - 720*log(c + d*x)*a**3*b**4*d**4*n**2*x**2 - 576*log(c + d*x)*a**3*b*
*4*d**4*n*x**3 - 216*log(c + d*x)*a**2*b**5*c*d**3*n**2*x**2 - 480*log(c +
 d*x)*a**2*b**5*d**4*n**2*x**3 - 144*log(c + d*x)*a**2*b**5*d**4*n*x**4 -
144*log(c + d*x)*a*b**6*c*d**3*n**2*x**3 - 120*log(c + d*x)*a*b**6*d**4*n*
*2*x**4 - 36*log(c + d*x)*b**7*c*d**3*n**2*x**4 + 432*log(((a + b*x)**n*e)
/(c + d*x)**n)**2*a**3*b**4*c**2*d**2 + 864*log(((a + b*x)**n*e)/(c + d*x)
**n)**2*a**3*b**4*c*d**3*x + 432*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**
3*b**4*d**4*x**2 - 576*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**5*...
```

3.168 $\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result 1892
Mathematica [B] (verified) 1893
Rubi [A] (verified) 1894
Maple [F] 1904
Fricas [F] 1904
Sympy [F(-1)] 1905
Maxima [B] (verification not implemented) 1906
Giac [F(-1)] 1907
Mupad [F(-1)] 1907
Reduce [F] 1907

Optimal result

Integrand size = 45, antiderivative size = 766

$$
\begin{aligned}
&\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx \\
&= \frac{3B^2(bc-ad)^5g^3i^2n^2x}{20b^2d^3} + \frac{B^2(bc-ad)^2g^3i^2n^2(a+bx)^4}{60b^3} \\
&\quad - \frac{3B^2(bc-ad)^4g^3i^2n^2(c+dx)^2}{40bd^4} + \frac{B^2(bc-ad)^3g^3i^2n^2(c+dx)^3}{60d^4} \\
&\quad - \frac{B(bc-ad)^3g^3i^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{90b^3d} \\
&\quad - \frac{B(bc-ad)^2g^3i^2n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^3} \\
&\quad - \frac{B(bc-ad)g^3i^2n(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^2} \\
&\quad + \frac{(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{60b^3} \\
&\quad + \frac{(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{15b^2} \\
&\quad + \frac{g^3i^2(a+bx)^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b} \\
&\quad + \frac{B(bc-ad)^4g^3i^2n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{180b^3d^2} \\
&\quad - \frac{B(bc-ad)^5g^3i^2n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{180b^3d^3} \\
&\quad - \frac{B(bc-ad)^6g^3i^2n\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{180b^3d^4} \\
&\quad - \frac{B^2(bc-ad)^6g^3i^2n^2\log(c+dx)}{20b^3d^4} - \frac{B^2(bc-ad)^6g^3i^2n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{30b^3d^4}
\end{aligned}
$$

output

```
3/20*B^2*(-a*d+b*c)^5*g^3*i^2*n^2*x/b^2/d^3+1/60*B^2*(-a*d+b*c)^2*g^3*i^2*
n^2*(b*x+a)^4/b^3-3/40*B^2*(-a*d+b*c)^4*g^3*i^2*n^2*(d*x+c)^2/b/d^4+1/60*B
^2*(-a*d+b*c)^3*g^3*i^2*n^2*(d*x+c)^3/d^4-1/90*B*(-a*d+b*c)^3*g^3*i^2*n*(b
*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d-1/20*B*(-a*d+b*c)^2*g^3*i^2*
n*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3-1/15*B*(-a*d+b*c)*g^3*i^2*
n*(b*x+a)^4*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2+1/60*(-a*d+b*c)^2*
g^3*i^2*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3+1/15*(-a*d+b*c)*g^
3*i^2*(b*x+a)^4*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2+1/6*g^3*i^2*
(b*x+a)^4*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/180*B*(-a*d+b*c)
^4*g^3*i^2*n*(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d^2-1/1
80*B*(-a*d+b*c)^5*g^3*i^2*n*(b*x+a)*(6*A+5*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^
n))/b^3/d^3-1/180*B*(-a*d+b*c)^6*g^3*i^2*n*(6*A+11*B*n+6*B*ln(e*((b*x+a)/(
d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^3/d^4-1/20*B^2*(-a*d+b*c)^6*g^3*i^2
*n^2*ln(d*x+c)/b^3/d^4-1/30*B^2*(-a*d+b*c)^6*g^3*i^2*n^2*polylog(2,d*(b*x+
a)/b/(d*x+c))/b^3/d^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1634 vs. $2(766) = 1532$.

Time = 1.58 (sec) , antiderivative size = 1634, normalized size of antiderivative = 2.13

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

input

```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2,x]
```

output

```
(g^3*i^2*(15*(b*c - a*d)^2*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2 + 24*d*(b*c - a*d)*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2 + 10*d^2*(a + b*x)^6*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (5*B*(
b*c - a*d)^3*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Lo
g[e*((a + b*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(b*c
 - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a
*d)^2*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n*((2*Lo
g[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)])))/d^4 + (2*B*(b*c - a*d)^2*n*(24*A*b*d*(b*c
- a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n]
 - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
 + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) -
6*d^4*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 24*B*(b*c - a*d
)^4*n*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[c + d*x] + 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)
^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*
x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d...
```

Rubi [A] (verified)

Time = 2.37 (sec) , antiderivative size = 1003, normalized size of antiderivative = 1.31, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.378$, Rules used = {2961, 2783, 2782, 27, 87, 49, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\downarrow 2961$$

$$g^3i^2(bc-ad)^6\int\frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$g^3 i^2 (bc -$$

$$ad)^6 \left(-\frac{Bn \int \frac{(a+bx)^3 \left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{3b} + \frac{\int \frac{(a+bx)^3 \left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{3b} + \frac{(a+bx)^4 \left(B \log\left(e\left(\frac{a}{c}\right.\right.\right.}{6b(c+dx)^4 \left(b-\right.}$$

$$\downarrow 2782$$

$$g^3 i^2 (bc -$$

$$ad)^6 \left(-\frac{Bn \left(-Bn \int \frac{(a+bx)^3 \left(5b-\frac{d(a+bx)}{c+dx}\right)}{20b^2 (c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx} + \frac{(a+bx)^4 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{20b^2 (c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{(a+bx)^4 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} \right)}{3b} -$$

$$\downarrow 27$$

$$g^3 i^2 (bc -$$

$$ad)^6 \left(-\frac{Bn \left(-\frac{Bn \int \frac{(a+bx)^3 \left(5b-\frac{d(a+bx)}{c+dx}\right)}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{20b^2} + \frac{(a+bx)^4 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{20b^2 (c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{(a+bx)^4 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} \right)}{3b} + \frac{\int (a}{}$$

$$\downarrow 87$$

$$g^3 i^2 (bc -$$

$$ad)^6 \left(-\frac{Bn \left(-\frac{Bn \left(\int \frac{(a+bx)^3}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx} + \frac{(a+bx)^4}{(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} \right)}{20b^2} + \frac{(a+bx)^4 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{20b^2 (c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{(a+bx)^4 \left(B \log\left(e\right.\right.\right.}{5b(c+dx)^4 \left(b-\right.} \right)}{3b}$$

$$\downarrow 49$$

$$g^3 i^2 (bc -$$

$$ad)^6 \left(-\frac{Bn\left(-\frac{Bn\left(\int \left(\frac{b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{3b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{3b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{1}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)} \right) d\frac{a+bx}{c+dx} + \frac{(a+bx)^4}{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} \right)}{20b^2} + \right.}{3b}$$

$$\downarrow 2009$$

$$g^3 i^2 (bc -$$

$$ad)^6 \left(\frac{\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{3b} - \frac{Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \right.}{3b}$$

$$\downarrow 2783$$

$$g^3 i^2 (bc -$$

$$ad)^6 \left(\frac{-\frac{2Bn\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \right.}{3b}$$

$$\downarrow 2773$$

$$g^3 i^2 (bc - ad)^6 \left(\frac{2Bn \left(\frac{(a+bx)^4 \left(B \log\left(e \left(\frac{a+bx}{c+dx} \right)^n \right) + A \right)}{4b(c+dx)^4 \left(b - \frac{d(a+bx)}{c+dx} \right)^4} - \frac{Bn \int \frac{(a+bx)^3}{(c+dx)^3 \left(b - \frac{d(a+bx)}{c+dx} \right)^4} d\frac{a+bx}{c+dx}}{4b} \right)}{5b} + \frac{\int \frac{(a+bx)^3 \left(A + B \log\left(e \left(\frac{a+bx}{c+dx} \right)^n \right) \right)^2}{(c+dx)^3 \left(b - \frac{d(a+bx)}{c+dx} \right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{(a+bx)^4}{5b(} \right)$$

$$\Big\downarrow 49$$

$$g^3 i^2 (bc - ad)^6 \left(\frac{2Bn \left(\frac{(a+bx)^4 \left(B \log\left(e \left(\frac{a+bx}{c+dx} \right)^n \right) + A \right)}{4b(c+dx)^4 \left(b - \frac{d(a+bx)}{c+dx} \right)^4} - \frac{Bn \int \left(\frac{b^3}{d^3 \left(b - \frac{d(a+bx)}{c+dx} \right)^4} - \frac{3b^2}{d^3 \left(b - \frac{d(a+bx)}{c+dx} \right)^3} + \frac{3b}{d^3 \left(b - \frac{d(a+bx)}{c+dx} \right)^2} - \frac{1}{d^3 \left(b - \frac{d(a+bx)}{c+dx} \right)} \right) d\frac{a+bx}{c+dx}}{4b} \right)}{5b} + \right.$$

$$\Big\downarrow 2009$$

$$g^3 i^2 (bc - ad)^6 \left(\frac{\int \frac{(a+bx)^3 \left(A + B \log\left(e \left(\frac{a+bx}{c+dx} \right)^n \right) \right)^2}{(c+dx)^3 \left(b - \frac{d(a+bx)}{c+dx} \right)^5} d\frac{a+bx}{c+dx}}{5b} - \frac{2Bn \left(\frac{(a+bx)^4 \left(B \log\left(e \left(\frac{a+bx}{c+dx} \right)^n \right) + A \right)}{4b(c+dx)^4 \left(b - \frac{d(a+bx)}{c+dx} \right)^4} - \frac{Bn \left(\frac{b^3}{3d^4 \left(b - \frac{d(a+bx)}{c+dx} \right)^3} - \frac{3b^2}{2d^4 \left(b - \frac{d(a+bx)}{c+dx} \right)^2} + \frac{}{d^4 \left(b - \right.} \right.} \right.}{4b}$$

$$\Big\downarrow 2781$$

$$g^3 i^2 (bc -$$

$$ad)^6 \left(\frac{\frac{(a+bx)^4 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn \int \frac{(a+bx)^3 \left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{2b}}{5b} - \frac{2Bn \left(\frac{(a+bx)^4 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn \left(\frac{b^3}{3d^4 \left(b-\frac{d(a}{c-}\right.\right.\right.}{3b}$$

$$\downarrow 2784$$

$$(bc -$$

$$ad)^6 g^3 i^2 \left(\frac{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{6b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{Bn \left(\frac{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{20b^2(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{5b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} \right.}{} \right.$$

$$\downarrow 2784$$

$$(bc-ad)^6 g^3 i^2 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} \right.$$

$$\downarrow 2784$$

$$(bc-ad)^6 g^3 i^2 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} \right.$$

$$\downarrow 2754$$

$$(bc-ad)^6 g^3 i^2 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} \right. \right.$$

↓ 2838

$$(bc-ad)^6 g^3 i^2 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} \right. \right.$$

input

```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2,x]
```

output

```
(b*c - a*d)^6*g^3*i^2*(((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2)/(6*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^6) - (B*n*(((a + b*x)^4
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*b*(c + d*x)^4*(b - (d*(a + b*x
))/(c + d*x))^5) + ((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2
0*b^2*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*((a + b*x)^4/((c
 + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) + b^3/(3*d^4*(b - (d*(a + b*x))
/(c + d*x))^3) - (3*b^2)/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (3*b)/(
d^4*(b - (d*(a + b*x))/(c + d*x))) + Log[b - (d*(a + b*x))/(c + d*x)]/d^4)
)/(20*b^2)))/(3*b) + (((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2)/(5*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*n*(((a + b*x)^
4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b*(c + d*x)^4*(b - (d*(a + b*
x))/(c + d*x))^4) - (B*n*(b^3/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^3) - (3
*b^2)/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (3*b)/(d^4*(b - (d*(a + b*
x))/(c + d*x))) + Log[b - (d*(a + b*x))/(c + d*x)]/d^4))/(4*b)))/(5*b) + (
((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*b*(c + d*x)^4*(b
 - (d*(a + b*x))/(c + d*x))^4) - (B*n*(((a + b*x)^3*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a
 + b*x)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)
^2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(6*A + 5*B*n + 6*B*Log[e
*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))))...
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)^3 (dix+ci)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3 (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (bgx+ag)^3 (dix+ci)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="fricas")
```

output
```
integral(A^2*b^3*d^2*g^3*i^2*x^5 + A^2*a^3*c^2*g^3*i^2 + (2*A^2*b^3*c*d +
3*A^2*a*b^2*d^2)*g^3*i^2*x^4 + (A^2*b^3*c^2 + 6*A^2*a*b^2*c*d + 3*A^2*a^2*
b*d^2)*g^3*i^2*x^3 + (3*A^2*a*b^2*c^2 + 6*A^2*a^2*b*c*d + A^2*a^3*d^2)*g^3
*i^2*x^2 + (3*A^2*a^2*b*c^2 + 2*A^2*a^3*c*d)*g^3*i^2*x + (B^2*b^3*d^2*g^3*
i^2*x^5 + B^2*a^3*c^2*g^3*i^2 + (2*B^2*b^3*c*d + 3*B^2*a*b^2*d^2)*g^3*i^2*
x^4 + (B^2*b^3*c^2 + 6*B^2*a*b^2*c*d + 3*B^2*a^2*b*d^2)*g^3*i^2*x^3 + (3*B
^2*a*b^2*c^2 + 6*B^2*a^2*b*c*d + B^2*a^3*d^2)*g^3*i^2*x^2 + (3*B^2*a^2*b*c
^2 + 2*B^2*a^3*c*d)*g^3*i^2*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b
^3*d^2*g^3*i^2*x^5 + A*B*a^3*c^2*g^3*i^2 + (2*A*B*b^3*c*d + 3*A*B*a*b^2*d^
2)*g^3*i^2*x^4 + (A*B*b^3*c^2 + 6*A*B*a*b^2*c*d + 3*A*B*a^2*b*d^2)*g^3*i^2
*x^3 + (3*A*B*a*b^2*c^2 + 6*A*B*a^2*b*c*d + A*B*a^3*d^2)*g^3*i^2*x^2 + (3*
A*B*a^2*b*c^2 + 2*A*B*a^3*c*d)*g^3*i^2*x)*log(e*((b*x + a)/(d*x + c))^n),
x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**
2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5952 vs. $2(735) = 1470$.

Time = 0.65 (sec) , antiderivative size = 5952, normalized size of antiderivative = 7.77

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="maxima")
```

output
```
1/3*A*B*b^3*d^2*g^3*i^2*x^6*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/6*A
^2*b^3*d^2*g^3*i^2*x^6 + 4/5*A*B*b^3*c*d*g^3*i^2*x^5*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n) + 6/5*A*B*a*b^2*d^2*g^3*i^2*x^5*log(e*(b*x/(d*x + c) + a
/(d*x + c))^n) + 2/5*A^2*b^3*c*d*g^3*i^2*x^5 + 3/5*A^2*a*b^2*d^2*g^3*i^2*x
^5 + 1/2*A*B*b^3*c^2*g^3*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
3*A*B*a*b^2*c*d*g^3*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A
*B*a^2*b*d^2*g^3*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*
b^3*c^2*g^3*i^2*x^4 + 3/2*A^2*a*b^2*c*d*g^3*i^2*x^4 + 3/4*A^2*a^2*b*d^2*g^
3*i^2*x^4 + 2*A*B*a*b^2*c^2*g^3*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n) + 4*A*B*a^2*b*c*d*g^3*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ 2/3*A*B*a^3*d^2*g^3*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2
*a*b^2*c^2*g^3*i^2*x^3 + 2*A^2*a^2*b*c*d*g^3*i^2*x^3 + 1/3*A^2*a^3*d^2*g^3
*i^2*x^3 + 3*A*B*a^2*b*c^2*g^3*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))
^n) + 2*A*B*a^3*c*d*g^3*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3
/2*A^2*a^2*b*c^2*g^3*i^2*x^2 + A^2*a^3*c*d*g^3*i^2*x^2 - 1/180*A*B*b^3*d^2
*g^3*i^2*n*(60*a^6*log(b*x + a)/b^6 - 60*c^6*log(d*x + c)/d^6 + (12*(b^5*c
*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*b^3*d^5)*x^4 + 20*(b^5*c^3*d
^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*d^5)*x^2 + 60*(b^5*c^5 - a^5
*d^5)*x)/(b^5*d^5)) + 1/15*A*B*b^3*c*d*g^3*i^2*n*(12*a^5*log(b*x + a)/b^5
- 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2...
```

Giac [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^3\,(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))
^2,x)
```

output
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))
^2, x)
```

Reduce [F]

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
(g**3*( - 12*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c
*x + b*d*x**2),x)*a**6*b**2*d**7*n + 72*int((log(((a + b*x)**n*e)/(c + d*x
)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**5*b**3*c*d**6*n - 180*int(
(log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)
*a**4*b**4*c**2*d**5*n + 240*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a
*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**5*c**3*d**4*n - 180*int((log(((a
 + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b*
*6*c**4*d**3*n + 72*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*
x + b*c*x + b*d*x**2),x)*a*b**7*c**5*d**2*n - 12*int((log(((a + b*x)**n*e)
/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**8*c**6*d*n - 12*l
og(c + d*x)*a**7*d**6*n + 72*log(c + d*x)*a**6*b*c*d**5*n - 4*log(c + d*x)
*a**6*b*d**6*n**2 - 180*log(c + d*x)*a**5*b**2*c**2*d**4*n + 24*log(c + d*
x)*a**5*b**2*c*d**5*n**2 + 240*log(c + d*x)*a**4*b**3*c**3*d**3*n - 60*log
(c + d*x)*a**4*b**3*c**2*d**4*n**2 - 180*log(c + d*x)*a**3*b**4*c**4*d**2*
n + 80*log(c + d*x)*a**3*b**4*c**3*d**3*n**2 + 72*log(c + d*x)*a**2*b**5*c
**5*d*n - 60*log(c + d*x)*a**2*b**5*c**4*d**2*n**2 - 12*log(c + d*x)*a*b**
6*c**6*n + 24*log(c + d*x)*a*b**6*c**5*d*n**2 - 4*log(c + d*x)*b**7*c**6*n
**2 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**5*b**2*c*d**5 - 30*log(((
a + b*x)**n*e)/(c + d*x)**n)**2*a**4*b**3*c**2*d**4 - 60*log(((a + b*x)**n
*e)/(c + d*x)**n)**2*a**3*b**4*c**3*d**3 - 360*log(((a + b*x)**n*e)/(c ...
```

3.169 $\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result 1910
Mathematica [A] (verified) 1911
Rubi [A] (verified) 1912
Maple [F] 1923
Fricas [F] 1923
Sympy [F(-1)] 1924
Maxima [B] (verification not implemented) 1924
Giac [F(-1)] 1925
Mupad [F(-1)] 1926
Reduce [F] 1926

Optimal result

Integrand size = 45, antiderivative size = 819

$$
\begin{aligned}
&\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx \\
&= -\frac{B^2(bc-ad)^4g^2i^2n^2x}{10b^2d^2} - \frac{B^2(bc-ad)^3g^2i^2n^2(c+dx)^2}{20bd^3} \\
&\quad + \frac{B^2(bc-ad)^2g^2i^2n^2(c+dx)^3}{30d^3} \\
&\quad - \frac{B(bc-ad)^3g^2i^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^3d} \\
&\quad - \frac{B(bc-ad)^2g^2i^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^3} \\
&\quad - \frac{B(bc-ad)^3g^2i^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5bd^3} \\
&\quad + \frac{4B(bc-ad)^2g^2i^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3} \\
&\quad - \frac{bB(bc-ad)g^2i^2n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^3} \\
&\quad + \frac{(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{30b^3} \\
&\quad + \frac{(bc-ad)g^2i^2(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2} \\
&\quad + \frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} \\
&\quad + \frac{B(bc-ad)^4g^2i^2n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^3d^2} \\
&\quad + \frac{B(bc-ad)^5g^2i^2n\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{30b^3d^3} \\
&\quad + \frac{B^2(bc-ad)^5g^2i^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3} + \frac{B^2(bc-ad)^5g^2i^2n^2\log(c+dx)}{10b^3d^3} \\
&\quad + \frac{B^2(bc-ad)^5g^2i^2n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{15b^3d^3}
\end{aligned}
$$

output

```
-1/10*B^2*(-a*d+b*c)^4*g^2*i^2*n^2*x/b^2/d^2-1/20*B^2*(-a*d+b*c)^3*g^2*i^2
*n^2*(d*x+c)^2/b/d^3+1/30*B^2*(-a*d+b*c)^2*g^2*i^2*n^2*(d*x+c)^3/d^3-1/30*
B*(-a*d+b*c)^3*g^2*i^2*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d-1
/15*B*(-a*d+b*c)^2*g^2*i^2*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3
-1/5*B*(-a*d+b*c)^3*g^2*i^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/
d^3+4/15*B*(-a*d+b*c)^2*g^2*i^2*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)/d^3-1/10*b*B*(-a*d+b*c)*g^2*i^2*n*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))/d^3+1/30*(-a*d+b*c)^2*g^2*i^2*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)^2/b^3+1/10*(-a*d+b*c)*g^2*i^2*(b*x+a)^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))^2/b^2+1/5*g^2*i^2*(b*x+a)^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))^2/b+1/30*B*(-a*d+b*c)^4*g^2*i^2*n*(b*x+a)*(2*A+B*n+2*B*ln(e*((b*x+a)/(
d*x+c))^n))/b^3/d^2+1/30*B*(-a*d+b*c)^5*g^2*i^2*n*(2*A+3*B*n+2*B*ln(e*((b*
x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^3/d^3+1/30*B^2*(-a*d+b*c)^5*g
^2*i^2*n^2*ln((b*x+a)/(d*x+c))/b^3/d^3+1/10*B^2*(-a*d+b*c)^5*g^2*i^2*n^2*l
n(d*x+c)/b^3/d^3+1/15*B^2*(-a*d+b*c)^5*g^2*i^2*n^2*polylog(2,d*(b*x+a)/b/(
d*x+c))/b^3/d^3
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 1254, normalized size of antiderivative = 1.53

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

input

```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2,x]
```

output

```
(g^2*i^2*(20*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^2 + 30*d^4*(b*c - a*d)*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^2 + 12*d^5*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 +
20*B*(b*c - a*d)^3*n*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*
Log[e*((a + b*x)/(c + d*x))^n] - d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(b*d*x + (-
(b*c) + a*d)*Log[c + d*x]) + B*(b*c - a*d)^2*n*((2*Log[(d*(a + b*x))/(-(b*
c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c
- a*d)])) - 10*B*(b*c - a*d)^2*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a
*d)^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a +
 b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*
c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c -
a*d)*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*
x]) + 3*B*(b*c - a*d)^2*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c
 - a*d)^3*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + B*(b*c - a*d)*n*(24*A*b
*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c +
d*x))^n] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c ...
```

Rubi [A] (verified)

Time = 2.23 (sec) , antiderivative size = 891, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2961, 2783, 2782, 27, 1195, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\downarrow \text{2961}$$

$$g^2i^2(bc-ad)^5\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(-\frac{2Bn \int \frac{(a+bx)^2 \left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{2 \int \frac{(a+bx)^2 \left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{(a+bx)^3 \left(B \log\left(e\right.\right.}{5b(c+dx)^3 \left(b\right.}$$

$$\downarrow 2782$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(-\frac{2Bn \left(-Bn \int \frac{(c+dx)\left(b^2 - \frac{4d(a+bx)b}{c+dx} + \frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{12d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx} + \frac{b^2\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2b\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \right.}{5b}$$

$$\downarrow 27$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(-\frac{2Bn \left(-\frac{Bn \int \frac{(c+dx)\left(b^2 - \frac{4d(a+bx)b}{c+dx} + \frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{12d^3} + \frac{b^2\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2b\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{B \log}{2d^3} \right.}{5b}$$

$$\downarrow 1195$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(-\frac{2Bn \left(-\frac{Bn \int \left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)} + \frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{5d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{3bd}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{c+dx}{b^2(a+bx)} \right) d\frac{a+bx}{c+dx}}{12d^3} + \frac{b^2\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} \right.}{5b}$$

$$\downarrow 2009$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(\frac{2 \int \frac{(a+bx)^2 \left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} - \frac{2Bn \left(\frac{b^2 \left(B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2b \left(B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{B \log\left(e \left(\frac{a}{c}\right.\right.\right.}{2d^3 \left(b-\right.}$$

$$\downarrow 2783$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(\frac{2 \left(-\frac{Bn \int \frac{(a+bx)^2 \left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{2b} + \frac{\int \frac{(a+bx)^2 \left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{4b} + \frac{(a+bx)^3 \left(B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} \right)}{5b}$$

$$\downarrow 2773$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(\frac{2 \left(-\frac{Bn \left(\frac{(a+bx)^3 \left(B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{Bn \int \frac{(a+bx)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^3} d\frac{a+bx}{c+dx}}{3b} \right)}{2b} + \frac{\int \frac{(a+bx)^2 \left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{4b} + \frac{(a+b}{} \right)}{5b}$$

$$\downarrow 49$$

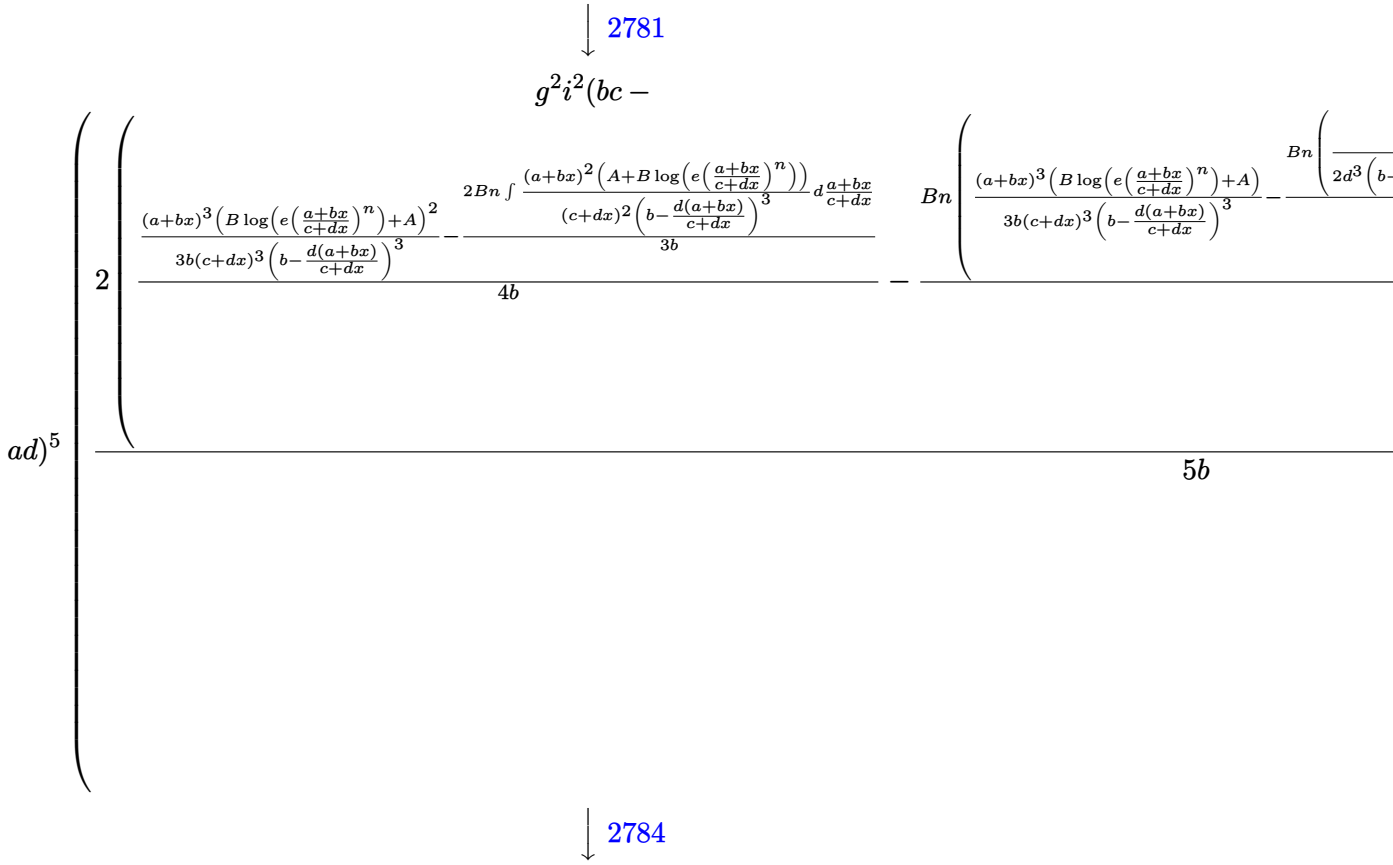
$$g^2 i^2 (bc -$$

$$ad)^5 \left(\frac{2\left(-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{Bn\int\left(\frac{b^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{1}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}}{3b}\right)}{2b} + \frac{\int \frac{(a+bx)^2\left(A+B\right.}{(c+dx)^2\left(\right.}}{5b} \right.$$

$$\downarrow 2009$$

$$g^2 i^2 (bc -$$

$$ad)^5 \left(\frac{2\left(\frac{\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b} - \frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{Bn\left(\frac{b^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\log\left(b\right.\right.}{3b}\right.}{2b}}{5b} \right.$$


2781
$g^2 i^2 (bc -$
$ad)^5$
2
$\frac{\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}}{4b}-$
$Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{}{2d^3\left(b-\right.\right.}\right.$
$5b$
2784

$$(bc-ad)^5 g^2 i^2 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{5b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2} \right.$$

↓ 2784

$$(bc-ad)^5 g^2 i^2 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{5b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2} \right.\right.$$

↓ 2754

$$(bc-ad)^5 g^2 i^2 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{5b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2} \right.$$

↓ 2838

$$(bc-ad)^5 g^2 i^2 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{5b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2} \right.$$

input

```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2,x]
```

output
```
(b*c - a*d)^5*g^2*i^2*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2)/(5*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*n*((b^2*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d^3*(b - (d*(a + b*x))/(c + d*x))^4)
 - (2*b*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d^3*(b - (d*(a + b*x))/
(c + d*x))^3) + (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(2*d^3*(b - (d*(a +
 b*x))/(c + d*x))^2) - (B*n*(b/(b - (d*(a + b*x))/(c + d*x))^3 - 5/(2*(b -
 (d*(a + b*x))/(c + d*x))^2) + 1/(b*(b - (d*(a + b*x))/(c + d*x))) + Log[(
a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(12*d^3))
)/(5*b) + (2*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*b*
(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*(((a + b*x)^3*(A + B*L
og[e*((a + b*x)/(c + d*x))^n]))/(3*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d
*x))^3) - (B*n*(b^2/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^2) - (2*b)/(d^3*(
b - (d*(a + b*x))/(c + d*x))) - Log[b - (d*(a + b*x))/(c + d*x)]/d^3))/(3*
b)))/(2*b) + (((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*b*
(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*n*(((a + b*x)^2*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c +
 d*x))^2) - (((a + b*x)*(2*A + B*n + 2*B*Log[e*((a + b*x)/(c + d*x))^n]))/
(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((2*A + 3*B*n + 2*B*Log[e
*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (2*B
*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d)))/(3*b))/(4*b))...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)^2 (dix+ci)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input

```
int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2 (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (bgx+ag)^2 (dix+ci)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="fricas")
```

output
```
integral(A^2*b^2*d^2*g^2*i^2*x^4 + A^2*a^2*c^2*g^2*i^2 + 2*(A^2*b^2*c*d +
A^2*a*b*d^2)*g^2*i^2*x^3 + (A^2*b^2*c^2 + 4*A^2*a*b*c*d + A^2*a^2*d^2)*g^2
*i^2*x^2 + 2*(A^2*a*b*c^2 + A^2*a^2*c*d)*g^2*i^2*x + (B^2*b^2*d^2*g^2*i^2*
x^4 + B^2*a^2*c^2*g^2*i^2 + 2*(B^2*b^2*c*d + B^2*a*b*d^2)*g^2*i^2*x^3 + (B
^2*b^2*c^2 + 4*B^2*a*b*c*d + B^2*a^2*d^2)*g^2*i^2*x^2 + 2*(B^2*a*b*c^2 + B
^2*a^2*c*d)*g^2*i^2*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^2*d^2*g
^2*i^2*x^4 + A*B*a^2*c^2*g^2*i^2 + 2*(A*B*b^2*c*d + A*B*a*b*d^2)*g^2*i^2*x
^3 + (A*B*b^2*c^2 + 4*A*B*a*b*c*d + A*B*a^2*d^2)*g^2*i^2*x^2 + 2*(A*B*a*b*
c^2 + A*B*a^2*c*d)*g^2*i^2*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**
2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4247 vs. $2(786) = 1572$.

Time $= 0.64$ (sec) , antiderivative size $= 4247$, normalized size of antiderivative $= 5.19$

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="maxima")
```

output

```
2/5*A*B*b^2*d^2*g^2*i^2*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A
^2*b^2*d^2*g^2*i^2*x^5 + A*B*b^2*c*d*g^2*i^2*x^4*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n) + A*B*a*b*d^2*g^2*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n) + 1/2*A^2*b^2*c*d*g^2*i^2*x^4 + 1/2*A^2*a*b*d^2*g^2*i^2*x^4 + 2/3*A*B
*b^2*c^2*g^2*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 8/3*A*B*a*b*
c*d*g^2*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2/3*A*B*a^2*d^2*g
^2*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*b^2*c^2*g^2*i^
2*x^3 + 4/3*A^2*a*b*c*d*g^2*i^2*x^3 + 1/3*A^2*a^2*d^2*g^2*i^2*x^3 + 2*A*B*
a*b*c^2*g^2*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*B*a^2*c*d
*g^2*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*b*c^2*g^2*i^2*
x^2 + A^2*a^2*c*d*g^2*i^2*x^2 + 1/30*A*B*b^2*d^2*g^2*i^2*n*(12*a^5*log(b*x
 + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(
b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c
^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/6*A*B*b^2*c*d*g^2*i^2*n*(6*a^4*log(b*x + a
)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c
^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/6*A*B*a*b*
d^2*g^2*i^2*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c
*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d
^3)*x)/(b^3*d^3)) + 1/3*A*B*b^2*c^2*g^2*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*
c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)...
```

Giac [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Timed out}$$

input

```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="giac")
```

output

```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))
^2,x)
```

output
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))
^2, x)
```

Reduce [F]

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
(g**2*( - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*
x + b*d*x**2),x)*a**5*b**2*d**6*n + 20*int((log(((a + b*x)**n*e)/(c + d*x)
**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**4*b**3*c*d**5*n - 40*int((l
og(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a
**3*b**4*c**2*d**4*n + 40*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c
+ a*d*x + b*c*x + b*d*x**2),x)*a**2*b**5*c**3*d**3*n - 20*int((log(((a + b
*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**6*c**4
*d**2*n + 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*
x + b*d*x**2),x)*b**7*c**5*d*n - 4*log(c + d*x)*a**6*d**5*n + 20*log(c + d
*x)*a**5*b*c*d**4*n - 40*log(c + d*x)*a**4*b**2*c**2*d**3*n + 40*log(c + d
*x)*a**3*b**3*c**3*d**2*n - 20*log(c + d*x)*a**2*b**4*c**4*d*n + 4*log(c +
 d*x)*a*b**5*c**5*n + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**4*b**2*c*
d**4 - 8*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*b**3*c**2*d**3 - 8*log
(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**4*c**3*d**2 - 60*log(((a + b*x)
**n*e)/(c + d*x)**n)**2*a**2*b**4*c**2*d**3*x - 60*log(((a + b*x)**n*e)/(c
 + d*x)**n)**2*a**2*b**4*c*d**4*x**2 - 20*log(((a + b*x)**n*e)/(c + d*x)**
n)**2*a**2*b**4*d**5*x**3 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**5
*c**4*d - 60*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**5*c**2*d**3*x**2 -
 80*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**5*c*d**4*x**3 - 30*log(((a
+ b*x)**n*e)/(c + d*x)**n)**2*a*b**5*d**5*x**4 - 20*log(((a + b*x)**n*e...
```

3.170 $\int (ag+bgx)(ci+dix)^2 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result 1929
Mathematica [A] (verified) 1930
Rubi [A] (verified) 1931
Maple [F] 1938
Fricas [F] 1938
Sympy [F] 1940
Maxima [B] (verification not implemented) 1941
Giac [F(-1)] 1942
Mupad [F(-1)] 1943
Reduce [F] 1943

Optimal result

Integrand size = 43, antiderivative size = 635

$$
\begin{aligned}
&\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx \\
&= \frac{B^2(bc-ad)^3gi^2n^2x}{12b^2d} + \frac{B^2(bc-ad)^2gi^2n^2(c+dx)^2}{12bd^2} \\
&\quad - \frac{B(bc-ad)^3gi^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^3d} \\
&\quad - \frac{B(bc-ad)^2gi^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^3} \\
&\quad + \frac{B(bc-ad)^2gi^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4bd^2} \\
&\quad - \frac{B(bc-ad)gi^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^2} \\
&\quad + \frac{(bc-ad)^2gi^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{12b^3} \\
&\quad + \frac{(bc-ad)gi^2(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b^2} \\
&\quad + \frac{gi^2(a+bx)^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} \\
&\quad - \frac{B(bc-ad)^4gi^2n\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{6b^3d^2} \\
&\quad - \frac{B^2(bc-ad)^4gi^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2} - \frac{B^2(bc-ad)^4gi^2n^2\log(c+dx)}{4b^3d^2} \\
&\quad - \frac{B^2(bc-ad)^4gi^2n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{6b^3d^2}
\end{aligned}
$$

output

```
1/12*B^2*(-a*d+b*c)^3*g*i^2*n^2*x/b^2/d+1/12*B^2*(-a*d+b*c)^2*g*i^2*n^2*(d
*x+c)^2/b/d^2-1/6*B*(-a*d+b*c)^3*g*i^2*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))/b^3/d-1/6*B*(-a*d+b*c)^2*g*i^2*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))/b^3+1/4*B*(-a*d+b*c)^2*g*i^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))/b/d^2-1/6*B*(-a*d+b*c)*g*i^2*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c)
)^n))/d^2+1/12*(-a*d+b*c)^2*g*i^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)^2/b^3+1/6*(-a*d+b*c)*g*i^2*(b*x+a)^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))^2/b^2+1/4*g*i^2*(b*x+a)^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2
/b-1/6*B*(-a*d+b*c)^4*g*i^2*n*(A+B*n+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d
+b*c)/b/(d*x+c))/b^3/d^2-1/12*B^2*(-a*d+b*c)^4*g*i^2*n^2*ln((b*x+a)/(d*x+c
))/b^3/d^2-1/4*B^2*(-a*d+b*c)^4*g*i^2*n^2*ln(d*x+c)/b^3/d^2-1/6*B^2*(-a*d+
b*c)^4*g*i^2*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^2
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 713, normalized size of antiderivative = 1.12

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$

$$=\frac{gi^2\left(-4(bc-ad)(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+3b(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{4B(bc-ad)^2r}{}\right.}{}$$

input

```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^2,x]
```

output

```
(g*i^2*(-4*(b*c - a*d)*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2 + 3*b*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (4*B*(b*c -
 a*d)^2*n*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Lo
g[a + b*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] +
 b^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*(b*c - a*d)^2*
Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*
Log[c + d*x] - B*(b*c - a*d)^2*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c
 + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3
 - (B*(b*c - a*d)*n*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x
+ (b*c - a*d)*Log[a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(
c + d*x)^2 + 2*(b*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)
*Log[e*((a + b*x)/(c + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]) + 6*(b*c - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[a
+ b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d
*(a + b*x))/(-(b*c) + a*d)])))/b^3))/(12*d^2)
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 702, normalized size of antiderivative = 1.11, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.326$, Rules used = {2961, 2783, 2782, 27, 86, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$

$$\downarrow 2961$$

$$gi^2(bc-ad)^4\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$gi^2(bc-$$

$$ad)^4\left(-\frac{Bn\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{4b(c+dx)^2\left(b-\frac{d(}{c}\right.}$$

$$\downarrow 2782$$

$$gi^2(bc-$$

$$ad)^4\left(-\frac{Bn\left(-Bn\int-\frac{(c+dx)\left(b-\frac{3d(a+bx)}{c+dx}\right)}{6d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}+\frac{\int\frac{(a+bx)\left(A+B\right.}{(c+dx)}\right.$$

$$\downarrow 27$$

$$gi^2(bc-$$

$$ad)^4\left(-\frac{Bn\left(\frac{Bn\int\frac{(c+dx)\left(b-\frac{3d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{6d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{(c+dx)\left(b-\frac{d(a+bx}{c+dx}\right.}$$

$$\downarrow 86$$

$$gi^2(bc-$$

$$ad)^4\left(-\frac{Bn\left(\frac{Bn\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{6d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{3d^2\left(b-\frac{d(a+}{c+}\right.}$$

$$\downarrow 2009$$

$$ad)^4\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}-\frac{gi^2(bc-\left(Bn\left(-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\right.\right.\right.}{2b}\right.$$

$$\downarrow 2783$$

$$ad)^4\left(\frac{-\frac{2Bn\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{2b}-\right.$$

$$\downarrow 2773$$

$$ad)^4\left(\frac{-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{a+bx}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2b}\right)}{3b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+bx)^2\left(\right.}{3b(c+}}{2b}\right.$$

$$\downarrow 49$$

$$ad)^4\left(\frac{gi^2(bc-}{2b}\right.$$

$$\left.-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{b}{d\left(\frac{d(a+bx)}{c+dx}-b\right)^2}+\frac{1}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{2b}\right)}{3b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}$$

$$\downarrow 2009$$

$$ad)^4\left(\frac{gi^2(bc-}{2b}\right.$$

$$\left.\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^2}\right)}{2b}\right)}{3b}+\frac{(a+b}{}$$

$$\downarrow 2781$$

$$ad)^4\left(\frac{gi^2(bc-}{2b}\right.$$

$$\left.\frac{\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{3b}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{b}{d^2\left(b-\frac{d(a+b}{c+d}\right.\right.\right.}{3b}$$

$$\downarrow 2784$$

$$gi^2(bc-$$

$$ad)^4\left(\frac{\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{b}}{3b}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a}{c}\right.\right.\right.\right.}{2b(c+dx)^2\left(b-\frac{d}{c}\right.}$$

$$\downarrow 2754$$

$$gi^2(bc-$$

$$ad)^4\left(\frac{\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.\right.}{d}}{b}}{3b}$$

$$\downarrow 2838$$

$$gi^2(bc-$$

$$ad)^4\left(-\frac{Bn\left(-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{1}{\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{6d^2}\right.}{2b}$$

input
```
Int[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
,x]
```

output
```
(b*c - a*d)^4*g*i^2*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
)/(4*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*((b*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]))/(3*d^2*(b - (d*(a + b*x))/(c + d*x))^3) - (A
+ B*Log[e*((a + b*x)/(c + d*x))^n])/(2*d^2*(b - (d*(a + b*x))/(c + d*x))^2
) + (B*n*(-(b - (d*(a + b*x))/(c + d*x))^(-2) + 1/(b*(b - (d*(a + b*x))/(c
 + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)
]/b^2))/(6*d^2)))/(2*b) + (((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^2)/(3*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*n*(((a +
b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*b*(c + d*x)^2*(b - (d*(a
 + b*x))/(c + d*x))^2) - (B*n*(b/(d^2*(b - (d*(a + b*x))/(c + d*x))) + Log
[b - (d*(a + b*x))/(c + d*x)]/d^2))/(2*b)))/(3*b) + (((a + b*x)^2*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])^2)/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c +
 d*x))^2) - (B*n*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c
 + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((A + B*n + B*Log[e*((a + b*x)
/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (B*n*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))])/d)/d))/b)/(3*b))/(2*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)\,(dix+ci)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (bgx+ag)(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="fricas")
```

output
```
integral(A^2*b*d^2*g*i^2*x^3 + A^2*a*c^2*g*i^2 + (2*A^2*b*c*d + A^2*a*d^2)
*g*i^2*x^2 + (A^2*b*c^2 + 2*A^2*a*c*d)*g*i^2*x + (B^2*b*d^2*g*i^2*x^3 + B^
2*a*c^2*g*i^2 + (2*B^2*b*c*d + B^2*a*d^2)*g*i^2*x^2 + (B^2*b*c^2 + 2*B^2*a
*c*d)*g*i^2*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b*d^2*g*i^2*x^3 +
 A*B*a*c^2*g*i^2 + (2*A*B*b*c*d + A*B*a*d^2)*g*i^2*x^2 + (A*B*b*c^2 + 2*A*
B*a*c*d)*g*i^2*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F]

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= gi^2\left(\int A^2ac^2\,dx+\int A^2ad^2x^2\,dx+\int A^2bc^2x\,dx+\int A^2bd^2x^3\,dx\right.$$
$$+\int B^2ac^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+\int 2ABac^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx+\int 2A^2acdx\,dx+\int 2A^2bcdx^2\,dx$$
$$+\int B^2ad^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+\int B^2bc^2x\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+\int B^2bd^2x^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+\int 2ABad^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx$$
$$+\int 2ABbc^2x\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx$$
$$+\int 2ABbd^2x^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx$$
$$+\int 2B^2acdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+\int 2B^2bcdx^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+\int 4ABacdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx$$
$$\left.+\int 4ABbcdx^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx\right)$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x
)
```

output

```
g*i**2*(Integral(A**2*a*c**2, x) + Integral(A**2*a*d**2*x**2, x) + Integra
l(A**2*b*c**2*x, x) + Integral(A**2*b*d**2*x**3, x) + Integral(B**2*a*c**2
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*a*c**2*lo
g(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Integral(2*A**2*a*c*d*x, x) +
Integral(2*A**2*b*c*d*x**2, x) + Integral(B**2*a*d**2*x**2*log(e*(a/(c + d
*x) + b*x/(c + d*x))**n)**2, x) + Integral(B**2*b*c**2*x*log(e*(a/(c + d*x
) + b*x/(c + d*x))**n)**2, x) + Integral(B**2*b*d**2*x**3*log(e*(a/(c + d*
x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*a*d**2*x**2*log(e*(a/(c +
d*x) + b*x/(c + d*x))**n), x) + Integral(2*A*B*b*c**2*x*log(e*(a/(c + d*x)
 + b*x/(c + d*x))**n), x) + Integral(2*A*B*b*d**2*x**3*log(e*(a/(c + d*x)
+ b*x/(c + d*x))**n), x) + Integral(2*B**2*a*c*d*x*log(e*(a/(c + d*x) + b*
x/(c + d*x))**n)**2, x) + Integral(2*B**2*b*c*d*x**2*log(e*(a/(c + d*x) +
b*x/(c + d*x))**n)**2, x) + Integral(4*A*B*a*c*d*x*log(e*(a/(c + d*x) + b*
x/(c + d*x))**n), x) + Integral(4*A*B*b*c*d*x**2*log(e*(a/(c + d*x) + b*x/
(c + d*x))**n), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2662 vs. $2(608) = 1216$.

Time $= 0.61$ (sec) , antiderivative size $= 2662$, normalized size of antiderivative $= 4.19$

$$\int (ag + bgx)(ci + dix)^2 \left(A + B \log\left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="maxima")
```

output

```
1/2*A*B*b*d^2*g*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*b
*d^2*g*i^2*x^4 + 4/3*A*B*b*c*d*g*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n) + 2/3*A*B*a*d^2*g*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2
/3*A^2*b*c*d*g*i^2*x^3 + 1/3*A^2*a*d^2*g*i^2*x^3 + A*B*b*c^2*g*i^2*x^2*log
(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*B*a*c*d*g*i^2*x^2*log(e*(b*x/(d*
x + c) + a/(d*x + c))^n) + 1/2*A^2*b*c^2*g*i^2*x^2 + A^2*a*c*d*g*i^2*x^2 -
 1/12*A*B*b*d^2*g*i^2*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 +
 (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c
^3 - a^3*d^3)*x)/(b^3*d^3)) + 2/3*A*B*b*c*d*g*i^2*n*(2*a^3*log(b*x + a)/b^
3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d
^2)*x)/(b^2*d^2)) + 1/3*A*B*a*d^2*g*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*
log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^
2*d^2)) - A*B*b*c^2*g*i^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 +
 (b*c - a*d)*x/(b*d)) - 2*A*B*a*c*d*g*i^2*n*(a^2*log(b*x + a)/b^2 - c^2*lo
g(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a*c^2*g*i^2*n*(a*log(b*x + a
)/b - c*log(d*x + c)/d) + 2*A*B*a*c^2*g*i^2*x*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n) + A^2*a*c^2*g*i^2*x - 1/12*(7*a^2*b*c^2*d^2*g*i^2*n^2 - 2*a^3*c
*d^3*g*i^2*n^2 + (g*i^2*n^2 - 2*g*i^2*n*log(e))*b^3*c^4 - 2*(3*g*i^2*n^2 -
 4*g*i^2*n*log(e))*a*b^2*c^3*d)*B^2*log(d*x + c)/(b^2*d^2) + 1/6*(b^4*c^4*
g*i^2*n^2 - 4*a*b^3*c^3*d*g*i^2*n^2 + 6*a^2*b^2*c^2*d^2*g*i^2*n^2 - 4*a...
```

Giac [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Timed out}$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="giac")
```

output

```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)\,(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2
,x)
```

output
```
int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2
, x)
```

Reduce [F]

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
(g*( - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x +
 b*d*x**2),x)*a**4*b**2*d**5*n + 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)
*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**3*c*d**4*n - 12*int((log((
(a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*
b**4*c**2*d**3*n + 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d
*x + b*c*x + b*d*x**2),x)*a*b**5*c**3*d**2*n - 2*int((log(((a + b*x)**n*e)
/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**6*c**4*d*n - 2*lo
g(c + d*x)*a**5*d**4*n + 8*log(c + d*x)*a**4*b*c*d**3*n + log(c + d*x)*a**
4*b*d**4*n**2 - 12*log(c + d*x)*a**3*b**2*c**2*d**2*n - 4*log(c + d*x)*a**
3*b**2*c*d**3*n**2 + 8*log(c + d*x)*a**2*b**3*c**3*d*n + 6*log(c + d*x)*a*
*2*b**3*c**2*d**2*n**2 - 2*log(c + d*x)*a*b**4*c**4*n - 4*log(c + d*x)*a*b
**4*c**3*d*n**2 + log(c + d*x)*b**5*c**4*n**2 + log(((a + b*x)**n*e)/(c +
d*x)**n)**2*a**3*b**2*c*d**3 - 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**
2*b**3*c**2*d**2 - log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**4*c**3*d - 1
2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**4*c**2*d**2*x - 12*log(((a +
b*x)**n*e)/(c + d*x)**n)**2*a*b**4*c*d**3*x**2 - 4*log(((a + b*x)**n*e)/(c
 + d*x)**n)**2*a*b**4*d**4*x**3 - 6*log(((a + b*x)**n*e)/(c + d*x)**n)**2*
b**5*c**2*d**2*x**2 - 8*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**5*c*d**3*
x**3 - 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**5*d**4*x**4 - 2*log(((a
+ b*x)**n*e)/(c + d*x)**n)*a**5*d**4 + 8*log(((a + b*x)**n*e)/(c + d*x)...
```

3.171 $\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result . . . 1945
Mathematica [A] (verified) . . . 1946
Rubi [A] (verified) . . . 1947
Maple [F] . . . 1953
Fricas [F] . . . 1954
Sympy [F] . . . 1954
Maxima [B] (verification not implemented) . . . 1955
Giac [F] . . . 1956
Mupad [F(-1)] . . . 1957
Reduce [F] . . . 1957

Optimal result

Integrand size = 35, antiderivative size = 361

$$\begin{aligned}
&\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^2i^2n^2x}{3b^2} - \frac{2B(bc-ad)^2i^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3} \\
&\quad - \frac{B(bc-ad)i^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd} \\
&\quad + \frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} \\
&\quad + \frac{B^2(bc-ad)^3i^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d} + \frac{B^2(bc-ad)^3i^2n^2\log(c+dx)}{b^3d} \\
&\quad + \frac{2B(bc-ad)^3i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d} \\
&\quad - \frac{2B^2(bc-ad)^3i^2n^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d}
\end{aligned}$$

output
```
1/3*B^2*(-a*d+b*c)^2*i^2*n^2*x/b^2-2/3*B*(-a*d+b*c)^2*i^2*n*(b*x+a)*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))/b^3-1/3*B*(-a*d+b*c)*i^2*n*(d*x+c)^2*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/b/d+1/3*i^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
^2/d+1/3*B^2*(-a*d+b*c)^3*i^2*n^2*ln((b*x+a)/(d*x+c))/b^3/d+B^2*(-a*d+b*c)
^3*i^2*n^2*ln(d*x+c)/b^3/d+2/3*B*(-a*d+b*c)^3*i^2*n*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/d-2/3*B^2*(-a*d+b*c)^3*i^2*n^2*pol
ylog(2,b*(d*x+c)/d/(b*x+a))/b^3/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.84

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \frac{i^2\left((c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - \frac{B(bc-ad)n\left(2Abd(bc-ad)x-B(bc-ad)n(bdx+(bc-ad)\log(a+bx))+2Bd(bc-ad)(a+b\right.}{}\right.}{}$$

input
```
Integrate[(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(i^2*((c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d
)*n*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a +
b*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + b^2*(
c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*(b*c - a*d)^2*Log[a
+ b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*Log[c
+ d*x] - B*(b*c - a*d)^2*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x
))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3))/(3*
d)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {2951, 2756, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)^2 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$

$$\downarrow 2951$$

$$i^2(bc-ad)^3 \int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}$$

$$\downarrow 2756$$

$$i^2(bc-ad)^3 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3} d\frac{a+bx}{c+dx}}{3d}\right)$$

$$\downarrow 2789$$

$$i^2(bc- ad)^3 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn\left(\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3} d\frac{a+bx}{c+dx}}{b} + \frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2} d\frac{a+bx}{c+dx}}{b}\right)}{3d}\right)$$

$$\downarrow 2756$$

$$ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}dx}{b}\right)}{3d}+\frac{i^2(bc-$$

$$\downarrow 54$$

$$ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}\right)}{3d}+\frac{i^2(bc-$$

$$\downarrow 2009$$

$$ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{i^2(bc-}{3d}\right.$$

$$2Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\right.\right.\right.}{b}\right.$$

$$\downarrow 2789$$

$$ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{i^2(bc-}{3d}\right.$$

$$2Bn\left(\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a}{c}\right.\right.\right.\right.}{2d\left(b-\right.}\right.$$

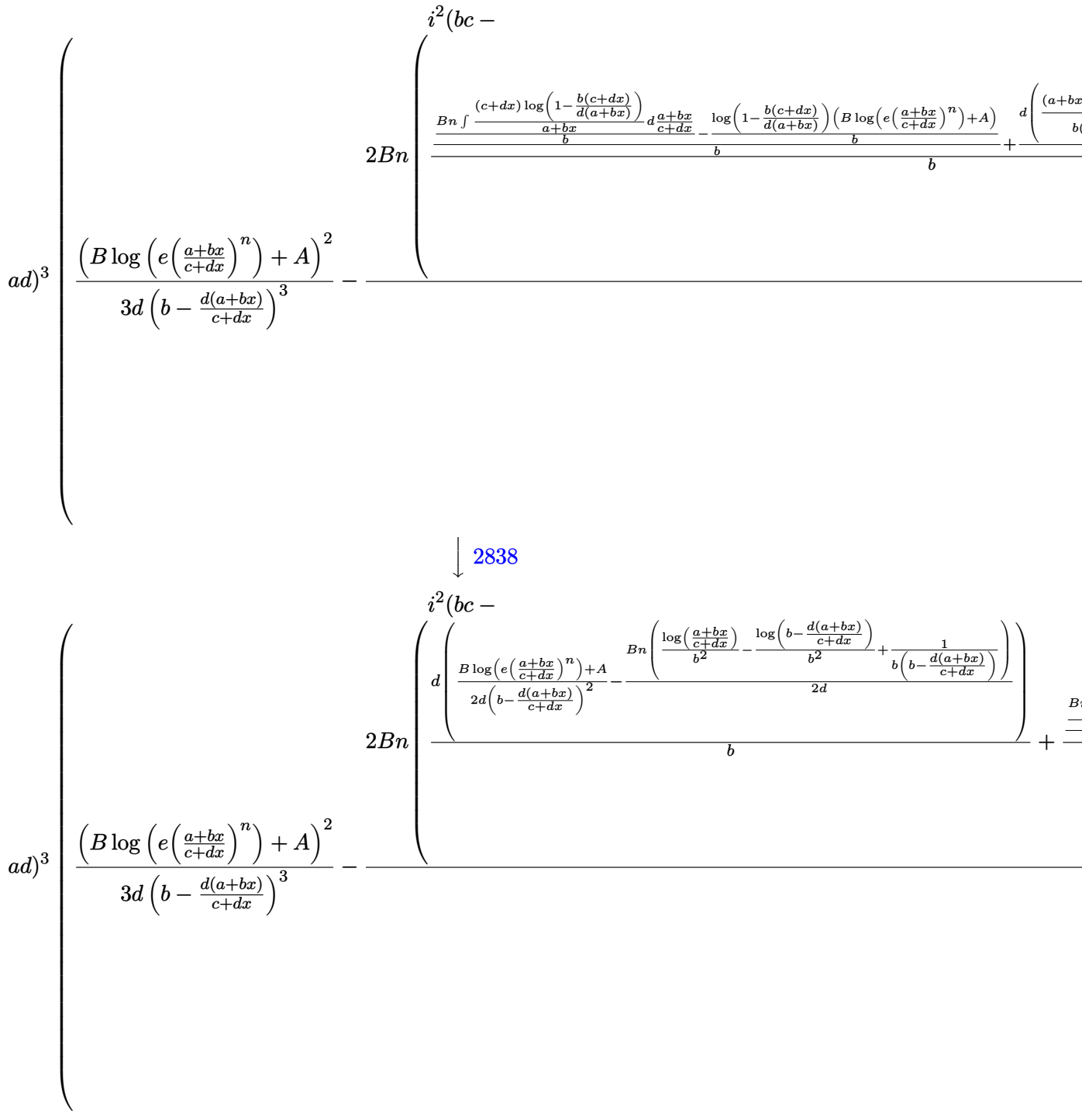
$$\downarrow 2751$$

$$ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{b}}{i^2(bc-}\right.$$

$$\downarrow 16$$

$$ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a}{c+}\right.\right.}{bd}}{b}}{3}\right.$$

$$\downarrow 2779$$



↓ 2838

input
```
Int[(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^3*i^2*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(3*d*(b - (d*(
a + b*x))/(c + d*x))^3) - (2*B*n*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n
])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*(1/(b*(b - (d*(a + b*x))/(
c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x
)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*n*Log[b - (d*(a + b*x)
)/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1
 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*(c + d*x))/(d*(a
+ b*x))])/b)/b)/b))/(3*d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [F]

$$\int (dix+ci)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

output
```
integral(A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x^2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F]

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= i^2\left(\int A^2c^2\,dx+\int A^2d^2x^2\,dx+\int B^2c^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx\right.$$
$$+\int 2ABc^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx+\int 2A^2cdx\,dx$$
$$+\int B^2d^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+\int 2ABd^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx$$
$$+\int 2B^2cdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$\left.+\int 4ABcdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx\right)$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output

```
i**2*(Integral(A**2*c**2, x) + Integral(A**2*d**2*x**2, x) + Integral(B**2
*c**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*c**2
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Integral(2*A**2*c*d*x, x) +
 Integral(B**2*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) +
Integral(2*A*B*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Int
egral(2*B**2*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integr
al(4*A*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1473 vs. $2(346) = 692$.

Time $= 0.55$ (sec) , antiderivative size $= 1473$, normalized size of antiderivative $= 4.08$

$$\int (ci + dix)^2 \left(A + B \log \left(e \left(\frac{a + bx}{c + dx} \right)^n \right) \right)^2 dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output

```
2/3*A*B*d^2*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*d^2*i
^2*x^3 + 2*A*B*c*d*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c*
d*i^2*x^2 + 1/3*A*B*d^2*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*
A*B*c*d*i^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x
/(b*d)) + 2*A*B*c^2*i^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c^
2*i^2*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c^2*i^2*x - 1/3*(5*a*
b*c^2*d*i^2*n^2 - 2*a^2*c*d^2*i^2*n^2 - (3*i^2*n^2 - 2*i^2*n*log(e))*b^2*c
^3)*B^2*log(d*x + c)/(b^2*d) - 2/3*(b^3*c^3*i^2*n^2 - 3*a*b^2*c^2*d*i^2*n^
2 + 3*a^2*b*c*d^2*i^2*n^2 - a^3*d^3*i^2*n^2)*(log(b*x + a)*log((b*d*x + a*
d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d) + 1/3
*(B^2*b^3*d^3*i^2*x^3*log(e)^2 + 2*B^2*b^3*c^3*i^2*n^2*log(b*x + a)*log(d*
x + c) - B^2*b^3*c^3*i^2*n^2*log(d*x + c)^2 + (a*b^2*d^3*i^2*n*log(e) - (i
^2*n*log(e) - 3*i^2*log(e)^2)*b^3*c*d^2)*B^2*x^2 - (3*a*b^2*c^2*d*i^2*n^2
- 3*a^2*b*c*d^2*i^2*n^2 + a^3*d^3*i^2*n^2)*B^2*log(b*x + a)^2 + ((i^2*n^2
- 4*i^2*n*log(e) + 3*i^2*log(e)^2)*b^3*c^2*d - 2*(i^2*n^2 - 3*i^2*n*log(e)
)*a*b^2*c*d^2 + (i^2*n^2 - 2*i^2*n*log(e))*a^2*b*d^3)*B^2*x - (2*(2*i^2*n^
2 - 3*i^2*n*log(e))*a*b^2*c^2*d - (7*i^2*n^2 - 6*i^2*n*log(e))*a^2*b*c*d^2
 + (3*i^2*n^2 - 2*i^2*n*log(e))*a^3*d^3)*B^2*log(b*x + a) + (B^2*b^3*d^3*i
^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x)*log((b*x + a)...
```

Giac [F]

$$\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (dix+ci)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output

```
integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (c\,i+d\,i\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [F]

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
( - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a**3*b**2*d**4*n + 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)
/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**3*c*d**3*n - 6*int((log(((a +
 b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**4*c*
*2*d**2*n + 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*
c*x + b*d*x**2),x)*b**5*c**3*d*n - 2*log(a + b*x)*a**4*d**3*n + 6*log(a +
b*x)*a**3*b*c*d**2*n + 3*log(a + b*x)*a**3*b*d**3*n**2 - 6*log(a + b*x)*a*
*2*b**2*c**2*d*n - 9*log(a + b*x)*a**2*b**2*c*d**2*n**2 + 2*log(a + b*x)*a
*b**3*c**3*n + 9*log(a + b*x)*a*b**3*c**2*d*n**2 - 3*log(a + b*x)*b**4*c**
3*n**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*c*d**2 - 2*log(((
a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3*c**2*d - 3*log(((a + b*x)**n*e)/(c
+ d*x)**n)**2*b**4*c**2*d*x - 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4
*c*d**2*x**2 - log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*d**3*x**3 + 2*lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*c*d**2*n + 2*log(((a + b*x)**n*
e)/(c + d*x)**n)*a**2*b**2*d**3*n*x - 2*log(((a + b*x)**n*e)/(c + d*x)**n)
*a*b**3*c**3 - 5*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c**2*d*n - 6*lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c**2*d*x - 6*log(((a + b*x)**n*e)/
(c + d*x)**n)*a*b**3*c*d**2*n*x - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b
**3*c*d**2*x**2 - log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*d**3*n*x**2 -
2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*d**3*x**3 + 3*log(((a + b*x...
```

3.172 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

Optimal result 1960
Mathematica [B] (verified) 1961
Rubi [A] (verified) 1962
Maple [F] 1970
Fricas [F] 1970
Sympy [F] 1971
Maxima [F] 1971
Giac [F] 1972
Mupad [F(-1)] 1973
Reduce [F] 1973

Optimal result

Integrand size = 45, antiderivative size = 572

$$
\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx \\
&= -\frac{Bd(bc-ad)i^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g} \\
&\quad + \frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g} \\
&\quad + \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg} \\
&\quad + \frac{2B(bc-ad)^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^3g} \\
&\quad + \frac{B^2(bc-ad)^2i^2n^2\log(c+dx)}{b^3g} \\
&\quad + \frac{B(bc-ad)^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad - \frac{(bc-ad)^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad + \frac{2B^2(bc-ad)^2i^2n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g} - \frac{B^2(bc-ad)^2i^2n^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad + \frac{2B(bc-ad)^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad + \frac{2B^2(bc-ad)^2i^2n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}
\end{aligned}
$$

output

```
-B*d*(-a*d+b*c)*i^2*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/g+d*(-a*
d+b*c)*i^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3/g+1/2*i^2*(d*x+c)
^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b/g+2*B*(-a*d+b*c)^2*i^2*n*(A+B*ln(e*
((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^3/g+B^2*(-a*d+b*c)^2*i^2*
n^2*ln(d*x+c)/b^3/g+B*(-a*d+b*c)^2*i^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*l
n(1-b*(d*x+c)/d/(b*x+a))/b^3/g-(-a*d+b*c)^2*i^2*(A+B*ln(e*((b*x+a)/(d*x+c)
)^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g+2*B^2*(-a*d+b*c)^2*i^2*n^2*polylog
(2,d*(b*x+a)/b/(d*x+c))/b^3/g-B^2*(-a*d+b*c)^2*i^2*n^2*polylog(2,b*(d*x+c)
/d/(b*x+a))/b^3/g+2*B*(-a*d+b*c)^2*i^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*p
olylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g+2*B^2*(-a*d+b*c)^2*i^2*n^2*polylog(3,b
*(d*x+c)/d/(b*x+a))/b^3/g
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2852 vs. $2(572) = 1144$.

Time = 3.97 (sec) , antiderivative size = 2852, normalized size of antiderivative = 4.99

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \text{Result too large to show}$$

input

```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x),x]
```

output

```
(i^2*(12*b*d*(2*b*c - a*d)*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*L
og[(a + b*x)/(c + d*x)])^2 + 6*b^2*d^2*x^2*(A + B*Log[e*((a + b*x)/(c + d*
x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 + 12*(b*c - a*d)^2*Log[a + b*x]*(
A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 - 2
4*b*B*c*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d
*x)])*(a*d*Log[a/b + x]^2 - 2*a*d*Log[a/b + x]*(1 + Log[a + b*x]) + 2*(-(b
*c) + a*d + Log[c/d + x]*(b*c + a*d*Log[a + b*x] - a*d*Log[(d*(a + b*x))/(
-(b*c) + a*d)]) + (-(b*d*x) + a*d*Log[a + b*x])*Log[(a + b*x)/(c + d*x)])
- 2*a*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 12*b^2*B*c^2*n*(A + B*Log
[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(Log[a/b + x]^
2 - 2*Log[a + b*x]*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)]
) - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c +
 d*x))/(b*c - a*d)])) + 6*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*
Log[(a + b*x)/(c + d*x)])*(-4*a*d^2*(a + b*x)*(-1 + Log[a/b + x]) + 2*a^2*
d^2*Log[a/b + x]^2 + 4*a*b*d*(c + d*x)*(-1 + Log[c/d + x]) + d^2*(b*x*(2*a
 - b*x) + 2*b^2*x^2*Log[a/b + x] - 2*a^2*Log[a + b*x]) - 2*d^2*(b*x*(-2*a
+ b*x) + 2*a^2*Log[a + b*x])*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/
(c + d*x)]) + b^2*(d*x*(-2*c + d*x) - 2*d^2*x^2*Log[c/d + x] + 2*c^2*Log[c
 + d*x]) - 4*a^2*d^2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Pol
yLog[2, (b*(c + d*x))/(b*c - a*d)])) - 8*b*B^2*c*n^2*(a*d*Log[a/b + x]^...
```

Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 529, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2961, 2789, 2756, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{ag+bgx}\,dx$$

$$\downarrow 2961$$

$$\frac{i^2(bc-ad)^2\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{g}$$

$$\downarrow 2789$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\int\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\downarrow 2756$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\downarrow 2789$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{d}\right)}{b}+\frac{d\int\frac{\left(A+B\log\right.}{\left(b-\right.}}{g}$$

$$\downarrow 2751$$

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{b}\right)}{d}\right)}{b}\right)}{g}$$

$$\downarrow 16$$

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}\right)}{d}\right)}{b}\right)}{g}$$

↓ 2755

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+bx}{c+dx}\right.\right.}{bd}\right.}{d}\right.}{b}\right.}{g}$$

↓ 2754

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+bx}{c+dx}\right.\right.}{bd}\right.}{d}\right.}{b}\right.}{}$$

↓ 2779

$$i^2(bc-ad)^2 \left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\right.}{b(c+d}\right.}{}\right.}{d}\right.}{b}\right.$$

$\Big\downarrow$ 2821

$$i^2(bc-ad)^2 \left(\frac{\frac{2Bn\left(\frac{\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int\frac{(c+dx)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}\right)-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}{b}}{b}\right.$$

$\Big\downarrow$ 2838

$$i^2(bc-ad)^2\left(\frac{\frac{2Bn\left(\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int\frac{(c+dx)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}\right)$$

$$\downarrow 7143$$

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{Bn\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.\right.}{b(c+dx)\left(b-\frac{d(a+b}{c+d}\right.}\right.}{d}\right.}{b}\right)$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x),x]
```

output
```
((b*c - a*d)^2*i^2*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(2*d*(b -
 (d*(a + b*x))/(c + d*x))^2) - (B*n*((d*(((a + b*x)*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*n*Log[b
 - (d*(a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*((a + b*x)/(c +
d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*(c
 + d*x))/(d*(a + b*x))])/b)/b))/d))/b + ((d*(((a + b*x)*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n])^2)/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (2*B
*n*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c
+ d*x))])/d) - (B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d))/b))/b + (
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a +
b*x))])/b) + (2*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (b*
(c + d*x))/(d*(a + b*x))] + B*n*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/
b)/b)/b))/g
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(dix+ci)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{bgx+ag}dx$$

input

```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

output

```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}dx$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x,
algorithm="fricas")
```

output

```
integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x
^2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(
A*B*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log(e*((b*x + a)/(d*x + c
))^n))/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$$

$$= \frac{i^2\left(\int \frac{A^2c^2}{a+bx}\,dx+\int \frac{A^2d^2x^2}{a+bx}\,dx+\int \frac{B^2c^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a+bx}\,dx+\int \frac{2ABc^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx+\int \frac{2A^2cdx}{a+bx}\,dx\right.}{}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g),x
)
```

output
```
i**2*(Integral(A**2*c**2/(a + b*x), x) + Integral(A**2*d**2*x**2/(a + b*x)
, x) + Integral(B**2*c**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a + 
b*x), x) + Integral(2*A*B*c**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a
 + b*x), x) + Integral(2*A**2*c*d*x/(a + b*x), x) + Integral(B**2*d**2*x**2
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a + b*x), x) + Integral(2*A*B
*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x) + Integra
l(2*B**2*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a + b*x), x) + 
Integral(4*A*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x)
)/g
```

Maxima [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x,
algorithm="maxima")
```

output

```
2*A^2*c*d*i^2*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + 1/2*A^2*d^2*i^2*(2*a^2*
log(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A^2*c^2*i^2*log(b*g*x +
a*g)/(b*g) + 1/2*(B^2*b^2*d^2*i^2*x^2 + 2*(2*b^2*c*d*i^2 - a*b*d^2*i^2)*B^
2*x + 2*(b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*B^2*log(b*x + a))*log(
(d*x + c)^n)^2/(b^3*g) - integrate(-(B^2*b^3*c^3*i^2*log(e)^2 + 2*A*B*b^3*
c^3*i^2*log(e) + (B^2*b^3*d^3*i^2*log(e)^2 + 2*A*B*b^3*d^3*i^2*log(e))*x^3
 + 3*(B^2*b^3*c*d^2*i^2*log(e)^2 + 2*A*B*b^3*c*d^2*i^2*log(e))*x^2 + (B^2*
b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x + B^2*b^
3*c^3*i^2)*log((b*x + a)^n)^2 + 3*(B^2*b^3*c^2*d*i^2*log(e)^2 + 2*A*B*b^3*
c^2*d*i^2*log(e))*x + 2*(B^2*b^3*c^3*i^2*log(e) + A*B*b^3*c^3*i^2 + (B^2*b
^3*d^3*i^2*log(e) + A*B*b^3*d^3*i^2)*x^3 + 3*(B^2*b^3*c*d^2*i^2*log(e) + A
*B*b^3*c*d^2*i^2)*x^2 + 3*(B^2*b^3*c^2*d*i^2*log(e) + A*B*b^3*c^2*d*i^2)*x
)*log((b*x + a)^n) - (2*B^2*b^3*c^3*i^2*log(e) + 2*A*B*b^3*c^3*i^2 + (2*A*
B*b^3*d^3*i^2 + (i^2*n + 2*i^2*log(e))*B^2*b^3*d^3)*x^3 + (6*A*B*b^3*c*d^2
*i^2 - (a*b^2*d^3*i^2*n - 2*(2*i^2*n + 3*i^2*log(e))*b^3*c*d^2)*B^2)*x^2 +
 2*(3*A*B*b^3*c^2*d*i^2 + (2*a*b^2*c*d^2*i^2*n - a^2*b*d^3*i^2*n + 3*b^3*c
^2*d*i^2*log(e))*B^2)*x + 2*((b^3*c^2*d*i^2*n - 2*a*b^2*c*d^2*i^2*n + a^2*
b*d^3*i^2*n)*B^2*x + (a*b^2*c^2*d*i^2*n - 2*a^2*b*c*d^2*i^2*n + a^3*d^3*i^
2*n)*B^2)*log(b*x + a) + 2*(B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2
+ 3*B^2*b^3*c^2*d*i^2*x + B^2*b^3*c^3*i^2)*log((b*x + a)^n))*log((d*x +...
```

Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x,
algorithm="giac")
```

output

```
integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x
+ a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x),x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x), x)
```

Reduce [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{-2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{bx+a}dx\right)b^5c^2 - 4\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{bx+a}dx\right)a\,b^4c^2 - 2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 x^2}{bx+a}dx\right)b^5d^2 - 4\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{bx+a}dx\right)a\,b^4d^2 - 8\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)x}{bx+a}dx\right)b^5cd - 2\log(a+bx)a^4d^2 + 4\log(a+bx)a^3b^2cd - 2\log(a+bx)a^2b^2c^2 + 2a^3b^2d^2x - 4a^2b^2cdx - a^2b^2d^2x^2}{2b^3g}$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

output
```
( - 2*int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(a + b*x),x)*b**5*c**2 - 4
*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a + b*x),x)*a*b**4*c**2 - 2*int((
log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(a + b*x),x)*b**5*d**2 - 4*int
((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a + b*x),x)*b**5*c*d - 4*int((
log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(a + b*x),x)*a*b**4*d**2 - 8*int(
(log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a + b*x),x)*a*b**4*c*d - 2*log(a +
 b*x)*a**4*d**2 + 4*log(a + b*x)*a**3*b*c*d - 2*log(a + b*x)*a**2*b**2*c**
2 + 2*a**3*b*d**2*x - 4*a**2*b**2*c*d*x - a**2*b**2*d**2*x**2)/(2*b**3*g)
```

3.173 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result 1975
Mathematica [B] (verified) 1976
Rubi [A] (verified) 1977
Maple [F] 1978
Fricas [F] 1979
Sympy [F(-1)] 1979
Maxima [F] 1980
Giac [F] 1980
Mupad [F(-1)] 1981
Reduce [F] 1981

Optimal result

Integrand size = 45, antiderivative size = 472

$$
\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx \\
&= -\frac{2B^2(bc-ad)i^2n^2(c+dx)}{b^2g^2(a+bx)} - \frac{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^2(a+bx)} \\
&\quad + \frac{d^2i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^2} \\
&\quad - \frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2g^2(a+bx)} \\
&\quad + \frac{2Bd(bc-ad)i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^3g^2} \\
&\quad - \frac{2d(bc-ad)i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&\quad + \frac{2B^2d(bc-ad)i^2n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g^2} \\
&\quad + \frac{4Bd(bc-ad)i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&\quad + \frac{4B^2d(bc-ad)i^2n^2\,\mathrm{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}
\end{aligned}
$$

output

```
-2*B^2*(-a*d+b*c)*i^2*n^2*(d*x+c)/b^2/g^2/(b*x+a)-2*B*(-a*d+b*c)*i^2*n*(d*
x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^2/(b*x+a)+d^2*i^2*(b*x+a)*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))^2/b^3/g^2-(-a*d+b*c)*i^2*(d*x+c)*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))^2/b^2/g^2/(b*x+a)+2*B*d*(-a*d+b*c)*i^2*n*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^3/g^2-2*d*(-a*d+b*c)*i^2*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^2+2*B^2*d*(-a
*d+b*c)*i^2*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/g^2+4*B*d*(-a*d+b*c)*i^
2*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g^2
+4*B^2*d*(-a*d+b*c)*i^2*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^3/g^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2885 vs. $2(472) = 944$.

Time = 5.59 (sec) , antiderivative size = 2885, normalized size of antiderivative = 6.11

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Result too large to show}$$

input

```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x)^2,x]
```

output

```
(i^2*(3*b*d^2*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/
(c + d*x)])^2 - (3*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B
*n*Log[(a + b*x)/(c + d*x)])^2)/(a + b*x) + 6*d*(b*c - a*d)*Log[a + b*x]*(
A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 + (
6*b^2*B*c^2*n*(-A - B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(a + b*x)/(
c + d*x)])*(-(d*(a + b*x)*Log[c/d + x]) + d*(a + b*x)*Log[(d*(a + b*x))/(-
(b*c) + a*d)] + (b*c - a*d)*(1 + Log[(a + b*x)/(c + d*x)])))/((b*c - a*d)*
(a + b*x)) + (3*b^2*B^2*c^2*n^2*(-2*b*c + 2*a*d - 2*d*(a + b*x)*Log[a + b*
x] - 2*(b*c - a*d)*Log[(a + b*x)/(c + d*x)] - 2*d*(a + b*x)*Log[a + b*x]*L
og[(a + b*x)/(c + d*x)] - (b*c - a*d)*Log[(a + b*x)/(c + d*x)]^2 + 2*d*(a
+ b*x)*Log[c + d*x] - 2*d*(a + b*x)*Log[(a + b*x)/(c + d*x)]*Log[(b*c - a*
d)/(b*c + b*d*x)] + d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c
+ d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + d*(a
 + b*x)*(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a
*d)])))/((b*c - a*d)*(a + b*x)) + 6*b*B*c*d*n*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(Log[a/b + x]^2 - 2*Log[a/b + x]
*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[a
 + b*x]*((a*d)/(b*c - a*d) + Log[c/d + x] + Log[(a + b*x)/(c + d*x)]) + 2*
a*((a + b*x)^(-1) + Log[(a + b*x)/(c + d*x)]/(a + b*x) + (d*Log[c + d*x...
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^2}\,dx$$

$$\downarrow 2961$$

$$\frac{i^2(bc-ad)\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g^2}$$

$$\downarrow 2795$$

$$\frac{i^2(bc-ad)\int\left(\frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2(a+bx)^2}+\frac{2d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right)d\frac{a+bx}{c+dx}}{g^2}$$

$$\downarrow 2009$$

$$i^2(bc-ad)\left(\frac{d^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^3(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{4Bdn\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3}+\frac{2Bdn\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\right.}{b^3}$$

input

```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x)^2,x]
```

output
```
((b*c - a*d)*i^2*((-2*B^2*n^2*(c + d*x))/(b^2*(a + b*x)) - (2*B*n*(c + d*x
)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^2*(a + b*x)) - ((c + d*x)*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(b^2*(a + b*x)) + (d^2*(a + b*x)*(A
  + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(b^3*(c + d*x)*(b - (d*(a + b*x))/
(c + d*x))) + (2*B*d*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(
a + b*x))/(b*(c + d*x))])/b^3 - (2*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b^3 + (2*B^2*d*n^2*PolyLog[2, (d
*(a + b*x))/(b*(c + d*x))])/b^3 + (4*B*d*n*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b^3 + (4*B^2*d*n^2*PolyLo
g[3, (b*(c + d*x))/(d*(a + b*x))])/b^3))/g^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(dix+ci)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(bgx+ag)^2}dx$$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

output
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x
, algorithm="fricas")
```

output
```
integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x
^2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(
A*B*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log(e*((b*x + a)/(d*x + c
))^n))/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**
2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x
, algorithm="maxima")
```

output

```
-2*A*B*c^2*i^2*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d
)*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - A^2*(a^2/(b^4*g^2*x + a*b
^3*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^2))*d^2*i^2 + 2*A^2*c*d*i^
2*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - 2*A*B*c^2*i^2*log
(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^2*g^2*x + a*b*g^2) - A^2*c^2*i^2/(b
^2*g^2*x + a*b*g^2) + (B^2*b^2*d^2*i^2*x^2 + B^2*a*b*d^2*i^2*x - (b^2*c^2*
i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*B^2 + 2*((b^2*c*d*i^2 - a*b*d^2*i^2)*B^
2*x + (a*b*c*d*i^2 - a^2*d^2*i^2)*B^2)*log(b*x + a))*log((d*x + c)^n)^2/(b
^4*g^2*x + a*b^3*g^2) - integrate(-(B^2*b^3*c^3*i^2*log(e)^2 + (B^2*b^3*d^
3*i^2*log(e)^2 + 2*A*B*b^3*d^3*i^2*log(e))*x^3 + 3*(B^2*b^3*c*d^2*i^2*log(
e)^2 + 2*A*B*b^3*c*d^2*i^2*log(e))*x^2 + (B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*
c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x + B^2*b^3*c^3*i^2)*log((b*x + a)^n)^
2 + (3*B^2*b^3*c^2*d*i^2*log(e)^2 + 4*A*B*b^3*c^2*d*i^2*log(e))*x + 2*(B^2
*b^3*c^3*i^2*log(e) + (B^2*b^3*d^3*i^2*log(e) + A*B*b^3*d^3*i^2)*x^3 + 3*(
B^2*b^3*c*d^2*i^2*log(e) + A*B*b^3*c*d^2*i^2)*x^2 + (3*B^2*b^3*c^2*d*i^2*l
og(e) + 2*A*B*b^3*c^2*d*i^2)*x)*log((b*x + a)^n) - 2*((A*B*b^3*d^3*i^2 + (
i^2*n + i^2*log(e))*B^2*b^3*d^3)*x^3 - (a*b^2*c^2*d*i^2*n - 2*a^2*b*c*d^2*
i^2*n + a^3*d^3*i^2*n - b^3*c^3*i^2*log(e))*B^2 + (3*A*B*b^3*c*d^2*i^2 + (
2*a*b^2*d^3*i^2*n + 3*b^3*c*d^2*i^2*log(e))*B^2)*x^2 + (2*A*B*b^3*c^2*d*i^
2 + (2*a*b^2*c*d^2*i^2*n - (i^2*n - 3*i^2*log(e))*b^3*c^2*d)*B^2)*x + 2...
```

Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x
, algorithm="giac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x
+ a*g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(a\,g+b\,g\,x)^2}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^2,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^2, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

output

```
( - int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(a**2 + 2*a*b*x + b**
2*x**2),x)*a**3*b**5*d**3 + int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**
2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**6*c*d**2 - int((log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**6*d**
3*x + int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(a**2 + 2*a*b*x + b
**2*x**2),x)*a*b**7*c*d**2*x - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)**
2*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**3*b**5*c*d**2 + 2*int((log(((a + b
*x)**n*e)/(c + d*x)**n)**2*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**6*c*
*2*d - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a**2 + 2*a*b*x + b
**2*x**2),x)*a**2*b**6*c*d**2*x + 2*int((log(((a + b*x)**n*e)/(c + d*x)**n
)**2*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a*b**7*c**2*d*x - 2*int((log(((a +
 b*x)**n*e)/(c + d*x)**n)*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**4*b**4*
d**3 + 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(a**2 + 2*a*b*x + b
**2*x**2),x)*a**3*b**5*c*d**2 - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*
x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**3*b**5*d**3*x + 2*int((log(((a +
b*x)**n*e)/(c + d*x)**n)*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**6*c
*d**2*x - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a**2 + 2*a*b*x + b
**2*x**2),x)*a**4*b**4*c*d**2 + 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*
x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**3*b**5*c**2*d - 4*int((log(((a + b*x
)**n*e)/(c + d*x)**n)*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**3*b**5*c*d*...
```

3.174 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result . . . 1983
Mathematica [B] (verified) . . . 1984
Rubi [A] (verified) . . . 1985
Maple [F] . . . 1990
Fricas [F] . . . 1990
Sympy [F] . . . 1991
Maxima [F] . . . 1992
Giac [F(-1)] . . . 1992
Mupad [F(-1)] . . . 1993
Reduce [F] . . . 1993

Optimal result

Integrand size = 45, antiderivative size = 417

$$
\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx \\
&= -\frac{2B^2di^2n^2(c+dx)}{b^2g^3(a+bx)} - \frac{B^2i^2n^2(c+dx)^2}{4bg^3(a+bx)^2} - \frac{2Bdi^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^3(a+bx)} \\
&\quad - \frac{Bi^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2} \\
&\quad - \frac{di^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2g^3(a+bx)} - \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a+bx)^2} \\
&\quad - \frac{d^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} \\
&\quad + \frac{2Bd^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} \\
&\quad + \frac{2B^2d^2i^2n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}
\end{aligned}
$$

output

```
-2*B^2*d*i^2*n^2*(d*x+c)/b^2/g^3/(b*x+a)-1/4*B^2*i^2*n^2*(d*x+c)^2/b/g^3/(
b*x+a)^2-2*B*d*i^2*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^3/(b*x+
a)-1/2*B*i^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^3/(b*x+a)^2-d
*i^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2/g^3/(b*x+a)-1/2*i^2*(d*
x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b/g^3/(b*x+a)^2-d^2*i^2*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^3+2*B*d^2*i^2*n*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g^3+2*B^
2*d^2*i^2*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^3/g^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4535 vs. $2(417) = 834$.

Time $= 7.32$ (sec) , antiderivative size $= 4535$, normalized size of antiderivative $= 10.88$

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Result too large to show}$$

input

```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x)^3,x]
```

output

```
(d^2*i^2*Log[a + b*x]*(A + B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a +
b*x)/(c + d*x)]))^2)/(b^3*g^3) + (2*(-(A^2*b*c*d*i^2) + a*A^2*d^2*i^2 - 2*
A*b*B*c*d*i^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]
) + 2*a*A*B*d^2*i^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c +
 d*x)]) - b*B^2*c*d*i^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/
(c + d*x)])^2 + a*B^2*d^2*i^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a +
 b*x)/(c + d*x)])^2))/(b^3*g^3*(a + b*x)) + (-(A^2*b^2*c^2*i^2) + 2*a*A^2*
b*c*d*i^2 - a^2*A^2*d^2*i^2 - 2*A*b^2*B*c^2*i^2*(Log[e*((a + b*x)/(c + d*x
))^n] - n*Log[(a + b*x)/(c + d*x)]) + 4*a*A*b*B*c*d*i^2*(Log[e*((a + b*x)/
(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) - 2*a^2*A*B*d^2*i^2*(Log[e*((a
 + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) - b^2*B^2*c^2*i^2*(Log
[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2 + 2*a*b*B^2*c*
d*i^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2 - a^
2*B^2*d^2*i^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]
)^2)/(2*b^3*g^3*(a + b*x)^2) + (2*B*c^2*i^2*n*(A + B*(Log[e*((a + b*x)/(c
+ d*x))^n] - n*Log[(a + b*x)/(c + d*x)]))*(-1/8*((a/b + x)*(2*Log[a/b + x]
 + 4*Log[a/b + x]^2))/((a + b*x)^3*Log[a/b + x]) - ((b*(c/d + x))/((-a + (
b*c)/d)^3*(1 - (b*(c/d + x))/(-a + (b*c)/d))) - ((b^2*(c/d + x)^2)/((-a +
(b*c)/d)^4*(1 - (b*(c/d + x))/(-a + (b*c)/d))^2) + (2*b*(c/d + x))/((-a +
(b*c)/d)^3*(1 - (b*(c/d + x))/(-a + (b*c)/d))))*Log[c/d + x] - Log[1 - ...
```

Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2961, 2780, 2742, 2741, 2780, 2742, 2741, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{i^2 \int \frac{(c+dx)^3 \left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)} d\frac{a+bx}{c+dx}}{g^3}$$

$$\downarrow 2780$$

$$\frac{i^2\left(\frac{\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^3}$$

$$\downarrow 2742$$

$$\frac{i^2\left(\frac{Bn\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}}{b}+\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^3}$$

$$\downarrow 2741$$

$$\frac{i^2\left(\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{Bn\left(-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}\right)-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}}{b}\right)}{g^3}$$

$$\downarrow 2780$$

$$\frac{i^2\left(\frac{d\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{Bn\left(-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+}{4(a+b}\right.\right.}{b}}{g^3}$$

$$\downarrow 2742$$

$$i^2\left(\frac{d\left(\frac{2Bn\int\frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}+\frac{d\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{Bn\left(-\frac{(c+}{}\right.}{g^3}$$

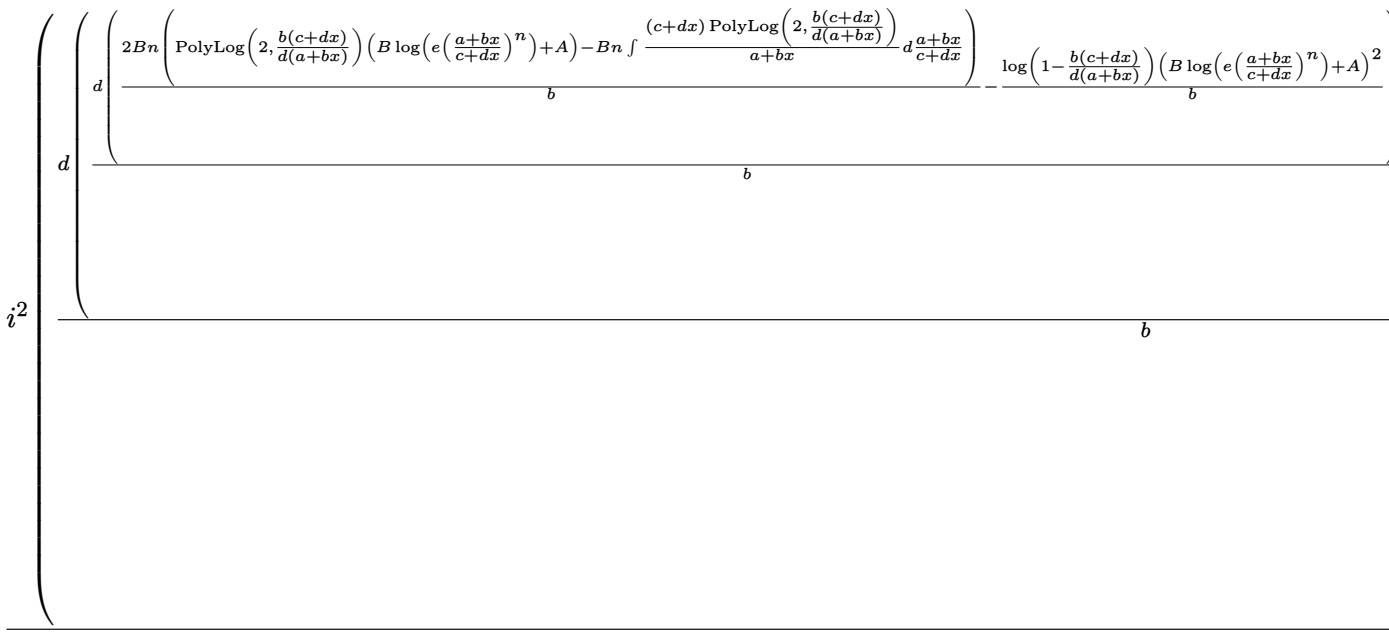
$$\downarrow 2741$$

$$i^2\left(\frac{d\left(\frac{d\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{2Bn\left(-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{b}\right)}{b}+\frac{Bn}{}\right.}{g^3}$$

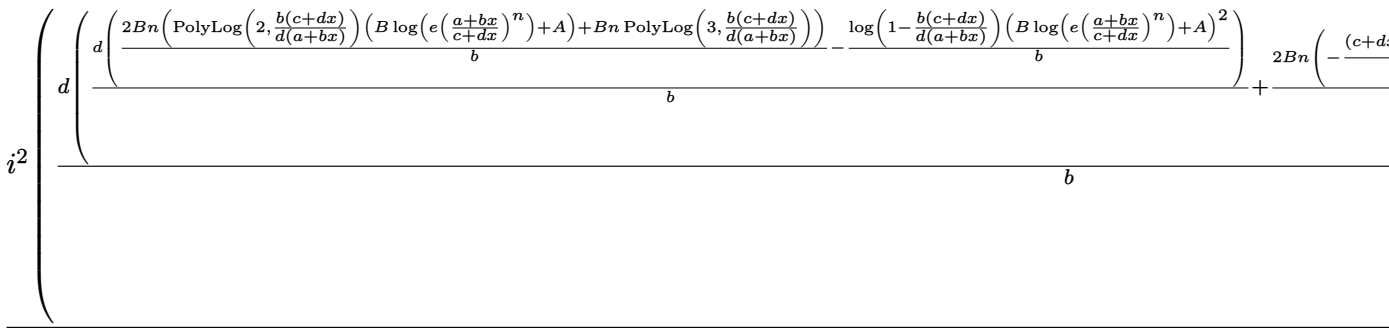
$$\downarrow 2779$$

$$i^2\left(\frac{d\left(\frac{d\left(\frac{2Bn\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}\right)}{b}+\frac{2Bn\left(-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{a+bx}\right.}{b}\right)$$

$$\downarrow 2821$$



↓ 7143



input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x)^3,x]
```

output
```
(i^2*((-1/2*((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*
x)^2 + B*n*(-1/4*(B*n*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2)))/b + (d*((-(((c + d*x)*(A + B
*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)) + 2*B*n*(-((B*n*(c + d*x))/
(a + b*x)) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x))
)/b + (d*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x)
)/(d*(a + b*x))])/b) + (2*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Poly
Log[2, (b*(c + d*x))/(d*(a + b*x))] + B*n*PolyLog[3, (b*(c + d*x))/(d*(a +
 b*x))]))/b))/b))/b))/g^3
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2821

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)  Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(dix+ci)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(bgx+ag)^3}dx$$

input

```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

output

```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x
, algorithm="fricas")
```

output
```
integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x
^2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(
A*B*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log(e*((b*x + a)/(d*x + c
))^n))/(b^3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$

$$= \frac{i^2\left(\int \frac{A^2c^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{A^2d^2x^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{B^2c^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{2ABc^2\log\left(e\left(\frac{a}{c+d}\right.\right.\right.}{a^3+3a^2bx+3ab}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**
3,x)
```

output
```
i**2*(Integral(A**2*c**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),
x) + Integral(A**2*d**2*x**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**
3), x) + Integral(B**2*c**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a*
*3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*c**2*log
(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 +
b**3*x**3), x) + Integral(2*A**2*c*d*x/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2
+ b**3*x**3), x) + Integral(B**2*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d
*x))**n)**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral
(2*A*B*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*
x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*B**2*c*d*x*log(e*(a/(c + d
*x) + b*x/(c + d*x))**n)**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3
), x) + Integral(4*A*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**3
 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x))/g**3
```

Maxima [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x
, algorithm="maxima")
```

output
```
-A*B*c*d*i^2*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c - a*b^4*
d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3)
+ 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*
g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*
d^2)*g^3)) + 1/2*A*B*c^2*i^2*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)
*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*
d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x
 + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) + 1/2*A^2*d^2*i^2*((4*a*b
*x + 3*a^2)/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(
b^3*g^3)) - 2*(2*b*x + a)*A*B*c*d*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - (2*b*x + a)*A^2*c*d*i^2/(
b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - A*B*c^2*i^2*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2*
c^2*i^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*(4*(b^2*c*d*i^2 -
a*b*d^2*i^2)*B^2*x + (b^2*c^2*i^2 + 2*a*b*c*d*i^2 - 3*a^2*d^2*i^2)*B^2 - 2
*(B^2*b^2*d^2*i^2*x^2 + 2*B^2*a*b*d^2*i^2*x + B^2*a^2*d^2*i^2)*log(b*x + a
))*log((d*x + c)^n)^2/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) - integr
ate(-(3*B^2*b^3*c^2*d*i^2*x*log(e)^2 + B^2*b^3*c^3*i^2*log(e)^2 + (B^2*b^3
*d^3*i^2*log(e)^2 + 2*A*B*b^3*d^3*i^2*log(e))*x^3 + (3*B^2*b^3*c*d^2*i^2*l
og(e)^2 + 2*A*B*b^3*c*d^2*i^2*log(e))*x^2 + (B^2*b^3*d^3*i^2*x^3 + 3*B^...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x
, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(a\,g+b\,g\,x)^3}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^3,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^3, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{too large to display}$$

input
```
int((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

output

```
( - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(a**3 + 3*a**2*b*x
+ 3*a*b**2*x**2 + b**3*x**3),x)*a**5*b**5*d**4 + 8*int((log(((a + b*x)**n*
e)/(c + d*x)**n)**2*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),
x)*a**4*b**6*c*d**3 - 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(
a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**4*b**6*d**4*x - 4*int
((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**
2*x**2 + b**3*x**3),x)*a**3*b**7*c**2*d**2 + 16*int((log(((a + b*x)**n*e)/
(c + d*x)**n)**2*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*
a**3*b**7*c*d**3*x - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(a
**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**3*b**7*d**4*x**2 - 8*i
nt((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(a**3 + 3*a**2*b*x + 3*a*b
**2*x**2 + b**3*x**3),x)*a**2*b**8*c**2*d**2*x + 8*int((log(((a + b*x)**n*
e)/(c + d*x)**n)**2*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),
x)*a**2*b**8*c*d**3*x**2 - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x*
*2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a*b**9*c**2*d**2*x*
*2 - 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(a**3 + 3*a**2*b*x +
3*a*b**2*x**2 + b**3*x**3),x)*a**6*b**4*d**4 + 16*int((log(((a + b*x)**n*e
)/(c + d*x)**n)*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a
**5*b**5*c*d**3 - 16*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(a**3 +
 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**5*b**5*d**4*x - 8*int((l...
```

3.175 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result . . . 1995
Mathematica [C] (verified) . . . 1996
Rubi [A] (verified) . . . 1997
Maple [B] (verified) . . . 1998
Fricas [B] (verification not implemented) . . . 1999
Sympy [F] . . . 2000
Maxima [B] (verification not implemented) . . . 2001
Giac [A] (verification not implemented) . . . 2002
Mupad [B] (verification not implemented) . . . 2003
Reduce [B] (verification not implemented) . . . 2004

Optimal result

Integrand size = 45, antiderivative size = 157

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{2B^2i^2n^2(c+dx)^3}{27(bc-ad)g^4(a+bx)^3} - \frac{2Bi^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)g^4(a+bx)^3} - \frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)g^4(a+bx)^3}$$

output

```
-2/27*B^2*i^2*n^2*(d*x+c)^3/(-a*d+b*c)/g^4/(b*x+a)^3-2/9*B*i^2*n*(d*x+c)^3
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/g^4/(b*x+a)^3-1/3*i^2*(d*x+c)^
3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/g^4/(b*x+a)^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.43 (sec) , antiderivative size = 1418, normalized size of antiderivative = 9.03

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input

```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x)^4,x]
```

output

```
-1/54*(i^2*(18*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 54
*d*(b*c - a*d)^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - 54*d
^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + B
*n*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3*n - 18*A*d*(b*c - a*d)^2*(a + b
*x) - 15*B*d*(b*c - a*d)^2*n*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2
+ 66*B*d^2*(b*c - a*d)*n*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] +
 66*B*d^3*n*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*n*(a + b*x)^3*Log[a + b*x]
^2 + 12*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n] - 18*B*d*(b*c - a*d
)^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^2*(b*c - a*d)*(a + b
*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*L
og[e*((a + b*x)/(c + d*x))^n] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d
^3*n*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*n*(a + b*x)^3*Log[(d*(a + b*x))/(
-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[e*((a + b*x)/(c + d
*x))^n]*Log[c + d*x] - 18*B*d^3*n*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*n*
(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*n*(a +
b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*n*(a + b*x)^3*P
olyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 54*B*d^2*n*(a + b*x)^2*(2*(b*c - a
*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d*(a + b*x)*Log[a + b*x]*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*d*(a + b*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])*Log[c + d*x] + 2*B*n*(b*c - a*d + d*(a + b*x)*Log[a +...
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^4}\,dx$$

$$\downarrow 2961$$

$$\frac{i^2\int \frac{(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)}$$

$$\downarrow 2742$$

$$\frac{i^2\left(\frac{2}{3}Bn\int \frac{(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}\right)}{g^4(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{i^2\left(\frac{2}{3}Bn\left(-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}-\frac{Bn(c+dx)^3}{9(a+bx)^3}\right)-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}\right)}{g^4(bc-ad)}$$

input

```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x)^4,x]
```

output

```
(i^2*(-1/3*((c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x
)^3 + (2*B*n*(-1/9*(B*n*(c + d*x)^3)/(a + b*x)^3 - ((c + d*x)^3*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^3)))/3))/((b*c - a*d)*g^4)
```

Defintions of rubi rules used

rule 2741

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 824 vs. 2(151) = 302.

Time = 10.30 (sec) , antiderivative size = 825, normalized size of antiderivative = 5.25

method	result
parallelrisch	$-\frac{18ABx^2ab^4d^4i^2n^2-54ABx^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^5cd^3i^2n-54ABx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^5c^2d^2i^2n-9B^2x^3\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2b^5d^4$

input

```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,method
=_RETURNVERBOSE)
```

output

```
-1/27*(18*A*B*x^2*a*b^4*d^4*i^2*n^2-54*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b
^5*c*d^3*i^2*n-54*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^2*d^2*i^2*n-9*B^2*
x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^5*d^4*i^2*n-6*B^2*x^3*ln(e*((b*x+a)/(d*x
+c))^n)*b^5*d^4*i^2*n^2+6*B^2*x^2*a*b^4*d^4*i^2*n^3-6*B^2*x^2*b^5*c*d^3*i^
2*n^3+6*B^2*x*a^2*b^3*d^4*i^2*n^3-6*B^2*x*b^5*c^2*d^2*i^2*n^3+27*A^2*x^2*a
*b^4*d^4*i^2*n-27*A^2*x^2*b^5*c*d^3*i^2*n+6*A*B*a^3*b^2*d^4*i^2*n^2-6*A*B*
b^5*c^3*d*i^2*n^2-18*A*B*x^2*b^5*c*d^3*i^2*n^2-27*B^2*x*ln(e*((b*x+a)/(d*x
+c))^n)^2*b^5*c^2*d^2*i^2*n-18*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^2*d^2
*i^2*n^2+18*A*B*x*a^2*b^3*d^4*i^2*n^2-18*A*B*x*b^5*c^2*d^2*i^2*n^2-18*A*B*
ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^3*d*i^2*n-18*A*B*x^3*ln(e*((b*x+a)/(d*x+c)
)^n)*b^5*d^4*i^2*n-27*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^5*c*d^3*i^2*n-
18*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d^3*i^2*n^2+2*B^2*a^3*b^2*d^4*i
^2*n^3-2*B^2*b^5*c^3*d*i^2*n^3+9*A^2*a^3*b^2*d^4*i^2*n-9*A^2*b^5*c^3*d*i^2
*n-9*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^5*c^3*d*i^2*n-6*B^2*ln(e*((b*x+a)/(
d*x+c))^n)*b^5*c^3*d*i^2*n^2+27*A^2*x*a^2*b^3*d^4*i^2*n-27*A^2*x*b^5*c^2*d
^2*i^2*n)/g^4/(b*x+a)^3/b^5/d/n/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(151) = 302$.

Time = 0.10 (sec) , antiderivative size = 975, normalized size of antiderivative = 6.21

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x
, algorithm="fricas")
```

output

```
-1/27*(2*(B^2*b^3*c^3 - B^2*a^3*d^3)*i^2*n^2 + 6*(A*B*b^3*c^3 - A*B*a^3*d^
3)*i^2*n + 9*(A^2*b^3*c^3 - A^2*a^3*d^3)*i^2 + 3*(2*(B^2*b^3*c*d^2 - B^2*a
*b^2*d^3)*i^2*n^2 + 6*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*i^2*n + 9*(A^2*b^3*c
*d^2 - A^2*a*b^2*d^3)*i^2)*x^2 + 9*(3*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*i^2*
x^2 + 3*(B^2*b^3*c^2*d - B^2*a^2*b*d^3)*i^2*x + (B^2*b^3*c^3 - B^2*a^3*d^3
)*i^2)*log(e)^2 + 9*(B^2*b^3*d^3*i^2*n^2*x^3 + 3*B^2*b^3*c*d^2*i^2*n^2*x^2
 + 3*B^2*b^3*c^2*d*i^2*n^2*x + B^2*b^3*c^3*i^2*n^2)*log((b*x + a)/(d*x + c
))^2 + 3*(2*(B^2*b^3*c^2*d - B^2*a^2*b*d^3)*i^2*n^2 + 6*(A*B*b^3*c^2*d - A
*B*a^2*b*d^3)*i^2*n + 9*(A^2*b^3*c^2*d - A^2*a^2*b*d^3)*i^2)*x + 6*((B^2*b
^3*c^3 - B^2*a^3*d^3)*i^2*n + 3*(A*B*b^3*c^3 - A*B*a^3*d^3)*i^2 + 3*((B^2*
b^3*c*d^2 - B^2*a*b^2*d^3)*i^2*n + 3*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*i^2)*
x^2 + 3*((B^2*b^3*c^2*d - B^2*a^2*b*d^3)*i^2*n + 3*(A*B*b^3*c^2*d - A*B*a^
2*b*d^3)*i^2)*x + 3*(B^2*b^3*d^3*i^2*n*x^3 + 3*B^2*b^3*c*d^2*i^2*n*x^2 + 3
*B^2*b^3*c^2*d*i^2*n*x + B^2*b^3*c^3*i^2*n)*log((b*x + a)/(d*x + c)))*log(
e) + 6*(B^2*b^3*c^3*i^2*n^2 + 3*A*B*b^3*c^3*i^2*n + (B^2*b^3*d^3*i^2*n^2 +
 3*A*B*b^3*d^3*i^2*n)*x^3 + 3*(B^2*b^3*c*d^2*i^2*n^2 + 3*A*B*b^3*c*d^2*i^2
*n)*x^2 + 3*(B^2*b^3*c^2*d*i^2*n^2 + 3*A*B*b^3*c^2*d*i^2*n)*x)*log((b*x +
a)/(d*x + c)))/((b^7*c - a*b^6*d)*g^4*x^3 + 3*(a*b^6*c - a^2*b^5*d)*g^4*x^
2 + 3*(a^2*b^5*c - a^3*b^4*d)*g^4*x + (a^3*b^4*c - a^4*b^3*d)*g^4)
```

Sympy [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$$

$$= i^2\left(\int \frac{A^2c^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \int \frac{A^2d^2x^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \int \frac{B^2c^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx\right.$$

input

```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**
4,x)
```

output

```
i**2*(Integral(A**2*c**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*
x**3 + b**4*x**4), x) + Integral(A**2*d**2*x**2/(a**4 + 4*a**3*b*x + 6*a**
2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(B**2*c**2*log(e*(a
/(c + d*x) + b*x/(c + d*x))**n)**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 +
 4*a*b**3*x**3 + b**4*x**4), x) + Integral(2*A*B*c**2*log(e*(a/(c + d*x) +
 b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3
+ b**4*x**4), x) + Integral(2*A**2*c*d*x/(a**4 + 4*a**3*b*x + 6*a**2*b**2*
x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(B**2*d**2*x**2*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n)**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4
*a*b**3*x**3 + b**4*x**4), x) + Integral(2*A*B*d**2*x**2*log(e*(a/(c + d*x
) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x*
*3 + b**4*x**4), x) + Integral(2*B**2*c*d*x*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*
x**4), x) + Integral(4*A*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(
a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x))/g**
4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5588 vs. $2(151) = 302$.

Time $= 0.36$ (sec) , antiderivative size $= 5588$, normalized size of antiderivative $= 35.59$

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x
, algorithm="maxima")
```

output

```
-1/9*A*B*d^2*i^2*n*((11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*c
^2 - 3*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a
^3*b*d^2)*x)/((b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c^2
 - 2*a^2*b^6*c*d + a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d +
 a^4*b^4*d^2)*g^4*x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6
*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^
2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 +
 a^2*d^3)*log(d*x + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b
^3*d^3)*g^4)) - 1/9*A*B*c^2*i^2*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d
+ 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^
4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(
a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^
2*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3
*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*
c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/9*A*B*c*d*i^2*n*((5*a*b^2*c
^2 - 22*a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c
^2 - 16*a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)
*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^
5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d
+ a^5*b^2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*...
```

Giac [A] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.38

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{1}{27}\left(\frac{9\,(dx+c)^3B^2i^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)^3g^4}+\frac{6\,(B^2i^2n^2+3\,B^2i^2n\log(e)+3\,ABi^2n)(dx+c)^3\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^3g^4}+\frac{(}{}\right.$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x
, algorithm="giac")
```

output
```
-1/27*(9*(d*x + c)^3*B^2*i^2*n^2*log((b*x + a)/(d*x + c))^2/((b*x + a)^3*g
^4) + 6*(B^2*i^2*n^2 + 3*B^2*i^2*n*log(e) + 3*A*B*i^2*n)*(d*x + c)^3*log((
b*x + a)/(d*x + c))/((b*x + a)^3*g^4) + (2*B^2*i^2*n^2 + 6*B^2*i^2*n*log(e
) + 9*B^2*i^2*log(e)^2 + 6*A*B*i^2*n + 18*A*B*i^2*log(e) + 9*A^2*i^2)*(d*x
 + c)^3/((b*x + a)^3*g^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 29.03 (sec) , antiderivative size = 1195, normalized size of antiderivative = 7.61

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^4,x)
```

output
```
- (x*(9*A^2*a*b*d^2*i^2 + 9*A^2*b^2*c*d*i^2 + 2*B^2*a*b*d^2*i^2*n^2 + 2*B^
2*b^2*c*d*i^2*n^2 + 6*A*B*a*b*d^2*i^2*n + 6*A*B*b^2*c*d*i^2*n) + x^2*(9*A^
2*b^2*d^2*i^2 + 2*B^2*b^2*d^2*i^2*n^2 + 6*A*B*b^2*d^2*i^2*n) + 3*A^2*a^2*d
^2*i^2 + 3*A^2*b^2*c^2*i^2 + (2*B^2*a^2*d^2*i^2*n^2)/3 + (2*B^2*b^2*c^2*i^
2*n^2)/3 + 3*A^2*a*b*c*d*i^2 + 2*A*B*a^2*d^2*i^2*n + 2*A*B*b^2*c^2*i^2*n +
 (2*B^2*a*b*c*d*i^2*n^2)/3 + 2*A*B*a*b*c*d*i^2*n)/(9*a^3*b^3*g^4 + 9*b^6*g
^4*x^3 + 27*a^2*b^4*g^4*x + 27*a*b^5*g^4*x^2) - log(e*((a + b*x)/(c + d*x)
)^n)*((a*(2*A*B*a*d^2*i^2 - B^2*a*d^2*i^2*n + B^2*b*c*d*i^2*n + 2*A*B*b*c*
d*i^2) + x*(b*(2*A*B*a*d^2*i^2 - B^2*a*d^2*i^2*n + B^2*b*c*d*i^2*n + 2*A*B
*b*c*d*i^2) + 4*A*B*a*b*d^2*i^2 + 4*A*B*b^2*c*d*i^2 - 2*B^2*a*b*d^2*i^2*n
+ 2*B^2*b^2*c*d*i^2*n) + 2*A*B*b^2*c^2*i^2 - 2*B^2*a^2*d^2*i^2*n + 6*A*B*b
^2*d^2*i^2*x^2 + 2*B^2*a*b*c*d*i^2*n)/(3*a^3*b^3*g^4 + 3*b^6*g^4*x^3 + 9*a
^2*b^4*g^4*x + 9*a*b^5*g^4*x^2) + (2*B^2*d^3*i^2*(x*(b*((a*b^3*g^4*n*(a*d
- b*c))/d + (b^3*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2)) + (2*a*b^4*g^4*
n*(a*d - b*c))/d + (b^4*g^4*n*(a*d - b*c)*(3*a*d - b*c))/d^2) + a*((a*b^3*
g^4*n*(a*d - b*c))/d + (b^3*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2)) + (3
*b^5*g^4*n*x^2*(a*d - b*c))/d + (b^3*g^4*n*(a*d - b*c)*(3*a^2*d^2 + b^2*c^
2 - 3*a*b*c*d))/d^3))/(3*b^3*g^4*(a*d - b*c)*(3*a^3*b^3*g^4 + 3*b^6*g^4*x^
3 + 9*a^2*b^4*g^4*x + 9*a*b^5*g^4*x^2))) - log(e*((a + b*x)/(c + d*x))^n)^
2*((a*((B^2*c*d*i^2)/(3*b^2) + (B^2*a*d^2*i^2)/(3*b^3)) + x*(b*((B^2*c*...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1161, normalized size of antiderivative = 7.39

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x)
```

output

```
( - 18*log(a + b*x)*a**4*b*c*d**2*n - 6*log(a + b*x)*a**3*b**2*c*d**2*n**2
 - 54*log(a + b*x)*a**3*b**2*c*d**2*n*x - 18*log(a + b*x)*a**2*b**3*c*d**2
*n**2*x - 54*log(a + b*x)*a**2*b**3*c*d**2*n*x**2 - 18*log(a + b*x)*a*b**4
*c*d**2*n**2*x**2 - 18*log(a + b*x)*a*b**4*c*d**2*n*x**3 - 6*log(a + b*x)*
b**5*c*d**2*n**2*x**3 + 18*log(c + d*x)*a**4*b*c*d**2*n + 6*log(c + d*x)*a
**3*b**2*c*d**2*n**2 + 54*log(c + d*x)*a**3*b**2*c*d**2*n*x + 18*log(c + d
*x)*a**2*b**3*c*d**2*n**2*x + 54*log(c + d*x)*a**2*b**3*c*d**2*n*x**2 + 18
*log(c + d*x)*a*b**4*c*d**2*n**2*x**2 + 18*log(c + d*x)*a*b**4*c*d**2*n*x*
*3 + 6*log(c + d*x)*b**5*c*d**2*n**2*x**3 - 9*log(((a + b*x)**n*e)/(c + d*
x)**n)**2*a*b**4*c**3 - 27*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**4*c*
*2*d*x - 27*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**4*c*d**2*x**2 - 9*l
og(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**4*d**3*x**3 + 18*log(((a + b*x)*
*n*e)/(c + d*x)**n)*a**4*b*c*d**2 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a
**3*b**2*c*d**2*n + 54*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**2*c*d**2
*x - 18*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*c**3 - 54*log(((a + b
*x)**n*e)/(c + d*x)**n)*a**2*b**3*c**2*d*x + 18*log(((a + b*x)**n*e)/(c +
d*x)**n)*a**2*b**3*c*d**2*n*x - 18*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2
*b**3*d**3*x**3 - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**4*c**3*n - 18*
log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**4*c**2*d*n*x + 18*log(((a + b*x)**
n*e)/(c + d*x)**n)*a*b**4*c*d**2*x**3 - 6*log(((a + b*x)**n*e)/(c + d*x...
```

3.176 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result 2005
Mathematica [C] (verified) 2006
Rubi [A] (verified) 2007
Maple [B] (verified) 2009
Fricas [B] (verification not implemented) 2010
Sympy [F(-1)] 2011
Maxima [B] (verification not implemented) 2012
Giac [A] (verification not implemented) 2013
Mupad [B] (verification not implemented) 2013
Reduce [B] (verification not implemented) 2014

Optimal result

Integrand size = 45, antiderivative size = 319

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx &= \frac{2B^2di^2n^2(c+dx)^3}{27(bc-ad)^2g^5(a+bx)^3}\\ &\quad-\frac{bB^2i^2n^2(c+dx)^4}{32(bc-ad)^2g^5(a+bx)^4}\\ &\quad+\frac{2Bdi^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^2g^5(a+bx)^3}\\ &\quad-\frac{bBi^2n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^2g^5(a+bx)^4}\\ &\quad+\frac{di^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^2g^5(a+bx)^3}\\ &\quad-\frac{bi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4(bc-ad)^2g^5(a+bx)^4}\end{aligned}$$

output

```
2/27*B^2*d*i^2*n^2*(d*x+c)^3/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/32*b*B^2*i^2*n^2
*(d*x+c)^4/(-a*d+b*c)^2/g^5/(b*x+a)^4+2/9*B*d*i^2*n*(d*x+c)^3*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/8*b*B*i^2*n*(d*x+c)^4*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/g^5/(b*x+a)^4+1/3*d*i^2*(d*x+c)^
3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/4*b*i^2*(
d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^5/(b*x+a)^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 3.69 (sec) , antiderivative size = 1787, normalized size of antiderivative = 5.60

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x)^5,x]
```

output

```
-1/864*(i^2*(216*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 -
576*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
+ 432*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2 + 32*B*d*n*(a + b*x)*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3*n - 18*A*d
*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*n*(a + b*x) + 36*A*d^2*(b*
c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*n*(a + b*x)^2 + 36*A*d^3*(a +
b*x)^3*Log[a + b*x] + 66*B*d^3*n*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*n*(a
+ b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n
] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d
^2*(b*c - a*d)*(a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^3*(a +
b*x)^3*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n] - 36*A*d^3*(a + b*x)^3*
Log[c + d*x] - 66*B*d^3*n*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*n*(a + b*x)^
3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Lo
g[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x] - 18*B*d^3*n*(a + b*x)^3*Log[c +
 d*x]^2 + 36*B*d^3*n*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d
)] + 36*B*d^3*n*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*
B*d^3*n*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 3*B*n*(36*A*(
b*c - a*d)^4 + 9*B*(b*c - a*d)^4*n + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 2
8*B*d*(-(b*c) + a*d)^3*n*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 +
78*B*d^2*(b*c - a*d)^2*n*(a + b*x)^2 + 144*A*d^3*(-(b*c) + a*d)*(a + b*...
```

Rubi [A] (verified)

Time $= 0.53$ (sec) , antiderivative size $= 240$, normalized size of antiderivative $= 0.75$, number of steps used $= 4$, number of rules used $= 3$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used $= \{2961, 2795, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^5}\,dx$$

$$\downarrow 2961$$

$$\frac{i^2\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^2}$$

$$\downarrow 2795$$

$$\frac{i^2\int\left(\frac{b(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^5}-\frac{d(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}\right)d\frac{a+bx}{c+dx}}{g^5(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{i^2\left(-\frac{b(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4(a+bx)^4}-\frac{bBn(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{8(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}+\frac{2Bdn(c+dx)^3}{}\right.}{g^5(bc-ad)^2}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x)^5,x]
```

output
```
(i^2*((2*B^2*d*n^2*(c + d*x)^3)/(27*(a + b*x)^3) - (b*B^2*n^2*(c + d*x)^4)
/(32*(a + b*x)^4) + (2*B*d*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(9*(a + b*x)^3) - (b*B*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]))/(8*(a + b*x)^4) + (d*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^2)/(3*(a + b*x)^3) - (b*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])^2)/(4*(a + b*x)^4)))/((b*c - a*d)^2*g^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1883 vs. $2(307) = 614$.

Time = 29.84 (sec) , antiderivative size = 1884, normalized size of antiderivative = 5.91

method	result	size
parallelrisch	Expression too large to display	1884

input

```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,method
=_RETURNVERBOSE)
```

output

```
1/864*(864*A^2*x^3*a^3*b^4*c^5*i^2*n+192*B^2*x*a^7*c^3*d^2*i^2*n^3+108*B^2
*x*a^5*b^2*c^5*i^2*n^3+27*B^2*x^4*a^2*b^5*c^5*i^2*n^3+64*B^2*x^3*a^7*c*d^4
*i^2*n^3+108*B^2*x^3*a^3*b^4*c^5*i^2*n^3+216*A^2*x^4*a^2*b^5*c^5*i^2*n+192
*B^2*x^2*a^7*c^2*d^3*i^2*n^3+162*B^2*x^2*a^4*b^3*c^5*i^2*n^3+288*A^2*x^3*a
^7*c*d^4*i^2*n+576*A*B*x*a^7*c^3*d^2*i^2*n^2+432*A*B*x*a^5*b^2*c^5*i^2*n^2
-1728*A^2*x*a^6*b*c^4*d*i^2*n+576*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^7*c^4*d*
i^2*n-432*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^5*i^2*n+576*A*B*x^2*a^7*c^
2*d^3*i^2*n^2+648*A*B*x^2*a^4*b^3*c^5*i^2*n^2+864*B^2*x*ln(e*((b*x+a)/(d*x
+c))^n)^2*a^7*c^3*d^2*i^2*n+576*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*c^3*d^
2*i^2*n^2-300*B^2*x*a^6*b*c^4*d*i^2*n^3-432*A^2*x^2*a^6*b*c^3*d^2*i^2*n-17
28*A^2*x^2*a^5*b^2*c^4*d*i^2*n+864*A^2*x^2*a^7*c^2*d^3*i^2*n+1296*A^2*x^2*
a^4*b^3*c^5*i^2*n+288*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^7*c^4*d*i^2*n-216*
B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b*c^5*i^2*n+192*B^2*ln(e*((b*x+a)/(d*x
+c))^n)*a^7*c^4*d*i^2*n^2-108*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^5*i^2*
n^2+864*A^2*x*a^7*c^3*d^2*i^2*n+864*A^2*x*a^5*b^2*c^5*i^2*n+144*A*B*x^4*ln
(e*((b*x+a)/(d*x+c))^n)*a^6*b*c*d^4*i^2*n-864*A*B*x^2*ln(e*((b*x+a)/(d*x+c
))^n)*a^6*b*c^3*d^2*i^2*n-1152*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^4*d
*i^2*n-1152*A*B*x^2*a^5*b^2*c^4*d*i^2*n^2-576*B^2*x*ln(e*((b*x+a)/(d*x+c))
^n)^2*a^6*b*c^4*d*i^2*n-240*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^4*d*i^
2*n^2+1728*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*c^3*d^2*i^2*n-1008*A*B*x...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1729 vs. $2(307) = 614$.

Time = 0.14 (sec) , antiderivative size = 1729, normalized size of antiderivative = 5.42

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x
, algorithm="fricas")
```

output

```
-1/864*((27*B^2*b^4*c^4 - 64*B^2*a*b^3*c^3*d + 37*B^2*a^4*d^4)*i^2*n^2 + 1
2*(9*A*B*b^4*c^4 - 16*A*B*a*b^3*c^3*d + 7*A*B*a^4*d^4)*i^2*n - 12*(7*(B^2*
b^4*c*d^3 - B^2*a*b^3*d^4)*i^2*n^2 + 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*i^
2*n)*x^3 + 72*(3*A^2*b^4*c^4 - 4*A^2*a*b^3*c^3*d + A^2*a^4*d^4)*i^2 - 6*((
5*B^2*b^4*c^2*d^2 + 32*B^2*a*b^3*c*d^3 - 37*B^2*a^2*b^2*d^4)*i^2*n^2 - 12*
(A*B*b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + 7*A*B*a^2*b^2*d^4)*i^2*n - 72*(A^2*
b^4*c^2*d^2 - 2*A^2*a*b^3*c*d^3 + A^2*a^2*b^2*d^4)*i^2)*x^2 + 72*(6*(B^2*b
^4*c^2*d^2 - 2*B^2*a*b^3*c*d^3 + B^2*a^2*b^2*d^4)*i^2*x^2 + 4*(2*B^2*b^4*c
^3*d - 3*B^2*a*b^3*c^2*d^2 + B^2*a^3*b*d^4)*i^2*x + (3*B^2*b^4*c^4 - 4*B^2
*a*b^3*c^3*d + B^2*a^4*d^4)*i^2)*log(e)^2 - 72*(B^2*b^4*d^4*i^2*n^2*x^4 +
4*B^2*a*b^3*d^4*i^2*n^2*x^3 - 6*(B^2*b^4*c^2*d^2 - 2*B^2*a*b^3*c*d^3)*i^2*
n^2*x^2 - 4*(2*B^2*b^4*c^3*d - 3*B^2*a*b^3*c^2*d^2)*i^2*n^2*x - (3*B^2*b^4
*c^4 - 4*B^2*a*b^3*c^3*d)*i^2*n^2)*log((b*x + a)/(d*x + c))^2 + 4*((11*B^2
*b^4*c^3*d - 48*B^2*a*b^3*c^2*d^2 + 37*B^2*a^3*b*d^4)*i^2*n^2 + 12*(5*A*B*
b^4*c^3*d - 12*A*B*a*b^3*c^2*d^2 + 7*A*B*a^3*b*d^4)*i^2*n + 72*(2*A^2*b^4*
c^3*d - 3*A^2*a*b^3*c^2*d^2 + A^2*a^3*b*d^4)*i^2)*x - 12*(12*(B^2*b^4*c*d^
3 - B^2*a*b^3*d^4)*i^2*n*x^3 - (9*B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d + 7*B^2
*a^4*d^4)*i^2*n - 12*(3*A*B*b^4*c^4 - 4*A*B*a*b^3*c^3*d + A*B*a^4*d^4)*i^2
 - 6*((B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + 7*B^2*a^2*b^2*d^4)*i^2*n + 12
*(A*B*b^4*c^2*d^2 - 2*A*B*a*b^3*c*d^3 + A*B*a^2*b^2*d^4)*i^2)*x^2 - 4*(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Timed out}$$

input

```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**
5,x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8087 vs. $2(307) = 614$.

Time $= 0.56$ (sec) , antiderivative size $= 8087$, normalized size of antiderivative $= 25.35$

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x
, algorithm="maxima")
```

output

```
1/24*A*B*c^2*i^2*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*
b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*
a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2
 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2
 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d
^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c
*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d
^2 - a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a
^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/(
(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4
)*g^5)) - 1/72*A*B*d^2*i^2*n*((13*a^2*b^3*c^3 - 75*a^3*b^2*c^2*d + 33*a^4*
b*c*d^2 - 7*a^5*d^3 - 12*(6*b^5*c^2*d - 4*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3 +
 6*(6*b^5*c^3 - 46*a*b^4*c^2*d + 29*a^2*b^3*c*d^2 - 7*a^3*b^2*d^3)*x^2 + 4
*(10*a*b^4*c^3 - 63*a^2*b^3*c^2*d + 33*a^3*b^2*c*d^2 - 7*a^4*b*d^3)*x)/((b
^10*c^3 - 3*a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*g^5*x^4 + 4*(a*b^
9*c^3 - 3*a^2*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*g^5*x^3 + 6*(a^2*
b^8*c^3 - 3*a^3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*g^5*x^2 + 4*(a^
3*b^7*c^3 - 3*a^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*g^5*x + (a^4*
b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3)*g^5) - 12*(6*b^
2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*log(b*x + a)/((b^7*c^4 - 4*a*b^6*c^3...
```

Giac [A] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.70

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{1}{864}\left(\frac{72\left(3\,B^2bi^2n^2-\frac{4\,(bx+a)B^2di^2n^2}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)^2}{\frac{(bx+a)^4bcg^5}{(dx+c)^4}-\frac{(bx+a)^4adg^5}{(dx+c)^4}}+\frac{12\left(9\,B^2bi^2n^2-\frac{16\,(bx+a)B^2di^2n^2}{dx+c}+36\,B^2bi^2n\log\left(\right.\right.\right.$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x
, algorithm="giac")
```

output
```
-1/864*(72*(3*B^2*b*i^2*n^2 - 4*(b*x + a)*B^2*d*i^2*n^2/(d*x + c))*log((b*
x + a)/(d*x + c))^2/((b*x + a)^4*b*c*g^5/(d*x + c)^4 - (b*x + a)^4*a*d*g^5
/(d*x + c)^4) + 12*(9*B^2*b*i^2*n^2 - 16*(b*x + a)*B^2*d*i^2*n^2/(d*x + c)
 + 36*B^2*b*i^2*n*log(e) - 48*(b*x + a)*B^2*d*i^2*n*log(e)/(d*x + c) + 36*
A*B*b*i^2*n - 48*(b*x + a)*A*B*d*i^2*n/(d*x + c))*log((b*x + a)/(d*x + c))
/((b*x + a)^4*b*c*g^5/(d*x + c)^4 - (b*x + a)^4*a*d*g^5/(d*x + c)^4) + (27
*B^2*b*i^2*n^2 - 64*(b*x + a)*B^2*d*i^2*n^2/(d*x + c) + 108*B^2*b*i^2*n*lo
g(e) - 192*(b*x + a)*B^2*d*i^2*n*log(e)/(d*x + c) + 216*B^2*b*i^2*log(e)^2
 - 288*(b*x + a)*B^2*d*i^2*log(e)^2/(d*x + c) + 108*A*B*b*i^2*n - 192*(b*x
 + a)*A*B*d*i^2*n/(d*x + c) + 432*A*B*b*i^2*log(e) - 576*(b*x + a)*A*B*d*i
^2*log(e)/(d*x + c) + 216*A^2*b*i^2 - 288*(b*x + a)*A^2*d*i^2/(d*x + c))/(
(b*x + a)^4*b*c*g^5/(d*x + c)^4 - (b*x + a)^4*a*d*g^5/(d*x + c)^4))*(b*c/(
b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 30.38 (sec) , antiderivative size = 1934, normalized size of antiderivative = 6.06

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^5,x)
```

output

```
- log(e*((a + b*x)/(c + d*x))^n)*((a*(A*B*a*d^2*i^2 - (B^2*a*d^2*i^2*n)/2
+ (B^2*b*c*d*i^2*n)/2 + 2*A*B*b*c*d*i^2) + x*(b*(A*B*a*d^2*i^2 - (B^2*a*d^
2*i^2*n)/2 + (B^2*b*c*d*i^2*n)/2 + 2*A*B*b*c*d*i^2) + 3*A*B*a*b*d^2*i^2 +
6*A*B*b^2*c*d*i^2 - (3*B^2*a*b*d^2*i^2*n)/2 + (3*B^2*b^2*c*d*i^2*n)/2) + 3
*A*B*b^2*c^2*i^2 - B^2*a^2*d^2*i^2*n + (B^2*b^2*c^2*i^2*n)/2 + 6*A*B*b^2*d
^2*i^2*x^2 + (B^2*a*b*c*d*i^2*n)/2)/(6*a^4*b^3*g^5 + 6*b^7*g^5*x^4 + 24*a^
3*b^4*g^5*x + 24*a*b^6*g^5*x^3 + 36*a^2*b^5*g^5*x^2) + (B^2*d^4*i^2*(x^2*(
b*(b*((3*a*b^3*g^5*n*(a*d - b*c))/(2*d) + (b^3*g^5*n*(a*d - b*c)*(4*a*d -
b*c))/(2*d^2)) + (3*a*b^4*g^5*n*(a*d - b*c))/d + (b^4*g^5*n*(a*d - b*c)*(4
*a*d - b*c))/d^2) + (9*a*b^5*g^5*n*(a*d - b*c))/(2*d) + (3*b^5*g^5*n*(a*d
- b*c)*(4*a*d - b*c))/(2*d^2)) + a*(a*((3*a*b^3*g^5*n*(a*d - b*c))/(2*d) +
 (b^3*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(2*d^2)) + (b^3*g^5*n*(a*d - b*c)*(
6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(2*d^3)) + x*(a*(b*((3*a*b^3*g^5*n*(a*d
- b*c))/(2*d) + (b^3*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(2*d^2)) + (3*a*b^4*
g^5*n*(a*d - b*c))/d + (b^4*g^5*n*(a*d - b*c)*(4*a*d - b*c))/d^2) + b*(a*(
(3*a*b^3*g^5*n*(a*d - b*c))/(2*d) + (b^3*g^5*n*(a*d - b*c)*(4*a*d - b*c))/
(2*d^2)) + (b^3*g^5*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(2*d^
3)) + (3*b^4*g^5*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(2*d^3))
 + (3*b^3*g^5*n*(a*d - b*c)*(4*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b
*c*d^2))/(2*d^4) + (6*b^6*g^5*n*x^3*(a*d - b*c))/d))/(6*b^3*g^5*(a^2*d^...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2302, normalized size of antiderivative = 7.22

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x)
```

output

```
( - 144*log(a + b*x)*a**6*b*d**4*n - 48*log(a + b*x)*a**5*b**2*d**4*n**2 -
 576*log(a + b*x)*a**5*b**2*d**4*n*x - 36*log(a + b*x)*a**4*b**3*c*d**3*n*
*2 - 192*log(a + b*x)*a**4*b**3*d**4*n**2*x - 864*log(a + b*x)*a**4*b**3*d
**4*n*x**2 - 144*log(a + b*x)*a**3*b**4*c*d**3*n**2*x - 288*log(a + b*x)*a
**3*b**4*d**4*n**2*x**2 - 576*log(a + b*x)*a**3*b**4*d**4*n*x**3 - 216*log
(a + b*x)*a**2*b**5*c*d**3*n**2*x**2 - 192*log(a + b*x)*a**2*b**5*d**4*n**
2*x**3 - 144*log(a + b*x)*a**2*b**5*d**4*n*x**4 - 144*log(a + b*x)*a*b**6*
c*d**3*n**2*x**3 - 48*log(a + b*x)*a*b**6*d**4*n**2*x**4 - 36*log(a + b*x)
*b**7*c*d**3*n**2*x**4 + 144*log(c + d*x)*a**6*b*d**4*n + 48*log(c + d*x)*
a**5*b**2*d**4*n**2 + 576*log(c + d*x)*a**5*b**2*d**4*n*x + 36*log(c + d*x
)*a**4*b**3*c*d**3*n**2 + 192*log(c + d*x)*a**4*b**3*d**4*n**2*x + 864*log
(c + d*x)*a**4*b**3*d**4*n*x**2 + 144*log(c + d*x)*a**3*b**4*c*d**3*n**2*x
 + 288*log(c + d*x)*a**3*b**4*d**4*n**2*x**2 + 576*log(c + d*x)*a**3*b**4*
d**4*n*x**3 + 216*log(c + d*x)*a**2*b**5*c*d**3*n**2*x**2 + 192*log(c + d*
x)*a**2*b**5*d**4*n**2*x**3 + 144*log(c + d*x)*a**2*b**5*d**4*n*x**4 + 144
*log(c + d*x)*a*b**6*c*d**3*n**2*x**3 + 48*log(c + d*x)*a*b**6*d**4*n**2*x
**4 + 36*log(c + d*x)*b**7*c*d**3*n**2*x**4 - 288*log(((a + b*x)**n*e)/(c
+ d*x)**n)**2*a**2*b**5*c**3*d - 864*log(((a + b*x)**n*e)/(c + d*x)**n)**2
*a**2*b**5*c**2*d**2*x - 864*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b*
*5*c*d**3*x**2 - 288*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**5*d*...
```

3.177 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx$

Optimal result . . . 2016
Mathematica [C] (verified) . . . 2017
Rubi [A] (verified) . . . 2018
Maple [B] (verified) . . . 2020
Fricas [B] (verification not implemented) . . . 2021
Sympy [F(-1)] . . . 2022
Maxima [B] (verification not implemented) . . . 2023
Giac [A] (verification not implemented) . . . 2024
Mupad [B] (verification not implemented) . . . 2025
Reduce [B] (verification not implemented) . . . 2026

Optimal result

Integrand size = 45, antiderivative size = 493

$$
\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx \\
&= -\frac{2B^2d^2i^2n^2(c+dx)^3}{27(bc-ad)^3g^6(a+bx)^3} + \frac{bB^2di^2n^2(c+dx)^4}{16(bc-ad)^3g^6(a+bx)^4} \\
&\quad - \frac{2b^2B^2i^2n^2(c+dx)^5}{125(bc-ad)^3g^6(a+bx)^5} - \frac{2Bd^2i^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^6(a+bx)^3} \\
&\quad + \frac{bBdi^2n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^3g^6(a+bx)^4} \\
&\quad - \frac{2b^2Bi^2n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{25(bc-ad)^3g^6(a+bx)^5} \\
&\quad - \frac{d^2i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^3g^6(a+bx)^3} \\
&\quad + \frac{bdi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3g^6(a+bx)^4} - \frac{b^2i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5(bc-ad)^3g^6(a+bx)^5}
\end{aligned}
$$

output

```
-2/27*B^2*d^2*i^2*n^2*(d*x+c)^3/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/16*b*B^2*d*i^
2*n^2*(d*x+c)^4/(-a*d+b*c)^3/g^6/(b*x+a)^4-2/125*b^2*B^2*i^2*n^2*(d*x+c)^5
/(-a*d+b*c)^3/g^6/(b*x+a)^5-2/9*B*d^2*i^2*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(
d*x+c))^n))/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/4*b*B*d*i^2*n*(d*x+c)^4*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^6/(b*x+a)^4-2/25*b^2*B*i^2*n*(d*x+c)
^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^6/(b*x+a)^5-1/3*d^2*i^2*
(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/2
*b*d*i^2*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^6/(b*x
+a)^4-1/5*b^2*i^2*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3
/g^6/(b*x+a)^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 3.46 (sec) , antiderivative size = 2112, normalized size of antiderivative = 4.28

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Result too large to show}$$

input

```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x)^6,x]
```

output

```
-1/54000*(i^2*(10800*(b*c - a*d)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2 + 27000*d*(b*c - a*d)^4*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2 - 18000*d^2*(-(b*c) + a*d)^3*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])^2 + 1000*B*d^2*n*(a + b*x)^2*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d
)^3*n - 18*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*n*(a + b*x)
+ 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*n*(a + b*x)^2 +
36*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*n*(a + b*x)^3*Log[a + b*x] -
18*B*d^3*n*(a + b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[e*((a + b*x
)/(c + d*x))^n] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x
))^n] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n] +
36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n] - 36*A*d^
3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*n*(a + b*x)^3*Log[c + d*x] + 36*B*d^
3*n*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*
(a + b*x)^3*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x] - 18*B*d^3*n*(a +
b*x)^3*Log[c + d*x]^2 + 36*B*d^3*n*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*
x))/(b*c - a*d)] + 36*B*d^3*n*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c)
 + a*d)] + 36*B*d^3*n*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) +
 375*B*d*n*(a + b*x)*(36*A*(b*c - a*d)^4 + 9*B*(b*c - a*d)^4*n + 48*A*d*(-
(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c) + a*d)^3*n*(a + b*x) + 72*A*d^2*
(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c - a*d)^2*n*(a + b*x)^2 + 144*...
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^6}\,dx$$

$$\downarrow 2961$$

$$\frac{i^2 \int \frac{(c+dx)^6 \left(b-\frac{d(a+bx)}{c+dx}\right)^2 \left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^6} d\frac{a+bx}{c+dx}}{g^6(bc-ad)^3}$$

$$\downarrow 2795$$

$$\frac{i^2\int\left(\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^6}{(a+bx)^6}-\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^5}{(a+bx)^5}+\frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^4}{(a+bx)^4}\right)d\frac{a+bx}{c+dx}}{g^6(bc-ad)^3}$$

$$\downarrow \text{2009}$$

$$\frac{i^2\left(-\frac{b^2(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5(a+bx)^5}-\frac{2b^2Bn(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{25(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}-\frac{2Bd^2n(c-}{}\right.}$$

input

```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x)^6,x]
```

output

```
(i^2*((-2*B^2*d^2*n^2*(c + d*x)^3)/(27*(a + b*x)^3) + (b*B^2*d*n^2*(c + d*
x)^4)/(16*(a + b*x)^4) - (2*b^2*B^2*n^2*(c + d*x)^5)/(125*(a + b*x)^5) - (
2*B*d^2*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(a + b*x)
^3) + (b*B*d*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*(a +
 b*x)^4) - (2*b^2*B*n*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(25*(a + b*x)^5) - (d^2*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2)/(3*(a + b*x)^3) + (b*d*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2)/(2*(a + b*x)^4) - (b^2*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2)/(5*(a + b*x)^5)))/((b*c - a*d)^3*g^6)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2527 vs. $2(475) = 950$.

Time = 61.60 (sec) , antiderivative size = 2528, normalized size of antiderivative = 5.13

method	result	size
parallelrisch	Expression too large to display	2528

input

```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^6,x,method
=_RETURNVERBOSE)
```

output

```
-1/54000*(-4000*B^2*a^2*b^7*c^3*d^3*i^2*n^3+9000*A*B*x^2*a*b^8*c^2*d^4*i^2
*n^2-54000*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^7*c^2*d^4*i^2*n+72000*B
^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^8*c^3*d^3*i^2*n-36000*B^2*x*ln(e*((b*
x+a)/(d*x+c))^n)*a^2*b^7*c^2*d^4*i^2*n^2+1489*B^2*a^5*b^4*d^6*i^2*n^3-864*
B^2*b^9*c^5*d*i^2*n^3+1800*A^2*a^5*b^4*d^6*i^2*n-10800*A^2*b^9*c^5*d*i^2*n
+13500*A*B*a*b^8*c^4*d^2*i^2*n^2+30000*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b
^8*c^3*d^3*i^2*n^2-54000*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^4*d^2*i^2*n
-36000*A*B*x*a^2*b^7*c^2*d^4*i^2*n^2+30000*A*B*x*a*b^8*c^3*d^3*i^2*n^2-180
00*A*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*d^6*i^2*n-18000*B^2*x^3*ln(e*((
b*x+a)/(d*x+c))^n)*a*b^8*c*d^5*i^2*n^2-36000*A*B*x^3*ln(e*((b*x+a)/(d*x+c)
)^n)*a^2*b^7*d^6*i^2*n-18000*A*B*x^3*a*b^8*c*d^5*i^2*n^2-54000*B^2*x^2*ln(
e*((b*x+a)/(d*x+c))^n)^2*a^2*b^7*c*d^5*i^2*n+54000*B^2*x^2*ln(e*((b*x+a)/(
d*x+c))^n)^2*a*b^8*c^2*d^4*i^2*n-36000*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a
^2*b^7*c*d^5*i^2*n^2+9000*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^2*d^4*
i^2*n^2-36000*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^3*d^3*i^2*n-36000*A*
B*x^2*a^2*b^7*c*d^5*i^2*n^2-36000*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*c^
3*d^3*i^2*n+54000*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^4*d^2*i^2*n+9000*A
^2*x*a^4*b^5*d^6*i^2*n-27000*A^2*x*b^9*c^4*d^2*i^2*n-12000*A*B*a^2*b^7*c^3
*d^3*i^2*n^2+3375*B^2*a*b^8*c^4*d^2*i^2*n^3+2820*A*B*a^5*b^4*d^6*i^2*n^2-4
320*A*B*b^9*c^5*d*i^2*n^2-18000*A^2*a^2*b^7*c^3*d^3*i^2*n+27000*A^2*a*b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2633 vs. $2(475) = 950$.

Time $= 0.19$ (sec) , antiderivative size $= 2633$, normalized size of antiderivative $= 5.34$

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^6,x
, algorithm="fricas")
```

output

```
-1/54000*((864*B^2*b^5*c^5 - 3375*B^2*a*b^4*c^4*d + 4000*B^2*a^2*b^3*c^3*d
^2 - 1489*B^2*a^5*d^5)*i^2*n^2 + 60*(47*(B^2*b^5*c*d^4 - B^2*a*b^4*d^5)*i^
2*n^2 + 60*(A*B*b^5*c*d^4 - A*B*a*b^4*d^5)*i^2*n)*x^4 + 60*(72*A*B*b^5*c^5
 - 225*A*B*a*b^4*c^4*d + 200*A*B*a^2*b^3*c^3*d^2 - 47*A*B*a^5*d^5)*i^2*n +
 30*((13*B^2*b^5*c^2*d^3 + 350*B^2*a*b^4*c*d^4 - 363*B^2*a^2*b^3*d^5)*i^2*
n^2 - 60*(A*B*b^5*c^2*d^3 - 10*A*B*a*b^4*c*d^4 + 9*A*B*a^2*b^3*d^5)*i^2*n)
*x^3 + 1800*(6*A^2*b^5*c^5 - 15*A^2*a*b^4*c^4*d + 10*A^2*a^2*b^3*c^3*d^2 -
 A^2*a^5*d^5)*i^2 - 10*((86*B^2*b^5*c^3*d^2 - 375*B^2*a*b^4*c^2*d^3 - 1200
*B^2*a^2*b^3*c*d^4 + 1489*B^2*a^3*b^2*d^5)*i^2*n^2 - 60*(2*A*B*b^5*c^3*d^2
 - 15*A*B*a*b^4*c^2*d^3 + 60*A*B*a^2*b^3*c*d^4 - 47*A*B*a^3*b^2*d^5)*i^2*n
 - 1800*(A^2*b^5*c^3*d^2 - 3*A^2*a*b^4*c^2*d^3 + 3*A^2*a^2*b^3*c*d^4 - A^2
*a^3*b^2*d^5)*i^2)*x^2 + 1800*(10*(B^2*b^5*c^3*d^2 - 3*B^2*a*b^4*c^2*d^3 +
 3*B^2*a^2*b^3*c*d^4 - B^2*a^3*b^2*d^5)*i^2*x^2 + 5*(3*B^2*b^5*c^4*d - 8*B
^2*a*b^4*c^3*d^2 + 6*B^2*a^2*b^3*c^2*d^3 - B^2*a^4*b*d^5)*i^2*x + (6*B^2*b
^5*c^5 - 15*B^2*a*b^4*c^4*d + 10*B^2*a^2*b^3*c^3*d^2 - B^2*a^5*d^5)*i^2)*l
og(e)^2 + 1800*(B^2*b^5*d^5*i^2*n^2*x^5 + 5*B^2*a*b^4*d^5*i^2*n^2*x^4 + 10
*B^2*a^2*b^3*d^5*i^2*n^2*x^3 + 10*(B^2*b^5*c^3*d^2 - 3*B^2*a*b^4*c^2*d^3 +
 3*B^2*a^2*b^3*c*d^4)*i^2*n^2*x^2 + 5*(3*B^2*b^5*c^4*d - 8*B^2*a*b^4*c^3*d
^2 + 6*B^2*a^2*b^3*c^2*d^3)*i^2*n^2*x + (6*B^2*b^5*c^5 - 15*B^2*a*b^4*c^4*
d + 10*B^2*a^2*b^3*c^3*d^2)*i^2*n^2)*log((b*x + a)/(d*x + c))^2 + 5*((1...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Timed out}$$

input

```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**
6,x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10936 vs. $2(475) = 950$.

Time $= 0.77$ (sec) , antiderivative size $= 10936$, normalized size of antiderivative $= 22.18$

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^6,x
, algorithm="maxima")
```

output

```
-1/150*A*B*c^2*i^2*n*((60*b^4*d^4*x^4 + 12*b^4*c^4 - 63*a*b^3*c^3*d + 137*
a^2*b^2*c^2*d^2 - 163*a^3*b*c*d^3 + 137*a^4*d^4 - 30*(b^4*c*d^3 - 9*a*b^3*
d^4)*x^3 + 10*(2*b^4*c^2*d^2 - 13*a*b^3*c*d^3 + 47*a^2*b^2*d^4)*x^2 - 5*(3
*b^4*c^3*d - 17*a*b^3*c^2*d^2 + 43*a^2*b^2*c*d^3 - 77*a^3*b*d^4)*x)/((b^10
*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*
g^6*x^5 + 5*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c
*d^3 + a^5*b^5*d^4)*g^6*x^4 + 10*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^
6*c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*g^6*x^3 + 10*(a^3*b^7*c^4 - 4*a
^4*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*g^6*x^2
+ 5*(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 +
 a^8*b^2*d^4)*g^6*x + (a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 -
 4*a^8*b^2*c*d^3 + a^9*b*d^4)*g^6) + 60*d^5*log(b*x + a)/((b^6*c^5 - 5*a*b
^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5
*b*d^5)*g^6) - 60*d^5*log(d*x + c)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*
c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6)) - 1/900*
A*B*d^2*i^2*n*((47*a^2*b^4*c^4 - 278*a^3*b^3*c^3*d + 822*a^4*b^2*c^2*d^2 -
 278*a^5*b*c*d^3 + 47*a^6*d^4 + 60*(10*b^6*c^2*d^2 - 5*a*b^5*c*d^3 + a^2*b
^4*d^4)*x^4 - 30*(10*b^6*c^3*d - 95*a*b^5*c^2*d^2 + 46*a^2*b^4*c*d^3 - 9*a
^3*b^3*d^4)*x^3 + 10*(20*b^6*c^4 - 140*a*b^5*c^3*d + 537*a^2*b^4*c^2*d^2 -
 248*a^3*b^3*c*d^3 + 47*a^4*b^2*d^4)*x^2 + 5*(35*a*b^5*c^4 - 218*a^2*b^...
```

Giac [A] (verification not implemented)

Time = 5.97 (sec) , antiderivative size = 931, normalized size of antiderivative = 1.89

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^6,x
, algorithm="giac")
```

output

```
-1/54000*(1800*(6*B^2*b^2*i^2*n^2 - 15*(b*x + a)*B^2*b*d*i^2*n^2/(d*x + c)
 + 10*(b*x + a)^2*B^2*d^2*i^2*n^2/(d*x + c)^2)*log((b*x + a)/(d*x + c))^2/
((b*x + a)^5*b^2*c^2*g^6/(d*x + c)^5 - 2*(b*x + a)^5*a*b*c*d*g^6/(d*x + c)
^5 + (b*x + a)^5*a^2*d^2*g^6/(d*x + c)^5) + 60*(72*B^2*b^2*i^2*n^2 - 225*(
b*x + a)*B^2*b*d*i^2*n^2/(d*x + c) + 200*(b*x + a)^2*B^2*d^2*i^2*n^2/(d*x
+ c)^2 + 360*B^2*b^2*i^2*n*log(e) - 900*(b*x + a)*B^2*b*d*i^2*n*log(e)/(d*
x + c) + 600*(b*x + a)^2*B^2*d^2*i^2*n*log(e)/(d*x + c)^2 + 360*A*B*b^2*i^
2*n - 900*(b*x + a)*A*B*b*d*i^2*n/(d*x + c) + 600*(b*x + a)^2*A*B*d^2*i^2*
n/(d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^5*b^2*c^2*g^6/(d*x + c)
^5 - 2*(b*x + a)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*x + a)^5*a^2*d^2*g^6/(d*x
+ c)^5) + (864*B^2*b^2*i^2*n^2 - 3375*(b*x + a)*B^2*b*d*i^2*n^2/(d*x + c)
+ 4000*(b*x + a)^2*B^2*d^2*i^2*n^2/(d*x + c)^2 + 4320*B^2*b^2*i^2*n*log(e)
 - 13500*(b*x + a)*B^2*b*d*i^2*n*log(e)/(d*x + c) + 12000*(b*x + a)^2*B^2*
d^2*i^2*n*log(e)/(d*x + c)^2 + 10800*B^2*b^2*i^2*log(e)^2 - 27000*(b*x + a
)*B^2*b*d*i^2*log(e)^2/(d*x + c) + 18000*(b*x + a)^2*B^2*d^2*i^2*log(e)^2/
(d*x + c)^2 + 4320*A*B*b^2*i^2*n - 13500*(b*x + a)*A*B*b*d*i^2*n/(d*x + c)
 + 12000*(b*x + a)^2*A*B*d^2*i^2*n/(d*x + c)^2 + 21600*A*B*b^2*i^2*log(e)
- 54000*(b*x + a)*A*B*b*d*i^2*log(e)/(d*x + c) + 36000*(b*x + a)^2*A*B*d^2
*i^2*log(e)/(d*x + c)^2 + 10800*A^2*b^2*i^2 - 27000*(b*x + a)*A^2*b*d*i^2/
(d*x + c) + 18000*(b*x + a)^2*A^2*d^2*i^2/(d*x + c)^2)/((b*x + a)^5*b^2...
```

Mupad [B] (verification not implemented)

Time = 32.53 (sec) , antiderivative size = 3296, normalized size of antiderivative = 6.69

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input

```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^6,x)
```

output

```
((1800*A^2*a^4*d^4*i^2 + 10800*A^2*b^4*c^4*i^2 + 1489*B^2*a^4*d^4*i^2*n^2
+ 864*B^2*b^4*c^4*i^2*n^2 - 16200*A^2*a*b^3*c^3*d*i^2 + 1800*A^2*a^3*b*c*d
^3*i^2 + 2820*A*B*a^4*d^4*i^2*n + 4320*A*B*b^4*c^4*i^2*n + 1800*A^2*a^2*b^
2*c^2*d^2*i^2 + 1489*B^2*a^2*b^2*c^2*d^2*i^2*n^2 - 2511*B^2*a*b^3*c^3*d*i^
2*n^2 + 1489*B^2*a^3*b*c*d^3*i^2*n^2 + 2820*A*B*a^2*b^2*c^2*d^2*i^2*n - 91
80*A*B*a*b^3*c^3*d*i^2*n + 2820*A*B*a^3*b*c*d^3*i^2*n)/(60*(a*d - b*c)) +
(x*(1800*A^2*a^3*b*d^4*i^2 + 5400*A^2*b^4*c^3*d*i^2 - 9000*A^2*a*b^3*c^2*d
^2*i^2 + 1800*A^2*a^2*b^2*c*d^3*i^2 + 1489*B^2*a^3*b*d^4*i^2*n^2 + 189*B^2
*b^4*c^3*d*i^2*n^2 + 1620*A*B*b^4*c^3*d*i^2*n - 911*B^2*a*b^3*c^2*d^2*i^2*
n^2 + 1489*B^2*a^2*b^2*c*d^3*i^2*n^2 + 2820*A*B*a^3*b*d^4*i^2*n - 4380*A*B
*a*b^3*c^2*d^2*i^2*n + 2820*A*B*a^2*b^2*c*d^3*i^2*n))/(12*(a*d - b*c)) + (
x^2*(1800*A^2*a^2*b^2*d^4*i^2 + 1800*A^2*b^4*c^2*d^2*i^2 - 3600*A^2*a*b^3*
c*d^3*i^2 + 1489*B^2*a^2*b^2*d^4*i^2*n^2 - 86*B^2*b^4*c^2*d^2*i^2*n^2 + 28
20*A*B*a^2*b^2*d^4*i^2*n + 120*A*B*b^4*c^2*d^2*i^2*n + 289*B^2*a*b^3*c*d^3
*i^2*n^2 - 780*A*B*a*b^3*c*d^3*i^2*n))/(6*(a*d - b*c)) + (x^3*(363*B^2*a*b
^3*d^4*i^2*n^2 + 13*B^2*b^4*c*d^3*i^2*n^2 - 60*A*B*b^4*c*d^3*i^2*n + 540*A
*B*a*b^3*d^4*i^2*n))/(2*(a*d - b*c)) + (d*x^4*(47*B^2*b^4*d^3*i^2*n^2 + 60
*A*B*b^4*d^3*i^2*n))/(a*d - b*c))/(x*(4500*a^4*b^5*c*g^6 - 4500*a^5*b^4*d*
g^6) - x^4*(4500*a^2*b^7*d*g^6 - 4500*a*b^8*c*g^6) + x^5*(900*b^9*c*g^6 -
900*a*b^8*d*g^6) + x^2*(9000*a^3*b^6*c*g^6 - 9000*a^4*b^5*d*g^6) + x^3*...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 3287, normalized size of antiderivative = 6.67

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input

```
int((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^6,x)
```

output

```
( - 3600*log(a + b*x)*a**7*b*d**5*n - 2100*log(a + b*x)*a**6*b**2*d**5*n**
2 - 18000*log(a + b*x)*a**6*b**2*d**5*n*x - 720*log(a + b*x)*a**5*b**3*c*d
**4*n**2 - 10500*log(a + b*x)*a**5*b**3*d**5*n**2*x - 36000*log(a + b*x)*a
**5*b**3*d**5*n*x**2 - 3600*log(a + b*x)*a**4*b**4*c*d**4*n**2*x - 21000*l
og(a + b*x)*a**4*b**4*d**5*n**2*x**2 - 36000*log(a + b*x)*a**4*b**4*d**5*n
*x**3 - 7200*log(a + b*x)*a**3*b**5*c*d**4*n**2*x**2 - 21000*log(a + b*x)*
a**3*b**5*d**5*n**2*x**3 - 18000*log(a + b*x)*a**3*b**5*d**5*n*x**4 - 7200
*log(a + b*x)*a**2*b**6*c*d**4*n**2*x**3 - 10500*log(a + b*x)*a**2*b**6*d*
*5*n**2*x**4 - 3600*log(a + b*x)*a**2*b**6*d**5*n*x**5 - 3600*log(a + b*x)
*a*b**7*c*d**4*n**2*x**4 - 2100*log(a + b*x)*a*b**7*d**5*n**2*x**5 - 720*l
og(a + b*x)*b**8*c*d**4*n**2*x**5 + 3600*log(c + d*x)*a**7*b*d**5*n + 2100
*log(c + d*x)*a**6*b**2*d**5*n**2 + 18000*log(c + d*x)*a**6*b**2*d**5*n*x
+ 720*log(c + d*x)*a**5*b**3*c*d**4*n**2 + 10500*log(c + d*x)*a**5*b**3*d*
*5*n**2*x + 36000*log(c + d*x)*a**5*b**3*d**5*n*x**2 + 3600*log(c + d*x)*a
**4*b**4*c*d**4*n**2*x + 21000*log(c + d*x)*a**4*b**4*d**5*n**2*x**2 + 360
00*log(c + d*x)*a**4*b**4*d**5*n*x**3 + 7200*log(c + d*x)*a**3*b**5*c*d**4
*n**2*x**2 + 21000*log(c + d*x)*a**3*b**5*d**5*n**2*x**3 + 18000*log(c + d
*x)*a**3*b**5*d**5*n*x**4 + 7200*log(c + d*x)*a**2*b**6*c*d**4*n**2*x**3 +
 10500*log(c + d*x)*a**2*b**6*d**5*n**2*x**4 + 3600*log(c + d*x)*a**2*b**6
*d**5*n*x**5 + 3600*log(c + d*x)*a*b**7*c*d**4*n**2*x**4 + 2100*log(c +...
```

3.178 $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result 2027
Mathematica [B] (verified) 2028
Rubi [A] (verified) 2029
Maple [F] 2045
Fricas [F] 2046
Sympy [F(-1)] 2046
Maxima [B] (verification not implemented) 2047
Giac [F(-1)] 2048
Mupad [F(-1)] 2048
Reduce [F] 2048

Optimal result

Integrand size = 45, antiderivative size = 1172

$$\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

output

```
5/84*B^2*(-a*d+b*c)^6*g^3*i^3*n^2*x/b^3/d^3+1/140*B^2*(-a*d+b*c)^3*g^3*i^3
*n^2*(b*x+a)^4/b^4-29/840*B^2*(-a*d+b*c)^5*g^3*i^3*n^2*(d*x+c)^2/b^2/d^4+4
7/1260*B^2*(-a*d+b*c)^4*g^3*i^3*n^2*(d*x+c)^3/b/d^4-13/420*B^2*(-a*d+b*c)^
3*g^3*i^3*n^2*(d*x+c)^4/d^4+1/105*b*B^2*(-a*d+b*c)^2*g^3*i^3*n^2*(d*x+c)^5
/d^4-1/210*B*(-a*d+b*c)^4*g^3*i^3*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))/b^4/d-3/140*B*(-a*d+b*c)^3*g^3*i^3*n*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x
+c))^n))/b^4-1/35*B*(-a*d+b*c)^2*g^3*i^3*n*(b*x+a)^4*(d*x+c)*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/b^3+2/21*B*(-a*d+b*c)^4*g^3*i^3*n*(d*x+c)^3*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/b/d^4-3/14*B*(-a*d+b*c)^3*g^3*i^3*n*(d*x+c)^4*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))/d^4+6/35*b*B*(-a*d+b*c)^2*g^3*i^3*n*(d*x+c)^5*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))/d^4-1/21*b^2*B*(-a*d+b*c)*g^3*i^3*n*(d*x+c)^6
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4+1/140*(-a*d+b*c)^3*g^3*i^3*(b*x+a)^4*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^4+1/35*(-a*d+b*c)^2*g^3*i^3*(b*x+a)^4*
(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3+1/14*(-a*d+b*c)*g^3*i^3*(b*x
+a)^4*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2+1/7*g^3*i^3*(b*x+a)^
4*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/420*B*(-a*d+b*c)^5*g^3*i
^3*n*(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/d^2-1/420*B*(-a
*d+b*c)^6*g^3*i^3*n*(b*x+a)*(6*A+5*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/
d^3-1/420*B*(-a*d+b*c)^7*g^3*i^3*n*(6*A+11*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^
n))*ln((-a*d+b*c)/b/(d*x+c))/b^4/d^4-1/210*B^2*(-a*d+b*c)^7*g^3*i^3*n^2...
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2448 vs. $2(1172) = 2344$.

Time = 3.51 (sec) , antiderivative size = 2448, normalized size of antiderivative = 2.09

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Result too large to show}$$

input

```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2,x]
```

output

```
(g^3*i^3*(35*(b*c - a*d)^3*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2 + 84*d*(b*c - a*d)^2*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2 + 70*d^2*(b*c - a*d)*(a + b*x)^6*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2 + 20*d^3*(a + b*x)^7*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (35*
B*(b*c - a*d)^4*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)
*Log[e*((a + b*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(
b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c
- a*d)^2*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n*((2
*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLo
g[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^4) + (7*B*(b*c - a*d)^3*n*(24*A*b*
d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d
*x))^n] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]) - 6*d^4*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 24*B*(b*
c - a*d)^4*n*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)/(c +
d*x))^n])*Log[c + d*x] + 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x - d^2*(a
 + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*n*(6*b*d*(b*c...
```

Rubi [A] (verified)

Time = 3.66 (sec) , antiderivative size = 1455, normalized size of antiderivative = 1.24, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.489$, Rules used = {2961, 2783, 2782, 27, 2123, 2009, 2783, 2782, 27, 87, 49, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$

$$\downarrow 2961$$

$$g^3i^3(bc-ad)^7\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^8}d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$g^3 i^3 (bc -$$

$$ad)^7 \left(-\frac{2Bn \int \frac{(a+bx)^3 \left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^7} d\frac{a+bx}{c+dx}}{7b} + \frac{3 \int \frac{(a+bx)^3 \left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^7} d\frac{a+bx}{c+dx}}{7b} + \frac{(a+bx)^4 \left(B \log\left(e\right.\right.}{7b(c+dx)^4 \left(b\right.} \right.$$

$$\downarrow 2782$$

$$g^3 i^3 (bc -$$

$$ad)^7 \left(-\frac{2Bn \left(-Bn \int -\frac{(c+dx)\left(b^3 - \frac{6d(a+bx)b^2}{c+dx} + \frac{15d^2(a+bx)^2 b}{(c+dx)^2} - \frac{20d^3(a+bx)^3}{(c+dx)^3}\right)}{60d^4(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx} + \frac{b^3\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3b^2\left(B \log\left(e\right.\right.}{5d^4\left(b-\right.} \right.}{7b}$$

$$\downarrow 27$$

$$g^3 i^3 (bc -$$

$$ad)^7 \left(-\frac{2Bn \left(\frac{Bn \int \frac{(c+dx)\left(b^3 - \frac{6d(a+bx)b^2}{c+dx} + \frac{15d^2(a+bx)^2 b}{(c+dx)^2} - \frac{20d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{60d^4} + \frac{b^3\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3b^2\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)} \right.}{7b}$$

$$\downarrow 2123$$

$$g^3 i^3 (bc -$$

$$ad)^7 \left(-\frac{2Bn \left(\frac{Bn \int \left(-\frac{10db^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6} + \frac{26db}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{19d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3 b} + \frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2 b^2} + \frac{c+dx}{(a+bx)b^3} + \frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)b^3} \right.}{60d^4} \right.}{7b}$$

$$\downarrow 2009$$

$$g^3 i^3 (bc -$$

$$ad)^7 \left(\frac{3 \int \frac{(a+bx)^3 \left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^7} d\frac{a+bx}{c+dx}}{7b} - \frac{2Bn \left(\frac{b^3 \left(B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3b^2 \left(B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3b\left(B \log\left(\right.\right.}{4d^4 \left(\right.}$$

$$\downarrow 2783$$

$$g^3 i^3 (bc -$$

$$ad)^7 \left(\frac{3 \left(- \frac{Bn \int \frac{(a+bx)^3 \left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{3b} + \frac{\int \frac{(a+bx)^3 \left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{3b} + \frac{(a+bx)^4 \left(B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{6b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} \right)}{7b}$$

$$\downarrow 2782$$

$$(bc -$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn \left(\frac{\left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) b^3}{6d^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) b^2}{5d^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+\right.}{4} \right.$$

$$\downarrow 27$$

$$(bc-$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+\right.\right.}{4}$$

$$\downarrow 87$$

$$(bc-$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+\right.\right.}{4}$$

$$\downarrow 49$$

$$(bc-$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+\right.}{4} \right.$$

$$\downarrow 2009$$

$$(bc-$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+\right.}{4} \right.$$

$$\downarrow 2783$$

$$(bc-$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+\right.}{4} \right.$$

$$\downarrow 2773$$

$$(bc-$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+\right.}{4} \right.$$

$$\downarrow 49$$

$$(bc-ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - 2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+\right.\right.\right.\right.$$

↓ 2009

$$(bc-ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+\right.\right.}{4}\right.$$

↓ 2781

$$(bc-$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+\right.\right.}{4}$$

↓ 2784

$$(bc -$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+\right.}{4}\right.}{} \right.$$

$\downarrow$ 2784

$$(bc-$$

$$ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+\right.\right.}{4} \right.$$

↓ 2784

$$(bc-ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+\right.\right.}{4}\right.$$

↓ 2754

$$(bc-ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+\right.\right.}{4}\right.$$

$$\downarrow 2838$$

$$(bc-ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+\right.\right.}{4}\right.$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2,x]
```

output
```
(b*c - a*d)^7*g^3*i^3*(((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2)/(7*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^7) - (2*B*n*((b^3*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*d^4*(b - (d*(a + b*x))/(c + d*x))^6)
 - (3*b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*d^4*(b - (d*(a + b*x)
)/(c + d*x))^5) + (3*b*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d^4*(b -
 (d*(a + b*x))/(c + d*x))^4) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*d
^4*(b - (d*(a + b*x))/(c + d*x))^3) + (B*n*((-2*b^2)/(b - (d*(a + b*x))/(c
 + d*x))^5 + (13*b)/(2*(b - (d*(a + b*x))/(c + d*x))^4) - 19/(3*(b - (d*(a
 + b*x))/(c + d*x))^3) + 1/(2*b*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^2*
(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^3 - Log[b - (d
*(a + b*x))/(c + d*x)]/b^3))/(60*d^4)))/(7*b) + (3*(((a + b*x)^4*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n])^2)/(6*b*(c + d*x)^4*(b - (d*(a + b*x))/(c +
d*x))^6) - (B*n*(((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*b
*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^5) + ((a + b*x)^4*(A + B*Log[e*
((a + b*x)/(c + d*x))^n]))/(20*b^2*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x
))^4) - (B*n*((a + b*x)^4/((c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) +
b^3/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^3) - (3*b^2)/(2*d^4*(b - (d*(a +
b*x))/(c + d*x))^2) + (3*b)/(d^4*(b - (d*(a + b*x))/(c + d*x))) + Log[b -
(d*(a + b*x))/(c + d*x)]/d^4))/(20*b^2)))/(3*b) + (((a + b*x)^4*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^2)/(5*b*(c + d*x)^4*(b - (d*(a + b*x))/(c ...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))  Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2754

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))  Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))  Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])  u, x] - Simp[b*n  Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)^3 (dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (bgx+ag)^3(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="fricas")
```

output
```
integral(A^2*b^3*d^3*g^3*i^3*x^6 + A^2*a^3*c^3*g^3*i^3 + 3*(A^2*b^3*c*d^2
+ A^2*a*b^2*d^3)*g^3*i^3*x^5 + 3*(A^2*b^3*c^2*d + 3*A^2*a*b^2*c*d^2 + A^2*
a^2*b*d^3)*g^3*i^3*x^4 + (A^2*b^3*c^3 + 9*A^2*a*b^2*c^2*d + 9*A^2*a^2*b*c*
d^2 + A^2*a^3*d^3)*g^3*i^3*x^3 + 3*(A^2*a*b^2*c^3 + 3*A^2*a^2*b*c^2*d + A^
2*a^3*c*d^2)*g^3*i^3*x^2 + 3*(A^2*a^2*b*c^3 + A^2*a^3*c^2*d)*g^3*i^3*x + (
B^2*b^3*d^3*g^3*i^3*x^6 + B^2*a^3*c^3*g^3*i^3 + 3*(B^2*b^3*c*d^2 + B^2*a*b
^2*d^3)*g^3*i^3*x^5 + 3*(B^2*b^3*c^2*d + 3*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3
)*g^3*i^3*x^4 + (B^2*b^3*c^3 + 9*B^2*a*b^2*c^2*d + 9*B^2*a^2*b*c*d^2 + B^2
*a^3*d^3)*g^3*i^3*x^3 + 3*(B^2*a*b^2*c^3 + 3*B^2*a^2*b*c^2*d + B^2*a^3*c*d
^2)*g^3*i^3*x^2 + 3*(B^2*a^2*b*c^3 + B^2*a^3*c^2*d)*g^3*i^3*x)*log(e*((b*x
 + a)/(d*x + c))^n)^2 + 2*(A*B*b^3*d^3*g^3*i^3*x^6 + A*B*a^3*c^3*g^3*i^3 +
 3*(A*B*b^3*c*d^2 + A*B*a*b^2*d^3)*g^3*i^3*x^5 + 3*(A*B*b^3*c^2*d + 3*A*B*
a*b^2*c*d^2 + A*B*a^2*b*d^3)*g^3*i^3*x^4 + (A*B*b^3*c^3 + 9*A*B*a*b^2*c^2*
d + 9*A*B*a^2*b*c*d^2 + A*B*a^3*d^3)*g^3*i^3*x^3 + 3*(A*B*a*b^2*c^3 + 3*A*
B*a^2*b*c^2*d + A*B*a^3*c*d^2)*g^3*i^3*x^2 + 3*(A*B*a^2*b*c^3 + A*B*a^3*c^
2*d)*g^3*i^3*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**
2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7845 vs. $2(1125) = 2250$.

Time = 0.71 (sec) , antiderivative size = 7845, normalized size of antiderivative = 6.69

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="maxima")
```

output
```
2/7*A*B*b^3*d^3*g^3*i^3*x^7*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/7*A
^2*b^3*d^3*g^3*i^3*x^7 + A*B*b^3*c*d^2*g^3*i^3*x^6*log(e*(b*x/(d*x + c) +
a/(d*x + c))^n) + A*B*a*b^2*d^3*g^3*i^3*x^6*log(e*(b*x/(d*x + c) + a/(d*x
+ c))^n) + 1/2*A^2*b^3*c*d^2*g^3*i^3*x^6 + 1/2*A^2*a*b^2*d^3*g^3*i^3*x^6 +
 6/5*A*B*b^3*c^2*d*g^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 18
/5*A*B*a*b^2*c*d^2*g^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 6/
5*A*B*a^2*b*d^3*g^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/5*A
^2*b^3*c^2*d*g^3*i^3*x^5 + 9/5*A^2*a*b^2*c*d^2*g^3*i^3*x^5 + 3/5*A^2*a^2*b
*d^3*g^3*i^3*x^5 + 1/2*A*B*b^3*c^3*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n) + 9/2*A*B*a*b^2*c^2*d*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n) + 9/2*A*B*a^2*b*c*d^2*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n) + 1/2*A*B*a^3*d^3*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n) + 1/4*A^2*b^3*c^3*g^3*i^3*x^4 + 9/4*A^2*a*b^2*c^2*d*g^3*i^3*x^4 + 9/4
*A^2*a^2*b*c*d^2*g^3*i^3*x^4 + 1/4*A^2*a^3*d^3*g^3*i^3*x^4 + 2*A*B*a*b^2*c
^3*g^3*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 6*A*B*a^2*b*c^2*d*
g^3*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*B*a^3*c*d^2*g^3*i
^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*b^2*c^3*g^3*i^3*x^3
+ 3*A^2*a^2*b*c^2*d*g^3*i^3*x^3 + A^2*a^3*c*d^2*g^3*i^3*x^3 + 3*A*B*a^2*b*
c^3*g^3*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3*A*B*a^3*c^2*d*g
^3*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A^2*a^2*b*c^3*g...
```

Giac [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^3\,(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))
^2,x)
```

output
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))
^2, x)
```

Reduce [F]

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
(g**3*i*(36*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*
x + b*d*x**2),x)*a**7*b**2*d**8*n - 252*int((log(((a + b*x)**n*e)/(c + d*x
)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**6*b**3*c*d**7*n + 756*int(
(log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)
*a**5*b**4*c**2*d**6*n - 1260*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(
a*c + a*d*x + b*c*x + b*d*x**2),x)*a**4*b**5*c**3*d**5*n + 1260*int((log((
(a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*
b**6*c**4*d**4*n - 756*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a
*d*x + b*c*x + b*d*x**2),x)*a**2*b**7*c**5*d**3*n + 252*int((log(((a + b*x
)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**8*c**6*d
**2*n - 36*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x
 + b*d*x**2),x)*b**9*c**7*d*n + 36*log(a + b*x)*a**8*d**7*n - 252*log(a +
b*x)*a**7*b*c*d**6*n + 756*log(a + b*x)*a**6*b**2*c**2*d**5*n - 1260*log(a
 + b*x)*a**5*b**3*c**3*d**4*n + 1260*log(a + b*x)*a**4*b**4*c**4*d**3*n -
756*log(a + b*x)*a**3*b**5*c**5*d**2*n + 252*log(a + b*x)*a**2*b**6*c**6*d
*n - 36*log(a + b*x)*a*b**7*c**7*n - 18*log(((a + b*x)**n*e)/(c + d*x)**n)
**2*a**6*b**2*c*d**6 + 108*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**5*b**3
*c**2*d**5 - 270*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**4*b**4*c**3*d**4
 - 270*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*b**5*c**4*d**3 - 2520*lo
g(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*b**5*c**3*d**4*x - 3780*log(((...
```

3.179 $\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result 2050
Mathematica [A] (verified) 2051
Rubi [A] (verified) 2052
Maple [F] 2066
Fricas [F] 2066
Sympy [F(-1)] 2067
Maxima [B] (verification not implemented) 2068
Giac [F(-1)] 2069
Mupad [F(-1)] 2069
Reduce [F] 2069

Optimal result

Integrand size = 45, antiderivative size = 976

$$\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

output

```
-7/180*B^2*(-a*d+b*c)^5*g^2*i^3*n^2*x/b^3/d^2-7/360*B^2*(-a*d+b*c)^4*g^2*i
^3*n^2*(d*x+c)^2/b^2/d^3-1/60*B^2*(-a*d+b*c)^3*g^2*i^3*n^2*(d*x+c)^3/b/d^3
+1/60*B^2*(-a*d+b*c)^2*g^2*i^3*n^2*(d*x+c)^4/d^3-1/60*B*(-a*d+b*c)^4*g^2*i
^3*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/d-1/30*B*(-a*d+b*c)^3*g
^2*i^3*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4-1/10*B*(-a*d+b*c)^4
*g^2*i^3*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d^3+1/45*B*(-a*d+
b*c)^3*g^2*i^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3+7/60*B*(-
a*d+b*c)^2*g^2*i^3*n*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3-1/15*b*
B*(-a*d+b*c)*g^2*i^3*n*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3+1/60*
(-a*d+b*c)^3*g^2*i^3*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^4+1/20*
(-a*d+b*c)^2*g^2*i^3*(b*x+a)^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b
^3+1/10*(-a*d+b*c)*g^2*i^3*(b*x+a)^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))^2/b^2+1/6*g^2*i^3*(b*x+a)^3*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
^2/b+1/60*B*(-a*d+b*c)^5*g^2*i^3*n*(b*x+a)*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x
+c))^n))/b^4/d^2+1/60*B*(-a*d+b*c)^6*g^2*i^3*n*(2*A+3*B*n+2*B*ln(e*((b*x+a
)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^4/d^3+1/36*B^2*(-a*d+b*c)^6*g^2*
i^3*n^2*ln((b*x+a)/(d*x+c))/b^4/d^3+11/180*B^2*(-a*d+b*c)^6*g^2*i^3*n^2*ln
(d*x+c)/b^4/d^3+1/30*B^2*(-a*d+b*c)^6*g^2*i^3*n^2*polylog(2,d*(b*x+a)/b/(d
*x+c))/b^4/d^3
```

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 1627, normalized size of antiderivative = 1.67

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

input

```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2,x]
```

output

```
(g^2*i^3*(15*(b*c - a*d)^2*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2 - 24*b*(b*c - a*d)*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2 + 10*b^2*(c + d*x)^6*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (5*B*(
b*c - a*d)^3*n*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*
c - a*d)*Log[a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d
*x)^2 + 2*(b*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[
e*((a + b*x)/(c + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]) + 6*(b*c - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[a + b*x
]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a +
 b*x))/(-(b*c) + a*d)])))/b^4 + (2*B*(b*c - a*d)^2*n*(24*A*b*d*(b*c - a*d)
^3*x - 12*B*(b*c - a*d)^3*n*(b*d*x + (b*c - a*d)*Log[a + b*x]) - 4*B*(b*c
- a*d)^2*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c - a*d)^2*Log[a
+ b*x]) - B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c +
d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]) + 24*B*d*(b*c -
 a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 12*b^2*(b*c - a*d)^2*(c
 + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 8*b^3*(b*c - a*d)*(c +
d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*b^4*(c + d*x)^4*(A + B*L
og[e*((a + b*x)/(c + d*x))^n]) + 24*(b*c - a*d)^4*Log[a + b*x]*(A + B*L...
```

Rubi [A] (verified)

Time = 3.28 (sec) , antiderivative size = 1266, normalized size of antiderivative = 1.30, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2961, 2783, 2782, 27, 1195, 2009, 2783, 2782, 27, 1195, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\downarrow \text{2961}$$

$$g^2i^3(bc-ad)^6\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(-\frac{Bn \int \frac{(a+bx)^2 \left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{3b} + \frac{\int \frac{(a+bx)^2 \left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{2b} + \frac{(a+bx)^3 \left(B \log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{6b(c+dx)^3 \left(b-\right.}$$

$$\downarrow 2782$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(-\frac{Bn \left(-Bn \int \frac{(c+dx)\left(b^2-\frac{5d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{30d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx} + \frac{b^2\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{b\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B}{3}\right.}{3b}$$

$$\downarrow 27$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(-\frac{Bn \left(-\frac{Bn \int \frac{(c+dx)\left(b^2-\frac{5d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{30d^3} + \frac{b^2\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{b\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B \log\left(\right.}{3d^3\left(\right.}\right.}{3b}$$

$$\downarrow 1195$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(-\frac{Bn \left(-\frac{Bn \int \left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)} + \frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} + \frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{9d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{6bd}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{c+dx}{b^3(a+bx)} \right) d\frac{a+bx}{c+dx}}{30d^3} + \frac{b^2\left(B\right.}{5}\right.}{3b}$$

$$\downarrow 2009$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(\frac{\int \frac{(a+bx)^2 \left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{2b} - \frac{Bn\left(\frac{b^2\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{b\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^3\left(b-\frac{d(a+b}{c+d}\right.\right.}{}$$

$$\downarrow 2783$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(\frac{-\frac{2Bn\int \frac{(a+bx)^2\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{2\int \frac{(a+bx)^2\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{(a+bx)^3\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}}{2b} -$$

$$\downarrow 2782$$

$$g^2 i^3 (bc -$$

$$ad)^6 \left(\frac{-\frac{2Bn\left(-Bn\int \frac{(c+dx)\left(b^2-\frac{4d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{12d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx} + \frac{b^2\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2b\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}}{5b}}{2b}$$

$$\downarrow 27$$

$$g^2i^3(bc-$$

$$ad)^6\left(\frac{-\frac{2Bn\left(-\frac{Bn\int\frac{(c+dx)\left(b^2-\frac{4d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{12d^3}+\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{2b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right.}{5b}}{2b}\right.$$

$\downarrow$ 1195

$$(bc-$$

$$ad)^6g^2i^3\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{A+B\log}{3d^3\left(b-\right.}\right.}{}\right.$$

$\downarrow$ 2009

$$(bc-$$

$$ad)^6g^2i^3\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{A+B\log}{3d^3\left(b-\right.}\right.}{}\right.$$

$$\downarrow 2783$$

$$(bc -$$

$$ad)^6 g^2 i^3 \left(\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{6b(c+dx)^3 \left(b - \frac{d(a+bx)}{c+dx}\right)^6} - \frac{Bn \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log(}{3d^3\left(b-\right.} \right.$$

$$\downarrow 2773$$

$$(bc -$$

$$ad)^6 g^2 i^3 \left(\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{6b(c+dx)^3 \left(b - \frac{d(a+bx)}{c+dx}\right)^6} - \frac{Bn \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log(}{3d^3\left(b-\right.} \right.$$

$$\downarrow 49$$

$$(bc -$$

$$ad)^6 g^2 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log(}{3d^3\left(b-\right.}\right.}{} \right.$$

$\downarrow$ 2009

$$(bc -$$

$$ad)^6 g^2 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log(}{3d^3\left(b-\right.}\right.}{} \right.$$

$\downarrow$ 2781

$$(bc -$$

$$ad)^6 g^2 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log(}{3d^3\left(b-\right.}\right.}{} \right.$$

$\downarrow$ 2784

$$(bc -$$

$$ad)^6 g^2 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log(}{3d^3\left(b-\right.}\right.}{} \right.$$

$\downarrow$ 2784

$$(bc-ad)^6 g^2 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log(\right.\right.$$

↓ 2754

$$(bc-ad)^6 g^2 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log(\right.\right.$$

↓ 2838

$$(bc-ad)^6 g^2 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log(}{3d^3\left(b-\right.}\right.\right.$$

input

```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2,x]
```

output
```
(b*c - a*d)^6*g^2*i^3*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2)/(6*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^6) - (B*n*((b^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(5*d^3*(b - (d*(a + b*x))/(c + d*x))^5) -
 (b*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d^3*(b - (d*(a + b*x))/(c +
 d*x))^4) + (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*d^3*(b - (d*(a + b*x
))/(c + d*x))^3) - (B*n*((3*b)/(2*(b - (d*(a + b*x))/(c + d*x))^4) - 3/(b
 - (d*(a + b*x))/(c + d*x))^3 + 1/(2*b*(b - (d*(a + b*x))/(c + d*x))^2) + 1
/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^3 - Log[
b - (d*(a + b*x))/(c + d*x)]/b^3))/(30*d^3)))/(3*b) + (((a + b*x)^3*(A + B
*Log[e*((a + b*x)/(c + d*x))^n])^2)/(5*b*(c + d*x)^3*(b - (d*(a + b*x))/(c
 + d*x))^5) - (2*B*n*((b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d^3*
(b - (d*(a + b*x))/(c + d*x))^4) - (2*b*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]))/(3*d^3*(b - (d*(a + b*x))/(c + d*x))^3) + (A + B*Log[e*((a + b*x)/(c
 + d*x))^n])/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*(b/(b - (d*(a
 + b*x))/(c + d*x))^3 - 5/(2*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b*(b - (
d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a +
b*x))/(c + d*x)]/b^2))/(12*d^3)))/(5*b) + (2*(((a + b*x)^3*(A + B*Log[e*((
a + b*x)/(c + d*x))^n])^2)/(4*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^
4) - (B*n*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b*(c +
d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (B*n*(b^2/(2*d^3*(b - (d*(a +...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)^2 (dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2 (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (bgx+ag)^2 (dix+ci)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="fricas")
```

output
```
integral(A^2*b^2*d^3*g^2*i^3*x^5 + A^2*a^2*c^3*g^2*i^3 + (3*A^2*b^2*c*d^2
+ 2*A^2*a*b*d^3)*g^2*i^3*x^4 + (3*A^2*b^2*c^2*d + 6*A^2*a*b*c*d^2 + A^2*a^
2*d^3)*g^2*i^3*x^3 + (A^2*b^2*c^3 + 6*A^2*a*b*c^2*d + 3*A^2*a^2*c*d^2)*g^2
*i^3*x^2 + (2*A^2*a*b*c^3 + 3*A^2*a^2*c^2*d)*g^2*i^3*x + (B^2*b^2*d^3*g^2*
i^3*x^5 + B^2*a^2*c^3*g^2*i^3 + (3*B^2*b^2*c*d^2 + 2*B^2*a*b*d^3)*g^2*i^3*
x^4 + (3*B^2*b^2*c^2*d + 6*B^2*a*b*c*d^2 + B^2*a^2*d^3)*g^2*i^3*x^3 + (B^2
*b^2*c^3 + 6*B^2*a*b*c^2*d + 3*B^2*a^2*c*d^2)*g^2*i^3*x^2 + (2*B^2*a*b*c^3
 + 3*B^2*a^2*c^2*d)*g^2*i^3*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b
^2*d^3*g^2*i^3*x^5 + A*B*a^2*c^3*g^2*i^3 + (3*A*B*b^2*c*d^2 + 2*A*B*a*b*d^
3)*g^2*i^3*x^4 + (3*A*B*b^2*c^2*d + 6*A*B*a*b*c*d^2 + A*B*a^2*d^3)*g^2*i^3
*x^3 + (A*B*b^2*c^3 + 6*A*B*a*b*c^2*d + 3*A*B*a^2*c*d^2)*g^2*i^3*x^2 + (2*
A*B*a*b*c^3 + 3*A*B*a^2*c^2*d)*g^2*i^3*x)*log(e*((b*x + a)/(d*x + c))^n),
x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**
2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5931 vs. $2(937) = 1874$.

Time = 0.66 (sec) , antiderivative size = 5931, normalized size of antiderivative = 6.08

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="maxima")
```

output
```
1/3*A*B*b^2*d^3*g^2*i^3*x^6*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/6*A
^2*b^2*d^3*g^2*i^3*x^6 + 6/5*A*B*b^2*c*d^2*g^2*i^3*x^5*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n) + 4/5*A*B*a*b*d^3*g^2*i^3*x^5*log(e*(b*x/(d*x + c) + a
/(d*x + c))^n) + 3/5*A^2*b^2*c*d^2*g^2*i^3*x^5 + 2/5*A^2*a*b*d^3*g^2*i^3*x
^5 + 3/2*A*B*b^2*c^2*d*g^2*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ 3*A*B*a*b*c*d^2*g^2*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2
*A*B*a^2*d^3*g^2*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/4*A^2*
b^2*c^2*d*g^2*i^3*x^4 + 3/2*A^2*a*b*c*d^2*g^2*i^3*x^4 + 1/4*A^2*a^2*d^3*g^
2*i^3*x^4 + 2/3*A*B*b^2*c^3*g^2*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n) + 4*A*B*a*b*c^2*d*g^2*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ 2*A*B*a^2*c*d^2*g^2*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3
*A^2*b^2*c^3*g^2*i^3*x^3 + 2*A^2*a*b*c^2*d*g^2*i^3*x^3 + A^2*a^2*c*d^2*g^2
*i^3*x^3 + 2*A*B*a*b*c^3*g^2*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
) + 3*A*B*a^2*c^2*d*g^2*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A
^2*a*b*c^3*g^2*i^3*x^2 + 3/2*A^2*a^2*c^2*d*g^2*i^3*x^2 - 1/180*A*B*b^2*d^3
*g^2*i^3*n*(60*a^6*log(b*x + a)/b^6 - 60*c^6*log(d*x + c)/d^6 + (12*(b^5*c
*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*b^3*d^5)*x^4 + 20*(b^5*c^3*d
^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*d^5)*x^2 + 60*(b^5*c^5 - a^5
*d^5)*x)/(b^5*d^5)) + 1/10*A*B*b^2*c*d^2*g^2*i^3*n*(12*a^5*log(b*x + a)/b^
5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c...
```

Giac [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))
^2,x)
```

output
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))
^2, x)
```

Reduce [F]

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
(g**2*i*(12*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*
x + b*d*x**2),x)*a**6*b**2*d**7*n - 72*int((log(((a + b*x)**n*e)/(c + d*x)
**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**5*b**3*c*d**6*n + 180*int((
log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*
a**4*b**4*c**2*d**5*n - 240*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*
c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**5*c**3*d**4*n + 180*int((log(((a
 + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b**
6*c**4*d**3*n - 72*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x
 + b*c*x + b*d*x**2),x)*a*b**7*c**5*d**2*n + 12*int((log(((a + b*x)**n*e)/
(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**8*c**6*d*n + 12*lo
g(a + b*x)*a**7*d**6*n - 72*log(a + b*x)*a**6*b*c*d**5*n - 4*log(a + b*x)*
a**6*b*d**6*n**2 + 180*log(a + b*x)*a**5*b**2*c**2*d**4*n + 24*log(a + b*x
)*a**5*b**2*c*d**5*n**2 - 240*log(a + b*x)*a**4*b**3*c**3*d**3*n - 60*log(
a + b*x)*a**4*b**3*c**2*d**4*n**2 + 180*log(a + b*x)*a**3*b**4*c**4*d**2*n
 + 80*log(a + b*x)*a**3*b**4*c**3*d**3*n**2 - 72*log(a + b*x)*a**2*b**5*c*
*5*d*n - 60*log(a + b*x)*a**2*b**5*c**4*d**2*n**2 + 12*log(a + b*x)*a*b**6
*c**6*n + 24*log(a + b*x)*a*b**6*c**5*d*n**2 - 4*log(a + b*x)*b**7*c**6*n*
*2 - 6*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**5*b**2*c*d**5 + 30*log(((a
 + b*x)**n*e)/(c + d*x)**n)**2*a**4*b**3*c**2*d**4 - 60*log(((a + b*x)**n*
e)/(c + d*x)**n)**2*a**3*b**4*c**3*d**3 - 30*log(((a + b*x)**n*e)/(c + ...
```

3.180 $\int (ag+bgx)(ci+dix)^3 \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result 2072
Mathematica [A] (verified) 2073
Rubi [A] (verified) 2074
Maple [F] 2088
Fricas [F] 2088
Sympy [F(-1)] 2089
Maxima [B] (verification not implemented) 2089
Giac [F(-1)] 2090
Mupad [F(-1)] 2091
Reduce [F] 2091

Optimal result

Integrand size = 43, antiderivative size = 786

$$
\begin{aligned}
&\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx \\
&= \frac{B^2(bc-ad)^4gi^3n^2x}{60b^3d} + \frac{B^2(bc-ad)^3gi^3n^2(c+dx)^2}{30b^2d^2} + \frac{B^2(bc-ad)^2gi^3n^2(c+dx)^3}{30bd^2} \\
&\quad - \frac{B(bc-ad)^4gi^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^4d} \\
&\quad - \frac{B(bc-ad)^3gi^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^4} \\
&\quad + \frac{3B(bc-ad)^3gi^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^2d^2} \\
&\quad + \frac{B(bc-ad)^2gi^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^2} \\
&\quad - \frac{B(bc-ad)gi^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^2} \\
&\quad + \frac{(bc-ad)^3gi^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^4} \\
&\quad + \frac{(bc-ad)^2gi^3(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^3} \\
&\quad + \frac{3(bc-ad)gi^3(a+bx)^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2} \\
&\quad + \frac{gi^3(a+bx)^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} \\
&\quad - \frac{B(bc-ad)^5gi^3n\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{10b^4d^2} \\
&\quad - \frac{B^2(bc-ad)^5gi^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d^2} - \frac{11B^2(bc-ad)^5gi^3n^2\log(c+dx)}{60b^4d^2} \\
&\quad - \frac{B^2(bc-ad)^5gi^3n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{10b^4d^2}
\end{aligned}
$$

output

```
1/60*B^2*(-a*d+b*c)^4*g*i^3*n^2*x/b^3/d+1/30*B^2*(-a*d+b*c)^3*g*i^3*n^2*(d
*x+c)^2/b^2/d^2+1/30*B^2*(-a*d+b*c)^2*g*i^3*n^2*(d*x+c)^3/b/d^2-1/10*B*(-a
*d+b*c)^4*g*i^3*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/d-1/10*B*(-a
*d+b*c)^3*g*i^3*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4+3/20*B*(-a
*d+b*c)^3*g*i^3*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d^2+1/30*B
*(-a*d+b*c)^2*g*i^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2-1/10
*B*(-a*d+b*c)*g*i^3*n*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^2+1/20*(
-a*d+b*c)^3*g*i^3*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^4+1/10*(-a
*d+b*c)^2*g*i^3*(b*x+a)^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3+3/
20*(-a*d+b*c)*g*i^3*(b*x+a)^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/
b^2+1/5*g*i^3*(b*x+a)^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b-1/10
*B*(-a*d+b*c)^5*g*i^3*n*(A+B*n+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/
b/(d*x+c))/b^4/d^2-1/12*B^2*(-a*d+b*c)^5*g*i^3*n^2*ln((b*x+a)/(d*x+c))/b^4
/d^2-11/60*B^2*(-a*d+b*c)^5*g*i^3*n^2*ln(d*x+c)/b^4/d^2-1/10*B^2*(-a*d+b*c
)^5*g*i^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^2
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 945, normalized size of antiderivative = 1.20

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input

```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^2,x]
```

output

```
(g*i^3*(-5*(b*c - a*d)*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2 + 4*b*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (5*B*(b*c -
 a*d)^2*n*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*c - a
*d)*Log[a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2
 + 2*(b*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a
 + b*x)/(c + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]) + 6*(b*c - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
 - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[a + b*x]*(Lo
g[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x)
)/(-(b*c) + a*d)])))/(3*b^4) - (B*(b*c - a*d)*n*(24*A*b*d*(b*c - a*d)^3*x
 - 12*B*(b*c - a*d)^3*n*(b*d*x + (b*c - a*d)*Log[a + b*x]) - 4*B*(b*c - a*d
)^2*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c - a*d)^2*Log[a + b*x
]) - B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c + d*x)^
2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]) + 24*B*d*(b*c - a*d)
^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 12*b^2*(b*c - a*d)^2*(c + d*
x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 8*b^3*(b*c - a*d)*(c + d*x)^
3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*b^4*(c + d*x)^4*(A + B*Log[e*
((a + b*x)/(c + d*x))^n]) + 24*(b*c - a*d)^4*Log[a + b*x]*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]) - 24*B*(b*c - a*d)^4*n*Log[c + d*x] - 12*B*(b*c ...
```

Rubi [A] (verified)

Time = 2.31 (sec) , antiderivative size = 1002, normalized size of antiderivative = 1.27, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.442$, Rules used = {2961, 2783, 2782, 27, 86, 2009, 2783, 2782, 27, 86, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$

$$\downarrow \text{2961}$$

$$gi^3(bc-ad)^5\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\downarrow 2783$$

$$gi^3(bc-ad)^5\left(-\frac{2Bn\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{3\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a}{c}\right.\right.\right.}{5b(c+dx)^2\left(b-\right.}\right.$$

$$\downarrow 2782$$

$$gi^3(bc-ad)^5\left(-\frac{2Bn\left(-Bn\int-\frac{(c+dx)\left(b-\frac{4d(a+bx)}{c+dx}\right)}{12d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{5b}+\frac{3\int\frac{(a+bx)\left(\right.}{(c-}\right.$$

$$\downarrow 27$$

$$gi^3(bc-ad)^5\left(-\frac{2Bn\left(\frac{Bn\int\frac{(c+dx)\left(b-\frac{4d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{12d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{5b}+\frac{3\int\frac{(a+bx)\left(A+B\log\left(e\right.\right.}{(c+dx)\left(b-\frac{d(a}{c+}\right.}{5b}$$

$$\downarrow 86$$

$$gi^3(bc-ad)^5\left(-\frac{2Bn\left(\frac{Bn\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a+bx}{c+dx}}{12d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right.}{5b}$$

$$\downarrow 2009$$

$$gi^3(bc-$$

$$ad)^5\left(\frac{3\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}-\frac{2Bn\left(-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}\right.}{5b}\right.\right.$$

$$\downarrow 2783$$

$$gi^3(bc-$$

$$ad)^5\left(\frac{3\left(-\frac{Bn\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{5b}\right.$$

$$\downarrow 2782$$

$$gi^3(bc-$$

$$ad)^5\left(\frac{3\left(-\frac{Bn\left(-Bn\int-\frac{(c+dx)\left(b-\frac{3d(a+bx)}{c+dx}\right)}{6d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^4}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}}{2b}\right.\right.$$

$$\downarrow 27$$

$$gi^3(bc-$$

$$ad)^5\left(\frac{3\left(-\frac{Bn\left(\frac{Bn\int\frac{(c+dx)\left(b-\frac{3d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{6d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\,c}{2b}\right)}{5b}$$

$$\downarrow 86$$

$$gi^3(bc-$$

$$ad)^5\left(\frac{3\left(-\frac{Bn\left(\frac{Bn\int\left(\frac{\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{c+dx}{b^2(a+bx)}}\right)d\frac{a+bx}{c+dx}}{6d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}\right)}{5b}$$

$$\downarrow 2009$$

$$gi^3(bc-$$

$$ad)^5\left(\frac{3\left(\frac{\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}-\frac{Bn\left(-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right.}{b^2}\right.}{}\right.}{2b}\right.}{5b}\right.$$

$\downarrow$ 2783

$$(bc-$$

$$ad)^5gi^3\left(\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{Bn\left(\frac{\log\left(\frac{a+}{c+}\right.}{b^3}\right.}{}\right.}{5}\right.$$

$\downarrow$ 2773

$$(bc-$$

$$ad)^5 gi^3 \left(\frac{(a+bx)^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn \left(-\frac{A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{Bn \left(\frac{\log\left(\frac{a+}{c+}\right.}{b^3} \right.}{5} \right.$$

$$\downarrow 49$$

$$(bc-$$

$$ad)^5 gi^3 \left(\frac{(a+bx)^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn \left(-\frac{A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{Bn \left(\frac{\log\left(\frac{a+}{c+}\right.}{b^3} \right.}{} \right.$$

↓ 2009

$$(bc -$$

$$ad)^5 gi^3 \left(\frac{(a+bx)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b(c+dx)^2 \left(b - \frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{Bn\left(\frac{\log\left(\frac{a+}{c+}\right.}{b^3}\right.}{}\right.}{5}\right.$$

$\downarrow$ 2781

$$(bc -$$

$$ad)^5 gi^3 \left(\frac{(a+bx)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b(c+dx)^2 \left(b - \frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{Bn\left(\frac{\log\left(\frac{a+}{c+}\right.}{b^3}\right.}{}\right.}{5}\right.$$

$\downarrow$ 2784

$$(bc-$$

$$ad)^5 gi^3 \left(\frac{(a+bx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{Bn\left(\frac{\log\left(\frac{a+}{c+}\right.}{b^3}\right.}{5}\right.$$

↓ 2754

$$(bc-$$

$$ad)^5 gi^3 \left(\frac{(a+bx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{Bn\left(\frac{\log\left(\frac{a+}{c+}\right.}{b^3}\right.}{5}\right.$$

$$\downarrow 2838$$

$$(bc - ad)^5 g i^3 \left(\frac{(a+bx)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b(c+dx)^2\left(b - \frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{Bn\left(\frac{\log\left(\frac{a+}{c+}\right.}{b^3}\right.}{}\right.}{5}\right.$$

input

```
Int[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
,x]
```

output

```
(b*c - a*d)^5*g*i^3*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
)/(5*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*n*((b*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/(4*d^2*(b - (d*(a + b*x))/(c + d*x))^4) - (
A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*d^2*(b - (d*(a + b*x))/(c + d*x))
^3) + (B*n*(-(b - (d*(a + b*x))/(c + d*x))^(-3) + 1/(2*b*(b - (d*(a + b*x)
)/(c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c
 + d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(12*d^2)))/(5*b) + (
3*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*b*(c + d*x)^2
*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*((b*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(3*d^2*(b - (d*(a + b*x))/(c + d*x))^3) - (A + B*Log[e*((a + b*
x)/(c + d*x))^n])/(2*d^2*(b - (d*(a + b*x))/(c + d*x))^2) + (B*n*(-(b - (d
*(a + b*x))/(c + d*x))^(-2) + 1/(b*(b - (d*(a + b*x))/(c + d*x))) + Log[(a
 + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(6*d^2)))/
(2*b) + (((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*b*(c +
d*x)^2*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*n*(((a + b*x)^2*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]))/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x)
)^2) - (B*n*(b/(d^2*(b - (d*(a + b*x))/(c + d*x))) + Log[b - (d*(a + b*x))
/(c + d*x)]/d^2))/(2*b)))/(3*b) + (((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2)/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*(
((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d...
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 86

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int (bgx+ag)\,(dix+ci)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (bgx+ag)(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="fricas")
```

output
```
integral(A^2*b*d^3*g*i^3*x^4 + A^2*a*c^3*g*i^3 + (3*A^2*b*c*d^2 + A^2*a*d^
3)*g*i^3*x^3 + 3*(A^2*b*c^2*d + A^2*a*c*d^2)*g*i^3*x^2 + (A^2*b*c^3 + 3*A^
2*a*c^2*d)*g*i^3*x + (B^2*b*d^3*g*i^3*x^4 + B^2*a*c^3*g*i^3 + (3*B^2*b*c*d
^2 + B^2*a*d^3)*g*i^3*x^3 + 3*(B^2*b*c^2*d + B^2*a*c*d^2)*g*i^3*x^2 + (B^2
*b*c^3 + 3*B^2*a*c^2*d)*g*i^3*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B
*b*d^3*g*i^3*x^4 + A*B*a*c^3*g*i^3 + (3*A*B*b*c*d^2 + A*B*a*d^3)*g*i^3*x^3
 + 3*(A*B*b*c^2*d + A*B*a*c*d^2)*g*i^3*x^2 + (A*B*b*c^3 + 3*A*B*a*c^2*d)*g
*i^3*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x
)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3724 vs. $2(753) = 1506$.

Time $= 0.62$ (sec) , antiderivative size $= 3724$, normalized size of antiderivative $= 4.74$

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="maxima")
```

output

```
2/5*A*B*b*d^3*g*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A^2*b
*d^3*g*i^3*x^5 + 3/2*A*B*b*c*d^2*g*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x +
 c))^n) + 1/2*A*B*a*d^3*g*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
 3/4*A^2*b*c*d^2*g*i^3*x^4 + 1/4*A^2*a*d^3*g*i^3*x^4 + 2*A*B*b*c^2*d*g*i^3
*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*B*a*c*d^2*g*i^3*x^3*log(
e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*b*c^2*d*g*i^3*x^3 + A^2*a*c*d^2*g
*i^3*x^3 + A*B*b*c^3*g*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3*
A*B*a*c^2*d*g*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*b*c
^3*g*i^3*x^2 + 3/2*A^2*a*c^2*d*g*i^3*x^2 + 1/30*A*B*b*d^3*g*i^3*n*(12*a^5*
log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^
4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12
*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/4*A*B*b*c*d^2*g*i^3*n*(6*a^4*log(b*
x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(
b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/12*A*
B*a*d^3*g*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3
*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3
*d^3)*x)/(b^3*d^3)) + A*B*b*c^2*d*g*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*
log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^
2*d^2)) + A*B*a*c*d^2*g*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) -...
```

Giac [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input

```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="giac")
```

output

```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)\,(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2
,x)
```

output
```
int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2
, x)
```

Reduce [F]

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
(g*i*(6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x +
b*d*x**2),x)*a**5*b**2*d**6*n - 30*int((log(((a + b*x)**n*e)/(c + d*x)**n)
*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**4*b**3*c*d**5*n + 60*int((log((
(a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*
b**4*c**2*d**4*n - 60*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*
d*x + b*c*x + b*d*x**2),x)*a**2*b**5*c**3*d**3*n + 30*int((log(((a + b*x)*
*n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a*b**6*c**4*d**
2*n - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x +
b*d*x**2),x)*b**7*c**5*d*n + 6*log(a + b*x)*a**6*d**5*n - 30*log(a + b*x)*
a**5*b*c*d**4*n - 5*log(a + b*x)*a**5*b*d**5*n**2 + 60*log(a + b*x)*a**4*b
**2*c**2*d**3*n + 25*log(a + b*x)*a**4*b**2*c*d**4*n**2 - 60*log(a + b*x)*
a**3*b**3*c**3*d**2*n - 50*log(a + b*x)*a**3*b**3*c**2*d**3*n**2 + 30*log(
a + b*x)*a**2*b**4*c**4*d*n + 50*log(a + b*x)*a**2*b**4*c**3*d**2*n**2 - 6
*log(a + b*x)*a*b**5*c**5*n - 25*log(a + b*x)*a*b**5*c**4*d*n**2 + 5*log(a
 + b*x)*b**6*c**5*n**2 - 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**4*b**2
*c*d**4 + 12*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*b**3*c**2*d**3 - 1
8*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**4*c**3*d**2 - 3*log(((a +
b*x)**n*e)/(c + d*x)**n)**2*a*b**5*c**4*d - 60*log(((a + b*x)**n*e)/(c + d
*x)**n)**2*a*b**5*c**3*d**2*x - 90*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a
*b**5*c**2*d**3*x**2 - 60*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**5*...
```

3.181 $\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result . . . 2093
Mathematica [A] (verified) . . . 2094
Rubi [A] (verified) . . . 2095
Maple [F] . . . 2103
Fricas [F] . . . 2104
Sympy [F] . . . 2105
Maxima [B] (verification not implemented) . . . 2106
Giac [F] . . . 2107
Mupad [F(-1)] . . . 2107
Reduce [F] . . . 2107

Optimal result

Integrand size = 35, antiderivative size = 454

$$
\begin{aligned}
&\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{5B^2(bc-ad)^3i^3n^2x}{12b^3} + \frac{B^2(bc-ad)^2i^3n^2(c+dx)^2}{12b^2d} \\
&\quad - \frac{B(bc-ad)^3i^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^4} \\
&\quad - \frac{B(bc-ad)^2i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2d} \\
&\quad - \frac{B(bc-ad)i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} \\
&\quad + \frac{i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} \\
&\quad + \frac{5B^2(bc-ad)^4i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d} + \frac{11B^2(bc-ad)^4i^3n^2\log(c+dx)}{12b^4d} \\
&\quad + \frac{B(bc-ad)^4i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d} \\
&\quad - \frac{B^2(bc-ad)^4i^3n^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d}
\end{aligned}
$$

output

```
5/12*B^2*(-a*d+b*c)^3*i^3*n^2*x/b^3+1/12*B^2*(-a*d+b*c)^2*i^3*n^2*(d*x+c)^
2/b^2/d-1/2*B*(-a*d+b*c)^3*i^3*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b
^4-1/4*B*(-a*d+b*c)^2*i^3*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/
d-1/6*B*(-a*d+b*c)*i^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/4
*i^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d+5/12*B^2*(-a*d+b*c)^4*i
^3*n^2*ln((b*x+a)/(d*x+c))/b^4/d+11/12*B^2*(-a*d+b*c)^4*i^3*n^2*ln(d*x+c)/
b^4/d+1/2*B*(-a*d+b*c)^4*i^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x
+c)/d/(b*x+a))/b^4/d-1/2*B^2*(-a*d+b*c)^4*i^3*n^2*polylog(2,b*(d*x+c)/d/(b
*x+a))/b^4/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.90

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$

$$= \frac{i^3\left((c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - \frac{B(bc-ad)n\left(6Abd(bc-ad)^2x-3B(bc-ad)^2n(bdx+(bc-ad)\log(a+bx))-B(bc-ad)n(2\right.}{}\right.}{}$$

input

```
Integrate[(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output

```
(i^3*((c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d
)*n*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*c - a*d)*Lo
g[a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(
b*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x
)/(c + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) +
6*(b*c - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*B*
(b*c - a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[a + b*x]*(Log[a +
b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b
*c) + a*d)])))/(3*b^4)))/(4*d)
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2951, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)^3 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$

$$\downarrow 2951$$

$$i^3(bc-ad)^4 \int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}$$

$$\downarrow 2756$$

$$i^3(bc-ad)^4 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{2d}\right)$$

$$\downarrow 2789$$

$$ad)^4 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{b} + \frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3} d\frac{a+bx}{c+dx}}{b}\right)}{2d}\right)$$

$$i^3(bc-$$

$$\downarrow 2756$$

$$i^3(bc - ad)^4 \left(\frac{\left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2}{4d\left(b - \frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{Bn\int \frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d} \right)}{b} + \frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx}{c+dx}\right.}}{b}\right.}{2d}\right.$$

$\downarrow$ 54

$$i^3(bc - ad)^4 \left(\frac{\left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2}{4d\left(b - \frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{Bn\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)} + \frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} + \frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{}{b^3}\right.}{3d}\right.}{b}\right.}{2d}\right.$$

$\downarrow$ 2009

$$ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{i^3(bc-}{2d}\right.$$

$$Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log}{}\right.\right.\right.$$

$$\left.\left.\left.\right)\right)\right)\right)$$

$$\downarrow 2789$$

$$ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{i^3(bc-}{2d}\right.$$

$$Bn\left(\frac{\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\right)\right)\right)\right)$$

$$\downarrow 2756$$

$$ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\right.$$

$$Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{b}$$

$$i^3(bc-$$

$$\downarrow 54$$

$$ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\right.$$

$$Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+$$

$$i^3(bc-$$

$$\downarrow 2009$$

$$ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{i^3(bc-\; Bn\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log(b-}{}\right.}{}\right.}{b}}{b}\right.}{}\right.$$

$\downarrow$ 2789

$$ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{i^3(bc-\; Bn\left(\frac{\frac{\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.}{2d\left(b-\frac{d(a+}{c+d}\right.}\right.}{b}}{}\right.}{}\right.$$

$\downarrow$ 2751

$$ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\right.$$

$$Bn\left(\frac{i^3(bc-}{\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}\right.$$

$$\downarrow 16$$

$$ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\right.$$

$$Bn\left(\frac{i^3(bc-}{\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+}{c+d}\right.\right.}{b}}{b}\right.$$

$$\downarrow 2779$$

$$ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-Bn\left(\frac{i^3(bc-\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)(}{b(c}\right.\right.\right.\right.$$

$$\downarrow 2838$$

$$ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-Bn\left(\frac{i^3(bc-\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{Bn\,\mathrm{Pol}}{}\right.\right.\right.\right.$$

input
```
Int[(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^4*i^3*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(4*d*(b - (d*(
a + b*x))/(c + d*x))^4) - (B*n*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n])
/(3*d*(b - (d*(a + b*x))/(c + d*x))^3) - (B*n*(1/(2*b*(b - (d*(a + b*x))/(
c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c +
d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(3*d)))/b + ((d*((A + B
*Log[e*((a + b*x)/(c + d*x))^n])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (
B*n*(1/(b*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 -
Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)
)) + (B*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*(
(a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*P
olyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b)/b)/b)/b))/(2*d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
 - Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [F]

$$\int (dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (dix+ci)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

output
```
integral(A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*c^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2*c^3*i^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F]

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= i^3\left(\int A^2c^3\,dx + \int A^2d^3x^3\,dx + \int B^2c^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx\right.$$
$$+ \int 2ABc^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx + \int 3A^2cd^2x^2\,dx + \int 3A^2c^2dx\,dx$$
$$+ \int B^2d^3x^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+ \int 2ABd^3x^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx$$
$$+ \int 3B^2cd^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+ \int 3B^2c^2dx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2 dx$$
$$+ \int 6ABcd^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx$$
$$\left.+ \int 6ABc^2dx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx\right)$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
i**3*(Integral(A**2*c**3, x) + Integral(A**2*d**3*x**3, x) + Integral(B**2
*c**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*c**3
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Integral(3*A**2*c*d**2*x**2
, x) + Integral(3*A**2*c**2*d*x, x) + Integral(B**2*d**3*x**3*log(e*(a/(c
+ d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*d**3*x**3*log(e*(a/(c
+ d*x) + b*x/(c + d*x))**n), x) + Integral(3*B**2*c*d**2*x**2*log(e*(a/(c
+ d*x) + b*x/(c + d*x))**n)**2, x) + Integral(3*B**2*c**2*d*x*log(e*(a/(c
+ d*x) + b*x/(c + d*x))**n)**2, x) + Integral(6*A*B*c*d**2*x**2*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n), x) + Integral(6*A*B*c**2*d*x*log(e*(a/(c +
d*x) + b*x/(c + d*x))**n), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2129 vs. $2(433) = 866$.

Time $= 0.57$ (sec) , antiderivative size $= 2129$, normalized size of antiderivative $= 4.69$

$$\int (ci + dix)^3 \left(A + B \log \left(e \left(\frac{a + bx}{c + dx} \right)^n \right) \right)^2 dx = \text{Too large to display}$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output

```
1/2*A*B*d^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*d^3*i
^3*x^4 + 2*A*B*c*d^2*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*
c*d^2*i^3*x^3 + 3*A*B*c^2*d*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
 + 3/2*A^2*c^2*d*i^3*x^2 - 1/12*A*B*d^3*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*
c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2
*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + A*B*c*d^2*i^3*n*(2*a^3
*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A*B*c^2*d*i^3*n*(a^2*log(b*x + a)/b^
2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*c^3*i^3*n*(a*log(b
*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c^3*i^3*x*log(e*(b*x/(d*x + c) + a/(
d*x + c))^n) + A^2*c^3*i^3*x - 1/12*(26*a*b^2*c^3*d*i^3*n^2 - 21*a^2*b*c^2
*d^2*i^3*n^2 + 6*a^3*c*d^3*i^3*n^2 - (11*i^3*n^2 - 6*i^3*n*log(e))*b^3*c^4
)*B^2*log(d*x + c)/(b^3*d) - 1/2*(b^4*c^4*i^3*n^2 - 4*a*b^3*c^3*d*i^3*n^2
+ 6*a^2*b^2*c^2*d^2*i^3*n^2 - 4*a^3*b*c*d^3*i^3*n^2 + a^4*d^4*i^3*n^2)*(lo
g(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
- a*d)))*B^2/(b^4*d) + 1/12*(3*B^2*b^4*d^4*i^3*x^4*log(e)^2 + 6*B^2*b^4*c^
4*i^3*n^2*log(b*x + a)*log(d*x + c) - 3*B^2*b^4*c^4*i^3*n^2*log(d*x + c)^2
 + 2*(a*b^3*d^4*i^3*n*log(e) - (i^3*n*log(e) - 6*i^3*log(e)^2)*b^4*c*d^3)*
B^2*x^3 + ((i^3*n^2 - 9*i^3*n*log(e) + 18*i^3*log(e)^2)*b^4*c^2*d^2 - 2*(i
^3*n^2 - 6*i^3*n*log(e))*a*b^3*c*d^3 + (i^3*n^2 - 3*i^3*n*log(e))*a^2*b...
```

Giac [F]

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (dix+ci)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (c\,i+d\,i\,x)^3 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

Reduce [F]

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{too large to display}$$

input
```
int((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
(i*(6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*
d*x**2),x)*a**4*b**2*d**5*n - 24*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x
)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*b**3*c*d**4*n + 36*int((log(((a
 + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b*
*4*c**2*d**3*n - 24*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a*c + a*d*
x + b*c*x + b*d*x**2),x)*a*b**5*c**3*d**2*n + 6*int((log(((a + b*x)**n*e)/
(c + d*x)**n)*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*b**6*c**4*d*n + 6*log
(c + d*x)*a**5*d**4*n - 24*log(c + d*x)*a**4*b*c*d**3*n - 11*log(c + d*x)*
a**4*b*d**4*n**2 + 36*log(c + d*x)*a**3*b**2*c**2*d**2*n + 44*log(c + d*x)
*a**3*b**2*c*d**3*n**2 - 24*log(c + d*x)*a**2*b**3*c**3*d*n - 66*log(c + d
*x)*a**2*b**3*c**2*d**2*n**2 + 6*log(c + d*x)*a*b**4*c**4*n + 44*log(c + d
*x)*a*b**4*c**3*d*n**2 - 11*log(c + d*x)*b**5*c**4*n**2 - 3*log(((a + b*x)
**n*e)/(c + d*x)**n)**2*a**3*b**2*c*d**3 + 9*log(((a + b*x)**n*e)/(c + d*x
)**n)**2*a**2*b**3*c**2*d**2 - 9*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b
**4*c**3*d - 12*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**5*c**3*d*x - 18*l
og(((a + b*x)**n*e)/(c + d*x)**n)**2*b**5*c**2*d**2*x**2 - 12*log(((a + b*
x)**n*e)/(c + d*x)**n)**2*b**5*c*d**3*x**3 - 3*log(((a + b*x)**n*e)/(c + d
*x)**n)**2*b**5*d**4*x**4 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**5*d**4
 - 24*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*b*c*d**3 - 11*log(((a + b*x)
**n*e)/(c + d*x)**n)*a**4*b*d**4*n + 36*log(((a + b*x)**n*e)/(c + d*x)*...
```

3.182 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

Optimal result 2110
Mathematica [B] (verified) 2111
Rubi [A] (verified) 2112
Maple [F] 2128
Fricas [F] 2129
Sympy [F] 2129
Maxima [F] 2130
Giac [F] 2131
Mupad [F(-1)] 2132
Reduce [F] 2132

Optimal result

Integrand size = 45, antiderivative size = 762

$$
\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx \\
&= \frac{B^2d(bc-ad)^2i^3n^2x}{3b^3g} - \frac{5Bd(bc-ad)^2i^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^4g} \\
&\quad - \frac{B(bc-ad)i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2g} \\
&\quad + \frac{d(bc-ad)^2i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4g} \\
&\quad + \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g} \\
&\quad + \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg} \\
&\quad + \frac{2B(bc-ad)^3i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^4g} \\
&\quad + \frac{B^2(bc-ad)^3i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^4g} + \frac{2B^2(bc-ad)^3i^3n^2\log(c+dx)}{b^4g} \\
&\quad + \frac{5B(bc-ad)^3i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g} \\
&\quad - \frac{(bc-ad)^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} \\
&\quad + \frac{2B^2(bc-ad)^3i^3n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g} - \frac{5B^2(bc-ad)^3i^3n^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g} \\
&\quad + \frac{2B(bc-ad)^3i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} \\
&\quad + \frac{2B^2(bc-ad)^3i^3n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}
\end{aligned}
$$

output

```
1/3*B^2*d*(-a*d+b*c)^2*i^3*n^2*x/b^3/g-5/3*B*d*(-a*d+b*c)^2*i^3*n*(b*x+a)*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/g-1/3*B*(-a*d+b*c)*i^3*n*(d*x+c)^2*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g+d*(-a*d+b*c)^2*i^3*(b*x+a)*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))^2/b^4/g+1/2*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*((b*x+a
)/(d*x+c))^n))^2/b^2/g+1/3*i^3*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2
/b/g+2*B*(-a*d+b*c)^3*i^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/
b/(d*x+c))/b^4/g+1/3*B^2*(-a*d+b*c)^3*i^3*n^2*ln((b*x+a)/(d*x+c))/b^4/g+2*
B^2*(-a*d+b*c)^3*i^3*n^2*ln(d*x+c)/b^4/g+5/3*B*(-a*d+b*c)^3*i^3*n*(A+B*ln(
e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g-(-a*d+b*c)^3*i^3*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g+2*B^2*(-a
*d+b*c)^3*i^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/g-5/3*B^2*(-a*d+b*c)^
3*i^3*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g+2*B*(-a*d+b*c)^3*i^3*n*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g+2*B^2*(-a
*d+b*c)^3*i^3*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^4/g
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4969 vs. $2(762) = 1524$.

Time $= 9.45$ (sec) , antiderivative size $= 4969$, normalized size of antiderivative $= 6.52$

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \text{Result too large to show}$$

input

```
Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x),x]
```

output

```
(i^3*(36*b*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x*(A + B*Log[e*((a + b*x)/(
c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 + 18*b^2*d^2*(3*b*c - a*d)*
x^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^
2 + 12*b^3*d^3*x^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*
x)/(c + d*x)])^2 + 36*(b*c - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(
c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 - 108*b^2*B*c^2*n*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(a*d*Log[a/b
 + x]^2 - 2*a*d*Log[a/b + x]*(1 + Log[a + b*x]) + 2*(-(b*c) + a*d + Log[c/d
 + x]*(b*c + a*d*Log[a + b*x] - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)]) + (
-(b*d*x) + a*d*Log[a + b*x])*Log[(a + b*x)/(c + d*x)]) - 2*a*d*PolyLog[2,
(b*(c + d*x))/(b*c - a*d)]) - 12*B*n*(-A - B*Log[e*((a + b*x)/(c + d*x))^n
] + B*n*Log[(a + b*x)/(c + d*x)])*(6*a^2*b*c*d^2 - 6*a^3*d^3 + 2*b^3*c^2*d
*x + 3*a*b^2*c*d^2*x - 5*a^2*b*d^3*x - b^3*c*d^2*x^2 + a*b^2*d^3*x^2 - 3*a
^3*d^3*Log[a/b + x]^2 - 6*a^2*b*c*d^2*Log[c/d + x] + 5*a^3*d^3*Log[a + b*x
] - 6*a^3*d^3*Log[c/d + x]*Log[a + b*x] + 6*a^3*d^3*Log[a/b + x]*(1 + Log[
a + b*x]) + 6*a^3*d^3*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 6*a
^2*b*d^3*x*Log[(a + b*x)/(c + d*x)] - 3*a*b^2*d^3*x^2*Log[(a + b*x)/(c + d
*x)] + 2*b^3*d^3*x^3*Log[(a + b*x)/(c + d*x)] - 6*a^3*d^3*Log[a + b*x]*Log
[(a + b*x)/(c + d*x)] - 2*b^3*c^3*Log[c + d*x] - 3*a*b^2*c^2*d*Log[c + d*x
] + 6*a^3*d^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 36*b^3*B*c^3*n*(...
```

Rubi [A] (verified)

Time = 3.30 (sec) , antiderivative size = 901, normalized size of antiderivative = 1.18, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.356$, Rules used = {2961, 2789, 2756, 2789, 2756, 54, 2009, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{ag+bgx}\,dx$$

$$\downarrow 2961$$

$$\frac{i^3(bc-ad)^3 \int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{g}$$

$$\downarrow 2789$$

$$\frac{i^3(bc-ad)^3\left(\frac{d\int\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

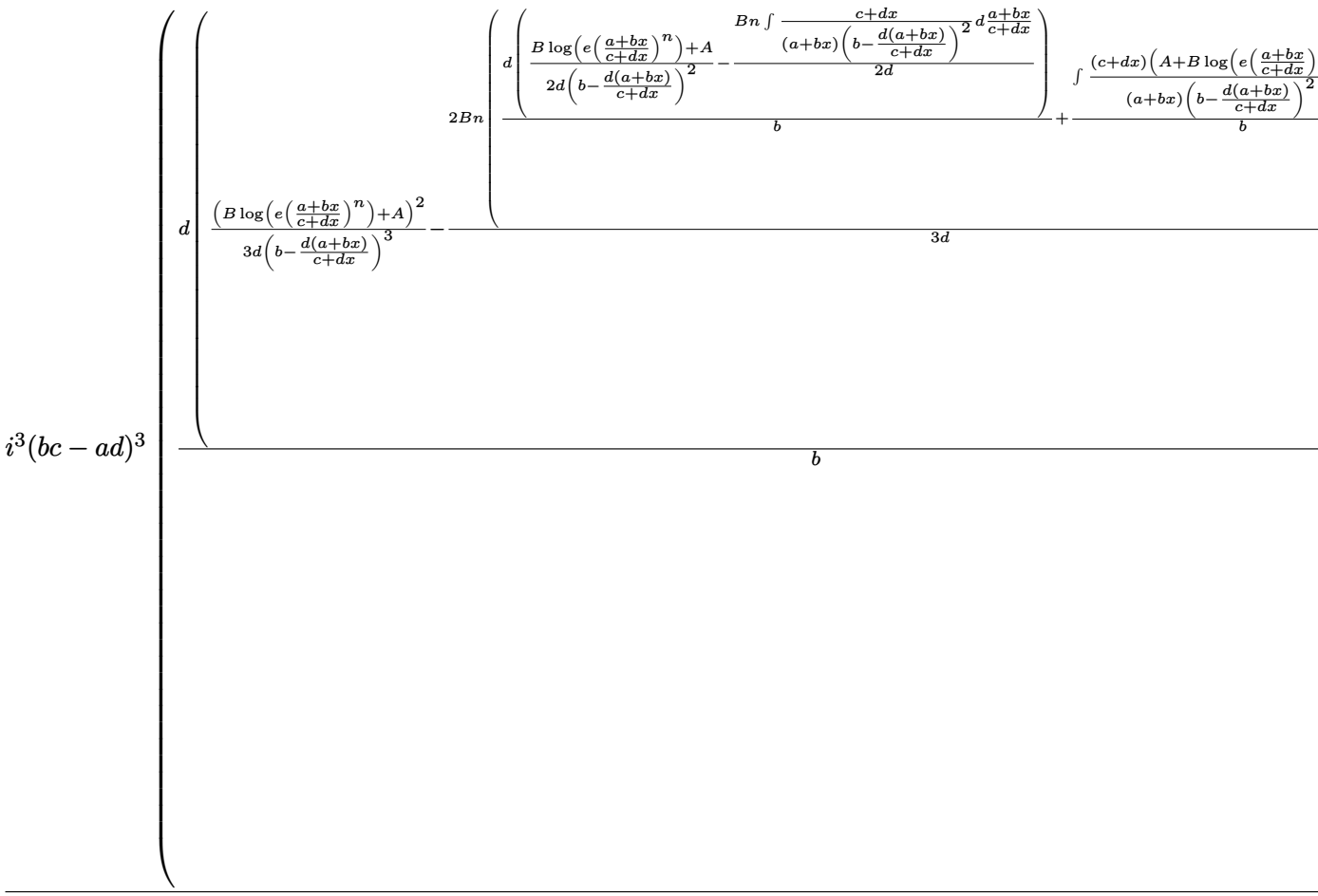
$$\downarrow 2756$$

$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\downarrow 2789$$

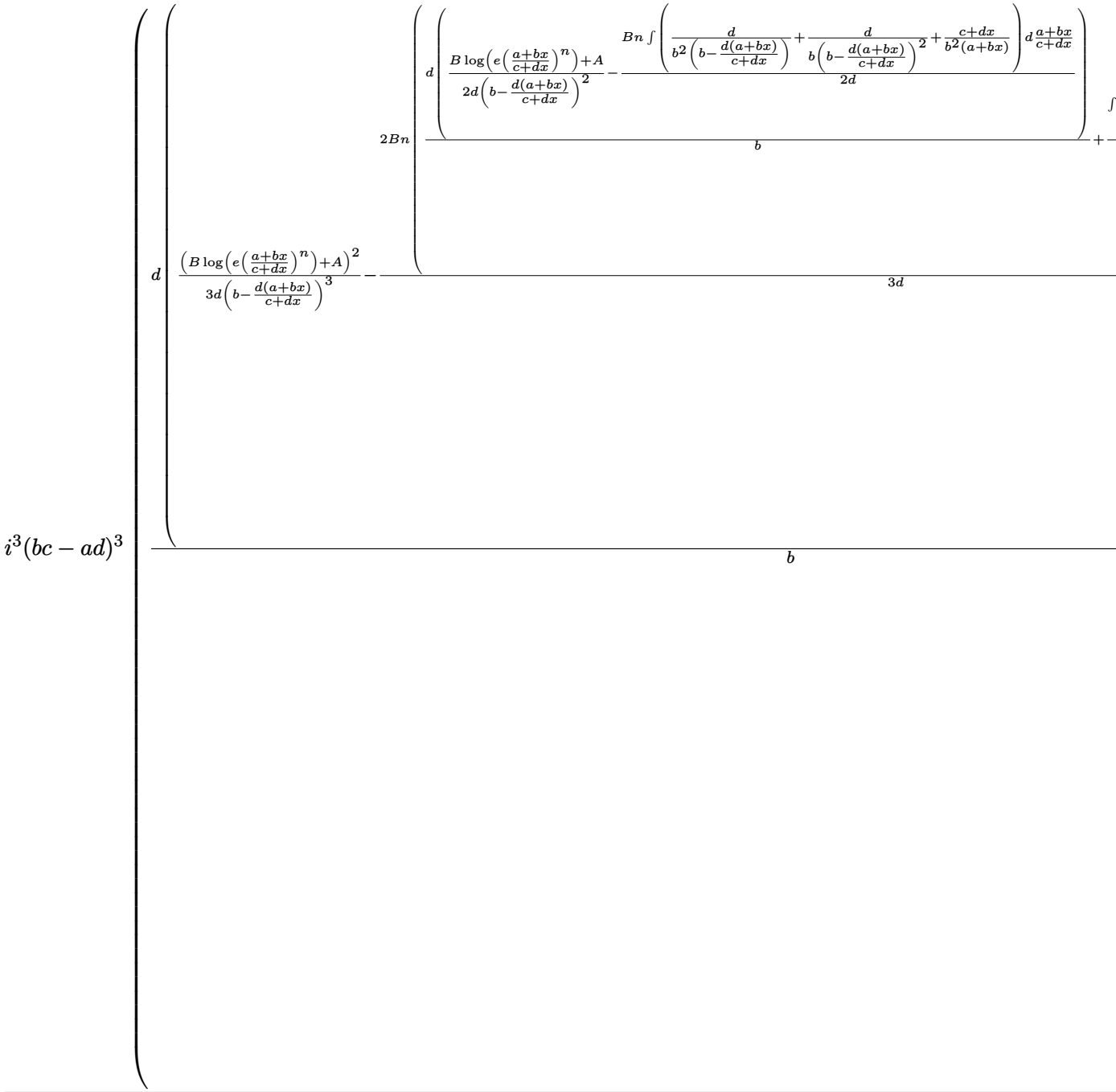
$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{3d}\right)}{b}+\frac{d\int\frac{\left(A+B\log\right.}{\left(b\right.}}{g}$$

$$\downarrow 2756$$

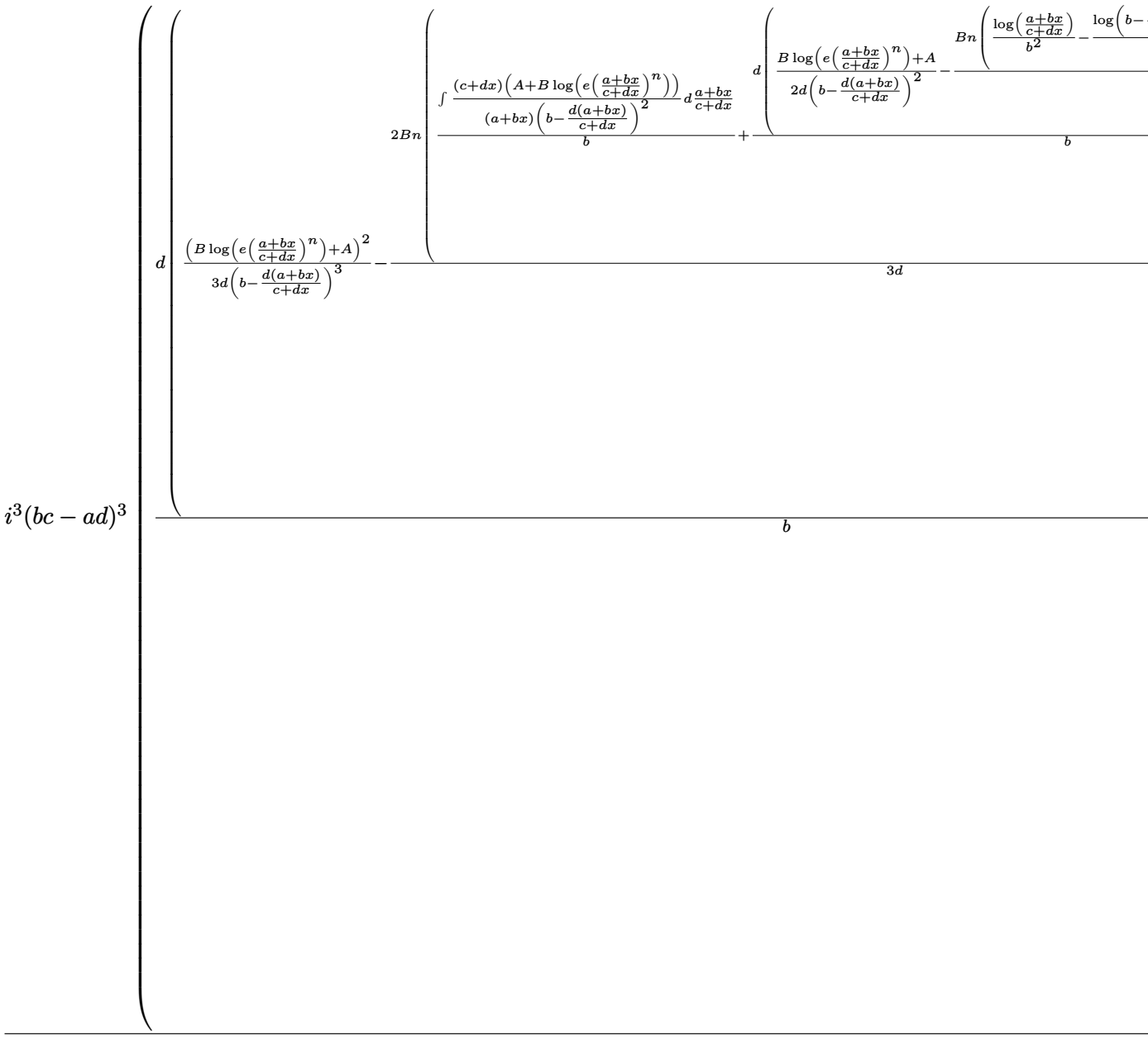


$$\frac{i^3(bc-ad)^3\left(d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}}{b}\right)}{3d}\right)}{b}}{g}$$

↓ 54



2009



2789

$$i^3(bc-ad)^3\left(d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{3d}+d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+b}{c+dx}\right)^2}\right.\right.\right.$$

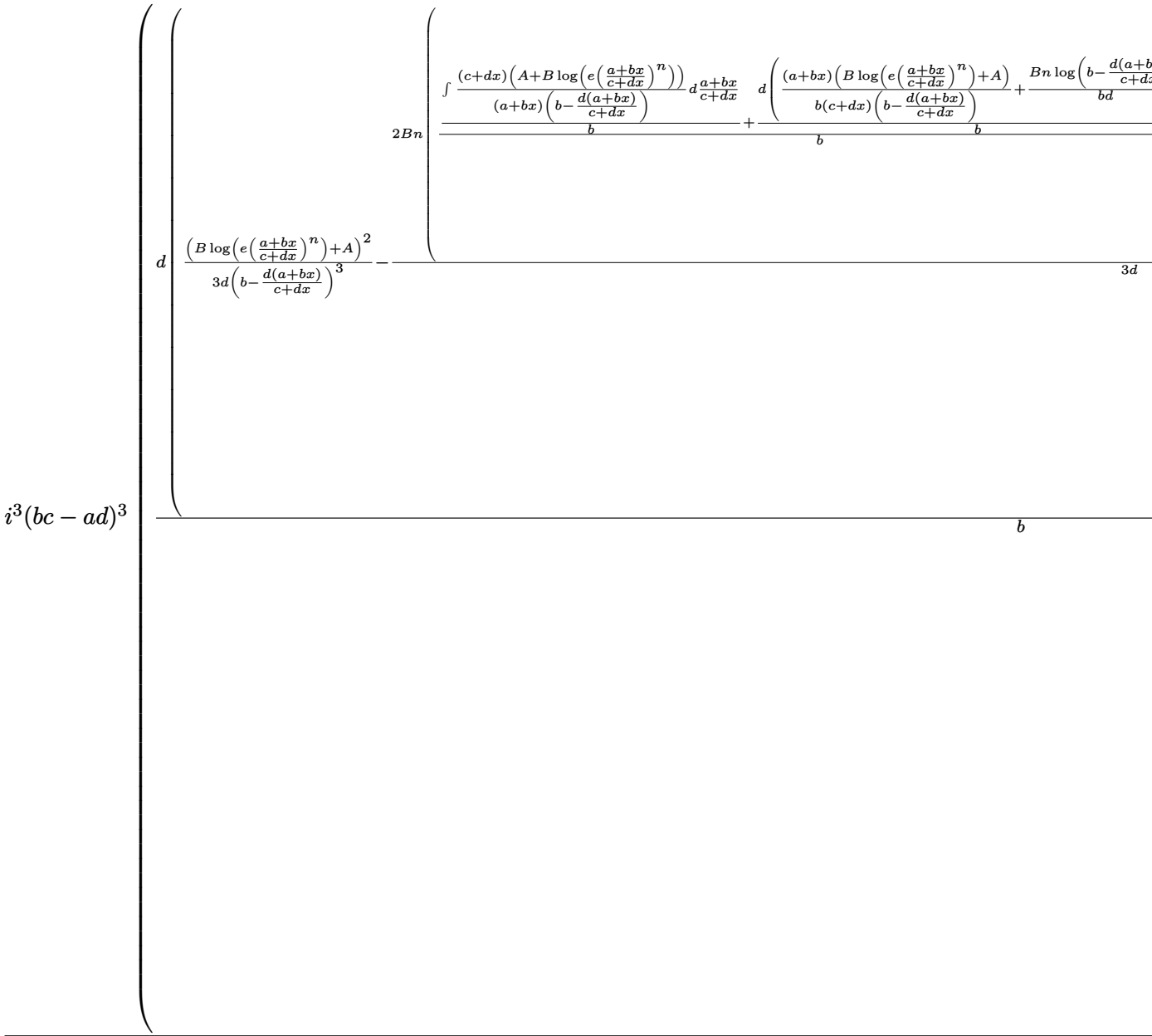
$$\left.\left.\left.\frac{}{b}\right)\right)\right)$$

↓ 2751

$$i^3(bc-ad)^3\left(d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{b}}{3d}\right.\right.$$

$$\left.\left.\frac{}{b}\right)\right)$$

↓ 16



↓ 2755

$$(bc-ad)^3 i^3 \left(d \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn\left(d\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)} \right)}{2d} \right) \right)}{b} + \frac{d\left(\frac{(a+bx}{b} \right.}{3d} \right.$$

↓ 2754

$$(bc-ad)^3 i^3 \left(d \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn\left(d\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)} \right)}{2d} \right) \right)}{b} + \frac{d\left(\frac{(a+bx}{b} \right.}{3d} \right.}{b} \right.$$

↓ 2779

$$(bc-ad)^3 i^3 \left(\frac{d \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn \left(\frac{d \left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d} \right)}{b} + \frac{d\left(\frac{(a+bx}{b}\right.}{} \right.}{} \right.}{} \right.$$

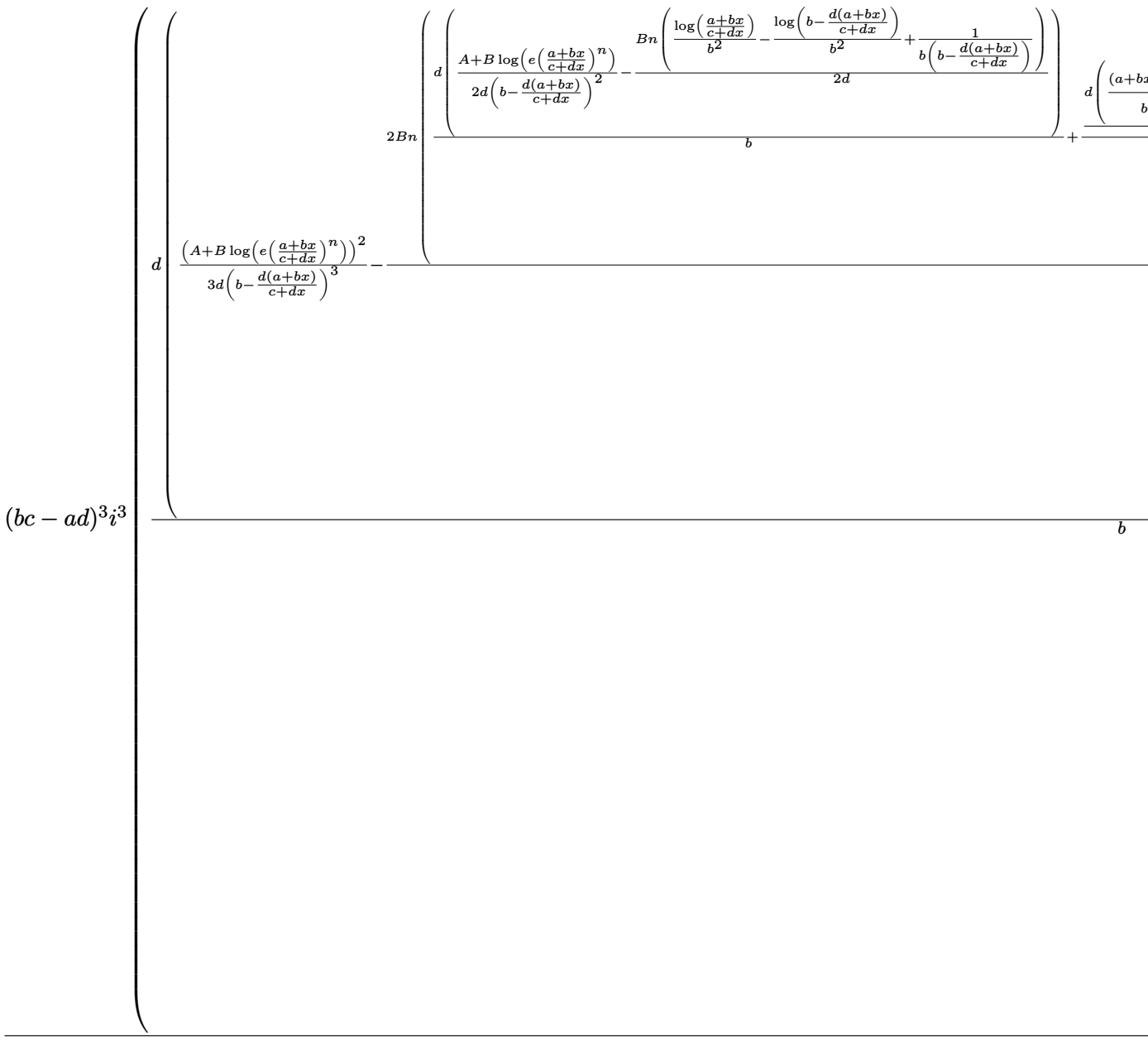
$\downarrow$ 2821

$$\frac{(bc-ad)^3 i^3 \left(\frac{d\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn\left(\frac{d\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b} + \frac{d\left(\frac{(a+bx\cdots}{b\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

↓ 2838

$$\frac{\left(d\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(d\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{d\left(\frac{(a+bx}{b}\right.}{b}\right)}{b}\right)}{(bc-ad)^3i^3}$$

↓ 7143



input
```
Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x),x]
```

output

```
((b*c - a*d)^3*i^3*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(3*d*(b -
 (d*(a + b*x))/(c + d*x))^3) - (2*B*n*((d*((A + B*Log[e*((a + b*x)/(c + d*
x))^n])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*(1/(b*(b - (d*(a + b*
x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c
 + d*x)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*n*Log[b - (d*(a +
 b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*
Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*(c + d*x))/(
d*(a + b*x))])/b)/b)/b))/(3*d)))/b + ((d*((A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*((d*(((a + b*x)*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d
*x))) + (B*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[
e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*
n*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b)/b))/d))/b + ((d*(((a + b*x)*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(b*(c + d*x)*(b - (d*(a + b*x))/
(c + d*x))) - (2*B*n*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d
*(a + b*x))/(b*(c + d*x))])/d) - (B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x
))])/d))/b))/b + (-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(
c + d*x))/(d*(a + b*x))])/b) + (2*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^
n])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + B*n*PolyLog[3, (b*(c + d*...
```

Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)  Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(dix+ci)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{bgx+ag}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x,
algorithm="fricas")
```

output
```
integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*
c^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2
*c^3*i^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*
d^2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log(e*((b*x + a)/(d*x + c))
^n))/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{i^3\left(\int \frac{A^2c^3}{a+bx}\,dx + \int \frac{A^2d^3x^3}{a+bx}\,dx + \int \frac{B^2c^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a+bx}\,dx + \int \frac{2ABc^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx + \int \frac{3A^2cd^2x^2}{a+bx}\,dx\right.}{}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g),x
)
```

output

```
i**3*(Integral(A**2*c**3/(a + b*x), x) + Integral(A**2*d**3*x**3/(a + b*x)
, x) + Integral(B**2*c**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a +
b*x), x) + Integral(2*A*B*c**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a
+ b*x), x) + Integral(3*A**2*c*d**2*x**2/(a + b*x), x) + Integral(3*A**2*c
**2*d*x/(a + b*x), x) + Integral(B**2*d**3*x**3*log(e*(a/(c + d*x) + b*x/(
c + d*x))**n)**2/(a + b*x), x) + Integral(2*A*B*d**3*x**3*log(e*(a/(c + d*
x) + b*x/(c + d*x))**n)/(a + b*x), x) + Integral(3*B**2*c*d**2*x**2*log(e*
(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a + b*x), x) + Integral(3*B**2*c**2*
d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a + b*x), x) + Integral(6*
A*B*c*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x) + In
tegral(6*A*B*c**2*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x
))/g
```

Maxima [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x,
algorithm="maxima")
```

output

```
3*A^2*c^2*d*i^3*(x/(b*g) - a*log(b*x + a)/(b^2*g)) - 1/6*A^2*d^3*i^3*(6*a^
3*log(b*x + a)/(b^4*g) - (2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/(b^3*g)) + 3/2*
A^2*c*d^2*i^3*(2*a^2*log(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A^2
*c^3*i^3*log(b*g*x + a*g)/(b*g) + 1/6*(2*B^2*b^3*d^3*i^3*x^3 + 3*(3*b^3*c*
d^2*i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 6*(3*b^3*c^2*d*i^3 - 3*a*b^2*c*d^2*i^3
+ a^2*b*d^3*i^3)*B^2*x + 6*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*d^
2*i^3 - a^3*d^3*i^3)*B^2*log(b*x + a))*log((d*x + c)^n)^2/(b^4*g) - integr
ate(-1/3*(3*B^2*b^4*c^4*i^3*log(e)^2 + 6*A*B*b^4*c^4*i^3*log(e) + 3*(B^2*b
^4*d^4*i^3*log(e)^2 + 2*A*B*b^4*d^4*i^3*log(e))*x^4 + 12*(B^2*b^4*c*d^3*i^
3*log(e)^2 + 2*A*B*b^4*c*d^3*i^3*log(e))*x^3 + 18*(B^2*b^4*c^2*d^2*i^3*log
(e)^2 + 2*A*B*b^4*c^2*d^2*i^3*log(e))*x^2 + 3*(B^2*b^4*d^4*i^3*x^4 + 4*B^2
*b^4*c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B
^2*b^4*c^4*i^3)*log((b*x + a)^n)^2 + 12*(B^2*b^4*c^3*d*i^3*log(e)^2 + 2*A*
B*b^4*c^3*d*i^3*log(e))*x + 6*(B^2*b^4*c^4*i^3*log(e) + A*B*b^4*c^4*i^3 +
(B^2*b^4*d^4*i^3*log(e) + A*B*b^4*d^4*i^3)*x^4 + 4*(B^2*b^4*c*d^3*i^3*log(
e) + A*B*b^4*c*d^3*i^3)*x^3 + 6*(B^2*b^4*c^2*d^2*i^3*log(e) + A*B*b^4*c^2*
d^2*i^3)*x^2 + 4*(B^2*b^4*c^3*d*i^3*log(e) + A*B*b^4*c^3*d*i^3)*x)*log((b*
x + a)^n) - (6*B^2*b^4*c^4*i^3*log(e) + 6*A*B*b^4*c^4*i^3 + 2*(3*A*B*b^4*d
^4*i^3 + (i^3*n + 3*i^3*log(e))*B^2*b^4*d^4)*x^4 + (24*A*B*b^4*c*d^3*i^3 -
 (a*b^3*d^4*i^3*n - 3*(3*i^3*n + 8*i^3*log(e))*b^4*c*d^3)*B^2)*x^3 + 3*...
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x,
algorithm="giac")
```

output

```
integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x
+ a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x),x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x), x)
```

Reduce [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{i\left(-6\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{bx+a}dx\right)b^6c^3 - 12\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{bx+a}dx\right)a\,b^5c^3 - 6\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 x^3}{bx+a}dx\right)b^6d^3 - 18\left(\int \text{lo}\right.\right.}{}$$

input
```
int((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

output
```
(i*( - 6*int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(a + b*x),x)*b**6*c**3
 - 12*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a + b*x),x)*a*b**5*c**3 - 6*i
nt((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(a + b*x),x)*b**6*d**3 - 1
8*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(a + b*x),x)*b**6*c*d**
2 - 18*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a + b*x),x)*b**6*c**
2*d - 12*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(a + b*x),x)*a*b**5
*d**3 - 36*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(a + b*x),x)*a*b*
*5*c*d**2 - 36*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(a + b*x),x)*a*b
**5*c**2*d + 6*log(a + b*x)*a**5*d**3 - 18*log(a + b*x)*a**4*b*c*d**2 + 18
*log(a + b*x)*a**3*b**2*c**2*d - 6*log(a + b*x)*a**2*b**3*c**3 - 6*a**4*b*
d**3*x + 18*a**3*b**2*c*d**2*x + 3*a**3*b**2*d**3*x**2 - 18*a**2*b**3*c**2
*d*x - 9*a**2*b**3*c*d**2*x**2 - 2*a**2*b**3*d**3*x**3))/(6*b**4*g)
```

3.183 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result 2134
Mathematica [B] (verified) 2135
Rubi [A] (verified) 2136
Maple [F] 2138
Fricas [F] 2138
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Giac [F(-1)] 2140
Mupad [F(-1)] 2140
Reduce [F] 2140

Optimal result

Integrand size = 45, antiderivative size = 739

$$
\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx \\
&= -\frac{2B^2(bc-ad)^2i^3n^2(c+dx)}{b^3g^2(a+bx)} - \frac{Bd^2(bc-ad)i^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4g^2} \\
&\quad - \frac{2B(bc-ad)^2i^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^2(a+bx)} \\
&\quad + \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4g^2} \\
&\quad - \frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^2(a+bx)} \\
&\quad + \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^2} \\
&\quad + \frac{4Bd(bc-ad)^2i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^4g^2} \\
&\quad + \frac{B^2d(bc-ad)^2i^3n^2\log(c+dx)}{b^4g^2} \\
&\quad + \frac{Bd(bc-ad)^2i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} \\
&\quad - \frac{3d(bc-ad)^2i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} \\
&\quad + \frac{4B^2d(bc-ad)^2i^3n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^2} \\
&\quad - \frac{B^2d(bc-ad)^2i^3n^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} \\
&\quad + \frac{6Bd(bc-ad)^2i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} \\
&\quad + \frac{6B^2d(bc-ad)^2i^3n^2\,\mathrm{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}
\end{aligned}
$$

output
```
-2*B^2*(-a*d+b*c)^2*i^3*n^2*(d*x+c)/b^3/g^2/(b*x+a)-B*d^2*(-a*d+b*c)*i^3*n
*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/g^2-2*B*(-a*d+b*c)^2*i^3*n*(d
*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/g^2/(b*x+a)+2*d^2*(-a*d+b*c)*i^3
*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^4/g^2-(-a*d+b*c)^2*i^3*(d*x+c
)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3/g^2/(b*x+a)+1/2*d*i^3*(d*x+c)^2*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2/g^2+4*B*d*(-a*d+b*c)^2*i^3*n*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^4/g^2+B^2*d*(-a*d+b*c)^2
*i^3*n^2*ln(d*x+c)/b^4/g^2+B*d*(-a*d+b*c)^2*i^3*n*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^2-3*d*(-a*d+b*c)^2*i^3*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^2+4*B^2*d*(-a*d+b*c
)^2*i^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/g^2-B^2*d*(-a*d+b*c)^2*i^3*
n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^2+6*B*d*(-a*d+b*c)^2*i^3*n*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^2+6*B^2*d*(
-a*d+b*c)^2*i^3*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^4/g^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5917 vs. $2(739) = 1478$.

Time = 7.54 (sec) , antiderivative size = 5917, normalized size of antiderivative = 8.01

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x)^2,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 624, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^2}\,dx$$

$$\downarrow 2961$$

$$\frac{i^3(bc-ad)^2\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{g^2}$$

$$\downarrow 2795$$

$$\frac{i^3(bc-ad)^2\int\left(\frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3(a+bx)^2}+\frac{3d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{2d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d^2\left(A+B\right.}{b^2(}\right.}{g^2}$$

$$\downarrow 2009$$

$$i^3(bc-ad)^2\left(\frac{2d^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bd^2n(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{6Bdn\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+b}{c+}\right.\right.\right.}{b^4}$$

input

```
Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x)^2,x]
```

output

```
((b*c - a*d)^2*i^3*((-2*B^2*n^2*(c + d*x))/(b^3*(a + b*x)) - (2*B*n*(c + d
*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^3*(a + b*x)) - (B*d^2*n*(a
 + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^4*(c + d*x)*(b - (d*(a +
 b*x))/(c + d*x))) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/
(b^3*(a + b*x)) + (d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b^2*(b -
 (d*(a + b*x))/(c + d*x))^2) + (2*d^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2)/(b^4*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (B^2*d*n^2
*Log[b - (d*(a + b*x))/(c + d*x)])/b^4 + (4*B*d*n*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/b^4 + (B*d*n*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b^4 -
 (3*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a
 + b*x))])/b^4 + (4*B^2*d*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/b^4
 - (B^2*d*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b^4 + (6*B*d*n*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))]
)/b^4 + (6*B^2*d*n^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/b^4))/g^2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(bgx+ag)^2}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}dx = \int \frac{(dix+ci)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x
, algorithm="fricas")
```

output
```
integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*
c^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2
*c^3*i^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*
d^2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log(e*((b*x + a)/(d*x + c))
^n))/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**
2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x
, algorithm="maxima")
```

output
```
-2*A*B*c^3*i^3*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d
)*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - 3*A^2*(a^2/(b^4*g^2*x + a
*b^3*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^2))*c*d^2*i^3 + 1/2*(2*a
^3/(b^5*g^2*x + a*b^4*g^2) + 6*a^2*log(b*x + a)/(b^4*g^2) + (b*x^2 - 4*a*x
)/(b^3*g^2))*A^2*d^3*i^3 + 3*A^2*c^2*d*i^3*(a/(b^3*g^2*x + a*b^2*g^2) + lo
g(b*x + a)/(b^2*g^2)) - 2*A*B*c^3*i^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)/(b^2*g^2*x + a*b*g^2) - A^2*c^3*i^3/(b^2*g^2*x + a*b*g^2) + 1/2*(B^2*b^
3*d^3*i^3*x^3 + 3*(2*b^3*c*d^2*i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 2*(3*a*b^2*c
*d^2*i^3 - 2*a^2*b*d^3*i^3)*B^2*x - 2*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3
*a^2*b*c*d^2*i^3 - a^3*d^3*i^3)*B^2 + 6*((b^3*c^2*d*i^3 - 2*a*b^2*c*d^2*i^
3 + a^2*b*d^3*i^3)*B^2*x + (a*b^2*c^2*d*i^3 - 2*a^2*b*c*d^2*i^3 + a^3*d^3*
i^3)*B^2)*log(b*x + a))*log((d*x + c)^n)^2/(b^5*g^2*x + a*b^4*g^2) - integ
rate(-(B^2*b^4*c^4*i^3*log(e)^2 + (B^2*b^4*d^4*i^3*log(e)^2 + 2*A*B*b^4*d^
4*i^3*log(e))*x^4 + 4*(B^2*b^4*c*d^3*i^3*log(e)^2 + 2*A*B*b^4*c*d^3*i^3*lo
g(e))*x^3 + 6*(B^2*b^4*c^2*d^2*i^3*log(e)^2 + 2*A*B*b^4*c^2*d^2*i^3*log(e)
)*x^2 + (B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2
*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B^2*b^4*c^4*i^3)*log((b*x + a)^n)^2 + 2
*(2*B^2*b^4*c^3*d*i^3*log(e)^2 + 3*A*B*b^4*c^3*d*i^3*log(e))*x + 2*(B^2*b^
4*c^4*i^3*log(e) + (B^2*b^4*d^4*i^3*log(e) + A*B*b^4*d^4*i^3)*x^4 + 4*(B^2
*b^4*c*d^3*i^3*log(e) + A*B*b^4*c*d^3*i^3)*x^3 + 6*(B^2*b^4*c^2*d^2*i^3...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x
, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(a\,g+b\,g\,x)^2}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^2,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^2, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \text{too large to display}$$

input
```
int((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

output

```
(i*( - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(a**2 + 2*a*b*x
+ b**2*x**2),x)*a**3*b**6*d**4 + 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)
**2*x**3)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**7*c*d**3 - 2*int((log(((
a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2
*b**7*d**4*x + 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(a**2 +
2*a*b*x + b**2*x**2),x)*a*b**8*c*d**3*x - 6*int((log(((a + b*x)**n*e)/(c +
 d*x)**n)**2*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**3*b**6*c*d**3 + 6*in
t((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(a**2 + 2*a*b*x + b**2*x**2
),x)*a**2*b**7*c**2*d**2 - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x*
*2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**7*c*d**3*x + 6*int((log(((a +
b*x)**n*e)/(c + d*x)**n)**2*x**2)/(a**2 + 2*a*b*x + b**2*x**2),x)*a*b**8*c
**2*d**2*x - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a**2 + 2*a*b
*x + b**2*x**2),x)*a**3*b**6*c**2*d**2 + 6*int((log(((a + b*x)**n*e)/(c +
d*x)**n)**2*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**2*b**7*c**3*d - 6*int((l
og(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a**2 + 2*a*b*x + b**2*x**2),x)*a*
*2*b**7*c**2*d**2*x + 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(a**
2 + 2*a*b*x + b**2*x**2),x)*a*b**8*c**3*d*x - 4*int((log(((a + b*x)**n*e)/
(c + d*x)**n)*x**3)/(a**2 + 2*a*b*x + b**2*x**2),x)*a**4*b**5*d**4 + 4*int
((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(a**2 + 2*a*b*x + b**2*x**2),x)
*a**3*b**6*c*d**3 - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(a*...
```

3.184 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result 2143
Mathematica [B] (verified) 2144
Rubi [A] (verified) 2144
Maple [F] 2146
Fricas [F] 2146
Sympy [F(-1)] 2147
Maxima [F] 2147
Giac [F(-1)] 2148
Mupad [F(-1)] 2149
Reduce [F] 2149

Optimal result

Integrand size = 45, antiderivative size = 644

$$
\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx \\
&= -\frac{4B^2d(bc-ad)i^3n^2(c+dx)}{b^3g^3(a+bx)} - \frac{B^2(bc-ad)i^3n^2(c+dx)^2}{4b^2g^3(a+bx)^2} \\
&\quad - \frac{4Bd(bc-ad)i^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^3(a+bx)} \\
&\quad - \frac{B(bc-ad)i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^3(a+bx)^2} \\
&\quad + \frac{d^3i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4g^3} \\
&\quad - \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^3(a+bx)} \\
&\quad - \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^3(a+bx)^2} \\
&\quad + \frac{2Bd^2(bc-ad)i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^4g^3} \\
&\quad - \frac{3d^2(bc-ad)i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} \\
&\quad + \frac{2B^2d^2(bc-ad)i^3n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^3} \\
&\quad + \frac{6Bd^2(bc-ad)i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} \\
&\quad + \frac{6B^2d^2(bc-ad)i^3n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}
\end{aligned}
$$

output
```
-4*B^2*d*(-a*d+b*c)*i^3*n^2*(d*x+c)/b^3/g^3/(b*x+a)-1/4*B^2*(-a*d+b*c)*i^3
*n^2*(d*x+c)^2/b^2/g^3/(b*x+a)^2-4*B*d*(-a*d+b*c)*i^3*n*(d*x+c)*(A+B*ln(e*
((b*x+a)/(d*x+c))^n))/b^3/g^3/(b*x+a)-1/2*B*(-a*d+b*c)*i^3*n*(d*x+c)^2*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^3/(b*x+a)^2+d^3*i^3*(b*x+a)*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))^2/b^4/g^3-2*d*(-a*d+b*c)*i^3*(d*x+c)*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))^2/b^3/g^3/(b*x+a)-1/2*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))^2/b^2/g^3/(b*x+a)^2+2*B*d^2*(-a*d+b*c)*i^3*n*(A+B*ln(
e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^4/g^3-3*d^2*(-a*d+b*c)*
i^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^3+2*
B^2*d^2*(-a*d+b*c)*i^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/g^3+6*B*d^2*
(-a*d+b*c)*i^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*
x+a))/b^4/g^3+6*B^2*d^2*(-a*d+b*c)*i^3*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/
b^4/g^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6938 vs. $2(644) = 1288$.

Time = 7.54 (sec) , antiderivative size = 6938, normalized size of antiderivative = 10.77

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x)^3,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 526, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{i^3(bc-ad)\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g^3}$$

$$\downarrow 2795$$

$$\frac{i^3(bc-ad)\int\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d^3}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d^2}{b^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d}{b^3(a+bx)^2}+\frac{(c+dx)^3}{}\right.}{g^3}$$

$$\downarrow 2009$$

$$\frac{i^3(bc-ad)\left(\frac{d^3(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{6Bd^2n\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4}+\frac{2Bd^2n\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\right.}{b^4}}$$

input

```
Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x)^3,x]
```

output

```
((b*c - a*d)*i^3*((-4*B^2*d*n^2*(c + d*x))/(b^3*(a + b*x)) - (B^2*n^2*(c +
 d*x)^2)/(4*b^2*(a + b*x)^2) - (4*B*d*n*(c + d*x)*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(b^3*(a + b*x)) - (B*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(2*b^2*(a + b*x)^2) - (2*d*(c + d*x)*(A + B*Log[e*((a + b
*x)/(c + d*x))^n])^2)/(b^3*(a + b*x)) - ((c + d*x)^2*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])^2)/(2*b^2*(a + b*x)^2) + (d^3*(a + b*x)*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])^2)/(b^4*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) +
(2*B*d^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b
*(c + d*x))])/b^4 - (3*d^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1
- (b*(c + d*x))/(d*(a + b*x))])/b^4 + (2*B^2*d^2*n^2*PolyLog[2, (d*(a + b*
x))/(b*(c + d*x))])/b^4 + (6*B*d^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b^4 + (6*B^2*d^2*n^2*PolyLog[3,
 (b*(c + d*x))/(d*(a + b*x))])/b^4))/g^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(dix+ci)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(bgx+ag)^3}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3}dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x
, algorithm="fricas")
```

output
```
integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*
c^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2
*c^3*i^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*
d^2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log(e*((b*x + a)/(d*x + c))
^n))/(b^3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**
3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x
, algorithm="maxima")
```

output

```
-3/2*A*B*c^2*d*i^3*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c -
a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)
*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2
*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^
2*b^2*d^2)*g^3)) + 1/2*A*B*c^3*i^3*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*
b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3
) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*l
og(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A^2*d^3*i^3*(
(6*a^2*b*x + 5*a^3)/(b^6*g^3*x^2 + 2*a*b^5*g^3*x + a^2*b^4*g^3) - 2*x/(b^3
*g^3) + 6*a*log(b*x + a)/(b^4*g^3)) + 3/2*A^2*c*d^2*i^3*((4*a*b*x + 3*a^2)
/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3)) -
 3*(2*b*x + a)*A*B*c^2*d*i^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g
^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - 3/2*(2*b*x + a)*A^2*c^2*d*i^3/(b^4
*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - A*B*c^3*i^3*log(e*(b*x/(d*x + c)
 + a/(d*x + c))^n)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2*c^3
*i^3/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 1/2*(2*B^2*b^3*d^3*i^3*x^
3 + 4*B^2*a*b^2*d^3*i^3*x^2 - 2*(3*b^3*c^2*d*i^3 - 6*a*b^2*c*d^2*i^3 + 2*a
^2*b*d^3*i^3)*B^2*x - (b^3*c^3*i^3 + 3*a*b^2*c^2*d*i^3 - 9*a^2*b*c*d^2*i^3
 + 5*a^3*d^3*i^3)*B^2 + 6*((b^3*c*d^2*i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 2*(a*
b^2*c*d^2*i^3 - a^2*b*d^3*i^3)*B^2*x + (a^2*b*c*d^2*i^3 - a^3*d^3*i^3)*...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Timed out}$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x
, algorithm="giac")
```

output

```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(a\,g+b\,g\,x)^3}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^3,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^3, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{too large to display}$$

input
```
int((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

output

```
(i*( - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(a**3 + 3*a**2*b
*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**5*b**6*d**5 + 8*int((log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**
3),x)*a**4*b**7*c*d**4 - 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3
)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**4*b**7*d**5*x - 4*
int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(a**3 + 3*a**2*b*x + 3*a*
b**2*x**2 + b**3*x**3),x)*a**3*b**8*c**2*d**3 + 16*int((log(((a + b*x)**n*
e)/(c + d*x)**n)**2*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),
x)*a**3*b**8*c*d**4*x - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)
/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**3*b**8*d**5*x**2 -
8*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(a**3 + 3*a**2*b*x + 3*
a*b**2*x**2 + b**3*x**3),x)*a**2*b**9*c**2*d**3*x + 8*int((log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**
3),x)*a**2*b**9*c*d**4*x**2 - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2
*x**3)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a*b**10*c**2*d**
3*x**2 - 12*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(a**3 + 3*a**
2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**5*b**6*c*d**4 + 24*int((log(((a +
 b*x)**n*e)/(c + d*x)**n)**2*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b*
*3*x**3),x)*a**4*b**7*c**2*d**3 - 24*int((log(((a + b*x)**n*e)/(c + d*x)**
n)**2*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**4*b**...
```

3.185 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result . . . 2151
Mathematica [B] (verified) . . . 2152
Rubi [A] (verified) . . . 2152
Maple [F] . . . 2160
Fricas [F] . . . 2160
Sympy [F(-1)] . . . 2161
Maxima [F] . . . 2161
Giac [F(-1)] . . . 2162
Mupad [F(-1)] . . . 2163
Reduce [F] . . . 2163

Optimal result

Integrand size = 45, antiderivative size = 561

$$
\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx \\
&= -\frac{2B^2d^2i^3n^2(c+dx)}{b^3g^4(a+bx)} - \frac{B^2di^3n^2(c+dx)^2}{4b^2g^4(a+bx)^2} - \frac{2B^2i^3n^2(c+dx)^3}{27bg^4(a+bx)^3} \\
&\quad - \frac{2Bd^2i^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^4(a+bx)} \\
&\quad - \frac{Bdi^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^4(a+bx)^2} \\
&\quad - \frac{2Bi^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9bg^4(a+bx)^3} - \frac{d^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^4(a+bx)} \\
&\quad - \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^4(a+bx)^2} - \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} \\
&\quad - \frac{d^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} \\
&\quad + \frac{2Bd^3i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} \\
&\quad + \frac{2B^2d^3i^3n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4}
\end{aligned}
$$

output
```
-2*B^2*d^2*i^3*n^2*(d*x+c)/b^3/g^4/(b*x+a)-1/4*B^2*d*i^3*n^2*(d*x+c)^2/b^2
/g^4/(b*x+a)^2-2/27*B^2*i^3*n^2*(d*x+c)^3/b/g^4/(b*x+a)^3-2*B*d^2*i^3*n*(d
*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/g^4/(b*x+a)-1/2*B*d*i^3*n*(d*x+c
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^4/(b*x+a)^2-2/9*B*i^3*n*(d*x+c)^
3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^4/(b*x+a)^3-d^2*i^3*(d*x+c)*(A+B*ln(
e*((b*x+a)/(d*x+c))^n))^2/b^3/g^4/(b*x+a)-1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))^2/b^2/g^4/(b*x+a)^2-1/3*i^3*(d*x+c)^3*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))^2/b/g^4/(b*x+a)^3-d^3*i^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^
2*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^4+2*B*d^3*i^3*n*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^4+2*B^2*d^3*i^3*n^2*polylog(3
,b*(d*x+c)/d/(b*x+a))/b^4/g^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 8570 vs. $2(561) = 1122$.

Time = 8.50 (sec) , antiderivative size = 8570, normalized size of antiderivative = 15.28

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x)^4,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 473, normalized size of antiderivative = 0.84, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {2961, 2780, 2742, 2741, 2780, 2742, 2741, 2780, 2742, 2741, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^4}\,dx$$

$$\downarrow 2961$$

$$\frac{i^3\int \frac{(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g^4}$$

$$\downarrow 2780$$

$$\frac{i^3\left(\frac{\int \frac{(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^4}$$

$$\downarrow 2742$$

$$\frac{i^3\left(\frac{\frac{2}{3}Bn\int \frac{(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}}{b}+\frac{d\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^4}$$

$$\downarrow 2741$$

$$\frac{i^3\left(\frac{d\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{\frac{2}{3}Bn\left(-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}-\frac{Bn(c+dx)^3}{9(a+bx)^3}\right)-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}}{b}\right)}{g^4}$$

$$\downarrow 2780$$

$$\frac{i^3\left(\frac{d\left(\frac{\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\frac{2}{3}Bn\left(-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}-\frac{Bn(}{b}\right.\right.}{g^4}$$

$$\downarrow 2742$$

$$i^3\left(\frac{d\left(\frac{Bn\int\frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{b}-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}+\frac{d\int\frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\frac{2}{3}Bn\left(-\right.}{g^4}$$

$$\downarrow 2741$$

$$i^3\left(\frac{d\left(\frac{d\int\frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{Bn\left(-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}\right)}{b}-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}\right)}{b}+\frac{}{g^4}$$

$$\downarrow 2780$$

$$i^3\left(\frac{d\left(\frac{\int\frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{Bn\left(-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+d}{4(a+bx}\right.}{g^4}$$

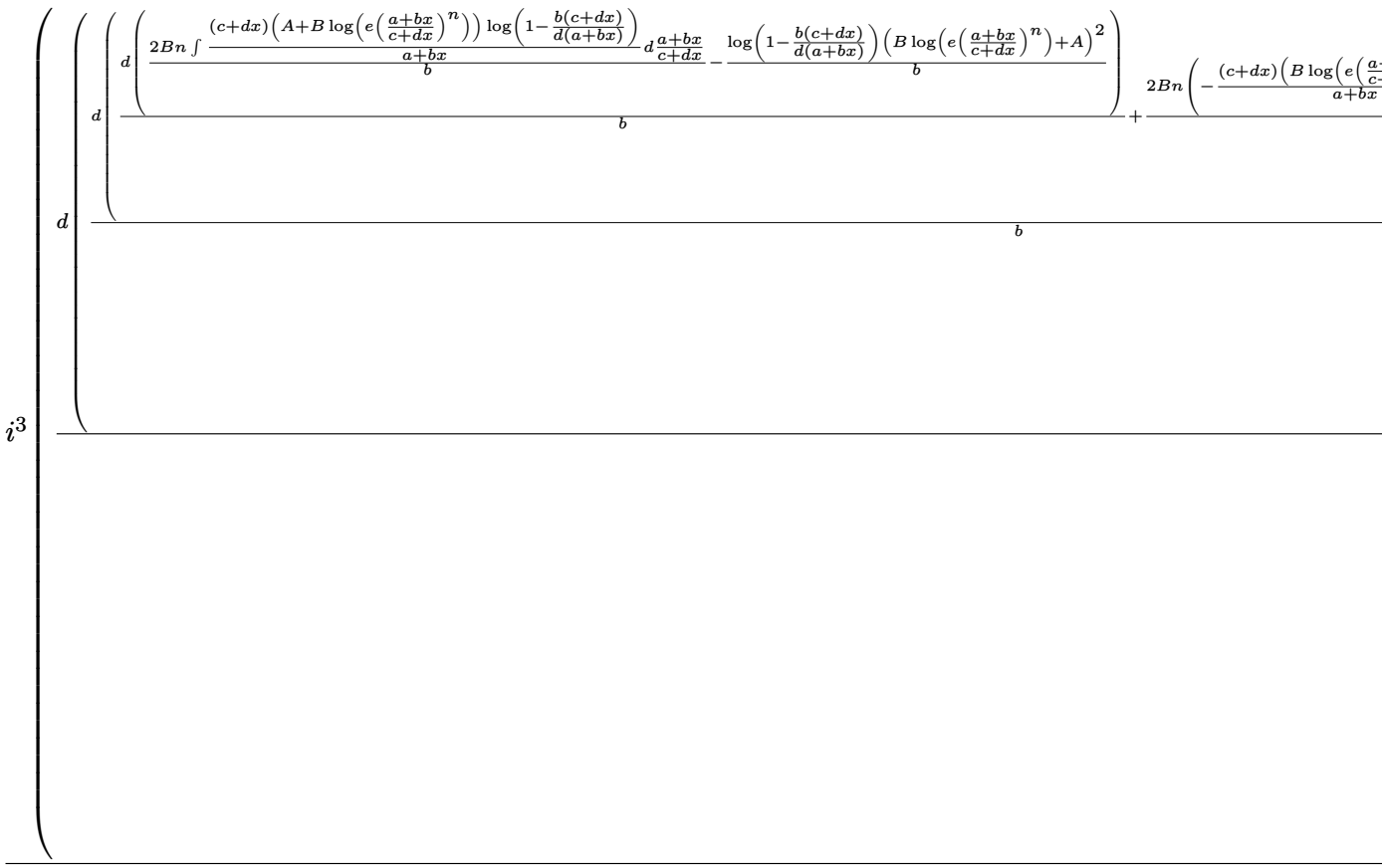
↓ 2742

$$i^3\left(d\left(\frac{d\left(\frac{2Bn\int\frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}+\frac{d\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{Bn\left(-\frac{(c+\right.}{}\right.\right.\right.$$

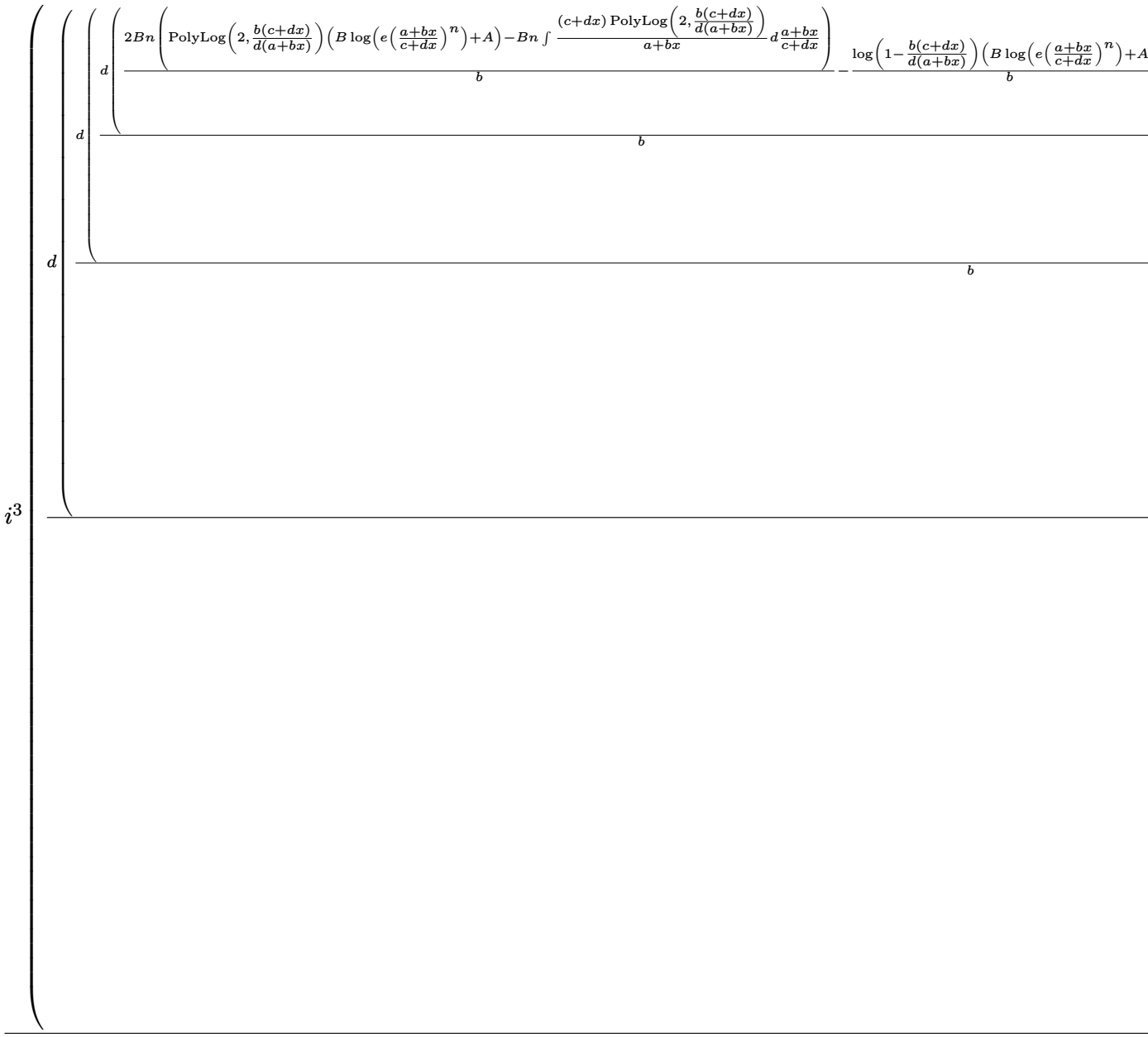
↓ 2741

$$i^3\left(d\left(\frac{d\left(\frac{d\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{2Bn\left(-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{b}\right)}{b}+\frac{Bn\left(\right.}{}\right.\right.\right.$$

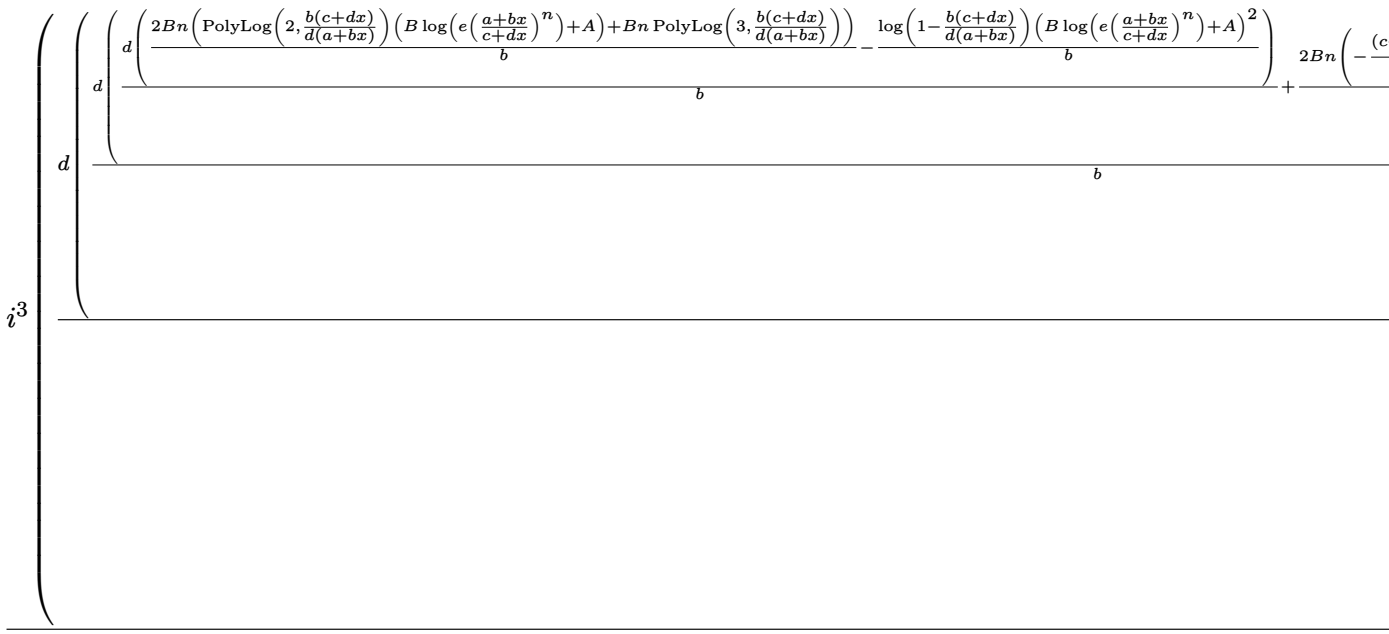
↓ 2779



2821



↓ 7143



input
```
Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x)^4,x]
```

output
```
(i^3*((-1/3*((c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*
x)^3 + (2*B*n*(-1/9*(B*n*(c + d*x)^3)/(a + b*x)^3 - ((c + d*x)^3*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^3)))/3)/b + (d*((-1/2*((c + d*
x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)^2 + B*n*(-1/4*(B*
n*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(2*(a + b*x)^2)))/b + (d*((-(((c + d*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2)/(a + b*x)) + 2*B*n*(-((B*n*(c + d*x))/(a + b*x)) - ((c + d
*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)))/b + (d*(-(((A + B*
Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b)
 + (2*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (b*(c + d*x))
/(d*(a + b*x))] + B*n*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/b))/b))/b)
)/b))/g^4
```

Defintions of rubi rules used

rule 2741

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2779

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2821

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(bgx+ag)^4}dx$$

input

```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x)
```

output

```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \int \frac{(dix+ci)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^4}\,dx$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x
, algorithm="fricas")
```

output

```
integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*
c^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2
*c^3*i^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*
d^2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log(e*((b*x + a)/(d*x + c))
^n))/(b^4*g^4*x^4 + 4*a*b^3*g^4*x^3 + 6*a^2*b^2*g^4*x^2 + 4*a^3*b*g^4*x +
a^4*g^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^4}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, algorithm="maxima")
```

output

```
-1/3*A*B*c*d^2*i^3*n*((11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4
*c^2 - 3*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2
*a^3*b*d^2)*x)/((b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c
^2 - 2*a^2*b^6*c*d + a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d
 + a^4*b^4*d^2)*g^4*x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) +
 6*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*
c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2
 + a^2*d^3)*log(d*x + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3
*b^3*d^3)*g^4)) - 1/9*A*B*c^3*i^3*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*
d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*
b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3
*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*
b^2*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d +
 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^
3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/6*A*B*c^2*d*i^3*n*((5*a*b
^2*c^2 - 22*a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b
^3*c^2 - 16*a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*
d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^
2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*
c*d + a^5*b^2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 ...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Timed out}$$

input

```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x
, algorithm="giac")
```

output

```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(a\,g+b\,g\,x)^4}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^4,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^4, x)
```

Reduce [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{too large to display}$$

input
```
int((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x)
```

output

```
(i*( - 108*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(a**4 + 4*a**3
*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4),x)*a**7*b**6*d**6 + 3
24*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(a**4 + 4*a**3*b*x + 6
*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4),x)*a**6*b**7*c*d**5 - 324*int
((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*
b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4),x)*a**6*b**7*d**6*x - 324*int((log(
((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x
**2 + 4*a*b**3*x**3 + b**4*x**4),x)*a**5*b**8*c**2*d**4 + 972*int((log(((a
 + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2
 + 4*a*b**3*x**3 + b**4*x**4),x)*a**5*b**8*c*d**5*x - 324*int((log(((a + b
*x)**n*e)/(c + d*x)**n)**2*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4
*a*b**3*x**3 + b**4*x**4),x)*a**5*b**8*d**6*x**2 + 108*int((log(((a + b*x)
**n*e)/(c + d*x)**n)**2*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*
b**3*x**3 + b**4*x**4),x)*a**4*b**9*c**3*d**3 - 972*int((log(((a + b*x)**n
*e)/(c + d*x)**n)**2*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**
3*x**3 + b**4*x**4),x)*a**4*b**9*c**2*d**4*x + 972*int((log(((a + b*x)**n*
e)/(c + d*x)**n)**2*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3
*x**3 + b**4*x**4),x)*a**4*b**9*c*d**5*x**2 - 108*int((log(((a + b*x)**n*e
)/(c + d*x)**n)**2*x**3)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*
x**3 + b**4*x**4),x)*a**4*b**9*d**6*x**3 + 324*int((log(((a + b*x)**n*e...
```

3.186 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$

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Optimal result

Integrand size = 45, antiderivative size = 768

$$
\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx \\
&= \frac{bB^2(bc-ad)^2g^3n^2x}{3d^3i} + \frac{7B(bc-ad)^2g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^3i} \\
&\quad - \frac{b^2B(bc-ad)g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^4i} \\
&\quad + \frac{3(bc-ad)^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i} \\
&\quad - \frac{3b^2(bc-ad)g^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^4i} \\
&\quad + \frac{b^3g^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d^4i} \\
&\quad + \frac{6B(bc-ad)^3g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i} \\
&\quad + \frac{(bc-ad)^3g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i} \\
&\quad + \frac{B^2(bc-ad)^3g^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{3d^4i} - \frac{2B^2(bc-ad)^3g^3n^2\log(c+dx)}{d^4i} \\
&\quad - \frac{7B(bc-ad)^3g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i} \\
&\quad + \frac{6B^2(bc-ad)^3g^3n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} \\
&\quad + \frac{2B(bc-ad)^3g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} \\
&\quad + \frac{7B^2(bc-ad)^3g^3n^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i} \\
&\quad - \frac{2B^2(bc-ad)^3g^3n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}
\end{aligned}
$$

output

```
1/3*b*B^2*(-a*d+b*c)^2*g^3*n^2*x/d^3/i+7/3*B*(-a*d+b*c)^2*g^3*n*(b*x+a)*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3/i-1/3*b^2*B*(-a*d+b*c)*g^3*n*(d*x+c)^2*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4/i+3*(-a*d+b*c)^2*g^3*(b*x+a)*(A+B*ln(e*
((b*x+a)/(d*x+c))^n))^2/d^3/i-3/2*b^2*(-a*d+b*c)*g^3*(d*x+c)^2*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))^2/d^4/i+1/3*b^3*g^3*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x
+c))^n))^2/d^4/i+6*B*(-a*d+b*c)^3*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln
((-a*d+b*c)/b/(d*x+c))/d^4/i+(-a*d+b*c)^3*g^3*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))^2*ln((-a*d+b*c)/b/(d*x+c))/d^4/i+1/3*B^2*(-a*d+b*c)^3*g^3*n^2*ln((b*x+
a)/(d*x+c))/d^4/i-2*B^2*(-a*d+b*c)^3*g^3*n^2*ln(d*x+c)/d^4/i-7/3*B*(-a*d+b
*c)^3*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/d^4/
i+6*B^2*(-a*d+b*c)^3*g^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i+2*B*(-a*
d+b*c)^3*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+
c))/d^4/i+7/3*B^2*(-a*d+b*c)^3*g^3*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/d^4/
i-2*B^2*(-a*d+b*c)^3*g^3*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/d^4/i
```

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 1072, normalized size of antiderivative = 1.40

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \text{Too large to display}$$

input

```
Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i
+ d*i*x),x]
```

output

```
(g^3*(6*b*d*(b*c - a*d)^2*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 3*d
^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 2
*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - 6*A^2*(b*c - a
*d)^3*Log[c + d*x] + 12*A*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n]*L
og[(b*c - a*d)/(b*c + b*d*x)] + 6*B^2*(b*c - a*d)^3*Log[e*((a + b*x)/(c +
d*x))^n]^2*Log[(b*c - a*d)/(b*c + b*d*x)] - 6*A*B*(b*c - a*d)^3*n*(Log[(b*
c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c -
a*d)/(b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 6*B*(b*c
 - a*d)^2*n*(2*a*d*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2
*b*c*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - a*B*d*n*(Log[a
+ b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d
*(a + b*x))/(-(b*c) + a*d)]) + b*B*c*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
) + 3*B*(b*c - a*d)^2*n*(2*A*b*d*x + 2*B*d*(a + b*x)*Log[e*((a + b*x)/(c +
 d*x))^n] - 2*B*(b*c - a*d)*n*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[e*((
a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*((2*Log[(d*(a + b*x
))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x
))/(b*c - a*d)])) + 2*B*(b*c - a*d)*n*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c
- a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - d^2*(a + b*x)^2*(A + B*L
og[e*((a + b*x)/(c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*Log[c + d*x] - 2*(...
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 679, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{ci+dix}\,dx$$

$$\downarrow 2961$$

$$\frac{g^3(bc-ad)^3 \int \frac{(a+bx)^3 \left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{i}$$

$$\downarrow 2795$$

$$\frac{g^3(bc-ad)^3\int\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^n}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\right.}{i}$$

$$\downarrow 2009$$

$$g^3(bc-ad)^3\left(\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{b^2Bn\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{2Bn\,\mathrm{PolyLog}\left(2,\frac{d(a+b}{b(c+d}\right.\right.$$

input

```
Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*
x),x]
```

output

```
((b*c - a*d)^3*g^3*((b*B^2*n^2)/(3*d^4*(b - (d*(a + b*x))/(c + d*x))) - (b
^2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d^4*(b - (d*(a + b*x))/(
c + d*x))^2) + (7*B*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3
*d^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (b^3*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])^2)/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^3) - (3*b^2*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*d^4*(b - (d*(a + b*x))/(c + d*x))
^2) + (3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^3*(c + d*x
)*(b - (d*(a + b*x))/(c + d*x))) + (B^2*n^2*Log[(a + b*x)/(c + d*x)])/(3*d
^4) + (2*B^2*n^2*Log[b - (d*(a + b*x))/(c + d*x)])/d^4 + (6*B*n*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 + (
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x
))])/d^4 - (7*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d
*x))/(d*(a + b*x))])/(3*d^4) + (6*B^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c +
 d*x))])/d^4 + (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d
*(a + b*x))/(b*(c + d*x))])/d^4 + (7*B^2*n^2*PolyLog[2, (b*(c + d*x))/(d*(
a + b*x))])/(3*d^4) - (2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/
d^4))/i
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{dix+ci}dx$$

input

```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

output

```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}dx$$

input

```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x,
algorithm="fricas")
```

output
```
integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*
a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2
*a^3*g^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*
b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log(e*((b*x + a)/(d*x + c))
^n))/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$

$$= g^3\left(\int \frac{A^2a^3}{c+dx}\,dx + \int \frac{A^2b^3x^3}{c+dx}\,dx + \int \frac{B^2a^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c+dx}\,dx + \int \frac{2ABa^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx + \int \frac{3A^2ab^2x^2}{c+dx}\,d\right.$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i),x
)
```

output
```
g**3*(Integral(A**2*a**3/(c + d*x), x) + Integral(A**2*b**3*x**3/(c + d*x)
, x) + Integral(B**2*a**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c +
d*x), x) + Integral(2*A*B*a**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c
+ d*x), x) + Integral(3*A**2*a*b**2*x**2/(c + d*x), x) + Integral(3*A**2*a
**2*b*x/(c + d*x), x) + Integral(B**2*b**3*x**3*log(e*(a/(c + d*x) + b*x/(
c + d*x))**n)**2/(c + d*x), x) + Integral(2*A*B*b**3*x**3*log(e*(a/(c + d*
x) + b*x/(c + d*x))**n)/(c + d*x), x) + Integral(3*B**2*a*b**2*x**2*log(e*
(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c + d*x), x) + Integral(3*B**2*a**2*
b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c + d*x), x) + Integral(6*
A*B*a*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x) + In
tegral(6*A*B*a**2*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x
))/i
```

Maxima [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x,
algorithm="maxima")
```

output
```
3*A^2*a^2*b*g^3*(x/(d*i) - c*log(d*x + c)/(d^2*i)) - 1/6*A^2*b^3*g^3*(6*c^
3*log(d*x + c)/(d^4*i) - (2*d^2*x^3 - 3*c*d*x^2 + 6*c^2*x)/(d^3*i)) + 3/2*
A^2*a*b^2*g^3*(2*c^2*log(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A^2
*a^3*g^3*log(d*i*x + c*i)/(d*i) + 1/6*(2*B^2*b^3*d^3*g^3*x^3 - 3*(b^3*c*d^
2*g^3 - 3*a*b^2*d^3*g^3)*B^2*x^2 + 6*(b^3*c^2*d*g^3 - 3*a*b^2*c*d^2*g^3 +
3*a^2*b*d^3*g^3)*B^2*x - 6*(b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^
2*g^3 - a^3*d^3*g^3)*B^2*log(d*x + c))*log((d*x + c)^n)^2/(d^4*i) - integr
ate(-1/3*(3*B^2*a^3*d^3*g^3*log(e)^2 + 6*A*B*a^3*d^3*g^3*log(e) + 3*(B^2*b
^3*d^3*g^3*log(e)^2 + 2*A*B*b^3*d^3*g^3*log(e))*x^3 + 9*(B^2*a*b^2*d^3*g^3
*log(e)^2 + 2*A*B*a*b^2*d^3*g^3*log(e))*x^2 + 3*(B^2*b^3*d^3*g^3*x^3 + 3*B
^2*a*b^2*d^3*g^3*x^2 + 3*B^2*a^2*b*d^3*g^3*x + B^2*a^3*d^3*g^3)*log((b*x +
 a)^n)^2 + 9*(B^2*a^2*b*d^3*g^3*log(e)^2 + 2*A*B*a^2*b*d^3*g^3*log(e))*x +
 6*(B^2*a^3*d^3*g^3*log(e) + A*B*a^3*d^3*g^3 + (B^2*b^3*d^3*g^3*log(e) + A
*B*b^3*d^3*g^3)*x^3 + 3*(B^2*a*b^2*d^3*g^3*log(e) + A*B*a*b^2*d^3*g^3)*x^2
 + 3*(B^2*a^2*b*d^3*g^3*log(e) + A*B*a^2*b*d^3*g^3)*x)*log((b*x + a)^n) -
(6*B^2*a^3*d^3*g^3*log(e) + 6*A*B*a^3*d^3*g^3 + 2*(3*A*B*b^3*d^3*g^3 + (g^
3*n + 3*g^3*log(e))*B^2*b^3*d^3)*x^3 - 6*(b^3*c^3*g^3*n - 3*a*b^2*c^2*d*g^
3*n + 3*a^2*b*c*d^2*g^3*n - a^3*d^3*g^3*n)*B^2*log(d*x + c) + 3*(6*A*B*a*b
^2*d^3*g^3 - (b^3*c*d^2*g^3*n - 3*(g^3*n + 2*g^3*log(e))*a*b^2*d^3)*B^2)*x
^2 + 6*(3*A*B*a^2*b*d^3*g^3 + (b^3*c^2*d*g^3*n - 3*a*b^2*c*d^2*g^3*n + ...
```

Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x,
algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x
+ c*i), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$
$$=\int \frac{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x),x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x), x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$
$$=\frac{g^3 i\left(-6\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{dx+c}dx\right)a^3b^2d^4-12\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{dx+c}dx\right)a^4b\,d^4-6\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 x^3}{dx+c}dx\right)b^5d^4-18\right.}{}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

output

```
(g**3*i*( - 6*int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(c + d*x),x)*a**3*
b**2*d**4 - 12*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(c + d*x),x)*a**4*b*
d**4 - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(c + d*x),x)*b**
5*d**4 - 18*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(c + d*x),x)*
a*b**4*d**4 - 18*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(c + d*x),x
)*a**2*b**3*d**4 - 12*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(c + d
*x),x)*a*b**4*d**4 - 36*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c +
 d*x),x)*a**2*b**3*d**4 - 36*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c
 + d*x),x)*a**3*b**2*d**4 - 6*log(c + d*x)*a**5*d**3 + 18*log(c + d*x)*a**
4*b*c*d**2 - 18*log(c + d*x)*a**3*b**2*c**2*d + 6*log(c + d*x)*a**2*b**3*c
**3 - 18*a**4*b*d**3*x + 18*a**3*b**2*c*d**2*x - 9*a**3*b**2*d**3*x**2 - 6
*a**2*b**3*c**2*d*x + 3*a**2*b**3*c*d**2*x**2 - 2*a**2*b**3*d**3*x**3))/(6
*d**4)
```

3.187 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$

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Optimal result

Integrand size = 45, antiderivative size = 573

$$
\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx \\
&= -\frac{B(bc-ad)g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d^2i} \\
&\quad -\frac{2(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i} \\
&\quad +\frac{b^2g^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^3i} \\
&\quad -\frac{4B(bc-ad)^2g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i} \\
&\quad -\frac{(bc-ad)^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i} \\
&\quad +\frac{B^2(bc-ad)^2g^2n^2\log(c+dx)}{d^3i} \\
&\quad +\frac{B(bc-ad)^2g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i} \\
&\quad -\frac{4B^2(bc-ad)^2g^2n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i} \\
&\quad -\frac{2B(bc-ad)^2g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i} \\
&\quad -\frac{B^2(bc-ad)^2g^2n^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i}+\frac{2B^2(bc-ad)^2g^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}
\end{aligned}
$$

output

```
-B*(-a*d+b*c)*g^2*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^2/i-2*(-a*d+
b*c)*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^2/i+1/2*b^2*g^2*(d*x+
c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^3/i-4*B*(-a*d+b*c)^2*g^2*n*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^3/i-(-a*d+b*c)^2*g^2*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/d^3/i+B^2*(-a*d
+b*c)^2*g^2*n^2*ln(d*x+c)/d^3/i+B*(-a*d+b*c)^2*g^2*n*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/d^3/i-4*B^2*(-a*d+b*c)^2*g^2*n^2*poly
log(2,d*(b*x+a)/b/(d*x+c))/d^3/i-2*B*(-a*d+b*c)^2*g^2*n*(A+B*ln(e*((b*x+a)
/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i-B^2*(-a*d+b*c)^2*g^2*n^
2*polylog(2,b*(d*x+c)/d/(b*x+a))/d^3/i+2*B^2*(-a*d+b*c)^2*g^2*n^2*polylog(
3,d*(b*x+a)/b/(d*x+c))/d^3/i
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 763, normalized size of antiderivative = 1.33

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$
$$=\frac{g^2\Big(-2bd(bc-ad)x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+2A^2(bc-ad)^2\log$$

input

```
Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i
+ d*i*x),x]
```

output

```
(g^2*(-2*b*d*(b*c - a*d)*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + d^2*
(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 2*A^2*(b*c - a*d)^2
*Log[c + d*x] - 4*A*B*(b*c - a*d)^2*Log[e*((a + b*x)/(c + d*x))^n]*Log[(b*
c - a*d)/(b*c + b*d*x)] - 2*B^2*(b*c - a*d)^2*Log[e*((a + b*x)/(c + d*x))^
n]^2*Log[(b*c - a*d)/(b*c + b*d*x)] + 2*A*B*(b*c - a*d)^2*n*(Log[(b*c - a*
d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(
b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 2*B*(b*c - a*d
)*n*(2*a*d*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b*c*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - a*B*d*n*(Log[a + b*x]*(
Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*
x))/(-(b*c) + a*d)]) + b*B*c*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log
[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - B*(b
*c - a*d)*n*(2*A*b*d*x + 2*B*d*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] -
2*B*(b*c - a*d)*n*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) +
 a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*
d)])) + 4*B^2*(b*c - a*d)^2*n*(-(Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))]) + n*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))]
)))/(2*d^3*i)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 507, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{ci+dix}\,dx$$

$$\downarrow 2961$$

$$\frac{g^2(bc-ad)^2\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{i}$$

$$\downarrow 2795$$

$$\frac{g^2(bc-ad)^2\int\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)d\frac{a+bx}{c+dx}}{i}$$

$$\downarrow 2009$$

$$g^2(bc-ad)^2\left(\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2Bn\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3}-\frac{4Bn\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.\right.}{d^3}$$

input

```
Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*
x),x]
```

output

```
((b*c - a*d)^2*g^2*(-((B*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
))/(d^2*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)))) + (b^2*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])^2)/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^2) - (2*(a +
 b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^2*(c + d*x)*(b - (d*(a
 + b*x))/(c + d*x))) - (B^2*n^2*Log[b - (d*(a + b*x))/(c + d*x)])/d^3 - (4*
B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d
*x))])/d^3 - ((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x
))/(b*(c + d*x))])/d^3 + (B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1
 - (b*(c + d*x))/(d*(a + b*x))])/d^3 - (4*B^2*n^2*PolyLog[2, (d*(a + b*x))
/(b*(c + d*x))])/d^3 - (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyL
og[2, (d*(a + b*x))/(b*(c + d*x))])/d^3 - (B^2*n^2*PolyLog[2, (b*(c + d*x)
)/(d*(a + b*x))])/d^3 + (2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))]
)/d^3))/i
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{dix+ci}dx$$

input

```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

output

```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x,
algorithm="fricas")
```

output
```
integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x
^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(
A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log(e*((b*x + a)/(d*x + c
))^n))/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{g^2\left(\int \frac{A^2a^2}{c+dx}\,dx + \int \frac{A^2b^2x^2}{c+dx}\,dx + \int \frac{B^2a^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c+dx}\,dx + \int \frac{2ABa^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx + \int \frac{2A^2abx}{c+dx}\,dx\right.}{}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i),x
)
```

output
```
g**2*(Integral(A**2*a**2/(c + d*x), x) + Integral(A**2*b**2*x**2/(c + d*x)
, x) + Integral(B**2*a**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c + 
d*x), x) + Integral(2*A*B*a**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c
 + d*x), x) + Integral(2*A**2*a*b*x/(c + d*x), x) + Integral(B**2*b**2*x**2
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c + d*x), x) + Integral(2*A*B
*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x) + Integra
l(2*B**2*a*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c + d*x), x) + 
Integral(4*A*B*a*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x)
)/i
```

Maxima [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x,
algorithm="maxima")
```

output

```
2*A^2*a*b*g^2*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + 1/2*A^2*b^2*g^2*(2*c^2*
log(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A^2*a^2*g^2*log(d*i*x +
c*i)/(d*i) + 1/2*(B^2*b^2*d^2*g^2*x^2 - 2*(b^2*c*d*g^2 - 2*a*b*d^2*g^2)*B^
2*x + 2*(b^2*c^2*g^2 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)*B^2*log(d*x + c))*log(
(d*x + c)^n)^2/(d^3*i) - integrate(-(B^2*a^2*d^2*g^2*log(e)^2 + 2*A*B*a^2*
d^2*g^2*log(e) + (B^2*b^2*d^2*g^2*log(e)^2 + 2*A*B*b^2*d^2*g^2*log(e))*x^2
 + (B^2*b^2*d^2*g^2*x^2 + 2*B^2*a*b*d^2*g^2*x + B^2*a^2*d^2*g^2)*log((b*x
+ a)^n)^2 + 2*(B^2*a*b*d^2*g^2*log(e)^2 + 2*A*B*a*b*d^2*g^2*log(e))*x + 2*
(B^2*a^2*d^2*g^2*log(e) + A*B*a^2*d^2*g^2 + (B^2*b^2*d^2*g^2*log(e) + A*B*
b^2*d^2*g^2)*x^2 + 2*(B^2*a*b*d^2*g^2*log(e) + A*B*a*b*d^2*g^2)*x)*log((b*
x + a)^n) - (2*B^2*a^2*d^2*g^2*log(e) + 2*A*B*a^2*d^2*g^2 + 2*(b^2*c^2*g^2
*n - 2*a*b*c*d*g^2*n + a^2*d^2*g^2*n)*B^2*log(d*x + c) + (2*A*B*b^2*d^2*g^
2 + (g^2*n + 2*g^2*log(e))*B^2*b^2*d^2)*x^2 + 2*(2*A*B*a*b*d^2*g^2 - (b^2*
c*d*g^2*n - 2*(g^2*n + g^2*log(e))*a*b*d^2)*B^2)*x + 2*(B^2*b^2*d^2*g^2*x^
2 + 2*B^2*a*b*d^2*g^2*x + B^2*a^2*d^2*g^2)*log((b*x + a)^n))*log((d*x + c)
^n))/(d^3*i*x + c*d^2*i), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x,
algorithm="giac")
```

output

```
integrate((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x
+ c*i), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$
$$=\int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x),x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x), x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$
$$=\frac{g^2i\left(-2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{dx+c}dx\right)a^2b^2d^3-4\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{dx+c}dx\right)a^3b\,d^3-2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 x^2}{dx+c}dx\right)b^4d^3-4\left(\int\right.\right.}{}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

output
```
(g**2*i*( - 2*int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(c + d*x),x)*a**2*
b**2*d**3 - 4*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(c + d*x),x)*a**3*b*d
**3 - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(c + d*x),x)*b**4
*d**3 - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(c + d*x),x)*a*b**
3*d**3 - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c + d*x),x)*a*b*
*3*d**3 - 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c + d*x),x)*a**2*b
**2*d**3 - 2*log(c + d*x)*a**4*d**2 + 4*log(c + d*x)*a**3*b*c*d - 2*log(c
+ d*x)*a**2*b**2*c**2 - 4*a**3*b*d**2*x + 2*a**2*b**2*c*d*x - a**2*b**2*d*
*2*x**2))/(2*d**3)
```

3.188 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$

Optimal result . . . 2184
Mathematica [B] (verified) . . . 2185
Rubi [A] (verified) . . . 2186
Maple [F] . . . 2188
Fricas [F] . . . 2188
Sympy [F] . . . 2189
Maxima [F] . . . 2189
Giac [F] . . . 2190
Mupad [F(-1)] . . . 2190
Reduce [F] . . . 2190

Optimal result

Integrand size = 43, antiderivative size = 303

$$
\begin{aligned}
&\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx \\
&= \frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{di} \\
&\quad + \frac{2B(bc-ad)gn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i} \\
&\quad + \frac{(bc-ad)g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i} \\
&\quad + \frac{2B^2(bc-ad)gn^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i} \\
&\quad + \frac{2B(bc-ad)gn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i} \\
&\quad - \frac{2B^2(bc-ad)gn^2\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}
\end{aligned}
$$

output
```
g*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d/i+2*B*(-a*d+b*c)*g*n*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^2/i+(-a*d+b*c)*g*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/d^2/i+2*B^2*(-a*d+b*
c)*g*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/d^2/i+2*B*(-a*d+b*c)*g*n*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^2/i-2*B^2*(-a*d+b*
c)*g*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/d^2/i
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1385 vs. $2(303) = 606$.

Time $= 1.00$ (sec) , antiderivative size $= 1385$, normalized size of antiderivative $= 4.57$

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \text{Too large to display}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i +
d*i*x),x]
```

output

```
(g*(3*b*d*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c +
 d*x)])^2 - 3*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[
(a + b*x)/(c + d*x)])^2*Log[c + d*x] - 3*a*B*d*n*(A + B*Log[e*((a + b*x)/(
c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(Log[c/d + x]^2 + 2*(Log[a/b
+ x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*Log[c + d*x] - 2*(Log[a/b
+ x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a
*d)])) - 3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(
c + d*x)])*(-2*d*(a + b*x)*(-1 + Log[a/b + x]) + 2*b*(c + d*x)*(-1 + Log[c
/d + x]) - b*c*Log[c/d + x]^2 + 2*b*(Log[a/b + x] - Log[c/d + x] - Log[(a
+ b*x)/(c + d*x)])*(d*x - c*Log[c + d*x]) + 2*b*c*(Log[a/b + x]*Log[(b*(c
+ d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) + a*B^2*
d*n^2*(Log[c/d + x]^3 + 3*Log[c/d + x]^2*(-Log[a/b + x] + Log[(d*(a + b*x)
)/(-(b*c) + a*d)]) + 3*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c +
d*x)])^2*Log[c + d*x] + 3*Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] +
6*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 3*(Log[a/b + x]
- Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*(Log[c/d + x]^2 - 2*(Log[a/b +
x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d
)])) + 6*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 6*PolyLog[3,
 (d*(a + b*x))/(-(b*c) + a*d)] - 6*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])
+ B^2*n^2*(3*d*(2*b*x - 2*(a + b*x)*Log[a/b + x] + (a + b*x)*Log[a/b + ...
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{ci+dix}\,dx$$

$$\downarrow 2961$$

$$\frac{g(bc-ad)\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i}$$

$$\downarrow 2795$$

$$\frac{g(bc-ad)\int\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}+\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d\left(\frac{d(a+bx)}{c+dx}-b\right)^2}\right)d\frac{a+bx}{c+dx}}{i}$$

$$\downarrow 2009$$

$$\frac{g(bc-ad)\left(\frac{2Bn\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^2}+\frac{2Bn\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^2}+\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\right.}{i}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*x)
,x]
```

output
```
((b*c - a*d)*g*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d*(c
 + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (2*B*n*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 + ((A + B*Log[e*((a
 + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 + (2*B^
2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^2 + (2*B*n*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^2 - (2
*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/d^2))/i
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{dix+ci}dx$$

input

```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

output

```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}dx = \int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}dx$$

input

```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, al
gorithm="fricas")
```

output

```
integral((A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log(e*((b*x + a)/(d*
x + c))^n)^2 + 2*(A*B*b*g*x + A*B*a*g)*log(e*((b*x + a)/(d*x + c))^n))/(d*
i*x + c*i), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$

$$= \frac{g\left(\int \frac{A^2a}{c+dx}\,dx+\int \frac{A^2bx}{c+dx}\,dx+\int \frac{B^2a\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c+dx}\,dx+\int \frac{2ABa\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx+\int \frac{B^2bx\log\left(e\left(\frac{a}{c+dx}\right.\right.\right.}{c+dx}$$

$$\frac{}{i}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i),x)
```

output
```
g*(Integral(A**2*a/(c + d*x), x) + Integral(A**2*b*x/(c + d*x), x) + Integ
ral(B**2*a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c + d*x), x) + Inte
gral(2*A*B*a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x) + Integ
ral(B**2*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c + d*x), x) + In
tegral(2*A*B*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x))/i
```

Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, al
gorithm="maxima")
```

output
```
A^2*b*g*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + A^2*a*g*log(d*i*x + c*i)/(d*i
) + (B^2*b*d*g*x - (b*c*g - a*d*g)*B^2*log(d*x + c))*log((d*x + c)^n)^2/(d
^2*i) - integrate(-(B^2*a*d*g*log(e)^2 + 2*A*B*a*d*g*log(e) + (B^2*b*d*g*x
 + B^2*a*d*g)*log((b*x + a)^n)^2 + (B^2*b*d*g*log(e)^2 + 2*A*B*b*d*g*log(e
))*x + 2*(B^2*a*d*g*log(e) + A*B*a*d*g + (B^2*b*d*g*log(e) + A*B*b*d*g)*x)
*log((b*x + a)^n) - 2*(B^2*a*d*g*log(e) + A*B*a*d*g - (b*c*g*n - a*d*g*n)*
B^2*log(d*x + c) + ((g*n + g*log(e))*B^2*b*d + A*B*b*d*g)*x + (B^2*b*d*g*x
 + B^2*a*d*g)*log((b*x + a)^n))*log((d*x + c)^n))/(d^2*i*x + c*d*i), x)
```

Giac [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, al
gorithm="giac")
```

output
```
integrate((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x +
c*i), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x)
,x)
```

output
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x)
, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{gi\left(-\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{dx+c}dx\right) a\,b^2d^2 - 2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{dx+c}dx\right) a^2b\,d^2 - \left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 x}{dx+c}dx\right) b^3d^2 - 2\left(\int \frac{\log(}{}\right.\right.}{d^2}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

output
```
(g*i*( - int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(c + d*x),x)*a*b**2*d**
2 - 2*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(c + d*x),x)*a**2*b*d**2 - in
t((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(c + d*x),x)*b**3*d**2 - 2*int
((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c + d*x),x)*a*b**2*d**2 - log(c +
 d*x)*a**3*d + log(c + d*x)*a**2*b*c - a**2*b*d*x))/d**2
```

3.189 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$

Optimal result . . . 2192
Mathematica [A] (verified) . . . 2193
Rubi [A] (verified) . . . 2193
Maple [F] . . . 2195
Fricas [F] . . . 2195
Sympy [F] . . . 2196
Maxima [F] . . . 2196
Giac [F] . . . 2197
Mupad [F(-1)] . . . 2197
Reduce [F] . . . 2197

Optimal result

Integrand size = 35, antiderivative size = 137

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{di} - \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{di} + \frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{di}$$

output

```
-(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/d/i-2*B*n*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d/i+2*B^2*n^2*p
olylog(3,d*(b*x+a)/b/(d*x+c))/d/i
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.96

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{A^2\log(c+dx)+2ABn\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log\left(\frac{bc-ad}{bc+bdx}\right)-2AB\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{bc+bdx}\right)-B^2\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{di}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x),x]
```

output
```
(A^2*Log[c + d*x] + 2*A*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[(b*c - a
*d)/(b*c + b*d*x)] - 2*A*B*Log[e*((a + b*x)/(c + d*x))^n]*Log[(b*c - a*d)/
(b*c + b*d*x)] - B^2*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*c
 + b*d*x)] + A*B*n*Log[(b*c - a*d)/(b*c + b*d*x)]^2 - 2*B^2*n*Log[e*((a +
b*x)/(c + d*x))^n]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*A*B*n*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)] + 2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(
c + d*x))])/(d*i)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2951, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{ci+dix}\,dx$$
$$\downarrow 2951$$
$$\frac{\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b-\frac{d(a+bx)}{c+dx}}\,d\frac{a+bx}{c+dx}}{i}$$
$$\downarrow 2754$$

$$\frac{\frac{2Bn\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d}}{i}$$

$$\downarrow 2821$$

$$\frac{\frac{2Bn\left(Bn\int\frac{(c+dx)\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d}}{i}$$

$$\downarrow 7143$$

$$\frac{\frac{2Bn\left(Bn\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)-\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d}}{i}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x),x]
```

output
```
(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c +
 d*x))])/d) + (2*B*n*(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2,
(d*(a + b*x))/(b*(c + d*x))]) + B*n*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))
]))/d)/i
```

Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)  Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2951

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m  Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{dix+ci}dx$$

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

output

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, algorithm="fri
cas")
```

output

```
integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d
*x + c))^n) + A^2)/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci + dix}\, dx$$
$$= \frac{\int \frac{A^2}{c+dx}\, dx + \int \frac{B^2 \log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)^2}{c+dx}\, dx + \int \frac{2AB \log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)}{c+dx}\, dx}{i}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i),x)
```

output
```
(Integral(A**2/(c + d*x), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c +
 d*x))**n)**2/(c + d*x), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c +
 d*x))**n)/(c + d*x), x))/i
```

Maxima [F]

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci + dix}\, dx = \int \frac{\left(B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}{dix + ci}\, dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, algorithm="max
ima")
```

output
```
B^2*log(d*x + c)*log((d*x + c)^n)^2/(d*i) + A^2*log(d*i*x + c*i)/(d*i) - i
ntegrate(-(B^2*log((b*x + a)^n)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*l
og(e) + A*B)*log((b*x + a)^n) - 2*(B^2*n*log(d*x + c) + B^2*log((b*x + a)^
n) + B^2*log(e) + A*B)*log((d*x + c)^n))/(d*i*x + c*i), x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, algorithm="gia
c")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x + c*i), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*i + d*i*x),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*i + d*i*x), x)
```

Reduce [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \frac{i\left(-\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{dx+c}dx\right)b^2d-2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{dx+c}dx\right)abd-\log(dx+c)\,a^2\right)}{d}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

output

```
(i*( - int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(c + d*x),x)*b**2*d - 2*i
nt(log(((a + b*x)**n*e)/(c + d*x)**n)/(c + d*x),x)*a*b*d - log(c + d*x)*a*
*2))/d
```

3.190 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$

Optimal result . . . 2199
Mathematica [A] (verified) . . . 2199
Rubi [A] (warning: unable to verify) . . . 2200
Maple [B] (verified) . . . 2201
Fricas [B] (verification not implemented) . . . 2202
Sympy [F] . . . 2202
Maxima [B] (verification not implemented) . . . 2203
Giac [B] (verification not implemented) . . . 2204
Mupad [B] (verification not implemented) . . . 2204
Reduce [B] (verification not implemented) . . . 2205

Optimal result

Integrand size = 45, antiderivative size = 50

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)gin}$$

output
```
1/3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)/g/i/n
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.80

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = \frac{3A^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3AB\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+B^2\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bcgin-3adgin}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d
*i*x)),x]
```

output
```
(3*A^2*Log[e*((a + b*x)/(c + d*x))^n] + 3*A*B*Log[e*((a + b*x)/(c + d*x))^
n]^2 + B^2*Log[e*((a + b*x)/(c + d*x))^n]^3)/(3*b*c*g*i*n - 3*a*d*g*i*n)
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)(ci+dix)}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx} d\frac{a+bx}{c+dx}}{gi(bc-ad)}$$

$$\downarrow 2739$$

$$\frac{\int \frac{(a+bx)^2}{(c+dx)^2} d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bgin(bc-ad)}$$

$$\downarrow 15$$

$$\frac{(a+bx)^3}{3Bgin(c+dx)^3(bc-ad)}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d*i*x))
,x]
```

output
```
(a + b*x)^3/(3*B*(b*c - a*d)*g*i*n*(c + d*x)^3)
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(48) = 96$.

Time = 1.75 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.36

method	result	size
parallelrisch	$-\frac{B^2a^2c^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^3+3AB\,a^2c^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2+3A^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^2c^2}{3ig\,a^2c^2n(da-bc)}$	118
default	$\frac{A^2\left(\frac{\ln(dx+c)}{da-bc}-\frac{\ln(bx+a)}{da-bc}\right)}{gi}-\frac{B^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^3}{3gin(da-bc)}-\frac{AB\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}{gin(da-bc)}$	135
parts	$\frac{A^2\left(\frac{\ln(dx+c)}{da-bc}-\frac{\ln(bx+a)}{da-bc}\right)}{gi}-\frac{B^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^3}{3gin(da-bc)}-\frac{AB\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}{gin(da-bc)}$	135

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i),x,method=_RE
TURNVERBOSE)
```

output

```
-1/3*(B^2*a^2*c^2*ln(e*((b*x+a)/(d*x+c))^n)^3+3*A*B*a^2*c^2*ln(e*((b*x+a)/
(d*x+c))^n)^2+3*A^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*c^2)/i/g/a^2/c^2/n/(a*d-
b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(48) = 96$.

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.98

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$$
$$= \frac{B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^3 + 3\,B^2\log(e)^2\log\left(\frac{bx+a}{dx+c}\right) + 3\,ABn\log\left(\frac{bx+a}{dx+c}\right)^2 + 3\,A^2\log\left(\frac{bx+a}{dx+c}\right) + 3\left(B^2n\log\left(\frac{bx+a}{dx+c}\right)^2 +$$
$$3\,(bc-ad)gi$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i),x, al
gorithm="fricas")
```

output
```
1/3*(B^2*n^2*log((b*x + a)/(d*x + c))^3 + 3*B^2*log(e)^2*log((b*x + a)/(d*
x + c)) + 3*A*B*n*log((b*x + a)/(d*x + c))^2 + 3*A^2*log((b*x + a)/(d*x +
c)) + 3*(B^2*n*log((b*x + a)/(d*x + c))^2 + 2*A*B*log((b*x + a)/(d*x + c))
)*log(e))/((b*c - a*d)*g*i)
```

Sympy [F]

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$$
$$= \frac{\int \frac{A^2}{ac+adx+bcx+bdx^2}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{ac+adx+bcx+bdx^2}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{ac+adx+bcx+bdx^2}\,dx}{gi}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)/(d*i*x+c*i),x)
```

output
```
(Integral(A**2/(a*c + a*d*x + b*c*x + b*d*x**2), x) + Integral(B**2*log(e*
(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a*c + a*d*x + b*c*x + b*d*x**2), x)
+ Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a*c + a*d*x + b*
c*x + b*d*x**2), x))/(g*i)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(48) = 96$.

Time $= 0.06$ (sec) , antiderivative size $= 407$, normalized size of antiderivative $= 8.14$

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$$

$$= B^2\left(\frac{\log(bx+a)}{(bc-ad)gi} - \frac{\log(dx+c)}{(bc-ad)gi}\right)\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)^2$$

$$+ 2AB\left(\frac{\log(bx+a)}{(bc-ad)gi} - \frac{\log(dx+c)}{(bc-ad)gi}\right)\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)$$

$$+ \frac{1}{3}\left(\frac{\left(\log(bx+a)^3 - 3\log(bx+a)^2\log(dx+c) + 3\log(bx+a)\log(dx+c)^2 - \log(dx+c)^3\right)n^2}{bcgi - adgi} - \frac{3}{}\right.$$

$$\left. - \frac{\left(\log(bx+a)^2 - 2\log(bx+a)\log(dx+c) + \log(dx+c)^2\right)ABn}{bcgi - adgi}\right.$$

$$+ A^2\left(\frac{\log(bx+a)}{(bc-ad)gi} - \frac{\log(dx+c)}{(bc-ad)gi}\right)$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i),x, al
gorithm="maxima")
```

output

```
B^2*(log(b*x + a)/((b*c - a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))*log(
e*(b*x/(d*x + c) + a/(d*x + c))^n)^2 + 2*A*B*(log(b*x + a)/((b*c - a*d)*g*
i) - log(d*x + c)/((b*c - a*d)*g*i))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
) + 1/3*((log(b*x + a)^3 - 3*log(b*x + a)^2*log(d*x + c) + 3*log(b*x + a)*
log(d*x + c)^2 - log(d*x + c)^3)*n^2/(b*c*g*i - a*d*g*i) - 3*(log(b*x + a)
^2 - 2*log(b*x + a)*log(d*x + c) + log(d*x + c)^2)*n*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/(b*c*g*i - a*d*g*i))*B^2 - (log(b*x + a)^2 - 2*log(b*x +
 a)*log(d*x + c) + log(d*x + c)^2)*A*B*n/(b*c*g*i - a*d*g*i) + A^2*(log(b*
x + a)/((b*c - a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(48) = 96$.

Time $= 0.40$ (sec) , antiderivative size $= 167$, normalized size of antiderivative $= 3.34$

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$$
$$= \frac{\left(B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^3 + 3\,B^2n\log(e)\log\left(\frac{bx+a}{dx+c}\right)^2 + 3\,B^2\log(e)^2\log\left(\frac{bx+a}{dx+c}\right) + 3\,ABn\log\left(\frac{bx+a}{dx+c}\right)^2 + 6\,AB\log\right.}{3\,gi}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i),x, al
gorithm="giac")
```

output
```
1/3*(B^2*n^2*log((b*x + a)/(d*x + c))^3 + 3*B^2*n*log(e)*log((b*x + a)/(d*
x + c))^2 + 3*B^2*log(e)^2*log((b*x + a)/(d*x + c)) + 3*A*B*n*log((b*x + a
)/(d*x + c))^2 + 6*A*B*log(e)*log((b*x + a)/(d*x + c)) + 3*A^2*log((b*x +
a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(g*i)
```

Mupad [B] (verification not implemented)

Time $= 27.09$ (sec) , antiderivative size $= 122$, normalized size of antiderivative $= 2.44$

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = -\frac{\frac{B^2\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}{3} + A\,B\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}{g\,i\,n\,(a\,d-b\,c)} + \frac{A^2\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g\,i\,(a\,d-b\,c)}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)*(c*i + d*i*x))
,x)
```

output
```
(A^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g*i*(a*d - b*c))
- ((B^2*log(e*((a + b*x)/(c + d*x))^n)^3)/3 + A*B*log(e*((a + b*x)/(c + d*
x))^n)^2)/(g*i*n*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.88

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$$
$$= \frac{i\left(3\log(bx+a)\,a^2n-3\log(dx+c)\,a^2n+\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)^3b^2+3\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)^2ab\right)}{3gn\,(ad-bc)}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i),x)
```

output
```
(i*(3*log(a + b*x)*a**2*n - 3*log(c + d*x)*a**2*n + log(((a + b*x)**n*e)/(
c + d*x)**n)**3*b**2 + 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b))/(3*g*
n*(a*d - b*c))
```

3.191 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$

Optimal result . . . 2206
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Reduce [B] (verification not implemented) . . . 2213

Optimal result

Integrand size = 45, antiderivative size = 199

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{2bB^2n^2(c+dx)}{(bc-ad)^2g^2i(a+bx)} - \frac{2bBn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{b(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2g^2i(a+bx)} - \frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^2g^2in}$$

output

```
-2*b*B^2*n^2*(d*x+c)/(-a*d+b*c)^2/g^2/i/(b*x+a)-2*b*B*n*(d*x+c)*(A+B*ln(e*
((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/g^2/i/(b*x+a)-b*(d*x+c)*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^2/i/(b*x+a)-1/3*d*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))^3/B/(-a*d+b*c)^2/g^2/i/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 793 vs. $2(199) = 398$.

Time $= 0.76$ (sec) , antiderivative size $= 793$, normalized size of antiderivative $= 3.98$

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$$

$$= -\frac{B^2dn^2\log^3\left(\frac{a+bx}{c+dx}\right)}{3(bc-ad)^2g^2i} + \frac{2Bn\log\left(\frac{a+bx}{c+dx}\right)\left(A+Bn+B\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)\right)}{(-bc+ad)g^2i(a+bx)}$$

$$+\frac{\log^2\left(\frac{a+bx}{c+dx}\right)\left(-aABdn-bB^2cn^2-AbBdnx-bB^2dn^2x-aB^2dn\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)-bB\right.}{(-bc+ad)^2g^2i(a+bx)}$$

$$+\frac{-A^2-2ABn-2B^2n^2-2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)-2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right.}{(bc-ad)g^2i(a+bx)}$$

$$-\frac{d\log(a+bx)\left(A^2+2ABn+2B^2n^2+2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)+2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-\right.\right.}{(bc-ad)^2g^2i}$$

$$+\frac{d\left(A^2+2ABn+2B^2n^2+2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)+2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right.\right.}{(bc-ad)^2g^2i}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i +
 d*i*x)),x]
```

output

```
-1/3*(B^2*d*n^2*Log[(a + b*x)/(c + d*x)]^3)/((b*c - a*d)^2*g^2*i) + (2*B*n
*Log[(a + b*x)/(c + d*x)]*(A + B*n + B*(Log[e*((a + b*x)/(c + d*x))^n] - n
*Log[(a + b*x)/(c + d*x)])))/((-(b*c) + a*d)*g^2*i*(a + b*x)) + (Log[(a +
b*x)/(c + d*x)]^2*(-(a*A*B*d*n) - b*B^2*c*n^2 - A*b*B*d*n*x - b*B^2*d*n^2*
x - a*B^2*d*n*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]
) - b*B^2*d*n*x*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x
)])))/((-(b*c) + a*d)^2*g^2*i*(a + b*x)) + (-A^2 - 2*A*B*n - 2*B^2*n^2 - 2
*A*B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) - 2*B^2
*n*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) - B^2*(Lo
g[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2)/((b*c - a*d)
*g^2*i*(a + b*x)) - (d*Log[a + b*x]*(A^2 + 2*A*B*n + 2*B^2*n^2 + 2*A*B*(Lo
g[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) + 2*B^2*n*(Log[
e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) + B^2*(Log[e*((a
+ b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2))/((b*c - a*d)^2*g^2*
i) + (d*(A^2 + 2*A*B*n + 2*B^2*n^2 + 2*A*B*(Log[e*((a + b*x)/(c + d*x))^n]
 - n*Log[(a + b*x)/(c + d*x)]) + 2*B^2*n*(Log[e*((a + b*x)/(c + d*x))^n] -
 n*Log[(a + b*x)/(c + d*x)]) + B^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log
[(a + b*x)/(c + d*x)])^2)*Log[c + d*x])/((b*c - a*d)^2*g^2*i)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}\,d\frac{a+bx}{c+dx}}{g^2 i(bc-ad)^2}$$

$$\downarrow 2795$$

$$\frac{\int \left(\frac{b(c+dx)^2 \left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2} - \frac{d(c+dx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx} \right) d\frac{a+bx}{c+dx}}{g^2 i (bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{-\frac{d\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3Bn} - \frac{b(c+dx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx} - \frac{2bBn(c+dx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx} - \frac{2bB^2n^2(c+dx)}{a+bx}}{g^2 i (bc-ad)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i + d*i*x
)),x]
```

output
```
((-2*b*B^2*n^2*(c + d*x))/(a + b*x) - (2*b*B*n*(c + d*x)*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]))/(a + b*x) - (b*(c + d*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2)/(a + b*x) - (d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(
3*B*n))/((b*c - a*d)^2*g^2*i)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(197) = 394.

Time = 5.10 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.56

method	result
parallelrisch	$-\frac{-6ABa\,b^3d^3n^2+6AB\,b^4c\,d^2n^2+3B^2x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2b^4d^3n+6B^2x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4d^3n^2+3ABx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2b^4d^3$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x,method=_
RETURNVERBOSE)
```

output
```
-1/3*(-6*A*B*a*b^3*d^3*n^2+6*A*B*b^4*c*d^2*n^2+3*B^2*x*ln(e*((b*x+a)/(d*x+
c))^n)^2*b^4*d^3*n+6*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^3*n^2+3*A*B*x*l
n(e*((b*x+a)/(d*x+c))^n)^2*b^4*d^3+3*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*c
*d^2*n+6*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^2*n^2+3*A*B*ln(e*((b*x+a)/(
d*x+c))^n)^2*a*b^3*d^3+6*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^3*n+6*A*B*l
n(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^2*n-6*B^2*a*b^3*d^3*n^3+6*B^2*b^4*c*d^2*n
^3-3*A^2*a*b^3*d^3*n+3*A^2*b^4*c*d^2*n+B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*b
^4*d^3+B^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a*b^3*d^3+3*A^2*x*ln(e*((b*x+a)/(d*
x+c))^n)*b^4*d^3+3*A^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*d^3)/i/g^2/(b*x+a)/
n/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(197) = 394.

Time = 0.09 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.15

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx =$$
$$-\frac{3\,A^2bc-3\,A^2ad+\left(B^2bdn^2x+B^2adn^2\right)\log\left(\frac{bx+a}{dx+c}\right)^3+6\left(B^2bc-B^2ad\right)n^2+3\left(B^2bc-B^2ad+\left(B^2bd\right.\right.}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x,
algorithm="fricas")
```

output
```
-1/3*(3*A^2*b*c - 3*A^2*a*d + (B^2*b*d*n^2*x + B^2*a*d*n^2)*log((b*x + a)/
(d*x + c))^3 + 6*(B^2*b*c - B^2*a*d)*n^2 + 3*(B^2*b*c - B^2*a*d + (B^2*b*d
*x + B^2*a*d)*log((b*x + a)/(d*x + c)))*log(e)^2 + 3*(B^2*b*c*n^2 + A*B*a*
d*n + (B^2*b*d*n^2 + A*B*b*d*n)*x)*log((b*x + a)/(d*x + c))^2 + 6*(A*B*b*c
 - A*B*a*d)*n + 3*(2*A*B*b*c - 2*A*B*a*d + (B^2*b*d*n*x + B^2*a*d*n)*log((
b*x + a)/(d*x + c))^2 + 2*(B^2*b*c - B^2*a*d)*n + 2*(B^2*b*c*n + A*B*a*d +
 (B^2*b*d*n + A*B*b*d)*x)*log((b*x + a)/(d*x + c)))*log(e) + 3*(2*B^2*b*c*
n^2 + 2*A*B*b*c*n + A^2*a*d + (2*B^2*b*d*n^2 + 2*A*B*b*d*n + A^2*b*d)*x)*l
og((b*x + a)/(d*x + c)))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^2*i*x + (a
*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*g^2*i)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2/(d*i*x+c*i),x
)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. $2(197) = 394$.

Time = 0.10 (sec) , antiderivative size = 1018, normalized size of antiderivative = 5.12

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x,
algorithm="maxima")
```

output

```
-B^2*(1/((b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)
/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*g^2*i))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2 - 2*A*B*
(1/((b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*g^2*i))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/3*(((b*d*x
+ a*d)*log(b*x + a)^3 - (b*d*x + a*d)*log(d*x + c)^3 - 3*(b*d*x + a*d)*log
(b*x + a)^2 - 3*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))*log(d*x + c)^2
+ 6*b*c - 6*a*d + 6*(b*d*x + a*d)*log(b*x + a) - 3*(2*b*d*x + (b*d*x + a*d
)*log(b*x + a)^2 + 2*a*d - 2*(b*d*x + a*d)*log(b*x + a))*log(d*x + c))*n^2
/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*i + a^3*d^2*g^2*i + (b^3*c^2*g^2*i - 2
*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*x) - 3*((b*d*x + a*d)*log(b*x + a)^2 +
 (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d - 2*(b*d*x + a*d)*log(b*x +
a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))*log(d*x + c))*n*log(e*(b
*x/(d*x + c) + a/(d*x + c))^n)/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*i + a^3*
d^2*g^2*i + (b^3*c^2*g^2*i - 2*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*x))*B^2
+ ((b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2
*a*d - 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b
*x + a))*log(d*x + c))*A*B*n/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*i + a^3*d^
2*g^2*i + (b^3*c^2*g^2*i - 2*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*x) - A^...
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2(dix+ci)}\,dx$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x,
algorithm="giac")
```

output

```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/((b*g*x + a*g)^2*(d*i*x
 + c*i)), x)
```

Mupad [B] (verification not implemented)

Time = 27.16 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.81

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$$
$$= \ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^2\left(\frac{B^2}{(ad-bc)\,(ag^2i+bg^2ix)}-\frac{Bd(A+Bn)}{g^2in(ad-bc)^2}\right)$$
$$+\frac{A^2+2ABn+2B^2n^2}{(ad-bc)\,(ag^2i+bg^2ix)}+\frac{2B\,\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\,(A+Bn)}{(ad-bc)\,(ag^2i+bg^2ix)}-\frac{B^2d\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}{3g^2in(ad-bc)^2}$$
$$+\frac{d\,\mathrm{atan}\left(\frac{d\left(2bdx+\frac{a^2d^2g^2i-b^2c^2g^2i}{g^2i(ad-bc)}\right)(A^2+2ABn+2B^2n^2)\,1\mathrm{i}}{(ad-bc)(dA^2+2dABn+2dB^2n^2)}\right)(A^2+2ABn+2B^2n^2)\,2\mathrm{i}}{g^2i(ad-bc)^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^2*(c*i + d*i*x
)),x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/((a*d - b*c)*(a*g^2*i + b*g^2*i*x))
 - (B*d*(A + B*n))/(g^2*i*n*(a*d - b*c)^2)) + (A^2 + 2*B^2*n^2 + 2*A*B*n)/(
(a*d - b*c)*(a*g^2*i + b*g^2*i*x)) + (2*B*log(e*((a + b*x)/(c + d*x))^n)*(
A + B*n))/((a*d - b*c)*(a*g^2*i + b*g^2*i*x)) + (d*atan((d*(2*b*d*x + (a^2
*d^2*g^2*i - b^2*c^2*g^2*i)/(g^2*i*(a*d - b*c)))*(A^2 + 2*B^2*n^2 + 2*A*B*
n)*1i)/((a*d - b*c)*(A^2*d + 2*B^2*d*n^2 + 2*A*B*d*n)))*(A^2 + 2*B^2*n^2 +
 2*A*B*n)*2i)/(g^2*i*(a*d - b*c)^2) - (B^2*d*log(e*((a + b*x)/(c + d*x))^n
)^3)/(3*g^2*i*n*(a*d - b*c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 612, normalized size of antiderivative = 3.08

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x)
```

output

```
(i*(3*log(a + b*x)*a**4*d*n + 3*log(a + b*x)*a**3*b*d*n*x + 6*log(a + b*x)
*a**2*b**2*c*n**2 + 6*log(a + b*x)*a*b**3*c*n**3 + 6*log(a + b*x)*a*b**3*c
*n**2*x + 6*log(a + b*x)*b**4*c*n**3*x - 3*log(c + d*x)*a**4*d*n - 3*log(c
 + d*x)*a**3*b*d*n*x - 6*log(c + d*x)*a**2*b**2*c*n**2 - 6*log(c + d*x)*a*
b**3*c*n**3 - 6*log(c + d*x)*a*b**3*c*n**2*x - 6*log(c + d*x)*b**4*c*n**3*
x + log(((a + b*x)**n*e)/(c + d*x)**n)**3*a**2*b**2*d + log(((a + b*x)**n*
e)/(c + d*x)**n)**3*a*b**3*d*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a
**3*b*d + 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*d*x + 3*log(((
a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3*c*n + 3*log(((a + b*x)**n*e)/(c + d
*x)**n)**2*a*b**3*d*n*x + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*d
*n*x - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c*n*x + 6*log(((a + b*x
)**n*e)/(c + d*x)**n)*a*b**3*d*n**2*x - 6*log(((a + b*x)**n*e)/(c + d*x)**
n)*b**4*c*n**2*x + 3*a**3*b*d*n*x - 3*a**2*b**2*c*n*x + 6*a**2*b**2*d*n**2
*x - 6*a*b**3*c*n**2*x + 6*a*b**3*d*n**3*x - 6*b**4*c*n**3*x))/(3*a*g**2*n
*(a**3*d**2 - 2*a**2*b*c*d + a**2*b*d**2*x + a*b**2*c**2 - 2*a*b**2*c*d*x
+ b**3*c**2*x))
```

3.192 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$

Optimal result . . . 2215
Mathematica [B] (verified) . . . 2216
Rubi [A] (verified) . . . 2217
Maple [B] (verified) . . . 2219
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Sympy [F(-1)] . . . 2221
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Giac [F(-1)] . . . 2223
Mupad [B] (verification not implemented) . . . 2223
Reduce [B] (verification not implemented) . . . 2224

Optimal result

Integrand size = 45, antiderivative size = 369

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = {}& \frac{4bB^2dn^2(c+dx)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2B^2n^2(c+dx)^2}{4(bc-ad)^3g^3i(a+bx)^2} \\ &+ \frac{4bBdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^3i(a+bx)} \\ &- \frac{b^2Bn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2} \\ &+ \frac{2bd(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^3i(a+bx)} \\ &- \frac{b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3g^3i(a+bx)^2} \\ &+ \frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^3g^3in}\end{aligned}$$

output

```
4*b*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^3/g^3/i/(b*x+a)-1/4*b^2*B^2*n^2*(d*x+c)^2
/(-a*d+b*c)^3/g^3/i/(b*x+a)^2+4*b*B*d*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c)
)^n))/(-a*d+b*c)^3/g^3/i/(b*x+a)-1/2*b^2*B*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))/(-a*d+b*c)^3/g^3/i/(b*x+a)^2+2*b*d*(d*x+c)*(A+B*ln(e*((b*x+a)
/(d*x+c))^n))^2/(-a*d+b*c)^3/g^3/i/(b*x+a)-1/2*b^2*(d*x+c)^2*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^3/i/(b*x+a)^2+1/3*d^2*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))^3/B/(-a*d+b*c)^3/g^3/i/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 975 vs. $2(369) = 738$.

Time $= 1.23$ (sec) , antiderivative size $= 975$, normalized size of antiderivative $= 2.64$

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^3(ci + dix)}\, dx = \text{Too large to display}$$

input

```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i +
 d*i*x)),x]
```

output

```
(4*B^2*d^2*n^2*(a + b*x)^2*Log[(a + b*x)/(c + d*x)]^3 + 6*B*n*Log[(a + b*x
)/(c + d*x)]^2*(2*a^2*A*d^2 - b^2*B*c^2*n + 4*a*b*B*c*d*n + 4*a*A*b*d^2*x
+ 2*b^2*B*c*d*n*x + 4*a*b*B*d^2*n*x + 2*A*b^2*d^2*x^2 + 3*b^2*B*d^2*n*x^2
+ 2*B*d^2*(a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n] - 2*B*d^2*n*(a + b*x)
^2*Log[(a + b*x)/(c + d*x)]) - 6*B*(b*c - a*d)*n*Log[(a + b*x)/(c + d*x)]*
(2*A*b*c - 6*a*A*d + b*B*c*n - 7*a*B*d*n - 4*A*b*d*x - 6*b*B*d*n*x + 2*B*(
-3*a*d + b*(c - 2*d*x))*Log[e*((a + b*x)/(c + d*x))^n] + 2*B*n*(-(b*c) + 3
*a*d + 2*b*d*x)*Log[(a + b*x)/(c + d*x)]) - 3*(b*c - a*d)^2*(2*A^2 + 2*A*B
*n + B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(2*A + B*n)*
Log[(a + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[
e*((a + b*x)/(c + d*x))^n]*(2*A + B*n - 2*B*n*Log[(a + b*x)/(c + d*x)])) +
 6*d*(b*c - a*d)*(a + b*x)*(2*A^2 + 6*A*B*n + 7*B^2*n^2 + 2*B^2*Log[e*((a
+ b*x)/(c + d*x))^n]^2 - 2*B*n*(2*A + 3*B*n)*Log[(a + b*x)/(c + d*x)] + 2*
B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(2
*A + 3*B*n - 2*B*n*Log[(a + b*x)/(c + d*x)])) + 6*d^2*(a + b*x)^2*Log[a +
b*x]*(2*A^2 + 6*A*B*n + 7*B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2
 - 2*B*n*(2*A + 3*B*n)*Log[(a + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/
(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(2*A + 3*B*n - 2*B*n*Log
[(a + b*x)/(c + d*x)])) - 6*d^2*(a + b*x)^2*(2*A^2 + 6*A*B*n + 7*B^2*n^2 +
 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(2*A + 3*B*n)*Log[(a + ...
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}\,d\frac{a+bx}{c+dx}}{g^3 i(bc-ad)^3}$$

$$\downarrow 2795$$

$$\frac{\int \left(\frac{b^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (c+dx)^3}{(a+bx)^3} - \frac{2bd \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (c+dx)^2}{(a+bx)^2} + \frac{d^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (c+dx)}{a+bx} \right) d\frac{a+bx}{c+dx}}{g^3 i (bc-ad)^3}$$

$$\downarrow \text{2009}$$

$$\frac{-\frac{b^2 (c+dx)^2 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2} - \frac{b^2 Bn (c+dx)^2 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2} + \frac{d^2 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3Bn} + \frac{2bd(c+dx) \left(B \log \left(e \left(\frac{a}{c}\right.\right.\right.\right.}{a+bx}}{g^3 i (bc-ad)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i + d*i*x
)),x]
```

output
```
((4*b*B^2*d*n^2*(c + d*x))/(a + b*x) - (b^2*B^2*n^2*(c + d*x)^2)/(4*(a + b
*x)^2) + (4*b*B*d*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a +
 b*x) - (b^2*B*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a
 + b*x)^2) + (2*b*d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a
 + b*x) - (b^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a
 + b*x)^2) + (d^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*B*n))/((b*c
 - a*d)^3*g^3*i)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1235 vs. $2(361) = 722$.

Time = 13.40 (sec) , antiderivative size = 1236, normalized size of antiderivative = 3.35

method	result	size
parallelrisch	Expression too large to display	1236

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x,method=_
RETURNVERBOSE)
```

output

```
-1/12*(4*B^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^6*c^2*d^2+12*A^2*ln(e*((b*x+a)/
(d*x+c))^n)*a^6*c^2*d^2+36*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^3*d*n
^2+24*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^5*b*c^2*d^2+48*A*B*x*a^5*b*c^2*d
^2*n^2-60*A*B*x*a^4*b^2*c^3*d*n^2+48*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b*c
^3*d*n+4*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^4*b^2*c^2*d^2+45*B^2*x^2*a^
4*b^2*c^2*d^2*n^3-48*B^2*x^2*a^3*b^3*c^3*d*n^3+6*A*B*x^2*a^2*b^4*c^4*n^2+8
*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^5*b*c^2*d^2+48*B^2*x*a^5*b*c^2*d^2*n^
3-54*B^2*x*a^4*b^2*c^3*d*n^3-36*A^2*x*a^4*b^2*c^3*d*n-12*A*B*ln(e*((b*x+a)
/(d*x+c))^n)*a^4*b^2*c^4*n+12*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^
2*d^2+18*A^2*x^2*a^4*b^2*c^2*d^2*n-24*A^2*x^2*a^3*b^3*c^3*d*n+12*A*B*x*a^3
*b^3*c^4*n^2+24*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^5*b*c^3*d*n+48*B^2*ln(e*
((b*x+a)/(d*x+c))^n)*a^5*b*c^3*d*n^2+24*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^
5*b*c^2*d^2+24*A^2*x*a^5*b*c^2*d^2*n+18*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^
2*a^4*b^2*c^2*d^2*n+42*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^2*d^2*n
^2+12*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b^2*c^2*d^2+42*A*B*x^2*a^4*b
^2*c^2*d^2*n^2-48*A*B*x^2*a^3*b^3*c^3*d*n^2+24*B^2*x*ln(e*((b*x+a)/(d*x+c)
)^n)^2*a^5*b*c^2*d^2*n+12*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b^2*c^3*d*
n+48*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b*c^2*d^2*n^2+3*B^2*x^2*a^2*b^4*c
^4*n^3+6*B^2*x*a^3*b^3*c^4*n^3+6*A^2*x^2*a^2*b^4*c^4*n-6*B^2*ln(e*((b*x+a)
/(d*x+c))^n)^2*a^4*b^2*c^4*n+36*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1068 vs. $2(361) = 722$.

Time $= 0.10$ (sec) , antiderivative size $= 1068$, normalized size of antiderivative $= 2.89$

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x,
algorithm="fricas")
```

output

```
-1/12*(6*A^2*b^2*c^2 - 24*A^2*a*b*c*d + 18*A^2*a^2*d^2 - 4*(B^2*b^2*d^2*n^
2*x^2 + 2*B^2*a*b*d^2*n^2*x + B^2*a^2*d^2*n^2)*log((b*x + a)/(d*x + c))^3
+ 3*(B^2*b^2*c^2 - 16*B^2*a*b*c*d + 15*B^2*a^2*d^2)*n^2 + 6*(B^2*b^2*c^2 -
 4*B^2*a*b*c*d + 3*B^2*a^2*d^2 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*x - 2*(B^2*
b^2*d^2*x^2 + 2*B^2*a*b*d^2*x + B^2*a^2*d^2)*log((b*x + a)/(d*x + c)))*log
(e)^2 - 6*(2*A*B*a^2*d^2*n - (B^2*b^2*c^2 - 4*B^2*a*b*c*d)*n^2 + (3*B^2*b^
2*d^2*n^2 + 2*A*B*b^2*d^2*n)*x^2 + 2*(2*A*B*a*b*d^2*n + (B^2*b^2*c*d + 2*B
^2*a*b*d^2)*n^2)*x)*log((b*x + a)/(d*x + c))^2 + 6*(A*B*b^2*c^2 - 8*A*B*a*
b*c*d + 7*A*B*a^2*d^2)*n - 6*(2*A^2*b^2*c*d - 2*A^2*a*b*d^2 + 7*(B^2*b^2*c
*d - B^2*a*b*d^2)*n^2 + 6*(A*B*b^2*c*d - A*B*a*b*d^2)*n)*x + 6*(2*A*B*b^2*
c^2 - 8*A*B*a*b*c*d + 6*A*B*a^2*d^2 - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*a*b*d^2
*n*x + B^2*a^2*d^2*n)*log((b*x + a)/(d*x + c))^2 + (B^2*b^2*c^2 - 8*B^2*a*
b*c*d + 7*B^2*a^2*d^2)*n - 2*(2*A*B*b^2*c*d - 2*A*B*a*b*d^2 + 3*(B^2*b^2*c
*d - B^2*a*b*d^2)*n)*x - 2*(2*A*B*a^2*d^2 + (3*B^2*b^2*d^2*n + 2*A*B*b^2*d
^2)*x^2 - (B^2*b^2*c^2 - 4*B^2*a*b*c*d)*n + 2*(2*A*B*a*b*d^2 + (B^2*b^2*c*
d + 2*B^2*a*b*d^2)*n)*x)*log((b*x + a)/(d*x + c)))*log(e) - 6*(2*A^2*a^2*d
^2 - (B^2*b^2*c^2 - 8*B^2*a*b*c*d)*n^2 + (7*B^2*b^2*d^2*n^2 + 6*A*B*b^2*d^
2*n + 2*A^2*b^2*d^2)*x^2 - 2*(A*B*b^2*c^2 - 4*A*B*a*b*c*d)*n + 2*(2*A^2*a*
b*d^2 + (3*B^2*b^2*c*d + 4*B^2*a*b*d^2)*n^2 + 2*(A*B*b^2*c*d + 2*A*B*a*b*d
^2)*n)*x)*log((b*x + a)/(d*x + c)))/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \text{Timed out}$$

input

```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3/(d*i*x+c*i),x
)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2126 vs. $2(361) = 722$.

Time = 0.19 (sec) , antiderivative size = 2126, normalized size of antiderivative = 5.76

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x,
algorithm="maxima")
```

output

```
1/2*B^2*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^
3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a^2*b^2*c^2
 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*
c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(e*(b*x/(d*x + c) + a/(
d*x + c))^n)^2 + A*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^
2*b^2*d^2)*g^3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x +
 (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/
((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(e*(b*x/(d
*x + c) + a/(d*x + c))^n) - 1/12*((3*b^2*c^2 - 48*a*b*c*d + 45*a^2*d^2 - 4
*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^3 + 4*(b^2*d^2*x^2 + 2
*a*b*d^2*x + a^2*d^2)*log(d*x + c)^3 + 18*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2
*d^2)*log(b*x + a)^2 + 6*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2
*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c)^2 - 42*(b^2*c
*d - a*b*d^2)*x - 42*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) +
6*(7*b^2*d^2*x^2 + 14*a*b*d^2*x + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x
 + a^2*d^2)*log(b*x + a)^2 - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b
*x + a))*log(d*x + c))*n^2/(a^2*b^3*c^3*g^3*i - 3*a^3*b^2*c^2*d*g^3*i + 3*
a^4*b*c*d^2*g^3*i - a^5*d^3*g^3*i + (b^5*c^3*g^3*i - 3*a*b^4*c^2*d*g^3*...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x,
algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 29.93 (sec) , antiderivative size = 1011, normalized size of antiderivative = 2.74

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^3*(c*i + d*i*x
)),x)
```

output

```
log(e*((a + b*x)/(c + d*x))^n)*((B^2*n)/(x^2*(b^3*c*g^3*i - a*b^2*d*g^3*i)
 + x*(2*a*b^2*c*g^3*i - 2*a^2*b*d*g^3*i) - a^3*d*g^3*i + a^2*b*c*g^3*i) -
(d^2*(3*B^2*n + 2*A*B)*((a*g^3*i*n*(a*d - b*c)^2)/(2*d) + (g^3*i*n*(a*d -
b*c)^2*(2*a*d - b*c))/(2*d^2) + (b*g^3*i*n*x*(a*d - b*c)^2)/d))/(g^3*i*n*(
a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(x^2*(b^3*c*g^3*i - a*b^2*d*g^3
*i) + x*(2*a*b^2*c*g^3*i - 2*a^2*b*d*g^3*i) - a^3*d*g^3*i + a^2*b*c*g^3*i)
)) - log(e*((a + b*x)/(c + d*x))^n)^2*((d^2*(3*B^2*n + 2*A*B))/(2*g^3*i*n*
(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (B^2*d^2*((g^3*i*n*(a*d - b
*c)*(2*a*d - b*c))/(2*d^2) + (a*g^3*i*n*(a*d - b*c))/(2*d) + (b*g^3*i*n*x*
(a*d - b*c))/d))/(g^3*i*n*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2
*g^3*i + b^2*g^3*i*x^2 + 2*a*b*g^3*i*x))) - ((6*A^2*a*d - 2*A^2*b*c + 15*B
^2*a*d*n^2 - B^2*b*c*n^2 + 14*A*B*a*d*n - 2*A*B*b*c*n)/(2*(a*d - b*c)) + (
x*(2*A^2*b*d + 7*B^2*b*d*n^2 + 6*A*B*b*d*n))/(a*d - b*c))/(x^2*(2*b^3*c*g^
3*i - 2*a*b^2*d*g^3*i) + x*(4*a*b^2*c*g^3*i - 4*a^2*b*d*g^3*i) - 2*a^3*d*g
^3*i + 2*a^2*b*c*g^3*i) + (d^2*atan((d^2*((a^3*d^3*g^3*i + b^3*c^3*g^3*i -
 a*b^2*c^2*d*g^3*i - a^2*b*c*d^2*g^3*i)/(a^2*d^2*g^3*i + b^2*c^2*g^3*i - 2
*a*b*c*d*g^3*i) + 2*b*d*x)*(A^2 + (7*B^2*n^2)/2 + 3*A*B*n)*(a^2*d^2*g^3*i
+ b^2*c^2*g^3*i - 2*a*b*c*d*g^3*i)*2i)/(g^3*i*(a*d - b*c)^3*(2*A^2*d^2 + 7
*B^2*d^2*n^2 + 6*A*B*d^2*n)))*(A^2 + (7*B^2*n^2)/2 + 3*A*B*n)*2i)/(g^3*i*(
a*d - b*c)^3) - (B^2*d^2*log(e*((a + b*x)/(c + d*x))^n)^3)/(3*g^3*i*n*(...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1609, normalized size of antiderivative = 4.36

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^3(ci + dix)}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x)
```

output

```
(i*(12*log(a + b*x)*a**5*d**2*n + 24*log(a + b*x)*a**4*b*d**2*n**2 + 24*lo
g(a + b*x)*a**4*b*d**2*n*x + 12*log(a + b*x)*a**3*b**2*c*d*n**2 + 24*log(a
 + b*x)*a**3*b**2*d**2*n**3 + 48*log(a + b*x)*a**3*b**2*d**2*n**2*x + 12*l
og(a + b*x)*a**3*b**2*d**2*n*x**2 + 18*log(a + b*x)*a**2*b**3*c*d*n**3 + 2
4*log(a + b*x)*a**2*b**3*c*d*n**2*x + 48*log(a + b*x)*a**2*b**3*d**2*n**3*
x + 24*log(a + b*x)*a**2*b**3*d**2*n**2*x**2 + 36*log(a + b*x)*a*b**4*c*d*
n**3*x + 12*log(a + b*x)*a*b**4*c*d*n**2*x**2 + 24*log(a + b*x)*a*b**4*d**
2*n**3*x**2 + 18*log(a + b*x)*b**5*c*d*n**3*x**2 - 12*log(c + d*x)*a**5*d*
*2*n - 24*log(c + d*x)*a**4*b*d**2*n**2 - 24*log(c + d*x)*a**4*b*d**2*n*x
- 12*log(c + d*x)*a**3*b**2*c*d*n**2 - 24*log(c + d*x)*a**3*b**2*d**2*n**3
 - 48*log(c + d*x)*a**3*b**2*d**2*n**2*x - 12*log(c + d*x)*a**3*b**2*d**2*
n*x**2 - 18*log(c + d*x)*a**2*b**3*c*d*n**3 - 24*log(c + d*x)*a**2*b**3*c*
d*n**2*x - 48*log(c + d*x)*a**2*b**3*d**2*n**3*x - 24*log(c + d*x)*a**2*b*
*3*d**2*n**2*x**2 - 36*log(c + d*x)*a*b**4*c*d*n**3*x - 12*log(c + d*x)*a*
b**4*c*d*n**2*x**2 - 24*log(c + d*x)*a*b**4*d**2*n**3*x**2 - 18*log(c + d*
x)*b**5*c*d*n**3*x**2 + 4*log(((a + b*x)**n*e)/(c + d*x)**n)**3*a**3*b**2*
d**2 + 8*log(((a + b*x)**n*e)/(c + d*x)**n)**3*a**2*b**3*d**2*x + 4*log(((
a + b*x)**n*e)/(c + d*x)**n)**3*a*b**4*d**2*x**2 + 12*log(((a + b*x)**n*e)
/(c + d*x)**n)**2*a**4*b*d**2 + 24*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a
**3*b**2*d**2*x + 24*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**3*c*...
```

3.193 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx$

Optimal result . . . 2226
Mathematica [B] (verified) . . . 2227
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Reduce [B] (verification not implemented) . . . 2235

Optimal result

Integrand size = 45, antiderivative size = 543

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = &-\frac{6bB^2d^2n^2(c+dx)}{(bc-ad)^4g^4i(a+bx)}+\frac{3b^2B^2dn^2(c+dx)^2}{4(bc-ad)^4g^4i(a+bx)^2}\\ &-\frac{2b^3B^2n^2(c+dx)^3}{27(bc-ad)^4g^4i(a+bx)^3}\\ &-\frac{6bBd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^4i(a+bx)}\\ &+\frac{3b^2Bdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2}\\ &-\frac{2b^3Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^4g^4i(a+bx)^3}\\ &-\frac{3bd^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^4i(a+bx)}\\ &+\frac{3b^2d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^4i(a+bx)^2}\\ &-\frac{b^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^4g^4i(a+bx)^3}\\ &-\frac{d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^4g^4in}\end{aligned}$$

output

```
-6*b*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/4*b^2*B^2*d*n^2*(d*x
+c)^2/(-a*d+b*c)^4/g^4/i/(b*x+a)^2-2/27*b^3*B^2*n^2*(d*x+c)^3/(-a*d+b*c)^4
/g^4/i/(b*x+a)^3-6*b*B*d^2*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d
+b*c)^4/g^4/i/(b*x+a)+3/2*b^2*B*d*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))/(-a*d+b*c)^4/g^4/i/(b*x+a)^2-2/9*b^3*B*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))/(-a*d+b*c)^4/g^4/i/(b*x+a)^3-3*b*d^2*(d*x+c)*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/2*b^2*d*(d*x+c)^2*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^4/i/(b*x+a)^2-1/3*b^3*(d*x+c)^3*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^4/i/(b*x+a)^3-1/3*d^3*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)^4/g^4/i/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1295 vs. $2(543) = 1086$.

Time $= 1.73$ (sec) , antiderivative size $= 1295$, normalized size of antiderivative $= 2.38$

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^4(ci + dix)}\, dx = \text{Too large to display}$$

input

```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i +
 d*i*x)),x]
```

output

```
-1/108*(36*B^2*d^3*n^2*(a + b*x)^3*Log[(a + b*x)/(c + d*x)]^3 + 18*B*n*Log
[(a + b*x)/(c + d*x)]^2*(6*a^3*A*d^3 + 2*b^3*B*c^3*n - 9*a*b^2*B*c^2*d*n +
 18*a^2*b*B*c*d^2*n + 18*a^2*A*b*d^3*x - 3*b^3*B*c^2*d*n*x + 18*a*b^2*B*c*
d^2*n*x + 18*a^2*b*B*d^3*n*x + 18*a*A*b^2*d^3*x^2 + 6*b^3*B*c*d^2*n*x^2 +
27*a*b^2*B*d^3*n*x^2 + 6*A*b^3*d^3*x^3 + 11*b^3*B*d^3*n*x^3 + 6*B*d^3*(a +
 b*x)^3*Log[e*((a + b*x)/(c + d*x))^n] - 6*B*d^3*n*(a + b*x)^3*Log[(a + b*
x)/(c + d*x)]) - 3*d*(b*c - a*d)^2*(a + b*x)*(18*A^2 + 30*A*B*n + 19*B^2*n
^2 + 18*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(6*A + 5*B*n)*Log[(a
 + b*x)/(c + d*x)] + 18*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*((a
 + b*x)/(c + d*x))^n]*(6*A + 5*B*n - 6*B*n*Log[(a + b*x)/(c + d*x)])) + 6*d
^2*(b*c - a*d)*(a + b*x)^2*(18*A^2 + 66*A*B*n + 85*B^2*n^2 + 18*B^2*Log[e*
((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(6*A + 11*B*n)*Log[(a + b*x)/(c + d*x)]
 + 18*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*((a + b*x)/(c + d*x))
^n]*(6*A + 11*B*n - 6*B*n*Log[(a + b*x)/(c + d*x)])) + 6*d^3*(a + b*x)^3*L
og[a + b*x]*(18*A^2 + 66*A*B*n + 85*B^2*n^2 + 18*B^2*Log[e*((a + b*x)/(c +
 d*x))^n]^2 - 6*B*n*(6*A + 11*B*n)*Log[(a + b*x)/(c + d*x)] + 18*B^2*n^2*L
og[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*((a + b*x)/(c + d*x))^n]*(6*A + 11*B
*n - 6*B*n*Log[(a + b*x)/(c + d*x)])) + 4*(b*c - a*d)^3*(9*A^2 + 6*A*B*n +
 2*B^2*n^2 + 9*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(3*A + B*n)*Lo
g[(a + b*x)/(c + d*x)] + 9*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log...
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^4(ci+dix)}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}\,d\frac{a+bx}{c+dx}}{g^4 i(bc-ad)^4}$$

$$\downarrow 2795$$

$$\int \left(\frac{b^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^4}{(a+bx)^4} - \frac{3b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^3}{(a+bx)^3} + \frac{3bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^2}{(a+bx)^2} - \frac{d^3\left(A+B\l}{g^4i(bc-ad)^4}$$

$$\downarrow 2009$$

$$-\frac{b^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3} - \frac{2b^3Bn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9(a+bx)^3} + \frac{3b^2d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2} + \frac{3b^2Bdn(c+}{}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i + d*i*x
)),x]
```

output
```
((-6*b*B^2*d^2*n^2*(c + d*x))/(a + b*x) + (3*b^2*B^2*d*n^2*(c + d*x)^2)/(4
*(a + b*x)^2) - (2*b^3*B^2*n^2*(c + d*x)^3)/(27*(a + b*x)^3) - (6*b*B*d^2*
n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (3*b^2*B*d
*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) - (
2*b^3*B*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(a + b*x)
^3) - (3*b*d^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*
x) + (3*b^2*d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a
+ b*x)^2) - (b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*
(a + b*x)^3) - (d^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*B*n))/((b
*c - a*d)^4*g^4*i)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2215 vs. 2(529) = 1058.

Time = 28.33 (sec) , antiderivative size = 2216, normalized size of antiderivative = 4.08

method	result	size
parallelrisch	Expression too large to display	2216

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x,method=_
RETURNVERBOSE)
```

output

```
-1/108*(-54*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^5*b^3*c^4*d*n+36*B^2*ln(e*
((b*x+a)/(d*x+c))^n)^3*a^8*c^2*d^3+108*A^2*ln(e*((b*x+a)/(d*x+c))^n)*a^8*c
^2*d^3+36*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^3*a^5*b^3*c^2*d^3+575*B^2*x^3*
a^5*b^3*c^2*d^3*n^3-648*B^2*x^3*a^4*b^4*c^3*d^2*n^3+81*B^2*x^3*a^3*b^5*c^4
*d*n^3-24*A*B*x^3*a^2*b^6*c^5*n^2+108*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*
a^6*b^2*c^2*d^3+1215*B^2*x^2*a^6*b^2*c^2*d^3*n^3-1434*B^2*x^2*a^5*b^3*c^3*
d^2*n^3+243*B^2*x^2*a^4*b^4*c^4*d*n^3+108*A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n
)*a^5*b^3*c^2*d^3+198*A^2*x^3*a^5*b^3*c^2*d^3*n-324*A^2*x^3*a^4*b^4*c^3*d^
2*n+162*A^2*x^3*a^3*b^5*c^4*d*n-72*A*B*x^2*a^3*b^5*c^5*n^2+108*B^2*x*ln(e*
((b*x+a)/(d*x+c))^n)^3*a^7*b*c^2*d^3+648*B^2*x*a^7*b*c^2*d^3*n^3-810*B^2*x
*a^6*b^2*c^3*d^2*n^3+186*B^2*x*a^5*b^3*c^4*d*n^3+324*A^2*x^2*ln(e*((b*x+a)
/(d*x+c))^n)*a^6*b^2*c^2*d^3+486*A^2*x^2*a^6*b^2*c^2*d^3*n-864*A^2*x^2*a^5
*b^3*c^3*d^2*n+486*A^2*x^2*a^4*b^4*c^4*d*n-72*A*B*x*a^4*b^4*c^5*n^2+324*B^
2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^7*b*c^3*d^2*n-162*B^2*ln(e*((b*x+a)/(d*x+c
))^n)^2*a^6*b^2*c^4*d*n+648*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^3*d^2*n^
2-162*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^4*d*n^2+324*A^2*x*ln(e*((b*x
+a)/(d*x+c))^n)*a^7*b*c^2*d^3+324*A^2*x*a^7*b*c^2*d^3*n-648*A^2*x*a^6*b^2*
c^3*d^2*n+432*A^2*x*a^5*b^3*c^4*d*n+72*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b
^3*c^5*n-90*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^3*c^4*d*n^2+324*A*B*x*ln
(e*((b*x+a)/(d*x+c))^n)^2*a^7*b*c^2*d^3+648*A*B*x*a^7*b*c^2*d^3*n^2-972...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1910 vs. $2(529) = 1058$.

Time $= 0.12$ (sec) , antiderivative size $= 1910$, normalized size of antiderivative $= 3.52$

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x,
algorithm="fricas")
```

output

```
-1/108*(36*A^2*b^3*c^3 - 162*A^2*a*b^2*c^2*d + 324*A^2*a^2*b*c*d^2 - 198*A
^2*a^3*d^3 + 36*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3*n^2*x^2 + 3*B^2*a^2
*b*d^3*n^2*x + B^2*a^3*d^3*n^2)*log((b*x + a)/(d*x + c))^3 + (8*B^2*b^3*c^
3 - 81*B^2*a*b^2*c^2*d + 648*B^2*a^2*b*c*d^2 - 575*B^2*a^3*d^3)*n^2 + 6*(1
8*A^2*b^3*c*d^2 - 18*A^2*a*b^2*d^3 + 85*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^
2 + 66*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*n)*x^2 + 18*(2*B^2*b^3*c^3 - 9*B^2*
a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2 - 11*B^2*a^3*d^3 + 6*(B^2*b^3*c*d^2 - B^2
*a*b^2*d^3)*x^2 - 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + 5*B^2*a^2*b*d^3)*
x + 6*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*a^3
*d^3)*log((b*x + a)/(d*x + c)))*log(e)^2 + 18*(6*A*B*a^3*d^3*n + (11*B^2*b
^3*d^3*n^2 + 6*A*B*b^3*d^3*n)*x^3 + (2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 1
8*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*B*a*b^2*d^3*n + (2*B^2*b^3*c*d^2 + 9*B^2*a
*b^2*d^3)*n^2)*x^2 + 3*(6*A*B*a^2*b*d^3*n - (B^2*b^3*c^2*d - 6*B^2*a*b^2*c
*d^2 - 6*B^2*a^2*b*d^3)*n^2)*x)*log((b*x + a)/(d*x + c))^2 + 6*(4*A*B*b^3*
c^3 - 27*A*B*a*b^2*c^2*d + 108*A*B*a^2*b*c*d^2 - 85*A*B*a^3*d^3)*n - 3*(18
*A^2*b^3*c^2*d - 108*A^2*a*b^2*c*d^2 + 90*A^2*a^2*b*d^3 + (19*B^2*b^3*c^2*
d - 378*B^2*a*b^2*c*d^2 + 359*B^2*a^2*b*d^3)*n^2 + 6*(5*A*B*b^3*c^2*d - 54
*A*B*a*b^2*c*d^2 + 49*A*B*a^2*b*d^3)*n)*x + 6*(12*A*B*b^3*c^3 - 54*A*B*a*b
^2*c^2*d + 108*A*B*a^2*b*c*d^2 - 66*A*B*a^3*d^3 + 6*(6*A*B*b^3*c*d^2 - 6*A
*B*a*b^2*d^3 + 11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Timed out}$$

input

```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4/(d*i*x+c*i),x
)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3445 vs. $2(529) = 1058$.

Time = 0.31 (sec) , antiderivative size = 3445, normalized size of antiderivative = 6.34

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x,
algorithm="maxima")
```

output

```
-1/6*B^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
 - 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3
)*g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d
^3)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b
*d^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*
g^4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
- 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x + c)/((b^4*c^4 - 4*a*b^3
*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i))*log(e*(b*x/(
d*x + c) + a/(d*x + c))^n)^2 - 1/3*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b
*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d +
 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d
 + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c
^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2
*d + 3*a^5*b*c*d^2 - a^6*d^3)*g^4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*
b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*lo
g(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
 a^4*d^4)*g^4*i))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/108*((8*b^3*c
^3 - 81*a*b^2*c^2*d + 648*a^2*b*c*d^2 - 575*a^3*d^3 + 36*(b^3*d^3*x^3 + 3*
a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^3 - 36*(b^3*d^3*x^3
+ 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^3 + 510*(b^3*...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x,
algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 31.81 (sec) , antiderivative size = 1921, normalized size of antiderivative = 3.54

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^4*(c*i + d*i*x
)),x)
```

output

```
((198*A^2*a^2*d^2 + 36*A^2*b^2*c^2 + 575*B^2*a^2*d^2*n^2 + 8*B^2*b^2*c^2*n
^2 - 126*A^2*a*b*c*d + 510*A*B*a^2*d^2*n + 24*A*B*b^2*c^2*n - 73*B^2*a*b*c
*d*n^2 - 138*A*B*a*b*c*d*n)/(6*(a*d - b*c)) + (x^2*(18*A^2*b^2*d^2 + 85*B^
2*b^2*d^2*n^2 + 66*A*B*b^2*d^2*n))/(a*d - b*c) + (x*(90*A^2*a*b*d^2 - 18*A
^2*b^2*c*d + 359*B^2*a*b*d^2*n^2 - 19*B^2*b^2*c*d*n^2 + 294*A*B*a*b*d^2*n
- 30*A*B*b^2*c*d*n))/(2*(a*d - b*c)))/(x*(54*a^4*b*d^2*g^4*i + 54*a^2*b^3*
c^2*g^4*i - 108*a^3*b^2*c*d*g^4*i) + x^2*(54*a*b^4*c^2*g^4*i + 54*a^3*b^2*
d^2*g^4*i - 108*a^2*b^3*c*d*g^4*i) + x^3*(18*b^5*c^2*g^4*i + 18*a^2*b^3*d^
2*g^4*i - 36*a*b^4*c*d*g^4*i) + 18*a^5*d^2*g^4*i + 18*a^3*b^2*c^2*g^4*i -
36*a^4*b*c*d*g^4*i) - log(e*((a + b*x)/(c + d*x))^n)^2*((d^3*(11*B^2*n + 6
*A*B))/(6*g^4*i*n*(a*d - b*c)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2)) - (B^2*d^3*(x*(b*((g^4*i*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2) + (
a*g^4*i*n*(a*d - b*c))/(3*d)) + (2*a*b*g^4*i*n*(a*d - b*c))/(3*d) + (b*g^4
*i*n*(a*d - b*c)*(3*a*d - b*c))/(3*d^2)) + a*((g^4*i*n*(a*d - b*c)*(3*a*d
- b*c))/(6*d^2) + (a*g^4*i*n*(a*d - b*c))/(3*d)) + (g^4*i*n*(a*d - b*c)*(3
*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/(3*d^3) + (b^2*g^4*i*n*x^2*(a*d - b*c))/d
))/(g^4*i*n*(a*d - b*c)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
)*(a^3*g^4*i + b^3*g^4*i*x^3 + 3*a^2*b*g^4*i*x + 3*a*b^2*g^4*i*x^2))) - lo
g(e*((a + b*x)/(c + d*x))^n)*((6*B^2*a*d*n - 3*B^2*b*c*n + 3*B^2*b*d*n*x)/
(x*(9*a^4*b*d^2*g^4*i + 9*a^2*b^3*c^2*g^4*i - 18*a^3*b^2*c*d*g^4*i) + x...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2656, normalized size of antiderivative = 4.89

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^4(ci + dix)}\, dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x)
```

output

```
(i*(108*log(a + b*x)*a**6*d**3*n + 324*log(a + b*x)*a**5*b*d**3*n**2 + 324
*log(a + b*x)*a**5*b*d**3*n*x + 72*log(a + b*x)*a**4*b**2*c*d**2*n**2 + 37
8*log(a + b*x)*a**4*b**2*d**3*n**3 + 972*log(a + b*x)*a**4*b**2*d**3*n**2*
x + 324*log(a + b*x)*a**4*b**2*d**3*n*x**2 + 132*log(a + b*x)*a**3*b**3*c*
d**2*n**3 + 216*log(a + b*x)*a**3*b**3*c*d**2*n**2*x + 1134*log(a + b*x)*a
**3*b**3*d**3*n**3*x + 972*log(a + b*x)*a**3*b**3*d**3*n**2*x**2 + 108*log
(a + b*x)*a**3*b**3*d**3*n*x**3 + 396*log(a + b*x)*a**2*b**4*c*d**2*n**3*x
 + 216*log(a + b*x)*a**2*b**4*c*d**2*n**2*x**2 + 1134*log(a + b*x)*a**2*b*
*4*d**3*n**3*x**2 + 324*log(a + b*x)*a**2*b**4*d**3*n**2*x**3 + 396*log(a
+ b*x)*a*b**5*c*d**2*n**3*x**2 + 72*log(a + b*x)*a*b**5*c*d**2*n**2*x**3 +
 378*log(a + b*x)*a*b**5*d**3*n**3*x**3 + 132*log(a + b*x)*b**6*c*d**2*n**
3*x**3 - 108*log(c + d*x)*a**6*d**3*n - 324*log(c + d*x)*a**5*b*d**3*n**2
- 324*log(c + d*x)*a**5*b*d**3*n*x - 72*log(c + d*x)*a**4*b**2*c*d**2*n**2
 - 378*log(c + d*x)*a**4*b**2*d**3*n**3 - 972*log(c + d*x)*a**4*b**2*d**3*
n**2*x - 324*log(c + d*x)*a**4*b**2*d**3*n*x**2 - 132*log(c + d*x)*a**3*b*
*3*c*d**2*n**3 - 216*log(c + d*x)*a**3*b**3*c*d**2*n**2*x - 1134*log(c + d
*x)*a**3*b**3*d**3*n**3*x - 972*log(c + d*x)*a**3*b**3*d**3*n**2*x**2 - 10
8*log(c + d*x)*a**3*b**3*d**3*n*x**3 - 396*log(c + d*x)*a**2*b**4*c*d**2*n
**3*x - 216*log(c + d*x)*a**2*b**4*c*d**2*n**2*x**2 - 1134*log(c + d*x)*a*
*2*b**4*d**3*n**3*x**2 - 324*log(c + d*x)*a**2*b**4*d**3*n**2*x**3 - 39...
```

3.194 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$

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Mathematica [B] (verified) . . . 2239
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Maple [F] . . . 2242
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Optimal result

Integrand size = 45, antiderivative size = 770

$$
\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx \\
&= \frac{2AB(bc-ad)^2g^3n(a+bx)}{d^3i^2(c+dx)} - \frac{2B^2(bc-ad)^2g^3n^2(a+bx)}{d^3i^2(c+dx)} \\
&\quad + \frac{2B^2(bc-ad)^2g^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^2(c+dx)} \\
&\quad - \frac{bB(bc-ad)g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d^3i^2} \\
&\quad - \frac{3b(bc-ad)g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^2} \\
&\quad - \frac{(bc-ad)^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^2(c+dx)} \\
&\quad + \frac{b^3g^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^4i^2} \\
&\quad - \frac{6bB(bc-ad)^2g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^2} \\
&\quad - \frac{3b(bc-ad)^2g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^2} \\
&\quad + \frac{bB^2(bc-ad)^2g^3n^2\log(c+dx)}{d^4i^2} \\
&\quad + \frac{bB(bc-ad)^2g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2} \\
&\quad - \frac{6bB^2(bc-ad)^2g^3n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2} \\
&\quad - \frac{6bB(bc-ad)^2g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2} \\
&\quad - \frac{bB^2(bc-ad)^2g^3n^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2} \\
&\quad + \frac{6bB^2(bc-ad)^2g^3n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}
\end{aligned}
$$

output

```
2*A*B*(-a*d+b*c)^2*g^3*n*(b*x+a)/d^3/i^2/(d*x+c)-2*B^2*(-a*d+b*c)^2*g^3*n^
2*(b*x+a)/d^3/i^2/(d*x+c)+2*B^2*(-a*d+b*c)^2*g^3*n*(b*x+a)*ln(e*((b*x+a)/(
d*x+c))^n)/d^3/i^2/(d*x+c)-b*B*(-a*d+b*c)*g^3*n*(b*x+a)*(A+B*ln(e*((b*x+a)
/(d*x+c))^n))/d^3/i^2-3*b*(-a*d+b*c)*g^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))^2/d^3/i^2-(-a*d+b*c)^2*g^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^
2/d^3/i^2/(d*x+c)+1/2*b^3*g^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/
d^4/i^2-6*b*B*(-a*d+b*c)^2*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+
b*c)/b/(d*x+c))/d^4/i^2-3*b*(-a*d+b*c)^2*g^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n
))^2*ln((-a*d+b*c)/b/(d*x+c))/d^4/i^2+b*B^2*(-a*d+b*c)^2*g^3*n^2*ln(d*x+c)
/d^4/i^2+b*B*(-a*d+b*c)^2*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*
x+c)/d/(b*x+a))/d^4/i^2-6*b*B^2*(-a*d+b*c)^2*g^3*n^2*polylog(2,d*(b*x+a)/b
/(d*x+c))/d^4/i^2-6*b*B*(-a*d+b*c)^2*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^2-b*B^2*(-a*d+b*c)^2*g^3*n^2*polylog
(2,b*(d*x+c)/d/(b*x+a))/d^4/i^2+6*b*B^2*(-a*d+b*c)^2*g^3*n^2*polylog(3,d*(
b*x+a)/b/(d*x+c))/d^4/i^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5850 vs. $2(770) = 1540$.

Time $= 7.73$ (sec) , antiderivative size $= 5850$, normalized size of antiderivative $= 7.60$

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \text{Result too large to show}$$

input

```
Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i
+ d*i*x)^2,x]
```

output

```
Result too large to show
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 640, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ci+dix)^2}\,dx$$

$$\downarrow 2961$$

$$\frac{g^3(bc-ad)^2\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{i^2}$$

$$\downarrow 2795$$

$$\frac{g^3(bc-ad)^2\int \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.^n}{d^3}\right.}{i^2}$$

$$\downarrow 2009$$

$$g^3(bc-ad)^2\left(\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{6bBn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^4}-\frac{3b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.}{d^4}\right.$$

input

```
Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*
x)^2,x]
```

output
```
((b*c - a*d)^2*g^3*((2*A*B*n*(a + b*x))/(d^3*(c + d*x)) - (2*B^2*n^2*(a +
b*x))/(d^3*(c + d*x)) + (2*B^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])
/(d^3*(c + d*x)) - (b*B*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
)/(d^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - ((a + b*x)*(A + B*Log[e*
((a + b*x)/(c + d*x))^n])^2)/(d^3*(c + d*x)) + (b^3*(A + B*Log[e*((a + b*x
)/(c + d*x))^n])^2)/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) - (3*b*(a + b*
x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^3*(c + d*x)*(b - (d*(a + b
*x))/(c + d*x))) - (b*B^2*n^2*Log[b - (d*(a + b*x))/(c + d*x)])/d^4 - (6*b
*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c +
d*x))])/d^4 - (3*b*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a
+ b*x))/(b*(c + d*x))])/d^4 + (b*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/d^4 - (6*b*B^2*n^2*PolyLog[2, (d*(
a + b*x))/(b*(c + d*x))])/d^4 - (6*b*B*n*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^4 - (b*B^2*n^2*PolyLog[2,
 (b*(c + d*x))/(d*(a + b*x))])/d^4 + (6*b*B^2*n^2*PolyLog[3, (d*(a + b*x))
/(b*(c + d*x))])/d^4))/i^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(dix+ci)^2}dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}dx = \int \frac{(bgx+ag)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x
, algorithm="fricas")
```

output
```
integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*
a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2
*a^3*g^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*
b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log(e*((b*x + a)/(d*x + c))
^n))/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**
2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x
, algorithm="maxima")
```

output
```
2*A*B*a^3*g^3*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)
*i^2) - b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) + 1/2*(2*c^3/(d^5*i^2*x + c*
d^4*i^2) + 6*c^2*log(d*x + c)/(d^4*i^2) + (d*x^2 - 4*c*x)/(d^3*i^2))*A^2*b
^3*g^3 - 3*A^2*a*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2) - x/(d^2*i^2) + 2*c*log(
d*x + c)/(d^3*i^2))*g^3 + 3*A^2*a^2*b*g^3*(c/(d^3*i^2*x + c*d^2*i^2) + log
(d*x + c)/(d^2*i^2)) - 2*A*B*a^3*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
)/(d^2*i^2*x + c*d*i^2) - A^2*a^3*g^3/(d^2*i^2*x + c*d*i^2) + 1/2*(B^2*b^3
*d^3*g^3*x^3 - 3*(b^3*c*d^2*g^3 - 2*a*b^2*d^3*g^3)*B^2*x^2 - 2*(2*b^3*c^2*
d*g^3 - 3*a*b^2*c*d^2*g^3)*B^2*x + 2*(b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*
a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*B^2 + 6*((b^3*c^2*d*g^3 - 2*a*b^2*c*d^2*g^3
 + a^2*b*d^3*g^3)*B^2*x + (b^3*c^3*g^3 - 2*a*b^2*c^2*d*g^3 + a^2*b*c*d^2*g
^3)*B^2)*log(d*x + c))*log((d*x + c)^n)^2/(d^5*i^2*x + c*d^4*i^2) - integr
ate(-(B^2*a^3*d^3*g^3*log(e)^2 + (B^2*b^3*d^3*g^3*log(e)^2 + 2*A*B*b^3*d^3
*g^3*log(e))*x^3 + 3*(B^2*a*b^2*d^3*g^3*log(e)^2 + 2*A*B*a*b^2*d^3*g^3*log
(e))*x^2 + (B^2*b^3*d^3*g^3*x^3 + 3*B^2*a*b^2*d^3*g^3*x^2 + 3*B^2*a^2*b*d^
3*g^3*x + B^2*a^3*d^3*g^3)*log((b*x + a)^n)^2 + 3*(B^2*a^2*b*d^3*g^3*log(e
)^2 + 2*A*B*a^2*b*d^3*g^3*log(e))*x + 2*(B^2*a^3*d^3*g^3*log(e) + (B^2*b^3
*d^3*g^3*log(e) + A*B*b^3*d^3*g^3)*x^3 + 3*(B^2*a*b^2*d^3*g^3*log(e) + A*B
*a*b^2*d^3*g^3)*x^2 + 3*(B^2*a^2*b*d^3*g^3*log(e) + A*B*a^2*b*d^3*g^3)*x)*
log((b*x + a)^n) - ((2*A*B*b^3*d^3*g^3 + (g^3*n + 2*g^3*log(e))*B^2*b^3...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x
, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(c\,i+d\,i\,x)^2}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^2,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^2, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

output

```
(g**3*( - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(c**2 + 2*c*d
*x + d**2*x**2),x)*a*b**5*c**2*d**5 - 2*int((log(((a + b*x)**n*e)/(c + d*x
)**n)**2*x**3)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**5*c*d**6*x + 2*int((lo
g(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(c**2 + 2*c*d*x + d**2*x**2),x)*
b**6*c**3*d**4 + 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(c**2
+ 2*c*d*x + d**2*x**2),x)*b**6*c**2*d**5*x - 6*int((log(((a + b*x)**n*e)/(
c + d*x)**n)**2*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b**4*c**2*d**5
- 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(c**2 + 2*c*d*x + d**
2*x**2),x)*a**2*b**4*c*d**6*x + 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)*
*2*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**5*c**3*d**4 + 6*int((log(((a
 + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**
5*c**2*d**5*x - 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(c**2 + 2*
c*d*x + d**2*x**2),x)*a**3*b**3*c**2*d**5 - 6*int((log(((a + b*x)**n*e)/(c
 + d*x)**n)**2*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**3*b**3*c*d**6*x + 6*i
nt((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(c**2 + 2*c*d*x + d**2*x**2),
x)*a**2*b**4*c**3*d**4 + 6*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(
c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b**4*c**2*d**5*x - 4*int((log(((a + b*
x)**n*e)/(c + d*x)**n)*x**3)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b**4*c**
2*d**5 - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**3)/(c**2 + 2*c*d*x +
 d**2*x**2),x)*a**2*b**4*c*d**6*x + 4*int((log(((a + b*x)**n*e)/(c + d*...
```

3.195 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$

Optimal result . . . 2246
Mathematica [B] (verified) . . . 2247
Rubi [A] (verified) . . . 2248
Maple [F] . . . 2250
Fricas [F] . . . 2250
Sympy [F(-1)] . . . 2251
Maxima [F] . . . 2251
Giac [F] . . . 2252
Mupad [F(-1)] . . . 2253
Reduce [F] . . . 2253

Optimal result

Integrand size = 45, antiderivative size = 500

$$
\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx \\
&= -\frac{2AB(bc-ad)g^2n(a+bx)}{d^2i^2(c+dx)} + \frac{2B^2(bc-ad)g^2n^2(a+bx)}{d^2i^2(c+dx)} \\
&\quad - \frac{2B^2(bc-ad)g^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^2(c+dx)} + \frac{bg^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^2} \\
&\quad + \frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^2(c+dx)} \\
&\quad + \frac{2bB(bc-ad)g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^2} \\
&\quad + \frac{2b(bc-ad)g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^2} \\
&\quad + \frac{2bB^2(bc-ad)g^2n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} \\
&\quad + \frac{4bB(bc-ad)g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} \\
&\quad - \frac{4bB^2(bc-ad)g^2n^2\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}
\end{aligned}
$$

output

```
-2*A*B*(-a*d+b*c)*g^2*n*(b*x+a)/d^2/i^2/(d*x+c)+2*B^2*(-a*d+b*c)*g^2*n^2*(
b*x+a)/d^2/i^2/(d*x+c)-2*B^2*(-a*d+b*c)*g^2*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c
))^n)/d^2/i^2/(d*x+c)+b*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^2/
i^2+(-a*d+b*c)*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^2/i^2/(d*x+
c)+2*b*B*(-a*d+b*c)*g^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/
(d*x+c))/d^3/i^2+2*b*(-a*d+b*c)*g^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((
-a*d+b*c)/b/(d*x+c))/d^3/i^2+2*b*B^2*(-a*d+b*c)*g^2*n^2*polylog(2,d*(b*x+a
)/b/(d*x+c))/d^3/i^2+4*b*B*(-a*d+b*c)*g^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^2-4*b*B^2*(-a*d+b*c)*g^2*n^2*polylo
g(3,d*(b*x+a)/b/(d*x+c))/d^3/i^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3522 vs. $2(500) = 1000$.

Time $= 7.26$ (sec) , antiderivative size $= 3522$, normalized size of antiderivative $= 7.04$

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \text{Result too large to show}$$

input

```
Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i
+ d*i*x)^2,x]
```

output

```
(b^2*g^2*x*(A + B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d
*x)]))^2)/(d^2*i^2) + (-(A^2*b^2*c^2*g^2) + 2*a*A^2*b*c*d*g^2 - a^2*A^2*d^
2*g^2 - 2*A*b^2*B*c^2*g^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x
)/(c + d*x)]) + 4*a*A*b*B*c*d*g^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[
(a + b*x)/(c + d*x)]) - 2*a^2*A*B*d^2*g^2*(Log[e*((a + b*x)/(c + d*x))^n]
 - n*Log[(a + b*x)/(c + d*x)]) - b^2*B^2*c^2*g^2*(Log[e*((a + b*x)/(c + d*x
))^n] - n*Log[(a + b*x)/(c + d*x)])^2 + 2*a*b*B^2*c*d*g^2*(Log[e*((a + b*x
)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2 - a^2*B^2*d^2*g^2*(Log[e*(
(a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2)/(d^3*i^2*(c + d*x
)) - (2*b*(b*c - a*d)*g^2*(A + B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(
a + b*x)/(c + d*x)]))^2*Log[c + d*x])/(d^3*i^2) + (2*a^2*B*g^2*n*(A + B*(L
og[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]))*(((c/d + x)*(
Log[c/d + x] + Log[c/d + x]^2))/((c + d*x)^2*Log[c/d + x]) + ((d*(a/b + x)
*Log[a/b + x])/((-c + (a*d)/b)^2*(1 - (d*(a/b + x))/(-c + (a*d)/b))) + Log
[1 - (d*(a/b + x))/(-c + (a*d)/b)]/(-c + (a*d)/b))/d - (-Log[a/b + x] + Lo
g[c/d + x] + Log[a/(c + d*x) + (b*x)/(c + d*x)])/(d*(c + d*x))))/i^2 + (2*
b^2*B*g^2*n*(A + B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c +
d*x)]))*(((a/b + x)*(-1 + Log[a/b + x]))/d^2 - ((c/d + x)*(-1 + Log[c/d +
x]))/d^2 + (c*Log[c/d + x]^2)/d^3 + (c^2*(1 + Log[c/d + x]))/(d^3*(c + d*x
)) + (c^2*(-(Log[a/b + x]/(d*(c + d*x))) + ((b*Log[a + b*x])/(b*c - a*d...
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ci+dix)^2}\,dx$$

$$\downarrow 2961$$

$$\frac{g^2(bc-ad)\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i^2}$$

$$\downarrow 2795$$

$$\frac{g^2(bc-ad)\int\left(\frac{2b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)}+\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2}+\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)^2}\right)d\frac{a+bx}{c+dx}}{i^2}$$

$$\downarrow \text{2009}$$

$$\frac{g^2(bc-ad)\left(\frac{4bBn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3}+\frac{2bBn\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3}+\frac{2b\log\left(1-\frac{d(a+bx}{b(c+dx}\right.\right.}{}$$

input

```
Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*
x)^2,x]
```

output

```
((b*c - a*d)*g^2*((-2*A*B*n*(a + b*x))/(d^2*(c + d*x)) + (2*B^2*n^2*(a + b
*x))/(d^2*(c + d*x)) - (2*B^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/
(d^2*(c + d*x)) + ((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^
2*(c + d*x)) + (b*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^2
*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (2*b*B*n*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 + (2*b*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^
3 + (2*b*B^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^3 + (4*b*B*n*(
A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x
))])/d^3 - (4*b*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/d^3))/i^2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(dix+ci)^2}dx$$

input

```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

output

```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x
, algorithm="fricas")
```

output

```
integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x
^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(
A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log(e*((b*x + a)/(d*x + c
))^n))/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x, algorithm="maxima")
```

output

```
2*A*B*a^2*g^2*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)
*i^2) - b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) - A^2*b^2*(c^2/(d^4*i^2*x +
c*d^3*i^2) - x/(d^2*i^2) + 2*c*log(d*x + c)/(d^3*i^2))*g^2 + 2*A^2*a*b*g^2
*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2)) - 2*A*B*a^2*g^2*log(
e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^2*i^2*x + c*d*i^2) - A^2*a^2*g^2/(d^
2*i^2*x + c*d*i^2) + (B^2*b^2*d^2*g^2*x^2 + B^2*b^2*c*d*g^2*x - (b^2*c^2*g
^2 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)*B^2 - 2*((b^2*c*d*g^2 - a*b*d^2*g^2)*B^2
*x + (b^2*c^2*g^2 - a*b*c*d*g^2)*B^2)*log(d*x + c))*log((d*x + c)^n)^2/(d^
4*i^2*x + c*d^3*i^2) - integrate(-(B^2*a^2*d^2*g^2*log(e)^2 + (B^2*b^2*d^2
*g^2*log(e)^2 + 2*A*B*b^2*d^2*g^2*log(e))*x^2 + (B^2*b^2*d^2*g^2*x^2 + 2*B
^2*a*b*d^2*g^2*x + B^2*a^2*d^2*g^2)*log((b*x + a)^n)^2 + 2*(B^2*a*b*d^2*g^
2*log(e)^2 + 2*A*B*a*b*d^2*g^2*log(e))*x + 2*(B^2*a^2*d^2*g^2*log(e) + (B^
2*b^2*d^2*g^2*log(e) + A*B*b^2*d^2*g^2)*x^2 + 2*(B^2*a*b*d^2*g^2*log(e) +
A*B*a*b*d^2*g^2)*x)*log((b*x + a)^n) + 2*((b^2*c^2*g^2*n - 2*a*b*c*d*g^2*n
 + (g^2*n - g^2*log(e))*a^2*d^2)*B^2 - (A*B*b^2*d^2*g^2 + (g^2*n + g^2*log
(e))*B^2*b^2*d^2)*x^2 - (2*A*B*a*b*d^2*g^2 + (b^2*c*d*g^2*n + 2*a*b*d^2*g^
2*log(e))*B^2)*x + 2*((b^2*c*d*g^2*n - a*b*d^2*g^2*n)*B^2*x + (b^2*c^2*g^2
*n - a*b*c*d*g^2*n)*B^2)*log(d*x + c) - (B^2*b^2*d^2*g^2*x^2 + 2*B^2*a*b*d
^2*g^2*x + B^2*a^2*d^2*g^2)*log((b*x + a)^n))*log((d*x + c)^n))/(d^4*i^2*x
^2 + 2*c*d^3*i^2*x + c^2*d^2*i^2), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x
, algorithm="giac")
```

output

```
integrate((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x
+ c*i)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$$
$$=\int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(c\,i+d\,i\,x)^2}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^2,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^2, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

output

```
(g**2*( - int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(c**2 + 2*c*d*x
 + d**2*x**2),x)*a*b**4*c**2*d**4 - int((log(((a + b*x)**n*e)/(c + d*x)**n
)**2*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**4*c*d**5*x + int((log(((a
+ b*x)**n*e)/(c + d*x)**n)**2*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**5*c
**3*d**3 + int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(c**2 + 2*c*d*
x + d**2*x**2),x)*b**5*c**2*d**4*x - 2*int((log(((a + b*x)**n*e)/(c + d*x)
**n)**2*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b**3*c**2*d**4 - 2*int((lo
g(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**
2*b**3*c*d**5*x + 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(c**2 +
2*c*d*x + d**2*x**2),x)*a*b**4*c**3*d**3 + 2*int((log(((a + b*x)**n*e)/(c
+ d*x)**n)**2*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**4*c**2*d**4*x - 2*in
t((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x
)*a**2*b**3*c**2*d**4 - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c
**2 + 2*c*d*x + d**2*x**2),x)*a**2*b**3*c*d**5*x + 2*int((log(((a + b*x)**
n*e)/(c + d*x)**n)*x**2)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**4*c**3*d**3
+ 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c**2 + 2*c*d*x + d**2*x
**2),x)*a*b**4*c**2*d**4*x - 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/
(c**2 + 2*c*d*x + d**2*x**2),x)*a**3*b**2*c**2*d**4 - 4*int((log(((a + b*x
)**n*e)/(c + d*x)**n)*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**3*b**2*c*d**5*
x + 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c**2 + 2*c*d*x + d**2...
```

3.196 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$

Optimal result . . . 2255
Mathematica [B] (verified) . . . 2256
Rubi [A] (verified) . . . 2257
Maple [F] . . . 2258
Fricas [F] . . . 2258
Sympy [F(-1)] . . . 2259
Maxima [F] . . . 2259
Giac [F] . . . 2260
Mupad [F(-1)] . . . 2260
Reduce [F] . . . 2261

Optimal result

Integrand size = 43, antiderivative size = 282

$$\begin{aligned}
&\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx \\
&= \frac{2ABgn(a+bx)}{di^2(c+dx)} - \frac{2B^2gn^2(a+bx)}{di^2(c+dx)} + \frac{2B^2gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{di^2(c+dx)} \\
&\quad - \frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{di^2(c+dx)} - \frac{bg\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i^2} \\
&\quad - \frac{2bBgn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2} \\
&\quad + \frac{2bB^2gn^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}
\end{aligned}$$

output

```
2*A*B*g*n*(b*x+a)/d/i^2/(d*x+c)-2*B^2*g*n^2*(b*x+a)/d/i^2/(d*x+c)+2*B^2*g*
n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/d/i^2/(d*x+c)-g*(b*x+a)*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))^2/d/i^2/(d*x+c)-b*g*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln
((-a*d+b*c)/b/(d*x+c))/d^2/i^2-2*b*B*g*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*p
olylog(2,d*(b*x+a)/b/(d*x+c))/d^2/i^2+2*b*B^2*g*n^2*polylog(3,d*(b*x+a)/b/
(d*x+c))/d^2/i^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1570 vs. $2(282) = 564$.

Time = 1.84 (sec) , antiderivative size = 1570, normalized size of antiderivative = 5.57

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i +
d*i*x)^2,x]
```

output

```
(g*((3*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*
x)/(c + d*x)])^2)/(c + d*x) + 3*b*(A + B*Log[e*((a + b*x)/(c + d*x))^n] -
B*n*Log[(a + b*x)/(c + d*x)])^2*Log[c + d*x] + (6*a*B*d*n*(-A - B*Log[e*((
a + b*x)/(c + d*x))^n] + B*n*Log[(a + b*x)/(c + d*x)])*(b*c - a*d + b*(c +
 d*x)*Log[a/b + x] + (-(b*c) + a*d)*Log[(a + b*x)/(c + d*x)] - b*c*Log[(b*
(c + d*x))/(b*c - a*d)] - b*d*x*Log[(b*(c + d*x))/(b*c - a*d)]))/((-(b*c)
+ a*d)*(c + d*x)) + 3*b*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Lo
g[(a + b*x)/(c + d*x)])*(-Log[c/d + x]^2 + 2*Log[c/d + x]*Log[c + d*x] + 2
*(-(c/(c + d*x)) + (b*c*Log[a + b*x])/(-(b*c) + a*d) + (b*c*Log[c + d*x])/
(b*c - a*d) - Log[a/b + x]*Log[c + d*x] + Log[(a + b*x)/(c + d*x)]*(c/(c +
 d*x) + Log[c + d*x]) + Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)]) + 2*P
olyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - (3*a*B^2*d*n^2*(2*b*c - 2*a*d +
 2*b*(c + d*x)*Log[a + b*x] - 2*(b*c - a*d)*Log[(a + b*x)/(c + d*x)] - 2*b
*(c + d*x)*Log[a + b*x]*Log[(a + b*x)/(c + d*x)] + (b*c - a*d)*Log[(a + b*
x)/(c + d*x)]^2 - 2*b*(c + d*x)*Log[c + d*x] - 2*b*(c + d*x)*Log[(a + b*x)
/(c + d*x)]*Log[(b*c - a*d)/(b*c + b*d*x)] + b*(c + d*x)*(Log[a + b*x]*(Lo
g[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x)
)/(-(b*c) + a*d)]) + b*(c + d*x)*(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d
*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*PolyLog[
2, (b*(c + d*x))/(b*c - a*d)])))/((b*c - a*d)*(c + d*x)) + (b*B^2*n^2*(...
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ci+dix)^2}\,dx$$

$$\downarrow 2961$$

$$\frac{g\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{i^2}$$

$$\downarrow 2795$$

$$\frac{g\int\left(-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d}-\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{i^2}$$

$$\downarrow 2009$$

$$\frac{g\left(-\frac{2bBn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^2}-\frac{b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^2}-\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right.}{d(c+dx)}\right.}{i^2}$$

input

```
Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*x)
^2,x]
```

output

```
(g*((2*A*B*n*(a + b*x))/(d*(c + d*x)) - (2*B^2*n^2*(a + b*x))/(d*(c + d*x)
) + (2*B^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d*(c + d*x)) - ((a
 + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d*(c + d*x)) - (b*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/
d^2 - (2*b*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b
*x))/(b*(c + d*x))])/d^2 + (2*b*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d
*x))])/d^2))/i^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(dix+ci)^2}dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}dx = \int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x,
algorithm="fricas")
```

output
```
integral((A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log(e*((b*x + a)/(d*
x + c))^n)^2 + 2*(A*B*b*g*x + A*B*a*g)*log(e*((b*x + a)/(d*x + c))^n))/(d^
2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**2,x
)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x,
algorithm="maxima")
```

output
```
2*A*B*a*g*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2
) - b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) + A^2*b*g*(c/(d^3*i^2*x + c*d^2*
i^2) + log(d*x + c)/(d^2*i^2)) - 2*A*B*a*g*log(e*(b*x/(d*x + c) + a/(d*x +
 c))^n)/(d^2*i^2*x + c*d*i^2) - A^2*a*g/(d^2*i^2*x + c*d*i^2) + ((b*c*g -
a*d*g)*B^2 + (B^2*b*d*g*x + B^2*b*c*g)*log(d*x + c))*log((d*x + c)^n)^2/(d
^3*i^2*x + c*d^2*i^2) - integrate(-(B^2*a*d*g*log(e)^2 + (B^2*b*d*g*x + B^
2*a*d*g)*log((b*x + a)^n)^2 + (B^2*b*d*g*log(e)^2 + 2*A*B*b*d*g*log(e))*x
+ 2*(B^2*a*d*g*log(e) + (B^2*b*d*g*log(e) + A*B*b*d*g)*x)*log((b*x + a)^n)
 - 2*((b*c*g*n - (g*n - g*log(e))*a*d)*B^2 + (B^2*b*d*g*log(e) + A*B*b*d*g
)*x + (B^2*b*d*g*n*x + B^2*b*c*g*n)*log(d*x + c) + (B^2*b*d*g*x + B^2*a*d*
g)*log((b*x + a)^n))*log((d*x + c)^n))/(d^3*i^2*x^2 + 2*c*d^2*i^2*x + c^2*
d*i^2), x)
```

Giac [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x,
algorithm="giac")
```

output
```
integrate((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x +
c*i)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(c\,i+d\,i\,x)^2}\,dx$$

input
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x)
^2,x)
```

output
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x)
^2, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

output
```
(g*( - int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(c**2 + 2*c*d*x + d**
2*x**2),x)*a*b**3*c**2*d**3 - int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x
)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b**3*c*d**4*x + int((log(((a + b*x)**n
*e)/(c + d*x)**n)**2*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**4*c**3*d**2 + i
nt((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x)/(c**2 + 2*c*d*x + d**2*x**2),
x)*b**4*c**2*d**3*x - 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c**2 +
 2*c*d*x + d**2*x**2),x)*a**2*b**2*c**2*d**3 - 2*int((log(((a + b*x)**n*e)
/(c + d*x)**n)*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b**2*c*d**4*x + 2*i
nt((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*
a*b**3*c**3*d**2 + 2*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x)/(c**2 + 2*
c*d*x + d**2*x**2),x)*a*b**3*c**2*d**3*x + 2*log(a + b*x)*a**3*b*c*d**2*n
+ 2*log(a + b*x)*a**3*b*d**3*n*x - 2*log(a + b*x)*a**2*b**2*c*d**2*n**2 -
2*log(a + b*x)*a**2*b**2*d**3*n**2*x - log(c + d*x)*a**3*b*c**2*d - 2*log(
c + d*x)*a**3*b*c*d**2*n - log(c + d*x)*a**3*b*c*d**2*x - 2*log(c + d*x)*a
**3*b*d**3*n*x + log(c + d*x)*a**2*b**2*c**3 + log(c + d*x)*a**2*b**2*c**2
*d*x + 2*log(c + d*x)*a**2*b**2*c*d**2*n**2 + 2*log(c + d*x)*a**2*b**2*d**
3*n**2*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*c*d**2 + log(((
a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3*c*d**2*x - 2*log(((a + b*x)**n*e)/(
c + d*x)**n)*a**3*b*d**3*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**
2*c*d**2*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*d**3*n*x - ...
```

3.197 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$

Optimal result . . . 2262
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Rubi [A] (verified) . . . 2263
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Optimal result

Integrand size = 35, antiderivative size = 163

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = -\frac{2ABn(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{2B^2n^2(a+bx)}{(bc-ad)i^2(c+dx)} - \frac{2B^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)i^2(c+dx)} + \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(c+dx)}$$

output

```
-2*A*B*n*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)+2*B^2*n^2*(b*x+a)/(-a*d+b*c)/i^2/(
d*x+c)-2*B^2*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/i^2/(d*x+c)+(b
*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/i^2/(d*x+c)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.43 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.03

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$$

$$=\frac{-\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2b(c+dx)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2b(c+dx)\left(A+\right.\right.}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x)^2,x]
```

output
```
(-(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]) + 2*b*(c + d*x)*Log[a + b*x]*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]) - 2*b*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])*Log[c + d*x] - 2*B*n*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d
*x)*Log[c + d*x]) - b*B*n*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b
*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) +
b*B*n*(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[
c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(d*i^2*
(c + d*x))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2951, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ci+dix)^2}\,dx$$

$$\downarrow 2951$$

$$\frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2733$$

$$\frac{\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2Bn\int\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2009$$

$$\frac{\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2Bn\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}-\frac{Bn(a+bx)}{c+dx}\right)}{i^2(bc-ad)}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x)^2,x]
```

output
```
(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c + d*x) - 2*B*n*((
A*(a + b*x))/(c + d*x) - (B*n*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[e*((
a + b*x)/(c + d*x))^n])/(c + d*x)))/((b*c - a*d)*i^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.80

method	result
parallelrisch	$-\frac{-2ABab\,d^3n^2+2AB\,b^2c\,d^2n^2-2B^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)ab\,d^3n^2+B^2x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2b^2d^3n-2B^2x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^2d^3n^2+}{i^2(}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x,method=_RETURNVERBOS
E)
```

output
```
-(-2*A*B*a*b*d^3*n^2+2*A*B*b^2*c*d^2*n^2-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a
*b*d^3*n^2+B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^2*d^3*n-2*B^2*x*ln(e*((b*x+
a)/(d*x+c))^n)*b^2*d^3*n^2+2*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^3*n+2*A
*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^3*n+2*B^2*a*b*d^3*n^3-2*B^2*b^2*c*d^2*n
^3+A^2*a*b*d^3*n-A^2*b^2*c*d^2*n+B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*d^3*n
)/i^2/(d*x+c)/b/d^3/n/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.61

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx =$$
$$-\frac{A^2bc-A^2ad+2\left(B^2bc-B^2ad\right)n^2+\left(B^2bc-B^2ad\right)\log\left(e\right)^2-\left(B^2bdn^2x+B^2adn^2\right)\log\left(\frac{bx+a}{dx+c}\right)^2-2\,(}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x, algorithm="f
ricas")
```

output
```
-(A^2*b*c - A^2*a*d + 2*(B^2*b*c - B^2*a*d)*n^2 + (B^2*b*c - B^2*a*d)*log(
e)^2 - (B^2*b*d*n^2*x + B^2*a*d*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(A*B*b
*c - A*B*a*d)*n + 2*(A*B*b*c - A*B*a*d - (B^2*b*c - B^2*a*d)*n - (B^2*b*d*
n*x + B^2*a*d*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*(B^2*a*d*n^2 - A*B*a
*d*n + (B^2*b*d*n^2 - A*B*b*d*n)*x)*log((b*x + a)/(d*x + c)))/((b*c*d^2 -
a*d^3)*i^2*x + (b*c^2*d - a*c*d^2)*i^2)
```

Sympy [F]

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= \frac{\int \frac{A^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^2+2cdx+d^2x^2}\,dx}{i^2}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**2,x)
```

output
```
(Integral(A**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(B**2*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integr
al(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**2 + 2*c*d*x + d**2*x*
*2), x))/i**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(163) = 326$.

Time $= 0.05$ (sec) , antiderivative size $= 428$, normalized size of antiderivative $= 2.63$

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= 2\,ABn\left(\frac{1}{d^2i^2x+cdi^2} + \frac{b\log(bx+a)}{(bcd-ad^2)i^2} - \frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right)$$
$$+\left(2\,n\left(\frac{1}{d^2i^2x+cdi^2} + \frac{b\log(bx+a)}{(bcd-ad^2)i^2} - \frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right)\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) - \frac{((bdx+bc)\log$$
$$-\frac{B^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)^2}{d^2i^2x+cdi^2} - \frac{2\,AB\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{d^2i^2x+cdi^2} - \frac{A^2}{d^2i^2x+cdi^2}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x, algorithm="m
axima")
```

output
```
2*A*B*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2) -
b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) + (2*n*(1/(d^2*i^2*x + c*d*i^2) + b*
log(b*x + a)/((b*c*d - a*d^2)*i^2) - b*log(d*x + c)/((b*c*d - a*d^2)*i^2))
*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - ((b*d*x + b*c)*log(b*x + a)^2 +
(b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*(b*d*x + b*c)*log(b*x + a
) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*log(d*x + c))*n^2/(b*c^2*
d*i^2 - a*c*d^2*i^2 + (b*c*d^2*i^2 - a*d^3*i^2)*x))*B^2 - B^2*log(e*(b*x/(
d*x + c) + a/(d*x + c))^n)^2/(d^2*i^2*x + c*d*i^2) - 2*A*B*log(e*(b*x/(d*x
 + c) + a/(d*x + c))^n)/(d^2*i^2*x + c*d*i^2) - A^2/(d^2*i^2*x + c*d*i^2)
```

Giac [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.07

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= \left(\frac{(bx+a)B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(dx+c)i^2} - \frac{2\left(B^2n^2 - B^2n\log(e) - ABn\right)(bx+a)\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)i^2} + \frac{\left(2\,B^2n^2 - 2\,B^2n\log\right.\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x, algorithm="g
iac")
```

output
```
((b*x + a)*B^2*n^2*log((b*x + a)/(d*x + c))^2/((d*x + c)*i^2) - 2*(B^2*n^2
 - B^2*n*log(e) - A*B*n)*(b*x + a)*log((b*x + a)/(d*x + c))/((d*x + c)*i^2
) + (2*B^2*n^2 - 2*B^2*n*log(e) + B^2*log(e)^2 - 2*A*B*n + 2*A*B*log(e) +
A^2)*(b*x + a)/((d*x + c)*i^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 27.82 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.45

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(\frac{2B^2n}{xd^2i^2+cdi^2}-\frac{2AB}{xd^2i^2+cdi^2}\right) - \ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^2\left(\frac{B^2}{d\left(ci^2+di^2x\right)} + \frac{B^2b}{di^2\left(ad-bc\right)}\right) - \frac{A^2-2ABn+2B^2n^2}{xd^2i^2+cdi^2} + \frac{Bbn\operatorname{atan}\left(\frac{\left(2bdx+\frac{ad^2i^2+bcdi^2}{di^2}\right)1\mathrm{i}}{ad-bc}\right)(A-Bn)\,4\mathrm{i}}{di^2\left(ad-bc\right)}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*i + d*i*x)^2,x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)*((2*B^2*n)/(d^2*i^2*x + c*d*i^2) - (2*A*B)/
(d^2*i^2*x + c*d*i^2)) - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(d*(c*i^2 +
 d*i^2*x)) + (B^2*b)/(d*i^2*(a*d - b*c))) - (A^2 + 2*B^2*n^2 - 2*A*B*n)/(d
^2*i^2*x + c*d*i^2) + (B*b*n*atan(((2*b*d*x + (a*d^2*i^2 + b*c*d*i^2)/(d*i
^2))*1i)/(a*d - b*c))*(A - B*n)*4i)/(d*i^2*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.27

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= \frac{2\log(bx+a)\,a^2bcn+2\log(bx+a)\,a^2bdnx-2\log(bx+a)\,ab^2cn^2-2\log(bx+a)\,ab^2dn^2x-2\log(dx+$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

output

```
(2*log(a + b*x)*a**2*b*c*n + 2*log(a + b*x)*a**2*b*d*n*x - 2*log(a + b*x)*
a*b**2*c*n**2 - 2*log(a + b*x)*a*b**2*d*n**2*x - 2*log(c + d*x)*a**2*b*c*n
 - 2*log(c + d*x)*a**2*b*d*n*x + 2*log(c + d*x)*a*b**2*c*n**2 + 2*log(c +
d*x)*a*b**2*d*n**2*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c + lo
g(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*x - 2*log(((a + b*x)**n*e)/(c +
 d*x)**n)*a**2*b*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x + 2
*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*n*x - 2*log(((a + b*x)**n*e)/
(c + d*x)**n)*b**3*c*n*x - a**3*d*x + a**2*b*c*x + 2*a**2*b*d*n*x - 2*a*b*
*2*c*n*x - 2*a*b**2*d*n**2*x + 2*b**3*c*n**2*x)/(c*(a*c*d + a*d**2*x - b*c
**2 - b*c*d*x))
```

3.198 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$

Optimal result . . . 2270
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Rubi [A] (warning: unable to verify) . . . 2272
Maple [B] (verified) . . . 2275
Fricas [A] (verification not implemented) . . . 2275
Sympy [F(-1)] . . . 2276
Maxima [B] (verification not implemented) . . . 2276
Giac [A] (verification not implemented) . . . 2277
Mupad [B] (verification not implemented) . . . 2278
Reduce [B] (verification not implemented) . . . 2279

Optimal result

Integrand size = 45, antiderivative size = 231

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx &= \frac{2ABdn(a+bx)}{(bc-ad)^2gi^2(c+dx)} - \frac{2B^2dn^2(a+bx)}{(bc-ad)^2gi^2(c+dx)} \\ &+ \frac{2B^2dn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^2gi^2(c+dx)} \\ &- \frac{d(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2gi^2(c+dx)} \\ &+ \frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^2gi^2n}\end{aligned}$$

output

```
2*A*B*d*n*(b*x+a)/(-a*d+b*c)^2/g/i^2/(d*x+c)-2*B^2*d*n^2*(b*x+a)/(-a*d+b*c
)^2/g/i^2/(d*x+c)+2*B^2*d*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^2
/g/i^2/(d*x+c)-d*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g/
i^2/(d*x+c)+1/3*b*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)^2/g/i^2/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 789 vs. $2(231) = 462$.

Time $= 0.76$ (sec) , antiderivative size $= 789$, normalized size of antiderivative $= 3.42$

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$

$$= \frac{bB^2n^2\log^3\left(\frac{a+bx}{c+dx}\right)}{3(bc-ad)^2gi^2} - \frac{2Bn\log\left(\frac{a+bx}{c+dx}\right)\left(-A+Bn-B\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)\right)}{(bc-ad)gi^2(c+dx)}$$

$$+\frac{\log^2\left(\frac{a+bx}{c+dx}\right)\left(AbBcn - aB^2dn^2 + AbBdnx - bB^2dn^2x + bB^2cn\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right) + bB^2\right.}{(bc-ad)^2gi^2(c+dx)}$$

$$+\frac{A^2 - 2ABn + 2B^2n^2 + 2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right) - 2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)}{(bc-ad)gi^2(c+dx)}$$

$$+\frac{b\log(a+bx)\left(A^2 - 2ABn + 2B^2n^2 + 2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right) - 2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-\right.\right.}{(bc-ad)^2gi^2}$$

$$-\frac{b\left(A^2 - 2ABn + 2B^2n^2 + 2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right) - 2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right.\right.}{(bc-ad)^2gi^2}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d
*i*x)^2),x]
```

output

```
(b*B^2*n^2*Log[(a + b*x)/(c + d*x)]^3)/(3*(b*c - a*d)^2*g*i^2) - (2*B*n*Lo
g[(a + b*x)/(c + d*x)]*(-A + B*n - B*(Log[e*((a + b*x)/(c + d*x))^n] - n*L
og[(a + b*x)/(c + d*x)])))/((b*c - a*d)*g*i^2*(c + d*x)) + (Log[(a + b*x)/
(c + d*x)]^2*(A*b*B*c*n - a*B^2*d*n^2 + A*b*B*d*n*x - b*B^2*d*n^2*x + b*B^
2*c*n*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) + b*B^
2*d*n*x*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])))/((
b*c - a*d)^2*g*i^2*(c + d*x)) + (A^2 - 2*A*B*n + 2*B^2*n^2 + 2*A*B*(Log[e*
((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) - 2*B^2*n*(Log[e*((
a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) + B^2*(Log[e*((a + b*
x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2)/((b*c - a*d)*g*i^2*(c +
d*x)) + (b*Log[a + b*x]*(A^2 - 2*A*B*n + 2*B^2*n^2 + 2*A*B*(Log[e*((a + b*
x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) - 2*B^2*n*(Log[e*((a + b*x)
/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) + B^2*(Log[e*((a + b*x)/(c +
d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2))/((b*c - a*d)^2*g*i^2) - (b*(A^2
 - 2*A*B*n + 2*B^2*n^2 + 2*A*B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a
+ b*x)/(c + d*x)]) - 2*B^2*n*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a +
b*x)/(c + d*x)]) + B^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(
c + d*x)])^2)*Log[c + d*x])/((b*c - a*d)^2*g*i^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2961, 2788, 2733, 2009, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}\,d\frac{a+bx}{c+dx}}{gi^2(bc-ad)^2}$$

$$\downarrow 2788$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx} - d\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d\frac{a+bx}{c+dx}}{gi^2(bc-ad)^2}$$

$$\downarrow 2733$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx} - d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx} - 2Bn\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d\frac{a+bx}{c+dx}\right)}{gi^2(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx} - d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx} - 2Bn\left(\frac{A(a+bx)}{c+dx} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} - \right.\right.}{gi^2(bc-ad)^2}$$

$$\downarrow 2739$$

$$\frac{b\int \frac{(a+bx)^2}{(c+dx)^2}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn} - d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx} - 2Bn\left(\frac{A(a+bx)}{c+dx} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} - \frac{Bn(a+}{c+dx}\right.\right.$$

$$\downarrow 15$$

$$\frac{\frac{b(a+bx)^3}{3Bn(c+dx)^3} - d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx} - 2Bn\left(\frac{A(a+bx)}{c+dx} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} - \frac{Bn(a+bx)}{c+dx}\right)\right)}{gi^2(bc-ad)^2}$$

input

```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d*i*x)^
2),x]
```

output

```
((b*(a + b*x)^3)/(3*B*n*(c + d*x)^3) - d*(((a + b*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])^2)/(c + d*x) - 2*B*n*((A*(a + b*x))/(c + d*x) - (B*n*(a
+ b*x))/(c + d*x) + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(c + d*x)
)))/((b*c - a*d)^2*g*i^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(229) = 458.

Time = 4.60 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.20

method	result
parallelrisch	$\frac{-6ABx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3d^4n-6AB\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)ab^2d^4n-6B^2ab^2d^4n^3+6B^2b^3cd^3n^3-3A^2ab^2d^4n+3A^2b^3cd^3n+B^2xl}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x,method=_
RETURNVERBOSE)
```

output
```
1/3*(-6*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^4*n-6*A*B*ln(e*((b*x+a)/(d*x
+c))^n)*a*b^2*d^4*n-6*B^2*a*b^2*d^4*n^3+6*B^2*b^3*c*d^3*n^3-3*A^2*a*b^2*d^
4*n+3*A^2*b^3*c*d^3*n+B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*b^3*d^4+B^2*ln(e*(
(b*x+a)/(d*x+c))^n)^3*b^3*c*d^3+3*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^4+
3*A^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d^3+6*A*B*a*b^2*d^4*n^2-6*A*B*b^3*c*
d^3*n^2-3*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*d^4*n+6*B^2*x*ln(e*((b*x+a
)/(d*x+c))^n)*b^3*d^4*n^2+3*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*d^4-3*B^
2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^2*d^4*n+6*B^2*ln(e*((b*x+a)/(d*x+c))^n)*
a*b^2*d^4*n^2+3*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*c*d^3)/i^2/g/(d*x+c)/(
a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/d^3/n
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.87

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{3\,A^2bc-3\,A^2ad+\left(B^2bdn^2x+B^2bcn^2\right)\log\left(\frac{bx+a}{dx+c}\right)^3+6\left(B^2bc-B^2ad\right)n^2+3\left(B^2bc-B^2ad+\left(B^2bdx+\right.\right.}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x,
algorithm="fricas")
```

output
```
1/3*(3*A^2*b*c - 3*A^2*a*d + (B^2*b*d*n^2*x + B^2*b*c*n^2)*log((b*x + a)/(
d*x + c))^3 + 6*(B^2*b*c - B^2*a*d)*n^2 + 3*(B^2*b*c - B^2*a*d + (B^2*b*d*
x + B^2*b*c)*log((b*x + a)/(d*x + c)))*log(e)^2 - 3*(B^2*a*d*n^2 - A*B*b*c
*n + (B^2*b*d*n^2 - A*B*b*d*n)*x)*log((b*x + a)/(d*x + c))^2 - 6*(A*B*b*c
- A*B*a*d)*n + 3*(2*A*B*b*c - 2*A*B*a*d + (B^2*b*d*n*x + B^2*b*c*n)*log((b
*x + a)/(d*x + c))^2 - 2*(B^2*b*c - B^2*a*d)*n - 2*(B^2*a*d*n - A*B*b*c +
(B^2*b*d*n - A*B*b*d)*x)*log((b*x + a)/(d*x + c)))*log(e) + 3*(2*B^2*a*d*n
^2 - 2*A*B*a*d*n + A^2*b*c + (2*B^2*b*d*n^2 - 2*A*B*b*d*n + A^2*b*d)*x)*lo
g((b*x + a)/(d*x + c)))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g*i^2*x + (b^
2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*g*i^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)(ci + dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)/(d*i*x+c*i)**2,x
)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. $2(229) = 458$.

Time = 0.10 (sec) , antiderivative size = 1014, normalized size of antiderivative = 4.39

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)(ci + dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x,
algorithm="maxima")
```

output

```
B^2*(1/((b*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/
((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*g*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2 + 2*A*B*(
1/((b*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*g*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*(((b*d*x +
 b*c)*log(b*x + a)^3 - (b*d*x + b*c)*log(d*x + c)^3 + 3*(b*d*x + b*c)*log(
b*x + a)^2 + 3*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*log(d*x + c)^2 +
 6*b*c - 6*a*d + 6*(b*d*x + b*c)*log(b*x + a) - 3*(2*b*d*x + (b*d*x + b*c)
*log(b*x + a)^2 + 2*b*c + 2*(b*d*x + b*c)*log(b*x + a))*log(d*x + c))*n^2/
(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 + a^2*c*d^2*g*i^2 + (b^2*c^2*d*g*i^2 -
2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x) - 3*((b*d*x + b*c)*log(b*x + a)^2 +
(b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*(b*d*x + b*c)*log(b*x + a
) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*log(d*x + c))*n*log(e*(b*
x/(d*x + c) + a/(d*x + c))^n)/(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 + a^2*c*d
^2*g*i^2 + (b^2*c^2*d*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x))*B^2 -
 ((b*d*x + b*c)*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*
a*d + 2*(b*d*x + b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*
x + a))*log(d*x + c))*A*B*n/(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 + a^2*c*d^2
*g*i^2 + (b^2*c^2*d*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x) + A^2...
```

Giac [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.59

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{1}{3}\left(\frac{B^2bn^2\log\left(\frac{bx+a}{dx+c}\right)^3}{bcgi^2 - adgi^2} - 3\left(\frac{(bx+a)B^2dn^2}{(bcgi^2 - adgi^2)(dx+c)} - \frac{B^2bn\log(e) + ABbn}{bcgi^2 - adgi^2}\right)\log\left(\frac{bx+a}{dx+c}\right)^2 + \frac{3\left(B^2b\right.}{}\right.$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x,
algorithm="giac")
```

output

```
1/3*(B^2*b*n^2*log((b*x + a)/(d*x + c))^3/(b*c*g*i^2 - a*d*g*i^2) - 3*((b*
x + a)*B^2*d*n^2/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)) - (B^2*b*n*log(e) + A
*B*b*n)/(b*c*g*i^2 - a*d*g*i^2))*log((b*x + a)/(d*x + c))^2 + 3*(B^2*b*log
(e)^2 + 2*A*B*b*log(e) + A^2*b)*log((b*x + a)/(d*x + c))/(b*c*g*i^2 - a*d*
g*i^2) + 6*(B^2*d*n^2 - B^2*d*n*log(e) - A*B*d*n)*(b*x + a)*log((b*x + a)/
(d*x + c))/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)) - 3*(2*B^2*d*n^2 - 2*B^2*d*
n*log(e) + B^2*d*log(e)^2 - 2*A*B*d*n + 2*A*B*d*log(e) + A^2*d)*(b*x + a)/
((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^
2)
```

Mupad [B] (verification not implemented)

Time = 27.23 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.58

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{B^2\,b\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}{3\,g\,i^2\,n\,(a\,d-b\,c)^2} - \frac{A^2-2\,A\,B\,n+2\,B^2\,n^2}{(a\,d-b\,c)\,(c\,g\,i^2+d\,g\,i^2\,x)} - \frac{2\,B\,\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\,(A-B\,n)}{(a\,d-b\,c)\,(c\,g\,i^2+d\,g\,i^2\,x)}$$
$$- \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{(a\,d-b\,c)\,(c\,g\,i^2+d\,g\,i^2\,x)} - \frac{B\,b\,(A-B\,n)}{g\,i^2\,n\,(a\,d-b\,c)^2}\right)$$
$$- \frac{b\,\mathrm{atan}\left(\frac{b\left(2\,b\,d\,x+\frac{a^2\,d^2\,g\,i^2-b^2\,c^2\,g\,i^2}{g\,i^2\,(a\,d-b\,c)}\right)\,\left(A^2-2\,A\,B\,n+2\,B^2\,n^2\right)\,1\mathrm{i}}{(a\,d-b\,c)\,(b\,A^2-2\,b\,A\,B\,n+2\,b\,B^2\,n^2)}\right)\,\left(A^2-2\,A\,B\,n+2\,B^2\,n^2\right)\,2\mathrm{i}}{g\,i^2\,(a\,d-b\,c)^2}$$

input

```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)*(c*i + d*i*x)^
2),x)
```

output

```
(B^2*b*log(e*((a + b*x)/(c + d*x))^n)^3)/(3*g*i^2*n*(a*d - b*c)^2) - (A^2
+ 2*B^2*n^2 - 2*A*B*n)/((a*d - b*c)*(c*g*i^2 + d*g*i^2*x)) - (2*B*log(e*((
a + b*x)/(c + d*x))^n)*(A - B*n))/((a*d - b*c)*(c*g*i^2 + d*g*i^2*x)) - (b
*atan((b*(2*b*d*x + (a^2*d^2*g*i^2 - b^2*c^2*g*i^2)/(g*i^2*(a*d - b*c)))*(
A^2 + 2*B^2*n^2 - 2*A*B*n)*1i)/((a*d - b*c)*(A^2*b + 2*B^2*b*n^2 - 2*A*B*b
*n)))*(A^2 + 2*B^2*n^2 - 2*A*B*n)*2i)/(g*i^2*(a*d - b*c)^2) - log(e*((a +
b*x)/(c + d*x))^n)^2*(B^2/((a*d - b*c)*(c*g*i^2 + d*g*i^2*x)) - (B*b*(A -
B*n))/(g*i^2*n*(a*d - b*c)^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 640, normalized size of antiderivative = 2.77

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x)
```

output

```
( - 3*log(a + b*x)*a**2*b*c**2*n + 6*log(a + b*x)*a**2*b*c*d*n**2 - 3*log(
a + b*x)*a**2*b*c*d*n*x + 6*log(a + b*x)*a**2*b*d**2*n**2*x - 6*log(a + b*
x)*a*b**2*c*d*n**3 - 6*log(a + b*x)*a*b**2*d**2*n**3*x + 3*log(c + d*x)*a*
*2*b*c**2*n - 6*log(c + d*x)*a**2*b*c*d*n**2 + 3*log(c + d*x)*a**2*b*c*d*n
*x - 6*log(c + d*x)*a**2*b*d**2*n**2*x + 6*log(c + d*x)*a*b**2*c*d*n**3 +
6*log(c + d*x)*a*b**2*d**2*n**3*x - log(((a + b*x)**n*e)/(c + d*x)**n)**3*
b**3*c**2 - log(((a + b*x)**n*e)/(c + d*x)**n)**3*b**3*c*d*x - 3*log(((a +
 b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c**2 + 3*log(((a + b*x)**n*e)/(c + d*x
)**n)**2*a*b**2*c*d*n - 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c*d
*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*d*n*x - 6*log(((a + b*
x)**n*e)/(c + d*x)**n)*a**2*b*d**2*n*x + 6*log(((a + b*x)**n*e)/(c + d*x)*
*n)*a*b**2*c*d*n*x + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d**2*n**2
*x - 6*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*d*n**2*x - 3*a**3*d**2*n*
x + 3*a**2*b*c*d*n*x + 6*a**2*b*d**2*n**2*x - 6*a*b**2*c*d*n**2*x - 6*a*b*
*2*d**2*n**3*x + 6*b**3*c*d*n**3*x)/(3*c*g*n*(a**2*c*d**2 + a**2*d**3*x -
2*a*b*c**2*d - 2*a*b*c*d**2*x + b**2*c**3 + b**2*c**2*d*x))
```

3.199 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx$

Optimal result . . . 2280
Mathematica [B] (verified) . . . 2281
Rubi [A] (verified) . . . 2282
Maple [B] (verified) . . . 2284
Fricas [B] (verification not implemented) . . . 2285
Sympy [F(-1)] . . . 2286
Maxima [B] (verification not implemented) . . . 2287
Giac [F(-1)] . . . 2288
Mupad [F(-1)] . . . 2288
Reduce [B] (verification not implemented) . . . 2288

Optimal result

Integrand size = 45, antiderivative size = 392

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx &= -\frac{2ABd^2n(a+bx)}{(bc-ad)^3g^2i^2(c+dx)}+\frac{2B^2d^2n^2(a+bx)}{(bc-ad)^3g^2i^2(c+dx)}\\ &\quad-\frac{2b^2B^2n^2(c+dx)}{(bc-ad)^3g^2i^2(a+bx)}\\ &\quad-\frac{2B^2d^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^3g^2i^2(c+dx)}\\ &\quad-\frac{2b^2Bn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^2i^2(a+bx)}\\ &\quad+\frac{d^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^2i^2(c+dx)}\\ &\quad-\frac{b^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^2i^2(a+bx)}\\ &\quad-\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^3g^2i^2n}\end{aligned}$$

output

```
-2*A*B*d^2*n*(b*x+a)/(-a*d+b*c)^3/g^2/i^2/(d*x+c)+2*B^2*d^2*n^2*(b*x+a)/(-
a*d+b*c)^3/g^2/i^2/(d*x+c)-2*b^2*B^2*n^2*(d*x+c)/(-a*d+b*c)^3/g^2/i^2/(b*x
+a)-2*B^2*d^2*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^3/g^2/i^2/(d*
x+c)-2*b^2*B*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^2/i^
2/(b*x+a)+d^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^2/i
^2/(d*x+c)-b^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^2/
i^2/(b*x+a)-2/3*b*d*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)^3/g^2/i
^2/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 870 vs. $2(392) = 784$.

Time $= 1.12$ (sec) , antiderivative size $= 870$, normalized size of antiderivative $= 2.22$

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx =$$

$$-\frac{2bB^2dn^2(a+bx)(c+dx)\log^3\left(\frac{a+bx}{c+dx}\right) + 3Bn\log^2\left(\frac{a+bx}{c+dx}\right)\left(2aAbcd + b^2Bc^2n - a^2Bd^2n + 2Ab^2cdx + 2\right.}{}$$

input

```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i +
 d*i*x)^2),x]
```

output

```
-1/3*(2*b*B^2*d*n^2*(a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^3 + 3*B*n
*Log[(a + b*x)/(c + d*x)]^2*(2*a*A*b*c*d + b^2*B*c^2*n - a^2*B*d^2*n + 2*A
*b^2*c*d*x + 2*a*A*b*d^2*x + 2*b^2*B*c*d*n*x - 2*a*b*B*d^2*n*x + 2*A*b^2*d
^2*x^2 + 2*b*B*d*(a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*b*
B*d*n*(a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]) + 6*B*(b*c - a*d)*n*Lo
g[(a + b*x)/(c + d*x)]*(A*b*c + a*A*d + b*B*c*n - a*B*d*n + 2*A*b*d*x + B*
(a*d + b*(c + 2*d*x))*Log[e*((a + b*x)/(c + d*x))^n] - B*n*(b*c + a*d + 2*
b*d*x)*Log[(a + b*x)/(c + d*x)]) + 6*b*d*(a + b*x)*(c + d*x)*Log[a + b*x]*
(A^2 + 2*B^2*n^2 + 2*A*B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)
/(c + d*x)]) + B^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c +
d*x)])^2) + 3*b*(b*c - a*d)*(c + d*x)*(A^2 + 2*A*B*n + 2*B^2*n^2 + B^2*Log
[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(A + B*n)*Log[(a + b*x)/(c + d*x)] +
 B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(
A + B*n - B*n*Log[(a + b*x)/(c + d*x)])) + 3*d*(b*c - a*d)*(a + b*x)*(A^2
- 2*A*B*n + 2*B^2*n^2 + B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-A +
 B*n)*Log[(a + b*x)/(c + d*x)] + B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*
Log[e*((a + b*x)/(c + d*x))^n]*(-A + B*n + B*n*Log[(a + b*x)/(c + d*x)]))
- 6*b*d*(a + b*x)*(c + d*x)*(A^2 + 2*B^2*n^2 + 2*A*B*(Log[e*((a + b*x)/(c
+ d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) + B^2*(Log[e*((a + b*x)/(c + d*x)
)^n] - n*Log[(a + b*x)/(c + d*x)])^2)*Log[c + d*x])/((b*c - a*d)^3*g^2*...
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}\,d\frac{a+bx}{c+dx}}{g^2i^2(bc-ad)^3}$$

$$\downarrow 2795$$

$$\frac{\int \left(d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}-\frac{2bd(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}\right)d\frac{a+bx}{c+dx}}{g^2i^2(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}-\frac{2b^2Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}+\frac{d^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-\frac{2ABd^2n(a+bx)}{c+dx}}{g^2i^2(bc-ad)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i + d*i*x
)^2),x]
```

output
```
((-2*A*B*d^2*n*(a + b*x))/(c + d*x) + (2*B^2*d^2*n^2*(a + b*x))/(c + d*x)
- (2*b^2*B^2*n^2*(c + d*x))/(a + b*x) - (2*B^2*d^2*n*(a + b*x)*Log[e*((a +
 b*x)/(c + d*x))^n])/(c + d*x) - (2*b^2*B*n*(c + d*x)*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(a + b*x) + (d^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n])^2)/(c + d*x) - (b^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2)/(a + b*x) - (2*b*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*
B*n))/((b*c - a*d)^3*g^2*i^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1306 vs. $2(390) = 780$.

Time = 9.77 (sec) , antiderivative size = 1307, normalized size of antiderivative = 3.33

method	result	size
parallelrisch	Expression too large to display	1307

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,method
=_RETURNVERBOSE)
```

output

```
1/3*(-12*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b*c^3*d^2*n+12*A*B*x*ln(e*((b
*x+a)/(d*x+c))^n)*a^3*b^2*c^4*d*n+12*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3
*b^2*c^3*d^2*n^2+6*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^2*c^3*d^2-6*A
*B*x^2*a^4*b*c^2*d^3*n^2+12*A*B*x^2*a^3*b^2*c^3*d^2*n^2-6*A*B*x^2*a^2*b^3*
c^4*d*n^2-6*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b*c^3*d^2*n+6*B^2*x*ln(e
*((b*x+a)/(d*x+c))^n)^2*a^3*b^2*c^4*d*n+12*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)
*a^4*b*c^3*d^2*n^2+12*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^4*d*n^2+6*
A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b*c^3*d^2+6*A*B*x*ln(e*((b*x+a)/(d*x
+c))^n)^2*a^3*b^2*c^4*d+6*A*B*x*a^4*b*c^3*d^2*n^2+6*A*B*x*a^3*b^2*c^4*d*n^
2+6*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^5*c^3*d^2*n^2+6*B^2*ln(e*((b*x+a)/(d*x
+c))^n)*a^3*b^2*c^5*n^2+3*A^2*x*a^5*c^2*d^3*n-3*A^2*x*a^2*b^3*c^5*n+6*A^2*
ln(e*((b*x+a)/(d*x+c))^n)*a^4*b*c^4*d+6*B^2*x*a^5*c^2*d^3*n^3-6*B^2*x*a^2*
b^3*c^5*n^3+2*B^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^4*b*c^4*d-3*B^2*ln(e*((b*x
+a)/(d*x+c))^n)^2*a^5*c^3*d^2*n+3*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^2*
c^5*n+2*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^3*b^2*c^3*d^2+6*B^2*x^2*a^4*
b*c^2*d^3*n^3-6*B^2*x^2*a^2*b^3*c^4*d*n^3+2*B^2*x*ln(e*((b*x+a)/(d*x+c))^n
)^3*a^4*b*c^3*d^2+2*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^3*b^2*c^4*d-6*B^2*
x*a^4*b*c^3*d^2*n^3+6*B^2*x*a^3*b^2*c^4*d*n^3+6*A^2*x^2*ln(e*((b*x+a)/(d*x
+c))^n)*a^3*b^2*c^3*d^2+3*A^2*x^2*a^4*b*c^2*d^3*n-3*A^2*x^2*a^2*b^3*c^4*d*
n-6*A*B*x*a^5*c^2*d^3*n^2-6*A*B*x*a^2*b^3*c^5*n^2+6*A^2*x*ln(e*((b*x+a)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(390) = 780$.

Time = 0.11 (sec) , antiderivative size = 983, normalized size of antiderivative = 2.51

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x
, algorithm="fricas")
```

output

```
-1/3*(3*A^2*b^2*c^2 - 3*A^2*a^2*d^2 + 2*(B^2*b^2*d^2*n^2*x^2 + B^2*a*b*c*d
*n^2 + (B^2*b^2*c*d + B^2*a*b*d^2)*n^2*x)*log((b*x + a)/(d*x + c))^3 + 6*(
B^2*b^2*c^2 - B^2*a^2*d^2)*n^2 + 3*(B^2*b^2*c^2 - B^2*a^2*d^2 + 2*(B^2*b^2
*c*d - B^2*a*b*d^2)*x + 2*(B^2*b^2*d^2*x^2 + B^2*a*b*c*d + (B^2*b^2*c*d +
B^2*a*b*d^2)*x)*log((b*x + a)/(d*x + c)))*log(e)^2 + 3*(2*A*B*b^2*d^2*n*x^
2 + 2*A*B*a*b*c*d*n + (B^2*b^2*c^2 - B^2*a^2*d^2)*n^2 + 2*((B^2*b^2*c*d -
B^2*a*b*d^2)*n^2 + (A*B*b^2*c*d + A*B*a*b*d^2)*n)*x)*log((b*x + a)/(d*x +
c))^2 + 6*(A*B*b^2*c^2 - 2*A*B*a*b*c*d + A*B*a^2*d^2)*n + 6*(A^2*b^2*c*d -
 A^2*a*b*d^2 + 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2)*x + 6*(A*B*b^2*c^2 - A*B
*a^2*d^2 + (B^2*b^2*d^2*n*x^2 + B^2*a*b*c*d*n + (B^2*b^2*c*d + B^2*a*b*d^2
)*n*x)*log((b*x + a)/(d*x + c))^2 + (B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2
*d^2)*n + 2*(A*B*b^2*c*d - A*B*a*b*d^2)*x + (2*A*B*b^2*d^2*x^2 + 2*A*B*a*b
*c*d + (B^2*b^2*c^2 - B^2*a^2*d^2)*n + 2*(A*B*b^2*c*d + A*B*a*b*d^2 + (B^2
*b^2*c*d - B^2*a*b*d^2)*n)*x)*log((b*x + a)/(d*x + c)))*log(e) + 6*(A^2*a*
b*c*d + (B^2*b^2*c^2 + B^2*a^2*d^2)*n^2 + (2*B^2*b^2*d^2*n^2 + A^2*b^2*d^2
)*x^2 + (A*B*b^2*c^2 - A*B*a^2*d^2)*n + (A^2*b^2*c*d + A^2*a*b*d^2 + 2*(B^
2*b^2*c*d + B^2*a*b*d^2)*n^2 + 2*(A*B*b^2*c*d - A*B*a*b*d^2)*n)*x)*log((b*
x + a)/(d*x + c)))/((b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b
*d^4)*g^2*i^2*x^2 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*g^
2*i^2*x + (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*g...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \text{Timed out}$$

input

```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2/(d*i*x+c*i)**
2,x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2006 vs. $2(390) = 780$.

Time $= 0.16$ (sec) , antiderivative size $= 2006$, normalized size of antiderivative $= 5.12$

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x
, algorithm="maxima")
```

output

```
-B^2*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i
^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*g^2*i^2*x + (a*b^
2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d*log(b*x + a)/((b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2) - 2*b*d*log(d*x + c)/
((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2))*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n)^2 - 2*A*B*((2*b*d*x + b*c + a*d)/((b^3*c^2*d -
 2*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c
*d^2 + a^3*d^3)*g^2*i^2*x + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^
2) + 2*b*d*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*g^2*i^2) - 2*b*d*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
 - a^3*d^3)*g^2*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 2/3*((3*b^2
*c^2 - 3*a^2*d^2 + (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x
 + a)^3 + 3*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)*l
og(d*x + c)^2 - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(d*x +
c)^3 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b
*d^2)*x)*log(b*x + a) - 3*(2*b^2*d^2*x^2 + 2*a*b*c*d + (b^2*d^2*x^2 + a*b*
c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)^2 + 2*(b^2*c*d + a*b*d^2)*x)*log
(d*x + c))*n^2/(a*b^3*c^4*g^2*i^2 - 3*a^2*b^2*c^3*d*g^2*i^2 + 3*a^3*b*c^2*
d^2*g^2*i^2 - a^4*c*d^3*g^2*i^2 + (b^4*c^3*d*g^2*i^2 - 3*a*b^3*c^2*d^2*g^2
*i^2 + 3*a^2*b^2*c*d^3*g^2*i^2 - a^3*b*d^4*g^2*i^2)*x^2 + (b^4*c^4*g^2*...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x
, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \int \frac{\left(A+B\,\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)^2}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^2*(c*i + d*i*x
)^2),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^2*(c*i + d*i*x
)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2172, normalized size of antiderivative = 5.54

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x)
```

output

```
( - 6*log(a + b*x)*a**4*b*c*d**2*n - 6*log(a + b*x)*a**4*b*d**3*n*x - 6*lo
g(a + b*x)*a**3*b**2*c**2*d*n + 12*log(a + b*x)*a**3*b**2*c*d**2*n**2 - 12
*log(a + b*x)*a**3*b**2*c*d**2*n*x + 12*log(a + b*x)*a**3*b**2*d**3*n**2*x
 - 6*log(a + b*x)*a**3*b**2*d**3*n*x**2 - 12*log(a + b*x)*a**2*b**3*c**2*d
*n**2 - 6*log(a + b*x)*a**2*b**3*c**2*d*n*x - 12*log(a + b*x)*a**2*b**3*c*
d**2*n**3 - 6*log(a + b*x)*a**2*b**3*c*d**2*n*x**2 - 12*log(a + b*x)*a**2*
b**3*d**3*n**3*x + 12*log(a + b*x)*a**2*b**3*d**3*n**2*x**2 - 12*log(a + b
*x)*a*b**4*c**2*d*n**3 - 12*log(a + b*x)*a*b**4*c**2*d*n**2*x - 24*log(a +
 b*x)*a*b**4*c*d**2*n**3*x - 12*log(a + b*x)*a*b**4*c*d**2*n**2*x**2 - 12*
log(a + b*x)*a*b**4*d**3*n**3*x**2 - 12*log(a + b*x)*b**5*c**2*d*n**3*x -
12*log(a + b*x)*b**5*c*d**2*n**3*x**2 + 6*log(c + d*x)*a**4*b*c*d**2*n + 6
*log(c + d*x)*a**4*b*d**3*n*x + 6*log(c + d*x)*a**3*b**2*c**2*d*n - 12*log
(c + d*x)*a**3*b**2*c*d**2*n**2 + 12*log(c + d*x)*a**3*b**2*c*d**2*n*x - 1
2*log(c + d*x)*a**3*b**2*d**3*n**2*x + 6*log(c + d*x)*a**3*b**2*d**3*n*x**
2 + 12*log(c + d*x)*a**2*b**3*c**2*d*n**2 + 6*log(c + d*x)*a**2*b**3*c**2*
d*n*x + 12*log(c + d*x)*a**2*b**3*c*d**2*n**3 + 6*log(c + d*x)*a**2*b**3*c
*d**2*n*x**2 + 12*log(c + d*x)*a**2*b**3*d**3*n**3*x - 12*log(c + d*x)*a**
2*b**3*d**3*n**2*x**2 + 12*log(c + d*x)*a*b**4*c**2*d*n**3 + 12*log(c + d*
x)*a*b**4*c**2*d*n**2*x + 24*log(c + d*x)*a*b**4*c*d**2*n**3*x + 12*log(c
+ d*x)*a*b**4*c*d**2*n**2*x**2 + 12*log(c + d*x)*a*b**4*d**3*n**3*x**2 ...
```

3.200 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx$

Optimal result . . . 2290
Mathematica [B] (verified) . . . 2291
Rubi [A] (verified) . . . 2292
Maple [B] (verified) . . . 2294
Fricas [B] (verification not implemented) . . . 2295
Sympy [F(-1)] . . . 2296
Maxima [B] (verification not implemented) . . . 2297
Giac [F(-1)] . . . 2298
Mupad [B] (verification not implemented) . . . 2298
Reduce [B] (verification not implemented) . . . 2299

Optimal result

Integrand size = 45, antiderivative size = 560

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx &= \frac{2ABd^3n(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} - \frac{2B^2d^3n^2(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} \\ &+ \frac{6b^2B^2dn^2(c+dx)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3B^2n^2(c+dx)^2}{4(bc-ad)^4g^3i^2(a+bx)^2} \\ &+ \frac{2B^2d^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^4g^3i^2(c+dx)} \\ &+ \frac{6b^2Bdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} \\ &- \frac{b^3Bn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} \\ &- \frac{d^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^3i^2(c+dx)} \\ &+ \frac{3b^2d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^3i^2(a+bx)} \\ &- \frac{b^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^3i^2(a+bx)^2} \\ &+ \frac{bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{B(bc-ad)^4g^3i^2n}\end{aligned}$$

output

```
2*A*B*d^3*n*(b*x+a)/(-a*d+b*c)^4/g^3/i^2/(d*x+c)-2*B^2*d^3*n^2*(b*x+a)/(-a
*d+b*c)^4/g^3/i^2/(d*x+c)+6*b^2*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^4/g^3/i^2/(b*
x+a)-1/4*b^3*B^2*n^2*(d*x+c)^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2+2*B^2*d^3*n*
(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+6*b^2*B*d*n
*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)-1/2*
b^3*B*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^3/i^2/(b*
x+a)^2-d^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^3/i^2/
(d*x+c)+3*b^2*d*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^3
/i^2/(b*x+a)-1/2*b^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c
)^4/g^3/i^2/(b*x+a)^2+b*d^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)
^4/g^3/i^2/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1340 vs. $2(560) = 1120$.

Time $= 1.66$ (sec) , antiderivative size $= 1340$, normalized size of antiderivative $= 2.39$

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^3(ci + dix)^2}\, dx = \text{Too large to display}$$

input

```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i +
 d*i*x)^2),x]
```

output

```
(4*b*B^2*d^2*n^2*(a + b*x)^2*(c + d*x)*Log[(a + b*x)/(c + d*x)]^3 + 2*B*n*
Log[(a + b*x)/(c + d*x)]^2*(6*a^2*A*b*c*d^2 - b^3*B*c^3*n + 6*a*b^2*B*c^2*
d*n - 2*a^3*B*d^3*n + 12*a*A*b^2*c*d^2*x + 6*a^2*A*b*d^3*x + 3*b^3*B*c^2*d
*n*x + 12*a*b^2*B*c*d^2*n*x - 6*a^2*b*B*d^3*n*x + 6*A*b^3*c*d^2*x^2 + 12*a
*A*b^2*d^3*x^2 + 9*b^3*B*c*d^2*n*x^2 + 6*A*b^3*d^3*x^3 + 3*b^3*B*d^3*n*x^3
 + 6*b*B*d^2*(a + b*x)^2*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n] - 6*b*B*
d^2*n*(a + b*x)^2*(c + d*x)*Log[(a + b*x)/(c + d*x)]) + 2*b*d*(b*c - a*d)*
(a + b*x)*(c + d*x)*(4*A^2 + 10*A*B*n + 11*B^2*n^2 + 4*B^2*Log[e*((a + b*x
)/(c + d*x))^n]^2 - 2*B*n*(4*A + 5*B*n)*Log[(a + b*x)/(c + d*x)] + 4*B^2*n
^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(4*A +
5*B*n - 4*B*n*Log[(a + b*x)/(c + d*x)])) - b*(b*c - a*d)^2*(c + d*x)*(2*A^
2 + 2*A*B*n + B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(2*
A + B*n)*Log[(a + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 +
 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(2*A + B*n - 2*B*n*Log[(a + b*x)/(c +
d*x)])) + 6*b*d^2*(a + b*x)^2*(c + d*x)*Log[a + b*x]*(2*A^2 + 2*A*B*n + 5*
B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(2*A + B*n)*Log[(
a + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a
 + b*x)/(c + d*x))^n]*(2*A + B*n - 2*B*n*Log[(a + b*x)/(c + d*x)])) + 2*B*
(b*c - a*d)*n*Log[(a + b*x)/(c + d*x)]*(2*b*d*(a + b*x)*(c + d*x)*(4*A + 5
*B*n + 4*B*Log[e*((a + b*x)/(c + d*x))^n] - 4*B*n*Log[(a + b*x)/(c + d*...
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}\,d\frac{a+bx}{c+dx}}{g^3i^2(bc-ad)^4}$$

$$\downarrow 2795$$

$$\int \left(-\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d^3 + \frac{3b(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d^2}{a+bx} - \frac{3b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d}{(a+bx)^2} + \frac{b^3(c+dx}{}$$

$$\frac{}{g^3 i^2 (bc-ad)^4}$$

$$\downarrow 2009$$

$$-\frac{b^3(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2} - \frac{b^3Bn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2} + \frac{3b^2d(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx} + \frac{6b^2Bdn(c+dx}{}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i + d*i*x
)^2),x]
```

output
```
((2*A*B*d^3*n*(a + b*x))/(c + d*x) - (2*B^2*d^3*n^2*(a + b*x))/(c + d*x) +
 (6*b^2*B^2*d*n^2*(c + d*x))/(a + b*x) - (b^3*B^2*n^2*(c + d*x)^2)/(4*(a +
 b*x)^2) + (2*B^2*d^3*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(c + d*x
) + (6*b^2*B*d*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*
x) - (b^3*B*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a +
b*x)^2) - (d^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c + d*
x) + (3*b^2*d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x
) - (b^3*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a + b*x
)^2) + (b*d^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(B*n))/((b*c - a*d
)^4*g^3*i^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1985 vs. $2(554) = 1108$.

Time = 30.55 (sec) , antiderivative size = 1986, normalized size of antiderivative = 3.55

method	result	size
parallelrisch	Expression too large to display	1986

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,method
=_RETURNVERBOSE)
```

output

```
1/4*(36*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c*d^5*n+48*A*B*x*ln(e*((b*x+
a)/(d*x+c))^n)*a*b^6*c*d^5*n+4*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^3*b^7*d^6
+12*A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^7*d^6-8*B^2*a^3*b^4*d^6*n^3-B^2*b^
7*c^3*d^3*n^3-4*A^2*a^3*b^4*d^6*n-2*A^2*b^7*c^3*d^3*n-30*A*B*a^2*b^5*c*d^5
*n^2+12*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^7*d^6*n+18*B^2*x^2*ln(e*((b*x+
a)/(d*x+c))^n)^2*b^7*c*d^5*n+48*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*d^
6*n^2+24*A*B*a*b^6*c^2*d^4*n^2+18*A*B*x*b^7*c^2*d^4*n^2+12*B^2*ln(e*((b*x+
a)/(d*x+c))^n)^2*a*b^6*c^2*d^4*n+24*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c^
2*d^4*n^2+24*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c*d^5+12*A^2*x*a*b^6*c*
d^5*n+12*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^5*c*d^5-8*A*B*ln(e*((b*x+a)
/(d*x+c))^n)*a^3*b^4*d^6*n-4*A*B*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^3*d^3*n+4
2*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c*d^5*n^2+24*A*B*x^2*ln(e*((b*x+a)
/(d*x+c))^n)^2*a*b^6*d^6+12*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^7*c*d^5-
12*A*B*x^2*a*b^6*d^6*n^2+12*A*B*x^2*b^7*c*d^5*n^2+8*B^2*x*ln(e*((b*x+a)/(d
*x+c))^n)^3*a*b^6*c*d^5-12*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^5*d^6*n
+6*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^7*c^2*d^4*n+24*B^2*x*ln(e*((b*x+a)/
(d*x+c))^n)*a^2*b^5*d^6*n^2+18*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^2*d^4
*n^2+18*B^2*x*a*b^6*c*d^5*n^3+12*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^5
*d^6-6*A*B*x*a^2*b^5*d^6*n^2-15*B^2*a^2*b^5*c*d^5*n^3+24*B^2*a*b^6*c^2*d^4
*n^3+8*A*B*a^3*b^4*d^6*n^2-2*A*B*b^7*c^3*d^3*n^2-6*A^2*a^2*b^5*c*d^5*n+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2052 vs. $2(554) = 1108$.

Time $= 0.13$ (sec) , antiderivative size $= 2052$, normalized size of antiderivative $= 3.66$

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x
, algorithm="fricas")
```

output

```
-1/4*(2*A^2*b^3*c^3 - 12*A^2*a*b^2*c^2*d + 6*A^2*a^2*b*c*d^2 + 4*A^2*a^3*d
^3 - 4*(B^2*b^3*d^3*n^2*x^3 + B^2*a^2*b*c*d^2*n^2 + (B^2*b^3*c*d^2 + 2*B^2
*a*b^2*d^3)*n^2*x^2 + (2*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*n^2*x)*log((b*x
+ a)/(d*x + c))^3 + (B^2*b^3*c^3 - 24*B^2*a*b^2*c^2*d + 15*B^2*a^2*b*c*d^2
 + 8*B^2*a^3*d^3)*n^2 - 6*(2*A^2*b^3*c*d^2 - 2*A^2*a*b^2*d^3 + 5*(B^2*b^3*
c*d^2 - B^2*a*b^2*d^3)*n^2 + 2*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*n)*x^2 + 2*
(B^2*b^3*c^3 - 6*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2 + 2*B^2*a^3*d^3 - 6*(
B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*x^2 - 3*(B^2*b^3*c^2*d + 2*B^2*a*b^2*c*d^2
- 3*B^2*a^2*b*d^3)*x - 6*(B^2*b^3*d^3*x^3 + B^2*a^2*b*c*d^2 + (B^2*b^3*c*d
^2 + 2*B^2*a*b^2*d^3)*x^2 + (2*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*x)*log((b*
x + a)/(d*x + c)))*log(e)^2 - 2*(6*A*B*a^2*b*c*d^2*n + 3*(B^2*b^3*d^3*n^2
+ 2*A*B*b^3*d^3*n)*x^3 - (B^2*b^3*c^3 - 6*B^2*a*b^2*c^2*d + 2*B^2*a^3*d^3)
*n^2 + 3*(3*B^2*b^3*c*d^2*n^2 + 2*(A*B*b^3*c*d^2 + 2*A*B*a*b^2*d^3)*n)*x^2
 + 3*((B^2*b^3*c^2*d + 4*B^2*a*b^2*c*d^2 - 2*B^2*a^2*b*d^3)*n^2 + 2*(2*A*B
*a*b^2*c*d^2 + A*B*a^2*b*d^3)*n)*x)*log((b*x + a)/(d*x + c))^2 + 2*(A*B*b^
3*c^3 - 12*A*B*a*b^2*c^2*d + 15*A*B*a^2*b*c*d^2 - 4*A*B*a^3*d^3)*n - 3*(2*
A^2*b^3*c^2*d + 4*A^2*a*b^2*c*d^2 - 6*A^2*a^2*b*d^3 + (7*B^2*b^3*c^2*d + 6
*B^2*a*b^2*c*d^2 - 13*B^2*a^2*b*d^3)*n^2 + 2*(3*A*B*b^3*c^2*d - 2*A*B*a*b^
2*c*d^2 - A*B*a^2*b*d^3)*n)*x + 2*(2*A*B*b^3*c^3 - 12*A*B*a*b^2*c^2*d + 6*
A*B*a^2*b*c*d^2 + 4*A*B*a^3*d^3 - 6*(2*A*B*b^3*c*d^2 - 2*A*B*a*b^2*d^3 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Timed out}$$

input

```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3/(d*i*x+c*i)**
2,x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4198 vs. $2(554) = 1108$.

Time = 0.34 (sec) , antiderivative size = 4198, normalized size of antiderivative = 7.50

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x
, algorithm="maxima")
```

output

```
1/2*B^2*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3
*a*b*d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4
)*g^3*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d
^3 - 2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2
*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3
*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^
4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2)
- 6*b*d^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a
^3*b*c*d^3 + a^4*d^4)*g^3*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2 +
 A*B*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*
b*d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*g
^3*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3
- 2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^
2*d^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d
+ 3*a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4 -
 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) - 6
*b*d^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)*g^3*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/4*
((b^3*c^3 - 24*a*b^2*c^2*d + 15*a^2*b*c*d^2 + 8*a^3*d^3 - 4*(b^3*d^3*x^3 +
 a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x
, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 31.67 (sec) , antiderivative size = 1784, normalized size of antiderivative = 3.19

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^3*(c*i + d*i*x
)^2),x)
```

output

```
(B^2*b*d^2*log(e*((a + b*x)/(c + d*x))^n)^3)/(g^3*i^2*n*(a*d - b*c)^4) - l
og(e*((a + b*x)/(c + d*x))^n)^2*(((B^2*(2*a*d + b*c))/(2*(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d)) + (3*B^2*b*d*x)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x*(a
^2*d*g^3*i^2 + 2*a*b*c*g^3*i^2) + x^2*(b^2*c*g^3*i^2 + 2*a*b*d*g^3*i^2) +
a^2*c*g^3*i^2 + b^2*d*g^3*i^2*x^3) - (3*B*b*d^2*(2*A + B*n))/(2*g^3*i^2*n*
(a*d - b*c)^4) + (3*B^2*b*d^2*(b*g^3*i^2*n*x^2*(a*d - b*c) + (a*c*g^3*i^2*
n*(a*d - b*c))/d + (g^3*i^2*n*x*(a*d + b*c)*(a*d - b*c))/d))/(g^3*i^2*n*(a
*d - b*c)^4*(x*(a^2*d*g^3*i^2 + 2*a*b*c*g^3*i^2) + x^2*(b^2*c*g^3*i^2 + 2*
a*b*d*g^3*i^2) + a^2*c*g^3*i^2 + b^2*d*g^3*i^2*x^3))) - ((4*A^2*a^2*d^2 -
2*A^2*b^2*c^2 + 8*B^2*a^2*d^2*n^2 - B^2*b^2*c^2*n^2 + 10*A^2*a*b*c*d - 8*A
*B*a^2*d^2*n - 2*A*B*b^2*c^2*n + 23*B^2*a*b*c*d*n^2 + 22*A*B*a*b*c*d*n)/(2
*(a*d - b*c)) + (3*x^2*(2*A^2*b^2*d^2 + 5*B^2*b^2*d^2*n^2 + 2*A*B*b^2*d^2*
n))/(a*d - b*c) + (3*x*(6*A^2*a*b*d^2 + 2*A^2*b^2*c*d + 13*B^2*a*b*d^2*n^2
 + 7*B^2*b^2*c*d*n^2 + 2*A*B*a*b*d^2*n + 6*A*B*b^2*c*d*n))/(2*(a*d - b*c))
)/(x*(2*a^4*d^3*g^3*i^2 + 4*a*b^3*c^3*g^3*i^2 - 6*a^2*b^2*c^2*d*g^3*i^2) +
 x^2*(2*b^4*c^3*g^3*i^2 + 4*a^3*b*d^3*g^3*i^2 - 6*a^2*b^2*c*d^2*g^3*i^2) +
 x^3*(2*a^2*b^2*d^3*g^3*i^2 + 2*b^4*c^2*d*g^3*i^2 - 4*a*b^3*c*d^2*g^3*i^2)
 + 2*a^2*b^2*c^3*g^3*i^2 + 2*a^4*c*d^2*g^3*i^2 - 4*a^3*b*c^2*d*g^3*i^2) -
(b*d^2*atan((b*d^2*(2*A^2 + 5*B^2*n^2 + 2*A*B*n)*(2*a^4*d^4*g^3*i^2 - 2*b^
4*c^4*g^3*i^2 + 4*a*b^3*c^3*d*g^3*i^2 - 4*a^3*b*c*d^3*g^3*i^2)*3i)/(2*g...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 3630, normalized size of antiderivative = 6.48

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x)
```

output

```
( - 24*log(a + b*x)*a**5*b*c*d**3*n - 24*log(a + b*x)*a**5*b*d**4*n*x - 12
*log(a + b*x)*a**4*b**2*c**2*d**2*n - 60*log(a + b*x)*a**4*b**2*c*d**3*n*x
 - 48*log(a + b*x)*a**4*b**2*d**4*n*x**2 - 36*log(a + b*x)*a**3*b**3*c**2*
d**2*n**2 - 24*log(a + b*x)*a**3*b**3*c**2*d**2*n*x - 48*log(a + b*x)*a**3
*b**3*c*d**3*n**3 - 36*log(a + b*x)*a**3*b**3*c*d**3*n**2*x - 48*log(a + b
*x)*a**3*b**3*c*d**3*n*x**2 - 48*log(a + b*x)*a**3*b**3*d**4*n**3*x - 24*l
og(a + b*x)*a**3*b**3*d**4*n*x**3 - 42*log(a + b*x)*a**2*b**4*c**2*d**2*n*
*3 - 72*log(a + b*x)*a**2*b**4*c**2*d**2*n**2*x - 12*log(a + b*x)*a**2*b**
4*c**2*d**2*n*x**2 - 138*log(a + b*x)*a**2*b**4*c*d**3*n**3*x - 72*log(a +
 b*x)*a**2*b**4*c*d**3*n**2*x**2 - 12*log(a + b*x)*a**2*b**4*c*d**3*n*x**3
 - 96*log(a + b*x)*a**2*b**4*d**4*n**3*x**2 - 84*log(a + b*x)*a*b**5*c**2*
d**2*n**3*x - 36*log(a + b*x)*a*b**5*c**2*d**2*n**2*x**2 - 132*log(a + b*x
)*a*b**5*c*d**3*n**3*x**2 - 36*log(a + b*x)*a*b**5*c*d**3*n**2*x**3 - 48*l
og(a + b*x)*a*b**5*d**4*n**3*x**3 - 42*log(a + b*x)*b**6*c**2*d**2*n**3*x*
*2 - 42*log(a + b*x)*b**6*c*d**3*n**3*x**3 + 24*log(c + d*x)*a**5*b*c*d**3
*n + 24*log(c + d*x)*a**5*b*d**4*n*x + 12*log(c + d*x)*a**4*b**2*c**2*d**2
*n + 60*log(c + d*x)*a**4*b**2*c*d**3*n*x + 48*log(c + d*x)*a**4*b**2*d**4
*n*x**2 + 36*log(c + d*x)*a**3*b**3*c**2*d**2*n**2 + 24*log(c + d*x)*a**3*
b**3*c**2*d**2*n*x + 48*log(c + d*x)*a**3*b**3*c*d**3*n**3 + 36*log(c + d*
x)*a**3*b**3*c*d**3*n**2*x + 48*log(c + d*x)*a**3*b**3*c*d**3*n*x**2 + ...
```

3.201 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx$

Optimal result 2302
Mathematica [B] (verified) 2303
Rubi [A] (verified) 2304
Maple [B] (verified) 2306
Fricas [B] (verification not implemented) 2307
Sympy [F(-1)] 2308
Maxima [B] (verification not implemented) 2308
Giac [F(-1)] 2309
Mupad [B] (verification not implemented) 2310
Reduce [B] (verification not implemented) 2311

Optimal result

Integrand size = 45, antiderivative size = 729

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx &= -\frac{2ABd^4n(a+bx)}{(bc-ad)^5g^4i^2(c+dx)}+\frac{2B^2d^4n^2(a+bx)}{(bc-ad)^5g^4i^2(c+dx)}\\ &-\frac{12b^2B^2d^2n^2(c+dx)}{(bc-ad)^5g^4i^2(a+bx)}+\frac{b^3B^2dn^2(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2}\\ &-\frac{2b^4B^2n^2(c+dx)^3}{27(bc-ad)^5g^4i^2(a+bx)^3}\\ &-\frac{2B^2d^4n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^5g^4i^2(c+dx)}\\ &-\frac{12b^2Bd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)}\\ &+\frac{2b^3Bdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2}\\ &-\frac{2b^4Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^5g^4i^2(a+bx)^3}\\ &+\frac{d^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(c+dx)}\\ &-\frac{6b^2d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)}\\ &+\frac{2b^3d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)^2}\\ &-\frac{b^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^5g^4i^2(a+bx)^3}\\ &-\frac{4bd^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^5g^4i^2n}\end{aligned}$$

output

```
-2*A*B*d^4*n*(b*x+a)/(-a*d+b*c)^5/g^4/i^2/(d*x+c)+2*B^2*d^4*n^2*(b*x+a)/(-
a*d+b*c)^5/g^4/i^2/(d*x+c)-12*b^2*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^5/g^4/i^2
/(b*x+a)+b^3*B^2*d*n^2*(d*x+c)^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-2/27*b^4*B
^2*n^2*(d*x+c)^3/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3-2*B^2*d^4*n*(b*x+a)*ln(e*(
(b*x+a)/(d*x+c))^n)/(-a*d+b*c)^5/g^4/i^2/(d*x+c)-12*b^2*B*d^2*n*(d*x+c)*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)+2*b^3*B*d*n*(d*
x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-2/9*
b^4*B*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*
x+a)^3+d^4*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^5/g^4/i^2/
(d*x+c)-6*b^2*d^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^5/g
^4/i^2/(b*x+a)+2*b^3*d*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b
*c)^5/g^4/i^2/(b*x+a)^2-1/3*b^4*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^
2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3-4/3*b*d^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
^3/B/(-a*d+b*c)^5/g^4/i^2/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1695 vs. $2(729) = 1458$.

Time = 2.57 (sec) , antiderivative size = 1695, normalized size of antiderivative = 2.33

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i +
 d*i*x)^2),x]
```

output

```
-1/27*(36*b*B^2*d^3*n^2*(a + b*x)^3*(c + d*x)*Log[(a + b*x)/(c + d*x)]^3 +
 9*B*n*Log[(a + b*x)/(c + d*x)]^2*(12*a^3*A*b*c*d^3 + b^4*B*c^4*n - 6*a*b^
3*B*c^3*d*n + 18*a^2*b^2*B*c^2*d^2*n - 3*a^4*B*d^4*n + 36*a^2*A*b^2*c*d^3*
x + 12*a^3*A*b*d^4*x - 2*b^4*B*c^3*d*n*x + 18*a*b^3*B*c^2*d^2*n*x + 36*a^2
*b^2*B*c*d^3*n*x - 12*a^3*b*B*d^4*n*x + 36*a*A*b^3*c*d^3*x^2 + 36*a^2*A*b^
2*d^4*x^2 + 6*b^4*B*c^2*d^2*n*x^2 + 54*a*b^3*B*c*d^3*n*x^2 + 12*A*b^4*c*d^
3*x^3 + 36*a*A*b^3*d^4*x^3 + 22*b^4*B*c*d^3*n*x^3 + 18*a*b^3*B*d^4*n*x^3 +
 12*A*b^4*d^4*x^4 + 10*b^4*B*d^4*n*x^4 + 12*b*B*d^3*(a + b*x)^3*(c + d*x)*
Log[e*((a + b*x)/(c + d*x))^n] - 12*b*B*d^3*n*(a + b*x)^3*(c + d*x)*Log[(a
 + b*x)/(c + d*x)]) + 3*b*d^2*(b*c - a*d)*(a + b*x)^2*(c + d*x)*(27*A^2 +
78*A*B*n + 92*B^2*n^2 + 27*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(9
*A + 13*B*n)*Log[(a + b*x)/(c + d*x)] + 27*B^2*n^2*Log[(a + b*x)/(c + d*x)
]^2 + 6*B*Log[e*((a + b*x)/(c + d*x))^n]*(9*A + 13*B*n - 9*B*n*Log[(a + b*
x)/(c + d*x)])) + 6*b*d^3*(a + b*x)^3*(c + d*x)*Log[a + b*x]*(18*A^2 + 30*
A*B*n + 55*B^2*n^2 + 18*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(6*A
+ 5*B*n)*Log[(a + b*x)/(c + d*x)] + 18*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2
+ 6*B*Log[e*((a + b*x)/(c + d*x))^n]*(6*A + 5*B*n - 6*B*n*Log[(a + b*x)/(c
 + d*x)])) + b*(b*c - a*d)^3*(c + d*x)*(9*A^2 + 6*A*B*n + 2*B^2*n^2 + 9*B^
2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(3*A + B*n)*Log[(a + b*x)/(c +
d*x)] + 9*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*((a + b*x)/(c ...
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 522, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}\,d\frac{a+bx}{c+dx}}{g^4i^2(bc-ad)^5}$$

$$\downarrow 2795$$

$$\int \frac{\left(\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d^4 - \frac{4b(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d^3}{a+bx} + \frac{6b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d^2}{(a+bx)^2} - \frac{4b^3(c+dx}{}\right.}{g^4 i^2 (bc-ad)^5}$$

$$\downarrow 2009$$

$$-\frac{b^4(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3} - \frac{2b^4Bn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9(a+bx)^3} + \frac{2b^3d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(a+bx)^2} + \frac{2b^3Bdn(c+}{}$$

input

```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i + d*i*x
)^2),x]
```

output

```
((-2*A*B*d^4*n*(a + b*x))/(c + d*x) + (2*B^2*d^4*n^2*(a + b*x))/(c + d*x)
- (12*b^2*B^2*d^2*n^2*(c + d*x))/(a + b*x) + (b^3*B^2*d*n^2*(c + d*x)^2)/(
a + b*x)^2 - (2*b^4*B^2*n^2*(c + d*x)^3)/(27*(a + b*x)^3) - (2*B^2*d^4*n*(
a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(c + d*x) - (12*b^2*B*d^2*n*(c +
d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (2*b^3*B*d*n*(c +
 d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 - (2*b^4*B*n*(
c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(a + b*x)^3) + (d^4*
(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c + d*x) - (6*b^2*d^2
*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) + (2*b^3*d*
(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)^2 - (b^4*(
c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*(a + b*x)^3) - (4*
b*d^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*B*n))/((b*c - a*d)^5*g^
4*i^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3055 vs. $2(721) = 1442$.

Time = 48.60 (sec) , antiderivative size = 3056, normalized size of antiderivative = 4.19

method	result	size
parallelrisch	Expression too large to display	3056

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,method
=_RETURNVERBOSE)
```

output

```
1/27*(36*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)^3*b^9*d^7+108*A^2*x^4*ln(e*((b*
x+a)/(d*x+c))^n)*b^9*d^7+180*A*B*x^3*b^9*c*d^6*n^2+108*B^2*x^2*ln(e*((b*x+
a)/(d*x+c))^n)^3*a*b^8*c*d^6+54*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^9*c^
2*d^5*n+648*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*d^7*n^2+198*B^2*x^2*
ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^2*d^5*n^2+480*B^2*x^2*a*b^8*c*d^6*n^3+324*
A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^7*d^7-342*A*B*x^2*a^2*b^7*d^7*n^
2+198*A*B*x^2*b^9*c^2*d^5*n^2+108*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^
7*c*d^6-108*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^6*d^7*n-18*B^2*x*ln(e*
((b*x+a)/(d*x+c))^n)^2*b^9*c^3*d^4*n+216*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a
^3*b^6*d^7*n^2-30*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^3*d^4*n^2-87*B^2*x
*a^2*b^7*c*d^6*n^3+567*B^2*x*a*b^8*c^2*d^5*n^3+324*A^2*x^2*ln(e*((b*x+a)/(
d*x+c))^n)*a*b^8*c*d^6+216*A^2*x^2*a*b^8*c*d^6*n+108*A*B*x*ln(e*((b*x+a)/(
d*x+c))^n)^2*a^3*b^6*d^7-114*A*B*x*a^3*b^6*d^7*n^2-30*A*B*x*b^9*c^3*d^4*n^
2-54*B^2*a^4*b^5*d^7*n^3+2*B^2*b^9*c^4*d^3*n^3-27*A^2*a^4*b^5*d^7*n+9*A^2*
b^9*c^4*d^3*n+972*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c*d^6*n+648*A*B*
x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*c*d^6*n-330*A*B*a^3*b^6*c*d^6*n^2+324*
A*B*a^2*b^7*c^2*d^5*n^2-54*A*B*a*b^8*c^3*d^4*n^2+162*B^2*x*ln(e*((b*x+a)/(
d*x+c))^n)^2*a*b^8*c^2*d^5*n+648*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*c
*d^6*n^2+486*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^2*d^5*n^2+324*A*B*x*l
n(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^7*c*d^6-216*A*B*x*ln(e*((b*x+a)/(d*x+c...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3183 vs. $2(721) = 1442$.

Time $= 0.17$ (sec) , antiderivative size $= 3183$, normalized size of antiderivative $= 4.37$

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x
, algorithm="fricas")
```

output

```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4/(d*i*x+c*i)**
2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6171 vs. $2(721) = 1442$.

Time = 0.49 (sec) , antiderivative size = 6171, normalized size of antiderivative = 8.47

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x
, algorithm="maxima")
```

output

```
-1/3*B^2*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 13*a^2*b*c*d^2 + 3*a
^3*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 8*a*b^2*c*d^2 -
11*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3
*b^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5
*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*
x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d
^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^
4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*
g^4*i^2*x + (a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c
^2*d^3 + a^7*c*d^4)*g^4*i^2) + 12*b*d^3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c
^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*
g^4*i^2) - 12*b*d^3*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^
3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2))*log(e*(b*x
/(d*x + c) + a/(d*x + c))^n)^2 - 2/3*A*B*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*
b^2*c^2*d + 13*a^2*b*c*d^2 + 3*a^3*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 -
 2*(b^3*c^2*d - 8*a*b^2*c*d^2 - 11*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3
*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b
^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3
*c*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a
^3*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Timed out}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x
, algorithm="giac")
```

output

```
Timed out
```

Mupad [B] (verification not implemented)

Time = 32.83 (sec) , antiderivative size = 3157, normalized size of antiderivative = 4.33

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^4*(c*i + d*i*x
)^2),x)
```

output

```
log(e*((a + b*x)/(c + d*x))^n)*((x*(8*A*B*b^2*c*d - 8*A*B*a*b*d^2 + 12*B^2
*b*d*n*(a*d + b*c) + (16*B^2*a*b*d^2*n)/3 - (16*B^2*b^2*c*d*n)/3) - 6*A*B*
a^2*d^2 + 2*A*B*b^2*c^2 + 6*B^2*a^2*d^2*n + (2*B^2*b^2*c^2*n)/3 + 12*B^2*b
^2*d^2*n*x^2 + 4*A*B*a*b*c*d + (16*B^2*a*b*c*d*n)/3)/(x*(3*a^6*d^4*g^4*i^2
 - 9*a^2*b^4*c^4*g^4*i^2 + 24*a^3*b^3*c^3*d*g^4*i^2 - 18*a^4*b^2*c^2*d^2*g
^4*i^2) - x^2*(9*a*b^5*c^4*g^4*i^2 - 9*a^5*b*d^4*g^4*i^2 - 18*a^2*b^4*c^3*
d*g^4*i^2 + 18*a^4*b^2*c*d^3*g^4*i^2) - x^3*(3*b^6*c^4*g^4*i^2 - 9*a^4*b^2
*d^4*g^4*i^2 + 24*a^3*b^3*c*d^3*g^4*i^2 - 18*a^2*b^4*c^2*d^2*g^4*i^2) + x^
4*(3*a^3*b^3*d^4*g^4*i^2 - 3*b^6*c^3*d*g^4*i^2 + 9*a*b^5*c^2*d^2*g^4*i^2 -
 9*a^2*b^4*c*d^3*g^4*i^2) - 3*a^3*b^3*c^4*g^4*i^2 + 3*a^6*c*d^3*g^4*i^2 +
9*a^4*b^2*c^3*d*g^4*i^2 - 9*a^5*b*c^2*d^2*g^4*i^2) - (4*d^3*(6*A*B*b + 5*B
^2*b*n)*(x*((a*d + b*c)*((3*a*g^4*i^2*n*(a*d - b*c)^4)/(2*d) + (3*g^4*i^2*
n*(a*d - b*c)^4*(2*a*d - b*c))/(2*d^2)) + (3*a*b*c*g^4*i^2*n*(a*d - b*c)^4
)/d) + x^2*(b*d*((3*a*g^4*i^2*n*(a*d - b*c)^4)/(2*d) + (3*g^4*i^2*n*(a*d -
 b*c)^4*(2*a*d - b*c))/(2*d^2)) + (3*b*g^4*i^2*n*(a*d + b*c)*(a*d - b*c)^4
)/d) + a*c*((3*a*g^4*i^2*n*(a*d - b*c)^4)/(2*d) + (3*g^4*i^2*n*(a*d - b*c)
^4*(2*a*d - b*c))/(2*d^2)) + 3*b^2*g^4*i^2*n*x^3*(a*d - b*c)^4))/(3*g^4*i^
2*n*(a*d - b*c)^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(x*(3*a^6*d^4*g^4*i^2 -
9*a^2*b^4*c^4*g^4*i^2 + 24*a^3*b^3*c^3*d*g^4*i^2 - 18*a^4*b^2*c^2*d^2*g^4*
i^2) - x^2*(9*a*b^5*c^4*g^4*i^2 - 9*a^5*b*d^4*g^4*i^2 - 18*a^2*b^4*c^3*...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 5699, normalized size of antiderivative = 7.82

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x)
```

output

```
( - 324*log(a + b*x)*a**6*b*c*d**4*n - 324*log(a + b*x)*a**6*b*d**5*n*x -
108*log(a + b*x)*a**5*b**2*c**2*d**3*n - 324*log(a + b*x)*a**5*b**2*c*d**4
*n**2 - 1080*log(a + b*x)*a**5*b**2*c*d**4*n*x - 324*log(a + b*x)*a**5*b**
2*d**5*n**2*x - 972*log(a + b*x)*a**5*b**2*d**5*n*x**2 - 396*log(a + b*x)*
a**4*b**3*c**2*d**3*n**2 - 324*log(a + b*x)*a**4*b**3*c**2*d**3*n*x - 810*
log(a + b*x)*a**4*b**3*c*d**4*n**3 - 1368*log(a + b*x)*a**4*b**3*c*d**4*n*
*2*x - 1296*log(a + b*x)*a**4*b**3*c*d**4*n*x**2 - 810*log(a + b*x)*a**4*b
**3*d**5*n**3*x - 972*log(a + b*x)*a**4*b**3*d**5*n**2*x**2 - 972*log(a +
b*x)*a**4*b**3*d**5*n*x**3 - 510*log(a + b*x)*a**3*b**4*c**2*d**3*n**3 - 1
188*log(a + b*x)*a**3*b**4*c**2*d**3*n**2*x - 324*log(a + b*x)*a**3*b**4*c
**2*d**3*n*x**2 - 2940*log(a + b*x)*a**3*b**4*c*d**4*n**3*x - 2160*log(a +
 b*x)*a**3*b**4*c*d**4*n**2*x**2 - 648*log(a + b*x)*a**3*b**4*c*d**4*n*x**
3 - 2430*log(a + b*x)*a**3*b**4*d**5*n**3*x**2 - 972*log(a + b*x)*a**3*b**
4*d**5*n**2*x**3 - 324*log(a + b*x)*a**3*b**4*d**5*n*x**4 - 1530*log(a + b
*x)*a**2*b**5*c**2*d**3*n**3*x - 1188*log(a + b*x)*a**2*b**5*c**2*d**3*n**
2*x**2 - 108*log(a + b*x)*a**2*b**5*c**2*d**3*n*x**3 - 3960*log(a + b*x)*a
**2*b**5*c*d**4*n**3*x**2 - 1512*log(a + b*x)*a**2*b**5*c*d**4*n**2*x**3 -
 108*log(a + b*x)*a**2*b**5*c*d**4*n*x**4 - 2430*log(a + b*x)*a**2*b**5*d*
*5*n**3*x**3 - 324*log(a + b*x)*a**2*b**5*d**5*n**2*x**4 - 1530*log(a + b*
x)*a*b**6*c**2*d**3*n**3*x**2 - 396*log(a + b*x)*a*b**6*c**2*d**3*n**2*...
```

3.202 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$

Optimal result 2313
Mathematica [B] (verified) 2314
Rubi [A] (verified) 2314
Maple [F] 2316
Fricas [F] 2316
Sympy [F(-1)] 2317
Maxima [F] 2317
Giac [F(-1)] 2318
Mupad [F(-1)] 2319
Reduce [F] 2319

Optimal result

Integrand size = 45, antiderivative size = 676

$$
\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx \\
&= \frac{B^2(bc-ad)g^3n^2(a+bx)^2}{4d^2i^3(c+dx)^2} - \frac{4AbB(bc-ad)g^3n(a+bx)}{d^3i^3(c+dx)} \\
&\quad + \frac{4bB^2(bc-ad)g^3n^2(a+bx)}{d^3i^3(c+dx)} - \frac{4bB^2(bc-ad)g^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^3(c+dx)} \\
&\quad - \frac{B(bc-ad)g^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^3(c+dx)^2} \\
&\quad + \frac{b^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^3} \\
&\quad + \frac{(bc-ad)g^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^2i^3(c+dx)^2} \\
&\quad + \frac{2b(bc-ad)g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^3(c+dx)} \\
&\quad + \frac{2b^2B(bc-ad)g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^3} \\
&\quad + \frac{3b^2(bc-ad)g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^3} \\
&\quad + \frac{2b^2B^2(bc-ad)g^3n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3} \\
&\quad + \frac{6b^2B(bc-ad)g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3} \\
&\quad - \frac{6b^2B^2(bc-ad)g^3n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}
\end{aligned}
$$

output
```
1/4*B^2*(-a*d+b*c)*g^3*n^2*(b*x+a)^2/d^2/i^3/(d*x+c)^2-4*A*b*B*(-a*d+b*c)*
g^3*n*(b*x+a)/d^3/i^3/(d*x+c)+4*b*B^2*(-a*d+b*c)*g^3*n^2*(b*x+a)/d^3/i^3/(
d*x+c)-4*b*B^2*(-a*d+b*c)*g^3*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/d^3/i^3/
(d*x+c)-1/2*B*(-a*d+b*c)*g^3*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
^2/i^3/(d*x+c)^2+b^2*g^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^3/i^3
+1/2*(-a*d+b*c)*g^3*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^2/i^3/(d
*x+c)^2+2*b*(-a*d+b*c)*g^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^3/i
^3/(d*x+c)+2*b^2*B*(-a*d+b*c)*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a
*d+b*c)/b/(d*x+c))/d^4/i^3+3*b^2*(-a*d+b*c)*g^3*(A+B*ln(e*((b*x+a)/(d*x+c)
)^n))^2*ln((-a*d+b*c)/b/(d*x+c))/d^4/i^3+2*b^2*B^2*(-a*d+b*c)*g^3*n^2*poly
log(2,d*(b*x+a)/b/(d*x+c))/d^4/i^3+6*b^2*B*(-a*d+b*c)*g^3*n*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^3-6*b^2*B^2*(-a*d+b
*c)*g^3*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/d^4/i^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6878 vs. $2(676) = 1352$.

Time $= 7.79$ (sec) , antiderivative size $= 6878$, normalized size of antiderivative $= 10.17$

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i
+ d*i*x)^3,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time $= 0.79$ (sec) , antiderivative size $= 545$, normalized size of antiderivative $= 0.81$, number of steps used $= 4$, number of rules used $= 3$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used $= \{2961, 2795, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ci+dix)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{g^3(bc-ad)\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i^3}$$

$$\downarrow 2795$$

$$\frac{g^3(bc-ad)\int\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b^3}{d^3\left(\frac{d(a+bx)}{c+dx}-b\right)^2}+\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b^2}{d^3\left(\frac{d(a+bx)}{c+dx}-b\right)}+\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b}{d^3}+\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a}{c+}\right.\right.\right.}{i^3}$$

$$\downarrow 2009$$

$$g^3(bc-ad)\left(\frac{6b^2Bn\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^4}+\frac{3b^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^4}+\frac{2b^2Bn\log\left(1-\frac{d}{b(}\right.\right.$$

input

```
Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*
x)^3,x]
```

output

```
((b*c - a*d)*g^3*((B^2*n^2*(a + b*x)^2)/(4*d^2*(c + d*x)^2) - (4*A*b*B*n*(
a + b*x))/(d^3*(c + d*x)) + (4*b*B^2*n^2*(a + b*x))/(d^3*(c + d*x)) - (4*b
*B^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d^3*(c + d*x)) - (B*n*(a
 + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d^2*(c + d*x)^2) + ((
a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*d^2*(c + d*x)^2) +
 (2*b*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^3*(c + d*x))
+ (b^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^3*(c + d*x)*
(b - (d*(a + b*x))/(c + d*x))) + (2*b^2*B*n*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 + (3*b^2*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 + (2*
b^2*B^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^4 + (6*b^2*B*n*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))]
)/d^4 - (6*b^2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/d^4))/i^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(dix+ci)^3}dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x
, algorithm="fricas")
```

output
```
integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*
a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2
*a^3*g^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*
b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log(e*((b*x + a)/(d*x + c))
^n))/(d^3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**
3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x
, algorithm="maxima")
```

output

```
3/2*A*B*a^2*b*g^3*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 -
 a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*
i^3) + 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*
d^4)*i^3) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 +
 a^2*d^4)*i^3)) + 1/2*A*B*a^3*g^3*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a
*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3)
 + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*lo
g(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*A^2*b^3*g^3*((
6*c^2*d*x + 5*c^3)/(d^6*i^3*x^2 + 2*c*d^5*i^3*x + c^2*d^4*i^3) - 2*x/(d^3*
i^3) + 6*c*log(d*x + c)/(d^4*i^3)) + 3/2*A^2*a*b^2*g^3*((4*c*d*x + 3*c^2)/
(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) + 2*log(d*x + c)/(d^3*i^3)) -
3*(2*d*x + c)*A*B*a^2*b*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^4*i^
3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - 3/2*(2*d*x + c)*A^2*a^2*b*g^3/(d^4*
i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - A*B*a^3*g^3*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A^2*a^3*
g^3/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) + 1/2*(2*B^2*b^3*d^3*g^3*x^3
 + 4*B^2*b^3*c*d^2*g^3*x^2 - 2*(2*b^3*c^2*d*g^3 - 6*a*b^2*c*d^2*g^3 + 3*a^
2*b*d^3*g^3)*B^2*x - (5*b^3*c^3*g^3 - 9*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^
3 + a^3*d^3*g^3)*B^2 - 6*((b^3*c*d^2*g^3 - a*b^2*d^3*g^3)*B^2*x^2 + 2*(b^3
*c^2*d*g^3 - a*b^2*c*d^2*g^3)*B^2*x + (b^3*c^3*g^3 - a*b^2*c^2*d*g^3)*B...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{Timed out}$$

input

```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x
, algorithm="giac")
```

output

```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$=\int \frac{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(c\,i+d\,i\,x)^3}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^3,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^3, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x)
```

output

```
(g**3*i*(4*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(c**3 + 3*c**2
*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*b**5*c**4*d**6 + 8*int((log(((a
+ b*x)**n*e)/(c + d*x)**n)**2*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d
**3*x**3),x)*a**2*b**5*c**3*d**7*x + 4*int((log(((a + b*x)**n*e)/(c + d*x)
**n)**2*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*b**5
*c**2*d**8*x**2 - 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(c**3
 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**6*c**5*d**5 - 16*int((l
og(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x
**2 + d**3*x**3),x)*a*b**6*c**4*d**6*x - 8*int((log(((a + b*x)**n*e)/(c +
d*x)**n)**2*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**
6*c**3*d**7*x**2 + 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(c**
3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**7*c**6*d**4 + 8*int((log
(((a + b*x)**n*e)/(c + d*x)**n)**2*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**
2 + d**3*x**3),x)*b**7*c**5*d**5*x + 4*int((log(((a + b*x)**n*e)/(c + d*x)
**n)**2*x**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**7*c**4
*d**6*x**2 + 12*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(c**3 + 3
*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**3*b**4*c**4*d**6 + 24*int((lo
g(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x*
*2 + d**3*x**3),x)*a**3*b**4*c**3*d**7*x + 12*int((log(((a + b*x)**n*e)/(c
 + d*x)**n)**2*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)...
```

3.203 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$

Optimal result . . . 2321
Mathematica [B] (verified) . . . 2322
Rubi [A] (verified) . . . 2323
Maple [F] . . . 2325
Fricas [F] . . . 2325
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Giac [F(-1)] . . . 2327
Mupad [F(-1)] . . . 2328
Reduce [F] . . . 2328

Optimal result

Integrand size = 45, antiderivative size = 441

$$
\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx \\
&= -\frac{B^2g^2n^2(a+bx)^2}{4di^3(c+dx)^2} + \frac{2AbBg^2n(a+bx)}{d^2i^3(c+dx)} - \frac{2bB^2g^2n^2(a+bx)}{d^2i^3(c+dx)} \\
&\quad + \frac{2bB^2g^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^3(c+dx)} + \frac{Bg^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^3(c+dx)^2} \\
&\quad - \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2di^3(c+dx)^2} - \frac{bg^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^3(c+dx)} \\
&\quad - \frac{b^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^3} \\
&\quad - \frac{2b^2Bg^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3} \\
&\quad + \frac{2b^2B^2g^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}
\end{aligned}
$$

output

```
-1/4*B^2*g^2*n^2*(b*x+a)^2/d/i^3/(d*x+c)^2+2*A*b*B*g^2*n*(b*x+a)/d^2/i^3/(
d*x+c)-2*b*B^2*g^2*n^2*(b*x+a)/d^2/i^3/(d*x+c)+2*b*B^2*g^2*n*(b*x+a)*ln(e*
((b*x+a)/(d*x+c))^n)/d^2/i^3/(d*x+c)+1/2*B*g^2*n*(b*x+a)^2*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))/d/i^3/(d*x+c)^2-1/2*g^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x
+c))^n))^2/d/i^3/(d*x+c)^2-b*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2
/d^2/i^3/(d*x+c)-b^2*g^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b
/(d*x+c))/d^3/i^3-2*b^2*B*g^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,
d*(b*x+a)/b/(d*x+c))/d^3/i^3+2*b^2*B^2*g^2*n^2*polylog(3,d*(b*x+a)/b/(d*x+
c))/d^3/i^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4493 vs. $2(441) = 882$.

Time $= 7.21$ (sec) , antiderivative size $= 4493$, normalized size of antiderivative $= 10.19$

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{Result too large to show}$$

input

```
Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i
+ d*i*x)^3,x]
```

output

```
(-2*(-(A^2*b^2*c*g^2) + a*A^2*b*d*g^2 - 2*A*b^2*B*c*g^2*(Log[e*((a + b*x)/
(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) + 2*a*A*b*B*d*g^2*(Log[e*((a +
 b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) - b^2*B^2*c*g^2*(Log[e*(
(a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2 + a*b*B^2*d*g^2*(L
og[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2))/(d^3*i^3*(
c + d*x)) + (-(A^2*b^2*c^2*g^2) + 2*a*A^2*b*c*d*g^2 - a^2*A^2*d^2*g^2 - 2*
A*b^2*B*c^2*g^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x
)]) + 4*a*A*b*B*c*d*g^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/
(c + d*x)]) - 2*a^2*A*B*d^2*g^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a
 + b*x)/(c + d*x)]) - b^2*B^2*c^2*g^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*
Log[(a + b*x)/(c + d*x)])^2 + 2*a*b*B^2*c*d*g^2*(Log[e*((a + b*x)/(c + d*x
))^n] - n*Log[(a + b*x)/(c + d*x)])^2 - a^2*B^2*d^2*g^2*(Log[e*((a + b*x)/
(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2)/(2*d^3*i^3*(c + d*x)^2) + (
b^2*g^2*(A + B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)
]))^2*Log[c + d*x])/(d^3*i^3) + (2*a^2*B*g^2*n*(A + B*(Log[e*((a + b*x)/(c
 + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]))*(((c/d + x)*(2*Log[c/d + x] + 4
*Log[c/d + x]^2))/(8*(c + d*x)^3*Log[c/d + x]) + ((d*(a/b + x))/((-c + (a*
d)/b)^3*(1 - (d*(a/b + x))/(-c + (a*d)/b))) - ((d^2*(a/b + x)^2)/((-c + (a
*d)/b)^4*(1 - (d*(a/b + x))/(-c + (a*d)/b))^2) + (2*d*(a/b + x))/((-c + (a
*d)/b)^3*(1 - (d*(a/b + x))/(-c + (a*d)/b))))*Log[a/b + x] - Log[1 - (d...
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ci+dix)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{g^2\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{i^3}$$

$$\downarrow 2795$$

$$\frac{g^2\int\left(-\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d(c+dx)}-\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)}-\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2}\right)d\frac{a+bx}{c+dx}}{i^3}$$

$$\downarrow 2009$$

$$g^2\left(-\frac{2b^2Bn\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3}-\frac{b^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^3}-\frac{b(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{d^2(c+dx)}$$

input

```
Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*
x)^3,x]
```

output

```
(g^2*(-1/4*(B^2*n^2*(a + b*x)^2)/(d*(c + d*x)^2) + (2*A*b*B*n*(a + b*x))/(
d^2*(c + d*x)) - (2*b*B^2*n^2*(a + b*x))/(d^2*(c + d*x)) + (2*b*B^2*n*(a +
 b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d^2*(c + d*x)) + (B*n*(a + b*x)^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)^2) - ((a + b*x)^2*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*d*(c + d*x)^2) - (b*(a + b*x)*(
A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^2*(c + d*x)) - (b^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 -
 (2*b^2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x)
)/(b*(c + d*x))])/d^3 + (2*b^2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*
x))])/d^3))/i^3
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bgx+ag)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(dix+ci)^3}dx$$

input

```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x)
```

output

```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x
, algorithm="fricas")
```

output

```
integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x
^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(
A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log(e*((b*x + a)/(d*x + c
))^n))/(d^3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$

$$= g^2\left(\int \frac{A^2a^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{A^2b^2x^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{B^2a^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2ABa^2\log\left(e\left(\frac{}{c+}\right.\right.\right.$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**
3,x)
```

output
```
g**2*(Integral(A**2*a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(A**2*b**2*x**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**
3), x) + Integral(B**2*a**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c*
*3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*a**2*log
(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
d**3*x**3), x) + Integral(2*A**2*a*b*x/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
+ d**3*x**3), x) + Integral(B**2*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d
*x))**n)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral
(2*A*B*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*
x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*B**2*a*b*x*log(e*(a/(c + d
*x) + b*x/(c + d*x))**n)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3
), x) + Integral(4*A*B*a*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**3
 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/i**3
```

Maxima [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x
, algorithm="maxima")
```

output

```
A*B*a*b*g^2*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 - a*d^5
)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3) +
 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i
^3) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d
^4)*i^3)) + 1/2*A*B*a^2*g^2*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*
i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b
^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x
 + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) + 1/2*A^2*b^2*g^2*((4*c*d*
x + 3*c^2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) + 2*log(d*x + c)/(d
^3*i^3)) - 2*(2*d*x + c)*A*B*a*b*g^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
)/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (2*d*x + c)*A^2*a*b*g^2/(d
^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - A*B*a^2*g^2*log(e*(b*x/(d*x +
c) + a/(d*x + c))^n)/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A^2*a
^2*g^2/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) + 1/2*(4*(b^2*c*d*g^2 - a
*b*d^2*g^2)*B^2*x + (3*b^2*c^2*g^2 - 2*a*b*c*d*g^2 - a^2*d^2*g^2)*B^2 + 2*
(B^2*b^2*d^2*g^2*x^2 + 2*B^2*b^2*c*d*g^2*x + B^2*b^2*c^2*g^2)*log(d*x + c)
)*log((d*x + c)^n)^2/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) - integra
te(-(2*B^2*a*b*d^2*g^2*x*log(e)^2 + B^2*a^2*d^2*g^2*log(e)^2 + (B^2*b^2*d^
2*g^2*log(e)^2 + 2*A*B*b^2*d^2*g^2*log(e))*x^2 + (B^2*b^2*d^2*g^2*x^2 + 2*
B^2*a*b*d^2*g^2*x + B^2*a^2*d^2*g^2)*log((b*x + a)^n)^2 + 2*(2*B^2*a*b*...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{Timed out}$$

input

```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x
, algorithm="giac")
```

output

```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$=\int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(c\,i+d\,i\,x)^3}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^3,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^3, x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{too large to display}$$

input
```
int((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x)
```

output

```
(g**2*i*(4*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(c**3 + 3*c**2
*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*b**4*c**4*d**5 + 8*int((log(((a
+ b*x)**n*e)/(c + d*x)**n)**2*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d
**3*x**3),x)*a**2*b**4*c**3*d**6*x + 4*int((log(((a + b*x)**n*e)/(c + d*x)
**n)**2*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*b**4
*c**2*d**7*x**2 - 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(c**3
 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**5*c**5*d**4 - 16*int((l
og(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x
**2 + d**3*x**3),x)*a*b**5*c**4*d**5*x - 8*int((log(((a + b*x)**n*e)/(c +
d*x)**n)**2*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b**
5*c**3*d**6*x**2 + 4*int((log(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(c**
3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**6*c**6*d**3 + 8*int((log
(((a + b*x)**n*e)/(c + d*x)**n)**2*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**
2 + d**3*x**3),x)*b**6*c**5*d**4*x + 4*int((log(((a + b*x)**n*e)/(c + d*x)
**n)**2*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**6*c**4
*d**5*x**2 + 8*int((log(((a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c**3 + 3*c**
2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**3*b**3*c**4*d**5 + 16*int((log(((
a + b*x)**n*e)/(c + d*x)**n)*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d*
*3*x**3),x)*a**3*b**3*c**3*d**6*x + 8*int((log(((a + b*x)**n*e)/(c + d*x)*
*n)*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**3*b**3*...
```

3.204 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$

Optimal result . . . 2330
Mathematica [C] (verified) . . . 2331
Rubi [A] (verified) . . . 2332
Maple [B] (verified) . . . 2333
Fricas [B] (verification not implemented) . . . 2334
Sympy [F] . . . 2335
Maxima [B] (verification not implemented) . . . 2336
Giac [A] (verification not implemented) . . . 2337
Mupad [B] (verification not implemented) . . . 2337
Reduce [B] (verification not implemented) . . . 2338

Optimal result

Integrand size = 43, antiderivative size = 151

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \frac{B^2gn^2(a+bx)^2}{4(bc-ad)i^3(c+dx)^2} - \frac{Bgn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)i^3(c+dx)^2} + \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)i^3(c+dx)^2}$$

output

```
1/4*B^2*g*n^2*(b*x+a)^2/(-a*d+b*c)/i^3/(d*x+c)^2-1/2*B*g*n*(b*x+a)^2*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/i^3/(d*x+c)^2+1/2*g*(b*x+a)^2*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/i^3/(d*x+c)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.15 (sec) , antiderivative size = 803, normalized size of antiderivative = 5.32

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{g\Big(2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - 4b(bc-ad)(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + 4bBn(c+dx)$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i +
d*i*x)^3,x]
```

output
```
(g*(2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - 4*b*(b*c -
a*d)*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 4*b*B*n*(c + d*x
)*(2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*b*(c + d*x)*Lo
g[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b*(c + d*x)*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*B*n*(b*c - a*d + b*(c + d*
x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]) - b*B*n*(c + d*x)*(Log[a + b*x
]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a +
 b*x))/(-(b*c) + a*d)]) + b*B*n*(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) +
a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d
)])) - B*n*(2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b*(
b*c - a*d)*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b^2*(c + d
*x)^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 4*b^2*(c + d*x
)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 4*b*B*n*(c + d*x
)*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]) - B*n*
((b*c - a*d)^2 + 2*b*(b*c - a*d)*(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x
] - 2*b^2*(c + d*x)^2*Log[c + d*x]) - 2*b^2*B*n*(c + d*x)^2*(Log[a + b*x]*
(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b
*x))/(-(b*c) + a*d)]) + 2*b^2*B*n*(c + d*x)^2*((2*Log[(d*(a + b*x))/(-(b*c
) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c -
 a*d)])))/(4*d^2*(b*c - a*d)*i^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ci+dix)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{g\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{c+dx}d\frac{a+bx}{c+dx}}{i^3(bc-ad)}$$

$$\downarrow 2742$$

$$\frac{g\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(c+dx)^2}-Bn\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{c+dx}d\frac{a+bx}{c+dx}\right)}{i^3(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{g\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(c+dx)^2}-Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(c+dx)^2}-\frac{Bn(a+bx)^2}{4(c+dx)^2}\right)\right)}{i^3(bc-ad)}$$

input

```
Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*x)
^3,x]
```

output

```
(g*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(c + d*x)^2)
 - B*n*(-1/4*(B*n*(a + b*x)^2)/(c + d*x)^2 + ((a + b*x)^2*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]))/(2*(c + d*x)^2))))/((b*c - a*d)*i^3)
```

Defintions of rubi rules used

rule 2741

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(145) = 290.

Time = 4.75 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.51

method	result
parallelrisch	$-\frac{8ABx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)ab^2d^4gn+4ABx^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3d^4gn+4B^2x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2ab^2d^4gn-4B^2x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a$

input

```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,method=_
RETURNVERBOSE)
```

output

```
-1/4*(8*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^4*g*n+4*A*B*x^2*ln(e*((b*x
+a)/(d*x+c))^n)*b^3*d^4*g*n+4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^2*d^4*
g*n-4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^4*g*n^2-4*A*B*x*a*b^2*d^4*g*
n^2+4*A*B*x*b^3*c*d^3*g*n^2+4*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*d^4*g*n+
2*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*d^4*g*n-2*B^2*x^2*ln(e*((b*x+a)/
(d*x+c))^n)*b^3*d^4*g*n^2+2*B^2*x*a*b^2*d^4*g*n^3-2*B^2*x*b^3*c*d^3*g*n^3+
2*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b*d^4*g*n-2*B^2*ln(e*((b*x+a)/(d*x+c
))^n)*a^2*b*d^4*g*n^2+4*A^2*x*a*b^2*d^4*g*n-4*A^2*x*b^3*c*d^3*g*n-2*A*B*a^
2*b*d^4*g*n^2+2*A*B*b^3*c^2*d^2*g*n^2+B^2*a^2*b*d^4*g*n^3-B^2*b^3*c^2*d^2*
g*n^3+2*A^2*a^2*b*d^4*g*n-2*A^2*b^3*c^2*d^2*g*n)/i^3/(d*x+c)^2/b/d^4/n/(a*
d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(145) = 290$.

Time $= 0.10$ (sec) , antiderivative size $= 600$, normalized size of antiderivative $= 3.97$

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx =$$

$$-\frac{(B^2b^2c^2-B^2a^2d^2)gn^2-2\left(ABb^2c^2-ABa^2d^2\right)gn+2\left(2\left(B^2b^2cd-B^2abd^2\right)gx+\left(B^2b^2c^2-B^2a^2d^2\right)g\right.}{}$$

input

```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,
algorithm="fricas")
```

output

```
-1/4*((B^2*b^2*c^2 - B^2*a^2*d^2)*g*n^2 - 2*(A*B*b^2*c^2 - A*B*a^2*d^2)*g*
n + 2*(2*(B^2*b^2*c*d - B^2*a*b*d^2)*g*x + (B^2*b^2*c^2 - B^2*a^2*d^2)*g)*
log(e)^2 - 2*(B^2*b^2*d^2*g*n^2*x^2 + 2*B^2*a*b*d^2*g*n^2*x + B^2*a^2*d^2*
g*n^2)*log((b*x + a)/(d*x + c))^2 + 2*(A^2*b^2*c^2 - A^2*a^2*d^2)*g + 2*((
B^2*b^2*c*d - B^2*a*b*d^2)*g*n^2 - 2*(A*B*b^2*c*d - A*B*a*b*d^2)*g*n + 2*(
A^2*b^2*c*d - A^2*a*b*d^2)*g)*x - 2*((B^2*b^2*c^2 - B^2*a^2*d^2)*g*n - 2*(
A*B*b^2*c^2 - A*B*a^2*d^2)*g + 2*((B^2*b^2*c*d - B^2*a*b*d^2)*g*n - 2*(A*B
*b^2*c*d - A*B*a*b*d^2)*g)*x + 2*(B^2*b^2*d^2*g*n*x^2 + 2*B^2*a*b*d^2*g*n*
x + B^2*a^2*d^2*g*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*(B^2*a^2*d^2*g*n
^2 - 2*A*B*a^2*d^2*g*n + (B^2*b^2*d^2*g*n^2 - 2*A*B*b^2*d^2*g*n)*x^2 + 2*(
B^2*a*b*d^2*g*n^2 - 2*A*B*a*b*d^2*g*n)*x)*log((b*x + a)/(d*x + c)))/((b*c*
d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*
d^3)*i^3)
```

Sympy [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$

$$= \frac{g\left(\int \frac{A^2a}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{A^2bx}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{B^2a\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2ABa\log\left(e\left(\frac{a}{c+dx}\right.\right.\right.}{c^3+3c^2dx+3cd^2x}$$

$$\frac{}{i^3}$$

input

```
integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**3,x
)
```

output

```
g*(Integral(A**2*a/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + I
ntegral(A**2*b*x/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Int
egral(B**2*a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**3 + 3*c**2*d*x
 + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*a*log(e*(a/(c + d*x) +
b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + I
ntegral(B**2*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**3 + 3*c**2
*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*b*x*log(e*(a/(c + d
*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x))/i**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1995 vs. $2(145) = 290$.

Time $= 0.13$ (sec) , antiderivative size $= 1995$, normalized size of antiderivative $= 13.21$

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,
algorithm="maxima")
```

output

```
1/2*A*B*b*g*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 - a*d^5
)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3) +
 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i
^3) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d
^4)*i^3)) + 1/2*A*B*a*g*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*
x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*l
og(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)
/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*(2*d*x + c)*B^2*b*g*log(
e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^
2*i^3) + 1/4*(2*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*
(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log(b*x +
 a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c
^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
 - (7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c)^
2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(
b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))*n^2/(b^2*c^4*d*i^3 - 2*a
*b*c^3*d^2*i^3 + a^2*c^2*d^3*i^3 + (b^2*c^2*d^3*i^3 - 2*a*b*c*d^4*i^3 + a^
2*d^5*i^3)*x^2 + 2*(b^2*c^3*d^2*i^3 - 2*a*b*c^2*d^3*i^3 + a^2*c*d^4*i^3...
```

Giac [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.29

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(\frac{2\,(bx+a)^2B^2gn^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(dx+c)^2i^3}-\frac{2\,(B^2gn^2-2\,B^2gn\log(e)-2\,ABgn)(bx+a)^2\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)^2i^3}+\frac{(B^2gn\right.$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,
algorithm="giac")
```

output
```
1/4*(2*(b*x + a)^2*B^2*g*n^2*log((b*x + a)/(d*x + c))^2/((d*x + c)^2*i^3)
- 2*(B^2*g*n^2 - 2*B^2*g*n*log(e) - 2*A*B*g*n)*(b*x + a)^2*log((b*x + a)/(
d*x + c))/((d*x + c)^2*i^3) + (B^2*g*n^2 - 2*B^2*g*n*log(e) + 2*B^2*g*log(
e)^2 - 2*A*B*g*n + 4*A*B*g*log(e) + 2*A^2*g)*(b*x + a)^2/((d*x + c)^2*i^3)
)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 29.01 (sec) , antiderivative size = 565, normalized size of antiderivative = 3.74

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$=-\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^2\left(\frac{\frac{B^2ag}{2d}+\frac{B^2bgx}{d}+\frac{B^2bcg}{2d^2}}{c^2i^3+2cdi^3x+d^2i^3x^2}+\frac{B^2b^2g}{2d^2i^3\,(ad-bc)}\right)$$
$$-\frac{x\,(2bdgA^2-2bdgABn+bdgB^2n^2)+A^2adg+A^2bcg+\frac{B^2adgn^2}{2}+\frac{B^2bcgn^2}{2}-ABadgn-A}{2c^2d^2i^3+4cd^3i^3x+2d^4i^3x^2}$$
$$-\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(\frac{ABadg+ABbcg-B^2adgn+B^2bcgn+2ABbdgx}{c^2d^2i^3+2cd^3i^3x+d^4i^3x^2}\right.$$
$$\left.-\frac{B^2b^2g\left(\frac{cd^2i^3n(ad-bc)}{2b}+\frac{d^3i^3nx(ad-bc)}{b}-\frac{d^2i^3n(ad-bc)(ad-2bc)}{2b^2}\right)}{d^2i^3\,(ad-bc)\,(c^2d^2i^3+2cd^3i^3x+d^4i^3x^2)}\right)$$
$$-\frac{Bb^2gn\operatorname{atan}\left(\frac{Bb^2gn(2A-Bn)\left(\frac{ad^3i^3+bcd^2i^3}{d^2i^3}+2bdx\right)1\mathrm{i}}{(ad-bc)(B^2b^2gn^2-2ABb^2gn)}\right)(2A-Bn)\,1\mathrm{i}}{d^2i^3\,(ad-bc)}$$

input
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x)
^3,x)
```

output
```
- log(e*((a + b*x)/(c + d*x))^n)^2*(((B^2*a*g)/(2*d) + (B^2*b*g*x)/d + (B^
2*b*c*g)/(2*d^2))/(c^2*i^3 + d^2*i^3*x^2 + 2*c*d*i^3*x) + (B^2*b^2*g)/(2*d
^2*i^3*(a*d - b*c))) - (x*(2*A^2*b*d*g + B^2*b*d*g*n^2 - 2*A*B*b*d*g*n) +
A^2*a*d*g + A^2*b*c*g + (B^2*a*d*g*n^2)/2 + (B^2*b*c*g*n^2)/2 - A*B*a*d*g*
n - A*B*b*c*g*n)/(2*c^2*d^2*i^3 + 2*d^4*i^3*x^2 + 4*c*d^3*i^3*x) - log(e*(
(a + b*x)/(c + d*x))^n)*((A*B*a*d*g + A*B*b*c*g - B^2*a*d*g*n + B^2*b*c*g*
n + 2*A*B*b*d*g*x)/(c^2*d^2*i^3 + d^4*i^3*x^2 + 2*c*d^3*i^3*x) - (B^2*b^2*
g*((c*d^2*i^3*n*(a*d - b*c))/(2*b) + (d^3*i^3*n*x*(a*d - b*c))/b - (d^2*i^
3*n*(a*d - b*c)*(a*d - 2*b*c))/(2*b^2)))/(d^2*i^3*(a*d - b*c)*(c^2*d^2*i^3
 + d^4*i^3*x^2 + 2*c*d^3*i^3*x))) - (B*b^2*g*n*atan((B*b^2*g*n*(2*A - B*n)
*((a*d^3*i^3 + b*c*d^2*i^3)/(d^2*i^3) + 2*b*d*x)*1i)/((a*d - b*c)*(B^2*b^2
*g*n^2 - 2*A*B*b^2*g*n)))*(2*A - B*n)*1i)/(d^2*i^3*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 761, normalized size of antiderivative = 5.04

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
int((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x)
```

output

```
(g*i*( - 4*log(a + b*x)*a**2*b**2*c**2*n - 8*log(a + b*x)*a**2*b**2*c*d*n*
x - 4*log(a + b*x)*a**2*b**2*d**2*n*x**2 + 2*log(a + b*x)*a*b**3*c**2*n**2
 + 4*log(a + b*x)*a*b**3*c*d*n**2*x + 2*log(a + b*x)*a*b**3*d**2*n**2*x**2
 + 4*log(c + d*x)*a**2*b**2*c**2*n + 8*log(c + d*x)*a**2*b**2*c*d*n*x + 4*
log(c + d*x)*a**2*b**2*d**2*n*x**2 - 2*log(c + d*x)*a*b**3*c**2*n**2 - 4*l
og(c + d*x)*a*b**3*c*d*n**2*x - 2*log(c + d*x)*a*b**3*d**2*n**2*x**2 - 2*l
og(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*c*d - 4*log(((a + b*x)**n*e
)/(c + d*x)**n)**2*a*b**3*c*d*x - 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*
b**4*c*d*x**2 - 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*c*d + 4*log(((
a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*c**2 + 2*log(((a + b*x)**n*e)/(c +
d*x)**n)*a**2*b**2*c*d*n + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*
d**2*x**2 - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c**2*n - 4*log(((a
 + b*x)**n*e)/(c + d*x)**n)*a*b**3*c*d*x**2 - 2*log(((a + b*x)**n*e)/(c +
d*x)**n)*a*b**3*d**2*n*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*c*
d*n*x**2 - 2*a**4*c*d + 2*a**3*b*c**2 + 2*a**3*b*c*d*n + 2*a**3*b*d**2*x**
2 - 2*a**2*b**2*c**2*n - a**2*b**2*c*d*n**2 - 2*a**2*b**2*c*d*x**2 - 2*a**
2*b**2*d**2*n*x**2 + a*b**3*c**2*n**2 + 2*a*b**3*c*d*n*x**2 + a*b**3*d**2*
n**2*x**2 - b**4*c*d*n**2*x**2))/(4*c*d*(a*c**2*d + 2*a*c*d**2*x + a*d**3*
x**2 - b*c**3 - 2*b*c**2*d*x - b*c*d**2*x**2))
```

3.205 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$

Optimal result . . . 2340
Mathematica [C] (verified) . . . 2341
Rubi [A] (verified) . . . 2342
Maple [B] (verified) . . . 2343
Fricas [B] (verification not implemented) . . . 2344
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Maxima [B] (verification not implemented) . . . 2346
Giac [A] (verification not implemented) . . . 2347
Mupad [B] (verification not implemented) . . . 2348
Reduce [B] (verification not implemented) . . . 2349

Optimal result

Integrand size = 35, antiderivative size = 317

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = &-\frac{B^2dn^2(a+bx)^2}{4(bc-ad)^2i^3(c+dx)^2}\\ &-\frac{2AbBn(a+bx)}{(bc-ad)^2i^3(c+dx)}+\frac{2bB^2n^2(a+bx)}{(bc-ad)^2i^3(c+dx)}\\ &-\frac{2bB^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^2i^3(c+dx)}\\ &+\frac{Bdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2i^3(c+dx)^2}\\ &-\frac{d(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2i^3(c+dx)^2}\\ &+\frac{b(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2i^3(c+dx)}\end{aligned}$$

output
```
-1/4*B^2*d*n^2*(b*x+a)^2/(-a*d+b*c)^2/i^3/(d*x+c)^2-2*A*b*B*n*(b*x+a)/(-a*
d+b*c)^2/i^3/(d*x+c)+2*b*B^2*n^2*(b*x+a)/(-a*d+b*c)^2/i^3/(d*x+c)-2*b*B^2*
n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^2/i^3/(d*x+c)+1/2*B*d*n*(b*
x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/i^3/(d*x+c)^2-1/2*d*(b
*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/i^3/(d*x+c)^2+b*(b*
x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/i^3/(d*x+c)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.46 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.46

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$

$$= \frac{-2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{Bn\left(2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b(bc-ad)(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b^2(c+dx)^2\,\mathrm{lo}\right.}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x)^3,x]
```

output
```
(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)^2*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b*(b*c - a*d)*(c + d*x)*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]) + 4*b^2*(c + d*x)^2*Log[a + b*x]*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]) - 4*b^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c +
d*x))^n])*Log[c + d*x] - 4*b*B*n*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log[a
+ b*x] - b*(c + d*x)*Log[c + d*x]) - B*n*((b*c - a*d)^2 + 2*b*(b*c - a*d)*
(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*x
]) - 2*b^2*B*n*(c + d*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x
))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^2*B*n
*(c + d*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
 d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(4*d*i^3
*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2951, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ci+dix)^3}\,dx$$

$$\downarrow 2951$$

$$\frac{\int \left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d\frac{a+bx}{c+dx}}{i^3(bc-ad)^2}$$

$$\downarrow 2767$$

$$\frac{\int \left(b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{d(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{c+dx}\right) d\frac{a+bx}{c+dx}}{i^3(bc-ad)^2}$$

$$\downarrow 2009$$

$$\frac{Bdn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(c+dx)^2}+\frac{b(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-\frac{d(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(c+dx)^2}-\frac{2AbBn(a+bx)}{c+dx}-\frac{2bI}{}$$

$$i^3(bc-ad)^2$$

input

```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x)^3,x]
```

output

```
(-1/4*(B^2*d*n^2*(a + b*x)^2)/(c + d*x)^2 - (2*A*b*B*n*(a + b*x))/(c + d*x
) + (2*b*B^2*n^2*(a + b*x))/(c + d*x) - (2*b*B^2*n*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n])/(c + d*x) + (B*d*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(2*(c + d*x)^2) - (d*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^2)/(2*(c + d*x)^2) + (b*(a + b*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2)/(c + d*x))/((b*c - a*d)^2*i^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(311) = 622$.

Time = 4.81 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.12

method	result
parallelrisch	$-\frac{B^2a^2bd^5n^3+7B^2b^3c^2d^3n^3+2A^2a^2bd^5n+2A^2b^3c^2d^3n-4ABxb^3cd^4n^2-4B^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2ab^2cd^4n+8B^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,method=_RETURNVERBOS
E)
```

output

```
-1/4*(B^2*a^2*b*d^5*n^3+7*B^2*b^3*c^2*d^3*n^3+2*A^2*a^2*b*d^5*n+2*A^2*b^3*
c^2*d^3*n-4*A*B*x*b^3*c*d^4*n^2-4*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^2*c*
d^4*n+8*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c*d^4*n^2+4*A*B*ln(e*((b*x+a)/
(d*x+c))^n)*a^2*b*d^5*n-8*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d^4*n-8*A*
B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c*d^4*n+8*A*B*a*b^2*c*d^4*n^2-4*A*B*x^2*
ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^5*n-4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^
3*c*d^4*n+4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^5*n^2+8*B^2*x*ln(e*((b
*x+a)/(d*x+c))^n)*b^3*c*d^4*n^2+4*A*B*x*a*b^2*d^5*n^2-2*B^2*x^2*ln(e*((b*x
+a)/(d*x+c))^n)^2*b^3*d^5*n+6*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^5*n^
2-6*B^2*x*a*b^2*d^5*n^3+6*B^2*x*b^3*c*d^4*n^3+2*B^2*ln(e*((b*x+a)/(d*x+c))
^n)^2*a^2*b*d^5*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*d^5*n^2-8*B^2*a*b^
2*c*d^4*n^3-2*A*B*a^2*b*d^5*n^2-6*A*B*b^3*c^2*d^3*n^2-4*A^2*a*b^2*c*d^4*n)
/i^3/(d*x+c)^2/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(311) = 622$.

Time $= 0.11$ (sec) , antiderivative size $= 654$, normalized size of antiderivative $= 2.06$

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx =$$

$$-\frac{2\,A^2b^2c^2 - 4\,A^2abcd + 2\,A^2a^2d^2 + (7\,B^2b^2c^2 - 8\,B^2abcd + B^2a^2d^2)n^2 + 2\,(B^2b^2c^2 - 2\,B^2abcd + B^2a^2$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x, algorithm="f
ricas")
```

output

```
-1/4*(2*A^2*b^2*c^2 - 4*A^2*a*b*c*d + 2*A^2*a^2*d^2 + (7*B^2*b^2*c^2 - 8*B
^2*a*b*c*d + B^2*a^2*d^2)*n^2 + 2*(B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d
^2)*log(e)^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*b^2*c*d*n^2*x + (2*B^2*a*b*c
*d - B^2*a^2*d^2)*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(3*A*B*b^2*c^2 - 4*A
*B*a*b*c*d + A*B*a^2*d^2)*n + 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2 - 2*(A*
B*b^2*c*d - A*B*a*b*d^2)*n)*x + 2*(2*A*B*b^2*c^2 - 4*A*B*a*b*c*d + 2*A*B*a
^2*d^2 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n*x - (3*B^2*b^2*c^2 - 4*B^2*a*b*c*
d + B^2*a^2*d^2)*n - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*b^2*c*d*n*x + (2*B^2*a*b
*c*d - B^2*a^2*d^2)*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*((4*B^2*a*b*c*
d - B^2*a^2*d^2)*n^2 + (3*B^2*b^2*d^2*n^2 - 2*A*B*b^2*d^2*n)*x^2 - 2*(2*A*
B*a*b*c*d - A*B*a^2*d^2)*n - 2*(2*A*B*b^2*c*d*n - (2*B^2*b^2*c*d + B^2*a*b
*d^2)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*
d^5)*i^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*i^3*x + (b^2*c^
4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*i^3)
```

Sympy [F]

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{\int \frac{A^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{i^3}$$

input

```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**3,x)
```

output

```
(Integral(A**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integ
ral(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/i**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. 2(311) = 622.

Time = 0.08 (sec) , antiderivative size = 861, normalized size of antiderivative = 2.72

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x, algorithm="m
axima")
```

output

```
1/2*A*B*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d
^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log(b*x + a)/((b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2
*a*b*c*d^2 + a^2*d^3)*i^3)) + 1/4*(2*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3
- a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i
^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2
*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3))*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n) - (7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2
 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))*n^2
/(b^2*c^4*d*i^3 - 2*a*b*c^3*d^2*i^3 + a^2*c^2*d^3*i^3 + (b^2*c^2*d^3*i^3 -
 2*a*b*c*d^4*i^3 + a^2*d^5*i^3)*x^2 + 2*(b^2*c^3*d^2*i^3 - 2*a*b*c^2*d^3*i
^3 + a^2*c*d^4*i^3)*x))*B^2 - 1/2*B^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)^2/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - A*B*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A^2/(d^3
*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3)
```

Giac [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.28

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(2\left(\frac{2\,(bx+a)B^2bn^2}{(bci^3-adi^3)(dx+c)}-\frac{(bx+a)^2B^2dn^2}{(bci^3-adi^3)(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)^2+2\left(\frac{(B^2dn^2-2\,B^2dn\log(e)-$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x, algorithm="g
iac")
```

output
```
1/4*(2*(2*(b*x + a)*B^2*b*n^2/((b*c*i^3 - a*d*i^3)*(d*x + c)) - (b*x + a)^
2*B^2*d*n^2/((b*c*i^3 - a*d*i^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c))^2
+ 2*((B^2*d*n^2 - 2*B^2*d*n*log(e) - 2*A*B*d*n)*(b*x + a)^2/((b*c*i^3 - a*
d*i^3)*(d*x + c)^2) - 4*(B^2*b*n^2 - B^2*b*n*log(e) - A*B*b*n)*(b*x + a)/(
(b*c*i^3 - a*d*i^3)*(d*x + c)))*log((b*x + a)/(d*x + c)) - (B^2*d*n^2 - 2*
B^2*d*n*log(e) + 2*B^2*d*log(e)^2 - 2*A*B*d*n + 4*A*B*d*log(e) + 2*A^2*d)*
(b*x + a)^2/((b*c*i^3 - a*d*i^3)*(d*x + c)^2) + 4*(2*B^2*b*n^2 - 2*B^2*b*n
*log(e) + B^2*b*log(e)^2 - 2*A*B*b*n + 2*A*B*b*log(e) + A^2*b)*(b*x + a)/(
(b*c*i^3 - a*d*i^3)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 27.73 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.59

$$
\begin{aligned}
&\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx \\
&= -\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^2\left(\frac{B^2}{2d\left(c^2i^3+2cdi^3x+d^2i^3x^2\right)}\right. \\
&\qquad\left. -\frac{B^2b^2}{2di^3\left(a^2d^2-2abcd+b^2c^2\right)}\right) \\
&\quad -\frac{\frac{2A^2ad-2A^2bc+B^2adn^2-7B^2bcn^2-2ABadn+6ABbcn}{2(ad-bc)}-\frac{bx\left(3B^2dn^2-2ABdn\right)}{ad-bc}}{2c^2di^3+4cd^2i^3x+2d^3i^3x^2} \\
&\quad -\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(\frac{AB}{c^2di^3+2cd^2i^3x+d^3i^3x^2}\right. \\
&\qquad\left. +\frac{B^2b^2\left(\frac{d^2i^3nx(ad-bc)}{b}-\frac{di^3n(ad-bc)(ad-2bc)}{2b^2}+\frac{cdi^3n(ad-bc)}{2b}\right)}{di^3\left(a^2d^2-2abcd+b^2c^2\right)\left(c^2di^3+2cd^2i^3x+d^3i^3x^2\right)}\right) \\
&\quad -\frac{Bb^2n\operatorname{atan}\left(\frac{\left(2bdx+\frac{2a^2d^3i^3-2b^2c^2di^3}{2di^3(ad-bc)}\right)1i}{ad-bc}\right)(2A-3Bn)\,1i}{di^3(ad-bc)^2}
\end{aligned}
$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*i + d*i*x)^3,x)
```

output
```
- log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(2*d*(c^2*i^3 + d^2*i^3*x^2 + 2*c*
d*i^3*x)) - (B^2*b^2)/(2*d*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2
*a*d - 2*A^2*b*c + B^2*a*d*n^2 - 7*B^2*b*c*n^2 - 2*A*B*a*d*n + 6*A*B*b*c*n
)/(2*(a*d - b*c)) - (b*x*(3*B^2*d*n^2 - 2*A*B*d*n))/(a*d - b*c))/(2*c^2*d*
i^3 + 2*d^3*i^3*x^2 + 4*c*d^2*i^3*x) - log(e*((a + b*x)/(c + d*x))^n)*((A*
B)/(c^2*d*i^3 + d^3*i^3*x^2 + 2*c*d^2*i^3*x) + (B^2*b^2*((d^2*i^3*n*x*(a*d
 - b*c))/b - (d*i^3*n*(a*d - b*c)*(a*d - 2*b*c))/(2*b^2) + (c*d*i^3*n*(a*d
 - b*c))/(2*b)))/(d*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(c^2*d*i^3 + d^3*i
^3*x^2 + 2*c*d^2*i^3*x))) - (B*b^2*n*atan(((2*b*d*x + (2*a^2*d^3*i^3 - 2*b
^2*c^2*d*i^3)/(2*d*i^3*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A - 3*B*n)*1i)/(d
*i^3*(a*d - b*c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 977, normalized size of antiderivative = 3.08

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x)
```

output

```
(i*(4*log(a + b*x)*a*b**3*c**3*n - 2*log(a + b*x)*a*b**3*c**2*d*n**2 + 8*l
og(a + b*x)*a*b**3*c**2*d*n*x - 4*log(a + b*x)*a*b**3*c*d**2*n**2*x + 4*lo
g(a + b*x)*a*b**3*c*d**2*n*x**2 - 2*log(a + b*x)*a*b**3*d**3*n**2*x**2 - 4
*log(a + b*x)*b**4*c**3*n**2 - 8*log(a + b*x)*b**4*c**2*d*n**2*x - 4*log(a
 + b*x)*b**4*c*d**2*n**2*x**2 - 4*log(c + d*x)*a*b**3*c**3*n + 2*log(c + d
*x)*a*b**3*c**2*d*n**2 - 8*log(c + d*x)*a*b**3*c**2*d*n*x + 4*log(c + d*x)
*a*b**3*c*d**2*n**2*x - 4*log(c + d*x)*a*b**3*c*d**2*n*x**2 + 2*log(c + d*
x)*a*b**3*d**3*n**2*x**2 + 4*log(c + d*x)*b**4*c**3*n**2 + 8*log(c + d*x)*
b**4*c**2*d*n**2*x + 4*log(c + d*x)*b**4*c*d**2*n**2*x**2 - 2*log(((a + b*
x)**n*e)/(c + d*x)**n)**2*a**2*b**2*c*d**2 + 4*log(((a + b*x)**n*e)/(c + d
*x)**n)**2*a*b**3*c**2*d + 4*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*c*
*2*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*c*d**2*x**2 - 4*log(
((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*c*d**2 + 8*log(((a + b*x)**n*e)/(c +
 d*x)**n)*a**2*b**2*c**2*d + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**
2*c*d**2*n - 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c**3 - 6*log(((a
+ b*x)**n*e)/(c + d*x)**n)*a*b**3*c**2*d*n + 2*log(((a + b*x)**n*e)/(c + d
*x)**n)*a*b**3*d**3*n*x**2 + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*c**
3*n - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*c*d**2*n*x**2 - 2*a**4*c*d
**2 + 4*a**3*b*c**2*d + 2*a**3*b*c*d**2*n - 2*a**2*b**2*c**3 - 6*a**2*b**2
*c**2*d*n - a**2*b**2*c*d**2*n**2 + 2*a**2*b**2*d**3*n*x**2 + 4*a*b**3*...
```

3.206 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$

Optimal result . . . 2350
Mathematica [B] (verified) . . . 2351
Rubi [A] (warning: unable to verify) . . . 2352
Maple [B] (verified) . . . 2355
Fricas [B] (verification not implemented) . . . 2356
Sympy [F(-1)] . . . 2357
Maxima [B] (verification not implemented) . . . 2358
Giac [A] (verification not implemented) . . . 2359
Mupad [B] (verification not implemented) . . . 2359
Reduce [B] (verification not implemented) . . . 2360

Optimal result

Integrand size = 45, antiderivative size = 402

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx &= \frac{B^2d^2n^2(a+bx)^2}{4(bc-ad)^3gi^3(c+dx)^2}+\frac{4AbBdn(a+bx)}{(bc-ad)^3gi^3(c+dx)}\\ &-\frac{4bB^2dn^2(a+bx)}{(bc-ad)^3gi^3(c+dx)}\\ &+\frac{4bB^2dn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^3gi^3(c+dx)}\\ &-\frac{Bd^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2}\\ &+\frac{d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3gi^3(c+dx)^2}\\ &-\frac{2bd(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3gi^3(c+dx)}\\ &+\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^3gi^3n}\end{aligned}$$

output

```
1/4*B^2*d^2*n^2*(b*x+a)^2/(-a*d+b*c)^3/g/i^3/(d*x+c)^2+4*A*b*B*d*n*(b*x+a)
/(-a*d+b*c)^3/g/i^3/(d*x+c)-4*b*B^2*d*n^2*(b*x+a)/(-a*d+b*c)^3/g/i^3/(d*x+
c)+4*b*B^2*d*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^3/g/i^3/(d*x+c
)-1/2*B*d^2*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g/i^3
/(d*x+c)^2+1/2*d^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^
3/g/i^3/(d*x+c)^2-2*b*d*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*
c)^3/g/i^3/(d*x+c)+1/3*b^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)^
3/g/i^3/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 971 vs. $2(402) = 804$.

Time $= 0.95$ (sec) , antiderivative size $= 971$, normalized size of antiderivative $= 2.42$

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)(ci + dix)^3}\, dx = \text{Too large to display}$$

input

```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d
*i*x)^3),x]
```

output

```
(4*b^2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^3 - (6*B*n*Log[(a + b*x)/(c + d*x)
]^2*(-2*A*b^2*c^2 + 4*a*b*B*c*d*n - a^2*B*d^2*n - 4*A*b^2*c*d*x + 4*b^2*B*
c*d*n*x + 2*a*b*B*d^2*n*x - 2*A*b^2*d^2*x^2 + 3*b^2*B*d^2*n*x^2 - 2*b^2*B*
(c + d*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + 2*b^2*B*n*(c + d*x)^2*Log[(a
 + b*x)/(c + d*x)]))/(c + d*x)^2 - (6*B*(b*c - a*d)*n*Log[(a + b*x)/(c + d*
x)]*(-6*A*b*c + 2*a*A*d + 7*b*B*c*n - a*B*d*n - 4*A*b*d*x + 6*b*B*d*n*x +
2*B*(-3*b*c + a*d - 2*b*d*x)*Log[e*((a + b*x)/(c + d*x))^n] + 2*B*n*(3*b*c
 - a*d + 2*b*d*x)*Log[(a + b*x)/(c + d*x)]))/(c + d*x)^2 + (3*(b*c - a*d)^
2*(2*A^2 - 2*A*B*n + B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*
B*n*(-2*A + B*n)*Log[(a + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d
*x)]^2 - 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(-2*A + B*n + 2*B*n*Log[(a + b
*x)/(c + d*x)])))/(c + d*x)^2 + (6*b*(b*c - a*d)*(2*A^2 - 6*A*B*n + 7*B^2*
n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-2*A + 3*B*n)*Log[(a
 + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*Log[e*((a
+ b*x)/(c + d*x))^n]*(-2*A + 3*B*n + 2*B*n*Log[(a + b*x)/(c + d*x)])))/(c
+ d*x) + 6*b^2*Log[a + b*x]*(2*A^2 - 6*A*B*n + 7*B^2*n^2 + 2*B^2*Log[e*((a
 + b*x)/(c + d*x))^n]^2 + 2*B*n*(-2*A + 3*B*n)*Log[(a + b*x)/(c + d*x)] +
2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*Log[e*((a + b*x)/(c + d*x))^n]*
(-2*A + 3*B*n + 2*B*n*Log[(a + b*x)/(c + d*x)])) - 6*b^2*(2*A^2 - 6*A*B*n
+ 7*B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-2*A + 3*...
```

Rubi [A] (warning: unable to verify)

Time = 0.96 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2961, 2788, 2767, 2009, 2788, 2733, 2009, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}\,d\frac{a+bx}{c+dx}}{gi^3(bc-ad)^3}$$

$$\downarrow 2788$$

$$\frac{b\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\int\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d\frac{a+bx}{c+dx}}{gi^3(bc-ad)^3}$$

$$\downarrow 2767$$

$$\frac{b\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\int\left(b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{d(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{c+dx}\right.}{gi^3(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{b\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\left(\frac{Bdn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(c+dx)^2}+\frac{b(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{c+dx}\right.}{gi^3(bc-}$$

$$\downarrow 2788$$

$$\frac{b\left(b\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\int\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d\frac{a+bx}{c+dx}\right)-d\left(\frac{Bdn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{2(c+dx)^2}\right.}$$

$$\downarrow 2733$$

$$\frac{b\left(b\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2Bn\int\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d\frac{a+b}{c+d}\right.\right.}$$

$$\downarrow 2009$$

$$\frac{b\left(b\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2Bn\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}\right.\right.\right.}$$

$$\downarrow 2739$$

$$\frac{b\left(\frac{b\int\frac{(a+bx)^2}{(c+dx)^2}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}-d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2Bn\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}-\frac{Bn(}{c}\right.\right.\right.}$$

$$\downarrow 15$$

$$b\left(\frac{b(a+bx)^3}{3Bn(c+dx)^3}-d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2Bn\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}-\frac{Bn(a+bx)}{c+dx}\right)\right)\right)-d\left(\frac{Ba}{}\right.$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d*i*x)^
3),x]
```

output
```
(-(d*(-1/4*(B^2*d*n^2*(a + b*x)^2)/(c + d*x)^2 - (2*A*b*B*n*(a + b*x))/(c
+ d*x) + (2*b*B^2*n^2*(a + b*x))/(c + d*x) - (2*b*B^2*n*(a + b*x)*Log[e*((
a + b*x)/(c + d*x))^n])/(c + d*x) + (B*d*n*(a + b*x)^2*(A + B*Log[e*((a +
b*x)/(c + d*x))^n]))/(2*(c + d*x)^2) - (d*(a + b*x)^2*(A + B*Log[e*((a + b
*x)/(c + d*x))^n])^2)/(2*(c + d*x)^2) + (b*(a + b*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])^2)/(c + d*x))) + b*((b*(a + b*x)^3)/(3*B*n*(c + d*x)^3)
- d*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c + d*x) - 2*B*
n*((A*(a + b*x))/(c + d*x) - (B*n*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[
e*((a + b*x)/(c + d*x))^n])/(c + d*x)))))/((b*c - a*d)^3*g*i^3)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2767

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2788

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1235 vs. $2(394) = 788$.

Time = 13.41 (sec) , antiderivative size = 1236, normalized size of antiderivative = 3.07

method	result	size
parallelrisch	Expression too large to display	1236

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x,method=_
RETURNVERBOSE)
```

output

```
-1/12*(4*B^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^2*c^6+12*A^2*ln(e*((b*x+a)/
(d*x+c))^n)*a^2*b^2*c^6-18*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c^4
*d^2*n+42*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^4*d^2*n^2+12*A*B*x^2
*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c^4*d^2-48*A*B*x^2*a^3*b*c^3*d^3*n^2+
42*A*B*x^2*a^2*b^2*c^4*d^2*n^2-12*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b*
c^4*d^2*n-24*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c^5*d*n+36*B^2*x*ln
(e*((b*x+a)/(d*x+c))^n)*a^3*b*c^4*d^2*n^2+48*B^2*x*ln(e*((b*x+a)/(d*x+c))^
n)*a^2*b^2*c^5*d*n^2+24*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c^5*d-60
*A*B*x*a^3*b*c^4*d^2*n^2+48*A*B*x*a^2*b^2*c^5*d*n^2-48*A*B*ln(e*((b*x+a)/(
d*x+c))^n)*a^3*b*c^5*d*n+8*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^2*c^5*d
+54*B^2*x*a^3*b*c^4*d^2*n^3-48*B^2*x*a^2*b^2*c^5*d*n^3+12*A^2*x^2*ln(e*((b
*x+a)/(d*x+c))^n)*a^2*b^2*c^4*d^2+24*A^2*x^2*a^3*b*c^3*d^3*n-18*A^2*x^2*a^
2*b^2*c^4*d^2*n+12*A*B*x*a^4*c^3*d^3*n^2+4*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^
n)^3*a^2*b^2*c^4*d^2+48*B^2*x^2*a^3*b*c^3*d^3*n^3-45*B^2*x^2*a^2*b^2*c^4*d
^2*n^3+6*A*B*x^2*a^4*c^2*d^4*n^2-24*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b*
c^5*d*n+48*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*c^5*d*n^2+24*A^2*x*ln(e*((b
*x+a)/(d*x+c))^n)*a^2*b^2*c^5*d+36*A^2*x*a^3*b*c^4*d^2*n-24*A^2*x*a^2*b^2*
c^5*d*n+12*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^4*c^4*d^2*n-36*A*B*x^2*ln(e*((b
*x+a)/(d*x+c))^n)*a^2*b^2*c^4*d^2*n-24*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3
*b*c^4*d^2*n-48*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^5*d*n-3*B^2*x...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(394) = 788$.

Time = 0.11 (sec) , antiderivative size = 1076, normalized size of antiderivative = 2.68

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)(ci + dix)^3}\, dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x,
algorithm="fricas")
```

output

```
1/12*(18*A^2*b^2*c^2 - 24*A^2*a*b*c*d + 6*A^2*a^2*d^2 + 4*(B^2*b^2*d^2*n^2
*x^2 + 2*B^2*b^2*c*d*n^2*x + B^2*b^2*c^2*n^2)*log((b*x + a)/(d*x + c))^3 +
 3*(15*B^2*b^2*c^2 - 16*B^2*a*b*c*d + B^2*a^2*d^2)*n^2 + 6*(3*B^2*b^2*c^2
- 4*B^2*a*b*c*d + B^2*a^2*d^2 + 2*(B^2*b^2*c*d - B^2*a*b*d^2)*x + 2*(B^2*b
^2*d^2*x^2 + 2*B^2*b^2*c*d*x + B^2*b^2*c^2)*log((b*x + a)/(d*x + c)))*log(
e)^2 + 6*(2*A*B*b^2*c^2*n - (4*B^2*a*b*c*d - B^2*a^2*d^2)*n^2 - (3*B^2*b^2
*d^2*n^2 - 2*A*B*b^2*d^2*n)*x^2 + 2*(2*A*B*b^2*c*d*n - (2*B^2*b^2*c*d + B^
2*a*b*d^2)*n^2)*x)*log((b*x + a)/(d*x + c))^2 - 6*(7*A*B*b^2*c^2 - 8*A*B*a
*b*c*d + A*B*a^2*d^2)*n + 6*(2*A^2*b^2*c*d - 2*A^2*a*b*d^2 + 7*(B^2*b^2*c*
d - B^2*a*b*d^2)*n^2 - 6*(A*B*b^2*c*d - A*B*a*b*d^2)*n)*x + 6*(6*A*B*b^2*c
^2 - 8*A*B*a*b*c*d + 2*A*B*a^2*d^2 + 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*b^2*c*d*
n*x + B^2*b^2*c^2*n)*log((b*x + a)/(d*x + c))^2 - (7*B^2*b^2*c^2 - 8*B^2*a
*b*c*d + B^2*a^2*d^2)*n + 2*(2*A*B*b^2*c*d - 2*A*B*a*b*d^2 - 3*(B^2*b^2*c*
d - B^2*a*b*d^2)*n)*x + 2*(2*A*B*b^2*c^2 - (3*B^2*b^2*d^2*n - 2*A*B*b^2*d^
2)*x^2 - (4*B^2*a*b*c*d - B^2*a^2*d^2)*n + 2*(2*A*B*b^2*c*d - (2*B^2*b^2*c
*d + B^2*a*b*d^2)*n)*x)*log((b*x + a)/(d*x + c)))*log(e) + 6*(2*A^2*b^2*c^
2 + (8*B^2*a*b*c*d - B^2*a^2*d^2)*n^2 + (7*B^2*b^2*d^2*n^2 - 6*A*B*b^2*d^2
*n + 2*A^2*b^2*d^2)*x^2 - 2*(4*A*B*a*b*c*d - A*B*a^2*d^2)*n + 2*(2*A^2*b^2
*c*d + (4*B^2*b^2*c*d + 3*B^2*a*b*d^2)*n^2 - 2*(2*A*B*b^2*c*d + A*B*a*b*d^
2)*n)*x)*log((b*x + a)/(d*x + c)))/((b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)(ci + dix)^3}\,dx = \text{Timed out}$$

input

```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)/(d*i*x+c*i)**3,x
)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2126 vs. $2(394) = 788$.

Time $= 0.18$ (sec) , antiderivative size $= 2126$, normalized size of antiderivative $= 5.29$

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)(ci + dix)^3}\, dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x,
algorithm="maxima")
```

output

```
1/2*B^2*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g*
i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b^2*c^4 - 2
*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*
c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(e*(b*x/(d*x + c) + a/(
d*x + c))^n)^2 + A*B*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3
+ a^2*d^4)*g*i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x +
 (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/
((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(e*(b*x/(d
*x + c) + a/(d*x + c))^n) + 1/12*((45*b^2*c^2 - 48*a*b*c*d + 3*a^2*d^2 + 4
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^3 - 4*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*log(d*x + c)^3 + 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*log(b*x + a)^2 + 6*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c)^2 + 42*(b^2*c
*d - a*b*d^2)*x + 42*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) -
6*(7*b^2*d^2*x^2 + 14*b^2*c*d*x + 7*b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*log(b*x + a)^2 + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b
*x + a))*log(d*x + c))*n^2/(b^3*c^5*g*i^3 - 3*a*b^2*c^4*d*g*i^3 + 3*a^2*b*
c^3*d^2*g*i^3 - a^3*c^2*d^3*g*i^3 + (b^3*c^3*d^2*g*i^3 - 3*a*b^2*c^2*d^...
```

Giac [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 747, normalized size of antiderivative = 1.86

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)(ci + dix)^3}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x,
algorithm="giac")
```

output
```
1/12*(4*B^2*b^2*n^2*log((b*x + a)/(d*x + c))^3/(b^2*c^2*g*i^3 - 2*a*b*c*d*
g*i^3 + a^2*d^2*g*i^3) - 6*(4*(b*x + a)*B^2*b*d*n^2/((b^2*c^2*g*i^3 - 2*a*
b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)) - (b*x + a)^2*B^2*d^2*n^2/((b^2*c^
2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)^2) - 2*(B^2*b^2*n*log
(e) + A*B*b^2*n)/(b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3))*log((b
*x + a)/(d*x + c))^2 - 6*((B^2*d^2*n^2 - 2*B^2*d^2*n*log(e) - 2*A*B*d^2*n)
*(b*x + a)^2/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)^
2) - 8*(B^2*b*d*n^2 - B^2*b*d*n*log(e) - A*B*b*d*n)*(b*x + a)/((b^2*c^2*g*
i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)))*log((b*x + a)/(d*x + c)
) + 12*(B^2*b^2*log(e)^2 + 2*A*B*b^2*log(e) + A^2*b^2)*log((b*x + a)/(d*x
+ c))/(b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3) + 3*(B^2*d^2*n^2 -
 2*B^2*d^2*n*log(e) + 2*B^2*d^2*log(e)^2 - 2*A*B*d^2*n + 4*A*B*d^2*log(e)
+ 2*A^2*d^2)*(b*x + a)^2/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3
)*(d*x + c)^2) - 24*(2*B^2*b*d*n^2 - 2*B^2*b*d*n*log(e) + B^2*b*d*log(e)^2
 - 2*A*B*b*d*n + 2*A*B*b*d*log(e) + A^2*b*d)*(b*x + a)/((b^2*c^2*g*i^3 - 2
*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c
- a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 29.96 (sec) , antiderivative size = 1007, normalized size of antiderivative = 2.50

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)(ci + dix)^3}\, dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)*(c*i + d*i*x)^
3),x)
```

output

```
log(e*((a + b*x)/(c + d*x))^n)^2*((b^2*(3*B^2*n - 2*A*B))/(2*g*i^3*n*(a*d
- b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B^2*b^2*((c*g*i^3*n*(a*d - b*c)
)/(2*b) - (g*i^3*n*(a*d - b*c)*(a*d - 2*b*c))/(2*b^2) + (d*g*i^3*n*x*(a*d
- b*c))/b))/(g*i^3*n*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(c^2*g*i^
3 + d^2*g*i^3*x^2 + 2*c*d*g*i^3*x))) - ((2*A^2*a*d - 6*A^2*b*c + B^2*a*d*n
^2 - 15*B^2*b*c*n^2 - 2*A*B*a*d*n + 14*A*B*b*c*n)/(2*(a*d - b*c)) - (x*(2*
A^2*b*d + 7*B^2*b*d*n^2 - 6*A*B*b*d*n))/(a*d - b*c))/(x^2*(2*a*d^3*g*i^3 -
 2*b*c*d^2*g*i^3) + x*(4*a*c*d^2*g*i^3 - 4*b*c^2*d*g*i^3) - 2*b*c^3*g*i^3
+ 2*a*c^2*d*g*i^3) - log(e*((a + b*x)/(c + d*x))^n)*((B^2*n)/(x^2*(a*d^3*g
*i^3 - b*c*d^2*g*i^3) + x*(2*a*c*d^2*g*i^3 - 2*b*c^2*d*g*i^3) - b*c^3*g*i^
3 + a*c^2*d*g*i^3) + (b^2*(3*B^2*n - 2*A*B)*((c*g*i^3*n*(a*d - b*c)^2)/(2*
b) - (g*i^3*n*(a*d - b*c)^2*(a*d - 2*b*c))/(2*b^2) + (d*g*i^3*n*x*(a*d - b
*c)^2)/b))/(g*i^3*n*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(x^2*(a*d^
3*g*i^3 - b*c*d^2*g*i^3) + x*(2*a*c*d^2*g*i^3 - 2*b*c^2*d*g*i^3) - b*c^3*g
*i^3 + a*c^2*d*g*i^3))) + (b^2*atan((b^2*((a^3*d^3*g*i^3 + b^3*c^3*g*i^3 -
 a*b^2*c^2*d*g*i^3 - a^2*b*c*d^2*g*i^3)/(a^2*d^2*g*i^3 + b^2*c^2*g*i^3 - 2
*a*b*c*d*g*i^3) + 2*b*d*x)*(A^2 + (7*B^2*n^2)/2 - 3*A*B*n)*(a^2*d^2*g*i^3
+ b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3)*2i)/(g*i^3*(a*d - b*c)^3*(2*A^2*b^2 + 7
*B^2*b^2*n^2 - 6*A*B*b^2*n)))*(A^2 + (7*B^2*n^2)/2 - 3*A*B*n)*2i)/(g*i^3*(
a*d - b*c)^3) - (B^2*b^2*log(e*((a + b*x)/(c + d*x))^n)^3)/(3*g*i^3*n*(...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1640, normalized size of antiderivative = 4.08

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)(ci + dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x)
```

output

```
(i*( - 12*log(a + b*x)*a**2*b**2*c**3*n + 12*log(a + b*x)*a**2*b**2*c**2*d
*n**2 - 24*log(a + b*x)*a**2*b**2*c**2*d*n*x + 24*log(a + b*x)*a**2*b**2*c
*d**2*n**2*x - 12*log(a + b*x)*a**2*b**2*c*d**2*n*x**2 + 12*log(a + b*x)*a
**2*b**2*d**3*n**2*x**2 + 24*log(a + b*x)*a*b**3*c**3*n**2 - 18*log(a + b*
x)*a*b**3*c**2*d*n**3 + 48*log(a + b*x)*a*b**3*c**2*d*n**2*x - 36*log(a +
b*x)*a*b**3*c*d**2*n**3*x + 24*log(a + b*x)*a*b**3*c*d**2*n**2*x**2 - 18*l
og(a + b*x)*a*b**3*d**3*n**3*x**2 - 24*log(a + b*x)*b**4*c**3*n**3 - 48*lo
g(a + b*x)*b**4*c**2*d*n**3*x - 24*log(a + b*x)*b**4*c*d**2*n**3*x**2 + 12
*log(c + d*x)*a**2*b**2*c**3*n - 12*log(c + d*x)*a**2*b**2*c**2*d*n**2 + 2
4*log(c + d*x)*a**2*b**2*c**2*d*n*x - 24*log(c + d*x)*a**2*b**2*c*d**2*n**
2*x + 12*log(c + d*x)*a**2*b**2*c*d**2*n*x**2 - 12*log(c + d*x)*a**2*b**2*
d**3*n**2*x**2 - 24*log(c + d*x)*a*b**3*c**3*n**2 + 18*log(c + d*x)*a*b**3
*c**2*d*n**3 - 48*log(c + d*x)*a*b**3*c**2*d*n**2*x + 36*log(c + d*x)*a*b*
*3*c*d**2*n**3*x - 24*log(c + d*x)*a*b**3*c*d**2*n**2*x**2 + 18*log(c + d*
x)*a*b**3*d**3*n**3*x**2 + 24*log(c + d*x)*b**4*c**3*n**3 + 48*log(c + d*x
)*b**4*c**2*d*n**3*x + 24*log(c + d*x)*b**4*c*d**2*n**3*x**2 - 4*log(((a +
 b*x)**n*e)/(c + d*x)**n)**3*b**4*c**3 - 8*log(((a + b*x)**n*e)/(c + d*x)*
*n)**3*b**4*c**2*d*x - 4*log(((a + b*x)**n*e)/(c + d*x)**n)**3*b**4*c*d**2
*x**2 - 6*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**2*c*d**2*n - 12*lo
g(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**3*c**3 + 24*log(((a + b*x)**n*...
```

3.207 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx$

Optimal result . . . 2362
Mathematica [B] (verified) . . . 2363
Rubi [A] (verified) . . . 2364
Maple [B] (verified) . . . 2366
Fricas [B] (verification not implemented) . . . 2367
Sympy [F(-1)] . . . 2368
Maxima [B] (verification not implemented) . . . 2369
Giac [F] . . . 2370
Mupad [B] (verification not implemented) . . . 2370
Reduce [B] (verification not implemented) . . . 2371

Optimal result

Integrand size = 45, antiderivative size = 562

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = &-\frac{B^2d^3n^2(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2}-\frac{6AbBd^2n(a+bx)}{(bc-ad)^4g^2i^3(c+dx)}\\ &+\frac{6bB^2d^2n^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)}-\frac{2b^3B^2n^2(c+dx)}{(bc-ad)^4g^2i^3(a+bx)}\\ &-\frac{6bB^2d^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^4g^2i^3(c+dx)}\\ &+\frac{Bd^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2}\\ &-\frac{2b^3Bn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^2i^3(a+bx)}\\ &-\frac{d^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^2i^3(c+dx)^2}\\ &+\frac{3bd^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^2i^3(c+dx)}\\ &-\frac{b^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^2i^3(a+bx)}\\ &-\frac{b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{B(bc-ad)^4g^2i^3n}\end{aligned}$$

output

```
-1/4*B^2*d^3*n^2*(b*x+a)^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2-6*A*b*B*d^2*n*(b
*x+a)/(-a*d+b*c)^4/g^2/i^3/(d*x+c)+6*b*B^2*d^2*n^2*(b*x+a)/(-a*d+b*c)^4/g^
2/i^3/(d*x+c)-2*b^3*B^2*n^2*(d*x+c)/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-6*b*B^2*d
^2*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^4/g^2/i^3/(d*x+c)+1/2*B*
d^3*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^2/i^3/(d*x+
c)^2-2*b^3*B*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^2/i^
3/(b*x+a)-1/2*d^3*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4
/g^2/i^3/(d*x+c)^2+3*b*d^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d
+b*c)^4/g^2/i^3/(d*x+c)-b^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*
d+b*c)^4/g^2/i^3/(b*x+a)-b^2*d*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b
*c)^4/g^2/i^3/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1334 vs. $2(562) = 1124$.

Time $= 1.63$ (sec) , antiderivative size $= 1334$, normalized size of antiderivative $= 2.37$

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^2(ci + dix)^3}\, dx = \text{Too large to display}$$

input

```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i +
 d*i*x)^3),x]
```

output

```
-1/4*(4*b^2*B^2*d*n^2*(a + b*x)*(c + d*x)^2*Log[(a + b*x)/(c + d*x)]^3 + 2
*B*n*Log[(a + b*x)/(c + d*x)]^2*(6*a*A*b^2*c^2*d + 2*b^3*B*c^3*n - 6*a^2*b
*B*c*d^2*n + a^3*B*d^3*n + 6*A*b^3*c^2*d*x + 12*a*A*b^2*c*d^2*x + 6*b^3*B*
c^2*d*n*x - 12*a*b^2*B*c*d^2*n*x - 3*a^2*b*B*d^3*n*x + 12*A*b^3*c*d^2*x^2
+ 6*a*A*b^2*d^3*x^2 - 9*a*b^2*B*d^3*n*x^2 + 6*A*b^3*d^3*x^3 - 3*b^3*B*d^3*
n*x^3 + 6*b^2*B*d*(a + b*x)*(c + d*x)^2*Log[e*((a + b*x)/(c + d*x))^n] - 6
*b^2*B*d*n*(a + b*x)*(c + d*x)^2*Log[(a + b*x)/(c + d*x)]) + 4*b^2*(b*c -
a*d)*(c + d*x)^2*(A^2 + 2*A*B*n + 2*B^2*n^2 + B^2*Log[e*((a + b*x)/(c + d*
x))^n]^2 - 2*B*n*(A + B*n)*Log[(a + b*x)/(c + d*x)] + B^2*n^2*Log[(a + b*x
)/(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(A + B*n - B*n*Log[(a
+ b*x)/(c + d*x)])) + 2*B*(b*c - a*d)*n*Log[(a + b*x)/(c + d*x)]*(2*b*d*(a
 + b*x)*(c + d*x)*(4*A - 5*B*n + 4*B*Log[e*((a + b*x)/(c + d*x))^n] - 4*B*
n*Log[(a + b*x)/(c + d*x)]) + d*(b*c - a*d)*(a + b*x)*(2*A - B*n + 2*B*Log
[e*((a + b*x)/(c + d*x))^n] - 2*B*n*Log[(a + b*x)/(c + d*x)]) + 4*b^2*(c +
 d*x)^2*(A + B*n + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c
 + d*x)])) + d*(b*c - a*d)^2*(a + b*x)*(2*A^2 - 2*A*B*n + B^2*n^2 + 2*B^2*
Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-2*A + B*n)*Log[(a + b*x)/(c + d
*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*Log[e*((a + b*x)/(c + d*
x))^n]*(-2*A + B*n + 2*B*n*Log[(a + b*x)/(c + d*x)])) + 6*b^2*d*(a + b*x)*
(c + d*x)^2*Log[a + b*x]*(2*A^2 - 2*A*B*n + 5*B^2*n^2 + 2*B^2*Log[e*((a...
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}\,d\frac{a+bx}{c+dx}}{g^2i^3(bc-ad)^4}$$

$$\downarrow 2795$$

$$\int \left(\frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^3}{(a+bx)^2} - \frac{3d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^2}{a+bx} + 3d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b - \frac{d^3(a+bx)(}{g^2 i^3 (bc-ad)^4}$$

$$\downarrow 2009$$

$$-\frac{b^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx} - \frac{2b^3Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx} - \frac{b^2d\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{Bn} - \frac{d^3(a+bx)^2\left(B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{2(c+dx)^2}$$

input

```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i + d*i*x
)^3),x]
```

output

```
(-1/4*(B^2*d^3*n^2*(a + b*x)^2)/(c + d*x)^2 - (6*A*b*B*d^2*n*(a + b*x))/(c
 + d*x) + (6*b*B^2*d^2*n^2*(a + b*x))/(c + d*x) - (2*b^3*B^2*n^2*(c + d*x)
)/(a + b*x) - (6*b*B^2*d^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(c
+ d*x) + (B*d^3*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(
c + d*x)^2) - (2*b^3*B*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
/(a + b*x) - (d^3*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2
*(c + d*x)^2) + (3*b*d^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2)/(c + d*x) - (b^3*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a
 + b*x) - (b^2*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(B*n))/((b*c -
a*d)^4*g^2*i^3)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1999 vs. $2(556) = 1112$.

Time = 30.48 (sec) , antiderivative size = 2000, normalized size of antiderivative = 3.56

method	result	size
parallelrisch	Expression too large to display	2000

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,method
=_RETURNVERBOSE)
```

output

```
-1/4*(B^2*a^3*b^3*d^7*n^3+8*B^2*b^6*c^3*d^4*n^3+2*A^2*a^3*b^3*d^7*n+4*A^2*
b^6*c^3*d^4*n+4*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^3*b^6*d^7+12*A^2*x^3*ln(
e*((b*x+a)/(d*x+c))^n)*b^6*d^7-6*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^4
*d^7*n+12*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^6*c^2*d^5*n+18*B^2*x*ln(e*((
b*x+a)/(d*x+c))^n)*a^2*b^4*d^7*n^2+24*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^6*
c^2*d^5*n^2-18*B^2*x*a*b^5*c*d^6*n^3+12*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*
b^6*c^2*d^5+18*A*B*x*a^2*b^4*d^7*n^2-6*A*B*x*b^6*c^2*d^5*n^2-12*B^2*ln(e*(
(b*x+a)/(d*x+c))^n)^2*a^2*b^4*c*d^6*n+24*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2
*b^4*c*d^6*n^2+24*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^6-12*A^2*x*a*b
^5*c*d^6*n+12*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^5*c^2*d^5+4*A*B*ln(e*((b
*x+a)/(d*x+c))^n)*a^3*b^3*d^7*n+8*A*B*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^3*d^
4*n-12*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^7*n-18*B^2*x^2*ln(e*((b*x+a
)/(d*x+c))^n)^2*a*b^5*d^7*n+42*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^7
*n^2+48*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c*d^6*n^2+12*A*B*x^2*ln(e*((
b*x+a)/(d*x+c))^n)^2*a*b^5*d^7+24*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^6*
c*d^6+12*A*B*x^2*a*b^5*d^7*n^2-12*A*B*x^2*b^6*c*d^6*n^2+8*B^2*x*ln(e*((b*x
+a)/(d*x+c))^n)^3*a*b^5*c*d^6-36*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d
^7*n-24*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^5*c*d^6*n+48*B^2*x*ln(e*((b*
x+a)/(d*x+c))^n)*a*b^5*c*d^6*n^2+24*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^
5*c*d^6-12*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*d^7*n+24*A*B*x*ln(e*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2057 vs. $2(556) = 1112$.

Time $= 0.13$ (sec) , antiderivative size $= 2057$, normalized size of antiderivative $= 3.66$

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x
, algorithm="fricas")
```

output

```
-1/4*(4*A^2*b^3*c^3 + 6*A^2*a*b^2*c^2*d - 12*A^2*a^2*b*c*d^2 + 2*A^2*a^3*d
^3 + 4*(B^2*b^3*d^3*n^2*x^3 + B^2*a*b^2*c^2*d*n^2 + (2*B^2*b^3*c*d^2 + B^2
*a*b^2*d^3)*n^2*x^2 + (B^2*b^3*c^2*d + 2*B^2*a*b^2*c*d^2)*n^2*x)*log((b*x
+ a)/(d*x + c))^3 + (8*B^2*b^3*c^3 + 15*B^2*a*b^2*c^2*d - 24*B^2*a^2*b*c*d
^2 + B^2*a^3*d^3)*n^2 + 6*(2*A^2*b^3*c*d^2 - 2*A^2*a*b^2*d^3 + 5*(B^2*b^3*
c*d^2 - B^2*a*b^2*d^3)*n^2 - 2*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*n)*x^2 + 2*
(2*B^2*b^3*c^3 + 3*B^2*a*b^2*c^2*d - 6*B^2*a^2*b*c*d^2 + B^2*a^3*d^3 + 6*(
B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*x^2 + 3*(3*B^2*b^3*c^2*d - 2*B^2*a*b^2*c*d^
2 - B^2*a^2*b*d^3)*x + 6*(B^2*b^3*d^3*x^3 + B^2*a*b^2*c^2*d + (2*B^2*b^3*c
*d^2 + B^2*a*b^2*d^3)*x^2 + (B^2*b^3*c^2*d + 2*B^2*a*b^2*c*d^2)*x)*log((b*
x + a)/(d*x + c)))*log(e)^2 + 2*(6*A*B*a*b^2*c^2*d*n - 3*(B^2*b^3*d^3*n^2
- 2*A*B*b^3*d^3*n)*x^3 + (2*B^2*b^3*c^3 - 6*B^2*a^2*b*c*d^2 + B^2*a^3*d^3)
*n^2 - 3*(3*B^2*a*b^2*d^3*n^2 - 2*(2*A*B*b^3*c*d^2 + A*B*a*b^2*d^3)*n)*x^2
 + 3*((2*B^2*b^3*c^2*d - 4*B^2*a*b^2*c*d^2 - B^2*a^2*b*d^3)*n^2 + 2*(A*B*b
^3*c^2*d + 2*A*B*a*b^2*c*d^2)*n)*x)*log((b*x + a)/(d*x + c))^2 + 2*(4*A*B*
b^3*c^3 - 15*A*B*a*b^2*c^2*d + 12*A*B*a^2*b*c*d^2 - A*B*a^3*d^3)*n + 3*(6*
A^2*b^3*c^2*d - 4*A^2*a*b^2*c*d^2 - 2*A^2*a^2*b*d^3 + (13*B^2*b^3*c^2*d -
6*B^2*a*b^2*c*d^2 - 7*B^2*a^2*b*d^3)*n^2 - 2*(A*B*b^3*c^2*d + 2*A*B*a*b^2*
c*d^2 - 3*A*B*a^2*b*d^3)*n)*x + 2*(4*A*B*b^3*c^3 + 6*A*B*a*b^2*c^2*d - 12*
A*B*a^2*b*c*d^2 + 2*A*B*a^3*d^3 + 6*(2*A*B*b^3*c*d^2 - 2*A*B*a*b^2*d^3 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Timed out}$$

input

```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2/(d*i*x+c*i)**
3,x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4199 vs. $2(556) = 1112$.

Time = 0.34 (sec) , antiderivative size = 4199, normalized size of antiderivative = 7.47

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x
, algorithm="maxima")
```

output

```
-1/2*B^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d
+ a*b*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^
5)*g^2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*
b*c*d^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^
2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*
d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c
^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3)
 - 6*b^2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4)*g^2*i^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2
- A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a
*b*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*
g^2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c
*d^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 +
 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*d +
 3*a^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c^4
- 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3) -
6*b^2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)*g^2*i^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/4
*((8*b^3*c^3 + 15*a*b^2*c^2*d - 24*a^2*b*c*d^2 + a^3*d^3 + 4*(b^3*d^3*x^3
+ a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*...
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x
, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/((b*g*x + a*g)^2*(d*i*x
 + c*i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 31.76 (sec) , antiderivative size = 1785, normalized size of antiderivative = 3.18

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^2*(c*i + d*i*x
)^3),x)
```

output

```
((4*A^2*b^2*c^2 - 2*A^2*a^2*d^2 - B^2*a^2*d^2*n^2 + 8*B^2*b^2*c^2*n^2 + 10
*A^2*a*b*c*d + 2*A*B*a^2*d^2*n + 8*A*B*b^2*c^2*n + 23*B^2*a*b*c*d*n^2 - 22
*A*B*a*b*c*d*n)/(2*(a*d - b*c)) + (3*x^2*(2*A^2*b^2*d^2 + 5*B^2*b^2*d^2*n^
2 - 2*A*B*b^2*d^2*n))/(a*d - b*c) + (3*x*(2*A^2*a*b*d^2 + 6*A^2*b^2*c*d +
7*B^2*a*b*d^2*n^2 + 13*B^2*b^2*c*d*n^2 - 6*A*B*a*b*d^2*n - 2*A*B*b^2*c*d*n
))/(2*(a*d - b*c)))/(x*(2*b^3*c^4*g^2*i^3 + 4*a^3*c*d^3*g^2*i^3 - 6*a^2*b*
c^2*d^2*g^2*i^3) + x^2*(2*a^3*d^4*g^2*i^3 + 4*b^3*c^3*d*g^2*i^3 - 6*a*b^2*
c^2*d^2*g^2*i^3) + x^3*(2*b^3*c^2*d^2*g^2*i^3 + 2*a^2*b*d^4*g^2*i^3 - 4*a*
b^2*c*d^3*g^2*i^3) + 2*a^3*c^2*d^2*g^2*i^3 + 2*a*b^2*c^4*g^2*i^3 - 4*a^2*b
*c^3*d*g^2*i^3) - log(e*((a + b*x)/(c + d*x))^n)^2*(((B^2*(a*d + 2*b*c))/(
2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (3*B^2*b*d*x)/(2*(a^2*d^2 + b^2*c^2 -
 2*a*b*c*d)))/(x*(b*c^2*g^2*i^3 + 2*a*c*d*g^2*i^3) + x^2*(a*d^2*g^2*i^3 +
2*b*c*d*g^2*i^3) + a*c^2*g^2*i^3 + b*d^2*g^2*i^3*x^3) + (3*B*b^2*d*(2*A -
B*n))/(2*g^2*i^3*n*(a*d - b*c)^4) - (3*B^2*b^2*d*(d*g^2*i^3*n*x^2*(a*d - b
*c) + (a*c*g^2*i^3*n*(a*d - b*c))/b + (g^2*i^3*n*x*(a*d + b*c)*(a*d - b*c)
)/b))/(g^2*i^3*n*(a*d - b*c)^4*(x*(b*c^2*g^2*i^3 + 2*a*c*d*g^2*i^3) + x^2*
(a*d^2*g^2*i^3 + 2*b*c*d*g^2*i^3) + a*c^2*g^2*i^3 + b*d^2*g^2*i^3*x^3))) -
 log(e*((a + b*x)/(c + d*x))^n)*((x*((3*B^2*b*d*n)/2 + 3*A*B*b*d) - (B^2*a
*d*n)/2 + 2*B^2*b*c*n + A*B*a*d + 2*A*B*b*c)/(x*(b^3*c^4*g^2*i^3 + 2*a^3*c
*d^3*g^2*i^3 - 3*a^2*b*c^2*d^2*g^2*i^3) + x^2*(a^3*d^4*g^2*i^3 + 2*b^3*...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 3638, normalized size of antiderivative = 6.47

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^2(ci + dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x)
```

output

```
(i*( - 12*log(a + b*x)*a**4*b**2*c**2*d**2*n - 24*log(a + b*x)*a**4*b**2*c
*d**3*n*x - 12*log(a + b*x)*a**4*b**2*d**4*n*x**2 - 24*log(a + b*x)*a**3*b
**3*c**3*d*n + 36*log(a + b*x)*a**3*b**3*c**2*d**2*n**2 - 60*log(a + b*x)*
a**3*b**3*c**2*d**2*n*x + 72*log(a + b*x)*a**3*b**3*c*d**3*n**2*x - 48*log
(a + b*x)*a**3*b**3*c*d**3*n*x**2 + 36*log(a + b*x)*a**3*b**3*d**4*n**2*x*
*2 - 12*log(a + b*x)*a**3*b**3*d**4*n*x**3 - 24*log(a + b*x)*a**2*b**4*c**
3*d*n*x - 42*log(a + b*x)*a**2*b**4*c**2*d**2*n**3 + 36*log(a + b*x)*a**2*
b**4*c**2*d**2*n**2*x - 48*log(a + b*x)*a**2*b**4*c**2*d**2*n*x**2 - 84*lo
g(a + b*x)*a**2*b**4*c*d**3*n**3*x + 72*log(a + b*x)*a**2*b**4*c*d**3*n**2
*x**2 - 24*log(a + b*x)*a**2*b**4*c*d**3*n*x**3 - 42*log(a + b*x)*a**2*b**
4*d**4*n**3*x**2 + 36*log(a + b*x)*a**2*b**4*d**4*n**2*x**3 - 48*log(a + b
*x)*a*b**5*c**3*d*n**3 - 138*log(a + b*x)*a*b**5*c**2*d**2*n**3*x - 132*lo
g(a + b*x)*a*b**5*c*d**3*n**3*x**2 - 42*log(a + b*x)*a*b**5*d**4*n**3*x**3
 - 48*log(a + b*x)*b**6*c**3*d*n**3*x - 96*log(a + b*x)*b**6*c**2*d**2*n**
3*x**2 - 48*log(a + b*x)*b**6*c*d**3*n**3*x**3 + 12*log(c + d*x)*a**4*b**2
*c**2*d**2*n + 24*log(c + d*x)*a**4*b**2*c*d**3*n*x + 12*log(c + d*x)*a**4
*b**2*d**4*n*x**2 + 24*log(c + d*x)*a**3*b**3*c**3*d*n - 36*log(c + d*x)*a
**3*b**3*c**2*d**2*n**2 + 60*log(c + d*x)*a**3*b**3*c**2*d**2*n*x - 72*log
(c + d*x)*a**3*b**3*c*d**3*n**2*x + 48*log(c + d*x)*a**3*b**3*c*d**3*n*x**
2 - 36*log(c + d*x)*a**3*b**3*d**4*n**2*x**2 + 12*log(c + d*x)*a**3*b**...
```

3.208 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx$

Optimal result 2374
Mathematica [B] (verified) 2375
Rubi [A] (verified) 2376
Maple [B] (verified) 2378
Fricas [B] (verification not implemented) 2379
Sympy [F(-1)] 2380
Maxima [B] (verification not implemented) 2380
Giac [F] 2381
Mupad [B] (verification not implemented) 2382
Reduce [B] (verification not implemented) 2383

Optimal result

Integrand size = 45, antiderivative size = 732

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx &= \frac{B^2d^4n^2(a+bx)^2}{4(bc-ad)^5g^3i^3(c+dx)^2}+\frac{8AbBd^3n(a+bx)}{(bc-ad)^5g^3i^3(c+dx)}\\ &-\frac{8bB^2d^3n^2(a+bx)}{(bc-ad)^5g^3i^3(c+dx)}+\frac{8b^3B^2dn^2(c+dx)}{(bc-ad)^5g^3i^3(a+bx)}\\ &-\frac{b^4B^2n^2(c+dx)^2}{4(bc-ad)^5g^3i^3(a+bx)^2}\\ &+\frac{8bB^2d^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^5g^3i^3(c+dx)}\\ &-\frac{Bd^4n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2}\\ &+\frac{8b^3Bdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^3i^3(a+bx)}\\ &-\frac{b^4Bn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2}\\ &+\frac{d^4(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^5g^3i^3(c+dx)^2}\\ &-\frac{4bd^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^3i^3(c+dx)}\\ &+\frac{4b^3d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^3i^3(a+bx)}\\ &-\frac{b^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^5g^3i^3(a+bx)^2}\\ &+\frac{2b^2d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{B(bc-ad)^5g^3i^3n}\end{aligned}$$

output

```
1/4*B^2*d^4*n^2*(b*x+a)^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2+8*A*b*B*d^3*n*(b*
x+a)/(-a*d+b*c)^5/g^3/i^3/(d*x+c)-8*b*B^2*d^3*n^2*(b*x+a)/(-a*d+b*c)^5/g^3
/i^3/(d*x+c)+8*b^3*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/4*b^4*
B^2*n^2*(d*x+c)^2/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+8*b*B^2*d^3*n*(b*x+a)*ln(
e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^5/g^3/i^3/(d*x+c)-1/2*B*d^4*n*(b*x+a)^2*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2+8*b^3*B*d*n
*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/2*
b^4*B*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(b*
x+a)^2+1/2*d^4*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^5/g^
3/i^3/(d*x+c)^2-4*b*d^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*
c)^5/g^3/i^3/(d*x+c)+4*b^3*d*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a
*d+b*c)^5/g^3/i^3/(b*x+a)-1/2*b^4*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)^2/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+2*b^2*d^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n
))^3/B/(-a*d+b*c)^5/g^3/i^3/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1653 vs. $2(732) = 1464$.

Time = 2.14 (sec) , antiderivative size = 1653, normalized size of antiderivative = 2.26

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i +
 d*i*x)^3),x]
```

output

```
(8*b^2*B^2*d^2*n^2*(a + b*x)^2*(c + d*x)^2*Log[(a + b*x)/(c + d*x)]^3 + 2*
B*n*Log[(a + b*x)/(c + d*x)]^2*(12*a^2*A*b^2*c^2*d^2 - b^4*B*c^4*n + 8*a*b
^3*B*c^3*d*n - 8*a^3*b*B*c*d^3*n + a^4*B*d^4*n + 24*a*A*b^3*c^2*d^2*x + 24
*a^2*A*b^2*c*d^3*x + 4*b^4*B*c^3*d*n*x + 24*a*b^3*B*c^2*d^2*n*x - 24*a^2*b
^2*B*c*d^3*n*x - 4*a^3*b*B*d^4*n*x + 12*A*b^4*c^2*d^2*x^2 + 48*a*A*b^3*c*d
^3*x^2 + 12*a^2*A*b^2*d^4*x^2 + 18*b^4*B*c^2*d^2*n*x^2 - 18*a^2*b^2*B*d^4*
n*x^2 + 24*A*b^4*c*d^3*x^3 + 24*a*A*b^3*d^4*x^3 + 12*b^4*B*c*d^3*n*x^3 - 1
2*a*b^3*B*d^4*n*x^3 + 12*A*b^4*d^4*x^4 + 12*b^2*B*d^2*(a + b*x)^2*(c + d*x
)^2*Log[e*((a + b*x)/(c + d*x))^n] - 12*b^2*B*d^2*n*(a + b*x)^2*(c + d*x)^
2*Log[(a + b*x)/(c + d*x)]) + 12*b^2*d^2*(a + b*x)^2*(c + d*x)^2*Log[a + b
*x]*(2*A^2 + 5*B^2*n^2 + 4*A*B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a
+ b*x)/(c + d*x)]) + 2*B^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*
x)/(c + d*x)])^2) + 2*b^2*d*(b*c - a*d)*(a + b*x)*(c + d*x)^2*(6*A^2 + 14*
A*B*n + 15*B^2*n^2 + 6*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(6*A +
 7*B*n)*Log[(a + b*x)/(c + d*x)] + 6*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 +
2*B*Log[e*((a + b*x)/(c + d*x))^n]*(6*A + 7*B*n - 6*B*n*Log[(a + b*x)/(c +
 d*x)])) - b^2*(b*c - a*d)^2*(c + d*x)^2*(2*A^2 + 2*A*B*n + B^2*n^2 + 2*B^
2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(2*A + B*n)*Log[(a + b*x)/(c +
d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d
*x))^n]*(2*A + B*n - 2*B*n*Log[(a + b*x)/(c + d*x)])) + 2*B*(b*c - a*d)...
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}\,d\frac{a+bx}{c+dx}}{g^3i^3(bc-ad)^5}$$

$$\downarrow 2795$$

$$\int \left(\frac{(c+dx)^3 \left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^4}{(a+bx)^3} - \frac{4d(c+dx)^2 \left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^3}{(a+bx)^2} + \frac{6d^2(c+dx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^2}{a+bx} - 4d^3\left(A + \right.$$

$$\frac{}{g^3 i^3 (bc-ad)^5}$$

$$\downarrow 2009$$

$$-\frac{b^4(c+dx)^2\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2} - \frac{b^4 Bn(c+dx)^2\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2} + \frac{4b^3 d(c+dx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx} + \frac{8b^3 Bdn(c+dx}{}$$

input

```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i + d*i*x
)^3),x]
```

output

```
((B^2*d^4*n^2*(a + b*x)^2)/(4*(c + d*x)^2) + (8*A*b*B*d^3*n*(a + b*x))/(c
 + d*x) - (8*b*B^2*d^3*n^2*(a + b*x))/(c + d*x) + (8*b^3*B^2*d*n^2*(c + d*x
))/(a + b*x) - (b^4*B^2*n^2*(c + d*x)^2)/(4*(a + b*x)^2) + (8*b*B^2*d^3*n*
(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(c + d*x) - (B*d^4*n*(a + b*x)^2
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(c + d*x)^2) + (8*b^3*B*d*n*(c
 + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - (b^4*B*n*(c +
d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) + (d^4*(a +
 b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(c + d*x)^2) - (4*b*d
^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c + d*x) + (4*b^3*
d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) - (b^4*(c
+ d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a + b*x)^2) + (2*b^
2*d^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(B*n))/((b*c - a*d)^5*g^3*
i^3)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2992 vs. 2(720) = 1440.

Time = 58.61 (sec) , antiderivative size = 2993, normalized size of antiderivative = 4.09

method	result	size
parallelrisch	Expression too large to display	2993

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,method
=_RETURNVERBOSE)
```

output

```
-1/4*(B^2*a^4*b^4*d^8*n^3-B^2*b^8*c^4*d^4*n^3+2*A^2*a^4*b^4*d^8*n-2*A^2*b^
8*c^4*d^4*n-16*A^2*a^3*b^5*c*d^7*n+16*A^2*a*b^7*c^3*d^5*n+8*B^2*x^4*ln(e*(
(b*x+a)/(d*x+c))^n)^3*b^8*d^8+24*A^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^8*d^8
+60*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^8*d^8*n^2+24*A*B*x^4*ln(e*((b*x+a)
/(d*x+c))^n)^2*b^8*d^8+16*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^3*a*b^7*d^8+16
*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^3*b^8*c*d^7-60*B^2*x^3*a*b^7*d^8*n^3+60
*B^2*x^3*b^8*c*d^7*n^3+8*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^6*d^8+8
*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*b^8*c^2*d^6-90*B^2*x^2*a^2*b^6*d^8*n^
3+90*B^2*x^2*b^8*c^2*d^6*n^3+48*A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*d^
8+48*A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^8*c*d^7-24*A^2*x^3*a*b^7*d^8*n+24
*A^2*x^3*b^8*c*d^7*n-28*B^2*x*a^3*b^5*d^8*n^3+28*B^2*x*b^8*c^3*d^5*n^3+24*
A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^6*d^8+24*A^2*x^2*ln(e*((b*x+a)/(d*
x+c))^n)*b^8*c^2*d^6-36*A^2*x^2*a^2*b^6*d^8*n+36*A^2*x^2*b^8*c^2*d^6*n+8*B
^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^6*c^2*d^6+2*B^2*ln(e*((b*x+a)/(d*x+c)
)^n)^2*a^4*b^4*d^8*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^8*c^4*d^4*n-2*B^2
*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^4*d^8*n^2-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)
*b^8*c^4*d^4*n^2-8*A^2*x*a^3*b^5*d^8*n+8*A^2*x*b^8*c^3*d^5*n+24*A^2*ln(e*(
(b*x+a)/(d*x+c))^n)*a^2*b^6*c^2*d^6+24*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2
*b^8*c*d^7*n+120*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*d^8*n^2+120*B^2*x
^3*ln(e*((b*x+a)/(d*x+c))^n)*b^8*c*d^7*n^2+48*A*B*x^3*ln(e*((b*x+a)/(d*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3062 vs. $2(720) = 1440$.

Time $= 0.17$ (sec) , antiderivative size $= 3062$, normalized size of antiderivative $= 4.18$

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x
, algorithm="fricas")
```

output

```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3/(d*i*x+c*i)**
3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5594 vs. $2(720) = 1440$.

Time = 0.39 (sec) , antiderivative size = 5594, normalized size of antiderivative = 7.64

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x
, algorithm="maxima")
```

output

```
1/2*B^2*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d
^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b
*d^3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c
*d^5 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b
^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3
+ (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 +
a^6*d^6)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2
+ 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^
6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g
^3*i^3) + 12*b^2*d^2*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c
^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d
^2*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^
2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3))*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n)^2 + A*B*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c
*d^2 - a^3*d^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c
*d^2 + a^2*b*d^3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 -
 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d
^2 + 2*a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*
g^3*i^3*x^3 + (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^
2*c^2*d^4 + a^6*d^6)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a...
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3(dix+ci)^3}\,dx$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x
, algorithm="giac")
```

output

```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/((b*g*x + a*g)^3*(d*i*x
 + c*i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 33.75 (sec) , antiderivative size = 2419, normalized size of antiderivative = 3.30

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^3*(c*i + d*i*x
)^3),x)
```

output

```
((3*x^2*(6*A^2*a*b^2*d^3 + 6*A^2*b^3*c*d^2 + 15*B^2*a*b^2*d^3*n^2 + 15*B^2
*b^3*c*d^2*n^2 - 4*A*B*a*b^2*d^3*n + 4*A*B*b^3*c*d^2*n))/(a*d - b*c) - (2*
A^2*a^3*d^3 + 2*A^2*b^3*c^3 + B^2*a^3*d^3*n^2 + B^2*b^3*c^3*n^2 - 14*A^2*a
*b^2*c^2*d - 14*A^2*a^2*b*c*d^2 - 2*A*B*a^3*d^3*n + 2*A*B*b^3*c^3*n - 31*B
^2*a*b^2*c^2*d*n^2 - 31*B^2*a^2*b*c*d^2*n^2 - 30*A*B*a*b^2*c^2*d*n + 30*A*
B*a^2*b*c*d^2*n)/(2*(a*d - b*c)) + (2*x*(2*A^2*a^2*b*d^3 + 2*A^2*b^3*c^2*d
 + 14*A^2*a*b^2*c*d^2 + 7*B^2*a^2*b*d^3*n^2 + 7*B^2*b^3*c^2*d*n^2 + 31*B^2
*a*b^2*c*d^2*n^2 - 6*A*B*a^2*b*d^3*n + 6*A*B*b^3*c^2*d*n))/(a*d - b*c) + (
6*x^3*(2*A^2*b^3*d^3 + 5*B^2*b^3*d^3*n^2))/(a*d - b*c))/(x^4*(2*a^3*b^2*d^
5*g^3*i^3 - 2*b^5*c^3*d^2*g^3*i^3 + 6*a*b^4*c^2*d^3*g^3*i^3 - 6*a^2*b^3*c*
d^4*g^3*i^3) - x*(4*a*b^4*c^5*g^3*i^3 - 4*a^5*c*d^4*g^3*i^3 - 8*a^2*b^3*c^
4*d*g^3*i^3 + 8*a^4*b*c^2*d^3*g^3*i^3) + x^3*(4*a^4*b*d^5*g^3*i^3 - 4*b^5*
c^4*d*g^3*i^3 + 8*a*b^4*c^3*d^2*g^3*i^3 - 8*a^3*b^2*c*d^4*g^3*i^3) + x^2*(
2*a^5*d^5*g^3*i^3 - 2*b^5*c^5*g^3*i^3 - 2*a*b^4*c^4*d*g^3*i^3 + 2*a^4*b*c*
d^4*g^3*i^3 + 16*a^2*b^3*c^3*d^2*g^3*i^3 - 16*a^3*b^2*c^2*d^3*g^3*i^3) - 2
*a^2*b^3*c^5*g^3*i^3 + 2*a^5*c^2*d^3*g^3*i^3 + 6*a^3*b^2*c^4*d*g^3*i^3 - 6
*a^4*b*c^3*d^2*g^3*i^3) + log(e*((a + b*x)/(c + d*x))^n)^2*((x*((3*B^2*b*d
*(a*d + b*c)^2)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2 - (B^2*b*d)/(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d) + (6*B^2*a*b^2*c*d^2)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^
2) - (B^2*(a*d + b*c))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (6*B^2*b^3...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 5363, normalized size of antiderivative = 7.33

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x)
```

output

```
(i*( - 24*log(a + b*x)*a**5*b**2*c**2*d**3*n - 48*log(a + b*x)*a**5*b**2*c
*d**4*n*x - 24*log(a + b*x)*a**5*b**2*d**5*n*x**2 - 24*log(a + b*x)*a**4*b
**3*c**3*d**2*n + 24*log(a + b*x)*a**4*b**3*c**2*d**3*n**2 - 96*log(a + b*
x)*a**4*b**3*c**2*d**3*n*x + 48*log(a + b*x)*a**4*b**3*c*d**4*n**2*x - 120
*log(a + b*x)*a**4*b**3*c*d**4*n*x**2 + 24*log(a + b*x)*a**4*b**3*d**5*n**
2*x**2 - 48*log(a + b*x)*a**4*b**3*d**5*n*x**3 - 24*log(a + b*x)*a**3*b**4
*c**3*d**2*n**2 - 48*log(a + b*x)*a**3*b**4*c**3*d**2*n*x - 60*log(a + b*x
)*a**3*b**4*c**2*d**3*n**3 - 120*log(a + b*x)*a**3*b**4*c**2*d**3*n*x**2 -
 120*log(a + b*x)*a**3*b**4*c*d**4*n**3*x + 72*log(a + b*x)*a**3*b**4*c*d*
*4*n**2*x**2 - 96*log(a + b*x)*a**3*b**4*c*d**4*n*x**3 - 60*log(a + b*x)*a
**3*b**4*d**5*n**3*x**2 + 48*log(a + b*x)*a**3*b**4*d**5*n**2*x**3 - 24*lo
g(a + b*x)*a**3*b**4*d**5*n*x**4 - 60*log(a + b*x)*a**2*b**5*c**3*d**2*n**
3 - 48*log(a + b*x)*a**2*b**5*c**3*d**2*n**2*x - 24*log(a + b*x)*a**2*b**5
*c**3*d**2*n*x**2 - 240*log(a + b*x)*a**2*b**5*c**2*d**3*n**3*x - 72*log(a
 + b*x)*a**2*b**5*c**2*d**3*n**2*x**2 - 48*log(a + b*x)*a**2*b**5*c**2*d**
3*n*x**3 - 300*log(a + b*x)*a**2*b**5*c*d**4*n**3*x**2 - 24*log(a + b*x)*a
**2*b**5*c*d**4*n*x**4 - 120*log(a + b*x)*a**2*b**5*d**5*n**3*x**3 + 24*lo
g(a + b*x)*a**2*b**5*d**5*n**2*x**4 - 120*log(a + b*x)*a*b**6*c**3*d**2*n*
*3*x - 24*log(a + b*x)*a*b**6*c**3*d**2*n**2*x**2 - 300*log(a + b*x)*a*b**
6*c**2*d**3*n**3*x**2 - 48*log(a + b*x)*a*b**6*c**2*d**3*n**2*x**3 - 24...
```

3.209 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx$

Optimal result 2385
Mathematica [B] (verified) 2386
Rubi [A] (verified) 2387
Maple [B] (verified) 2389
Fricas [B] (verification not implemented) 2390
Sympy [F(-1)] 2391
Maxima [B] (verification not implemented) 2391
Giac [F] 2392
Mupad [B] (verification not implemented) 2393
Reduce [B] (verification not implemented) 2394

Optimal result

Integrand size = 45, antiderivative size = 908

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = &-\frac{B^2d^5n^2(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2}-\frac{10AbBd^4n(a+bx)}{(bc-ad)^6g^4i^3(c+dx)}\\&+\frac{10bB^2d^4n^2(a+bx)}{(bc-ad)^6g^4i^3(c+dx)}-\frac{20b^3B^2d^2n^2(c+dx)}{(bc-ad)^6g^4i^3(a+bx)}\\&+\frac{5b^4B^2dn^2(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2}\\&-\frac{2b^5B^2n^2(c+dx)^3}{27(bc-ad)^6g^4i^3(a+bx)^3}\\&-\frac{10bB^2d^4n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^6g^4i^3(c+dx)}\\&+\frac{Bd^5n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2}\\&-\frac{20b^3Bd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(a+bx)}\\&+\frac{5b^4Bdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2}\\&-\frac{2b^5Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^6g^4i^3(a+bx)^3}\\&-\frac{d^5(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^6g^4i^3(c+dx)^2}\\&+\frac{5bd^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^6g^4i^3(c+dx)}\\&-\frac{10b^3d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^6g^4i^3(a+bx)}\\&+\frac{5b^4d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^6g^4i^3(a+bx)^2}\\&-\frac{b^5(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^6g^4i^3(a+bx)^3}\\&-\frac{10b^2d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^6g^4i^3n}\end{aligned}$$

output

```
-1/4*B^2*d^5*n^2*(b*x+a)^2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2-10*A*b*B*d^4*n*(
b*x+a)/(-a*d+b*c)^6/g^4/i^3/(d*x+c)+10*b*B^2*d^4*n^2*(b*x+a)/(-a*d+b*c)^6/
g^4/i^3/(d*x+c)-20*b^3*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/
4*b^4*B^2*d*n^2*(d*x+c)^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-2/27*b^5*B^2*n^2*
(d*x+c)^3/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^3-10*b*B^2*d^4*n*(b*x+a)*ln(e*((b*x
+a)/(d*x+c))^n)/(-a*d+b*c)^6/g^4/i^3/(d*x+c)+1/2*B*d^5*n*(b*x+a)^2*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2-20*b^3*B*d^2*n*(d*
x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/2*b^4*
B*d*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(b*x+
a)^2-2/9*b^5*B*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^
4/i^3/(b*x+a)^3-1/2*d^5*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+
b*c)^6/g^4/i^3/(d*x+c)^2+5*b*d^4*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2
/(-a*d+b*c)^6/g^4/i^3/(d*x+c)-10*b^3*d^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/2*b^4*d*(d*x+c)^2*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-1/3*b^5*(d*x+c)^3*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^3-10/3*b^2*d^3*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)^6/g^4/i^3/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2138 vs. $2(908) = 1816$.

Time = 3.52 (sec) , antiderivative size = 2138, normalized size of antiderivative = 2.35

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Result too large to show}$$

input

```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i +
 d*i*x)^3),x]
```

output

```
-1/108*(360*b^2*B^2*d^3*n^2*(a + b*x)^3*(c + d*x)^2*Log[(a + b*x)/(c + d*x
)]^3 + 18*B*n*Log[(a + b*x)/(c + d*x)]^2*(60*a^3*A*b^2*c^2*d^3 + 2*b^5*B*c
^5*n - 15*a*b^4*B*c^4*d*n + 60*a^2*b^3*B*c^3*d^2*n - 30*a^4*b*B*c*d^4*n +
3*a^5*B*d^5*n + 180*a^2*A*b^3*c^2*d^3*x + 120*a^3*A*b^2*c*d^4*x - 5*b^5*B*
c^4*d*n*x + 60*a*b^4*B*c^3*d^2*n*x + 180*a^2*b^3*B*c^2*d^3*n*x - 120*a^3*b
^2*B*c*d^4*n*x - 15*a^4*b*B*d^5*n*x + 180*a*A*b^4*c^2*d^3*x^2 + 360*a^2*A*
b^3*c*d^4*x^2 + 60*a^3*A*b^2*d^5*x^2 + 20*b^5*B*c^3*d^2*n*x^2 + 270*a*b^4*
B*c^2*d^3*n*x^2 - 90*a^3*b^2*B*d^5*n*x^2 + 60*A*b^5*c^2*d^3*x^3 + 360*a*A*
b^4*c*d^4*x^3 + 180*a^2*A*b^3*d^5*x^3 + 110*b^5*B*c^2*d^3*n*x^3 + 180*a*b^
4*B*c*d^4*n*x^3 - 90*a^2*b^3*B*d^5*n*x^3 + 120*A*b^5*c*d^4*x^4 + 180*a*A*b
^4*d^5*x^4 + 100*b^5*B*c*d^4*n*x^4 + 60*A*b^5*d^5*x^5 + 20*b^5*B*d^5*n*x^5
 + 60*b^2*B*d^3*(a + b*x)^3*(c + d*x)^2*Log[e*((a + b*x)/(c + d*x))^n] - 6
0*b^2*B*d^3*n*(a + b*x)^3*(c + d*x)^2*Log[(a + b*x)/(c + d*x)]) + 6*b^2*d^
2*(b*c - a*d)*(a + b*x)^2*(c + d*x)^2*(108*A^2 + 282*A*B*n + 319*B^2*n^2 +
 108*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(36*A + 47*B*n)*Log[(a +
 b*x)/(c + d*x)] + 108*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*((a
+ b*x)/(c + d*x))^n]*(36*A + 47*B*n - 36*B*n*Log[(a + b*x)/(c + d*x)])) -
3*b^2*d*(b*c - a*d)^2*(a + b*x)*(c + d*x)^2*(54*A^2 + 66*A*B*n + 37*B^2*n^
2 + 54*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(18*A + 11*B*n)*Log[(a
 + b*x)/(c + d*x)] + 54*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*...
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 653, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}\,d\frac{a+bx}{c+dx}}{g^4i^3(bc-ad)^6}$$

$$\downarrow 2795$$

$$\int \left(\frac{(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^5}{(a+bx)^4}-\frac{5d(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^4}{(a+bx)^3}+\frac{10d^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^3}{(a+bx)^2}-\frac{10d^3(c+}{}\right.$$

$$g^4 i^3(bc-ad)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^5(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}-\frac{2b^5Bn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9(a+bx)^3}+\frac{5b^4d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}+\frac{5b^4Bdn(c+}{}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i + d*i*x
)^3),x]
```

output
```
(-1/4*(B^2*d^5*n^2*(a + b*x)^2)/(c + d*x)^2 - (10*A*b*B*d^4*n*(a + b*x))/(
c + d*x) + (10*b*B^2*d^4*n^2*(a + b*x))/(c + d*x) - (20*b^3*B^2*d^2*n^2*(c
 + d*x))/(a + b*x) + (5*b^4*B^2*d*n^2*(c + d*x)^2)/(4*(a + b*x)^2) - (2*b^
5*B^2*n^2*(c + d*x)^3)/(27*(a + b*x)^3) - (10*b*B^2*d^4*n*(a + b*x)*Log[e*
((a + b*x)/(c + d*x))^n])/(c + d*x) + (B*d^5*n*(a + b*x)^2*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]))/(2*(c + d*x)^2) - (20*b^3*B*d^2*n*(c + d*x)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (5*b^4*B*d*n*(c + d*x)^2*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) - (2*b^5*B*n*(c + d*
x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(a + b*x)^3) - (d^5*(a + b
*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(c + d*x)^2) + (5*b*d^4
*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c + d*x) - (10*b^3*d
^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) + (5*b^4*
d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a + b*x)^2) -
(b^5*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*(a + b*x)^3)
 - (10*b^2*d^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*B*n))/((b*c -
a*d)^6*g^4*i^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4485 vs. 2(888) = 1776.

Time = 210.38 (sec) , antiderivative size = 4486, normalized size of antiderivative = 4.94

method	result	size
parallelrisch	Expression too large to display	4486

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,method
=_RETURNVERBOSE)
```

output

```
-1/108*(27*B^2*a^5*b^5*d^9*n^3+8*B^2*b^10*c^5*d^4*n^3+54*A^2*a^5*b^5*d^9*n
+36*A^2*b^10*c^5*d^4*n+6480*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*c^2*d^
7*n^2+1080*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^10*c^2*d^7+1320*B^2*x^2*l
n(e*((b*x+a)/(d*x+c))^n)*b^10*c^3*d^6*n^2-5880*B^2*x^2*a^2*b^8*c*d^8*n^3+9
210*B^2*x^2*a*b^9*c^2*d^7*n^3+6480*A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9
*c*d^8+1080*A^2*x^3*a*b^9*c*d^8*n+1080*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2
*a^3*b^7*d^9+840*A*B*x^2*a^3*b^7*d^9*n^2+1320*A*B*x^2*b^10*c^3*d^6*n^2+720
*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^3*b^7*c*d^8+1080*B^2*x*ln(e*((b*x+a)/
(d*x+c))^n)^3*a^2*b^8*c^2*d^7-270*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b^
6*d^9*n-90*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^10*c^4*d^5*n+810*B^2*x*ln(e
*((b*x+a)/(d*x+c))^n)*a^4*b^6*d^9*n^2-150*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*
b^10*c^4*d^5*n^2-6280*B^2*x*a^3*b^7*c*d^8*n^3+3540*B^2*x*a^2*b^8*c^2*d^7*n
^3+3780*B^2*x*a*b^9*c^3*d^6*n^3+6480*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2
*b^8*c*d^8+3240*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c^2*d^7-2160*A^2*x
^2*a^2*b^8*c*d^8*n+3780*A^2*x^2*a*b^9*c^2*d^7*n+810*A*B*x*a^4*b^6*d^9*n^2-
150*A*B*x*b^10*c^4*d^5*n^2-540*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b^6*c*d
^8*n+1080*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^8*c^3*d^6*n-270*B^2*ln(e*(
(b*x+a)/(d*x+c))^n)^2*a*b^9*c^4*d^5*n+1080*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a
^4*b^6*c*d^8*n^2+2160*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*c^3*d^6*n^2-27
0*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c^4*d^5*n^2+2160*A^2*x*ln(e*((b*x...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4725 vs. $2(888) = 1776$.

Time $= 0.25$ (sec) , antiderivative size $= 4725$, normalized size of antiderivative $= 5.20$

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x
, algorithm="fricas")
```

output

```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4/(d*i*x+c*i)**
3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9293 vs. $2(888) = 1776$.

Time = 0.81 (sec) , antiderivative size = 9293, normalized size of antiderivative = 10.23

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x
, algorithm="maxima")
```

output

```
-1/6*B^2*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2*b^2*c^2*d^
2 + 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 10*(
2*b^4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3*d - 11*a
*b^3*c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/((b^8*c^5*d^2 - 5*a*b^7*
c^4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*
b^3*d^7)*g^4*i^3*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3
+ 10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d
^7)*g^4*i^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5
*c^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*
b*d^7)*g^4*i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25
*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 -
 a^8*d^7)*g^4*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5
*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*
d^6)*g^4*i^3*x + (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*
a^6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i^3) + 60*b^2*d^3*log
(b*x + a)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*
d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3) - 60*b^2*d^3*
log(d*x + c)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c
^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3))*log(e*(b*
x/(d*x + c) + a/(d*x + c))^n)^2 - 1/3*A*B*((60*b^4*d^4*x^4 + 2*b^4*c^4 ...
```

Giac [F]

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^4(dix+ci)^3}\,dx$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x
, algorithm="giac")
```

output

```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/((b*g*x + a*g)^4*(d*i*x
 + c*i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 35.04 (sec) , antiderivative size = 4649, normalized size of antiderivative = 5.12

$$\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^4*(c*i + d*i*x
)^3),x)
```

output

```
log(e*((a + b*x)/(c + d*x))^n)*((x*((a*d + b*c)*(20*A*B*a*b*d^2 + 10*A*B*b
^2*c*d - (70*B^2*a*b*d^2*n)/3 + (10*B^2*b^2*c*d*n)/3) + a*c*(30*A*B*b^2*d^
2 - 20*B^2*b^2*d^2*n) + (5*B^2*a^2*b*d^3*n)/6 + (5*B^2*b^3*c^2*d*n)/6 - 5*
A*B*a^2*b*d^3 - 5*A*B*b^3*c^2*d + 10*A*B*a*b^2*c*d^2 - (5*B^2*a*b^2*c*d^2*
n)/3) + x^2*((a*d + b*c)*(30*A*B*b^2*d^2 - 20*B^2*b^2*d^2*n) + b*d*(20*A*B
*a*b*d^2 + 10*A*B*b^2*c*d - (70*B^2*a*b*d^2*n)/3 + (10*B^2*b^2*c*d*n)/3))
+ a*c*(20*A*B*a*b*d^2 + 10*A*B*b^2*c*d - (70*B^2*a*b*d^2*n)/3 + (10*B^2*b^
2*c*d*n)/3) - 3*A*B*a^3*d^3 - 2*A*B*b^3*c^3 + b*d*x^3*(30*A*B*b^2*d^2 - 20
*B^2*b^2*d^2*n) + (3*B^2*a^3*d^3*n)/2 - (2*B^2*b^3*c^3*n)/3 + A*B*a*b^2*c^
2*d + 4*A*B*a^2*b*c*d^2 + (17*B^2*a*b^2*c^2*d*n)/6 - (11*B^2*a^2*b*c*d^2*n
)/3)/(x^5*(3*a^4*b^3*d^6*g^4*i^3 + 3*b^7*c^4*d^2*g^4*i^3 - 12*a*b^6*c^3*d^
3*g^4*i^3 - 12*a^3*b^4*c*d^5*g^4*i^3 + 18*a^2*b^5*c^2*d^4*g^4*i^3) + x*(9*
a^2*b^5*c^6*g^4*i^3 + 6*a^7*c*d^5*g^4*i^3 - 30*a^3*b^4*c^5*d*g^4*i^3 - 15*
a^6*b*c^2*d^4*g^4*i^3 + 30*a^4*b^3*c^4*d^2*g^4*i^3) + x^2*(3*a^7*d^6*g^4*i
^3 + 9*a*b^6*c^6*g^4*i^3 + 6*a^6*b*c*d^5*g^4*i^3 - 18*a^2*b^5*c^5*d*g^4*i^
3 - 15*a^3*b^4*c^4*d^2*g^4*i^3 + 60*a^4*b^3*c^3*d^3*g^4*i^3 - 45*a^5*b^2*c
^2*d^4*g^4*i^3) + x^3*(3*b^7*c^6*g^4*i^3 + 9*a^6*b*d^6*g^4*i^3 + 6*a*b^6*c
^5*d*g^4*i^3 - 18*a^5*b^2*c*d^5*g^4*i^3 - 45*a^2*b^5*c^4*d^2*g^4*i^3 + 60*
a^3*b^4*c^3*d^3*g^4*i^3 - 15*a^4*b^3*c^2*d^4*g^4*i^3) + x^4*(9*a^5*b^2*d^6
*g^4*i^3 + 6*b^7*c^5*d*g^4*i^3 - 15*a*b^6*c^4*d^2*g^4*i^3 - 30*a^4*b^3*...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 7917, normalized size of antiderivative = 8.72

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x)
```

output

```
(i*( - 3240*log(a + b*x)*a**6*b**2*c**2*d**4*n - 6480*log(a + b*x)*a**6*b*
*2*c*d**5*n*x - 3240*log(a + b*x)*a**6*b**2*d**6*n*x**2 - 2160*log(a + b*x
)*a**5*b**3*c**3*d**3*n - 14040*log(a + b*x)*a**5*b**3*c**2*d**4*n*x - 216
00*log(a + b*x)*a**5*b**3*c*d**5*n*x**2 - 9720*log(a + b*x)*a**5*b**3*d**6
*n*x**3 - 3600*log(a + b*x)*a**4*b**4*c**3*d**3*n**2 - 6480*log(a + b*x)*a
**4*b**4*c**3*d**3*n*x - 8100*log(a + b*x)*a**4*b**4*c**2*d**4*n**3 - 7200
*log(a + b*x)*a**4*b**4*c**2*d**4*n**2*x - 22680*log(a + b*x)*a**4*b**4*c*
*2*d**4*n*x**2 - 16200*log(a + b*x)*a**4*b**4*c*d**5*n**3*x - 3600*log(a +
 b*x)*a**4*b**4*c*d**5*n**2*x**2 - 25920*log(a + b*x)*a**4*b**4*c*d**5*n*x
**3 - 8100*log(a + b*x)*a**4*b**4*d**6*n**3*x**2 - 9720*log(a + b*x)*a**4*
b**4*d**6*n*x**4 - 6600*log(a + b*x)*a**3*b**5*c**3*d**3*n**3 - 10800*log(
a + b*x)*a**3*b**5*c**3*d**3*n**2*x - 6480*log(a + b*x)*a**3*b**5*c**3*d**
3*n*x**2 - 37500*log(a + b*x)*a**3*b**5*c**2*d**4*n**3*x - 21600*log(a + b
*x)*a**3*b**5*c**2*d**4*n**2*x**2 - 16200*log(a + b*x)*a**3*b**5*c**2*d**4
*n*x**3 - 55200*log(a + b*x)*a**3*b**5*c*d**5*n**3*x**2 - 10800*log(a + b*
x)*a**3*b**5*c*d**5*n**2*x**3 - 12960*log(a + b*x)*a**3*b**5*c*d**5*n*x**4
 - 24300*log(a + b*x)*a**3*b**5*d**6*n**3*x**3 - 3240*log(a + b*x)*a**3*b*
*5*d**6*n*x**5 - 19800*log(a + b*x)*a**2*b**6*c**3*d**3*n**3*x - 10800*log
(a + b*x)*a**2*b**6*c**3*d**3*n**2*x**2 - 2160*log(a + b*x)*a**2*b**6*c**3
*d**3*n*x**3 - 63900*log(a + b*x)*a**2*b**6*c**2*d**4*n**3*x**2 - 21600...
```

3.210 $\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^1$

Optimal result . . . 2395
Mathematica [F] . . . 2395
Rubi [A] (verified) . . . 2396
Maple [F] . . . 2397
Fricas [F] . . . 2398
Sympy [F(-1)] . . . 2398
Maxima [F] . . . 2398
Giac [F] . . . 2399
Mupad [F(-1)] . . . 2399
Reduce [F] . . . 2400

Optimal result

Integrand size = 49, antiderivative size = 189

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx$$

$$= \frac{e^{-\frac{A(1+m)}{Bn}} (a+bx)(g(a+bx))^m \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}} (i(c+dx))^{-m} \Gamma\left(1+p, -\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right) (A+}{(bc-ad)i^2(1+m)(c+dx)}$$

output

```
(b*x+a)*(g*(b*x+a))^m*GAMMA(p+1,-(1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n
)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p/(-a*d+b*c)/exp(A*(1+m)/B/n)/i^2/(1+m)/
((e*((b*x+a)/(d*x+c))^n)^((1+m)/n))/(d*x+c)/((i*(d*x+c))^m)/((-(1+m)*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))/B/n)^p)
```

Mathematica [F]

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx$$

$$= \int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx$$

input
```
Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^p,x]
```

output
```
Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^p, x]
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2963, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^m (ci+dix)^{-m-2} \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^p dx$$

$$\downarrow 2963$$

$$\frac{(g(a+bx))^m (i(c+dx))^{-m} \left(\frac{a+bx}{c+dx}\right)^{-m} \int \left(\frac{a+bx}{c+dx}\right)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2747$$

$$\frac{(a+bx)(g(a+bx))^m (i(c+dx))^{-m} \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}} \int \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\log\left(e\left(\frac{a+}{c+}\right.\right.}{i^2 n(c+dx)(bc-ad)}$$

$$\downarrow 2612$$

$$\frac{(a+bx)e^{-\frac{A(m+1)}{Bn}} (g(a+bx))^m (i(c+dx))^{-m} \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}} \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^p \left(-\frac{(m+1)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.\right.}{Bn}}{i^2(m+1)(c+dx)(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^p,x]
```

output
```
((a + b*x)*(g*(a + b*x))^m*Gamma[1 + p, -(((1 + m)*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(B*n))]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p)/((b*c -
 a*d)*E^((A*(1 + m))/(B*n))*i^2*(1 + m)*(e*((a + b*x)/(c + d*x))^n)^((1 +
m)/n)*(c + d*x)*(i*(c + d*x))^m*(-(((1 + m)*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]))/(B*n)))^p)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

Maple [F]

$$\int (bgx+ag)^m (dix+ci)^{-2-m} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^p dx$$

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p,x)
```

output
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p,x)
```

Fricas [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx$$
$$= \int (bgx+ag)^m(dix+ci)^{-m-2}\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^p dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^p,x, algorithm="fricas")
```

output
```
integral((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)*(B*log(e*((b*x + a)/(d*x +
 c))^n) + A)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx$$
$$= \int (bgx+ag)^m(dix+ci)^{-m-2}\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^p dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^p,x, algorithm="maxima")
```

output
```
integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)*(B*log(e*((b*x + a)/(d*x
+ c))^n) + A)^p, x)
```

Giac [F]

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx$$
$$= \int (bgx+ag)^m (dix+ci)^{-m-2} \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^p dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^p,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)*(B*log(e*((b*x + a)/(d*x
+ c))^n) + A)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx$$
$$= \int \frac{(a\,g+b\,g\,x)^m \left(A + B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^p}{(c\,i+d\,i\,x)^{m+2}}\,dx$$

input
```
int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^p)/(c*i + d*i*
x)^(m + 2),x)
```

output
```
int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^p)/(c*i + d*i*
x)^(m + 2), x)
```

Reduce [F]

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx$$
$$= -\left(\int \frac{(bgx+ag)^m \left(\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) b+a\right)^p}{(dix+ci)^m c^2 + 2(dix+ci)^m cdx + (dix+ci)^m d^2x^2} dx\right)$$

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^p,x)
```

output
```
 - int(((a*g + b*g*x)**m*(log(((a + b*x)**n*e)/(c + d*x)**n)*b + a)**p)/((
c*i + d*i*x)**m*c**2 + 2*(c*i + d*i*x)**m*c*d*x + (c*i + d*i*x)**m*d**2*x*
*2),x)
```

3.211 $\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p$

Optimal result . . . 2401
Mathematica [F] . . . 2401
Rubi [A] (verified) . . . 2402
Maple [F] . . . 2403
Fricas [F] . . . 2404
Sympy [F(-1)] . . . 2404
Maxima [F] . . . 2404
Giac [F] . . . 2405
Mupad [F(-1)] . . . 2405
Reduce [F] . . . 2406

Optimal result

Integrand size = 49, antiderivative size = 190

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx =$$

$$-\frac{e^{\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1+m}{n}}(i(c+dx))^{2+m}\Gamma\left(1+p,\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)(A}{(bc-ad)i^2(1+m)(c+dx)}$$

output

```
-exp(A*(1+m)/B/n)*(b*x+a)*(g*(b*x+a))^(-2-m)*(e*((b*x+a)/(d*x+c))^n)^((1+m
)/n)*(i*(d*x+c))^(2+m)*GAMMA(p+1,(1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n
)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p/(-a*d+b*c)/i^2/(1+m)/(d*x+c)/(((1+m)*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)^p)
```

Mathematica [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx$$
$$= \int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx$$

input
```
Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^p,x]
```

output
```
Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^p, x]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2963, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^{-m-2}(ci+dix)^m \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^p dx$$

$$\downarrow 2963$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\int \left(\frac{a+bx}{c+dx}\right)^{-m-2}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2747$$

$$\frac{(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\int \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\log\left(\right.}{i^2n(c+dx)(bc-ad)}$$

$$\downarrow 2612$$

$$-\frac{(a+bx)e^{\frac{A(m+1)}{Bn}}(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^p\left(\frac{(m+1)\left(B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.\right.}{Bn}}{i^2(m+1)(c+dx)(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^p,x]
```

output
```
-((E^((A*(1 + m))/(B*n))*(a + b*x)*(g*(a + b*x))^(-2 - m)*(e*((a + b*x)/(c
 + d*x))^n)^((1 + m)/n)*(i*(c + d*x))^(2 + m)*Gamma[1 + p, ((1 + m)*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p)/((b*c - a*d)*i^2*(1 + m)*(c + d*x)*(((1 + m)*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(B*n))^p))
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

Maple [F]

$$\int (bgx+ag)^{-2-m}\,(dix+ci)^{m}\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{p}dx$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p,x)
```

output
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p,x)
```

Fricas [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx$$
$$= \int (bgx+ag)^{-m-2}(dix+ci)^m \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^p dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^p,x, algorithm="fricas")
```

output
```
integral((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m*(B*log(e*((b*x + a)/(d*x +
 c))^n) + A)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m*(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx$$
$$= \int (bgx+ag)^{-m-2}(dix+ci)^m \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^p dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^p,x, algorithm="maxima")
```

output
```
integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m*(B*log(e*((b*x + a)/(d*x
+ c))^n) + A)^p, x)
```

Giac [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx$$
$$= \int (bgx+ag)^{-m-2}(dix+ci)^m\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^p dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^p,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m*(B*log(e*((b*x + a)/(d*x
+ c))^n) + A)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx$$
$$= \int \frac{(ci+dix)^m\left(A+B\,\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p}{(ag+bgx)^{m+2}} dx$$

input
```
int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^p)/(a*g + b*g*
x)^(m + 2),x)
```

output
```
int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^p)/(a*g + b*g*
x)^(m + 2), x)
```

Reduce [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx$$
$$= \frac{\int \frac{(dix+ci)^m\left(\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b+a\right)^p}{(bgx+ag)^m a^2+2(bgx+ag)^m abx+(bgx+ag)^m b^2x^2}dx}{g^2}$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n))^p,x)
```

output
```
int(((c*i + d*i*x)**m*(log(((a + b*x)**n*e)/(c + d*x)**n)*b + a)**p)/((a*g
 + b*g*x)**m*a**2 + 2*(a*g + b*g*x)**m*a*b*x + (a*g + b*g*x)**m*b**2*x**2)
,x)/g**2
```

3.212 $\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3$

Optimal result . . . 2407
Mathematica [A] (verified) . . . 2408
Rubi [A] (verified) . . . 2408
Maple [F] . . . 2410
Fricas [B] (verification not implemented) . . . 2411
Sympy [F(-2)] . . . 2412
Maxima [F] . . . 2412
Giac [F] . . . 2413
Mupad [F(-1)] . . . 2413
Reduce [F] . . . 2414

Optimal result

Integrand size = 49, antiderivative size = 292

$$\begin{aligned}
&\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx \\
&= -\frac{6B^3 n^3 (a+bx)(g(a+bx))^m (i(c+dx))^{-m}}{(bc-ad)i^2(1+m)^4(c+dx)} \\
&\quad + \frac{6B^2 n^2 (a+bx)(g(a+bx))^m (i(c+dx))^{-m} \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)^3(c+dx)} \\
&\quad - \frac{3Bn(a+bx)(g(a+bx))^m (i(c+dx))^{-m} \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)^2(c+dx)} \\
&\quad + \frac{(a+bx)(g(a+bx))^m (i(c+dx))^{-m} \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)i^2(1+m)(c+dx)}
\end{aligned}$$

output

```
-6*B^3*n^3*(b*x+a)*(g*(b*x+a))^m/(-a*d+b*c)/i^2/(1+m)^4/(d*x+c)/((i*(d*x+c
))^m)+6*B^2*n^2*(b*x+a)*(g*(b*x+a))^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*
d+b*c)/i^2/(1+m)^3/(d*x+c)/((i*(d*x+c))^m)-3*B*n*(b*x+a)*(g*(b*x+a))^m*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/i^2/(1+m)^2/(d*x+c)/((i*(d*x+c))
^m)+(b*x+a)*(g*(b*x+a))^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/(-a*d+b*c)/i^2
/(1+m)/(d*x+c)/((i*(d*x+c))^m)
```

Mathematica [A] (verified)

Time = 7.72 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.71

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx$$
$$= \frac{(a+bx)(g(a+bx))^m (i(c+dx))^{-1-m} \left(A^3(1+m)^3 - 3A^2B(1+m)^2n + 6AB^2(1+m)n^2 - 6B^3n^3 + 3B$$

input
```
Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^3,x]
```

output
```
((a + b*x)*(g*(a + b*x))^m*(i*(c + d*x))^(-1 - m)*(A^3*(1 + m)^3 - 3*A^2*B
*(1 + m)^2*n + 6*A*B^2*(1 + m)*n^2 - 6*B^3*n^3 + 3*B*(1 + m)*(A^2*(1 + m)^
2 - 2*A*B*(1 + m)*n + 2*B^2*n^2)*Log[e*((a + b*x)/(c + d*x))^n] + 3*B^2*(1
 + m)^2*(A + A*m - B*n)*Log[e*((a + b*x)/(c + d*x))^n]^2 + B^3*(1 + m)^3*L
og[e*((a + b*x)/(c + d*x))^n]^3))/((b*c - a*d)*i*(1 + m)^4)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.082$, Rules used = {2963, 2742, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^m (ci+dix)^{-m-2} \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^3 dx$$
$$\downarrow 2963$$
$$\frac{(g(a+bx))^m (i(c+dx))^{-m} \left(\frac{a+bx}{c+dx}\right)^{-m} \int \left(\frac{a+bx}{c+dx}\right)^m \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$
$$\downarrow 2742$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{m+1}-\frac{3Bn\int\left(\frac{a+bx}{c+dx}\right)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d\frac{a+b}{c+d}}{m+1}\right.}{i^2(bc-ad)}$$

$$\downarrow 2742$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{m+1}-\frac{3Bn\left(\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{m+1}-\frac{2B}{m+1}\right.}{i^2(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{m+1}-\frac{3Bn\left(\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{m+1}-\frac{2B}{}\right.}{i^2(bc-ad)}$$

input

```
Int[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^3,x]
```

output

```
((g*(a + b*x))^m*((((a + b*x)/(c + d*x))^(1 + m)*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^3)/(1 + m) - (3*B*n*((((a + b*x)/(c + d*x))^(1 + m)*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])^2)/(1 + m) - (2*B*n*(-((B*n*((a + b*x)/(c +
 d*x))^(1 + m))/(1 + m)^2) + (((a + b*x)/(c + d*x))^(1 + m)*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]))/(1 + m)))/(1 + m)))/(1 + m)))/((b*c - a*d)*i^2*(
(a + b*x)/(c + d*x))^m*(i*(c + d*x))^m)
```

Defintions of rubi rules used

rule 2741

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2963

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

Maple [F]

$$\int (bgx+ag)^m (dix+ci)^{-2-m} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^3 dx$$

input

```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

output

```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2680 vs. $2(292) = 584$.

Time = 0.19 (sec) , antiderivative size = 2680, normalized size of antiderivative = 9.18

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="fricas")
```

output

```
(A^3*a*c*m^3 - 6*B^3*a*c*n^3 + 3*A^3*a*c*m^2 + 3*A^3*a*c*m + A^3*a*c + (B^
3*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m + B^3*a*c + (B^3*b*d*m^3 + 3*B^3*b
*d*m^2 + 3*B^3*b*d*m + B^3*b*d)*x^2 + (B^3*b*c + B^3*a*d + (B^3*b*c + B^3*
a*d)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B^3*b*c + B^3*a*d)*m)*x)*log(e)^
3 + ((B^3*b*d*m^3 + 3*B^3*b*d*m^2 + 3*B^3*b*d*m + B^3*b*d)*n^3*x^2 + (B^3*
b*c + B^3*a*d + (B^3*b*c + B^3*a*d)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B
^3*b*c + B^3*a*d)*m)*n^3*x + (B^3*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m +
B^3*a*c)*n^3)*log((b*x + a)/(d*x + c))^3 + 6*(A*B^2*a*c*m + A*B^2*a*c)*n^2
 + (A^3*b*d*m^3 - 6*B^3*b*d*n^3 + 3*A^3*b*d*m^2 + 3*A^3*b*d*m + A^3*b*d +
6*(A*B^2*b*d*m + A*B^2*b*d)*n^2 - 3*(A^2*B*b*d*m^2 + 2*A^2*B*b*d*m + A^2*B
*b*d)*n)*x^2 + 3*(A*B^2*a*c*m^3 + 3*A*B^2*a*c*m^2 + 3*A*B^2*a*c*m + A*B^2*
a*c + (A*B^2*b*d*m^3 + 3*A*B^2*b*d*m^2 + 3*A*B^2*b*d*m + A*B^2*b*d - (B^3*
b*d*m^2 + 2*B^3*b*d*m + B^3*b*d)*n)*x^2 - (B^3*a*c*m^2 + 2*B^3*a*c*m + B^3
*a*c)*n + (A*B^2*b*c + A*B^2*a*d + (A*B^2*b*c + A*B^2*a*d)*m^3 + 3*(A*B^2*
b*c + A*B^2*a*d)*m^2 + 3*(A*B^2*b*c + A*B^2*a*d)*m - (B^3*b*c + B^3*a*d +
(B^3*b*c + B^3*a*d)*m^2 + 2*(B^3*b*c + B^3*a*d)*m)*n)*x + ((B^3*b*d*m^3 +
3*B^3*b*d*m^2 + 3*B^3*b*d*m + B^3*b*d)*n*x^2 + (B^3*b*c + B^3*a*d + (B^3*b
*c + B^3*a*d)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B^3*b*c + B^3*a*d)*m)*n
*x + (B^3*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m + B^3*a*c)*n)*log((b*x + a
)/(d*x + c)))*log(e)^2 - 3*((B^3*a*c*m^2 + 2*B^3*a*c*m + B^3*a*c)*n^3 -...
```

Sympy [F(-2)]

Exception generated.

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^3 (bgx+ag)^m (dix+ci)^{-m-2}\, dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="maxima")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^3*(b*g*x + a*g)^m*(d*i*x
+ c*i)^(-m - 2), x)
```

Giac [F]

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3 (bgx+ag)^m (dix+ci)^{-m-2}\, dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^3*(b*g*x + a*g)^m*(d*i*x
+ c*i)^(-m - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx$$
$$= \int \frac{(a\,g+b\,g\,x)^m \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^3}{(c\,i+d\,i\,x)^{m+2}}\, dx$$

input
```
int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^3)/(c*i + d*i*
x)^(m + 2),x)
```

output
```
int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^3)/(c*i + d*i*
x)^(m + 2), x)
```

Reduce [F]

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx$$
$$= -\left(\int \frac{(bgx+ag)^m}{(dix+ci)^m c^2 + 2(dix+ci)^m cdx + (dix+ci)^m d^2x^2} dx\right) a^3$$
$$- \left(\int \frac{(bgx+ag)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3}{(dix+ci)^m c^2 + 2(dix+ci)^m cdx + (dix+ci)^m d^2x^2} dx\right) b^3$$
$$- 3\left(\int \frac{(bgx+ag)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{(dix+ci)^m c^2 + 2(dix+ci)^m cdx + (dix+ci)^m d^2x^2} dx\right) a\,b^2$$
$$- 3\left(\int \frac{(bgx+ag)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{(dix+ci)^m c^2 + 2(dix+ci)^m cdx + (dix+ci)^m d^2x^2} dx\right) a^2 b$$

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^3,x)
```

output
```
 - int((a*g + b*g*x)**m/((c*i + d*i*x)**m*c**2 + 2*(c*i + d*i*x)**m*c*d*x
+ (c*i + d*i*x)**m*d**2*x**2),x)*a**3 - int(((a*g + b*g*x)**m*log(((a + b*
x)**n*e)/(c + d*x)**n)**3)/((c*i + d*i*x)**m*c**2 + 2*(c*i + d*i*x)**m*c*d
*x + (c*i + d*i*x)**m*d**2*x**2),x)*b**3 - 3*int(((a*g + b*g*x)**m*log(((a
 + b*x)**n*e)/(c + d*x)**n)**2)/((c*i + d*i*x)**m*c**2 + 2*(c*i + d*i*x)**
m*c*d*x + (c*i + d*i*x)**m*d**2*x**2),x)*a*b**2 - 3*int(((a*g + b*g*x)**m*
log(((a + b*x)**n*e)/(c + d*x)**n))/((c*i + d*i*x)**m*c**2 + 2*(c*i + d*i*
x)**m*c*d*x + (c*i + d*i*x)**m*d**2*x**2),x)*a**2*b
```

3.213 $\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2$

Optimal result . . . 2415
Mathematica [A] (verified) . . . 2416
Rubi [A] (verified) . . . 2416
Maple [B] (verified) . . . 2418
Fricas [B] (verification not implemented) . . . 2419
Sympy [F(-2)] . . . 2420
Maxima [F] . . . 2421
Giac [F] . . . 2421
Mupad [F(-1)] . . . 2422
Reduce [F] . . . 2422

Optimal result

Integrand size = 49, antiderivative size = 210

$$\begin{aligned}
&\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{2B^2 n^2 (a+bx)(g(a+bx))^m (i(c+dx))^{-m}}{(bc-ad)i^2(1+m)^3(c+dx)} \\
&\quad - \frac{2Bn(a+bx)(g(a+bx))^m (i(c+dx))^{-m}\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)^2(c+dx)} \\
&\quad + \frac{(a+bx)(g(a+bx))^m (i(c+dx))^{-m}\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)(c+dx)}
\end{aligned}$$

output

```
2*B^2*n^2*(b*x+a)*(g*(b*x+a))^m/(-a*d+b*c)/i^2/(1+m)^3/(d*x+c)/((i*(d*x+c)
)^m)-2*B*n*(b*x+a)*(g*(b*x+a))^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c
)/i^2/(1+m)^2/(d*x+c)/((i*(d*x+c))^m)+(b*x+a)*(g*(b*x+a))^m*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))^2/(-a*d+b*c)/i^2/(1+m)/(d*x+c)/((i*(d*x+c))^m)
```

Mathematica [A] (verified)

Time = 3.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.64

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \frac{(a+bx)(g(a+bx))^m (i(c+dx))^{-1-m}\left(A^2(1+m)^2 - 2AB(1+m)n + 2B^2n^2 + 2B(1+m)(A+Am-\right.}{(bc-ad)i(1+m)^3}$$

input
```
Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^2,x]
```

output
```
((a + b*x)*(g*(a + b*x))^m*(i*(c + d*x))^(-1 - m)*(A^2*(1 + m)^2 - 2*A*B*(
1 + m)*n + 2*B^2*n^2 + 2*B*(1 + m)*(A + A*m - B*n)*Log[e*((a + b*x)/(c + d
*x))^n] + B^2*(1 + m)^2*Log[e*((a + b*x)/(c + d*x))^n]^2))/((b*c - a*d)*i*
(1 + m)^3)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2963, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^m (ci+dix)^{-m-2}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$
$$\downarrow 2963$$
$$\frac{(g(a+bx))^m (i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\int \left(\frac{a+bx}{c+dx}\right)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$
$$\downarrow 2742$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{m+1}-\frac{2Bn\int\left(\frac{a+bx}{c+dx}\right)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d\frac{a+bx}{c+dx}}{m+1}\right.}{i^2(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{m+1}-\frac{2Bn\left(\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{m+1}-\frac{Bn\left(\right.\right.}{m+1}\right.}{i^2(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^2,x]
```

output
```
((g*(a + b*x))^m*((((a + b*x)/(c + d*x))^(1 + m)*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^2)/(1 + m) - (2*B*n*(-((B*n*((a + b*x)/(c + d*x))^(1 + m))/(
1 + m)^2) + (((a + b*x)/(c + d*x))^(1 + m)*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]))/(1 + m)))/(1 + m)))/((b*c - a*d)*i^2*((a + b*x)/(c + d*x))^m*(i*(
c + d*x))^m)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2963

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2628 vs. 2(210) = 420.

Time = 148.28 (sec) , antiderivative size = 2629, normalized size of antiderivative = 12.52

method	result	size
parallelrisch	Expression too large to display	2629

input

```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x,m
ethod=_RETURNVERBOSE)
```

output

```
-(2*A*B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d
^2*m^2*n+2*A*B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n
)*b^2*c*d*m^2*n+A^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*c*d*n+2*B^2*x*(
g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*d^2*n^3+2*B^2*x*(g*(b*x+a))^m*(i*(d*x+
c))^(-2-m)*b^2*c*d*n^3+2*B^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*c*d*n^3+
A^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*d^2*n+4*A*B*x*(g*(b*x+a))^m*(i*
(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*m*n+4*A*B*x*(g*(b*x+a))^
m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*m*n+2*A*B*(g*(b*x+a
))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*m^2*n+4*A*B*(g*(
b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*m*n+B^2*x*(
g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*d^2*m^2*n+
B^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*b^2*c*d
*m^2*n+4*A*B*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n
)*b^2*d^2*m*n+2*B^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+
c))^n)^2*a*b*d^2*m*n+2*B^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a
)/(d*x+c))^n)^2*b^2*c*d*m*n-2*B^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*
((b*x+a)/(d*x+c))^n)*a*b*d^2*m*n^2-2*B^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m
)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*m*n^2+B^2*(g*(b*x+a))^m*(i*(d*x+c))^(-
2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*c*d*m^2*n-2*A*B*x*(g*(b*x+a))^m*(i*(d
*x+c))^(-2-m)*a*b*d^2*m*n^2-2*A*B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 991 vs. $2(210) = 420$.

Time = 0.13 (sec) , antiderivative size = 991, normalized size of antiderivative = 4.72

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="fricas")
```

output

```
(A^2*a*c*m^2 + 2*B^2*a*c*n^2 + 2*A^2*a*c*m + A^2*a*c + (A^2*b*d*m^2 + 2*B^
2*b*d*n^2 + 2*A^2*b*d*m + A^2*b*d - 2*(A*B*b*d*m + A*B*b*d)*n)*x^2 + (B^2*
a*c*m^2 + 2*B^2*a*c*m + B^2*a*c + (B^2*b*d*m^2 + 2*B^2*b*d*m + B^2*b*d)*x^
2 + (B^2*b*c + B^2*a*d + (B^2*b*c + B^2*a*d)*m^2 + 2*(B^2*b*c + B^2*a*d)*m
)*x)*log(e)^2 + ((B^2*b*d*m^2 + 2*B^2*b*d*m + B^2*b*d)*n^2*x^2 + (B^2*b*c
+ B^2*a*d + (B^2*b*c + B^2*a*d)*m^2 + 2*(B^2*b*c + B^2*a*d)*m)*n^2*x + (B^
2*a*c*m^2 + 2*B^2*a*c*m + B^2*a*c)*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(A*
B*a*c*m + A*B*a*c)*n + (A^2*b*c + A^2*a*d + (A^2*b*c + A^2*a*d)*m^2 + 2*(B
^2*b*c + B^2*a*d)*n^2 + 2*(A^2*b*c + A^2*a*d)*m - 2*(A*B*b*c + A*B*a*d + (
A*B*b*c + A*B*a*d)*m)*n)*x + 2*(A*B*a*c*m^2 + 2*A*B*a*c*m + A*B*a*c + (A*B
*b*d*m^2 + 2*A*B*b*d*m + A*B*b*d - (B^2*b*d*m + B^2*b*d)*n)*x^2 - (B^2*a*c
*m + B^2*a*c)*n + (A*B*b*c + A*B*a*d + (A*B*b*c + A*B*a*d)*m^2 + 2*(A*B*b*
c + A*B*a*d)*m - (B^2*b*c + B^2*a*d + (B^2*b*c + B^2*a*d)*m)*n)*x + ((B^2*
b*d*m^2 + 2*B^2*b*d*m + B^2*b*d)*n*x^2 + (B^2*b*c + B^2*a*d + (B^2*b*c + B
^2*a*d)*m^2 + 2*(B^2*b*c + B^2*a*d)*m)*n*x + (B^2*a*c*m^2 + 2*B^2*a*c*m +
B^2*a*c)*n)*log((b*x + a)/(d*x + c)))*log(e) - 2*((B^2*a*c*m + B^2*a*c)*n^
2 + ((B^2*b*d*m + B^2*b*d)*n^2 - (A*B*b*d*m^2 + 2*A*B*b*d*m + A*B*b*d)*n)*
x^2 - (A*B*a*c*m^2 + 2*A*B*a*c*m + A*B*a*c)*n + ((B^2*b*c + B^2*a*d + (B^2
*b*c + B^2*a*d)*m)*n^2 - (A*B*b*c + A*B*a*d + (A*B*b*c + A*B*a*d)*m^2 + 2*
(A*B*b*c + A*B*a*d)*m)*n)*x)*log((b*x + a)/(d*x + c)))*(b*g*x + a*g)^m*...
```

Sympy [F(-2)]

Exception generated.

$$\int (ag + bgx)^m (ci + dix)^{-2-m} \left(A + B \log\left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right)^2 dx$$

$$= \text{Exception raised: HeuristicGCDFailed}$$

input

```
integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**2,x)
```

output

```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2 (bgx+ag)^m (dix+ci)^{-m-2}\, dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="maxima")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2*(b*g*x + a*g)^m*(d*i*x
+ c*i)^(-m - 2), x)
```

Giac [F]

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2 (bgx+ag)^m (dix+ci)^{-m-2}\, dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2*(b*g*x + a*g)^m*(d*i*x
+ c*i)^(-m - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int \frac{(a\,g+b\,g\,x)^m \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(c\,i+d\,i\,x)^{m+2}}\,dx$$

input
```
int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^(m + 2),x)
```

output
```
int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^(m + 2), x)
```

Reduce [F]

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= -\left(\int \frac{(bgx+ag)^m}{(dix+ci)^m c^2 + 2(dix+ci)^m cdx + (dix+ci)^m d^2x^2}dx\right) a^2$$
$$-\left(\int \frac{(bgx+ag)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{(dix+ci)^m c^2 + 2(dix+ci)^m cdx + (dix+ci)^m d^2x^2}dx\right) b^2$$
$$-2\left(\int \frac{(bgx+ag)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{(dix+ci)^m c^2 + 2(dix+ci)^m cdx + (dix+ci)^m d^2x^2}dx\right) ab$$

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
 - int((a*g + b*g*x)**m/((c*i + d*i*x)**m*c**2 + 2*(c*i + d*i*x)**m*c*d*x
+ (c*i + d*i*x)**m*d**2*x**2),x)*a**2 - int(((a*g + b*g*x)**m*log(((a + b*
x)**n*e)/(c + d*x)**n)**2)/((c*i + d*i*x)**m*c**2 + 2*(c*i + d*i*x)**m*c*d
*x + (c*i + d*i*x)**m*d**2*x**2),x)*b**2 - 2*int(((a*g + b*g*x)**m*log(((a
 + b*x)**n*e)/(c + d*x)**n))/((c*i + d*i*x)**m*c**2 + 2*(c*i + d*i*x)**m*c
*d*x + (c*i + d*i*x)**m*d**2*x**2),x)*a*b
```

3.214 $\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)$

Optimal result . . . 2424
Mathematica [A] (verified) . . . 2424
Rubi [A] (verified) . . . 2425
Maple [B] (verified) . . . 2426
Fricas [B] (verification not implemented) . . . 2427
Sympy [F(-2)] . . . 2428
Maxima [F] . . . 2428
Giac [F] . . . 2429
Mupad [F(-1)] . . . 2429
Reduce [F] . . . 2430

Optimal result

Integrand size = 47, antiderivative size = 128

$$\begin{aligned}
&\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= -\frac{Bn(a+bx)(g(a+bx))^m (i(c+dx))^{-m}}{(bc-ad)i^2(1+m)^2(c+dx)} \\
&\quad + \frac{(a+bx)(g(a+bx))^m (i(c+dx))^{-m} \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)(c+dx)}
\end{aligned}$$

output

```
-B*n*(b*x+a)*(g*(b*x+a))^m/(-a*d+b*c)/i^2/(1+m)^2/(d*x+c)/((i*(d*x+c))^m)+
(b*x+a)*(g*(b*x+a))^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/i^2/(1+m)
/(d*x+c)/((i*(d*x+c))^m)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.61

$$\begin{aligned}
&\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= \frac{(a+bx)(g(a+bx))^m (i(c+dx))^{-1-m} \left(A + Am - Bn + B(1+m) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i(1+m)^2}
\end{aligned}$$

input
```
Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]),x]
```

output
```
((a + b*x)*(g*(a + b*x))^m*(i*(c + d*x))^(-1 - m)*(A + A*m - B*n + B*(1 +
m)*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)*i*(1 + m)^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2963, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^m (ci+dix)^{-m-2} \left(B \log\left(e \left(\frac{a+bx}{c+dx} \right)^n \right) + A \right) dx$$

$$\downarrow 2963$$

$$\frac{(g(a+bx))^m (i(c+dx))^{-m} \left(\frac{a+bx}{c+dx} \right)^{-m} \int \left(\frac{a+bx}{c+dx} \right)^m \left(A + B \log\left(e \left(\frac{a+bx}{c+dx} \right)^n \right) \right) d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{(g(a+bx))^m (i(c+dx))^{-m} \left(\frac{a+bx}{c+dx} \right)^{-m} \left(\frac{\left(\frac{a+bx}{c+dx} \right)^{m+1} \left(B \log\left(e \left(\frac{a+bx}{c+dx} \right)^n \right) + A \right)}{m+1} - \frac{Bn \left(\frac{a+bx}{c+dx} \right)^{m+1}}{(m+1)^2} \right)}{i^2(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]),x]
```

output
```
((g*(a + b*x))^m*(-((B*n*((a + b*x)/(c + d*x))^(1 + m))/(1 + m)^2) + (((a
+ b*x)/(c + d*x))^(1 + m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(1 + m))
)/((b*c - a*d)*i^2*((a + b*x)/(c + d*x))^m*(i*(c + d*x))^m)
```

Defintions of rubi rules used

rule 2741

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2963

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(128) = 256$.

Time = 36.54 (sec) , antiderivative size = 826, normalized size of antiderivative = 6.45

method	result
parallelrisch	$-\frac{-Bx(g(bx+a))^m(i(dx+c))^{-2-m}b^2cdn^2+Ax(g(bx+a))^m(i(dx+c))^{-2-m}abd^2n+Ax(g(bx+a))^m(i(dx+c))^{-2-m}b^2cdn-$

input

```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,met
hod=_RETURNVERBOSE)
```

output

```
-(-B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*c*d*n^2+A*x*(g*(b*x+a))^m*(i*(
d*x+c))^(-2-m)*a*b*d^2*n+A*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*c*d*n-B*
(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*c*d*n^2+A*(g*(b*x+a))^m*(i*(d*x+c))^(
-2-m)*a*b*c*d*n+B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c)
)^n)*a*b*d^2*m*n+B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c
))^n)*b^2*c*d*m*n+B*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c)
)^n)*a*b*c*d*m*n+A*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*d^2*m*n-B*x^2*
(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*d^2*n^2+A*x^2*(g*(b*x+a))^m*(i*(d*x+c
))^(-2-m)*b^2*d^2*n+B*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(
d*x+c))^n)*b^2*d^2*m*n+A*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*d^2*m*n+A*
x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*c*d*m*n+B*x*(g*(b*x+a))^m*(i*(d*x+c
))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*n+B*x*(g*(b*x+a))^m*(i*(d*x+c)
)^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*n+A*(g*(b*x+a))^m*(i*(d*x+c))^(
-2-m)*a*b*c*d*m*n+B*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c)
)^n)*a*b*c*d*n+B*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c
))^n)*b^2*d^2*n-B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*d^2*n^2)/n/(a*d*m
-b*c*m+a*d-b*c)/(1+m)/b/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(128) = 256$.

Time = 0.11 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.14

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$

$$= \frac{(Aacm - Bacn + Aac + (Abdm - Bbdn + Abd)x^2 + (Abc + Aad + (Abc + Aad)m - (Bbc + Bad)n)x}{}$$

input

```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="fricas")
```

output
```
(A*a*c*m - B*a*c*n + A*a*c + (A*b*d*m - B*b*d*n + A*b*d)*x^2 + (A*b*c + A*
a*d + (A*b*c + A*a*d)*m - (B*b*c + B*a*d)*n)*x + (B*a*c*m + B*a*c + (B*b*d
*m + B*b*d)*x^2 + (B*b*c + B*a*d + (B*b*c + B*a*d)*m)*x)*log(e) + ((B*b*d*
m + B*b*d)*n*x^2 + (B*b*c + B*a*d + (B*b*c + B*a*d)*m)*n*x + (B*a*c*m + B*
a*c)*n)*log((b*x + a)/(d*x + c)))*(b*g*x + a*g)^m*e^(-(m + 2)*log(b*g*x +
a*g) + (m + 2)*log((b*x + a)/(d*x + c)) - (m + 2)*log(i/g))/((b*c - a*d)*m
^2 + b*c - a*d + 2*(b*c - a*d)*m)
```

Sympy [F(-2)]

Exception generated.

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))**
n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)(bgx+ag)^m (dix+ci)^{-m-2}\, dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="maxima")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(b*g*x + a*g)^m*(d*i*x +
c*i)^(-m - 2), x)
```

Giac [F]

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)(bgx+ag)^m (dix+ci)^{-m-2}\, dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(b*g*x + a*g)^m*(d*i*x +
c*i)^(-m - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$= \int \frac{(a\,g+b\,g\,x)^m \left(A + B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(c\,i+d\,i\,x)^{m+2}}\, dx$$

input
```
int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^(m + 2),x)
```

output
```
int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^(m + 2), x)
```

Reduce [F]

$$\begin{aligned}
&\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= -\left(\int \frac{(bgx+ag)^m}{(dix+ci)^m c^2 + 2(dix+ci)^m cdx + (dix+ci)^m d^2x^2} dx\right) a \\
&\quad - \left(\int \frac{(bgx+ag)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{(dix+ci)^m c^2 + 2(dix+ci)^m cdx + (dix+ci)^m d^2x^2} dx\right) b
\end{aligned}$$

input

```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output

```
 - (int((a*g + b*g*x)**m/((c*i + d*i*x)**m*c**2 + 2*(c*i + d*i*x)**m*c*d*x
 + (c*i + d*i*x)**m*d**2*x**2),x)*a + int(((a*g + b*g*x)**m*log(((a + b*x)
**n*e)/(c + d*x)**n))/((c*i + d*i*x)**m*c**2 + 2*(c*i + d*i*x)**m*c*d*x +
(c*i + d*i*x)**m*d**2*x**2),x)*b)
```

3.215 $\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result . . . 2431
Mathematica [F] . . . 2431
Rubi [A] (verified) . . . 2432
Maple [F] . . . 2433
Fricas [A] (verification not implemented) . . . 2434
Sympy [F(-2)] . . . 2434
Maxima [F] . . . 2435
Giac [F] . . . 2435
Mupad [F(-1)] . . . 2435
Reduce [F] . . . 2436

Optimal result

Integrand size = 49, antiderivative size = 125

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$
$$= \frac{e^{-\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^m\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}}(i(c+dx))^{-m}\,\text{ExpIntegralEi}\left(\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B(bc-ad)i^2n(c+dx)}$$

output
```
(b*x+a)*(g*(b*x+a))^m*Ei((1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*
d+b*c)/exp(A*(1+m)/B/n)/i^2/n/((e*((b*x+a)/(d*x+c))^n)^((1+m)/n))/(d*x+c)/
((i*(d*x+c))^m)
```

Mathematica [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)
/(c + d*x))^n]),x]
```

output
```
Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)
/(c + d*x))^n]), x]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2963, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^m (ci+dix)^{-m-2}}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}\,dx$$

$$\downarrow 2963$$

$$\frac{(g(a+bx))^m (i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\int \frac{\left(\frac{a+bx}{c+dx}\right)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2747$$

$$\frac{(a+bx)(g(a+bx))^m (i(c+dx))^{-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{i^2 n(c+dx)(bc-ad)}$$

$$\downarrow 2609$$

$$\frac{(a+bx)e^{-\frac{A(m+1)}{Bn}}(g(a+bx))^m (i(c+dx))^{-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\text{ExpIntegralEi}\left(\frac{(m+1)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{Bi^2 n(c+dx)(bc-ad)}$$

input
```
Int[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)/(c +
d*x))^n]),x]
```

output
```
((a + b*x)*(g*(a + b*x))^m*ExpIntegralEi[((1 + m)*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(B*n)])/(B*(b*c - a*d)*E^((A*(1 + m))/(B*n))*i^2*n*(e*((a
+ b*x)/(c + d*x))^n)^((1 + m)/n)*(c + d*x)*(i*(c + d*x))^m)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))  Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))  Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

Maple [F]

$$\int \frac{(bgx+ag)^m (dix+ci)^{-2-m}}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\mathrm{Ei}\left(\frac{(Bm+B)n\log\left(\frac{bx+a}{dx+c}\right)+Am+(Bm+B)\log(e)+A}{Bn}\right)e^{\left(-\frac{(Bm+2B)n\log\left(\frac{i}{g}\right)+Am+(Bm+B)\log(e)+A}{Bn}\right)}}{(Bbc-Bad)g^2n}$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="fricas")
```

output
```
Ei(((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)/(B*
n))*e^(-((B*m + 2*B)*n*log(i/g) + A*m + (B*m + B)*log(e) + A)/(B*n))/((B*b
*c - B*a*d)*g^2*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))**
n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bgx+ag)^m(dix+ci)^{-m-2}}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="maxima")
```

output
```
integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)/(B*log(e*((b*x + a)/(d*x
+ c))^n) + A), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bgx+ag)^m(dix+ci)^{-m-2}}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)/(B*log(e*((b*x + a)/(d*x
+ c))^n) + A), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(a\,g+b\,g\,x)^m}{(c\,i+d\,i\,x)^{m+2}\,\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

input
```
int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))),x)
```

output
```
int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))), x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^m (ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx =$$

$$-\left(\int \frac{(bgx+ag)^m}{(dix+ci)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) b\,c^2 + 2\,(dix+ci)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) bcdx + (dix+ci)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) b\,d\right.$$

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
 - int((a*g + b*g*x)**m/((c*i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**
n)*b*c**2 + 2*(c*i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*b*c*d*x
+ (c*i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*b*d**2*x**2 + (c*i +
 d*i*x)**m*a*c**2 + 2*(c*i + d*i*x)**m*a*c*d*x + (c*i + d*i*x)**m*a*d**2*x
**2),x)
```

3.216 $\int \frac{(ag+bgx)^m (ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result . . . 2437
Mathematica [F] . . . 2438
Rubi [A] (verified) . . . 2438
Maple [F] . . . 2440
Fricas [A] (verification not implemented) . . . 2440
Sympy [F(-2)] . . . 2441
Maxima [F] . . . 2441
Giac [F] . . . 2442
Mupad [F(-1)] . . . 2442
Reduce [F] . . . 2443

Optimal result

Integrand size = 49, antiderivative size = 206

$$\int \frac{(ag+bgx)^m (ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

$$= \frac{e^{-\frac{A(1+m)}{Bn}}(1+m)(a+bx)(g(a+bx))^m \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}} (i(c+dx))^{-m}\,\text{ExpIntegralEi}\left(\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{Bn}$$

$$B^2(bc-ad)i^2n^2(c+dx)$$

$$-\frac{(a+bx)(g(a+bx))^m (i(c+dx))^{-m}}{B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

output

```
(1+m)*(b*x+a)*(g*(b*x+a))^m*Ei((1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/
B^2/(-a*d+b*c)/exp(A*(1+m)/B/n)/i^2/n^2/((e*((b*x+a)/(d*x+c))^n)^((1+m)/n)
)/(d*x+c)/((i*(d*x+c))^m)-(b*x+a)*(g*(b*x+a))^m/B/(-a*d+b*c)/i^2/n/(d*x+c)
/((i*(d*x+c))^m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Mathematica [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2,x]
```

output
```
Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2, x]
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.082$, Rules used = {2963, 2743, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-m-2}}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}\,dx$$

$$\downarrow 2963$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\int \frac{\left(\frac{a+bx}{c+dx}\right)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2743$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{(m+1)\int \frac{\left(\frac{a+bx}{c+dx}\right)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn}-\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}}{Bn\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}\right)}{i^2(bc-ad)}$$

$$\downarrow 2747$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{(m+1)\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\int\frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn^2}-\frac{}{Bn\left(}\right)}{i^2(bc-ad)}$$

$$\downarrow 2609$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{(m+1)e^{-\frac{A(m+1)}{Bn}}\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\text{ExpIntegralEi}\left(\frac{(m+1)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.\right.}{Bn}}{B^2n^2}}{i^2(bc-ad)}$$

input
```
Int[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
((g*(a + b*x))^m*(((1 + m)*((a + b*x)/(c + d*x))^(1 + m)*ExpIntegralEi[((1 + m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)])/(B^2*E^((A*(1 + m))/(B*n))*n^2*(e*((a + b*x)/(c + d*x))^n)^((1 + m)/n)) - ((a + b*x)/(c + d*x))^(1 + m)/(B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/((b*c - a*d)*i^2*((a + b*x)/(c + d*x))^m*(i*(c + d*x))^m)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2743
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] - Simp[(m + 1)/(b*n*(p + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2747

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

Maple [F]

$$\int \frac{(bgx+ag)^m\,(dix+ci)^{-2-m}}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}\,dx$$

input

```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.42

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx =$$

$$-\frac{(Bbdg^2nx^2+Bacg^2n+(Bbc+Bad)g^2nx)(bgx+ag)^m e^{\left(-(m+2)\log(bgx+ag)+(m+2)\log\left(\frac{bx+a}{dx+c}\right)-(m+2)\log\left(\frac{i}{g}\right)\right.}}{(B^3bc-B^3ad)g^2n^3\log(}$$

input

```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="fricas")
```

output
```
-((B*b*d*g^2*n*x^2 + B*a*c*g^2*n + (B*b*c + B*a*d)*g^2*n*x)*(b*g*x + a*g)^
m*e^(-(m + 2)*log(b*g*x + a*g) + (m + 2)*log((b*x + a)/(d*x + c)) - (m + 2
)*log(i/g)) - ((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(
e) + A)*Ei(((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e)
+ A)/(B*n))*e^(-((B*m + 2*B)*n*log(i/g) + A*m + (B*m + B)*log(e) + A)/(B*n
)))/((B^3*b*c - B^3*a*d)*g^2*n^3*log((b*x + a)/(d*x + c)) + (B^3*b*c - B^3
*a*d)*g^2*n^2*log(e) + (A*B^2*b*c - A*B^2*a*d)*g^2*n^2)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ag+bgx)^m (ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ag+bgx)^m (ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(bgx+ag)^m (dix+ci)^{-m-2}}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="maxima")
```

output
```
-g^m*(m + 1)*integrate(-(b*x + a)^m/((B^2*d^2*i^(m + 2)*n*x^2 + 2*B^2*c*d*
i^(m + 2)*n*x + B^2*c^2*i^(m + 2)*n)*(d*x + c)^m*log((b*x + a)^n) - (B^2*d
^2*i^(m + 2)*n*x^2 + 2*B^2*c*d*i^(m + 2)*n*x + B^2*c^2*i^(m + 2)*n)*(d*x +
 c)^m*log((d*x + c)^n) + (B^2*c^2*i^(m + 2)*n*log(e) + A*B*c^2*i^(m + 2)*n
 + (B^2*d^2*i^(m + 2)*n*log(e) + A*B*d^2*i^(m + 2)*n)*x^2 + 2*(B^2*c*d*i^(
m + 2)*n*log(e) + A*B*c*d*i^(m + 2)*n)*x)*(d*x + c)^m), x) - (b*g^m*x + a*
g^m)*(b*x + a)^m/(((b*c*d*i^(m + 2)*n - a*d^2*i^(m + 2)*n)*B^2*x + (b*c^2*
i^(m + 2)*n - a*c*d*i^(m + 2)*n)*B^2)*(d*x + c)^m*log((b*x + a)^n) - ((b*c
*d*i^(m + 2)*n - a*d^2*i^(m + 2)*n)*B^2*x + (b*c^2*i^(m + 2)*n - a*c*d*i^(
m + 2)*n)*B^2)*(d*x + c)^m*log((d*x + c)^n) + ((b*c^2*i^(m + 2)*n - a*c*d*
i^(m + 2)*n)*A*B + (b*c^2*i^(m + 2)*n*log(e) - a*c*d*i^(m + 2)*n*log(e))*B
^2 + ((b*c*d*i^(m + 2)*n - a*d^2*i^(m + 2)*n)*A*B + (b*c*d*i^(m + 2)*n*log
(e) - a*d^2*i^(m + 2)*n*log(e))*B^2)*x)*(d*x + c)^m)
```

Giac [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(bgx+ag)^m(dix+ci)^{-m-2}}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)/(B*log(e*((b*x + a)/(d*x
+ c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(a\,g+b\,g\,x)^m}{(c\,i+d\,i\,x)^{m+2}\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))^2),x)
```

output
```
int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))^2), x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^m (ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx =$$

$$-\left(\int \frac{}{(dix+ci)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 b^2c^2 + 2\,(dix+ci)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 b^2cdx + (dix+ci)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right.\right.$$

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
 - int((a*g + b*g*x)**m/((c*i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**
n)**2*b**2*c**2 + 2*(c*i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)**2
*b**2*c*d*x + (c*i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**2*
d**2*x**2 + 2*(c*i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c**2
 + 4*(c*i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c*d*x + 2*(c*
i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d**2*x**2 + (c*i + d*
i*x)**m*a**2*c**2 + 2*(c*i + d*i*x)**m*a**2*c*d*x + (c*i + d*i*x)**m*a**2*
d**2*x**2),x)
```

3.217 $\int \frac{(ag+bgx)^m (ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$

Optimal result . . . 2444
Mathematica [F] . . . 2445
Rubi [A] (verified) . . . 2445
Maple [F] . . . 2447
Fricas [B] (verification not implemented) . . . 2448
Sympy [F(-1)] . . . 2449
Maxima [F] . . . 2449
Giac [F] . . . 2450
Mupad [F(-1)] . . . 2451
Reduce [F] . . . 2451

Optimal result

Integrand size = 49, antiderivative size = 295

$$\int \frac{(ag+bgx)^m (ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$$

$$= \frac{e^{-\frac{A(1+m)}{Bn}}(1+m)^2(a+bx)(g(a+bx))^m\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}}(i(c+dx))^{-m}\,\text{ExpIntegralEi}\left(\frac{(1+m)\left(A+B\log\left(e\left(\frac{a}{c}\right.\right.\right.\right.}{Bn}}{2B^3(bc-ad)i^2n^3(c+dx)}$$

$$-\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}$$

$$-\frac{(1+m)(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{2B^2(bc-ad)i^2n^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

output

```
1/2*(1+m)^2*(b*x+a)*(g*(b*x+a))^m*Ei((1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
/B/n)/B^3/(-a*d+b*c)/exp(A*(1+m)/B/n)/i^2/n^3/((e*((b*x+a)/(d*x+c))^n)^((1
+m)/n))/(d*x+c)/((i*(d*x+c))^m)-1/2*(b*x+a)*(g*(b*x+a))^m/B/(-a*d+b*c)/i^2
/n/(d*x+c)/((i*(d*x+c))^m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2-1/2*(1+m)*(b*
x+a)*(g*(b*x+a))^m/B^2/(-a*d+b*c)/i^2/n^2/(d*x+c)/((i*(d*x+c))^m)/(A+B*ln(
e*((b*x+a)/(d*x+c))^n))
```

Mathematica [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$$

input
```
Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^3,x]
```

output
```
Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^3, x]
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used = {2963, 2743, 2743, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-m-2}}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}\,dx$$

$$\downarrow 2963$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\int \frac{\left(\frac{a+bx}{c+dx}\right)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2743$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{(m+1)\int \frac{\left(\frac{a+bx}{c+dx}\right)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{2Bn}-\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}}{2Bn\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}\right)}{i^2(bc-ad)}$$

$$\downarrow 2743$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{(m+1)\left(\frac{(m+1)\int\frac{\left(\frac{a+bx}{c+dx}\right)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn}-\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}}{Bn\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}\right)}{2Bn}-\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}}{2Bn\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}\right)}{i^2(bc-ad)}$$

$$\downarrow 2747$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{(m+1)\left(\frac{(m+1)\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\int\frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn^2}-\frac{}{Bn}\right)}{2Bn}\right)}{i^2(bc-ad)}$$

$$\downarrow 2609$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{(m+1)\left(\frac{(m+1)e^{-\frac{A(m+1)}{Bn}}\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\text{ExpIntegralEi}\left(\frac{(m+1)\left(A+B\log\left(e\left(\frac{}{}\right.\right.\right.\right.}{B^2n^2}\right.}{Bn}\right)}{2Bn}\right)}{i^2(bc-ad)}$$

input

```
Int[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3,x]
```

output

```
((g*(a + b*x))^m*(-1/2*((a + b*x)/(c + d*x))^(1 + m)/(B*n*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])^2) + ((1 + m)*(((1 + m)*((a + b*x)/(c + d*x))^(1 +
m)*ExpIntegralEi[((1 + m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)])/
(B^2*E^((A*(1 + m))/(B*n))*n^2*(e*((a + b*x)/(c + d*x))^n)^((1 + m)/n)) -
((a + b*x)/(c + d*x))^(1 + m)/(B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
))/(2*B*n)))/((b*c - a*d)*i^2*((a + b*x)/(c + d*x))^m*(i*(c + d*x))^m)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2743
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Simp[(m + 1)/(b*n*(p + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

Maple [F]

$$\int \frac{(bgx+ag)^m (dix+ci)^{-2-m}}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^3}dx$$

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

output
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(289) = 578$.

Time = 0.12 (sec) , antiderivative size = 818, normalized size of antiderivative = 2.77

$$\int \frac{(ag+bgx)^m (ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="fricas")
```

output

```
-1/2*((B^2*a*c*g^2*n^2 + (B^2*b*d*g^2*n^2 + (A*B*b*d*g^2*m + A*B*b*d*g^2)*
n)*x^2 + (A*B*a*c*g^2*m + A*B*a*c*g^2)*n + ((B^2*b*c + B^2*a*d)*g^2*n^2 +
((A*B*b*c + A*B*a*d)*g^2*m + (A*B*b*c + A*B*a*d)*g^2)*n)*x + ((B^2*b*d*g^2
*m + B^2*b*d*g^2)*n*x^2 + ((B^2*b*c + B^2*a*d)*g^2*m + (B^2*b*c + B^2*a*d)
*g^2)*n*x + (B^2*a*c*g^2*m + B^2*a*c*g^2)*n)*log(e) + ((B^2*b*d*g^2*m + B^
2*b*d*g^2)*n^2*x^2 + ((B^2*b*c + B^2*a*d)*g^2*m + (B^2*b*c + B^2*a*d)*g^2)
*n^2*x + (B^2*a*c*g^2*m + B^2*a*c*g^2)*n^2)*log((b*x + a)/(d*x + c)))*(b*g
*x + a*g)^m*e^(-(m + 2)*log(b*g*x + a*g) + (m + 2)*log((b*x + a)/(d*x + c)
) - (m + 2)*log(i/g)) - ((B^2*m^2 + 2*B^2*m + B^2)*n^2*log((b*x + a)/(d*x
+ c))^2 + A^2*m^2 + 2*A^2*m + (B^2*m^2 + 2*B^2*m + B^2)*log(e)^2 + 2*(A*B*
m^2 + 2*A*B*m + A*B)*n*log((b*x + a)/(d*x + c)) + A^2 + 2*(A*B*m^2 + 2*A*B
*m + (B^2*m^2 + 2*B^2*m + B^2)*n*log((b*x + a)/(d*x + c)) + A*B)*log(e))*E
i(((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)/(B*n
))*e^(-((B*m + 2*B)*n*log(i/g) + A*m + (B*m + B)*log(e) + A)/(B*n)))/((B^5
*b*c - B^5*a*d)*g^2*n^5*log((b*x + a)/(d*x + c))^2 + (B^5*b*c - B^5*a*d)*g
^2*n^3*log(e)^2 + 2*(A*B^4*b*c - A*B^4*a*d)*g^2*n^4*log((b*x + a)/(d*x + c
)) + (A^2*B^3*b*c - A^2*B^3*a*d)*g^2*n^3 + 2*((B^5*b*c - B^5*a*d)*g^2*n^4*
log((b*x + a)/(d*x + c)) + (A*B^4*b*c - A*B^4*a*d)*g^2*n^3)*log(e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \int \frac{(bgx+ag)^m(dix+ci)^{-m-2}}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="maxima")
```

output

```
-(m^2 + 2*m + 1)*g^m*integrate(-1/2*(b*x + a)^m/((B^3*d^2*i^(m + 2)*n^2*x^
2 + 2*B^3*c*d*i^(m + 2)*n^2*x + B^3*c^2*i^(m + 2)*n^2)*(d*x + c)^m*log((b*
x + a)^n) - (B^3*d^2*i^(m + 2)*n^2*x^2 + 2*B^3*c*d*i^(m + 2)*n^2*x + B^3*c
^2*i^(m + 2)*n^2)*(d*x + c)^m*log((d*x + c)^n) + (B^3*c^2*i^(m + 2)*n^2*lo
g(e) + A*B^2*c^2*i^(m + 2)*n^2 + (B^3*d^2*i^(m + 2)*n^2*log(e) + A*B^2*d^2
*i^(m + 2)*n^2)*x^2 + 2*(B^3*c*d*i^(m + 2)*n^2*log(e) + A*B^2*c*d*i^(m + 2
)*n^2)*x)*(d*x + c)^m), x) - 1/2*((B*b*g^m*(m + 1)*x + B*a*g^m*(m + 1))*(b
*x + a)^m*log((b*x + a)^n) - (B*b*g^m*(m + 1)*x + B*a*g^m*(m + 1))*(b*x +
a)^m*log((d*x + c)^n) + (A*a*g^m*(m + 1) + (g^m*(m + 1)*log(e) + g^m*n)*B*
a + (A*b*g^m*(m + 1) + (g^m*(m + 1)*log(e) + g^m*n)*B*b)*x)*(b*x + a)^m)/(
((b*c*d*i^(m + 2)*n^2 - a*d^2*i^(m + 2)*n^2)*B^4*x + (b*c^2*i^(m + 2)*n^2
- a*c*d*i^(m + 2)*n^2)*B^4)*(d*x + c)^m*log((b*x + a)^n)^2 + ((b*c*d*i^(m
+ 2)*n^2 - a*d^2*i^(m + 2)*n^2)*B^4*x + (b*c^2*i^(m + 2)*n^2 - a*c*d*i^(m
+ 2)*n^2)*B^4)*(d*x + c)^m*log((d*x + c)^n)^2 + 2*((b*c^2*i^(m + 2)*n^2 -
a*c*d*i^(m + 2)*n^2)*A*B^3 + (b*c^2*i^(m + 2)*n^2*log(e) - a*c*d*i^(m + 2)
*n^2*log(e))*B^4 + ((b*c*d*i^(m + 2)*n^2 - a*d^2*i^(m + 2)*n^2)*A*B^3 + (b
*c*d*i^(m + 2)*n^2*log(e) - a*d^2*i^(m + 2)*n^2*log(e))*B^4)*x)*(d*x + c)^
m*log((b*x + a)^n) + ((b*c^2*i^(m + 2)*n^2 - a*c*d*i^(m + 2)*n^2)*A^2*B^2
+ 2*(b*c^2*i^(m + 2)*n^2*log(e) - a*c*d*i^(m + 2)*n^2*log(e))*A*B^3 + (b*c
^2*i^(m + 2)*n^2*log(e)^2 - a*c*d*i^(m + 2)*n^2*log(e)^2)*B^4 + ((b*c*d...
```

Giac [F]

$$\int \frac{(ag+bgx)^m (ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \int \frac{(bgx+ag)^m (dix+ci)^{-m-2}}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3}\,dx$$

input

```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="giac")
```

output

```
integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)/(B*log(e*((b*x + a)/(d*x
+ c))^n) + A)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \int \frac{(a\,g+b\,g\,x)^m}{(c\,i+d\,i\,x)^{m+2}\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^3}\,dx$$

input
```
int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))^3),x)
```

output
```
int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))^3), x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx =$$

$$-\left(\int \frac{}{(dix+ci)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3 b^3c^2 + 2\,(dix+ci)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3 b^3cdx + (dix+ci)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right.\right.$$

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^3,x)
```

output
```
 - int((a*g + b*g*x)**m/((c*i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**
n)**3*b**3*c**2 + 2*(c*i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)**3
*b**3*c*d*x + (c*i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)**3*b**3*
d**2*x**2 + 3*(c*i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**
2*c**2 + 6*(c*i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c
*d*x + 3*(c*i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*d**
2*x**2 + 3*(c*i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c**2
 + 6*(c*i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c*d*x + 3*
(c*i + d*i*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d**2*x**2 + (c*
i + d*i*x)**m*a**3*c**2 + 2*(c*i + d*i*x)**m*a**3*c*d*x + (c*i + d*i*x)**m
*a**3*d**2*x**2),x)
```

3.218 $\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3$

Optimal result . . . 2452
Mathematica [A] (verified) . . . 2453
Rubi [A] (verified) . . . 2453
Maple [F] . . . 2455
Fricas [B] (verification not implemented) . . . 2456
Sympy [F(-1)] . . . 2457
Maxima [F] . . . 2457
Giac [F] . . . 2457
Mupad [F(-1)] . . . 2458
Reduce [F] . . . 2458

Optimal result

Integrand size = 49, antiderivative size = 309

$$\begin{aligned}
&\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx \\
&= -\frac{6B^3n^3(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{(bc-ad)i^2(1+m)^4(c+dx)} \\
&\quad -\frac{6B^2n^2(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)^3(c+dx)} \\
&\quad -\frac{3Bn(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)^2(c+dx)} \\
&\quad -\frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)i^2(1+m)(c+dx)}
\end{aligned}$$

output

```
-6*B^3*n^3*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)/(-a*d+b*c)/i^2/(1+
m)^4/(d*x+c)-6*B^2*n^2*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/i^2/(1+m)^3/(d*x+c)-3*B*n*(b*x+a)*(g*
(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+
b*c)/i^2/(1+m)^2/(d*x+c)-(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))^3/(-a*d+b*c)/i^2/(1+m)/(d*x+c)
```

Mathematica [A] (verified)

Time = 7.97 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.67

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx =$$
$$-\frac{(g(a+bx))^{-1-m}(c+dx)(i(c+dx))^m\left(A^3(1+m)^3+3A^2B(1+m)^2n+6AB^2(1+m)n^2+6B^3n^3+\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^3,x]
```

output
```
-(((g*(a + b*x))^(-1 - m)*(c + d*x)*(i*(c + d*x))^m*(A^3*(1 + m)^3 + 3*A^2
*B*(1 + m)^2*n + 6*A*B^2*(1 + m)*n^2 + 6*B^3*n^3 + 3*B*(1 + m)*(A^2*(1 + m
)^2 + 2*A*B*(1 + m)*n + 2*B^2*n^2)*Log[e*((a + b*x)/(c + d*x))^n] + 3*B^2*
(1 + m)^2*(A + A*m + B*n)*Log[e*((a + b*x)/(c + d*x))^n]^2 + B^3*(1 + m)^3
*Log[e*((a + b*x)/(c + d*x))^n]^3))/((b*c - a*d)*g*(1 + m)^4))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.082$, Rules used = {2963, 2742, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^{-m-2}(ci+dix)^m\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3\,dx$$
$$\downarrow 2963$$
$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\int\left(\frac{a+bx}{c+dx}\right)^{-m-2}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$
$$\downarrow 2742$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(\frac{3Bn\int\left(\frac{a+bx}{c+dx}\right)^{-m-2}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}d\frac{a+bx}{c+dx}}{m+1}-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(B\log\left(e\left(\frac{a}{c}\right.\right.\right.\right.}{m+1}}{i^2(bc-ad)}$$

$$\downarrow 2742$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(\frac{3Bn\left(\frac{2Bn\int\left(\frac{a+bx}{c+dx}\right)^{-m-2}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)d\frac{a+bx}{c+dx}}{m+1}-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.\right.}{m+1}\right)}{m+1}}{i^2(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(\frac{3Bn\left(\frac{2Bn\left(-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)+A\right)}{m+1}-\frac{Bn\left(\frac{a+bx}{c+dx}\right)^{-m-1}}{(m+1)^2}\right)}{m+1}-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m}}{m+1}\right)}{m+1}}{i^2(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^3,x]
```

output
```
((g*(a + b*x))^(-2 - m)*((a + b*x)/(c + d*x))^(2 + m)*(i*(c + d*x))^(2 + m
)*(-((((a + b*x)/(c + d*x))^(-1 - m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^3)/(1 + m)) + (3*B*n*(-((((a + b*x)/(c + d*x))^(-1 - m)*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])^2)/(1 + m)) + (2*B*n*(-((B*n*((a + b*x)/(c + d*x))^
(-1 - m))/(1 + m)^2) - (((a + b*x)/(c + d*x))^(-1 - m)*(A + B*Log[e*((a +
b*x)/(c + d*x))^n]))/(1 + m)))/(1 + m)))/(1 + m)))/((b*c - a*d)*i^2)
```

Defintions of rubi rules used

rule 2741

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2963

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

Maple [F]

$$\int (bgx+ag)^{-2-m}\,(dix+ci)^{m}\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{3}dx$$

input

```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

output

```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2669 vs. 2(309) = 618.

Time = 0.22 (sec) , antiderivative size = 2669, normalized size of antiderivative = 8.64

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="fricas")
```

output

```
-(A^3*a*c*m^3 + 6*B^3*a*c*n^3 + 3*A^3*a*c*m^2 + 3*A^3*a*c*m + A^3*a*c + (B
^3*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m + B^3*a*c + (B^3*b*d*m^3 + 3*B^3*
b*d*m^2 + 3*B^3*b*d*m + B^3*b*d)*x^2 + (B^3*b*c + B^3*a*d + (B^3*b*c + B^3
*a*d)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B^3*b*c + B^3*a*d)*m)*x)*log(e)
^3 + ((B^3*b*d*m^3 + 3*B^3*b*d*m^2 + 3*B^3*b*d*m + B^3*b*d)*n^3*x^2 + (B^3
*b*c + B^3*a*d + (B^3*b*c + B^3*a*d)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(
B^3*b*c + B^3*a*d)*m)*n^3*x + (B^3*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m +
 B^3*a*c)*n^3)*log((b*x + a)/(d*x + c))^3 + 6*(A*B^2*a*c*m + A*B^2*a*c)*n^
2 + (A^3*b*d*m^3 + 6*B^3*b*d*n^3 + 3*A^3*b*d*m^2 + 3*A^3*b*d*m + A^3*b*d +
 6*(A*B^2*b*d*m + A*B^2*b*d)*n^2 + 3*(A^2*B*b*d*m^2 + 2*A^2*B*b*d*m + A^2*
B*b*d)*n)*x^2 + 3*(A*B^2*a*c*m^3 + 3*A*B^2*a*c*m^2 + 3*A*B^2*a*c*m + A*B^2
*a*c + (A*B^2*b*d*m^3 + 3*A*B^2*b*d*m^2 + 3*A*B^2*b*d*m + A*B^2*b*d + (B^3
*b*d*m^2 + 2*B^3*b*d*m + B^3*b*d)*n)*x^2 + (B^3*a*c*m^2 + 2*B^3*a*c*m + B^
3*a*c)*n + (A*B^2*b*c + A*B^2*a*d + (A*B^2*b*c + A*B^2*a*d)*m^3 + 3*(A*B^2
*b*c + A*B^2*a*d)*m^2 + 3*(A*B^2*b*c + A*B^2*a*d)*m + (B^3*b*c + B^3*a*d +
 (B^3*b*c + B^3*a*d)*m^2 + 2*(B^3*b*c + B^3*a*d)*m)*n)*x + ((B^3*b*d*m^3 +
 3*B^3*b*d*m^2 + 3*B^3*b*d*m + B^3*b*d)*n*x^2 + (B^3*b*c + B^3*a*d + (B^3*
b*c + B^3*a*d)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B^3*b*c + B^3*a*d)*m)*
n*x + (B^3*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m + B^3*a*c)*n)*log((b*x +
a)/(d*x + c)))*log(e)^2 + 3*((B^3*a*c*m^2 + 2*B^3*a*c*m + B^3*a*c)*n^3 ...
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m*(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3 (bgx+ag)^{-m-2}(dix+ci)^m\, dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="maxima")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^3*(b*g*x + a*g)^(-m - 2)*
(d*i*x + c*i)^m, x)
```

Giac [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3 (bgx+ag)^{-m-2}(dix+ci)^m\, dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^3*(b*g*x + a*g)^(-m - 2)*
(d*i*x + c*i)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx$$
$$=\int \frac{(c\,i+d\,i\,x)^m\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^3}{(a\,g+b\,g\,x)^{m+2}}\,dx$$

input
```
int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^3)/(a*g + b*g*
x)^(m + 2),x)
```

output
```
int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^3)/(a*g + b*g*
x)^(m + 2), x)
```

Reduce [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx$$
$$=\frac{\left(\int \frac{(dix+ci)^m}{(bgx+ag)^m a^2+2(bgx+ag)^m abx+(bgx+ag)^m b^2x^2}dx\right)a^3+\left(\int \frac{(dix+ci)^m\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3}{(bgx+ag)^m a^2+2(bgx+ag)^m abx+(bgx+ag)^m b^2x^2}dx\right)b^3+3\left(\right.}{g^2}$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n))^3,x)
```

output

```
(int((c*i + d*i*x)**m/((a*g + b*g*x)**m*a**2 + 2*(a*g + b*g*x)**m*a*b*x +
(a*g + b*g*x)**m*b**2*x**2),x)*a**3 + int(((c*i + d*i*x)**m*log(((a + b*x)
**n*e)/(c + d*x)**n)**3)/((a*g + b*g*x)**m*a**2 + 2*(a*g + b*g*x)**m*a*b*x
 + (a*g + b*g*x)**m*b**2*x**2),x)*b**3 + 3*int(((c*i + d*i*x)**m*log(((a +
 b*x)**n*e)/(c + d*x)**n)**2)/((a*g + b*g*x)**m*a**2 + 2*(a*g + b*g*x)**m*
a*b*x + (a*g + b*g*x)**m*b**2*x**2),x)*a*b**2 + 3*int(((c*i + d*i*x)**m*lo
g(((a + b*x)**n*e)/(c + d*x)**n))/((a*g + b*g*x)**m*a**2 + 2*(a*g + b*g*x)
**m*a*b*x + (a*g + b*g*x)**m*b**2*x**2),x)*a**2*b)/g**2
```

3.219 $\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2$

Optimal result . . . 2460
Mathematica [A] (verified) . . . 2461
Rubi [A] (verified) . . . 2461
Maple [B] (verified) . . . 2463
Fricas [B] (verification not implemented) . . . 2464
Sympy [F(-2)] . . . 2465
Maxima [F] . . . 2466
Giac [F] . . . 2466
Mupad [F(-1)] . . . 2467
Reduce [F] . . . 2467

Optimal result

Integrand size = 49, antiderivative size = 223

$$\begin{aligned}
&\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= -\frac{2B^2n^2(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{(bc-ad)i^2(1+m)^3(c+dx)} \\
&\quad -\frac{2Bn(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)^2(c+dx)} \\
&\quad -\frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)(c+dx)}
\end{aligned}$$

output

```
-2*B^2*n^2*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)/(-a*d+b*c)/i^2/(1+
m)^3/(d*x+c)-2*B*n*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(A+B*ln(e*
((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/i^2/(1+m)^2/(d*x+c)-(b*x+a)*(g*(b*x+a))^(
-2-m)*(i*(d*x+c))^(2+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/i^2/(
1+m)/(d*x+c)
```

Mathematica [A] (verified)

Time = 3.00 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.60

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx =$$
$$-\frac{(g(a+bx))^{-1-m}(c+dx)(i(c+dx))^m\left(A^2(1+m)^2+2AB(1+m)n+2B^2n^2+2B(1+m)(A+Am\right.}{(bc-ad)g(1+m)^3}$$

input
```
Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^2,x]
```

output
```
-(((g*(a + b*x))^(-1 - m)*(c + d*x)*(i*(c + d*x))^m*(A^2*(1 + m)^2 + 2*A*B
*(1 + m)*n + 2*B^2*n^2 + 2*B*(1 + m)*(A + A*m + B*n)*Log[e*((a + b*x)/(c +
 d*x))^n] + B^2*(1 + m)^2*Log[e*((a + b*x)/(c + d*x))^n]^2))/((b*c - a*d)*
g*(1 + m)^3))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2963, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^{-m-2}(ci+dix)^m\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$
$$\downarrow \text{2963}$$
$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\int\left(\frac{a+bx}{c+dx}\right)^{-m-2}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$
$$\downarrow \text{2742}$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(\frac{2Bn\int\left(\frac{a+bx}{c+dx}\right)^{-m-2}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)d\frac{a+bx}{c+dx}}{m+1}-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)+A\right)}{m+1}\right.}{i^2(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(\frac{2Bn\left(-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)+A\right)}{m+1}-\frac{Bn\left(\frac{a+bx}{c+dx}\right)^{-m-1}}{(m+1)^2}\right)}{m+1}-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}}{}\right.}{i^2(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^2,x]
```

output
```
((g*(a + b*x))^(-2 - m)*((a + b*x)/(c + d*x))^(2 + m)*(i*(c + d*x))^(2 + m
)*(-((((a + b*x)/(c + d*x))^(-1 - m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2)/(1 + m)) + (2*B*n*(-((B*n*((a + b*x)/(c + d*x))^(-1 - m))/(1 + m)^2)
- (((a + b*x)/(c + d*x))^(-1 - m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(1 + m)))/(1 + m)))/((b*c - a*d)*i^2)
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2963

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2618 vs. $2(223) = 446$.

Time = 148.90 (sec) , antiderivative size = 2619, normalized size of antiderivative = 11.74

method	result	size
parallelrisch	Expression too large to display	2619

input

```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x,m
ethod=_RETURNVERBOSE)
```

output

```
(2*A*B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*
d*n+2*A*B*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*c*d*m*n^2+2*A*B*(i*(d*x+c))
^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*n+2*A*B*x^2*(i*(d*
x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*m^2*n+B^2*x*(
i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*d^2*m^2*n+
2*B^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*c*d
*m*n+2*B^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*
c*d*m*n^2+2*A*B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^
n)*a*b*d^2*n+2*A*B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*c*d*m*n^2+B^2*(i
*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*c*d*m^2*n+2
*A*B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*d^2*m*n^2+2*A^2*(i*(d*x+c))^m*
(g*(b*x+a))^(-2-m)*a*b*c*d*m*n+2*A*B*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*
c*d*n^2+4*A*B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)
*b^2*c*d*m*n+4*A*B*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))
^n)*a*b*c*d*m*n+2*A*B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*
x+c))^n)*a*b*d^2*m^2*n+2*A*B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x
+a)/(d*x+c))^n)*b^2*c*d*m^2*n+4*A*B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(
e*((b*x+a)/(d*x+c))^n)*a*b*d^2*m*n+A^2*x^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m
)*b^2*d^2*m^2*n+2*A*B*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+
c))^n)*a*b*c*d*m^2*n+B^2*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(223) = 446$.

Time = 0.13 (sec) , antiderivative size = 983, normalized size of antiderivative = 4.41

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input

```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="fricas")
```

output

```
-(A^2*a*c*m^2 + 2*B^2*a*c*n^2 + 2*A^2*a*c*m + A^2*a*c + (A^2*b*d*m^2 + 2*B
^2*b*d*n^2 + 2*A^2*b*d*m + A^2*b*d + 2*(A*B*b*d*m + A*B*b*d)*n)*x^2 + (B^2
*a*c*m^2 + 2*B^2*a*c*m + B^2*a*c + (B^2*b*d*m^2 + 2*B^2*b*d*m + B^2*b*d)*x
^2 + (B^2*b*c + B^2*a*d + (B^2*b*c + B^2*a*d)*m^2 + 2*(B^2*b*c + B^2*a*d)*
m)*x)*log(e)^2 + ((B^2*b*d*m^2 + 2*B^2*b*d*m + B^2*b*d)*n^2*x^2 + (B^2*b*c
 + B^2*a*d + (B^2*b*c + B^2*a*d)*m^2 + 2*(B^2*b*c + B^2*a*d)*m)*n^2*x + (B
^2*a*c*m^2 + 2*B^2*a*c*m + B^2*a*c)*n^2)*log((b*x + a)/(d*x + c))^2 + 2*(A
*B*a*c*m + A*B*a*c)*n + (A^2*b*c + A^2*a*d + (A^2*b*c + A^2*a*d)*m^2 + 2*(
B^2*b*c + B^2*a*d)*n^2 + 2*(A^2*b*c + A^2*a*d)*m + 2*(A*B*b*c + A*B*a*d +
(A*B*b*c + A*B*a*d)*m)*n)*x + 2*(A*B*a*c*m^2 + 2*A*B*a*c*m + A*B*a*c + (A*
B*b*d*m^2 + 2*A*B*b*d*m + A*B*b*d + (B^2*b*d*m + B^2*b*d)*n)*x^2 + (B^2*a*
c*m + B^2*a*c)*n + (A*B*b*c + A*B*a*d + (A*B*b*c + A*B*a*d)*m^2 + 2*(A*B*b
*c + A*B*a*d)*m + (B^2*b*c + B^2*a*d + (B^2*b*c + B^2*a*d)*m)*n)*x + ((B^2
*b*d*m^2 + 2*B^2*b*d*m + B^2*b*d)*n*x^2 + (B^2*b*c + B^2*a*d + (B^2*b*c +
B^2*a*d)*m^2 + 2*(B^2*b*c + B^2*a*d)*m)*n*x + (B^2*a*c*m^2 + 2*B^2*a*c*m +
 B^2*a*c)*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*((B^2*a*c*m + B^2*a*c)*n
^2 + ((B^2*b*d*m + B^2*b*d)*n^2 + (A*B*b*d*m^2 + 2*A*B*b*d*m + A*B*b*d)*n)
*x^2 + (A*B*a*c*m^2 + 2*A*B*a*c*m + A*B*a*c)*n + ((B^2*b*c + B^2*a*d + (B^
2*b*c + B^2*a*d)*m)*n^2 + (A*B*b*c + A*B*a*d + (A*B*b*c + A*B*a*d)*m^2 + 2
*(A*B*b*c + A*B*a*d)*m)*n)*x)*log((b*x + a)/(d*x + c)))*(b*g*x + a*g)^(...
```

Sympy [F(-2)]

Exception generated.

$$\int (ag + bgx)^{-2-m}(ci + dix)^m \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$

= Exception raised: HeuristicGCDFailed

input

```
integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m*(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**2,x)
```

output

```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 (bgx+ag)^{-m-2}(dix+ci)^m\, dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="maxima")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2*(b*g*x + a*g)^(-m - 2)*
(d*i*x + c*i)^m, x)
```

Giac [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 (bgx+ag)^{-m-2}(dix+ci)^m\, dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2*(b*g*x + a*g)^(-m - 2)*
(d*i*x + c*i)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int \frac{(c\,i+d\,i\,x)^m\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(a\,g+b\,g\,x)^{m+2}}\,dx$$

input
```
int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^(m + 2),x)
```

output
```
int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^(m + 2), x)
```

Reduce [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\frac{\left(\int \frac{(dix+ci)^m}{(bgx+ag)^m a^2+2(bgx+ag)^m abx+(bgx+ag)^m b^2x^2}dx\right)a^2+\left(\int \frac{(dix+ci)^m\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{(bgx+ag)^m a^2+2(bgx+ag)^m abx+(bgx+ag)^m b^2x^2}dx\right)b^2+2\left(\right.}{g^2}$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
(int((c*i + d*i*x)**m/((a*g + b*g*x)**m*a**2 + 2*(a*g + b*g*x)**m*a*b*x +
(a*g + b*g*x)**m*b**2*x**2),x)*a**2 + int(((c*i + d*i*x)**m*log(((a + b*x)
**n*e)/(c + d*x)**n)**2)/((a*g + b*g*x)**m*a**2 + 2*(a*g + b*g*x)**m*a*b*x
 + (a*g + b*g*x)**m*b**2*x**2),x)*b**2 + 2*int(((c*i + d*i*x)**m*log(((a +
 b*x)**n*e)/(c + d*x)**n))/((a*g + b*g*x)**m*a**2 + 2*(a*g + b*g*x)**m*a*b
*x + (a*g + b*g*x)**m*b**2*x**2),x)*a*b)/g**2
```

3.220 $\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)$

Optimal result . . . 2468
Mathematica [A] (verified) . . . 2468
Rubi [A] (verified) . . . 2469
Maple [B] (verified) . . . 2470
Fricas [A] (verification not implemented) . . . 2471
Sympy [F(-2)] . . . 2472
Maxima [F] . . . 2472
Giac [F] . . . 2472
Mupad [F(-1)] . . . 2473
Reduce [F] . . . 2473

Optimal result

Integrand size = 47, antiderivative size = 137

$$\begin{aligned}&\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\ &=-\frac{Bn(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{(bc-ad)i^2(1+m)^2(c+dx)}\\ &\quad-\frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)(c+dx)}\end{aligned}$$

output

```
-B*n*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)/(-a*d+b*c)/i^2/(1+m)^2/(
d*x+c)-(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))/(-a*d+b*c)/i^2/(1+m)/(d*x+c)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57

$$\begin{aligned}&\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\\ &\quad-\frac{(g(a+bx))^{-1-m}(c+dx)(i(c+dx))^m\left(A+Am+Bn+B(1+m)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)g(1+m)^2}\end{aligned}$$

input
```
Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]),x]
```

output
```
-(((g*(a + b*x))^(-1 - m)*(c + d*x)*(i*(c + d*x))^m*(A + A*m + B*n + B*(1
+ m)*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)*g*(1 + m)^2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2963, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^{-m-2}(ci+dix)^m \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\downarrow 2963$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\int \left(\frac{a+bx}{c+dx}\right)^{-m-2}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2741$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{m+1}-\frac{Bn\left(\frac{a+bx}{c+dx}\right)^{-m-1}}{(m+1)^2}\right)}{i^2(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]),x]
```

output
```
((g*(a + b*x))^(-2 - m)*((a + b*x)/(c + d*x))^(2 + m)*(i*(c + d*x))^(2 + m
)*(-((B*n*((a + b*x)/(c + d*x))^(-1 - m))/(1 + m)^2) - (((a + b*x)/(c + d*
x))^(-1 - m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(1 + m)))/((b*c - a*d
)*i^2)
```

Defintions of rubi rules used

rule 2741

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2963

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(137) = 274$.

Time = 37.20 (sec) , antiderivative size = 812, normalized size of antiderivative = 5.93

method	result
parallelrisch	$\frac{Ax(i(dx+c))^m(g(bx+a))^{-2-m}b^2cdn+B(i(dx+c))^m(g(bx+a))^{-2-m}abcdn^2+A(i(dx+c))^m(g(bx+a))^{-2-m}abcdn+Ax^2(i($

input

```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,met
hod=_RETURNVERBOSE)
```

output

```
(A*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*c*d*n+B*(i*(d*x+c))^m*(g*(b*x+a)
)^(-2-m)*a*b*c*d*n^2+A*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*c*d*n+A*x^2*(i
*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*d^2*m*n+B*x^2*(i*(d*x+c))^m*(g*(b*x+a))
^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*n+B*x*(i*(d*x+c))^m*(g*(b*x+a))^
(-2-m)*a*b*d^2*n^2+B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*c*d*n^2+A*x*(i
*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*d^2*n+B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2
-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*m*n+A*(i*(d*x+c))^m*(g*(b*x+a))^(-2-
m)*a*b*c*d*m*n+B*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n
)*a*b*c*d*n+B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)
*b^2*c*d*m*n+B*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*
a*b*c*d*m*n+B*x^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*d^2*n^2+A*x^2*(i*(d
*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*d^2*n+B*x^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-
m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*m*n+A*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2
-m)*a*b*d^2*m*n+A*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*c*d*m*n+B*x*(i*(d
*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*n+B*x*(i*(d*
x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*n)/(a*d-b*c)/
n/d/b/(1+m)^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.96

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx =$$

$$-\frac{(Aacm+Bacn+Aac+(Abdm+Bbdn+Abd)x^2+(Abc+Aad+(Abc+Aad)m+(Bbc+Bad)n$$

input

```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="fricas")
```

output

```
-(A*a*c*m + B*a*c*n + A*a*c + (A*b*d*m + B*b*d*n + A*b*d)*x^2 + (A*b*c + A
*a*d + (A*b*c + A*a*d)*m + (B*b*c + B*a*d)*n)*x + (B*a*c*m + B*a*c + (B*b*
d*m + B*b*d)*x^2 + (B*b*c + B*a*d + (B*b*c + B*a*d)*m)*x)*log(e) + ((B*b*d
*m + B*b*d)*n*x^2 + (B*b*c + B*a*d + (B*b*c + B*a*d)*m)*n*x + (B*a*c*m + B
*a*c)*n)*log((b*x + a)/(d*x + c)))*(b*g*x + a*g)^(-m - 2)*e^(m*log(b*g*x +
 a*g) - m*log((b*x + a)/(d*x + c)) + m*log(i/g))/((b*c - a*d)*m^2 + b*c -
a*d + 2*(b*c - a*d)*m)
```

Sympy [F(-2)]

Exception generated.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m*(A+B*ln(e*((b*x+a)/(d*x+c))**
n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)(bgx+ag)^{-m-2}(dix+ci)^m\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="maxima")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(b*g*x + a*g)^(-m - 2)*(d
*i*x + c*i)^m, x)
```

Giac [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)(bgx+ag)^{-m-2}(dix+ci)^m\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(b*g*x + a*g)^(-m - 2)*(d
*i*x + c*i)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\int \frac{(c\,i+d\,i\,x)^m\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(a\,g+b\,g\,x)^{m+2}}\,dx$$

input
```
int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^(m + 2),x)
```

output
```
int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^(m + 2), x)
```

Reduce [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{\left(\int \frac{(dix+ci)^m}{(bgx+ag)^m a^2+2(bgx+ag)^m abx+(bgx+ag)^m b^2x^2}\,dx\right)a+\left(\int \frac{(dix+ci)^m\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{(bgx+ag)^m a^2+2(bgx+ag)^m abx+(bgx+ag)^m b^2x^2}\,dx\right)b}{g^2}$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output

```
(int((c*i + d*i*x)**m/((a*g + b*g*x)**m*a**2 + 2*(a*g + b*g*x)**m*a*b*x +
(a*g + b*g*x)**m*b**2*x**2),x)*a + int(((c*i + d*i*x)**m*log(((a + b*x)**n
*e)/(c + d*x)**n))/((a*g + b*g*x)**m*a**2 + 2*(a*g + b*g*x)**m*a*b*x + (a*
g + b*g*x)**m*b**2*x**2),x)*b)/g**2
```

3.221 $\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result . . . 2475
Mathematica [F] . . . 2475
Rubi [A] (verified) . . . 2476
Maple [F] . . . 2477
Fricas [A] (verification not implemented) . . . 2478
Sympy [F(-2)] . . . 2478
Maxima [F] . . . 2479
Giac [F] . . . 2479
Mupad [F(-1)] . . . 2479
Reduce [F] . . . 2480

Optimal result

Integrand size = 49, antiderivative size = 128

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$
$$= \frac{e^{\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1+m}{n}}(i(c+dx))^{2+m}\,\text{ExpIntegralEi}\left(-\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right.}{B(bc-ad)i^2n(c+dx)}$$

output
```
exp(A*(1+m)/B/n)*(b*x+a)*(g*(b*x+a))^(-2-m)*(e*((b*x+a)/(d*x+c))^n)^((1+m)
/n)*(i*(d*x+c))^(2+m)*Ei(-(1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a
*d+b*c)/i^2/n/(d*x+c)
```

Mathematica [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)
/(c + d*x))^n]),x]
```

output
```
Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)
/(c + d*x))^n]), x]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2963, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^{-m-2}(ci+dix)^m}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}\,dx$$

$$\downarrow 2963$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\int \frac{\left(\frac{a+bx}{c+dx}\right)^{-m-2}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2747$$

$$\frac{(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{i^2n(c+dx)(bc-ad)}$$

$$\downarrow 2609$$

$$\frac{(a+bx)e^{\frac{A(m+1)}{Bn}}(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\text{ExpIntegralEi}\left(-\frac{(m+1)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{Bi^2n(c+dx)(bc-ad)}$$

input
```
Int[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)/(c +
d*x))^n]),x]
```

output
```
(E^((A*(1 + m))/(B*n))*(a + b*x)*(g*(a + b*x))^(-2 - m)*(e*((a + b*x)/(c +
 d*x))^n)^((1 + m)/n)*(i*(c + d*x))^(2 + m)*ExpIntegralEi[-(((1 + m)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n))])/(B*(b*c - a*d)*i^2*n*(c + d*x)
)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

Maple [F]

$$\int \frac{(bgx+ag)^{-2-m}(dix+ci)^m}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.73

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\mathrm{Ei}\left(-\frac{(Bm+B)n\log\left(\frac{bx+a}{dx+c}\right)+Am+(Bm+B)\log(e)+A}{Bn}\right)e^{\left(\frac{Bmn\log\left(\frac{i}{g}\right)+Am+(Bm+B)\log(e)+A}{Bn}\right)}}{(Bbc-Bad)g^2n}$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="fricas")
```

output
```
Ei(-((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)/(B
*n))*e^((B*m*n*log(i/g) + A*m + (B*m + B)*log(e) + A)/(B*n))/((B*b*c - B*a
*d)*g^2*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m/(A+B*ln(e*((b*x+a)/(d*x+c))**
n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="maxima")
```

output
```
integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m/(B*log(e*((b*x + a)/(d*x
+ c))^n) + A), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m/(B*log(e*((b*x + a)/(d*x
+ c))^n) + A), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(ci+dix)^m}{(ag+bgx)^{m+2}\left(A+B\,\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

input
```
int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))),x)
```

output
```
int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))), x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$
$$= \frac{\int \frac{(dix+ci)^m}{(bgx+ag)^m\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a^2b+2(bgx+ag)^m\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a\,b^2x+(bgx+ag)^m\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b^3x^2+(bgx+ag)^m a^3+2(bgx+ag)^m a^2bx+(bgx}}{g^2}$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((c*i + d*i*x)**m/((a*g + b*g*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*
a**2*b + 2*(a*g + b*g*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*x +
(a*g + b*g*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*x**2 + (a*g + b*g
*x)**m*a**3 + 2*(a*g + b*g*x)**m*a**2*b*x + (a*g + b*g*x)**m*a*b**2*x**2),
x)/g**2
```

3.222 $\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result . . . 2481
Mathematica [F] . . . 2482
Rubi [A] (verified) . . . 2482
Maple [F] . . . 2484
Fricas [A] (verification not implemented) . . . 2484
Sympy [F(-2)] . . . 2485
Maxima [F] . . . 2485
Giac [F] . . . 2486
Mupad [F(-1)] . . . 2486
Reduce [F] . . . 2487

Optimal result

Integrand size = 49, antiderivative size = 214

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{e^{\frac{A(1+m)}{Bn}}(1+m)(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1+m}{n}}(i(c+dx))^{2+m}\,\text{ExpIntegralEi}\left(-\frac{(1+m)\left(A+B\log\right.}{B^2(bc-ad)i^2n^2(c+dx)}$$
$$-\frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

output

```
-exp(A*(1+m)/B/n)*(1+m)*(b*x+a)*(g*(b*x+a))^(-2-m)*(e*((b*x+a)/(d*x+c))^n)
^((1+m)/n)*(i*(d*x+c))^(2+m)*Ei(-(1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n
)/B^2/(-a*d+b*c)/i^2/n^2/(d*x+c)-(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2
+m)/B/(-a*d+b*c)/i^2/n/(d*x+c)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Mathematica [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2,x]
```

output
```
Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2, x]
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.082$, Rules used = {2963, 2743, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^{-m-2}(ci+dix)^m}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}\,dx$$

$$\downarrow 2963$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\int \frac{\left(\frac{a+bx}{c+dx}\right)^{-m-2}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2743$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(-\frac{(m+1)\int \frac{\left(\frac{a+bx}{c+dx}\right)^{-m-2}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn}-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}}{Bn\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}\right)}{i^2(bc-ad)}$$

$$\downarrow 2747$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(-\frac{(m+1)\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)^{\frac{m+1}{n}}\int\frac{\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)^{-\frac{m+1}{n}}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{Bn^2}\right.}{i^2(bc-ad)}$$

$$\downarrow 2609$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(-\frac{(m+1)e^{\frac{A(m+1)}{Bn}}\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)^{\frac{m+1}{n}}\text{ExpIntegralEi}\left(-\frac{(m+1)\left(A+B\log\right.}{B^2n^2}\right.}{i^2(bc-ad)}$$

input
```
Int[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
((g*(a + b*x))^(-2 - m)*((a + b*x)/(c + d*x))^(2 + m)*(i*(c + d*x))^(2 + m)*(-((E^((A*(1 + m))/(B*n))*(1 + m)*(e*((a + b*x)/(c + d*x))^n)^((1 + m)/n)*((a + b*x)/(c + d*x))^(-1 - m)*ExpIntegralEi[-(((1 + m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n))])/(B^2*n^2)) - ((a + b*x)/(c + d*x))^(-1 - m)/(B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/((b*c - a*d)*i^2)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2743
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] - Simp[(m + 1)/(b*n*(p + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2747

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

Maple [F]

$$\int \frac{(bgx+ag)^{-2-m}\,(dix+ci)^m}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}\,dx$$

input

```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output

```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.33

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx =$$

$$-\frac{(Bbdg^2nx^2+Bacg^2n+(Bbc+Bad)g^2nx)(bgx+ag)^{-m-2}e^{\left(m\log(bgx+ag)-m\log\left(\frac{bx+a}{dx+c}\right)+m\log\left(\frac{i}{g}\right)\right)}+((Br}{(B^3bc-B^3ad)g^2n^3\log\left(\frac{bx+a}{dx+c}\right)}$$

input

```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="fricas")
```

output
```
-((B*b*d*g^2*n*x^2 + B*a*c*g^2*n + (B*b*c + B*a*d)*g^2*n*x)*(b*g*x + a*g)^
(-m - 2)*e^(m*log(b*g*x + a*g) - m*log((b*x + a)/(d*x + c)) + m*log(i/g))
+ ((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)*Ei(-
((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)/(B*n))
*e^((B*m*n*log(i/g) + A*m + (B*m + B)*log(e) + A)/(B*n)))/((B^3*b*c - B^3*
a*d)*g^2*n^3*log((b*x + a)/(d*x + c)) + (B^3*b*c - B^3*a*d)*g^2*n^2*log(e)
 + (A*B^2*b*c - A*B^2*a*d)*g^2*n^2)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m/(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="maxima")
```

output
```
i^m*(m + 1)*integrate(-(d*x + c)^m/((B^2*b^2*g^(m + 2)*n*x^2 + 2*B^2*a*b*g
^(m + 2)*n*x + B^2*a^2*g^(m + 2)*n)*(b*x + a)^m*log((b*x + a)^n) - (B^2*b^
2*g^(m + 2)*n*x^2 + 2*B^2*a*b*g^(m + 2)*n*x + B^2*a^2*g^(m + 2)*n)*(b*x +
a)^m*log((d*x + c)^n) + (B^2*a^2*g^(m + 2)*n*log(e) + A*B*a^2*g^(m + 2)*n
+ (B^2*b^2*g^(m + 2)*n*log(e) + A*B*b^2*g^(m + 2)*n)*x^2 + 2*(B^2*a*b*g^(m
 + 2)*n*log(e) + A*B*a*b*g^(m + 2)*n)*x)*(b*x + a)^m), x) - (d*i^m*x + c*i
^m)*(d*x + c)^m/(((b^2*c*g^(m + 2)*n - a*b*d*g^(m + 2)*n)*B^2*x + (a*b*c*g
^(m + 2)*n - a^2*d*g^(m + 2)*n)*B^2)*(b*x + a)^m*log((b*x + a)^n) - ((b^2*
c*g^(m + 2)*n - a*b*d*g^(m + 2)*n)*B^2*x + (a*b*c*g^(m + 2)*n - a^2*d*g^(m
 + 2)*n)*B^2)*(b*x + a)^m*log((d*x + c)^n) + ((a*b*c*g^(m + 2)*n - a^2*d*g
^(m + 2)*n)*A*B + (a*b*c*g^(m + 2)*n*log(e) - a^2*d*g^(m + 2)*n*log(e))*B^
2 + ((b^2*c*g^(m + 2)*n - a*b*d*g^(m + 2)*n)*A*B + (b^2*c*g^(m + 2)*n*log(
e) - a*b*d*g^(m + 2)*n*log(e))*B^2)*x)*(b*x + a)^m)
```

Giac [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m/(B*log(e*((b*x + a)/(d*x
+ c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(c\,i+d\,i\,x)^m}{(a\,g+b\,g\,x)^{m+2}\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))^2),x)
```

output
```
int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))^2), x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$= \frac{\int \frac{(dix+ci)^m}{(bgx+ag)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 a^2b^2+2(bgx+ag)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 a\, b^3x+(bgx+ag)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 b^4x^2+2(bgx+ag)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) a^3b+4}}{g^2}$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((c*i + d*i*x)**m/((a*g + b*g*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*
*2*a**2*b**2 + 2*(a*g + b*g*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*
b**3*x + (a*g + b*g*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**4*x**2
+ 2*(a*g + b*g*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b + 4*(a*g +
b*g*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*x + 2*(a*g + b*g*x)
**m*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*x**2 + (a*g + b*g*x)**m*a**4
 + 2*(a*g + b*g*x)**m*a**3*b*x + (a*g + b*g*x)**m*a**2*b**2*x**2),x)/g**2
```

3.223 $\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$

Optimal result . . . 2488
Mathematica [F] . . . 2489
Rubi [A] (verified) . . . 2489
Maple [F] . . . 2492
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Sympy [F(-1)] . . . 2493
Maxima [F] . . . 2494
Giac [F] . . . 2494
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Reduce [F] . . . 2495

Optimal result

Integrand size = 49, antiderivative size = 306

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$$

$$= \frac{e^{\frac{A(1+m)}{Bn}}(1+m)^2(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1+m}{n}}(i(c+dx))^{2+m}\,\text{ExpIntegralEi}\left(-\frac{(1+m)\left(A+B\log\left(\right.\right.\right.}{Bn}}{2B^3(bc-ad)i^2n^3(c+dx)}$$

$$-\frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}$$

$$+\frac{(1+m)(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{2B^2(bc-ad)i^2n^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

output

```
1/2*exp(A*(1+m)/B/n)*(1+m)^2*(b*x+a)*(g*(b*x+a))^(-2-m)*(e*((b*x+a)/(d*x+c
))^n)^((1+m)/n)*(i*(d*x+c))^(2+m)*Ei(-(1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)/B/n)/B^3/(-a*d+b*c)/i^2/n^3/(d*x+c)-1/2*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d
*x+c))^(2+m)/B/(-a*d+b*c)/i^2/n/(d*x+c)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2+
1/2*(1+m)*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)/B^2/(-a*d+b*c)/i^2/
n^2/(d*x+c)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Mathematica [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$$

input
```
Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^3,x]
```

output
```
Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^3, x]
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used = {2963, 2743, 2743, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^{-m-2}(ci+dix)^m}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}\,dx$$

$$\downarrow 2963$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\int \frac{\left(\frac{a+bx}{c+dx}\right)^{-m-2}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2743$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(-\frac{(m+1)\int \frac{\left(\frac{a+bx}{c+dx}\right)^{-m-2}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{2Bn}-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}}{2Bn\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}\right)}{i^2(bc-ad)}$$

$$\downarrow 2743$$

$$(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(-\frac{(m+1)\left(-\frac{(m+1)\int\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-2}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn}-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}}{Bn\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}\right)}{2Bn}-\right.$$

$$i^2(bc-ad)$$

$$\downarrow 2747$$

$$(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(-\frac{(m+1)\left(-\frac{(m+1)\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\int\frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+\right.\right.\right.\right.}{Bn^2}\right)}{2Bn}-\right.$$

$$i^2(bc-ad)$$

$$\downarrow 2609$$

$$(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(-\frac{(m+1)\left(-\frac{(m+1)e^{\frac{A(m+1)}{Bn}}\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\operatorname{ExpIntegralEi}\left(-\frac{(m+1)\left(\right.\right.\right.\right.}{B^2n^2}\right)}{2Bn}-\right.$$

$$i^2(bc-ad)$$

input

```
Int[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3,x]
```

output
```
((g*(a + b*x))^(-2 - m)*((a + b*x)/(c + d*x))^(2 + m)*(i*(c + d*x))^(2 + m
)*(-1/2*((a + b*x)/(c + d*x))^(-1 - m)/(B*n*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])^2) - ((1 + m)*(-((E^((A*(1 + m))/(B*n))*(1 + m)*(e*((a + b*x)/(c
+ d*x))^n)^((1 + m)/n)*((a + b*x)/(c + d*x))^(-1 - m)*ExpIntegralEi[-(((1
+ m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n))])/(B^2*n^2)) - ((a + b
*x)/(c + d*x))^(-1 - m)/(B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(2*
B*n)))/((b*c - a*d)*i^2)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2743
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Simp[(m + 1)/(b*n*(p + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

Maple [F]

$$\int \frac{(bgx+ag)^{-2-m}\,(dix+ci)^m}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^3}dx$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

output
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 815 vs. $2(299) = 598$.

Time $= 0.12$ (sec) , antiderivative size $= 815$, normalized size of antiderivative $= 2.66$

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="fricas")
```

output

```
-1/2*((B^2*a*c*g^2*n^2 + (B^2*b*d*g^2*n^2 - (A*B*b*d*g^2*m + A*B*b*d*g^2)*
n)*x^2 - (A*B*a*c*g^2*m + A*B*a*c*g^2)*n + ((B^2*b*c + B^2*a*d)*g^2*n^2 -
((A*B*b*c + A*B*a*d)*g^2*m + (A*B*b*c + A*B*a*d)*g^2)*n)*x - ((B^2*b*d*g^2
*m + B^2*b*d*g^2)*n*x^2 + ((B^2*b*c + B^2*a*d)*g^2*m + (B^2*b*c + B^2*a*d)
*g^2)*n*x + (B^2*a*c*g^2*m + B^2*a*c*g^2)*n)*log(e) - ((B^2*b*d*g^2*m + B^
2*b*d*g^2)*n^2*x^2 + ((B^2*b*c + B^2*a*d)*g^2*m + (B^2*b*c + B^2*a*d)*g^2)
*n^2*x + (B^2*a*c*g^2*m + B^2*a*c*g^2)*n^2)*log((b*x + a)/(d*x + c)))*(b*g
*x + a*g)^(-m - 2)*e^(m*log(b*g*x + a*g) - m*log((b*x + a)/(d*x + c)) + m*
log(i/g)) - ((B^2*m^2 + 2*B^2*m + B^2)*n^2*log((b*x + a)/(d*x + c))^2 + A^
2*m^2 + 2*A^2*m + (B^2*m^2 + 2*B^2*m + B^2)*log(e)^2 + 2*(A*B*m^2 + 2*A*B*
m + A*B)*n*log((b*x + a)/(d*x + c)) + A^2 + 2*(A*B*m^2 + 2*A*B*m + (B^2*m^
2 + 2*B^2*m + B^2)*n*log((b*x + a)/(d*x + c)) + A*B)*log(e))*Ei(-((B*m + B
)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)/(B*n))*e^((B*m*
n*log(i/g) + A*m + (B*m + B)*log(e) + A)/(B*n)))/((B^5*b*c - B^5*a*d)*g^2*
n^5*log((b*x + a)/(d*x + c))^2 + (B^5*b*c - B^5*a*d)*g^2*n^3*log(e)^2 + 2*
(A*B^4*b*c - A*B^4*a*d)*g^2*n^4*log((b*x + a)/(d*x + c)) + (A^2*B^3*b*c -
A^2*B^3*a*d)*g^2*n^3 + 2*((B^5*b*c - B^5*a*d)*g^2*n^4*log((b*x + a)/(d*x +
 c)) + (A*B^4*b*c - A*B^4*a*d)*g^2*n^3)*log(e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \text{Timed out}$$

input

```
integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m/(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**3,x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3}\,dx$$

input

```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="maxima")
```

output

```
-(m^2 + 2*m + 1)*i^m*integrate(-1/2*(d*x + c)^m/((B^3*b^2*g^(m + 2)*n^2*x^
2 + 2*B^3*a*b*g^(m + 2)*n^2*x + B^3*a^2*g^(m + 2)*n^2)*(b*x + a)^m*log((b*
x + a)^n) - (B^3*b^2*g^(m + 2)*n^2*x^2 + 2*B^3*a*b*g^(m + 2)*n^2*x + B^3*a
^2*g^(m + 2)*n^2)*(b*x + a)^m*log((d*x + c)^n) + (B^3*a^2*g^(m + 2)*n^2*lo
g(e) + A*B^2*a^2*g^(m + 2)*n^2 + (B^3*b^2*g^(m + 2)*n^2*log(e) + A*B^2*b^2
*g^(m + 2)*n^2)*x^2 + 2*(B^3*a*b*g^(m + 2)*n^2*log(e) + A*B^2*a*b*g^(m + 2
)*n^2)*x)*(b*x + a)^m), x) + 1/2*((B*d*i^m*(m + 1)*x + B*c*i^m*(m + 1))*(d
*x + c)^m*log((b*x + a)^n) - (B*d*i^m*(m + 1)*x + B*c*i^m*(m + 1))*(d*x +
c)^m*log((d*x + c)^n) + (A*c*i^m*(m + 1) + (i^m*(m + 1)*log(e) - i^m*n)*B*
c + (A*d*i^m*(m + 1) + (i^m*(m + 1)*log(e) - i^m*n)*B*d)*x)*(d*x + c)^m)/(
((b^2*c*g^(m + 2)*n^2 - a*b*d*g^(m + 2)*n^2)*B^4*x + (a*b*c*g^(m + 2)*n^2
- a^2*d*g^(m + 2)*n^2)*B^4)*(b*x + a)^m*log((b*x + a)^n)^2 + ((b^2*c*g^(m
+ 2)*n^2 - a*b*d*g^(m + 2)*n^2)*B^4*x + (a*b*c*g^(m + 2)*n^2 - a^2*d*g^(m
+ 2)*n^2)*B^4)*(b*x + a)^m*log((d*x + c)^n)^2 + 2*((a*b*c*g^(m + 2)*n^2 -
a^2*d*g^(m + 2)*n^2)*A*B^3 + (a*b*c*g^(m + 2)*n^2*log(e) - a^2*d*g^(m + 2)
*n^2*log(e))*B^4 + ((b^2*c*g^(m + 2)*n^2 - a*b*d*g^(m + 2)*n^2)*A*B^3 + (b
^2*c*g^(m + 2)*n^2*log(e) - a*b*d*g^(m + 2)*n^2*log(e))*B^4)*x)*(b*x + a)^
m*log((b*x + a)^n) + ((a*b*c*g^(m + 2)*n^2 - a^2*d*g^(m + 2)*n^2)*A^2*B^2
+ 2*(a*b*c*g^(m + 2)*n^2*log(e) - a^2*d*g^(m + 2)*n^2*log(e))*A*B^3 + (a*b
*c*g^(m + 2)*n^2*log(e)^2 - a^2*d*g^(m + 2)*n^2*log(e)^2)*B^4 + ((b^2*c...
```

Giac [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m/(B*log(e*((b*x + a)/(d*x
+ c))^n) + A)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \int \frac{(c\,i+d\,i\,x)^m}{(a\,g+b\,g\,x)^{m+2}\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^3}\,dx$$

input
```
int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))^3),x)
```

output
```
int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))^3), x)
```

Reduce [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$$
$$= \frac{\int}{(bgx+ag)^m\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3 a^2b^3+2(bgx+ag)^m\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3 a\,b^4x+(bgx+ag)^m\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3 b^5x^2+3(bgx+ag)^m\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 a^3b^2+}$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n))^3,x)
```

output

```
int((c*i + d*i*x)**m/((a*g + b*g*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*
*3*a**2*b**3 + 2*(a*g + b*g*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)**3*a*
b**4*x + (a*g + b*g*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)**3*b**5*x**2
+ 3*(a*g + b*g*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**3*b**2 + 6*(
a*g + b*g*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b**3*x + 3*(a*g
 + b*g*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**4*x**2 + 3*(a*g +
b*g*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*b + 6*(a*g + b*g*x)**m*l
og(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b**2*x + 3*(a*g + b*g*x)**m*log(((a
 + b*x)**n*e)/(c + d*x)**n)*a**2*b**3*x**2 + (a*g + b*g*x)**m*a**5 + 2*(a*
g + b*g*x)**m*a**4*b*x + (a*g + b*g*x)**m*a**3*b**2*x**2),x)/g**2
```

3.224 $\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$

Optimal result . . . 2497
Mathematica [A] (verified) . . . 2497
Rubi [A] (verified) . . . 2498
Maple [A] (verified) . . . 2499
Fricas [A] (verification not implemented) . . . 2500
Sympy [F(-1)] . . . 2500
Maxima [F] . . . 2500
Giac [A] (verification not implemented) . . . 2501
Mupad [F(-1)] . . . 2501
Reduce [B] (verification not implemented) . . . 2501

Optimal result

Integrand size = 35, antiderivative size = 41

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \frac{\log^{1+p}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n(1+p)}$$

output
```
ln(e*((b*x+a)/(d*x+c))^n)^(p+1)/(-a*d+b*c)/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \frac{\log^{1+p}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bcn-adn)(1+p)}$$

input
```
Integrate[Log[e*((a + b*x)/(c + d*x))^n]^p/((a + b*x)*(c + d*x)),x]
```

output
```
Log[e*((a + b*x)/(c + d*x))^n]^(1 + p)/((b*c*n - a*d*n)*(1 + p))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx} d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2739$$

$$\frac{\int \log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

$$\downarrow 15$$

$$\frac{\log^{p+1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(p+1)(bc-ad)}$$

input
```
Int[Log[e*((a + b*x)/(c + d*x))^n]^p/((a + b*x)*(c + d*x)),x]
```

output
```
Log[e*((a + b*x)/(c + d*x))^n]^(1 + p)/((b*c - a*d)*n*(1 + p))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [A] (verified)

Time = 8.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^{p+1}}{n(da-bc)(p+1)}$	43
default	$-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^{p+1}}{n(da-bc)(p+1)}$	43
parallelrisch	$-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^p}{n(adp-bcp+da-bc)}$	63

input
```
int(ln(e*((b*x+a)/(d*x+c))^n)^p/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/n/(a*d-b*c)*ln(e*((b*x+a)/(d*x+c))^n)^(p+1)/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \frac{\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)^p}{(bc-ad)np+(bc-ad)n}$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
(n*log((b*x + a)/(d*x + c)) + log(e))*(n*log((b*x + a)/(d*x + c)) + log(e))^p/((b*c - a*d)*n*p + (b*c - a*d)*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*((b*x+a)/(d*x+c))**n)**p/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^p}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(log(e*((b*x + a)/(d*x + c))^n)^p/((b*x + a)*(d*x + c)), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \frac{\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)^{p+1}}{(bcn-adn)(p+1)}$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(b*x+a)/(d*x+c),x, algorithm="giac"
)
```

output
```
(n*log((b*x + a)/(d*x + c)) + log(e))^(p + 1)/((b*c*n - a*d*n)*(p + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^p}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(log(e*((a + b*x)/(c + d*x))^n)^p/((a + b*x)*(c + d*x)),x)
```

output
```
int(log(e*((a + b*x)/(c + d*x))^n)^p/((a + b*x)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.56

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = -\frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^p \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{n\,(adp-bcp+ad-bc)}$$

input
```
int(log(e*((b*x+a)/(d*x+c))^n)^p/(b*x+a)/(d*x+c),x)
```

output
```
( - log(((a + b*x)**n*e)/(c + d*x)**n)**p*log(((a + b*x)**n*e)/(c + d*x)**
n))/(n*(a*d*p + a*d - b*c*p - b*c))
```

3.225 $\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx$

Optimal result . . . 2502
Mathematica [A] (verified) . . . 2502
Rubi [A] (verified) . . . 2503
Maple [A] (verified) . . . 2504
Fricas [A] (verification not implemented) . . . 2505
Sympy [F(-1)] . . . 2505
Maxima [F] . . . 2506
Giac [A] (verification not implemented) . . . 2506
Mupad [F(-1)] . . . 2506
Reduce [B] (verification not implemented) . . . 2507

Optimal result

Integrand size = 42, antiderivative size = 41

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \frac{\log^{1+p}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n(1+p)}$$

output
```
ln(e*((b*x+a)/(d*x+c))^n)^(p+1)/(-a*d+b*c)/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \frac{\log^{1+p}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bcn-adn)(1+p)}$$

input
```
Integrate[Log[e*((a + b*x)/(c + d*x))^n]^p/(a*c + (b*c + a*d)*x + b*d*x^2)
,x]
```

output
```
Log[e*((a + b*x)/(c + d*x))^n]^(1 + p)/((b*c*n - a*d*n)*(1 + p))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2974, 2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(ad+bc)+ac+bdx^2}\,dx$$

$$\downarrow 2974$$

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2739$$

$$\frac{\int \log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

$$\downarrow 15$$

$$\frac{\log^{p+1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(p+1)(bc-ad)}$$

input
```
Int[Log[e*((a + b*x)/(c + d*x))^n]^p/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
Log[e*((a + b*x)/(c + d*x))^n]^(1 + p)/((b*c - a*d)*n*(1 + p))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2974
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_) + (h_.)*(x_)^2)^(m_.), x_Symbol] :> Simp[h^
m/(b^m*d^m)   Int[(a + b*x)^m*(c + d*x)^m*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, n, p}, x] && EqQ[b*
d*f - a*c*h, 0] && EqQ[b*d*g - h*(b*c + a*d), 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 11.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

method	result	size
parallelrisch	$-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^p}{n(adp-bcp+da-bc)}$	63

input
```
int(ln(e*((b*x+a)/(d*x+c))^n)^p/(a*c+(a*d+b*c)*x+b*d*x^2),x,method=_RETURN
VERBOSE)
```

output
```
-ln(e*((b*x+a)/(d*x+c))^n)*ln(e*((b*x+a)/(d*x+c))^n)^p/n/(a*d*p-b*c*p+a*d-
b*c)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \frac{\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)^p}{(bc-ad)np+(bc-ad)n}$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(a*c+(a*d+b*c)*x+b*d*x^2),x, algori
thm="fricas")
```

output
```
(n*log((b*x + a)/(d*x + c)) + log(e))*(n*log((b*x + a)/(d*x + c)) + log(e)
)^p/((b*c - a*d)*n*p + (b*c - a*d)*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*((b*x+a)/(d*x+c))**n)**p/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^p}{bdx^2+ac+(bc+ad)x}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="maxima")
```

output
```
integrate(log(e*((b*x + a)/(d*x + c))^n)^p/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \frac{\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)^{p+1}}{(bcn-adn)(p+1)}$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")
```

output
```
(n*log((b*x + a)/(d*x + c)) + log(e))^(p + 1)/((b*c*n - a*d*n)*(p + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \int \frac{\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^p}{b\,d\,x^2+(a\,d+b\,c)\,x+a\,c}\,dx$$

input
```
int(log(e*((a + b*x)/(c + d*x))^n)^p/(a*c + x*(a*d + b*c) + b*d*x^2),x)
```

output
```
int(log(e*((a + b*x)/(c + d*x))^n)^p/(a*c + x*(a*d + b*c) + b*d*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.56

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = -\frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^p \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{n\left(adp-bcp+ad-bc\right)}$$

input
```
int(log(e*((b*x+a)/(d*x+c))^n)^p/(a*c+(a*d+b*c)*x+b*d*x^2),x)
```

output
```
( - log(((a + b*x)**n*e)/(c + d*x)**n)**p*log(((a + b*x)**n*e)/(c + d*x)**
n))/(n*(a*d*p + a*d - b*c*p - b*c))
```

3.226 $\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B \log\left(e(a+bx)^n (c+dx)^{-n}\right)\right)^p dx$

Optimal result 2508
Mathematica [F] 2508
Rubi [A] (warning: unable to verify) 2509
Maple [F] 2511
Fricas [F] 2511
Sympy [F(-1)] 2511
Maxima [F] 2512
Giac [F(-2)] 2512
Mupad [F(-1)] 2513
Reduce [F] 2513

Optimal result

Integrand size = 50, antiderivative size = 193

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B \log\left(e(a+bx)^n (c+dx)^{-n}\right)\right)^p dx$$

$$= \frac{e^{-\frac{A(1+m)}{Bn}} (a+bx)(g(a+bx))^m (i(c+dx))^{-m} \left(e(a+bx)^n (c+dx)^{-n}\right)^{-\frac{1+m}{n}} \Gamma\left(1+p, -\frac{(1+m)(A+B\log(e(a+bx)^n}{Bn}\right.}{(bc-ad)i^2(1+m)(c+}$$

output

```
(b*x+a)*(g*(b*x+a))^m*GAMMA(p+1,-(1+m)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B
/n)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(-a*d+b*c)/exp(A*(1+m)/B/n)/i^2/(1
+m)/(d*x+c)/((i*(d*x+c))^m)/((e*(b*x+a)^n/((d*x+c)^n))^((1+m)/n))/((-(1+m)
*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/n)^p)
```

Mathematica [F]

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B \log\left(e(a+bx)^n (c+dx)^{-n}\right)\right)^p dx$$
$$= \int (ag+bgx)^m (ci+dix)^{-2-m} \left(A + B \log\left(e(a+bx)^n (c+dx)^{-n}\right)\right)^p dx$$

input

```
Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])^p,x]
```

output
```
Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])^p, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2973, 2963, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^m (ci+dix)^{-m-2} \left(B \log\left(e(a+bx)^n (c+dx)^{-n}\right)+A\right)^p dx$$

$$\downarrow 2973$$

$$\int (ag+bgx)^m (ci+dix)^{-m-2} \left(B \log\left(e(a+bx)^n (c+dx)^{-n}\right)+A\right)^p dx$$

$$\downarrow 2963$$

$$\frac{(g(a+bx))^m (i(c+dx))^{-m} \left(\frac{a+bx}{c+dx}\right)^{-m} \int \left(\frac{a+bx}{c+dx}\right)^m \left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2747$$

$$\frac{(a+bx)(g(a+bx))^m (i(c+dx))^{-m} \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}} \int \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}} \left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\log\left(e\left(\frac{a+}{c+}\right.\right.}{i^2 n(c+dx)(bc-ad)}$$

$$\downarrow 2612$$

$$\frac{(a+bx)e^{-\frac{A(m+1)}{Bn}} (g(a+bx))^m (i(c+dx))^{-m} \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}} \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^p \left(-\frac{(m+1)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.\right.}{Bn}}{i^2(m+1)(c+dx)(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[(e*(a + b*x)^n)/(c +
 d*x)^n])^p,x]
```

output
```
((a + b*x)*(g*(a + b*x))^m*Gamma[1 + p, -(((1 + m)*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(B*n))]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p)/((b*c -
 a*d)*E^((A*(1 + m))/(B*n))*i^2*(1 + m)*(e*((a + b*x)/(c + d*x))^n)^((1 +
m)/n)*(c + d*x)*(i*(c + d*x))^m*(-(((1 + m)*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]))/(B*n)))^p)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [F]

$$\int (bgx+ag)^m (dix+ci)^{-2-m} \left(A+B\ln\left(e(bx+a)^n (dx+c)^{-n}\right)\right)^p dx$$

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p,x
)
```

output
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p,x
)
```

Fricas [F]

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e(a+bx)^n (c+dx)^{-n}\right)\right)^p \, dx$$
$$= \int (bgx+ag)^m (dix+ci)^{-m-2} \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^p dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n
)))^p,x, algorithm="fricas")
```

output
```
integral((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)*(B*log((b*x + a)^n*e/(d*x
+ c)^n) + A)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e(a+bx)^n (c+dx)^{-n}\right)\right)^p \, dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)*(A+B*ln(e*(b*x+a)**n/((d*x+c)
**n)))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$=\int (bgx+ag)^m(dix+ci)^{-m-2}\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^p dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n
)))^p,x, algorithm="maxima")
```

output
```
integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)*(B*log((b*x + a)^n*e/(d*x
 + c)^n) + A)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$=\text{Exception raised: RuntimeError}$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n
)))^p,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,0,5,5,0,2,2,3,3,0,0,0,2]%%%}+%%%{-2,[0,0,5,4,1,3,1,3,3,0,0,0,2]%%%}+%%%{1
,[0,0,5,3,2,
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p dx$$
$$= \int \frac{(a\,g+b\,g\,x)^m \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^p}{(c\,i+d\,i\,x)^{m+2}}\,dx$$

input
```
int(((a*g + b*g*x)^m*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p)/(c*i + d*
i*x)^(m + 2),x)
```

output
```
int(((a*g + b*g*x)^m*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p)/(c*i + d*
i*x)^(m + 2), x)
```

Reduce [F]

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p dx$$
$$= -\left(\int \frac{(bgx+ag)^m \left(\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) b+a\right)^p}{(dix+ci)^m c^2+2\,(dix+ci)^m\,cdx+(dix+ci)^m\,d^2x^2}dx\right)$$

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p,
x)
```

output
```
 - int(((a*g + b*g*x)**m*(log(((a + b*x)**n*e)/(c + d*x)**n)*b + a)**p)/((
c*i + d*i*x)**m*c**2 + 2*(c*i + d*i*x)**m*c*d*x + (c*i + d*i*x)**m*d**2*x*
*2),x)
```

3.227 $\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p dx$

Optimal result . . . 2514
Mathematica [F] . . . 2514
Rubi [A] (warning: unable to verify) . . . 2515
Maple [F] . . . 2517
Fricas [F] . . . 2517
Sympy [F(-1)] . . . 2517
Maxima [F] . . . 2518
Giac [F(-2)] . . . 2518
Mupad [F(-1)] . . . 2519
Reduce [F] . . . 2519

Optimal result

Integrand size = 50, antiderivative size = 194

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p dx =$$

$$-\frac{e^{\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1+m}{n}}\Gamma\left(1+p,\frac{(1+m)(A+B\log(e(a+b}{Bn}}{(bc-ad)i^2(1+m)(c}$$

output

```
-exp(A*(1+m)/B/n)*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(e*(b*x+a)^
n/((d*x+c)^n))^((1+m)/n)*GAMMA(p+1,(1+m)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
/B/n)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(-a*d+b*c)/i^2/(1+m)/(d*x+c)/(((
1+m)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/n)^p)
```

Mathematica [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p dx$$
$$= \int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p dx$$

input

```
Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])^p,x]
```

output
```
Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])^p, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.80 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2973, 2963, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^{-m-2}(ci+dix)^m \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^p dx$$

$$\downarrow 2973$$

$$\int (ag+bgx)^{-m-2}(ci+dix)^m \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^p dx$$

$$\downarrow 2963$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\int \left(\frac{a+bx}{c+dx}\right)^{-m-2}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\downarrow 2747$$

$$\frac{(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\int \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\log\left(\right.}{i^2n(c+dx)(bc-ad)}$$

$$\downarrow 2612$$

$$-\frac{(a+bx)e^{\frac{A(m+1)}{Bn}}(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^p\left(\frac{(m+1)\left(B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.\right.}{Bn}}{i^2(m+1)(c+dx)(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c +
 d*x)^n])^p,x]
```

output
```
-((E^((A*(1 + m))/(B*n))*(a + b*x)*(g*(a + b*x))^(-2 - m)*(e*((a + b*x)/(c
 + d*x))^n)^((1 + m)/n)*(i*(c + d*x))^(2 + m)*Gamma[1 + p, ((1 + m)*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p)/((b*c - a*d)*i^2*(1 + m)*(c + d*x)*(((1 + m)*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(B*n))^p))
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [F]

$$\int (bgx+ag)^{-2-m}(dix+ci)^m\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^p dx$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p,x
)
```

output
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p,x
)
```

Fricas [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$=\int (bgx+ag)^{-m-2}(dix+ci)^m\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^p dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*(b*x+a)^n/((d*x+c)^n
)))^p,x, algorithm="fricas")
```

output
```
integral((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m*(B*log((b*x + a)^n*e/(d*x
+ c)^n) + A)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m*(A+B*ln(e*(b*x+a)**n/((d*x+c)
**n)))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p dx$$
$$= \int (bgx+ag)^{-m-2}(dix+ci)^m \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^p dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*(b*x+a)^n/((d*x+c)^n
)))^p,x, algorithm="maxima")
```

output
```
integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m*(B*log((b*x + a)^n*e/(d*x
 + c)^n) + A)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p dx$$
$$= \text{Exception raised: RuntimeError}$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*(b*x+a)^n/((d*x+c)^n
)))^p,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,0,5,5,0,2,2,3,3,0,0,0,2]%%%}+%%%{-2,[0,0,5,4,1,3,1,3,3,0,0,0,2]%%%}+%%%{1
,[0,0,5,3,2,
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$=\int \frac{(c\,i+d\,i\,x)^m\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^p}{(a\,g+b\,g\,x)^{m+2}}\,dx$$

input
```
int(((c*i + d*i*x)^m*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p)/(a*g + b*
g*x)^(m + 2),x)
```

output
```
int(((c*i + d*i*x)^m*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p)/(a*g + b*
g*x)^(m + 2), x)
```

Reduce [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$=\frac{\int \frac{(dix+ci)^m\left(\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b+a\right)^p}{(bgx+ag)^m a^2+2(bgx+ag)^m abx+(bgx+ag)^m b^2x^2}dx}{g^2}$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p,
x)
```

output
```
int(((c*i + d*i*x)**m*(log(((a + b*x)**n*e)/(c + d*x)**n)*b + a)**p)/((a*g
 + b*g*x)**m*a**2 + 2*(a*g + b*g*x)**m*a*b*x + (a*g + b*g*x)**m*b**2*x**2)
,x)/g**2
```

3.228 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)(c+dx)}\,dx$

Optimal result . . . 2520
Mathematica [A] (verified) . . . 2520
Rubi [A] (warning: unable to verify) . . . 2521
Maple [A] (verified) . . . 2522
Fricas [B] (verification not implemented) . . . 2523
Sympy [F(-1)] . . . 2524
Maxima [B] (verification not implemented) . . . 2524
Giac [F] . . . 2525
Mupad [B] (verification not implemented) . . . 2526
Reduce [B] (verification not implemented) . . . 2526

Optimal result

Integrand size = 40, antiderivative size = 45

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)(c+dx)}\,dx = \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^4}{4B(bc-ad)n}$$

output
```
1/4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^4/B/(-a*d+b*c)/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)(c+dx)}\,dx = \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^4}{4(bBcn-aBdn)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/((a + b*x)*(c + d*x))
,x]
```

output
```
(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^4/(4*(b*B*c*n - a*B*d*n))
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2973, 2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B \log(e(a+bx)^n(c+dx)^{-n}) + A)^3}{(a+bx)(c+dx)}\,dx$$

$$\downarrow 2973$$

$$\int \frac{(B \log(e(a+bx)^n(c+dx)^{-n}) + A)^3}{(a+bx)(c+dx)}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{a+bx}\,d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2739$$

$$\frac{\int \frac{(a+bx)^3}{(c+dx)^3}\,d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn(bc-ad)}$$

$$\downarrow 15$$

$$\frac{(a+bx)^4}{4Bn(c+dx)^4(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/((a + b*x)*(c + d*x)),x]
```

output
```
(a + b*x)^4/(4*B*(b*c - a*d)*n*(c + d*x)^4)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [A] (verified)

Time = 39.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^4}{4n(da-bc)B}$
default	$-\frac{\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^4}{4n(da-bc)B}$
parallelrisch	$-\frac{B^3\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^4b^2d^2+4AB^2\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^3b^2d^2+6A^2B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^2b^2d^2+4}{4nb^2d^2(da-bc)}$
parts	$\frac{A^3\ln(dx+c)}{da-bc}-\frac{A^3\ln(bx+a)}{da-bc}-\frac{B^3\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^4}{4n(da-bc)}-\frac{AB^2\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^3}{n(da-bc)}-\frac{3A^2B\ln\left(e(b}{2}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)/(d*x+c),x,method=_RETURNVE
RBOSE)
```

output
```
-1/4/n/(a*d-b*c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^4/B
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(43) = 86$.

Time $= 0.10$ (sec) , antiderivative size $= 375$, normalized size of antiderivative $= 8.33$

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)(c+dx)}\,dx$$
$$= \frac{B^3n^3\log(bx+a)^4 + B^3n^3\log(dx+c)^4 + 4\left(B^3n^2\log(e) + AB^2n^2\right)\log(bx+a)^3 - 4\left(B^3n^3\log(bx+a)\right.}{}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)/(d*x+c),x, algorith
m="fricas")
```

output
```
1/4*(B^3*n^3*log(b*x + a)^4 + B^3*n^3*log(d*x + c)^4 + 4*(B^3*n^2*log(e) +
 A*B^2*n^2)*log(b*x + a)^3 - 4*(B^3*n^3*log(b*x + a) + B^3*n^2*log(e) + A*
B^2*n^2)*log(d*x + c)^3 + 6*(B^3*n*log(e)^2 + 2*A*B^2*n*log(e) + A^2*B*n)*
log(b*x + a)^2 + 6*(B^3*n^3*log(b*x + a)^2 + B^3*n*log(e)^2 + 2*A*B^2*n*lo
g(e) + A^2*B*n + 2*(B^3*n^2*log(e) + A*B^2*n^2)*log(b*x + a))*log(d*x + c)
^2 + 4*(B^3*log(e)^3 + 3*A*B^2*log(e)^2 + 3*A^2*B*log(e) + A^3)*log(b*x +
a) - 4*(B^3*n^3*log(b*x + a)^3 + B^3*log(e)^3 + 3*A*B^2*log(e)^2 + 3*A^2*B
*log(e) + A^3 + 3*(B^3*n^2*log(e) + A*B^2*n^2)*log(b*x + a)^2 + 3*(B^3*n*l
og(e)^2 + 2*A*B^2*n*log(e) + A^2*B*n)*log(b*x + a))*log(d*x + c))/(b*c - a
*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B \log (e(a + bx)^n (c + dx)^{-n}))^3}{(a + bx)(c + dx)}\, dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(43) = 86$.

Time = 0.10 (sec) , antiderivative size = 766, normalized size of antiderivative = 17.02

$$\int \frac{(A + B \log (e(a + bx)^n (c + dx)^{-n}))^3}{(a + bx)(c + dx)}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output

```
B^3*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log((b*x + a)^n*
e/(d*x + c)^n)^3 + 3*A*B^2*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c -
 a*d))*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*A^2*B*(log(b*x + a)/(b*c - a*d
) - log(d*x + c)/(b*c - a*d))*log((b*x + a)^n*e/(d*x + c)^n) - 1/4*B^3*(6*
(e*n*log(b*x + a)^2 - 2*e*n*log(b*x + a)*log(d*x + c) + e*n*log(d*x + c)^2
)*log((b*x + a)^n*e/(d*x + c)^n)^2/((b*c - a*d)*e) - (4*(e^2*n^2*log(b*x +
 a)^3 - 3*e^2*n^2*log(b*x + a)^2*log(d*x + c) + 3*e^2*n^2*log(b*x + a)*log
(d*x + c)^2 - e^2*n^2*log(d*x + c)^3)*log((b*x + a)^n*e/(d*x + c)^n)/((b*c
 - a*d)*e) - (e^3*n^3*log(b*x + a)^4 - 4*e^3*n^3*log(b*x + a)^3*log(d*x +
c) + 6*e^3*n^3*log(b*x + a)^2*log(d*x + c)^2 - 4*e^3*n^3*log(b*x + a)*log(
d*x + c)^3 + e^3*n^3*log(d*x + c)^4)/((b*c - a*d)*e^2))/e) + A^3*(log(b*x
+ a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)) - A*B^2*(3*(e*n*log(b*x + a)^
2 - 2*e*n*log(b*x + a)*log(d*x + c) + e*n*log(d*x + c)^2)*log((b*x + a)^n*
e/(d*x + c)^n)/((b*c - a*d)*e) - (e^2*n^2*log(b*x + a)^3 - 3*e^2*n^2*log(b
*x + a)^2*log(d*x + c) + 3*e^2*n^2*log(b*x + a)*log(d*x + c)^2 - e^2*n^2*l
og(d*x + c)^3)/((b*c - a*d)*e^2)) - 3/2*(e*n*log(b*x + a)^2 - 2*e*n*log(b*
x + a)*log(d*x + c) + e*n*log(d*x + c)^2)*A^2*B/((b*c - a*d)*e)
```

Giac [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)(c+dx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{(bx+a)(dx+c)}\,dx$$

input

```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)/(d*x+c),x, algorith
m="giac")
```

output

```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/((b*x + a)*(d*x + c)),
x)
```

Mupad [B] (verification not implemented)

Time = 28.67 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.13

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)(c+dx)}\,dx$$
$$= -\frac{\frac{3A^2B\ln\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)^2}{2} + AB^2\ln\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)^3 + \frac{B^3\ln\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)^4}{4}}{n\,(ad-bc)}$$
$$+ \frac{A^3\,\mathrm{atan}\left(\frac{ad\,1i+bc\,1i+bdx\,2i}{ad-bc}\right)\,2i}{ad-bc}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/((a + b*x)*(c + d*x)),x)
```

output
```
(A^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(a*d - b*c) - ((B^
3*log((e*(a + b*x)^n)/(c + d*x)^n)^4)/4 + (3*A^2*B*log((e*(a + b*x)^n)/(c
 + d*x)^n)^2)/2 + A*B^2*log((e*(a + b*x)^n)/(c + d*x)^n)^3)/(n*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.67

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)(c+dx)}\,dx$$
$$= \frac{-4\log(bx+a)\,a^3n + 4\log(dx+c)\,a^3n - \log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)^4 b^3 - 4\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)^3 a\,b^2 - 6\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)^2 a^2b}{4n\,(ad-bc)}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)/(d*x+c),x)
```

output
```
( - 4*log(a + b*x)*a**3*n + 4*log(c + d*x)*a**3*n - log(((a + b*x)**n*e)/(
c + d*x)**n)**4*b**3 - 4*log(((a + b*x)**n*e)/(c + d*x)**n)**3*a*b**2 - 6*
log(((a + b*x)**n*e)/(c + d*x)**n)**2*a**2*b)/(4*n*(a*d - b*c))
```

3.229 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)(c+dx)}\,dx$

Optimal result . . . 2527
Mathematica [A] (verified) . . . 2527
Rubi [A] (warning: unable to verify) . . . 2528
Maple [A] (verified) . . . 2529
Fricas [B] (verification not implemented) . . . 2530
Sympy [F(-1)] . . . 2530
Maxima [B] (verification not implemented) . . . 2531
Giac [F] . . . 2532
Mupad [B] (verification not implemented) . . . 2532
Reduce [B] (verification not implemented) . . . 2533

Optimal result

Integrand size = 40, antiderivative size = 45

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)(c+dx)}\,dx = \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3B(bc-ad)n}$$

output
```
1/3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/B/(-a*d+b*c)/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)(c+dx)}\,dx = \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3(bBcn-aBdn)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/((a + b*x)*(c + d*x))
,x]
```

output
```
(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(3*(b*B*c*n - a*B*d*n))
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2973, 2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B \log(e(a+bx)^n(c+dx)^{-n}) + A)^2}{(a+bx)(c+dx)}\,dx$$

$$\downarrow 2973$$

$$\int \frac{(B \log(e(a+bx)^n(c+dx)^{-n}) + A)^2}{(a+bx)(c+dx)}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}\,d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2739$$

$$\frac{\int \frac{(a+bx)^2}{(c+dx)^2}\,d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn(bc-ad)}$$

$$\downarrow 15$$

$$\frac{(a+bx)^3}{3Bn(c+dx)^3(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/((a + b*x)*(c + d*x)),x]
```

output
```
(a + b*x)^3/(3*B*(b*c - a*d)*n*(c + d*x)^3)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [A] (verified)

Time = 6.62 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

method	result	siz
derivativedivides	$-\frac{\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^3}{3n(da-bc)B}$	44
default	$-\frac{\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^3}{3n(da-bc)B}$	44
parallelrisch	$-\frac{B^2\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^3b^2d^2+3AB\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^2b^2d^2+3A^2\ln\left(e(bx+a)^n(dx+c)^{-n}\right)b^2d^2}{3n\,b^2d^2(da-bc)}$	11
parts	$\frac{A^2\ln(dx+c)}{da-bc}-\frac{A^2\ln(bx+a)}{da-bc}-\frac{B^2\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^3}{3n(da-bc)}-\frac{AB\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^2}{n(da-bc)}$	12
risch	Expression too large to display	11

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)/(d*x+c),x,method=_RETURNVE
RBOSE)
```

output
```
-1/3/n/(a*d-b*c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/B
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(43) = 86$.

Time $= 0.09$ (sec) , antiderivative size $= 188$, normalized size of antiderivative $= 4.18$

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx$$
$$= \frac{B^2n^2\log(bx+a)^3 - B^2n^2\log(dx+c)^3 + 3\left(B^2n\log(e) + ABn\right)\log(bx+a)^2 + 3\left(B^2n^2\log(bx+a) +\right.}{}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)/(d*x+c),x, algorith
m="fricas")
```

output
```
1/3*(B^2*n^2*log(b*x + a)^3 - B^2*n^2*log(d*x + c)^3 + 3*(B^2*n*log(e) + A
*B*n)*log(b*x + a)^2 + 3*(B^2*n^2*log(b*x + a) + B^2*n*log(e) + A*B*n)*log
(d*x + c)^2 + 3*(B^2*log(e)^2 + 2*A*B*log(e) + A^2)*log(b*x + a) - 3*(B^2*
n^2*log(b*x + a)^2 + B^2*log(e)^2 + 2*A*B*log(e) + A^2 + 2*(B^2*n*log(e) +
 A*B*n)*log(b*x + a))*log(d*x + c))/(b*c - a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(43) = 86$.

Time $= 0.06$ (sec) , antiderivative size $= 387$, normalized size of antiderivative $= 8.60$

$$\int \frac{\left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)(c+dx)}\,dx$$
$$= B^2\left(\frac{\log(bx+a)}{bc-ad} - \frac{\log(dx+c)}{bc-ad}\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2$$
$$+ 2AB\left(\frac{\log(bx+a)}{bc-ad} - \frac{\log(dx+c)}{bc-ad}\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)$$
$$+ A^2\left(\frac{\log(bx+a)}{bc-ad} - \frac{\log(dx+c)}{bc-ad}\right)$$
$$-\frac{1}{3}B^2\left(\frac{3\left(en\log(bx+a)^2 - 2en\log(bx+a)\log(dx+c) + en\log(dx+c)^2\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{(bc-ad)e} - \frac{e^2n^2\log}{}\right.$$
$$\left. - \frac{\left(en\log(bx+a)^2 - 2en\log(bx+a)\log(dx+c) + en\log(dx+c)^2\right)AB}{(bc-ad)e}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
B^2*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log((b*x + a)^n*e/(d*x + c)^n) + A^2*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)) - 1/3*B^2*(3*(e*n*log(b*x + a)^2 - 2*e*n*log(b*x + a)*log(d*x + c) + e*n*log(d*x + c)^2)*log((b*x + a)^n*e/(d*x + c)^n)/((b*c - a*d)*e) - (e^2*n^2*log(b*x + a)^3 - 3*e^2*n^2*log(b*x + a)^2*log(d*x + c) + 3*e^2*n^2*log(b*x + a)*log(d*x + c)^2 - e^2*n^2*log(d*x + c)^3)/((b*c - a*d)*e^2)) - (e*n*log(b*x + a)^2 - 2*e*n*log(b*x + a)*log(d*x + c) + e*n*log(d*x + c)^2)*A*B/((b*c - a*d)*e)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(bx+a)(dx+c)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/((b*x + a)*(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 27.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.22

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx$$
$$= -\frac{-6\mathrm{i}\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,A^2+3\,A\,B\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2+B^2\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^3}{3\,n\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/((a + b*x)*(c + d*x)),x)
```

output
```
-(B^2*log((e*(a + b*x)^n)/(c + d*x)^n)^3 + 3*A*B*log((e*(a + b*x)^n)/(c + d*x)^n)^2 - A^2*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*6i)/(3*n*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.02

$$\int \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^2}{(a+bx)(c+dx)}\,dx$$
$$= \frac{-3\log(bx+a)\,a^2n+3\log(dx+c)\,a^2n-\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)^3b^2-3\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)^2ab}{3n\,(ad-bc)}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)/(d*x+c),x)
```

output
```
( - 3*log(a + b*x)*a**2*n + 3*log(c + d*x)*a**2*n - log(((a + b*x)**n*e)/(
c + d*x)**n)**3*b**2 - 3*log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b)/(3*n*(
a*d - b*c))
```

3.230 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx$

Optimal result 2534
Mathematica [A] (verified) 2534
Rubi [A] (warning: unable to verify) 2535
Maple [A] (verified) 2536
Fricas [A] (verification not implemented) 2537
Sympy [F(-1)] 2537
Maxima [B] (verification not implemented) 2537
Giac [F] 2538
Mupad [B] (verification not implemented) 2539
Reduce [B] (verification not implemented) 2539

Optimal result

Integrand size = 38, antiderivative size = 45

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx = \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2B(bc-ad)n}$$

output
```
1/2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/B/(-a*d+b*c)/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx = \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bBcn-aBdn)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((a + b*x)*(c + d*x)),x
]
```

output
```
(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(2*(b*B*c*n - a*B*d*n))
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2973, 2961, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(a+bx)(c+dx)}\,dx$$

$$\downarrow 2973$$

$$\int \frac{B \log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(a+bx)(c+dx)}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}\,d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2738$$

$$\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2Bn(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((a + b*x)*(c + d*x)),x]
```

output
```
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(2*B*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$-\frac{\frac{B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^2}{2}+\ln\left(e(bx+a)^n(dx+c)^{-n}\right)A}{n(da-bc)}$	62
default	$-\frac{\frac{B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^2}{2}+\ln\left(e(bx+a)^n(dx+c)^{-n}\right)A}{n(da-bc)}$	62
parts	$\frac{A\ln(dx+c)}{da-bc}-\frac{\ln(bx+a)A}{da-bc}-\frac{B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^2}{2n(da-bc)}$	76
parallelrisch	$-\frac{B\,a^2c^2\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^2+2A\ln\left(e(bx+a)^n(dx+c)^{-n}\right)a^2c^2}{2a^2c^2n(da-bc)}$	80
risch	Expression too large to display	1153

input

```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)/(d*x+c),x,method=_RETURNVERB
OSE)
```

output

```
-1/n/(a*d-b*c)*(1/2*B*ln(e*(b*x+a)^n/((d*x+c)^n))^2+ln(e*(b*x+a)^n/((d*x+c
)^n))*A)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx$$
$$= \frac{Bn\log(bx+a)^2 + Bn\log(dx+c)^2 + 2\left(B\log(e)+A\right)\log(bx+a) - 2\left(Bn\log(bx+a) + B\log(e) + \right.}{2(bc-ad)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)/(d*x+c),x, algorithm=
"fricas")
```

output
```
1/2*(B*n*log(b*x + a)^2 + B*n*log(d*x + c)^2 + 2*(B*log(e) + A)*log(b*x +
a) - 2*(B*n*log(b*x + a) + B*log(e) + A)*log(d*x + c))/(b*c - a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(43) = 86$.

Time = 0.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.36

$$\begin{aligned}
&\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx \\
&= B\left(\frac{\log(bx+a)}{bc-ad} - \frac{\log(dx+c)}{bc-ad}\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) \\
&\quad + A\left(\frac{\log(bx+a)}{bc-ad} - \frac{\log(dx+c)}{bc-ad}\right) \\
&\quad - \frac{\left(en\log(bx+a)^2 - 2\,en\log(bx+a)\log(dx+c) + en\log(dx+c)^2\right)B}{2(bc-ad)e}
\end{aligned}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)/(d*x+c),x, algorithm=
"maxima")
```

output
```
B*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log((b*x + a)^n*e/
(d*x + c)^n) + A*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)) - 1
/2*(e*n*log(b*x + a)^2 - 2*e*n*log(b*x + a)*log(d*x + c) + e*n*log(d*x + c
)^2)*B/((b*c - a*d)*e)
```

Giac [F]

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{(bx+a)(dx+c)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)/(d*x+c),x, algorithm=
"giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/((b*x + a)*(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 26.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx = -\frac{B\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2 - A\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,4\mathrm{i}}{2\,n\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/((a + b*x)*(c + d*x)),x)
```

output
```
-(B*log((e*(a + b*x)^n)/(c + d*x)^n)^2 - A*n*atan((b*c*2i + b*d*x*2i)/(a*d
 - b*c) + 1i)*4i)/(2*n*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx$$
$$= \frac{-2\log(bx+a)\,an + 2\log(dx+c)\,an - \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 b}{2n\,(ad-bc)}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)/(d*x+c),x)
```

output
```
( - 2*log(a + b*x)*a*n + 2*log(c + d*x)*a*n - log(((a + b*x)**n*e)/(c + d*
x)**n)**2*b)/(2*n*(a*d - b*c))
```

3.231 $\int \frac{1}{(a+bx)(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

Optimal result . . . 2540
Mathematica [A] (verified) . . . 2540
Rubi [A] (warning: unable to verify) . . . 2541
Maple [A] (verified) . . . 2542
Fricas [A] (verification not implemented) . . . 2543
Sympy [F(-1)] . . . 2543
Maxima [A] (verification not implemented) . . . 2544
Giac [A] (verification not implemented) . . . 2544
Mupad [B] (verification not implemented) . . . 2545
Reduce [B] (verification not implemented) . . . 2545

Optimal result

Integrand size = 40, antiderivative size = 41

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{B(bc-ad)n}$$

output
```
ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/(-a*d+b*c)/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bBcn-aBdn}$$

input
```
Integrate[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))
,x]
```

output
```
Log[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]]/(b*B*c*n - a*B*d*n)
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2973, 2961, 2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}\,dx$$

$$\downarrow 2973$$

$$\int \frac{1}{(a+bx)(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{\frac{c+dx}{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}}{bc-ad}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2739$$

$$\frac{\int \frac{c+dx}{a+bx}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn(bc-ad)}$$

$$\downarrow 14$$

$$\frac{\log\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{Bn(bc-ad)}$$

input
```
Int[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

output
```
Log[A + B*Log[e*((a + b*x)/(c + d*x))^n]]/(B*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [A] (verified)

Time = 24.73 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)}{n(da-bc)B}$
default	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)}{n(da-bc)B}$
parallelrisch	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)}{n(da-bc)B}$
risch	$-\frac{\ln\left(\ln((bx+a)^{n})+\frac{-iB\pi\ \mathrm{csgn}\left(i(dx+c)^{-n}\right)\mathrm{csgn}\left(i(bx+a)^{n}\right)\mathrm{csgn}\left(i(dx+c)^{-n}(bx+a)^{n}\right)+iB\pi\ \mathrm{csgn}\left(i(dx+c)^{-n}\right)\mathrm{csgn}\left(i(dx+c)\right.}{}\right.}{}$

input
```
int(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVE
RBOSE)
```

output
```
-1/n/(a*d-b*c)*ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-Bn\log\left(bx+a\right)+Bn\log\left(dx+c\right)-B\log\left(e\right)-A\right)}{(Bbc-Bad)n}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorith
m="fricas")
```

output
```
log(-B*n*log(b*x + a) + B*n*log(d*x + c) - B*log(e) - A)/((B*b*c - B*a*d)*
n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-\frac{B\log((bx+a)^n)-B\log((dx+c)^n)+B\log(e)+A}{B}\right)}{(bcn-adn)B}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima")
```

output
```
log(-(B*log((b*x + a)^n) - B*log((d*x + c)^n) + B*log(e) + A)/B)/((b*c*n - a*d*n)*B)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(Bn\log\left(bx+a\right)-Bn\log\left(dx+c\right)+B\log\left(e\right)+A\right)}{Bbcn-Badn}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

output
```
log(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)/(B*b*c*n - B*a*d*n)
```

Mupad [B] (verification not implemented)

Time = 25.79 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = -\frac{\ln\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)}{B\,a\,d\,n-B\,b\,c\,n}$$

input
```
int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)*(c + d*x)),x)
```

output
```
-log(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(B*a*d*n - B*b*c*n)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = -\frac{\log\left(\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b+a\right)}{bn\,(ad-bc)}$$

input
```
int(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output
```
( - log(log(((a + b*x)**n*e)/(c + d*x)**n)*b + a))/(b*n*(a*d - b*c))
```

3.232 $\int \frac{1}{(a+bx)(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}\,dx$

Optimal result . . . 2546
Mathematica [A] (verified) . . . 2546
Rubi [A] (warning: unable to verify) . . . 2547
Maple [A] (verified) . . . 2548
Fricas [A] (verification not implemented) . . . 2549
Sympy [F(-1)] . . . 2549
Maxima [A] (verification not implemented) . . . 2550
Giac [B] (verification not implemented) . . . 2550
Mupad [B] (verification not implemented) . . . 2551
Reduce [B] (verification not implemented) . . . 2551

Optimal result

Integrand size = 40, antiderivative size = 43

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = -\frac{1}{B(bc-ad)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}$$

output
```
-1/B/(-a*d+b*c)/n/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = -\frac{1}{(bBcn-aBdn)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}$$

input
```
Integrate[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^
2),x]
```

output
```
-(1/((b*B*c*n - a*B*d*n)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])))
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2973, 2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}\,dx$$

$$\downarrow 2973$$

$$\int \frac{1}{(a+bx)(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{c+dx}{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2739$$

$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn(bc-ad)}$$

$$\downarrow 15$$

$$-\frac{c+dx}{Bn(a+bx)(bc-ad)}$$

input
```
Int[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2),x]
```

output
```
-((c + d*x)/(B*(b*c - a*d)*n*(a + b*x)))
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [A] (verified)

Time = 72.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{1}{\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)Bn(da-bc)}$
default	$\frac{1}{\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)Bn(da-bc)}$
parallelrisch	$\frac{1}{\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)Bn(da-bc)}$
risch	$\frac{}{Bn(da-bc)\left(2A+2B\ln(e)-2B\ln((dx+c)^n)+2B\ln((bx+a)^n)-iB\pi\ \mathrm{csgn}\left(i(dx+c)^{-n}\right)\mathrm{csgn}(i(bx+a)^n)\mathrm{csgn}\left(i(dx+c)\right.\right.$

input
```
int(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURN
VERBOSE)
```

output
```
1/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/n/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.00

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx =$$
$$-\frac{1}{(B^2bc-B^2ad)n^2\log(bx+a)-(B^2bc-B^2ad)n^2\log(dx+c)+(B^2bc-B^2ad)n\log(e)+(ABbc-A$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algori
thm="fricas")
```

output
```
-1/((B^2*b*c - B^2*a*d)*n^2*log(b*x + a) - (B^2*b*c - B^2*a*d)*n^2*log(d*x
 + c) + (B^2*b*c - B^2*a*d)*n*log(e) + (A*B*b*c - A*B*a*d)*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.88

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = -\frac{1}{(bcn-adn)B^2\log\left((bx+a)^n\right)-(bcn-adn)B^2\log\left((dx+c)^n\right)+(bcn-adn)AB+(bcn\log(e)-ad}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algori
thm="maxima")
```

output
```
-1/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x +
c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(43) = 86.

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.30

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = -\frac{1}{B^2bcn^2\log(bx+a)-B^2adn^2\log(bx+a)-B^2bcn^2\log(dx+c)+B^2adn^2\log(dx+c)+B^2bcn\log(}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algori
thm="giac")
```

output
```
-1/(B^2*b*c*n^2*log(b*x + a) - B^2*a*d*n^2*log(b*x + a) - B^2*b*c*n^2*log(
d*x + c) + B^2*a*d*n^2*log(d*x + c) + B^2*b*c*n*log(e) - B^2*a*d*n*log(e)
+ A*B*b*c*n - A*B*a*d*n)
```

Mupad [B] (verification not implemented)

Time = 25.79 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \frac{1}{B\,n\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)\,(a\,d-b\,c)}$$

input
```
int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)*(c + d*x)),x)
```

output
```
1/(B*n*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.05

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = -\frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{an\left(\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)abd-\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b^2c+a^2d-abc\right)}$$

input
```
int(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

output
```
( - log(((a + b*x)**n*e)/(c + d*x)**n))/(a*n*(log(((a + b*x)**n*e)/(c + d*
x)**n)*a*b*d - log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c + a**2*d - a*b*c)
)
```

3.233 $\int \frac{1}{(a+bx)(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}\,dx$

Optimal result . . . 2552
Mathematica [A] (verified) . . . 2552
Rubi [A] (warning: unable to verify) . . . 2553
Maple [A] (verified) . . . 2554
Fricas [B] (verification not implemented) . . . 2555
Sympy [F(-1)] . . . 2555
Maxima [B] (verification not implemented) . . . 2556
Giac [B] (verification not implemented) . . . 2556
Mupad [B] (verification not implemented) . . . 2557
Reduce [B] (verification not implemented) . . . 2557

Optimal result

Integrand size = 40, antiderivative size = 45

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx = -\frac{1}{2B(bc-ad)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}$$

output
```
-1/2/B/(-a*d+b*c)/n/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx = -\frac{1}{2(bBcn-aBdn)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}$$

input
```
Integrate[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^
3),x]
```

output
```
-1/2*1/((b*B*c*n - a*B*d*n)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2973, 2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}\,dx$$

$$\downarrow 2973$$

$$\int \frac{1}{(a+bx)(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{\frac{c+dx}{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}d\frac{a+bx}{c+dx}}{bc-ad}}{}$$

$$\downarrow 2739$$

$$\frac{\int \frac{(c+dx)^3}{(a+bx)^3}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn(bc-ad)}$$

$$\downarrow 15$$

$$-\frac{(c+dx)^2}{2Bn(a+bx)^2(bc-ad)}$$

input
```
Int[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3),x]
```

output
```
-1/2*(c + d*x)^2/(B*(b*c - a*d)*n*(a + b*x)^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [A] (verified)

Time = 181.95 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{1}{2\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^2 Bn(da-bc)}$
default	$\frac{1}{2\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^2 Bn(da-bc)}$
parallelrisch	$\frac{1}{2\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^2 Bn(da-bc)}$
risch	$\frac{}{Bn(da-bc)\left(2A+2B\ln(e)-2B\ln((dx+c)^n)+2B\ln((bx+a)^n)-iB\pi\ \mathrm{csgn}\left(i(dx+c)^{-n}\right)\mathrm{csgn}(i(bx+a)^n)\mathrm{csgn}\left(i(dx+c\right.\right.$

input
```
int(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x,method=_RETURN
VERBOSE)
```

output
```
1/2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/B/n/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(43) = 86$.

Time $= 0.08$ (sec) , antiderivative size $= 238$, normalized size of antiderivative $= 5.29$

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx =$$

$$-\frac{}{2\left((B^3bc-B^3ad)n^3\log(bx+a)^2+(B^3bc-B^3ad)n^3\log(dx+c)^2+(B^3bc-B^3ad)n\log(e)^2+2(A$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algori
thm="fricas")
```

output
```
-1/2/((B^3*b*c - B^3*a*d)*n^3*log(b*x + a)^2 + (B^3*b*c - B^3*a*d)*n^3*log
(d*x + c)^2 + (B^3*b*c - B^3*a*d)*n*log(e)^2 + 2*(A*B^2*b*c - A*B^2*a*d)*n
*log(e) + (A^2*B*b*c - A^2*B*a*d)*n + 2*((B^3*b*c - B^3*a*d)*n^2*log(e) +
(A*B^2*b*c - A*B^2*a*d)*n^2)*log(b*x + a) - 2*((B^3*b*c - B^3*a*d)*n^3*log
(b*x + a) + (B^3*b*c - B^3*a*d)*n^2*log(e) + (A*B^2*b*c - A*B^2*a*d)*n^2)*
log(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(43) = 86$.

Time = 0.23 (sec) , antiderivative size = 220, normalized size of antiderivative = 4.89

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx =$$

$$-\frac{}{2\left((bcn-adn)B^3\log\left((bx+a)^n\right)^2+(bcn-adn)B^3\log\left((dx+c)^n\right)^2+(bcn-adn)A^2B+2\,(bcn\log($$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algori
thm="maxima")
```

output
```
-1/2/((b*c*n - a*d*n)*B^3*log((b*x + a)^n)^2 + (b*c*n - a*d*n)*B^3*log((d*
x + c)^n)^2 + (b*c*n - a*d*n)*A^2*B + 2*(b*c*n*log(e) - a*d*n*log(e))*A*B^
2 + (b*c*n*log(e)^2 - a*d*n*log(e)^2)*B^3 + 2*((b*c*n - a*d*n)*A*B^2 + (b*
c*n*log(e) - a*d*n*log(e))*B^3)*log((b*x + a)^n) - 2*((b*c*n - a*d*n)*B^3*
log((b*x + a)^n) + (b*c*n - a*d*n)*A*B^2 + (b*c*n*log(e) - a*d*n*log(e))*B
^3)*log((d*x + c)^n))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(43) = 86$.

Time = 0.13 (sec) , antiderivative size = 321, normalized size of antiderivative = 7.13

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx =$$

$$-\frac{}{2\left(B^3bcn^3\log\left(bx+a\right)^2-B^3adn^3\log\left(bx+a\right)^2-2\,B^3bcn^3\log\left(bx+a\right)\log\left(dx+c\right)+2\,B^3adn^3\log\left(b\right.\right.}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algori
thm="giac")
```

output
```
-1/2/(B^3*b*c*n^3*log(b*x + a)^2 - B^3*a*d*n^3*log(b*x + a)^2 - 2*B^3*b*c*
n^3*log(b*x + a)*log(d*x + c) + 2*B^3*a*d*n^3*log(b*x + a)*log(d*x + c) +
B^3*b*c*n^3*log(d*x + c)^2 - B^3*a*d*n^3*log(d*x + c)^2 + 2*B^3*b*c*n^2*lo
g(b*x + a)*log(e) - 2*B^3*a*d*n^2*log(b*x + a)*log(e) - 2*B^3*b*c*n^2*log(
d*x + c)*log(e) + 2*B^3*a*d*n^2*log(d*x + c)*log(e) + 2*A*B^2*b*c*n^2*log(
b*x + a) - 2*A*B^2*a*d*n^2*log(b*x + a) - 2*A*B^2*b*c*n^2*log(d*x + c) + 2
*A*B^2*a*d*n^2*log(d*x + c) + B^3*b*c*n*log(e)^2 - B^3*a*d*n*log(e)^2 + 2*
A*B^2*b*c*n*log(e) - 2*A*B^2*a*d*n*log(e) + A^2*B*b*c*n - A^2*B*a*d*n)
```

Mupad [B] (verification not implemented)

Time = 25.73 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx = \frac{1}{2\,B\,n\,(a\,d-b\,c)\left(A^2+2\,A\,B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)+B^2\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2\right)}$$

input
```
int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)*(c + d*x)),x)
```

output
```
1/(2*B*n*(a*d - b*c)*(B^2*log((e*(a + b*x)^n)/(c + d*x)^n)^2 + A^2 + 2*A*B
*log((e*(a + b*x)^n)/(c + d*x)^n)))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.87

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx = \frac{1}{2bn\left(\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 a\,b^2d-\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 b^3c+2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a^2bd-2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)a\,b^2c+a^3d-a\right.}$$

input
```
int(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output

```
1/(2*b*n*(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*d - log(((a + b*x)*
**n*e)/(c + d*x)**n)**2*b**3*c + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*
b*d - 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c + a**3*d - a**2*b*c))
```

3.234 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p}{(a+bx)(c+dx)}\,dx$

Optimal result . . . 2559
Mathematica [A] (verified) . . . 2559
Rubi [A] (warning: unable to verify) . . . 2560
Maple [A] (verified) . . . 2561
Fricas [A] (verification not implemented) . . . 2562
Sympy [F(-1)] . . . 2562
Maxima [F] . . . 2563
Giac [A] (verification not implemented) . . . 2563
Mupad [F(-1)] . . . 2564
Reduce [B] (verification not implemented) . . . 2564

Optimal result

Integrand size = 40, antiderivative size = 49

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx = \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^{1+p}}{B(bc-ad)n(1+p)}$$

output
```
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^(p+1)/B/(-a*d+b*c)/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx = \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^{1+p}}{(bBcn-aBdn)(1+p)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/((a + b*x)*(c + d*x))
,x]
```

output
```
(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^(1 + p)/((b*B*c*n - a*B*d*n)*(1 +
 p))
```

Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2973, 2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B \log (e(a+bx)^n (c+dx)^{-n}) + A)^p}{(a+bx)(c+dx)}\, dx$$

$$\downarrow 2973$$

$$\int \frac{(B \log (e(a+bx)^n (c+dx)^{-n}) + A)^p}{(a+bx)(c+dx)} dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p}{a+bx} d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2739$$

$$\frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn(bc-ad)}$$

$$\downarrow 15$$

$$\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^{p+1}}{Bn(p+1)(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/((a + b*x)*(c + d*x)),x]
```

output
```
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(1 + p)/(B*(b*c - a*d)*n*(1 + p))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [A] (verified)

Time = 272.59 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

method	result	s
derivativedivides	$-\frac{\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^{p+1}}{n(da-bc)B(p+1)}$	5
default	$-\frac{\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^{p+1}}{n(da-bc)B(p+1)}$	5
parallelrisch	$-\frac{B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^p a^2c^2+A\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^p a^2c^2}{(adp-bcp+da-bc)B\,a^2c^2n}$	1

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*x+a)/(d*x+c),x,method=_RETURNVE
RBOSE)
```

output
```
-1/n/(a*d-b*c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^(p+1)/B/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.65

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx$$
$$= \frac{(Bn\log(bx+a) - Bn\log(dx+c) + B\log(e) + A)(Bn\log(bx+a) - Bn\log(dx+c) + B\log(e) + A}{(Bbc - Bad)np + (Bbc - Bad)n}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)*(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)^p/((B*b*c - B*a*d)*n*p + (B*b*c - B*a*d)*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**p/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^p}{(bx+a)(dx+c)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^p/((b*x + a)*(d*x + c)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx = \frac{(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)^{p+1}}{(Bbcn-Badn)(p+1)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)^(p + 1)/((B*b*c*n - B*a*d*n)*(p + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^p}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/((a + b*x)*(c + d*x)),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/((a + b*x)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx = -\frac{\left(\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b+a\right)^p\left(\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b+a\right)}{bn\,(adp-bcp+ad-bc)}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*x+a)/(d*x+c),x)
```

output
```
( - (log(((a + b*x)**n*e)/(c + d*x)**n)*b + a)**p*(log(((a + b*x)**n*e)/(c
 + d*x)**n)*b + a))/(b*n*(a*d*p + a*d - b*c*p - b*c))
```

3.235 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p}{(af+bfx)(cg+dgx)}\,dx$

Optimal result . . . 2565
Mathematica [A] (verified) . . . 2565
Rubi [A] (warning: unable to verify) . . . 2566
Maple [A] (verified) . . . 2567
Fricas [A] (verification not implemented) . . . 2568
Sympy [F(-1)] . . . 2568
Maxima [F] . . . 2569
Giac [A] (verification not implemented) . . . 2569
Mupad [F(-1)] . . . 2570
Reduce [B] (verification not implemented) . . . 2570

Optimal result

Integrand size = 46, antiderivative size = 55

$$\int \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^p}{(af+bfx)(cg+dgx)}\,dx = \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^{1+p}}{B(bc-ad)fgn(1+p)}$$

output
```
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^(p+1)/B/(-a*d+b*c)/f/g/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^p}{(af+bfx)(cg+dgx)}\,dx = \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^{1+p}}{(bBcfgn-aBdfgn)(1+p)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/((a*f + b*f*x)*(c*g +
 d*g*x)),x]
```

output
```
(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^(1 + p)/((b*B*c*f*g*n - a*B*d*f*g
*n)*(1 + p))
```

Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2973, 2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B \log (e(a+bx)^n (c+dx)^{-n}) + A)^p}{(af+bfx)(cg+dgx)}\,dx$$

$$\downarrow 2973$$

$$\int \frac{(B \log (e(a+bx)^n (c+dx)^{-n}) + A)^p}{(af+bfx)(cg+dgx)}dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p}{a+bx} d\frac{a+bx}{c+dx}}{fg(bc-ad)}$$

$$\downarrow 2739$$

$$\frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bfgn(bc-ad)}$$

$$\downarrow 15$$

$$\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^{p+1}}{Bfgn(p+1)(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/((a*f + b*f*x)*(c*g + d*g*x
)),x]
```

output
```
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(1 + p)/(B*(b*c - a*d)*f*g*n*(1 + p
))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$-\frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{p+1}}{fgn\left(da-bc\right)B\left(p+1\right)}$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*f*x+a*f)/(d*g*x+c*g),x)
```

output
```
-1/f/g/n/(a*d-b*c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^(p+1)/B/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(af+bfx)(cg+dgx)}\,dx$$
$$= \frac{(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)}{(Bbc-Bad)fgnp+(Bbc-Bad)fgn}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*f*x+a*f)/(d*g*x+c*g),x,
algorithm="fricas")
```

output
```
(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)*(B*n*log(b*x + a) - B
*n*log(d*x + c) + B*log(e) + A)^p/((B*b*c - B*a*d)*f*g*n*p + (B*b*c - B*a*
d)*f*g*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(af+bfx)(cg+dgx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**p/(b*f*x+a*f)/(d*g*x+c*g),x
)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(af+bfx)(cg+dgx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^p}{(bfx+af)(dgx+cg)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*f*x+a*f)/(d*g*x+c*g),x,
algorithm="maxima")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^p/((b*f*x + a*f)*(d*g*x +
 c*g)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(af+bfx)(cg+dgx)}\,dx = \frac{(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)^{p+1}}{(Bbcfgn-Badfgn)(p+1)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*f*x+a*f)/(d*g*x+c*g),x,
algorithm="giac")
```

output
```
(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)^(p + 1)/((B*b*c*f*g*n
 - B*a*d*f*g*n)*(p + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(af+bfx)(cg+dgx)}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^p}{(a\,f+b\,f\,x)\,(c\,g+d\,g\,x)}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/((a*f + b*f*x)*(c*g + d*g*x
)),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/((a*f + b*f*x)*(c*g + d*g*x
)), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.47

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(af+bfx)(cg+dgx)}\,dx = -\frac{\left(\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b+a\right)^p\left(\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b+a\right)}{bfgn\,(adp-bcp+ad-bc)}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*f*x+a*f)/(d*g*x+c*g),x)
```

output
```
( - (log(((a + b*x)**n*e)/(c + d*x)**n)*b + a)**p*(log(((a + b*x)**n*e)/(c
 + d*x)**n)*b + a))/(b*f*g*n*(a*d*p + a*d - b*c*p - b*c))
```

3.236 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p}{acf+(bc+ad)fx+bdfx^2}\,dx$

Optimal result . . . 2571
Mathematica [A] (verified) . . . 2571
Rubi [A] (warning: unable to verify) . . . 2572
Maple [F] . . . 2573
Fricas [A] (verification not implemented) . . . 2574
Sympy [F(-1)] . . . 2574
Maxima [F] . . . 2575
Giac [A] (verification not implemented) . . . 2575
Mupad [F(-1)] . . . 2576
Reduce [B] (verification not implemented) . . . 2576

Optimal result

Integrand size = 50, antiderivative size = 52

$$\int \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^p}{acf+(bc+ad)fx+bdfx^2}\,dx = \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^{1+p}}{B(bc-ad)fn(1+p)}$$

output
```
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^(p+1)/B/(-a*d+b*c)/f/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^p}{acf+(bc+ad)fx+bdfx^2}\,dx = \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^{1+p}}{f(bBcn-aBdn)(1+p)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/(a*c*f + (b*c + a*d)*
f*x + b*d*f*x^2),x]
```

output
```
(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^(1 + p)/(f*(b*B*c*n - a*B*d*n)*(1
 + p))
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2973, 2974, 2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B \log(e(a+bx)^n(c+dx)^{-n}) + A)^p}{fx(ad+bc) + acf + bdfx^2}\,dx$$

$$\downarrow 2973$$

$$\int \frac{(B \log(e(a+bx)^n(c+dx)^{-n}) + A)^p}{fx(ad+bc) + acf + bdfx^2}dx$$

$$\downarrow 2974$$

$$\frac{\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}dx}{f}$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p}{a+bx}d\frac{a+bx}{c+dx}}{f(bc-ad)}$$

$$\downarrow 2739$$

$$\frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bfn(bc-ad)}$$

$$\downarrow 15$$

$$\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^{p+1}}{Bfn(p+1)(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/(a*c*f + (b*c + a*d)*f*x + b*d*f*x^2),x]
```

output
```
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(1 + p)/(B*(b*c - a*d)*f*n*(1 + p))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 2974
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_) + (h_.)*(x_)^2)^(m_.), x_Symbol] :> Simp[h^
m/(b^m*d^m)   Int[(a + b*x)^m*(c + d*x)^m*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, n, p}, x] && EqQ[b*
d*f - a*c*h, 0] && EqQ[b*d*g - h*(b*c + a*d), 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\left(A + B \ln\left(e(bx+a)^n (dx+c)^{-n}\right)\right)^p}{acf + (da+bc) fx + bdf\, x^2} dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2),x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.60

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf + (bc+ad)fx + bdfx^2}\,dx$$
$$= \frac{(Bn\log(bx+a) - Bn\log(dx+c) + B\log(e) + A)(Bn\log(bx+a) - Bn\log(dx+c) + B\log(e) + A)}{(Bbc - Bad)fnp + (Bbc - Bad)fn}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(a*c*f+(a*d+b*c)*f*x+b*d*f*
x^2),x, algorithm="fricas")
```

output
```
(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)*(B*n*log(b*x + a) - B
*n*log(d*x + c) + B*log(e) + A)^p/((B*b*c - B*a*d)*f*n*p + (B*b*c - B*a*d)
*f*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf + (bc+ad)fx + bdfx^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**p/(a*c*f+(a*d+b*c)*f*x+b*d*
f*x**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^p}{bdfx^2+acf+(bc+ad)fx}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(a*c*f+(a*d+b*c)*f*x+b*d*f*
x^2),x, algorithm="maxima")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^p/(b*d*f*x^2 + a*c*f + (b
*c + a*d)*f*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx = \frac{(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)^{p+1}}{(Bbcfn-Badfn)(p+1)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(a*c*f+(a*d+b*c)*f*x+b*d*f*
x^2),x, algorithm="giac")
```

output
```
(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)^(p + 1)/((B*b*c*f*n -
 B*a*d*f*n)*(p + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^p}{b\,d\,f\,x^2+f\,\left(a\,d+b\,c\right)\,x+a\,c\,f}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/(a*c*f + f*x*(a*d + b*c) +
b*d*f*x^2),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/(a*c*f + f*x*(a*d + b*c) +
b*d*f*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.50

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx = -\frac{\left(\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b+a\right)^p\left(\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b+a\right)}{bfn\,(adp-bcp+ad-bc)}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2),x
)
```

output
```
( - (log(((a + b*x)**n*e)/(c + d*x)**n)*b + a)**p*(log(((a + b*x)**n*e)/(c
 + d*x)**n)*b + a))/(b*f*n*(a*d*p + a*d - b*c*p - b*c))
```

3.237 $\int \frac{1}{(a+bx)(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

Optimal result . . . 2577
Mathematica [A] (verified) . . . 2577
Rubi [A] (warning: unable to verify) . . . 2578
Maple [A] (verified) . . . 2579
Fricas [A] (verification not implemented) . . . 2580
Sympy [F(-1)] . . . 2580
Maxima [A] (verification not implemented) . . . 2581
Giac [A] (verification not implemented) . . . 2581
Mupad [B] (verification not implemented) . . . 2582
Reduce [B] (verification not implemented) . . . 2582

Optimal result

Integrand size = 40, antiderivative size = 41

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{B(bc-ad)n}$$

output
```
ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/(-a*d+b*c)/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bBcn-aBdn}$$

input
```
Integrate[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))
,x]
```

output
```
Log[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]]/(b*B*c*n - a*B*d*n)
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2973, 2961, 2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}\,dx$$

$$\downarrow 2973$$

$$\int \frac{1}{(a+bx)(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{\frac{c+dx}{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}}{bc-ad}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2739$$

$$\frac{\int \frac{c+dx}{a+bx}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn(bc-ad)}$$

$$\downarrow 14$$

$$\frac{\log\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{Bn(bc-ad)}$$

input
```
Int[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

output
```
Log[A + B*Log[e*((a + b*x)/(c + d*x))^n]]/(B*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [A] (verified)

Time = 24.60 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)}{n(da-bc)B}$
default	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)}{n(da-bc)B}$
parallelrisch	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)}{n(da-bc)B}$
risch	$-\frac{\ln\left(\ln((bx+a)^{n})+\frac{-iB\pi\ \mathrm{csgn}\left(i(dx+c)^{-n}\right)\mathrm{csgn}\left(i(bx+a)^{n}\right)\mathrm{csgn}\left(i(dx+c)^{-n}(bx+a)^{n}\right)+iB\pi\ \mathrm{csgn}\left(i(dx+c)^{-n}\right)\mathrm{csgn}\left(i(dx+c)\right.}{}\right.}{}$

input
```
int(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVE
RBOSE)
```

output
```
-1/n/(a*d-b*c)*ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-Bn\log\left(bx+a\right)+Bn\log\left(dx+c\right)-B\log\left(e\right)-A\right)}{(Bbc-Bad)n}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorith
m="fricas")
```

output
```
log(-B*n*log(b*x + a) + B*n*log(d*x + c) - B*log(e) - A)/((B*b*c - B*a*d)*
n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-\frac{B\log((bx+a)^n)-B\log((dx+c)^n)+B\log(e)+A}{B}\right)}{(bcn-adn)B}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima")
```

output
```
log(-(B*log((b*x + a)^n) - B*log((d*x + c)^n) + B*log(e) + A)/B)/((b*c*n - a*d*n)*B)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(Bn\log\left(bx+a\right)-Bn\log\left(dx+c\right)+B\log\left(e\right)+A\right)}{Bbcn-Badn}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

output
```
log(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)/(B*b*c*n - B*a*d*n)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = -\frac{\ln\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)}{B\,a\,d\,n-B\,b\,c\,n}$$

input
```
int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)*(c + d*x)),x)
```

output
```
-log(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(B*a*d*n - B*b*c*n)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = -\frac{\log\left(\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b+a\right)}{bn\,(ad-bc)}$$

input
```
int(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output
```
( - log(log(((a + b*x)**n*e)/(c + d*x)**n)*b + a))/(b*n*(a*d - b*c))
```

3.238 $\int \frac{1}{(af+bfx)(cg+dgx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

Optimal result . . . 2583
Mathematica [A] (verified) . . . 2583
Rubi [A] (warning: unable to verify) . . . 2584
Maple [A] (verified) . . . 2585
Fricas [A] (verification not implemented) . . . 2586
Sympy [F(-1)] . . . 2586
Maxima [A] (verification not implemented) . . . 2587
Giac [A] (verification not implemented) . . . 2587
Mupad [B] (verification not implemented) . . . 2588
Reduce [B] (verification not implemented) . . . 2588

Optimal result

Integrand size = 46, antiderivative size = 47

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{B(bc-ad)fgn}$$

output
```
ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/(-a*d+b*c)/f/g/n
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bBcfgn-aBdfgn}$$

input
```
Integrate[1/((a*f + b*f*x)*(c*g + d*g*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d
*x)^n])),x]
```

output
```
Log[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]]/(b*B*c*f*g*n - a*B*d*f*g*n)
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2973, 2961, 2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

$$\downarrow 2973$$

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{\frac{c+dx}{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}}{fg(bc-ad)}d\frac{a+bx}{c+dx}}{fg(bc-ad)}$$

$$\downarrow 2739$$

$$\frac{\int \frac{c+dx}{a+bx}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bfgn(bc-ad)}$$

$$\downarrow 14$$

$$\frac{\log\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{Bfgn(bc-ad)}$$

input
```
Int[1/((a*f + b*f*x)*(c*g + d*g*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]
)),x]
```

output
```
Log[A + B*Log[e*((a + b*x)/(c + d*x))^n]]/(B*(b*c - a*d)*f*g*n)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [A] (verified)

Time = 77.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

method	result
default	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)}{fgn(da-bc)B}$
parallelrisch	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)}{fgn(da-bc)B}$
risch	$-\frac{\ln\left(\ln((bx+a)^n)+\frac{-iB\pi\ \mathrm{csgn}\left(i(dx+c)^{-n}\right)\mathrm{csgn}\left(i(bx+a)^n\right)\mathrm{csgn}\left(i(dx+c)^{-n}(bx+a)^n\right)+iB\pi\ \mathrm{csgn}\left(i(dx+c)^{-n}\right)\mathrm{csgn}\left(i(dx+c)^{-n}(bx+$

input
```
int(1/(b*f*x+a*f)/(d*g*x+c*g)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_
RETURNVERBOSE)
```

output
```
-1/f/g/n/(a*d-b*c)*ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-Bn\log\left(bx+a\right)+Bn\log\left(dx+c\right)-B\log\left(e\right)-A\right)}{(Bbc-Bad)fgn}$$

input
```
integrate(1/(b*f*x+a*f)/(d*g*x+c*g)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas")
```

output
```
log(-B*n*log(b*x + a) + B*n*log(d*x + c) - B*log(e) - A)/((B*b*c - B*a*d)*f*g*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*f*x+a*f)/(d*g*x+c*g)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-\frac{B\log((bx+a)^n)-B\log((dx+c)^n)+B\log(e)+A}{B}\right)}{(bcfgn-adfgn)B}$$

input
```
integrate(1/(b*f*x+a*f)/(d*g*x+c*g)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x,
algorithm="maxima")
```

output
```
log(-(B*log((b*x + a)^n) - B*log((d*x + c)^n) + B*log(e) + A)/B)/((b*c*f*g
*n - a*d*f*g*n)*B)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(Bn\log\left(bx+a\right)-Bn\log\left(dx+c\right)+B\log\left(e\right)+A\right)}{Bbcfgn-Badfgn}$$

input
```
integrate(1/(b*f*x+a*f)/(d*g*x+c*g)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x,
algorithm="giac")
```

output
```
log(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)/(B*b*c*f*g*n - B*a
*d*f*g*n)
```

Mupad [B] (verification not implemented)

Time = 26.41 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = -\frac{\ln\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)}{B\,a\,d\,f\,g\,n-B\,b\,c\,f\,g\,n}$$

input
```
int(1/((a*f + b*f*x)*(c*g + d*g*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)
)),x)
```

output
```
-log(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(B*a*d*f*g*n - B*b*c*f*g*n)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = -\frac{\log\left(\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b+a\right)}{bfgn\,(ad-bc)}$$

input
```
int(1/(b*f*x+a*f)/(d*g*x+c*g)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output
```
( - log(log(((a + b*x)**n*e)/(c + d*x)**n)*b + a))/(b*f*g*n*(a*d - b*c))
```

3.239 $\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

Optimal result . . . 2589
Mathematica [A] (verified) . . . 2589
Rubi [A] (warning: unable to verify) . . . 2590
Maple [A] (verified) . . . 2592
Fricas [A] (verification not implemented) . . . 2592
Sympy [F(-1)] . . . 2593
Maxima [A] (verification not implemented) . . . 2593
Giac [A] (verification not implemented) . . . 2593
Mupad [B] (verification not implemented) . . . 2594
Reduce [B] (verification not implemented) . . . 2594

Optimal result

Integrand size = 50, antiderivative size = 44

$$\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{B(bc-ad)fn}$$

output
```
ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/(-a*d+b*c)/f/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{f(bBcn-aBdn)}$$

input
```
Integrate[1/((a*c*f + (b*c + a*d)*f*x + b*d*f*x^2)*(A + B*Log[(e*(a + b*x)
^n)/(c + d*x)^n])),x]
```

output
```
Log[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]]/(f*(b*B*c*n - a*B*d*n))
```

Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2973, 2974, 2961, 2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(fx(ad+bc)+acf+bdfx^2)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

$$\downarrow 2973$$

$$\int \frac{1}{(fx(ad+bc)+acf+bdfx^2)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

$$\downarrow 2974$$

$$\frac{\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}dx}{f}$$

$$\downarrow 2961$$

$$\frac{\int \frac{c+dx}{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}d\frac{a+bx}{c+dx}}{f(bc-ad)}$$

$$\downarrow 2739$$

$$\frac{\int \frac{c+dx}{a+bx}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bfn(bc-ad)}$$

$$\downarrow 14$$

$$\frac{\log\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{Bfn(bc-ad)}$$

input
```
Int[1/((a*c*f + (b*c + a*d)*f*x + b*d*f*x^2)*(A + B*Log[(e*(a + b*x)^n)/(c
 + d*x)^n])),x]
```

output
```
Log[A + B*Log[e*((a + b*x)/(c + d*x))^n]]/(B*(b*c - a*d)*f*n)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 2974
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_) + (h_.)*(x_)^2)^(m_.), x_Symbol] :> Simp[h^
m/(b^m*d^m)   Int[(a + b*x)^m*(c + d*x)^m*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, n, p}, x] && EqQ[b*
d*f - a*c*h, 0] && EqQ[b*d*g - h*(b*c + a*d), 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 62.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

method	result
parallelrisch	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)}{Bfn(da-bc)}$
risch	$-\frac{\ln\left(\ln((bx+a)^n)+\frac{-iB\pi\ \mathrm{csgn}\left(i(dx+c)^{-n}\right)\mathrm{csgn}\left(i(bx+a)^n\right)\mathrm{csgn}\left(i(dx+c)^{-n}(bx+a)^n\right)+iB\pi\ \mathrm{csgn}\left(i(dx+c)^{-n}\right)\mathrm{csgn}\left(i(dx+c)^{-n}(bx+$

input
```
int(1/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,
method=_RETURNVERBOSE)
```

output
```
-ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/f/n/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$
$$=\frac{\log\left(-Bn\log(bx+a)+Bn\log(dx+c)-B\log(e)-A\right)}{(Bbc-Bad)fn}$$

input
```
integrate(1/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2)/(A+B*log(e*(b*x+a)^n/((d*x+c)^
n))),x, algorithm="fricas")
```

output
```
log(-B*n*log(b*x + a) + B*n*log(d*x + c) - B*log(e) - A)/((B*b*c - B*a*d)*
f*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(acf + (bc + ad)fx + bdfx^2)\left(A + B\log\left(e(a + bx)^n(c + dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*c*f+(a*d+b*c)*f*x+b*d*f*x**2)/(A+B*ln(e*(b*x+a)**n/((d*x+c)
**n))),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{1}{(acf + (bc + ad)fx + bdfx^2)\left(A + B\log\left(e(a + bx)^n(c + dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-\frac{B\log((bx+a)^n) - B\log((dx+c)^n) + B\log(e) + A}{B}\right)}{(bcfn - adfn)B}$$

input
```
integrate(1/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2)/(A+B*log(e*(b*x+a)^n/((d*x+c)^
n))),x, algorithm="maxima")
```

output
```
log(-(B*log((b*x + a)^n) - B*log((d*x + c)^n) + B*log(e) + A)/B)/((b*c*f*n
 - a*d*f*n)*B)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{1}{(acf + (bc + ad)fx + bdfx^2)\left(A + B\log\left(e(a + bx)^n(c + dx)^{-n}\right)\right)}\,dx = \frac{\log\left(Bn\log(bx + a) - Bn\log(dx + c) + B\log(e) + A\right)}{Bbcfn - Badfn}$$

input
```
integrate(1/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2)/(A+B*log(e*(b*x+a)^n/((d*x+c)^
n))),x, algorithm="giac")
```

output
```
log(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)/(B*b*c*f*n - B*a*d
*f*n)
```

Mupad [B] (verification not implemented)

Time = 26.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = -\frac{\ln\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)}{B\,a\,d\,f\,n-B\,b\,c\,f\,n}$$

input
```
int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a*c*f + f*x*(a*d + b*c) +
 b*d*f*x^2)),x)
```

output
```
-log(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(B*a*d*f*n - B*b*c*f*n)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = -\frac{\log\left(\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)b+a\right)}{bfn\,(ad-bc)}$$

input
```
int(1/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x
)
```

output
```
( - log(log(((a + b*x)**n*e)/(c + d*x)**n)*b + a))/(b*f*n*(a*d - b*c))
```

3.240 $\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log(e(a+bx)^n(c+dx)^{-n})}\,dx$

Optimal result . . . 2595
Mathematica [F] . . . 2595
Rubi [A] (warning: unable to verify) . . . 2596
Maple [F] . . . 2598
Fricas [F] . . . 2598
Sympy [F(-1)] . . . 2598
Maxima [F] . . . 2599
Giac [F] . . . 2599
Mupad [F(-1)] . . . 2599
Reduce [F] . . . 2600

Optimal result

Integrand size = 40, antiderivative size = 88

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}\,dx = \frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(e(a+bx)^n(c+dx)^{-n}\right)^{-\frac{1+m}{n}}\operatorname{ExpIntegralEi}\left(\frac{(1+m)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{n}\right)}{(bc-ad)n}$$

output

```
(b*x+a)^(1+m)*(d*x+c)^(-1-m)*Ei((1+m)*ln(e*(b*x+a)^n/((d*x+c)^n))/n)/(-a*d
+b*c)/n/((e*(b*x+a)^n/((d*x+c)^n))^((1+m)/n))
```

Mathematica [F]

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}\,dx = \int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}\,dx$$

input

```
Integrate[((a + b*x)^m*(c + d*x)^(-2 - m))/Log[(e*(a + b*x)^n)/(c + d*x)^n
],x]
```

output
```
Integrate[((a + b*x)^m*(c + d*x)^(-2 - m))/Log[(e*(a + b*x)^n)/(c + d*x)^n
], x]
```

Rubi [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2973, 2963, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^m (c+dx)^{-m-2}}{\log\left(e(a+bx)^n (c+dx)^{-n}\right)}\,dx$$

$$\downarrow 2973$$

$$\int \frac{(a+bx)^m (c+dx)^{-m-2}}{\log\left(e(a+bx)^n (c+dx)^{-n}\right)}dx$$

$$\downarrow 2963$$

$$\frac{(a+bx)^m (c+dx)^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\int \frac{\left(\frac{a+bx}{c+dx}\right)^m}{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2747$$

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}(c+dx)}{a+bx}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

$$\downarrow 2609$$

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\text{ExpIntegralEi}\left(\frac{(m+1)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(bc-ad)}$$

input
```
Int[((a + b*x)^m*(c + d*x)^(-2 - m))/Log[(e*(a + b*x)^n)/(c + d*x)^n],x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^(-1 - m)*ExpIntegralEi[((1 + m)*Log[e*((a + b
*x)/(c + d*x))^n])/n])/((b*c - a*d)*n*(e*((a + b*x)/(c + d*x))^n)^((1 + m)
/n))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [F]

$$\int \frac{(bx+a)^m (dx+c)^{-2-m}}{\ln\left(e\,(bx+a)^n (dx+c)^{-n}\right)} dx$$

input
```
int((b*x+a)^m*(d*x+c)^(-2-m)/ln(e*(b*x+a)^n/((d*x+c)^n)),x)
```

output
```
int((b*x+a)^m*(d*x+c)^(-2-m)/ln(e*(b*x+a)^n/((d*x+c)^n)),x)
```

Fricas [F]

$$\int \frac{(a+bx)^m (c+dx)^{-2-m}}{\log\left(e(a+bx)^n (c+dx)^{-n}\right)} dx = \int \frac{(bx+a)^m (dx+c)^{-m-2}}{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)} dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-2-m)/log(e*(b*x+a)^n/((d*x+c)^n)),x, algorit
hm="fricas")
```

output
```
integral((b*x + a)^m*(d*x + c)^(-m - 2)/log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m (c+dx)^{-2-m}}{\log\left(e(a+bx)^n (c+dx)^{-n}\right)} dx = \text{Timed out}$$

input
```
integrate((b*x+a)**m*(d*x+c)**(-2-m)/ln(e*(b*x+a)**n/((d*x+c)**n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}\,dx = \int \frac{(bx+a)^m(dx+c)^{-m-2}}{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-2-m)/log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^m*(d*x + c)^(-m - 2)/log((b*x + a)^n*e/(d*x + c)^n), x)
```

Giac [F]

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}\,dx = \int \frac{(bx+a)^m(dx+c)^{-m-2}}{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-2-m)/log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="giac")
```

output
```
integrate((b*x + a)^m*(d*x + c)^(-m - 2)/log((b*x + a)^n*e/(d*x + c)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}\,dx = \int \frac{(a+b\,x)^m}{\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\,(c+d\,x)^{m+2}}\,dx$$

input
```
int((a + b*x)^m/(log((e*(a + b*x)^n)/(c + d*x)^n)*(c + d*x)^(m + 2)),x)
```

output
```
int((a + b*x)^m/(log((e*(a + b*x)^n)/(c + d*x)^n)*(c + d*x)^(m + 2)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^m (c+dx)^{-2-m}}{\log\left(e(a+bx)^n (c+dx)^{-n}\right)}\,dx$$
$$= \int \frac{(bx+a)^m}{(dx+c)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) c^2 + 2(dx+c)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) cdx + (dx+c)^m \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) d^2x^2}\,dx$$

input
```
int((b*x+a)^m*(d*x+c)^(-2-m)/log(e*(b*x+a)^n/((d*x+c)^n)),x)
```

output
```
int((a + b*x)**m/((c + d*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*c**2 + 2
*(c + d*x)**m*log(((a + b*x)**n*e)/(c + d*x)**n)*c*d*x + (c + d*x)**m*log(
((a + b*x)**n*e)/(c + d*x)**n)*d**2*x**2),x)
```

3.241 $\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result . . . 2601
Mathematica [A] (verified) . . . 2601
Rubi [A] (verified) . . . 2602
Maple [F] . . . 2603
Fricas [A] (verification not implemented) . . . 2604
Sympy [F(-1)] . . . 2604
Maxima [F] . . . 2604
Giac [F] . . . 2605
Mupad [F(-1)] . . . 2605
Reduce [F] . . . 2605

Optimal result

Integrand size = 35, antiderivative size = 75

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{(a+bx)^4 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-4/n} \text{ExpIntegralEi}\left(\frac{4\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^4}$$

output

```
(b*x+a)^4*Ei(4*ln(e*((b*x+a)/(d*x+c))^n)/n)/(-a*d+b*c)/n/((e*((b*x+a)/(d*x
+c))^n)^(4/n))/(d*x+c)^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{(a+bx)^4 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-4/n} \text{ExpIntegralEi}\left(\frac{4\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^4}$$

input
```
Integrate[(a + b*x)^3/((c + d*x)^5*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((a + b*x)^4*ExpIntegralEi[(4*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c -
a*d)*n*(e*((a + b*x)/(c + d*x))^n)^(4/n)*(c + d*x)^4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2961, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(a+bx)^3}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2747$$

$$\frac{(a+bx)^4 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-4/n} \int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{4/n}(c+dx)}{a+bx} d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(c+dx)^4(bc-ad)}$$

$$\downarrow 2609$$

$$\frac{(a+bx)^4 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-4/n} \text{ExpIntegralEi}\left(\frac{4\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(c+dx)^4(bc-ad)}$$

input
```
Int[(a + b*x)^3/((c + d*x)^5*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((a + b*x)^4*ExpIntegralEi[(4*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c -
a*d)*n*(e*((a + b*x)/(c + d*x))^n)^(4/n)*(c + d*x)^4)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bx+a)^3}{(dx+c)^5 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)} dx$$

input
```
int((b*x+a)^3/(d*x+c)^5/ln(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
int((b*x+a)^3/(d*x+c)^5/ln(e*((b*x+a)/(d*x+c))^n),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\text{log\_integral}\left(\frac{(b^4x^4+4ab^3x^3+6a^2b^2x^2+4a^3bx+a^4)e^{\frac{4}{n}}}{d^4x^4+4cd^3x^3+6c^2d^2x^2+4c^3dx+c^4}\right)}{(bc-ad)e^{\frac{4}{n}}n}$$

input
```
integrate((b*x+a)^3/(d*x+c)^5/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")
```

output
```
log_integral((b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*e^(4/n)/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4))/((b*c - a*d)*e^(4/n)*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**3/(d*x+c)**5/ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bx+a)^3}{(dx+c)^5 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((b*x+a)^3/(d*x+c)^5/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^3/((d*x + c)^5*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Giac [F]

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bx+a)^3}{(dx+c)^5 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((b*x+a)^3/(d*x+c)^5/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="gia
c")
```

output
```
integrate((b*x + a)^3/((d*x + c)^5*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(a+b\,x)^3}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(c+d\,x)^5}\,dx$$

input
```
int((a + b*x)^3/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^5),x)
```

output
```
int((a + b*x)^3/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^5), x)
```

Reduce [F]

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{too large to display}$$

input
```
int((b*x+a)^3/(d*x+c)^5/log(e*((b*x+a)/(d*x+c))^n),x)
```

output

```
(4*int(x**3/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*c**5 + 5*log(((a + b*x)*
*n*e)/(c + d*x)**n)*a*c**4*d*x + 10*log(((a + b*x)**n*e)/(c + d*x)**n)*a*c
**3*d**2*x**2 + 10*log(((a + b*x)**n*e)/(c + d*x)**n)*a*c**2*d**3*x**3 + 5
*log(((a + b*x)**n*e)/(c + d*x)**n)*a*c*d**4*x**4 + log(((a + b*x)**n*e)/(
c + d*x)**n)*a*d**5*x**5 + log(((a + b*x)**n*e)/(c + d*x)**n)*b*c**5*x + 5
*log(((a + b*x)**n*e)/(c + d*x)**n)*b*c**4*d*x**2 + 10*log(((a + b*x)**n*e
)/(c + d*x)**n)*b*c**3*d**2*x**3 + 10*log(((a + b*x)**n*e)/(c + d*x)**n)*b
*c**2*d**3*x**4 + 5*log(((a + b*x)**n*e)/(c + d*x)**n)*b*c*d**4*x**5 + log
(((a + b*x)**n*e)/(c + d*x)**n)*b*d**5*x**6),x)*a**2*b**3*d**5*n - 8*int(x
**3/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*c**5 + 5*log(((a + b*x)**n*e)/(c
 + d*x)**n)*a*c**4*d*x + 10*log(((a + b*x)**n*e)/(c + d*x)**n)*a*c**3*d**2
*x**2 + 10*log(((a + b*x)**n*e)/(c + d*x)**n)*a*c**2*d**3*x**3 + 5*log(((a
 + b*x)**n*e)/(c + d*x)**n)*a*c*d**4*x**4 + log(((a + b*x)**n*e)/(c + d*x)
**n)*a*d**5*x**5 + log(((a + b*x)**n*e)/(c + d*x)**n)*b*c**5*x + 5*log(((a
 + b*x)**n*e)/(c + d*x)**n)*b*c**4*d*x**2 + 10*log(((a + b*x)**n*e)/(c + d
*x)**n)*b*c**3*d**2*x**3 + 10*log(((a + b*x)**n*e)/(c + d*x)**n)*b*c**2*d*
*3*x**4 + 5*log(((a + b*x)**n*e)/(c + d*x)**n)*b*c*d**4*x**5 + log(((a + b
*x)**n*e)/(c + d*x)**n)*b*d**5*x**6),x)*a*b**4*c*d**4*n + 4*int(x**3/(log(
((a + b*x)**n*e)/(c + d*x)**n)*a*c**5 + 5*log(((a + b*x)**n*e)/(c + d*x)**
n)*a*c**4*d*x + 10*log(((a + b*x)**n*e)/(c + d*x)**n)*a*c**3*d**2*x**2 ...
```

3.242 $\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result . . . 2607
Mathematica [A] (verified) . . . 2607
Rubi [A] (verified) . . . 2608
Maple [F] . . . 2609
Fricas [A] (verification not implemented) . . . 2610
Sympy [F(-1)] . . . 2610
Maxima [F] . . . 2610
Giac [F] . . . 2611
Mupad [F(-1)] . . . 2611
Reduce [F] . . . 2611

Optimal result

Integrand size = 35, antiderivative size = 75

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{(a+bx)^3 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-3/n} \text{ExpIntegralEi}\left(\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^3}$$

output

```
(b*x+a)^3*Ei(3*ln(e*((b*x+a)/(d*x+c))^n)/n)/(-a*d+b*c)/n/((e*((b*x+a)/(d*x
+c))^n)^(3/n))/(d*x+c)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{(a+bx)^3 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-3/n} \text{ExpIntegralEi}\left(\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^3}$$

input
```
Integrate[(a + b*x)^2/((c + d*x)^4*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((a + b*x)^3*ExpIntegralEi[(3*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c -
a*d)*n*(e*((a + b*x)/(c + d*x))^n)^(3/n)*(c + d*x)^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2961, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(a+bx)^2}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2747$$

$$\frac{(a+bx)^3 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-3/n} \int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{3/n}(c+dx)}{a+bx} d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(c+dx)^3(bc-ad)}$$

$$\downarrow 2609$$

$$\frac{(a+bx)^3 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-3/n} \text{ExpIntegralEi}\left(\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(c+dx)^3(bc-ad)}$$

input
```
Int[(a + b*x)^2/((c + d*x)^4*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((a + b*x)^3*ExpIntegralEi[(3*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c -
a*d)*n*(e*((a + b*x)/(c + d*x))^n)^(3/n)*(c + d*x)^3)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(bx+a)^2}{(dx+c)^4 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)} dx$$

input
```
int((b*x+a)^2/(d*x+c)^4/ln(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
int((b*x+a)^2/(d*x+c)^4/ln(e*((b*x+a)/(d*x+c))^n),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\text{log\_integral}\left(\frac{(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)e^{\frac{3}{n}}}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)}{(bc-ad)e^{\frac{3}{n}}n}$$

input
```
integrate((b*x+a)^2/(d*x+c)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")
```

output
```
log_integral((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*e^(3/n)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/((b*c - a*d)*e^(3/n)*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**2/(d*x+c)**4/ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bx+a)^2}{(dx+c)^4 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((b*x+a)^2/(d*x+c)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^2/((d*x + c)^4*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Giac [F]

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bx+a)^2}{(dx+c)^4 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((b*x+a)^2/(d*x+c)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="gia
c")
```

output
```
integrate((b*x + a)^2/((d*x + c)^4*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(a+b\,x)^2}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(c+d\,x)^4}\,dx$$

input
```
int((a + b*x)^2/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^4),x)
```

output
```
int((a + b*x)^2/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^4), x)
```

Reduce [F]

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{too large to display}$$

input
```
int((b*x+a)^2/(d*x+c)^4/log(e*((b*x+a)/(d*x+c))^n),x)
```

output

```
(3*int(x**2/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*c**4 + 4*log(((a + b*x)*
*n*e)/(c + d*x)**n)*a*c**3*d*x + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*c*
*2*d**2*x**2 + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a*c*d**3*x**3 + log(((
a + b*x)**n*e)/(c + d*x)**n)*a*d**4*x**4 + log(((a + b*x)**n*e)/(c + d*x)*
*n)*b*c**4*x + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*b*c**3*d*x**2 + 6*log(
((a + b*x)**n*e)/(c + d*x)**n)*b*c**2*d**2*x**3 + 4*log(((a + b*x)**n*e)/(
c + d*x)**n)*b*c*d**3*x**4 + log(((a + b*x)**n*e)/(c + d*x)**n)*b*d**4*x**
5),x)*a**2*b**2*d**4*n - 6*int(x**2/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*
c**4 + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a*c**3*d*x + 6*log(((a + b*x)*
*n*e)/(c + d*x)**n)*a*c**2*d**2*x**2 + 4*log(((a + b*x)**n*e)/(c + d*x)**n
)*a*c*d**3*x**3 + log(((a + b*x)**n*e)/(c + d*x)**n)*a*d**4*x**4 + log(((a
 + b*x)**n*e)/(c + d*x)**n)*b*c**4*x + 4*log(((a + b*x)**n*e)/(c + d*x)**n
)*b*c**3*d*x**2 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*b*c**2*d**2*x**3 +
4*log(((a + b*x)**n*e)/(c + d*x)**n)*b*c*d**3*x**4 + log(((a + b*x)**n*e)/
(c + d*x)**n)*b*d**4*x**5),x)*a*b**3*c*d**3*n + 3*int(x**2/(log(((a + b*x)
**n*e)/(c + d*x)**n)*a*c**4 + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a*c**3*
d*x + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a*c**2*d**2*x**2 + 4*log(((a +
b*x)**n*e)/(c + d*x)**n)*a*c*d**3*x**3 + log(((a + b*x)**n*e)/(c + d*x)**n
)*a*d**4*x**4 + log(((a + b*x)**n*e)/(c + d*x)**n)*b*c**4*x + 4*log(((a +
b*x)**n*e)/(c + d*x)**n)*b*c**3*d*x**2 + 6*log(((a + b*x)**n*e)/(c + d*...
```

3.243 $\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result . . . 2613
Mathematica [A] (verified) . . . 2613
Rubi [A] (verified) . . . 2614
Maple [F] . . . 2615
Fricas [A] (verification not implemented) . . . 2615
Sympy [F(-1)] . . . 2616
Maxima [F] . . . 2616
Giac [F] . . . 2617
Mupad [F(-1)] . . . 2617
Reduce [F] . . . 2617

Optimal result

Integrand size = 33, antiderivative size = 75

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{(a+bx)^2 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n} \operatorname{ExpIntegralEi}\left(\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^2}$$

output

```
(b*x+a)^2*Ei(2*ln(e*((b*x+a)/(d*x+c))^n)/n)/(-a*d+b*c)/n/((e*((b*x+a)/(d*x
+c))^n)^(2/n))/(d*x+c)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{(a+bx)^2 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n} \operatorname{ExpIntegralEi}\left(\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^2}$$

input
```
Integrate[(a + b*x)/((c + d*x)^3*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((a + b*x)^2*ExpIntegralEi[(2*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c -
a*d)*n*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2961, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{\frac{a+bx}{c+dx}}{(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2747$$

$$\frac{(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)}{a+bx}\,d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(c+dx)^2(bc-ad)}$$

$$\downarrow 2609$$

$$\frac{(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(c+dx)^2(bc-ad)}$$

input
```
Int[(a + b*x)/((c + d*x)^3*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((a + b*x)^2*ExpIntegralEi[(2*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c -
a*d)*n*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))  Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{bx+a}{(dx+c)^3 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((b*x+a)/(d*x+c)^3/ln(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
int((b*x+a)/(d*x+c)^3/ln(e*((b*x+a)/(d*x+c))^n),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\text{log\_integral}\left(\frac{(b^2x^2+2\,abx+a^2)e^{\frac{2}{n}}}{d^2x^2+2\,cdx+c^2}\right)}{(bc-ad)e^{\frac{2}{n}}n}$$

input
```
integrate((b*x+a)/(d*x+c)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")
```

output
```
log_integral((b^2*x^2 + 2*a*b*x + a^2)*e^(2/n)/(d^2*x^2 + 2*c*d*x + c^2))/((b*c - a*d)*e^(2/n)*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)/(d*x+c)**3/ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{bx+a}{(dx+c)^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((b*x+a)/(d*x+c)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")
```

output
```
integrate((b*x + a)/((d*x + c)^3*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Giac [F]

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{bx+a}{(dx+c)^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((b*x+a)/(d*x+c)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac"
)
```

output
```
integrate((b*x + a)/((d*x + c)^3*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{a+b\,x}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(c+d\,x)^3}\,dx$$

input
```
int((a + b*x)/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^3),x)
```

output
```
int((a + b*x)/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^3), x)
```

Reduce [F]

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{too large to display}$$

input
```
int((b*x+a)/(d*x+c)^3/log(e*((b*x+a)/(d*x+c))^n),x)
```

output

```
(2*int(x/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*c**3 + 3*log(((a + b*x)**n*
e)/(c + d*x)**n)*a*c**2*d*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a*c*d**
2*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*a*d**3*x**3 + log(((a + b*x)**
n*e)/(c + d*x)**n)*b*c**3*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*b*c**2*
d*x**2 + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*b*c*d**2*x**3 + log(((a + b*
x)**n*e)/(c + d*x)**n)*b*d**3*x**4),x)*a**2*b*d**3*n - 4*int(x/(log(((a +
b*x)**n*e)/(c + d*x)**n)*a*c**3 + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a*c
**2*d*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a*c*d**2*x**2 + log(((a + b
*x)**n*e)/(c + d*x)**n)*a*d**3*x**3 + log(((a + b*x)**n*e)/(c + d*x)**n)*b
*c**3*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*b*c**2*d*x**2 + 3*log(((a +
 b*x)**n*e)/(c + d*x)**n)*b*c*d**2*x**3 + log(((a + b*x)**n*e)/(c + d*x)**
n)*b*d**3*x**4),x)*a*b**2*c*d**2*n + 2*int(x/(log(((a + b*x)**n*e)/(c + d*
x)**n)*a*c**3 + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a*c**2*d*x + 3*log(((
a + b*x)**n*e)/(c + d*x)**n)*a*c*d**2*x**2 + log(((a + b*x)**n*e)/(c + d*x
)**n)*a*d**3*x**3 + log(((a + b*x)**n*e)/(c + d*x)**n)*b*c**3*x + 3*log(((
a + b*x)**n*e)/(c + d*x)**n)*b*c**2*d*x**2 + 3*log(((a + b*x)**n*e)/(c + d
*x)**n)*b*c*d**2*x**3 + log(((a + b*x)**n*e)/(c + d*x)**n)*b*d**3*x**4),x)
*b**3*c**2*d*n + int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*c**3 + 3*log(
((a + b*x)**n*e)/(c + d*x)**n)*a*c**2*d*x + 3*log(((a + b*x)**n*e)/(c + d*
x)**n)*a*c*d**2*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*a*d**3*x**3 +...
```

3.244 $\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result . . . 2619
Mathematica [A] (verified) . . . 2619
Rubi [A] (verified) . . . 2620
Maple [F] . . . 2621
Fricas [A] (verification not implemented) . . . 2621
Sympy [F(-1)] . . . 2622
Maxima [F] . . . 2622
Giac [F] . . . 2622
Mupad [F(-1)] . . . 2623
Reduce [F] . . . 2623

Optimal result

Integrand size = 28, antiderivative size = 72

$$\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n} \operatorname{ExpIntegralEi}\left(\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)}$$

output
```
(b*x+a)*Ei(ln(e*((b*x+a)/(d*x+c))^n)/n)/(-a*d+b*c)/n/((e*((b*x+a)/(d*x+c))
^n)^(1/n))/(d*x+c)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n} \operatorname{ExpIntegralEi}\left(\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)}$$

input
```
Integrate[1/((c + d*x)^2*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((a + b*x)*ExpIntegralEi[Log[e*((a + b*x)/(c + d*x))^n]/n])/((b*c - a*d)*n
*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2951, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

$$\downarrow 2951$$

$$\frac{\int \frac{1}{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2737$$

$$\frac{(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n} \int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)}{a+bx} d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(c+dx)(bc-ad)}$$

$$\downarrow 2609$$

$$\frac{(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n} \text{ExpIntegralEi}\left(\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(c+dx)(bc-ad)}$$

input
```
Int[1/((c + d*x)^2*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((a + b*x)*ExpIntegralEi[Log[e*((a + b*x)/(c + d*x))^n]/n])/((b*c - a*d)*n
*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

Maple [F]

$$\int \frac{1}{(dx+c)^2 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)} dx$$

input
```
int(1/(d*x+c)^2/ln(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
int(1/(d*x+c)^2/ln(e*((b*x+a)/(d*x+c))^n),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{\log\_\text{integral}\left(\frac{(bx+a)e^{\left(\frac{1}{n}\right)}}{dx+c}\right)}{(bc-ad)e^{\left(\frac{1}{n}\right)}n}$$

input
```
integrate(1/(d*x+c)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")
```

output
```
log_integral((b*x + a)*e^(1/n)/(d*x + c))/((b*c - a*d)*e^(1/n)*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)**2/ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{(dx+c)^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate(1/(d*x+c)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)^2*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{(dx+c)^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate(1/(d*x+c)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(c+d\,x)^2}\,dx$$

input
```
int(1/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^2),x)
```

output
```
int(1/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^2/log(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
(int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*c**2 + 2*log(((a + b*x)**n*e)
/(c + d*x)**n)*a*c*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*a*d**2*x**2 +
log(((a + b*x)**n*e)/(c + d*x)**n)*b*c**2*x + 2*log(((a + b*x)**n*e)/(c +
d*x)**n)*b*c*d*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b*d**2*x**3),x)*a
**2*d**2*n - 2*int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*c**2 + 2*log(((
a + b*x)**n*e)/(c + d*x)**n)*a*c*d*x + log(((a + b*x)**n*e)/(c + d*x)**n)*
a*d**2*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b*c**2*x + 2*log(((a + b*
x)**n*e)/(c + d*x)**n)*b*c*d*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b*d
**2*x**3),x)*a*b*c*d*n + int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*c**2
+ 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*c*d*x + log(((a + b*x)**n*e)/(c +
 d*x)**n)*a*d**2*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b*c**2*x + 2*lo
g(((a + b*x)**n*e)/(c + d*x)**n)*b*c*d*x**2 + log(((a + b*x)**n*e)/(c + d*
x)**n)*b*d**2*x**3),x)*b**2*c**2*n - log(log(((a + b*x)**n*e)/(c + d*x)**n
))*b)/(d*n*(a*d - b*c))
```

3.245 $\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result . . . 2624
Mathematica [A] (verified) . . . 2624
Rubi [A] (verified) . . . 2625
Maple [A] (verified) . . . 2626
Fricas [A] (verification not implemented) . . . 2627
Sympy [B] (verification not implemented) . . . 2627
Maxima [A] (verification not implemented) . . . 2628
Giac [B] (verification not implemented) . . . 2628
Mupad [B] (verification not implemented) . . . 2629
Reduce [B] (verification not implemented) . . . 2629

Optimal result

Integrand size = 35, antiderivative size = 33

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\log\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)n}$$

output
```
ln(ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/n
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = -\frac{\log\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(-bc+ad)n}$$

input
```
Integrate[1/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
-(Log[Log[e*((a + b*x)/(c + d*x))^n]]/((-(b*c) + a*d)*n))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2961, 2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{c+dx}{(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2739$$

$$\frac{\int \frac{c+dx}{a+bx}\,d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

$$\downarrow 14$$

$$\frac{\log\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{n(bc-ad)}$$

input
```
Int[1/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
Log[Log[e*((a + b*x)/(c + d*x))^n]]/((b*c - a*d)*n)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\frac{\ln\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{n(da-bc)}$	35
default	$-\frac{\ln\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{n(da-bc)}$	35
parallelrisch	$-\frac{\ln\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{n(da-bc)}$	35

input
```
int(1/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n),x,method=_RETURNVERBOSE)
```

output
```
-1/n/(a*d-b*c)*ln(ln(e*((b*x+a)/(d*x+c))^n))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\log\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)}{(bc-ad)n}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")
```

output
```
log(n*log((b*x + a)/(d*x + c)) + log(e))/((b*c - a*d)*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(24) = 48.

Time = 93.17 (sec) , antiderivative size = 146, normalized size of antiderivative = 4.42

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$
$$= \begin{cases} -\frac{1}{(bc+bdx)\log(e)} & \text{for } a = \frac{bc}{d} \wedge n = 0 \\ -\frac{1}{bc\log\left(e\left(\frac{bc}{cd+d^2x}+\frac{bx}{c+dx}\right)^n\right)+bdx\log\left(e\left(\frac{bc}{cd+d^2x}+\frac{bx}{c+dx}\right)^n\right)} & \text{for } a = \frac{bc}{d} \\ \frac{-\frac{\log\left(\frac{a}{b}+x\right)}{ad-bc}+\frac{\log\left(\frac{c}{d}+x\right)}{ad-bc}}{\log(e)} & \text{for } n = 0 \\ -\frac{\log\left(\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)}{adn-bcn} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
Piecewise((-1/((b*c + b*d*x)*log(e)), Eq(n, 0) & Eq(a, b*c/d)), (-1/(b*c*log(e*(b*c/(c*d + d**2*x) + b*x/(c + d*x))**n) + b*d*x*log(e*(b*c/(c*d + d**2*x) + b*x/(c + d*x))**n)), Eq(a, b*c/d)), ((-log(a/b + x)/(a*d - b*c) + log(c/d + x)/(a*d - b*c))/log(e), Eq(n, 0)), (-log(log(e*(a/(c + d*x) + b*x/(c + d*x))**n))/(a*d*n - b*c*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\log\left(-\log\left((bx+a)^n\right)+\log\left((dx+c)^n\right)-\log(e)\right)}{bcn-adn}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxim
a")
```

output
```
log(-log((b*x + a)^n) + log((d*x + c)^n) - log(e))/(b*c*n - a*d*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(33) = 66.

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.67

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)\log\left(\frac{1}{4}\left(\pi(\operatorname{sgn}(bx+a)\operatorname{sgn}(dx+c)-1)n+\pi(\operatorname{sgn}(e)-1)\right)^2+\left(n\log\left(\frac{|bx+a|}{|dx+c|}\right)+\log\right.\right.}{2n}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac"
)
```

output
```
1/2*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)*log(1/4*(pi*(sgn(b*x + a)*sgn(
d*x + c) - 1)*n + pi*(sgn(e) - 1))^2 + (n*log(abs(b*x + a)/abs(d*x + c)) +
 log(abs(e)))^2)/n
```

Mupad [B] (verification not implemented)

Time = 27.72 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = -\frac{\ln\left(\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a\,d\,n-b\,c\,n}$$

input
```
int(1/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)*(c + d*x)),x)
```

output
```
-log(log(e*((a + b*x)/(c + d*x))^n))/(a*d*n - b*c*n)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = -\frac{\log\left(\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)\right)}{n\,(ad-bc)}$$

input
```
int(1/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
( - log(log(((a + b*x)**n*e)/(c + d*x)**n)))/(n*(a*d - b*c))
```

3.246 $\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result . . . 2630
Mathematica [A] (verified) . . . 2630
Rubi [A] (verified) . . . 2631
Maple [F] . . . 2632
Fricas [A] (verification not implemented) . . . 2632
Sympy [F(-1)] . . . 2633
Maxima [F] . . . 2633
Giac [F] . . . 2633
Mupad [F(-1)] . . . 2634
Reduce [F] . . . 2634

Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}} (c+dx)\,\text{ExpIntegralEi}\left(-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)}$$

output
```
(e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei(-ln(e*((b*x+a)/(d*x+c))^n)/n)/(-a
*d+b*c)/n/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}} (c+dx)\,\text{ExpIntegralEi}\left(-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)}$$

input
```
Integrate[1/((a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-(Log[e*((a +
b*x)/(c + d*x))^n]/n)])/((b*c - a*d)*n*(a + b*x))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2949, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

$$\downarrow 2949$$

$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2747$$

$$\frac{(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}} \int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}(c+dx)}{a+bx} d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(a+bx)(bc-ad)}$$

$$\downarrow 2609$$

$$\frac{(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}} \text{ExpIntegralEi}\left(-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(a+bx)(bc-ad)}$$

input
```
Int[1/((a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-(Log[e*((a +
b*x)/(c + d*x))^n]/n)])/((b*c - a*d)*n*(a + b*x))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

Maple [F]

$$\int \frac{1}{(bx+a)^2 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)} dx$$

input
```
int(1/(b*x+a)^2/ln(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
int(1/(b*x+a)^2/ln(e*((b*x+a)/(d*x+c))^n),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{e^{\left(\frac{1}{n}\right)} \log\_\text{integral}\left(\frac{dx+c}{(bx+a)e^{\left(\frac{1}{n}\right)}}\right)}{(bc-ad)n}$$

input
```
integrate(1/(b*x+a)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")
```

output
```
e^(1/n)*log_integral((d*x + c)/((b*x + a)*e^(1/n)))/((b*c - a*d)*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)**2/ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{(bx+a)^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate(1/(b*x+a)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^2*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{(bx+a)^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate(1/(b*x+a)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^2*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(a+b\,x)^2}\,dx$$

input
```
int(1/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^2),x)
```

output
```
int(1/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(b*x+a)^2/log(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
( - int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*c + log(((a + b*x)**n*e
)/(c + d*x)**n)*a**2*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c*x +
2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x**2 + log(((a + b*x)**n*e)/(c
+ d*x)**n)*b**2*c*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*x**3),x
)*a**2*d**2*n + 2*int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*c + log((
(a + b*x)**n*e)/(c + d*x)**n)*a**2*d*x + 2*log(((a + b*x)**n*e)/(c + d*x)*
*n)*a*b*c*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x**2 + log(((a +
b*x)**n*e)/(c + d*x)**n)*b**2*c*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*
b**2*d*x**3),x)*a*b*c*d*n - int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a**2
*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*d*x + 2*log(((a + b*x)**n*e)/
(c + d*x)**n)*a*b*c*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*x**2 +
log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*x**2 + log(((a + b*x)**n*e)/(c +
 d*x)**n)*b**2*d*x**3),x)*b**2*c**2*n - log(log(((a + b*x)**n*e)/(c + d*x)
**n))*d)/(b*n*(a*d - b*c))
```

3.247 $\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result . . . 2635
Mathematica [A] (verified) . . . 2635
Rubi [A] (verified) . . . 2636
Maple [F] . . . 2637
Fricas [A] (verification not implemented) . . . 2637
Sympy [F(-1)] . . . 2638
Maxima [F] . . . 2638
Giac [F] . . . 2639
Mupad [F(-1)] . . . 2639
Reduce [F] . . . 2639

Optimal result

Integrand size = 33, antiderivative size = 75

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)^2\,\text{ExpIntegralEi}\left(-\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)^2}$$

output
```
(e*((b*x+a)/(d*x+c))^n)^(2/n)*(d*x+c)^2*Ei(-2*ln(e*((b*x+a)/(d*x+c))^n)/n)
/(-a*d+b*c)/n/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)^2\,\text{ExpIntegralEi}\left(-\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)^2}$$

input
```
Integrate[(c + d*x)/((a + b*x)^3*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2*ExpIntegralEi[(-2*Log[e*((a
 + b*x)/(c + d*x))^n])/n])/((b*c - a*d)*n*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2961, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^3}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2747$$

$$\frac{(c+dx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n} \int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}(c+dx)}{a+bx} d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(a+bx)^2(bc-ad)}$$

$$\downarrow 2609$$

$$\frac{(c+dx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n} \text{ExpIntegralEi}\left(-\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(a+bx)^2(bc-ad)}$$

input
```
Int[(c + d*x)/((a + b*x)^3*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2*ExpIntegralEi[(-2*Log[e*((a
 + b*x)/(c + d*x))^n])/n])/((b*c - a*d)*n*(a + b*x)^2)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))  Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q  Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{dx+c}{(bx+a)^3 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)} dx$$

input
```
int((d*x+c)/(b*x+a)^3/ln(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
int((d*x+c)/(b*x+a)^3/ln(e*((b*x+a)/(d*x+c))^n),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{e^{\frac{2}{n}} \operatorname{log\_integral}\left(\frac{d^2x^2+2\,cdx+c^2}{(b^2x^2+2\,abx+a^2)e^{\frac{2}{n}}}\right)}{(bc-ad)n}$$

input
```
integrate((d*x+c)/(b*x+a)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")
```

output
```
e^(2/n)*log_integral((d^2*x^2 + 2*c*d*x + c^2)/((b^2*x^2 + 2*a*b*x + a^2)*e^(2/n)))/((b*c - a*d)*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)/(b*x+a)**3/ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{dx+c}{(bx+a)^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((d*x+c)/(b*x+a)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/((b*x + a)^3*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Giac [F]

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{dx+c}{(bx+a)^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((d*x+c)/(b*x+a)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac"
)
```

output
```
integrate((d*x + c)/((b*x + a)^3*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{c+d\,x}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(a+b\,x)^3}\,dx$$

input
```
int((c + d*x)/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^3),x)
```

output
```
int((c + d*x)/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^3), x)
```

Reduce [F]

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{too large to display}$$

input
```
int((d*x+c)/(b*x+a)^3/log(e*((b*x+a)/(d*x+c))^n),x)
```

output

```
( - 2*int(x/(log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*c + log(((a + b*x)**n
*e)/(c + d*x)**n)*a**3*d*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c
*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*x**2 + 3*log(((a + b*x)
**n*e)/(c + d*x)**n)*a*b**2*c*x**2 + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*
a*b**2*d*x**3 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*x**3 + log(((a +
 b*x)**n*e)/(c + d*x)**n)*b**3*d*x**4),x)*a**2*b*d**3*n + 4*int(x/(log(((a
 + b*x)**n*e)/(c + d*x)**n)*a**3*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a*
*3*d*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c*x + 3*log(((a + b*x
)**n*e)/(c + d*x)**n)*a**2*b*d*x**2 + 3*log(((a + b*x)**n*e)/(c + d*x)**n)
*a*b**2*c*x**2 + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**3 + log(
((a + b*x)**n*e)/(c + d*x)**n)*b**3*c*x**3 + log(((a + b*x)**n*e)/(c + d*x
)**n)*b**3*d*x**4),x)*a*b**2*c*d**2*n - 2*int(x/(log(((a + b*x)**n*e)/(c +
 d*x)**n)*a**3*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*d*x + 3*log(((a
 + b*x)**n*e)/(c + d*x)**n)*a**2*b*c*x + 3*log(((a + b*x)**n*e)/(c + d*x)*
*n)*a**2*b*d*x**2 + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c*x**2 + 3
*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*x**3 + log(((a + b*x)**n*e)/(
c + d*x)**n)*b**3*c*x**3 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**3*d*x**4)
,x)*b**3*c**2*d*n - int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*c + log
(((a + b*x)**n*e)/(c + d*x)**n)*a**3*d*x + 3*log(((a + b*x)**n*e)/(c + d*x
)**n)*a**2*b*c*x + 3*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*x**2 +...
```

3.248 $\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result . . . 2641
Mathematica [A] (verified) . . . 2641
Rubi [A] (verified) . . . 2642
Maple [F] . . . 2643
Fricas [A] (verification not implemented) . . . 2644
Sympy [F(-1)] . . . 2644
Maxima [F] . . . 2644
Giac [F] . . . 2645
Mupad [F(-1)] . . . 2645
Reduce [F] . . . 2645

Optimal result

Integrand size = 35, antiderivative size = 75

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{3/n}(c+dx)^3\,\text{ExpIntegralEi}\left(-\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)^3}$$

output

```
(e*((b*x+a)/(d*x+c))^n)^(3/n)*(d*x+c)^3*Ei(-3*ln(e*((b*x+a)/(d*x+c))^n)/n)
/(-a*d+b*c)/n/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{3/n}(c+dx)^3\,\text{ExpIntegralEi}\left(-\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)^3}$$

input
```
Integrate[(c + d*x)^2/((a + b*x)^4*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((e*((a + b*x)/(c + d*x))^n)^(3/n)*(c + d*x)^3*ExpIntegralEi[(-3*Log[e*((a
 + b*x)/(c + d*x))^n])/n])/((b*c - a*d)*n*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2961, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

$$\downarrow 2961$$

$$\frac{\int \frac{(c+dx)^4}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\downarrow 2747$$

$$\frac{(c+dx)^3 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{3/n} \int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-3/n}(c+dx)}{a+bx} d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(a+bx)^3(bc-ad)}$$

$$\downarrow 2609$$

$$\frac{(c+dx)^3 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{3/n} \text{ExpIntegralEi}\left(-\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(a+bx)^3(bc-ad)}$$

input
```
Int[(c + d*x)^2/((a + b*x)^4*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((e*((a + b*x)/(c + d*x))^n)^(3/n)*(c + d*x)^3*ExpIntegralEi[(-3*Log[e*((a
 + b*x)/(c + d*x))^n])/n])/((b*c - a*d)*n*(a + b*x)^3)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Maple [F]

$$\int \frac{(dx+c)^2}{(bx+a)^4 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)} dx$$

input
```
int((d*x+c)^2/(b*x+a)^4/ln(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
int((d*x+c)^2/(b*x+a)^4/ln(e*((b*x+a)/(d*x+c))^n),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{e^{\frac{3}{n}}\,\text{log\_integral}\left(\frac{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}{(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)e^{\frac{3}{n}}}\right)}{(bc-ad)n}$$

input
```
integrate((d*x+c)^2/(b*x+a)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")
```

output
```
e^(3/n)*log_integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)/((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*e^(3/n)))/((b*c - a*d)*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2/(b*x+a)**4/ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(dx+c)^2}{(bx+a)^4 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((d*x+c)^2/(b*x+a)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2/((b*x + a)^4*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Giac [F]

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(dx+c)^2}{(bx+a)^4 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((d*x+c)^2/(b*x+a)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/((b*x + a)^4*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(c+d\,x)^2}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(a+b\,x)^4}\,dx$$

input
```
int((c + d*x)^2/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^4),x)
```

output
```
int((c + d*x)^2/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^4), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^2/(b*x+a)^4/log(e*((b*x+a)/(d*x+c))^n),x)
```

output

```
( - 3*int(x**2/(log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*c + log(((a + b*x)
**n*e)/(c + d*x)**n)*a**4*d*x + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*
b*c*x + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*d*x**2 + 6*log(((a + b
*x)**n*e)/(c + d*x)**n)*a**2*b**2*c*x**2 + 6*log(((a + b*x)**n*e)/(c + d*x
)**n)*a**2*b**2*d*x**3 + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c*x**
3 + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*d*x**4 + log(((a + b*x)**n
*e)/(c + d*x)**n)*b**4*c*x**4 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*d*
x**5),x)*a**2*b**2*d**4*n + 6*int(x**2/(log(((a + b*x)**n*e)/(c + d*x)**n)
*a**4*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*d*x + 4*log(((a + b*x)**
n*e)/(c + d*x)**n)*a**3*b*c*x + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*
b*d*x**2 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*c*x**2 + 6*log((
(a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*d*x**3 + 4*log(((a + b*x)**n*e)/(c
 + d*x)**n)*a*b**3*c*x**3 + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*d*
x**4 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**4*c*x**4 + log(((a + b*x)**n*
e)/(c + d*x)**n)*b**4*d*x**5),x)*a*b**3*c*d**3*n - 3*int(x**2/(log(((a + b
*x)**n*e)/(c + d*x)**n)*a**4*c + log(((a + b*x)**n*e)/(c + d*x)**n)*a**4*d
*x + 4*log(((a + b*x)**n*e)/(c + d*x)**n)*a**3*b*c*x + 4*log(((a + b*x)**n
*e)/(c + d*x)**n)*a**3*b*d*x**2 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**
2*b**2*c*x**2 + 6*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b**2*d*x**3 + 4*
log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**3*c*x**3 + 4*log(((a + b*x)**n*...
```

3.249 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^4}{(f+gx)(ah+bhx)}\,dx$

Optimal result . . . 2647
Mathematica [F] . . . 2648
Rubi [A] (warning: unable to verify) . . . 2648
Maple [F] . . . 2651
Fricas [F] . . . 2651
Sympy [F(-1)] . . . 2652
Maxima [F] . . . 2652
Giac [F] . . . 2653
Mupad [F(-1)] . . . 2653
Reduce [F] . . . 2654

Optimal result

Integrand size = 43, antiderivative size = 361

$$
\begin{aligned}
&\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^4}{(f+gx)(ah+bhx)}\,dx \\
&= -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^4\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad + \frac{4Bn\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad + \frac{12B^2n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\operatorname{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad + \frac{24B^3n^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{PolyLog}\left(4,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad + \frac{24B^4n^4\operatorname{PolyLog}\left(5,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}
\end{aligned}
$$

output
```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^4*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b
*x+a))/(-a*g+b*f)/h+4*B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3*polylog(2,(-
a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+12*B^2*n^2*(A+B*ln(e*(b*
x+a)^n/((d*x+c)^n)))^2*polylog(3,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-
a*g+b*f)/h+24*B^3*n^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(4,(-a*g+b*
f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+24*B^4*n^4*polylog(5,(-a*g+b*f
)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h
```

Mathematica [F]

$$\int \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^4}{(f+gx)(ah+bhx)}\,dx = \int \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^4}{(f+gx)(ah+bhx)}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^4/((f + g*x)*(a*h + b*h
*x)),x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^4/((f + g*x)*(a*h + b*h
*x)), x]
```

Rubi [A] (warning: unable to verify)

Time = 1.21 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.186$, Rules used = {2973, 2967, 27, 2779, 2821, 2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A)^4}{(f+gx)(ah+bhx)}\,dx$$
$$\downarrow \text{2973}$$
$$\int \frac{(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A)^4}{(f+gx)(ah+bhx)}dx$$

$$\downarrow 2967$$

$$(bc-ad)\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^4}{(bc-ad)h(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\downarrow 27$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^4}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{h}$$

$$\downarrow 2779$$

$$\frac{4Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{bf-ag} - \frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^4}{bf-ag}}{h}$$

$$\downarrow 2821$$

$$\frac{4Bn\left(\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3-3Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{bf-ag} - \frac{\log($$

$$\downarrow 2830$$

$$\frac{4Bn\left(\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3-3Bn\left(2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\text{Poly}\right.\right.}{bf-ag}$$

$$\downarrow 2830$$

$$\frac{4Bn\left(\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3-3Bn\left(2Bn\left(Bn\int \frac{(c+dx)\operatorname{PolyLog}\left(4,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\operatorname{PolyLog}\left(4,\frac{(bf-ag)(c\right.\right.\right.}{bf-ag}$$

$$\downarrow 7143$$

$$\frac{4Bn\left(\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3-3Bn\left(2Bn\left(-\left(\operatorname{PolyLog}\left(4,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)-Bn\operatorname{PolyLo}\right.\right.}{bf-ag}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^4/((f + g*x)*(a*h + b*h*x)),x
]
```

output
```
(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^4*Log[1 - ((b*f - a*g)*(c + d*x
))/((d*f - c*g)*(a + b*x))])/(b*f - a*g)) + (4*B*n*((A + B*Log[e*((a + b*x
)/(c + d*x))^n])^3*PolyLog[2, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*
x))] - 3*B*n*(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*PolyLog[3, ((b*f
 - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]) + 2*B*n*(-((A + B*Log[e*((a +
 b*x)/(c + d*x))^n])*PolyLog[4, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b
*x))]) - B*n*PolyLog[5, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]))
))/(b*f - a*g))/h
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2967

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)   Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*(b*h - a*i
- (d*h - c*i)*x)^q*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a
+ b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && IntegersQ[m, q] && IGtQ[p, 0]
```

rule 2973

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^n\left(dx+c\right)^{-n}\right)\right)^4}{\left(gx+f\right)\left(bhx+ah\right)}dx$$

input

```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^4/(g*x+f)/(b*h*x+a*h),x)
```

output

```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^4/(g*x+f)/(b*h*x+a*h),x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^4}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^4}{(bhx+ah)(gx+f)}\,dx$$

input

```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^4/(g*x+f)/(b*h*x+a*h),x, algo
rithm="fricas")
```

output
```
integral((B^4*log((b*x + a)^n*e/(d*x + c)^n)^4 + 4*A*B^3*log((b*x + a)^n*e
/(d*x + c)^n)^3 + 6*A^2*B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 4*A^3*B*log
((b*x + a)^n*e/(d*x + c)^n) + A^4)/(b*g*h*x^2 + a*f*h + (b*f + a*g)*h*x),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B \log (e(a + bx)^n (c + dx)^{-n}))^4}{(f + gx)(ah + bhx)}\, dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**4/(g*x+f)/(b*h*x+a*h),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + B \log (e(a + bx)^n (c + dx)^{-n}))^4}{(f + gx)(ah + bhx)}\, dx = \int \frac{\left(B \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^4}{(bhx + ah)(gx + f)}\, dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^4/(g*x+f)/(b*h*x+a*h),x, algo
rithm="maxima")
```

output
```
A^4*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) + integr
ate((B^4*log((b*x + a)^n)^4 + B^4*log((d*x + c)^n)^4 + B^4*log(e)^4 + 4*A*
B^3*log(e)^3 + 6*A^2*B^2*log(e)^2 + 4*A^3*B*log(e) + 4*(B^4*log(e) + A*B^3
)*log((b*x + a)^n)^3 - 4*(B^4*log((b*x + a)^n) + B^4*log(e) + A*B^3)*log((
d*x + c)^n)^3 + 6*(B^4*log(e)^2 + 2*A*B^3*log(e) + A^2*B^2)*log((b*x + a)^
n)^2 + 6*(B^4*log((b*x + a)^n)^2 + B^4*log(e)^2 + 2*A*B^3*log(e) + A^2*B^2
 + 2*(B^4*log(e) + A*B^3)*log((b*x + a)^n))*log((d*x + c)^n)^2 + 4*(B^4*lo
g(e)^3 + 3*A*B^3*log(e)^2 + 3*A^2*B^2*log(e) + A^3*B)*log((b*x + a)^n) - 4
*(B^4*log((b*x + a)^n)^3 + B^4*log(e)^3 + 3*A*B^3*log(e)^2 + 3*A^2*B^2*log
(e) + A^3*B + 3*(B^4*log(e) + A*B^3)*log((b*x + a)^n)^2 + 3*(B^4*log(e)^2
+ 2*A*B^3*log(e) + A^2*B^2)*log((b*x + a)^n))*log((d*x + c)^n))/(b*g*h*x^2
 + a*f*h + (b*f*h + a*g*h)*x), x)
```

Giac [F]

$$\int \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^4}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^4}{(bhx+ah)(gx+f)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^4/(g*x+f)/(b*h*x+a*h),x, algo
rithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^4/((b*h*x + a*h)*(g*x + f
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^4}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^4}{(f+g\,x)\,(a\,h+b\,h\,x)}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^4/((f + g*x)*(a*h + b*h*x)),x
)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^4/((f + g*x)*(a*h + b*h*x)),
x)
```

Reduce [F]

$$\int \frac{(A + B \log (e(a+bx)^n (c+dx)^{-n}))^4}{(f+gx)(ah+bhx)}\,dx = \text{too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^4/(g*x+f)/(b*h*x+a*h),x)
```

output
```
(5*int(log(((a + b*x)**n*e)/(c + d*x)**n)**4/(a*c*f + a*c*g*x + a*d*f*x +
a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**2*b**4*
c*d*g**2*n - 5*int(log(((a + b*x)**n*e)/(c + d*x)**n)**4/(a*c*f + a*c*g*x
+ a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x
)*a**2*b**4*d**2*f*g*n - 5*int(log(((a + b*x)**n*e)/(c + d*x)**n)**4/(a*c*
f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b
*d*g*x**3),x)*a*b**5*c**2*g**2*n + 5*int(log(((a + b*x)**n*e)/(c + d*x)**n
)**4/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*
f*x**2 + b*d*g*x**3),x)*a*b**5*d**2*f**2*n + 5*int(log(((a + b*x)**n*e)/(c
 + d*x)**n)**4/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x
**2 + b*d*f*x**2 + b*d*g*x**3),x)*b**6*c**2*f*g*n - 5*int(log(((a + b*x)**
n*e)/(c + d*x)**n)**4/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x +
b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*b**6*c*d*f**2*n + 20*int(log(((a
+ b*x)**n*e)/(c + d*x)**n)**3/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*
c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**3*b**3*c*d*g**2*n - 20
*int(log(((a + b*x)**n*e)/(c + d*x)**n)**3/(a*c*f + a*c*g*x + a*d*f*x + a*
d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**3*b**3*d*
*2*f*g*n - 20*int(log(((a + b*x)**n*e)/(c + d*x)**n)**3/(a*c*f + a*c*g*x +
 a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)
*a**2*b**4*c**2*g**2*n + 20*int(log(((a + b*x)**n*e)/(c + d*x)**n)**3/(...
```

3.250 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(f+gx)(ah+bhx)}\,dx$

Optimal result . . . 2655
Mathematica [F] . . . 2656
Rubi [A] (warning: unable to verify) . . . 2656
Maple [F] . . . 2659
Fricas [F] . . . 2659
Sympy [F(-1)] . . . 2660
Maxima [F] . . . 2660
Giac [F] . . . 2661
Mupad [F(-1)] . . . 2661
Reduce [F] . . . 2661

Optimal result

Integrand size = 43, antiderivative size = 282

$$\begin{aligned}
&\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(f+gx)(ah+bhx)}\,dx \\
&= -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad+\frac{3Bn\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad+\frac{6B^2n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad+\frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}
\end{aligned}$$

output

```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b
*x+a))/(-a*g+b*f)/h+3*B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,(-
a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+6*B^2*n^2*(A+B*ln(e*(b*x
+a)^n/((d*x+c)^n)))*polylog(3,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g
+b*f)/h+6*B^3*n^3*polylog(4,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b
*f)/h
```

Mathematica [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx = \int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/((f + g*x)*(a*h + b*h
*x)),x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/((f + g*x)*(a*h + b*h
*x)), x]
```

Rubi [A] (warning: unable to verify)

Time = 1.09 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {2973, 2967, 27, 2779, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(f+gx)(ah+bhx)}\,dx$$

$$\downarrow 2973$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(f+gx)(ah+bhx)}dx$$

$$\downarrow 2967$$

$$(bc-ad)\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)h(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\downarrow 27$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{h}$$

$$\downarrow 2779$$

$$\frac{3Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{bf-ag} - \frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{bf-ag}}{h}$$

$$\downarrow 2821$$

$$\frac{3Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{bf-ag} - \frac{\log(1}{h}$$

$$\downarrow 2830$$

$$\frac{3Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\left(Bn\int\frac{(c+dx)\,\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\right.\right.}{bf-ag}}{h}$$

$$\downarrow 7143$$

$$\frac{3Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\left(-\left(\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)-Bn\,\text{PolyLog}\left(4,\right.\right.\right.}{bf-ag}}{h}$$

input

```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/((f + g*x)*(a*h + b*h*x)),x
]
```

output

```
(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^3*Log[1 - ((b*f - a*g)*(c + d*x
))/((d*f - c*g)*(a + b*x))])/(b*f - a*g)) + (3*B*n*((A + B*Log[e*((a + b*x
)/(c + d*x))^n])^2*PolyLog[2, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*
x))] - 2*B*n*(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[3, ((b*f -
a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]) - B*n*PolyLog[4, ((b*f - a*g)*(c
 + d*x))/((d*f - c*g)*(a + b*x))])))/(b*f - a*g))/h
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2779

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2967

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)   Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*(b*h - a*i
- (d*h - c*i)*x)^q*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a
+ b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && IntegersQ[m, q] && IGtQ[p, 0]
```

rule 2973

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^n\left(dx+c\right)^{-n}\right)\right)^3}{\left(gx+f\right)\left(bhx+ah\right)}dx$$

input

```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(g*x+f)/(b*h*x+a*h),x)
```

output

```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(g*x+f)/(b*h*x+a*h),x)
```

Fricas [F]

$$\int \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^3}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bhx+ah)(gx+f)}\,dx$$

input

```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(g*x+f)/(b*h*x+a*h),x, algo
rithm="fricas")
```

output

```
integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e
/(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(b*g*h*x^2
 + a*f*h + (b*f + a*g)*h*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(g*x+f)/(b*h*x+a*h),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bhx+ah)(gx+f)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(g*x+f)/(b*h*x+a*h),x, algo
rithm="maxima")
```

output
```
A^3*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) - integr
ate(-(B^3*log((b*x + a)^n)^3 - B^3*log((d*x + c)^n)^3 + B^3*log(e)^3 + 3*A
*B^2*log(e)^2 + 3*A^2*B*log(e) + 3*(B^3*log(e) + A*B^2)*log((b*x + a)^n)^2
 + 3*(B^3*log((b*x + a)^n) + B^3*log(e) + A*B^2)*log((d*x + c)^n)^2 + 3*(B
^3*log(e)^2 + 2*A*B^2*log(e) + A^2*B)*log((b*x + a)^n) - 3*(B^3*log((b*x +
 a)^n)^2 + B^3*log(e)^2 + 2*A*B^2*log(e) + A^2*B + 2*(B^3*log(e) + A*B^2)*
log((b*x + a)^n))*log((d*x + c)^n))/(b*g*h*x^2 + a*f*h + (b*f*h + a*g*h)*x
), x)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bhx+ah)(gx+f)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(g*x+f)/(b*h*x+a*h),x, algo
rithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/((b*h*x + a*h)*(g*x + f
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{(f+g\,x)\,(a\,h+b\,h\,x)}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/((f + g*x)*(a*h + b*h*x)),x
)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/((f + g*x)*(a*h + b*h*x)),
x)
```

Reduce [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx = \text{too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(g*x+f)/(b*h*x+a*h),x)
```

output

```
(4*int(log(((a + b*x)**n*e)/(c + d*x)**n)**3/(a*c*f + a*c*g*x + a*d*f*x +
a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**2*b**3*
c*d*g**2*n - 4*int(log(((a + b*x)**n*e)/(c + d*x)**n)**3/(a*c*f + a*c*g*x
+ a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x
)*a**2*b**3*d**2*f*g*n - 4*int(log(((a + b*x)**n*e)/(c + d*x)**n)**3/(a*c*
f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b
*d*g*x**3),x)*a*b**4*c**2*g**2*n + 4*int(log(((a + b*x)**n*e)/(c + d*x)**n
)**3/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*
f*x**2 + b*d*g*x**3),x)*a*b**4*d**2*f**2*n + 4*int(log(((a + b*x)**n*e)/(c
 + d*x)**n)**3/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x
**2 + b*d*f*x**2 + b*d*g*x**3),x)*b**5*c**2*f*g*n - 4*int(log(((a + b*x)**
n*e)/(c + d*x)**n)**3/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x +
b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*b**5*c*d*f**2*n + 12*int(log(((a
+ b*x)**n*e)/(c + d*x)**n)**2/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*
c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**3*b**2*c*d*g**2*n - 12
*int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(a*c*f + a*c*g*x + a*d*f*x + a*
d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**3*b**2*d*
*2*f*g*n - 12*int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(a*c*f + a*c*g*x +
 a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)
*a**2*b**3*c**2*g**2*n + 12*int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(...
```

3.251 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(f+gx)(ah+bhx)}\,dx$

Optimal result . . . 2663
Mathematica [B] (verified) . . . 2664
Rubi [A] (warning: unable to verify) . . . 2665
Maple [F] . . . 2667
Fricas [F] . . . 2667
Sympy [F(-1)] . . . 2668
Maxima [F] . . . 2668
Giac [F] . . . 2669
Mupad [F(-1)] . . . 2669
Reduce [F] . . . 2669

Optimal result

Integrand size = 43, antiderivative size = 203

$$\begin{aligned}
&\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(f+gx)(ah+bhx)}\,dx \\
&= -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad + \frac{2Bn\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad + \frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}
\end{aligned}$$

output

```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b
*x+a))/(-a*g+b*f)/h+2*B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,(-a*
g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+2*B^2*n^2*polylog(3,(-a*g+
b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1415 vs. $2(203) = 406$.

Time = 1.44 (sec) , antiderivative size = 1415, normalized size of antiderivative = 6.97

$$\int \frac{(A + B \log (e(a+bx)^n (c+dx)^{-n}))^2}{(f+gx)(ah+bhx)}\, dx = \text{Too large to display}$$

input

```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/((f + g*x)*(a*h + b*h
*x)),x]
```

output

```
(3*Log[a + b*x]*(A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b
*x)^n)/(c + d*x)^n]))^2 - 3*(A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + L
og[(e*(a + b*x)^n)/(c + d*x)^n]))^2*Log[f + g*x] + 3*B*n*(A + B*(-(n*Log[a
 + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]))*(Log[a + b*
x]^2 - 2*(Log[a + b*x]*Log[(b*(f + g*x))/(b*f - a*g)] + PolyLog[2, (g*(a +
 b*x))/(-(b*f) + a*g)])) - 6*A*B*n*(Log[c + d*x]*(Log[(d*(a + b*x))/(-(b*c
) + a*d)] - Log[(d*(f + g*x))/(d*f - c*g)]) + PolyLog[2, (b*(c + d*x))/(b*
c - a*d)] - PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)]) + 6*B^2*n*(n*Log[a +
 b*x] - n*Log[c + d*x] - Log[(e*(a + b*x)^n)/(c + d*x)^n])*(Log[c + d*x]*(
Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[(d*(f + g*x))/(d*f - c*g)]) + Poly
Log[2, (b*(c + d*x))/(b*c - a*d)] - PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g
)]) + B^2*n^2*(Log[a + b*x]^2*(Log[a + b*x] - 3*Log[(b*(f + g*x))/(b*f - a
*g)]) - 6*Log[a + b*x]*PolyLog[2, (g*(a + b*x))/(-(b*f) + a*g)] + 6*PolyLo
g[3, (g*(a + b*x))/(-(b*f) + a*g)]) + 3*B^2*n^2*(Log[(d*(a + b*x))/(-(b*c)
 + a*d)]*Log[c + d*x]^2 - Log[c + d*x]^2*Log[(d*(f + g*x))/(d*f - c*g)] +
2*Log[c + d*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*Log[c + d*x]*Poly
Log[2, (g*(c + d*x))/(-(d*f) + c*g)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a
*d)] + 2*PolyLog[3, (g*(c + d*x))/(-(d*f) + c*g)]) - 6*B^2*n^2*((Log[a + b
*x]^2*(Log[c + d*x] - Log[(b*(c + d*x))/(b*c - a*d)]))/2 - Log[a + b*x]*Lo
g[c + d*x]*Log[(b*(f + g*x))/(b*f - a*g)] - (Log[(g*(c + d*x))/(-(d*f) ...
```

Rubi [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {2973, 2967, 27, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(e(a+bx)^n (c+dx)^{-n}\right)+A\right)^2}{(f+gx)(ah+bhx)}\,dx$$

$$\downarrow 2973$$

$$\int \frac{\left(B \log\left(e(a+bx)^n (c+dx)^{-n}\right)+A\right)^2}{(f+gx)(ah+bhx)}\,dx$$

$$\downarrow 2967$$

$$(bc-ad)\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)h(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\,d\frac{a+bx}{c+dx}$$

$$\downarrow 27$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\,d\frac{a+bx}{c+dx}}{h}$$

$$\downarrow 2779$$

$$\frac{2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}\,d\frac{a+bx}{c+dx}}{bf-ag} - \frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{bf-ag}}{h}$$

$$\downarrow 2821$$

$$\frac{2Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int \frac{(c+dx)\,\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}\,d\frac{a+bx}{c+dx}\right)}{bf-ag} - \frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log}{bf-ag}}{h}$$

$$\downarrow 7143$$

$$\frac{\frac{2Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+Bn\,\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\right)}{bf-ag} - \frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{bf-ag}}{h}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/((f + g*x)*(a*h + b*h*x)),x
]
```

output
```
(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - ((b*f - a*g)*(c + d*x
))/((d*f - c*g)*(a + b*x))])/(b*f - a*g)) + (2*B*n*((A + B*Log[e*((a + b*x
)/(c + d*x))^n])*PolyLog[2, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x)
)] + B*n*PolyLog[3, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]))/(b*
f - a*g))/h
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2967

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)   Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*(b*h - a*i
- (d*h - c*i)*x)^q*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a
+ b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && IntegersQ[m, q] && IGtQ[p, 0]
```

rule 2973

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(A + B \ln\left(e(bx+a)^n (dx+c)^{-n}\right)\right)^2}{(gx+f)(bhx+ah)} dx$$

input

```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(g*x+f)/(b*h*x+a*h),x)
```

output

```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(g*x+f)/(b*h*x+a*h),x)
```

Fricas [F]

$$\int \frac{(A + B\log(e(a+bx)^n (c+dx)^{-n}))^2}{(f+gx)(ah+bhx)} dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{(bhx+ah)(gx+f)} dx$$

input

```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(g*x+f)/(b*h*x+a*h),x, algo
rithm="fricas")
```

output
```
integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(
d*x + c)^n) + A^2)/(b*g*h*x^2 + a*f*h + (b*f + a*g)*h*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B \log (e(a + bx)^n (c + dx)^{-n}))^2}{(f + gx)(ah + bhx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(g*x+f)/(b*h*x+a*h),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + B \log (e(a + bx)^n (c + dx)^{-n}))^2}{(f + gx)(ah + bhx)}\,dx = \int \frac{\left(B \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{(bhx + ah)(gx + f)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(g*x+f)/(b*h*x+a*h),x, algo
rithm="maxima")
```

output
```
A^2*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) + integr
ate((B^2*log((b*x + a)^n)^2 + B^2*log((d*x + c)^n)^2 + B^2*log(e)^2 + 2*A*
B*log(e) + 2*(B^2*log(e) + A*B)*log((b*x + a)^n) - 2*(B^2*log((b*x + a)^n)
 + B^2*log(e) + A*B)*log((d*x + c)^n))/(b*g*h*x^2 + a*f*h + (b*f*h + a*g*h
)*x), x)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(bhx+ah)(gx+f)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(g*x+f)/(b*h*x+a*h),x, algo
rithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/((b*h*x + a*h)*(g*x + f
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{(f+g\,x)\,(a\,h+b\,h\,x)}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/((f + g*x)*(a*h + b*h*x)),x
)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/((f + g*x)*(a*h + b*h*x)),
x)
```

Reduce [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(f+gx)(ah+bhx)}\,dx = \text{Too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(g*x+f)/(b*h*x+a*h),x)
```

output

```
(3*int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(a*c*f + a*c*g*x + a*d*f*x +
a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**2*b**2*
c*d*g**2*n - 3*int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(a*c*f + a*c*g*x
+ a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x
)*a**2*b**2*d**2*f*g*n - 3*int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(a*c*
f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b
*d*g*x**3),x)*a*b**3*c**2*g**2*n + 3*int(log(((a + b*x)**n*e)/(c + d*x)**n
)**2/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*
f*x**2 + b*d*g*x**3),x)*a*b**3*d**2*f**2*n + 3*int(log(((a + b*x)**n*e)/(c
 + d*x)**n)**2/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x
**2 + b*d*f*x**2 + b*d*g*x**3),x)*b**4*c**2*f*g*n - 3*int(log(((a + b*x)**
n*e)/(c + d*x)**n)**2/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x +
b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*b**4*c*d*f**2*n + 6*int(log(((a +
 b*x)**n*e)/(c + d*x)**n)/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*
x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**3*b*c*d*g**2*n - 6*int(log
(((a + b*x)**n*e)/(c + d*x)**n)/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 +
b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**3*b*d**2*f*g*n - 6*i
nt(log(((a + b*x)**n*e)/(c + d*x)**n)/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x
**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**2*b**2*c**2*g*
*2*n + 6*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a*c*f + a*c*g*x + a*d*...
```

3.252 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx$

Optimal result . . . 2671
Mathematica [B] (verified) . . . 2672
Rubi [A] (warning: unable to verify) . . . 2672
Maple [C] (warning: unable to verify) . . . 2674
Fricas [F] . . . 2675
Sympy [F(-1)] . . . 2676
Maxima [F] . . . 2676
Giac [F] . . . 2676
Mupad [F(-1)] . . . 2677
Reduce [F] . . . 2677

Optimal result

Integrand size = 41, antiderivative size = 123

$$\begin{aligned}&\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx\\&=-\frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}\\&\quad+\frac{Bn\,\mathrm{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}\end{aligned}$$

output

```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x
+a))/(-a*g+b*f)/h+B*n*polylog(2,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a
*g+b*f)/h
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 304 vs. $2(123) = 246$.

Time $= 0.46$ (sec) , antiderivative size $= 304$, normalized size of antiderivative $= 2.47$

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx =$$

$$-\frac{-2A\log(a+bx) + Bn\log^2(a+bx) - 2Bn\log(a+bx)\log(c+dx) + 2Bn\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log(c+dx) -$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((f + g*x)*(a*h + b*h*x
)),x]
```

output
```
-1/2*(-2*A*Log[a + b*x] + B*n*Log[a + b*x]^2 - 2*B*n*Log[a + b*x]*Log[c +
d*x] + 2*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 2*B*Log[a +
b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*A*Log[f + g*x] - 2*B*n*Log[a + b
*x]*Log[f + g*x] + 2*B*n*Log[c + d*x]*Log[f + g*x] + 2*B*Log[(e*(a + b*x)^
n)/(c + d*x)^n]*Log[f + g*x] + 2*B*n*Log[a + b*x]*Log[(b*(f + g*x))/(b*f -
 a*g)] - 2*B*n*Log[c + d*x]*Log[(d*(f + g*x))/(d*f - c*g)] + 2*B*n*PolyLog
[2, (g*(a + b*x))/(-(b*f) + a*g)] + 2*B*n*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)] - 2*B*n*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)])/((b*f - a*g)*h)
```

Rubi [A] (warning: unable to verify)

Time $= 0.63$ (sec) , antiderivative size $= 120$, normalized size of antiderivative $= 0.98$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used $= \{2973, 2967, 27, 2779, 2838\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right) + A}{(f+gx)(ah+bhx)}\,dx$$

$$\downarrow \text{2973}$$

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right) + A}{(f+gx)(ah+bhx)}dx$$

$$\downarrow 2967$$

$$(bc-ad)\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)h(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\downarrow 27$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{h}$$

$$\downarrow 2779$$

$$\frac{Bn\int \frac{(c+dx)\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{bf-ag}-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bf-ag}}{h}$$

$$\downarrow 2838$$

$$\frac{\frac{Bn\,\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{bf-ag}-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bf-ag}}{h}$$

input

```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((f + g*x)*(a*h + b*h*x)),x]
```

output

```
(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((b*f - a*g)*(c + d*x))
/((d*f - c*g)*(a + b*x))])/(b*f - a*g)) + (B*n*PolyLog[2, ((b*f - a*g)*(c
+ d*x))/((d*f - c*g)*(a + b*x))])/(b*f - a*g))/h
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2779

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2967
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)   Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*(b*h - a*i
- (d*h - c*i)*x)^q*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a
+ b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && IntegersQ[m, q] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 27.07 (sec) , antiderivative size = 1447, normalized size of antiderivative = 11.76

method	result	size
risch	Expression too large to display	1447

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(g*x+f)/(b*h*x+a*h),x,method=_RETURN
VERBOSE)
```

output

```
-1/h*B*ln((b*x+a)^n)/(a*g-b*f)*ln(b*x+a)-1/h*B*n/(a*g-b*f)*dilog((b*(g*x+f
)+a*g-b*f)/(a*g-b*f))+1/2/h*B*n/(a*g-b*f)*ln(b*x+a)^2+1/h*B*n/(a*g-b*f)*ln
(g*x+f)*ln((d*(g*x+f)+c*g-d*f)/(c*g-d*f))-1/h*B*n/(a*g-b*f)*ln(b*x+a)*ln((
-d*a+b*c+d*(b*x+a))/(-a*d+b*c))+1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I/((
d*x+c)^n))*csgn(I/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*P
i*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I/h/(a*g-b*f)*ln(g
*x+f)*B*Pi*csgn(I/((d*x+c)^n)*(b*x+a)^n)*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2
+1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2*csgn(I
*e)-1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I/((d*x+c)^n))*csgn(I/((d*x+c)^n
)*(b*x+a)^n)^2-1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/(
(d*x+c)^n)*(b*x+a)^n)^2-1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I/((d*x+c)^n
)*(b*x+a)^n)*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2-1/2*I/h/(a*g-b*f)*ln(b*x+a)
*B*Pi*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2*csgn(I*e)-1/2*I/h/(a*g-b*f)*ln(g*x
+f)*B*Pi*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^3+1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*P
i*csgn(I/((d*x+c)^n)*(b*x+a)^n)^3+1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*
e*(b*x+a)^n/((d*x+c)^n))^3-1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I/((d*x+c
)^n)*(b*x+a)^n)^3-1/h*B*n/(a*g-b*f)*ln(g*x+f)*ln((b*(g*x+f)+a*g-b*f)/(a*g-
b*f))+1/h/(a*g-b*f)*ln(g*x+f)*A-1/h/(a*g-b*f)*ln(b*x+a)*A+1/h*B/(a*g-b*f)*
ln(g*x+f)*ln((b*x+a)^n)-1/h*B/(a*g-b*f)*ln(g*x+f)*ln((d*x+c)^n)+1/h*B*ln((
d*x+c)^n)/(a*g-b*f)*ln(b*x+a)+1/h*B*n/(a*g-b*f)*dilog((d*(g*x+f)+c*g-d*...
```

Fricas [F]

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{(bhx+ah)(gx+f)}\,dx$$

input

```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(g*x+f)/(b*h*x+a*h),x, algori
thm="fricas")
```

output

```
integral((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*g*h*x^2 + a*f*h + (b*f
+ a*g)*h*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \log\left(e(a+bx)^n (c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(g*x+f)/(b*h*x+a*h),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B \log\left(e(a+bx)^n (c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx = \int \frac{B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{(bhx+ah)(gx+f)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(g*x+f)/(b*h*x+a*h),x, algori
thm="maxima")
```

output
```
A*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) - B*integr
ate(-(log((b*x + a)^n) - log((d*x + c)^n) + log(e))/(b*g*h*x^2 + a*f*h + (
b*f*h + a*g*h)*x), x)
```

Giac [F]

$$\int \frac{A + B \log\left(e(a+bx)^n (c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx = \int \frac{B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{(bhx+ah)(gx+f)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(g*x+f)/(b*h*x+a*h),x, algori
thm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/((b*h*x + a*h)*(g*x + f))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx = \int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{(f+g\,x)\,(a\,h+b\,h\,x)}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/((f + g*x)*(a*h + b*h*x)),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/((f + g*x)*(a*h + b*h*x)), x)
```

Reduce [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx$$
$$= \frac{2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{bdg\,x^3+adg\,x^2+bcg\,x^2+bdf\,x^2+acgx+adfx+bcfx+acf}dx\right)a^2bcd\,g^2n - 2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{bdg\,x^3+adg\,x^2+bcg\,x^2+bdf\,x^2+acgx+adfx+bcf}\right.}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(g*x+f)/(b*h*x+a*h),x)
```

output

```
(2*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a*c*f + a*c*g*x + a*d*f*x + a*d
*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**2*b*c*d*g*
*2*n - 2*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a*c*f + a*c*g*x + a*d*f*x
 + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**2*b*
d**2*f*g*n - 2*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a*c*f + a*c*g*x + a
*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a
*b**2*c**2*g**2*n + 2*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a*c*f + a*c*
g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**
3),x)*a*b**2*d**2*f**2*n + 2*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a*c*f
 + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*
d*g*x**3),x)*b**3*c**2*f*g*n - 2*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a
*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2
+ b*d*g*x**3),x)*b**3*c*d*f**2*n - 2*log(a + b*x)*a**2*d*g*n + 2*log(a + b
*x)*a*b*c*g*n + 2*log(f + g*x)*a**2*d*g*n - 2*log(f + g*x)*a*b*c*g*n - log
(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b*d*g + log(((a + b*x)**n*e)/(c + d*x
)**n)**2*b**2*d*f)/(2*g*h*n*(a**2*d*g - a*b*c*g - a*b*d*f + b**2*c*f))
```

3.253 $\int \frac{1}{(f+gx)(ah+bhx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

Optimal result . . . 2679
Mathematica [N/A] . . . 2679
Rubi [N/A] . . . 2680
Maple [N/A] . . . 2681
Fricas [N/A] . . . 2681
Sympy [F(-1)] . . . 2682
Maxima [N/A] . . . 2682
Giac [N/A] . . . 2683
Mupad [N/A] . . . 2683
Reduce [N/A] . . . 2684

Optimal result

Integrand size = 43, antiderivative size = 43

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Subst}\left(\text{Int}\left(\frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right),e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

output

```
Defer(Subst)(Defer(Int)(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*((b*x+a)/(d*x+c))^
n)),x),e*((b*x+a)/(d*x+c))^n,e*(b*x+a)^n/((d*x+c)^n))
```

Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^
n])),x]
```

output
```
Integrate[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^
n])), x]
```

Rubi [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2973, 2969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)(ah+bhx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}\,dx$$
$$\downarrow 2973$$
$$\int \frac{1}{(f+gx)(ah+bhx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$
$$\downarrow 2969$$
$$\int \frac{1}{(f+gx)(ah+bhx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x
]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2969

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Unintegrable[(f + g*x)^m*(h + i*x)^q*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
```

rule 2973

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)(bhx+ah)\left(A+B\ln\left(e\left(bx+a\right)^{n}\left(dx+c\right)^{-n}\right)\right)}dx$$

input

```
int(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output

```
int(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

$$\begin{aligned}&\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right)\right)}\,dx\\&=\int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)}\,dx\end{aligned}$$

input
```
integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algo
rithm="fricas")
```

output
```
integral(1/(A*b*g*h*x^2 + A*a*f*h + (A*b*f + A*a*g)*h*x + (B*b*g*h*x^2 + B
*a*f*h + (B*b*f + B*a*g)*h*x)*log((b*x + a)^n*e/(d*x + c)^n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algo
rithm="maxima")
```

output
```
integrate(1/((b*h*x + a*h)*(g*x + f)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A
)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algo
rithm="giac")
```

output
```
integrate(1/((b*h*x + a*h)*(g*x + f)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A
)), x)
```

Mupad [N/A]

Not integrable

Time = 28.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(f+g\,x)\,(a\,h+b\,h\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)}\,dx$$

input
```
int(1/((f + g*x)*(a*h + b*h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))),x
)
```

output
```
int(1/((f + g*x)*(a*h + b*h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))),
x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 1185, normalized size of antiderivative = 27.56

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Too large to display}$$

input

```
int(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output

```
(int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c*f + log(((a + b*x)**n*e)/
(c + d*x)**n)*a*b*c*g*x + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*f*x + l
og(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*g*x**2 + log(((a + b*x)**n*e)/(c +
 d*x)**n)*b**2*c*f*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*g*x**2 +
log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*f*x**2 + log(((a + b*x)**n*e)/(c
 + d*x)**n)*b**2*d*g*x**3 + a**2*c*f + a**2*c*g*x + a**2*d*f*x + a**2*d*g*
x**2 + a*b*c*f*x + a*b*c*g*x**2 + a*b*d*f*x**2 + a*b*d*g*x**3),x)*a*b*c*d*
g*n - int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c*f + log(((a + b*x)**
n*e)/(c + d*x)**n)*a*b*c*g*x + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*f*
x + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*g*x**2 + log(((a + b*x)**n*e)
/(c + d*x)**n)*b**2*c*f*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*c*g*x*
*2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*f*x**2 + log(((a + b*x)**n*
e)/(c + d*x)**n)*b**2*d*g*x**3 + a**2*c*f + a**2*c*g*x + a**2*d*f*x + a**2
*d*g*x**2 + a*b*c*f*x + a*b*c*g*x**2 + a*b*d*f*x**2 + a*b*d*g*x**3),x)*a*b
*d**2*f*n - int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*c*f + log(((a +
b*x)**n*e)/(c + d*x)**n)*a*b*c*g*x + log(((a + b*x)**n*e)/(c + d*x)**n)*a*
b*d*f*x + log(((a + b*x)**n*e)/(c + d*x)**n)*a*b*d*g*x**2 + log(((a + b*x)
**n*e)/(c + d*x)**n)*b**2*c*f*x + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*
c*g*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)*b**2*d*f*x**2 + log(((a + b*
x)**n*e)/(c + d*x)**n)*b**2*d*g*x**3 + a**2*c*f + a**2*c*g*x + a**2*d*f...
```

3.254 $\int \frac{1}{(f+gx)(ah+bhx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}\,dx$

Optimal result . . . 2685
Mathematica [N/A] . . . 2685
Rubi [N/A] . . . 2686
Maple [N/A] . . . 2687
Fricas [N/A] . . . 2687
Sympy [F(-1)] . . . 2688
Maxima [N/A] . . . 2688
Giac [N/A] . . . 2689
Mupad [N/A] . . . 2690
Reduce [N/A] . . . 2690

Optimal result

Integrand size = 43, antiderivative size = 43

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \text{Subst}\left(\text{Int}\left(\frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right),e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

output
```
Defer(Subst)(Defer(Int)(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*((b*x+a)/(d*x+c))^
n))^2,x),e*((b*x+a)/(d*x+c))^n,e*(b*x+a)^n/((d*x+c)^n))
```

Mathematica [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^
n])^2),x]
```

output
```
Integrate[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^
n])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2973, 2969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)(ah+bhx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}\,dx$$
$$\downarrow 2973$$
$$\int \frac{1}{(f+gx)(ah+bhx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}dx$$
$$\downarrow 2969$$
$$\int \frac{1}{(f+gx)(ah+bhx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)
,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2969

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Unintegrable[(f + g*x)^m*(h + i*x)^q*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
```

rule 2973

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)(bhx+ah)\left(A+B\ln\left(e\left(bx+a\right)^{n}\left(dx+c\right)^{-n}\right)\right)^{2}}dx$$

input

```
int(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

output

```
int(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.37

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right)\right)^{2}}\,dx$$
$$=\int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{2}}\,dx$$

input
```
integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, al
gorithm="fricas")
```

output
```
integral(1/(A^2*b*g*h*x^2 + A^2*a*f*h + (A^2*b*f + A^2*a*g)*h*x + (B^2*b*g
*h*x^2 + B^2*a*f*h + (B^2*b*f + B^2*a*g)*h*x)*log((b*x + a)^n*e/(d*x + c)^
n)^2 + 2*(A*B*b*g*h*x^2 + A*B*a*f*h + (A*B*b*f + A*B*a*g)*h*x)*log((b*x +
a)^n*e/(d*x + c)^n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 506, normalized size of antiderivative = 11.77

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, al
gorithm="maxima")
```

output
```
(d*f - c*g)*integrate(1/((b*c*f^2*h*n - a*d*f^2*h*n)*A*B + (b*c*f^2*h*n*lo
g(e) - a*d*f^2*h*n*log(e))*B^2 + ((b*c*g^2*h*n - a*d*g^2*h*n)*A*B + (b*c*g
^2*h*n*log(e) - a*d*g^2*h*n*log(e))*B^2)*x^2 + 2*((b*c*f*g*h*n - a*d*f*g*h
*n)*A*B + (b*c*f*g*h*n*log(e) - a*d*f*g*h*n*log(e))*B^2)*x + ((b*c*g^2*h*n
 - a*d*g^2*h*n)*B^2*x^2 + 2*(b*c*f*g*h*n - a*d*f*g*h*n)*B^2*x + (b*c*f^2*h
*n - a*d*f^2*h*n)*B^2)*log((b*x + a)^n) - ((b*c*g^2*h*n - a*d*g^2*h*n)*B^2
*x^2 + 2*(b*c*f*g*h*n - a*d*f*g*h*n)*B^2*x + (b*c*f^2*h*n - a*d*f^2*h*n)*B
^2)*log((d*x + c)^n)), x) - (d*x + c)/((b*c*f*h*n - a*d*f*h*n)*A*B + (b*c*
f*h*n*log(e) - a*d*f*h*n*log(e))*B^2 + ((b*c*g*h*n - a*d*g*h*n)*A*B + (b*c
*g*h*n*log(e) - a*d*g*h*n*log(e))*B^2)*x + ((b*c*g*h*n - a*d*g*h*n)*B^2*x
+ (b*c*f*h*n - a*d*f*h*n)*B^2)*log((b*x + a)^n) - ((b*c*g*h*n - a*d*g*h*n)
*B^2*x + (b*c*f*h*n - a*d*f*h*n)*B^2)*log((d*x + c)^n))
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx$$
$$= \int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, al
gorithm="giac")
```

output
```
integrate(1/((b*h*x + a*h)*(g*x + f)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A
)^2), x)
```

Mupad [N/A]

Not integrable

Time = 26.95 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx$$
$$=\int \frac{1}{(f+g\,x)\,(a\,h+b\,h\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)*(a*h + b*h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2)
,x)
```

output
```
int(1/((f + g*x)*(a*h + b*h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2)
, x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 4555, normalized size of antiderivative = 105.93

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

output

```
(int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c*f + log(((a + b*x)*
*n*e)/(c + d*x)**n)**2*a*b**2*c*g*x + log(((a + b*x)**n*e)/(c + d*x)**n)**
2*a*b**2*d*f*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*d*g*x**2 + l
og(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*f*x + log(((a + b*x)**n*e)/(c
 + d*x)**n)**2*b**3*c*g*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d
*f*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*g*x**3 + 2*log(((a
 + b*x)**n*e)/(c + d*x)**n)*a**2*b*c*f + 2*log(((a + b*x)**n*e)/(c + d*x)**
n)*a**2*b*c*g*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*f*x + 2*lo
g(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*d*g*x**2 + 2*log(((a + b*x)**n*e)/
(c + d*x)**n)*a*b**2*c*f*x + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*c
*g*x**2 + 2*log(((a + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*f*x**2 + 2*log(((a
 + b*x)**n*e)/(c + d*x)**n)*a*b**2*d*g*x**3 + a**3*c*f + a**3*c*g*x + a**3
*d*f*x + a**3*d*g*x**2 + a**2*b*c*f*x + a**2*b*c*g*x**2 + a**2*b*d*f*x**2
 + a**2*b*d*g*x**3),x)*log(((a + b*x)**n*e)/(c + d*x)**n)*a**2*b*c*d*g*n -
 int(1/(log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*c*f + log(((a + b*x)**
n*e)/(c + d*x)**n)**2*a*b**2*c*g*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2
*a*b**2*d*f*x + log(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b**2*d*g*x**2 + lo
g(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*c*f*x + log(((a + b*x)**n*e)/(c +
 d*x)**n)**2*b**3*c*g*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*
f*x**2 + log(((a + b*x)**n*e)/(c + d*x)**n)**2*b**3*d*g*x**3 + 2*log(((...
```

3.255 $\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx$

Optimal result . . . 2692
Mathematica [B] (verified) . . . 2692
Rubi [A] (verified) . . . 2693
Maple [A] (verified) . . . 2695
Fricas [A] (verification not implemented) . . . 2695
Sympy [F(-1)] . . . 2696
Maxima [B] (verification not implemented) . . . 2696
Giac [F] . . . 2697
Mupad [F(-1)] . . . 2697
Reduce [F] . . . 2698

Optimal result

Integrand size = 40, antiderivative size = 33

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx = -\frac{\text{PolyLog}\left(2,1-\frac{c+dx}{a+bx}\right)}{(bc-ad)h}$$

output
```
-polylog(2,1-(d*x+c)/(b*x+a))/(-a*d+b*c)/h
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 298 vs. 2(33) = 66.

Time = 0.24 (sec) , antiderivative size = 298, normalized size of antiderivative = 9.03

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx$$
$$= \frac{-\log^2\left(\frac{-bc+ad}{d(a+bx)}\right)+2\log\left(\frac{(b-d)(a+bx)}{bc-ad}\right)\log(a-c+bx-dx)-2\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)-2\log(a-c$$

input
```
Integrate[Log[(c + d*x)/(a + b*x)]/((a + b*x)*((a - c)*h + (b - d)*h*x)),x
]
```

output

```
(-Log[(-(b*c) + a*d)/(d*(a + b*x))]^2 + 2*Log[((b - d)*(a + b*x))/(b*c - a
*d)]*Log[a - c + b*x - d*x] - 2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(b*(
c + d*x))/(b*c - a*d)] - 2*Log[a - c + b*x - d*x]*Log[((b - d)*(c + d*x))/
(b*c - a*d)] + 2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(c + d*x)/(a + b*x)
] + 2*Log[a - c + b*x - d*x]*Log[(c + d*x)/(a + b*x)] + 2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)] + 2*PolyLog[2, -((b*(a - c + b*x - d*x))/(b*c - a
*d))] - 2*PolyLog[2, -((d*(-a + c - b*x + d*x))/(-(b*c) + a*d))])/((2*b*c
- 2*a*d)*h)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2965, 25, 27, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)(h(a-c)+hx(b-d))}\,dx$$

$$\downarrow 2965$$

$$\int -\frac{\log\left(\frac{c+dx}{a+bx}\right)}{h\left(-\frac{(c+dx)(bc-ad)}{a+bx}-ad+bc\right)}\,d\frac{c+dx}{a+bx}$$

$$\downarrow 25$$

$$-\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{h\left(bc-ad-\frac{(bc-ad)(c+dx)}{a+bx}\right)}\,d\frac{c+dx}{a+bx}$$

$$\downarrow 27$$

$$-\frac{\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{bc-ad-\frac{(bc-ad)(c+dx)}{a+bx}}\,d\frac{c+dx}{a+bx}}{h}$$

$$\downarrow 2752$$

$$-\frac{\text{PolyLog}\left(2,1-\frac{c+dx}{a+bx}\right)}{h(bc-ad)}$$

input
```
Int[Log[(c + d*x)/(a + b*x)]/((a + b*x)*((a - c)*h + (b - d)*h*x)),x]
```

output
```
-(PolyLog[2, 1 - (c + d*x)/(a + b*x)]/((b*c - a*d)*h))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2965
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(q + 1)*(i/d)^q   Subst[Int[(b*f - a*g - (d*f - c*g)*
x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*
x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ[b*c - a*d,
 0] && IntegersQ[m, q] && IGtQ[p, 0] && EqQ[d*h - c*i, 0]
```

Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{\text{dilog}\left(-\frac{da-bc}{b(bx+a)}+\frac{d}{b}\right)}{h(da-bc)}$
default	$\frac{\text{dilog}\left(-\frac{da-bc}{b(bx+a)}+\frac{d}{b}\right)}{h(da-bc)}$
risch	$\frac{\text{dilog}\left(-\frac{da-bc}{b(bx+a)}+\frac{d}{b}\right)}{h(da-bc)}$
parts	$-\frac{\ln\left(\frac{dx+c}{bx+a}\right)\ln(bx-dx+a-c)b}{h(da-bc)(b-d)}+\frac{\ln\left(\frac{dx+c}{bx+a}\right)\ln(bx-dx+a-c)d}{h(da-bc)(b-d)}+\frac{\ln\left(\frac{dx+c}{bx+a}\right)\ln(bx+a)}{h(da-bc)}-\frac{\text{dilog}\left(\frac{-da+bc+d(bx+a)}{-da+bc}\right)}{da-bc}$

input
```
int(ln((d*x+c)/(b*x+a))/(b*x+a)/((a-c)*h+(b-d)*h*x),x,method=_RETURNVERBOS
E)
```

output
```
dilog(-1/b*(a*d-b*c)/(b*x+a)+d/b)/h/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx = -\frac{\text{Li}_2\left(-\frac{dx+c}{bx+a}+1\right)}{(bc-ad)h}$$

input
```
integrate(log((d*x+c)/(b*x+a))/(b*x+a)/((a-c)*h+(b-d)*h*x),x, algorithm="f
ricas")
```

output
```
-dilog(-(d*x + c)/(b*x + a) + 1)/((b*c - a*d)*h)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx = \text{Timed out}$$

input
```
integrate(ln((d*x+c)/(b*x+a))/(b*x+a)/((a-c)*h+(b-d)*h*x),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(32) = 64$.

Time = 0.05 (sec) , antiderivative size = 357, normalized size of antiderivative = 10.82

$$\begin{aligned}
&\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx \\
&= \left(\frac{\log(-(b-d)x-a+c)}{(bc-ad)h} - \frac{\log(bx+a)}{(bc-ad)h}\right)\log\left(\frac{dx+c}{bx+a}\right) \\
&\quad + \frac{2\log(-(b-d)x-a+c)\log(bx+a) - \log(bx+a)^2}{2(bch-adh)} \\
&\quad + \frac{\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)}{bch-adh} \\
&\quad - \frac{\log(bx+a)\log\left(-\frac{a(b-d)+(b^2-bd)x}{bc-ad}+1\right) + \mathrm{Li}_2\left(\frac{a(b-d)+(b^2-bd)x}{bc-ad}\right)}{bch-adh} \\
&\quad - \frac{\log(-(b-d)x-a+c)\log\left(\frac{ad-cd+(bd-d^2)x}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{ad-cd+(bd-d^2)x}{bc-ad}\right)}{bch-adh}
\end{aligned}$$

input
```
integrate(log((d*x+c)/(b*x+a))/(b*x+a)/((a-c)*h+(b-d)*h*x),x, algorithm="m
axima")
```

output
```
(log(-(b - d)*x - a + c)/((b*c - a*d)*h) - log(b*x + a)/((b*c - a*d)*h))*l
og((d*x + c)/(b*x + a)) + 1/2*(2*log(-(b - d)*x - a + c)*log(b*x + a) - lo
g(b*x + a)^2)/(b*c*h - a*d*h) + (log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d
) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*c*h - a*d*h) - (log(b*x + a
)*log(-(a*(b - d) + (b^2 - b*d)*x)/(b*c - a*d) + 1) + dilog((a*(b - d) + (
b^2 - b*d)*x)/(b*c - a*d)))/(b*c*h - a*d*h) - (log(-(b - d)*x - a + c)*log
((a*d - c*d + (b*d - d^2)*x)/(b*c - a*d) + 1) + dilog(-(a*d - c*d + (b*d -
 d^2)*x)/(b*c - a*d)))/(b*c*h - a*d*h)
```

Giac [F]

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx = \int \frac{\log\left(\frac{dx+c}{bx+a}\right)}{((b-d)hx+(a-c)h)(bx+a)}\,dx$$

input
```
integrate(log((d*x+c)/(b*x+a))/(b*x+a)/((a-c)*h+(b-d)*h*x),x, algorithm="g
iac")
```

output
```
integrate(log((d*x + c)/(b*x + a))/(((b - d)*h*x + (a - c)*h)*(b*x + a)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx = \int \frac{\ln\left(\frac{c+d\,x}{a+b\,x}\right)}{(h\,(a-c)+h\,x\,(b-d))\,(a+b\,x)}\,dx$$

input
```
int(log((c + d*x)/(a + b*x))/((h*(a - c) + h*x*(b - d))*(a + b*x)),x)
```

output
```
int(log((c + d*x)/(a + b*x))/((h*(a - c) + h*x*(b - d))*(a + b*x)), x)
```

Reduce [F]

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx = \text{Too large to display}$$

input
```
int(log((d*x+c)/(b*x+a))/(b*x+a)/((a-c)*h+(b-d)*h*x),x)
```

output
```
( - 2*int(log((c + d*x)/(a + b*x))/(a**2*b*c + a**2*b*d*x - a**2*c*d - a**
2*d**2*x + 2*a*b**2*c*x + 2*a*b**2*d*x**2 - a*b*c**2 - 4*a*b*c*d*x - 3*a*b
*d**2*x**2 + a*c**2*d + 2*a*c*d**2*x + a*d**3*x**2 + b**3*c*x**2 + b**3*d*
x**3 - b**2*c**2*x - 3*b**2*c*d*x**2 - 2*b**2*d**2*x**3 + b*c**2*d*x + 2*b
*c*d**2*x**2 + b*d**3*x**3),x)*a**2*b*d**2 + 2*int(log((c + d*x)/(a + b*x)
)/(a**2*b*c + a**2*b*d*x - a**2*c*d - a**2*d**2*x + 2*a*b**2*c*x + 2*a*b**
2*d*x**2 - a*b*c**2 - 4*a*b*c*d*x - 3*a*b*d**2*x**2 + a*c**2*d + 2*a*c*d**
2*x + a*d**3*x**2 + b**3*c*x**2 + b**3*d*x**3 - b**2*c**2*x - 3*b**2*c*d*x
**2 - 2*b**2*d**2*x**3 + b*c**2*d*x + 2*b*c*d**2*x**2 + b*d**3*x**3),x)*a*
*2*d**3 + 4*int(log((c + d*x)/(a + b*x))/(a**2*b*c + a**2*b*d*x - a**2*c*d
 - a**2*d**2*x + 2*a*b**2*c*x + 2*a*b**2*d*x**2 - a*b*c**2 - 4*a*b*c*d*x -
 3*a*b*d**2*x**2 + a*c**2*d + 2*a*c*d**2*x + a*d**3*x**2 + b**3*c*x**2 + b
**3*d*x**3 - b**2*c**2*x - 3*b**2*c*d*x**2 - 2*b**2*d**2*x**3 + b*c**2*d*x
 + 2*b*c*d**2*x**2 + b*d**3*x**3),x)*a*b**2*c*d - 4*int(log((c + d*x)/(a +
 b*x))/(a**2*b*c + a**2*b*d*x - a**2*c*d - a**2*d**2*x + 2*a*b**2*c*x + 2*
a*b**2*d*x**2 - a*b*c**2 - 4*a*b*c*d*x - 3*a*b*d**2*x**2 + a*c**2*d + 2*a*
c*d**2*x + a*d**3*x**2 + b**3*c*x**2 + b**3*d*x**3 - b**2*c**2*x - 3*b**2*
c*d*x**2 - 2*b**2*d**2*x**3 + b*c**2*d*x + 2*b*c*d**2*x**2 + b*d**3*x**3),
x)*a*b*c*d**2 - 2*int(log((c + d*x)/(a + b*x))/(a**2*b*c + a**2*b*d*x - a*
*2*c*d - a**2*d**2*x + 2*a*b**2*c*x + 2*a*b**2*d*x**2 - a*b*c**2 - 4*a*...
```

3.256 $\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$

Optimal result . . . 2699
Mathematica [B] (verified) . . . 2699
Rubi [A] (verified) . . . 2700
Maple [A] (verified) . . . 2701
Fricas [A] (verification not implemented) . . . 2702
Sympy [F(-1)] . . . 2702
Maxima [B] (verification not implemented) . . . 2702
Giac [F] . . . 2703
Mupad [F(-1)] . . . 2704
Reduce [F] . . . 2704

Optimal result

Integrand size = 38, antiderivative size = 27

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{\text{PolyLog}\left(2, \frac{g(c+dx)}{a+bx}\right)}{bc-ad}$$

output
```
polylog(2,g*(d*x+c)/(b*x+a))/(-a*d+b*c)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 375 vs. 2(27) = 54.

Time = 0.38 (sec) , antiderivative size = 375, normalized size of antiderivative = 13.89

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$$
$$= \frac{-\log^2\left(\frac{a}{b}+x\right)+2\log\left(\frac{a}{b}+x\right)\log(a+bx)-2\log\left(\frac{a-cg}{b-dg}+x\right)\log(a+bx)+2\log\left(\frac{a-cg}{b-dg}+x\right)\log\left(\frac{(b-dg)}{(bc-}\right.$$

input
```
Integrate[Log[(a - c*g + (b - d*g)*x)/(a + b*x)]/((a + b*x)*(c + d*x)),x]
```

output
```
(-Log[a/b + x]^2 + 2*Log[a/b + x]*Log[a + b*x] - 2*Log[(a - c*g)/(b - d*g)
 + x]*Log[a + b*x] + 2*Log[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(a + b*
x))/((b*c - a*d)*g)] - 2*Log[a/b + x]*Log[c + d*x] + 2*Log[(a - c*g)/(b -
d*g) + x]*Log[c + d*x] + 2*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2
*Log[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(c + d*x))/(b*c - a*d)] + 2*L
og[a + b*x]*Log[(a - c*g + b*x - d*g*x)/(a + b*x)] - 2*Log[c + d*x]*Log[(a
 - c*g + b*x - d*g*x)/(a + b*x)] + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*
d)] + 2*PolyLog[2, -((b*(a - c*g + b*x - d*g*x))/((b*c - a*d)*g))] - 2*Pol
yLog[2, -((d*(-a + c*g - b*x + d*g*x))/(-(b*c) + a*d))])/(2*b*c - 2*a*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2965, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{a+x(b-dg)-cg}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$$

$$\downarrow 2965$$

$$\int \frac{\log\left(\frac{a+x(b-dg)-cg}{a+bx}\right)}{-\frac{(bc-ad)(a+x(b-dg)-cg)}{a+bx} - ad + bc} d\frac{a+x(b-dg)-cg}{a+bx}$$

$$\downarrow 2752$$

$$\frac{\text{PolyLog}\left(2, 1-\frac{a-cg+(b-dg)x}{a+bx}\right)}{bc-ad}$$

input
```
Int[Log[(a - c*g + (b - d*g)*x)/(a + b*x)]/((a + b*x)*(c + d*x)),x]
```

output
```
PolyLog[2, 1 - (a - c*g + (b - d*g)*x)/(a + b*x)]/(b*c - a*d)
```

Defintions of rubi rules used

rule 2752

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2965

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(q + 1)*(i/d)^q  Subst[Int[(b*f - a*g - (d*f - c*g)*
x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*
x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ[b*c - a*d,
 0] && IntegersQ[m, q] && IGtQ[p, 0] && EqQ[d*h - c*i, 0]
```

Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

method	result
derivativedivides	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{da-bc}$
default	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{da-bc}$
risch	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{da-bc}$
parts	$\frac{\ln\left(\frac{a-cg+(-dg+b)x}{bx+a}\right)\ln(dx+c)}{da-bc}-\frac{\ln\left(\frac{a-cg+(-dg+b)x}{bx+a}\right)\ln(bx+a)}{da-bc}-g\left(-\frac{\left(\frac{\operatorname{dilog}\left(\frac{(-dg+b)(bx+a)+adg-bcg}{adg-bcg}\right)}{-dg+b}+\frac{\ln(bx+$

input

```
int(ln((a-c*g+(-d*g+b)*x)/(b*x+a))/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE
)
```

output

```
-dilog((a*d*g-b*c*g)/b/(b*x+a)+(-d*g+b)/b)/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{\mathrm{Li}_2\left(\frac{cg+(dg-b)x-a}{bx+a}+1\right)}{bc-ad}$$

input
```
integrate(log((a-c*g+(-d*g+b)*x)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
dilog((c*g + (d*g - b)*x - a)/(b*x + a) + 1)/(b*c - a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(ln((a-c*g+(-d*g+b)*x)/(b*x+a))/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(26) = 52.

Time = 0.05 (sec) , antiderivative size = 344, normalized size of antiderivative = 12.74

$$\begin{aligned}
&\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx \\
&= \left(\frac{\log(bx+a)}{bc-ad} - \frac{\log(dx+c)}{bc-ad}\right)\log\left(-\frac{cg+(dg-b)x-a}{bx+a}\right) \\
&\quad + \frac{\log(bx+a)^2 - 2\log(bx+a)\log(dx+c)}{2(bc-ad)} \\
&\quad - \frac{\log(bx+a)\log\left(\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}+1\right) + \mathrm{Li}_2\left(-\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}\right)}{bc-ad} \\
&\quad + \frac{\log(dx+c)\log\left(\frac{cdg-bc+(d^2g-bd)x}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{cdg-bc+(d^2g-bd)x}{bc-ad}\right)}{bc-ad} \\
&\quad + \frac{\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)}{bc-ad}
\end{aligned}$$

input
```
integrate(log((a-c*g+(-d*g+b)*x)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log(-(c*g + (d*g - b)*x - a)/(b*x + a)) + 1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(d*x + c))/(b*c - a*d) - (log(b*x + a)*log(((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*d*g) + 1) + dilog(-((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*d*g)))/(b*c - a*d) + (log(d*x + c)*log((c*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d) + 1) + dilog(-(c*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d)))/(b*c - a*d) + (log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*c - a*d)
```

Giac [F]

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(-\frac{cg+(dg-b)x-a}{bx+a}\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log((a-c*g+(-d*g+b)*x)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(log(-(c*g + (d*g - b)*x - a)/(b*x + a))/((b*x + a)*(d*x + c)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\ln\left(\frac{a-c\,g+x\,(b-d\,g)}{a+b\,x}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(log((a - c*g + x*(b - d*g))/(a + b*x))/((a + b*x)*(c + d*x)),x)
```

output
```
int(log((a - c*g + x*(b - d*g))/(a + b*x))/((a + b*x)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$$
$$= \frac{2\left(\int \frac{\log\left(\frac{-dgx+bx-cg+a}{bx+a}\right)}{-b\,d^2g\,x^3-a\,d^2g\,x^2+b^2d\,x^3-2bcdg\,x^2+2abd\,x^2-2acdgx+b^2c\,x^2-b\,c^2gx+a^2dx+2abcx-a\,c^2g+a^2c}dx\right)a^2d^2g-4\left(\int \frac{}{-b\,d^2g\,x^3-}\right.}{}$$

input
```
int(log((a-c*g+(-d*g+b)*x)/(b*x+a))/(b*x+a)/(d*x+c),x)
```

output

```
(2*int(log((a + b*x - c*g - d*g*x)/(a + b*x))/(a**2*c + a**2*d*x + 2*a*b*c
*x + 2*a*b*d*x**2 - a*c**2*g - 2*a*c*d*g*x - a*d**2*g*x**2 + b**2*c*x**2 +
 b**2*d*x**3 - b*c**2*g*x - 2*b*c*d*g*x**2 - b*d**2*g*x**3),x)*a**2*d**2*g
 - 4*int(log((a + b*x - c*g - d*g*x)/(a + b*x))/(a**2*c + a**2*d*x + 2*a*b
*c*x + 2*a*b*d*x**2 - a*c**2*g - 2*a*c*d*g*x - a*d**2*g*x**2 + b**2*c*x**2
 + b**2*d*x**3 - b*c**2*g*x - 2*b*c*d*g*x**2 - b*d**2*g*x**3),x)*a*b*c*d*g
 + 2*int(log((a + b*x - c*g - d*g*x)/(a + b*x))/(a**2*c + a**2*d*x + 2*a*b
*c*x + 2*a*b*d*x**2 - a*c**2*g - 2*a*c*d*g*x - a*d**2*g*x**2 + b**2*c*x**2
 + b**2*d*x**3 - b*c**2*g*x - 2*b*c*d*g*x**2 - b*d**2*g*x**3),x)*b**2*c**2
*g - log((a + b*x - c*g - d*g*x)/(a + b*x))**2*b + log((a + b*x - c*g - d*
g*x)/(a + b*x))**2*d*g)/(2*d*g*(a*d - b*c))
```

3.257 $\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$

Optimal result . . . 2706
Mathematica [B] (verified) . . . 2706
Rubi [A] (verified) . . . 2707
Maple [A] (verified) . . . 2708
Fricas [A] (verification not implemented) . . . 2709
Sympy [F(-1)] . . . 2709
Maxima [B] (verification not implemented) . . . 2710
Giac [F] . . . 2711
Mupad [F(-1)] . . . 2711
Reduce [F] . . . 2711

Optimal result

Integrand size = 33, antiderivative size = 27

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{\text{PolyLog}\left(2,\frac{g(c+dx)}{a+bx}\right)}{bc-ad}$$

output
```
polylog(2,g*(d*x+c)/(b*x+a))/(-a*d+b*c)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 375 vs. 2(27) = 54.

Time = 0.35 (sec) , antiderivative size = 375, normalized size of antiderivative = 13.89

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$$
$$= \frac{-\log^2\left(\frac{a}{b}+x\right)+2\log\left(\frac{a}{b}+x\right)\log(a+bx)-2\log\left(\frac{a-cg}{b-dg}+x\right)\log(a+bx)+2\log\left(\frac{a-cg}{b-dg}+x\right)\log\left(\frac{(b-dg)}{(bc-}\right.$$

input
```
Integrate[Log[1 - (g*(c + d*x))/(a + b*x)]/((a + b*x)*(c + d*x)),x]
```

output

```
(-Log[a/b + x]^2 + 2*Log[a/b + x]*Log[a + b*x] - 2*Log[(a - c*g)/(b - d*g)
 + x]*Log[a + b*x] + 2*Log[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(a + b*
x))/((b*c - a*d)*g)] - 2*Log[a/b + x]*Log[c + d*x] + 2*Log[(a - c*g)/(b -
d*g) + x]*Log[c + d*x] + 2*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2
*Log[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(c + d*x))/(b*c - a*d)] + 2*L
og[a + b*x]*Log[(a - c*g + b*x - d*g*x)/(a + b*x)] - 2*Log[c + d*x]*Log[(a
 - c*g + b*x - d*g*x)/(a + b*x)] + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*
d)] + 2*PolyLog[2, -((b*(a - c*g + b*x - d*g*x))/((b*c - a*d)*g))] - 2*Pol
yLog[2, -((d*(-a + c*g - b*x + d*g*x))/(-(b*c) + a*d))])/(2*b*c - 2*a*d)
```

Rubi [A] (verified)

Time $= 0.35$ (sec) , antiderivative size $= 38$, normalized size of antiderivative $= 1.41$, number of steps used $= 4$, number of rules used $= 3$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used $= \{2997, 2965, 2752\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$$

$$\downarrow 2997$$

$$\int \frac{\log\left(\frac{a+x(b-dg)-cg}{a+bx}\right)}{(a+bx)(c+dx)}dx$$

$$\downarrow 2965$$

$$\int \frac{\log\left(\frac{a+x(b-dg)-cg}{a+bx}\right)}{-\frac{(bc-ad)(a+x(b-dg)-cg)}{a+bx}-ad+bc}d\frac{a+x(b-dg)-cg}{a+bx}$$

$$\downarrow 2752$$

$$\frac{\text{PolyLog}\left(2,1-\frac{a-cg+(b-dg)x}{a+bx}\right)}{bc-ad}$$

input

```
Int[Log[1 - (g*(c + d*x))/(a + b*x)]/((a + b*x)*(c + d*x)),x]
```

output
```
PolyLog[2, 1 - (a - c*g + (b - d*g)*x)/(a + b*x)]/(b*c - a*d)
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2965
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(q + 1)*(i/d)^q  Subst[Int[(b*f - a*g - (d*f - c*g)*
x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*
x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ[b*c - a*d,
 0] && IntegersQ[m, q] && IGtQ[p, 0] && EqQ[d*h - c*i, 0]
```

rule 2997
```
Int[Log[(e_.)*((f_.)*((g_) + (v_.)/(w_)))^(r_.)]^(s_.)*(u_.), x_Symbol] :>
Int[u*Log[e*(f*(ExpandToSum[v + g*w, x]/ExpandToSum[w, x]))^r]^s, x] /; Fre
eQ[{e, f, g, r, s}, x] && LinearQ[w, x] && (FreeQ[v, x] || LinearQ[v, x]) &
& AlgebraicFunctionQ[u, x]
```

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

method	result
derivativedivides	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{da-bc}$
default	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{da-bc}$
risch	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{da-bc}$
parts	$\frac{\ln\left(1-\frac{g(dx+c)}{bx+a}\right)\ln(dx+c)}{da-bc}-\frac{\ln\left(1-\frac{g(dx+c)}{bx+a}\right)\ln(bx+a)}{da-bc}-\frac{g\left(-\frac{\left(\frac{\operatorname{dilog}\left(\frac{(-dg+b)(bx+a)+adg-bcg}{adg-bcg}\right)}{-dg+b}+\frac{\ln(bx+a)\ln\left(\frac{(-dg+b)(bx+a)+adg-bcg}{adg-bcg}\right)}{-dg+b}\right)(da-bc)}{g}\right.$

input
```
int(ln(1-g*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-dilog((a*d*g-b*c*g)/b/(b*x+a)+(-d*g+b)/b)/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{\mathrm{Li}_2\left(\frac{cg+(dg-b)x-a}{bx+a}+1\right)}{bc-ad}$$

input
```
integrate(log(1-g*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
dilog((c*g + (d*g - b)*x - a)/(b*x + a) + 1)/(b*c - a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(ln(1-g*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(26) = 52$.

Time = 0.05 (sec) , antiderivative size = 336, normalized size of antiderivative = 12.44

$$\begin{aligned}
&\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx \\
&= \left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right)\log\left(-\frac{(dx+c)g}{bx+a}+1\right) \\
&\quad+\frac{\log(bx+a)^2-2\log(bx+a)\log(dx+c)}{2(bc-ad)} \\
&\quad-\frac{\log(bx+a)\log\left(\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}+1\right)+\mathrm{Li}_2\left(-\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}\right)}{bc-ad} \\
&\quad+\frac{\log(dx+c)\log\left(\frac{cdg-bc+(d^2g-bd)x}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{cdg-bc+(d^2g-bd)x}{bc-ad}\right)}{bc-ad} \\
&\quad+\frac{\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)}{bc-ad}
\end{aligned}$$

input
```
integrate(log(1-g*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log(-(d*x + c)*g/(b*
x + a) + 1) + 1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(d*x + c))/(b*c - a*
d) - (log(b*x + a)*log(((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*d*g) + 1
) + dilog(-((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*d*g)))/(b*c - a*d) +
 (log(d*x + c)*log((c*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d) + 1) + dilo
g(-(c*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d)))/(b*c - a*d) + (log(b*x +
a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))
/(b*c - a*d)
```

Giac [F]

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(-\frac{(dx+c)g}{bx+a}+1\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log(1-g*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(log(-(d*x + c)*g/(b*x + a) + 1)/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\ln\left(1-\frac{g\,(c+d\,x)}{a+b\,x}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(log(1 - (g*(c + d*x))/(a + b*x))/((a + b*x)*(c + d*x)),x)
```

output
```
int(log(1 - (g*(c + d*x))/(a + b*x))/((a + b*x)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$$
$$= \frac{2\left(\int \frac{\log\left(\frac{-dgx+bx-cg+a}{bx+a}\right)}{-b\,d^2g\,x^3-a\,d^2g\,x^2+b^2d\,x^3-2bcdg\,x^2+2abd\,x^2-2acdgx+b^2c\,x^2-b\,c^2gx+a^2dx+2abcx-a\,c^2g+a^2c}dx\right)a^2d^2g-4\left(\int \frac{}{-b\,d^2g\,x^3-}\right.}{}$$

input
```
int(log(1-g*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x)
```

output

```
(2*int(log((a + b*x - c*g - d*g*x)/(a + b*x))/(a**2*c + a**2*d*x + 2*a*b*c
*x + 2*a*b*d*x**2 - a*c**2*g - 2*a*c*d*g*x - a*d**2*g*x**2 + b**2*c*x**2 +
 b**2*d*x**3 - b*c**2*g*x - 2*b*c*d*g*x**2 - b*d**2*g*x**3),x)*a**2*d**2*g
 - 4*int(log((a + b*x - c*g - d*g*x)/(a + b*x))/(a**2*c + a**2*d*x + 2*a*b
*c*x + 2*a*b*d*x**2 - a*c**2*g - 2*a*c*d*g*x - a*d**2*g*x**2 + b**2*c*x**2
 + b**2*d*x**3 - b*c**2*g*x - 2*b*c*d*g*x**2 - b*d**2*g*x**3),x)*a*b*c*d*g
 + 2*int(log((a + b*x - c*g - d*g*x)/(a + b*x))/(a**2*c + a**2*d*x + 2*a*b
*c*x + 2*a*b*d*x**2 - a*c**2*g - 2*a*c*d*g*x - a*d**2*g*x**2 + b**2*c*x**2
 + b**2*d*x**3 - b*c**2*g*x - 2*b*c*d*g*x**2 - b*d**2*g*x**3),x)*b**2*c**2
*g - log((a + b*x - c*g - d*g*x)/(a + b*x))**2*b + log((a + b*x - c*g - d*
g*x)/(a + b*x))**2*d*g)/(2*d*g*(a*d - b*c))
```

3.258 $\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$

Optimal result . . . 2713
Mathematica [B] (verified) . . . 2713
Rubi [A] (verified) . . . 2714
Maple [A] (verified) . . . 2715
Fricas [A] (verification not implemented) . . . 2716
Sympy [F(-1)] . . . 2716
Maxima [B] (verification not implemented) . . . 2717
Giac [F] . . . 2718
Mupad [F(-1)] . . . 2718
Reduce [F] . . . 2718

Optimal result

Integrand size = 38, antiderivative size = 27

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{\text{PolyLog}\left(2, \frac{g(c+dx)}{a+bx}\right)}{bc-ad}$$

output
```
polylog(2,g*(d*x+c)/(b*x+a))/(-a*d+b*c)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 375 vs. 2(27) = 54.

Time = 0.10 (sec) , antiderivative size = 375, normalized size of antiderivative = 13.89

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$$
$$= \frac{-\log^2\left(\frac{a}{b}+x\right) + 2\log\left(\frac{a}{b}+x\right)\log(a+bx) - 2\log\left(\frac{a-cg}{b-dg}+x\right)\log(a+bx) + 2\log\left(\frac{a-cg}{b-dg}+x\right)\log\left(\frac{(b-dg)}{(bc-}\right.$$

input
```
Integrate[Log[(a - c*g + b*x - d*g*x)/(a + b*x)]/((a + b*x)*(c + d*x)),x]
```

output

```
(-Log[a/b + x]^2 + 2*Log[a/b + x]*Log[a + b*x] - 2*Log[(a - c*g)/(b - d*g)
 + x]*Log[a + b*x] + 2*Log[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(a + b*
x))/((b*c - a*d)*g)] - 2*Log[a/b + x]*Log[c + d*x] + 2*Log[(a - c*g)/(b -
d*g) + x]*Log[c + d*x] + 2*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2
*Log[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(c + d*x))/(b*c - a*d)] + 2*L
og[a + b*x]*Log[(a - c*g + b*x - d*g*x)/(a + b*x)] - 2*Log[c + d*x]*Log[(a
 - c*g + b*x - d*g*x)/(a + b*x)] + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*
d)] + 2*PolyLog[2, -((b*(a - c*g + b*x - d*g*x))/((b*c - a*d)*g))] - 2*Pol
yLog[2, -((d*(-a + c*g - b*x + d*g*x))/(-(b*c) + a*d))])/(2*b*c - 2*a*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2971, 2965, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{a+bx-cg-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$$

$$\downarrow 2971$$

$$\int \frac{\log\left(\frac{a+x(b-dg)-cg}{a+bx}\right)}{(a+bx)(c+dx)}dx$$

$$\downarrow 2965$$

$$\int \frac{\log\left(\frac{a+x(b-dg)-cg}{a+bx}\right)}{-\frac{(bc-ad)(a+x(b-dg)-cg)}{a+bx}-ad+bc}d\frac{a+x(b-dg)-cg}{a+bx}$$

$$\downarrow 2752$$

$$\frac{\text{PolyLog}\left(2,1-\frac{a-cg+(b-dg)x}{a+bx}\right)}{bc-ad}$$

input

```
Int[Log[(a - c*g + b*x - d*g*x)/(a + b*x)]/((a + b*x)*(c + d*x)),x]
```

output
```
PolyLog[2, 1 - (a - c*g + (b - d*g)*x)/(a + b*x)]/(b*c - a*d)
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2965
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(q + 1)*(i/d)^q  Subst[Int[(b*f - a*g - (d*f - c*g)*
x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*
x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ[b*c - a*d,
 0] && IntegersQ[m, q] && IGtQ[p, 0] && EqQ[d*h - c*i, 0]
```

rule 2971
```
Int[((A_.) + Log[(e_.)*((u_)/(v_))^(n_.)]*(B_.))^(p_.)*(w_)^(m_.)*(y_)^(q_.
), x_Symbol] :> Int[ExpandToSum[w, x]^m*ExpandToSum[y, x]^q*(A + B*Log[e*(E
xpandToSum[u, x]/ExpandToSum[v, x])^n])^p, x] /; FreeQ[{e, A, B, m, n, p, q
}, x] && LinearQ[{u, v, w, y}, x] &&  !LinearMatchQ[{u, v, w, y}, x]
```

Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

method	result
derivativedivides	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{da-bc}$
default	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{da-bc}$
risch	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{da-bc}$
parts	$\frac{\ln\left(\frac{-xdg+bx-cg+a}{bx+a}\right)\ln(dx+c)}{da-bc}-\frac{\ln\left(\frac{-xdg+bx-cg+a}{bx+a}\right)\ln(bx+a)}{da-bc}-g\left(-\frac{\left(\frac{\operatorname{dilog}\left(\frac{(-dg+b)(bx+a)+adg-bcg}{adg-bcg}\right)}{-dg+b}+\frac{\ln(bx+a}{g(da-}\right.\right.$

input
```
int(ln((-d*g*x+b*x-c*g+a)/(b*x+a))/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE
)
```

output
```
-dilog((a*d*g-b*c*g)/b/(b*x+a)+(-d*g+b)/b)/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{\mathrm{Li}_2\left(\frac{cg+(dg-b)x-a}{bx+a}+1\right)}{bc-ad}$$

input
```
integrate(log((-d*g*x+b*x-c*g+a)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="fr
icas")
```

output
```
dilog((c*g + (d*g - b)*x - a)/(b*x + a) + 1)/(b*c - a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(ln((-d*g*x+b*x-c*g+a)/(b*x+a))/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(26) = 52$.

Time = 0.05 (sec) , antiderivative size = 343, normalized size of antiderivative = 12.70

$$\begin{aligned}
&\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx \\
&= \left(\frac{\log(bx+a)}{bc-ad} - \frac{\log(dx+c)}{bc-ad}\right)\log\left(-\frac{dgx+cg-bx-a}{bx+a}\right) \\
&\quad + \frac{\log(bx+a)^2 - 2\log(bx+a)\log(dx+c)}{2(bc-ad)} \\
&\quad - \frac{\log(bx+a)\log\left(\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}+1\right) + \mathrm{Li}_2\left(-\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}\right)}{bc-ad} \\
&\quad + \frac{\log(dx+c)\log\left(\frac{cdg-bc+(d^2g-bd)x}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{cdg-bc+(d^2g-bd)x}{bc-ad}\right)}{bc-ad} \\
&\quad + \frac{\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)}{bc-ad}
\end{aligned}$$

input
```
integrate(log((-d*g*x+b*x-c*g+a)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log(-(d*g*x + c*g - b*x - a)/(b*x + a)) + 1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(d*x + c))/(b*c - a*d) - (log(b*x + a)*log(((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*d*g) + 1) + dilog(-((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*d*g)))/(b*c - a*d) + (log(d*x + c)*log((c*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d) + 1) + dilog(-(c*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d)))/(b*c - a*d) + (log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*c - a*d)
```

Giac [F]

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(-\frac{dgx+cg-bx-a}{bx+a}\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log((-d*g*x+b*x-c*g+a)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(log(-(d*g*x + c*g - b*x - a)/(b*x + a))/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\ln\left(\frac{a-c\,g+b\,x-d\,g\,x}{a+b\,x}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(log((a - c*g + b*x - d*g*x)/(a + b*x))/((a + b*x)*(c + d*x)),x)
```

output
```
int(log((a - c*g + b*x - d*g*x)/(a + b*x))/((a + b*x)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$$
$$= \frac{2\left(\int \frac{\log\left(\frac{-dgx+bx-cg+a}{bx+a}\right)}{-b\,d^2g\,x^3-a\,d^2g\,x^2+b^2d\,x^3-2bcdg\,x^2+2abd\,x^2-2acdgx+b^2c\,x^2-b\,c^2gx+a^2dx+2abcx-a\,c^2g+a^2c}dx\right)a^2d^2g - 4\left(\int \frac{}{-b\,d^2g\,x^3-}\right.}{}$$

input
```
int(log((-d*g*x+b*x-c*g+a)/(b*x+a))/(b*x+a)/(d*x+c),x)
```

output

```
(2*int(log((a + b*x - c*g - d*g*x)/(a + b*x))/(a**2*c + a**2*d*x + 2*a*b*c
*x + 2*a*b*d*x**2 - a*c**2*g - 2*a*c*d*g*x - a*d**2*g*x**2 + b**2*c*x**2 +
 b**2*d*x**3 - b*c**2*g*x - 2*b*c*d*g*x**2 - b*d**2*g*x**3),x)*a**2*d**2*g
 - 4*int(log((a + b*x - c*g - d*g*x)/(a + b*x))/(a**2*c + a**2*d*x + 2*a*b
*c*x + 2*a*b*d*x**2 - a*c**2*g - 2*a*c*d*g*x - a*d**2*g*x**2 + b**2*c*x**2
 + b**2*d*x**3 - b*c**2*g*x - 2*b*c*d*g*x**2 - b*d**2*g*x**3),x)*a*b*c*d*g
 + 2*int(log((a + b*x - c*g - d*g*x)/(a + b*x))/(a**2*c + a**2*d*x + 2*a*b
*c*x + 2*a*b*d*x**2 - a*c**2*g - 2*a*c*d*g*x - a*d**2*g*x**2 + b**2*c*x**2
 + b**2*d*x**3 - b*c**2*g*x - 2*b*c*d*g*x**2 - b*d**2*g*x**3),x)*b**2*c**2
*g - log((a + b*x - c*g - d*g*x)/(a + b*x))**2*b + log((a + b*x - c*g - d*
g*x)/(a + b*x))**2*d*g)/(2*d*g*(a*d - b*c))
```

3.259 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{afh+bghx^2+h(bfx+agx)}\,dx$

Optimal result . . . 2720
Mathematica [F] . . . 2721
Rubi [A] (warning: unable to verify) . . . 2721
Maple [F] . . . 2724
Fricas [F] . . . 2724
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Maxima [F] . . . 2725
Giac [F] . . . 2726
Mupad [F(-1)] . . . 2726
Reduce [F] . . . 2726

Optimal result

Integrand size = 51, antiderivative size = 282

$$\begin{aligned}
&\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{afh+bghx^2+h(bfx+agx)}\,dx \\
&= -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad+\frac{3Bn\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad+\frac{6B^2n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad+\frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}
\end{aligned}$$

output

```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b
*x+a))/(-a*g+b*f)/h+3*B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,(-
a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+6*B^2*n^2*(A+B*ln(e*(b*x
+a)^n/((d*x+c)^n)))*polylog(3,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g
+b*f)/h+6*B^3*n^3*polylog(4,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b
*f)/h
```

Mathematica [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx = \int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a*f*h + b*g*h*x^2 +
h*(b*f*x + a*g*x)),x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a*f*h + b*g*h*x^2 +
h*(b*f*x + a*g*x)), x]
```

Rubi [A] (warning: unable to verify)

Time = 1.16 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.137$, Rules used = {2973, 2976, 2026, 2779, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{h(agx+bfx)+afh+bghx^2}\,dx$$

$$\downarrow 2973$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{h(agx+bfx)+afh+bghx^2}dx$$

$$\downarrow 2976$$

$$(bc-ad)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{\frac{(bc-ad)(bf-ag)h(a+bx)}{c+dx}-\frac{(bc-ad)(df-cg)h(a+bx)^2}{(c+dx)^2}}d\frac{a+bx}{c+dx}$$

$$\downarrow 2026$$

$$(bc-ad)\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)\left((bc-ad)(bf-ag)h-\frac{(bc-ad)(df-cg)h(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\downarrow 2779$$

$$(bc-ad)\left(\frac{3Bn\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{h(bc-ad)(bf-ag)}-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{h(bc-ad)(bf-ag)}$$

$$\downarrow 2821$$

$$(bc-ad)\left(\frac{3Bn\left(\mathrm{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\mathrm{PolyLog}\left(2,\frac{(b}{(}\right.}{a+bx}}{h(bc-ad)(bf-ag)}$$

$$\downarrow 2830$$

$$(bc-ad)\left(\frac{3Bn\left(\mathrm{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\left(Bn\int\frac{(c+dx)\,\mathrm{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+}{c+}\right.\right.}{h(bc-ad)(bf-ag)}$$

$$\downarrow 7143$$

$$(bc-ad)\left(\frac{3Bn\left(\mathrm{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\left(-\left(\mathrm{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\right.\right.\right.\right.}{h(bc-ad)(bf-ag)}$$

input

```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a*f*h + b*g*h*x^2 + h*(b*f
*x + a*g*x)),x]
```

output

```
(b*c - a*d)*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^3*Log[1 - ((b*f - a
*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])/((b*c - a*d)*(b*f - a*g)*h)) + (3
*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*PolyLog[2, ((b*f - a*g)*(c
+ d*x))/((d*f - c*g)*(a + b*x))] - 2*B*n*(-((A + B*Log[e*((a + b*x)/(c + d
*x))^n])*PolyLog[3, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]) - B*
n*PolyLog[4, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])))/((b*c - a
*d)*(b*f - a*g)*h))
```

Defintions of rubi rules used

rule 2026

```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2779

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))  Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)  Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)  Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2973

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 2976

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)  Subst[Int[(b^2*f -
a*b*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^
2*h)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/
(c + d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] &
& NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^n\left(dx+c\right)^{-n}\right)\right)^3}{afh+bghx^2+h\left(agx+bfx\right)}dx$$

input

```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x)),
x)
```

output

```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x)),
x)
```

Fricas [F]

$$\int \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^3}{afh+bghx^2+h(bfx+agx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{bghx^2+afh+(bfx+agx)h}\,dx$$

input

```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(a*f*h+b*g*h*x^2+h*(a*g*x+b
*f*x)),x, algorithm="fricas")
```

output

```
integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e
/(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(b*g*h*x^2
 + a*f*h + (b*f + a*g)*h*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(a*f*h+b*g*h*x**2+h*(a*g*
x+b*f*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{bghx^2+afh+(bfx+agx)h}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(a*f*h+b*g*h*x^2+h*(a*g*x+b
*f*x)),x, algorithm="maxima")
```

output
```
A^3*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) - integr
ate(-(B^3*log((b*x + a)^n)^3 - B^3*log((d*x + c)^n)^3 + B^3*log(e)^3 + 3*A
*B^2*log(e)^2 + 3*A^2*B*log(e) + 3*(B^3*log(e) + A*B^2)*log((b*x + a)^n)^2
 + 3*(B^3*log((b*x + a)^n) + B^3*log(e) + A*B^2)*log((d*x + c)^n)^2 + 3*(B
^3*log(e)^2 + 2*A*B^2*log(e) + A^2*B)*log((b*x + a)^n) - 3*(B^3*log((b*x +
 a)^n)^2 + B^3*log(e)^2 + 2*A*B^2*log(e) + A^2*B + 2*(B^3*log(e) + A*B^2)*
log((b*x + a)^n))*log((d*x + c)^n))/(b*g*h*x^2 + a*f*h + (b*f*h + a*g*h)*x
), x)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{bghx^2+afh+(bfx+agx)h}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(a*f*h+b*g*h*x^2+h*(a*g*x+b
*f*x)),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*g*h*x^2 + a*f*h + (b
*f*x + a*g*x)*h), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{h\,(a\,g\,x+b\,f\,x)+a\,f\,h+b\,g\,h\,x^2}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(h*(a*g*x + b*f*x) + a*f*h
+ b*g*h*x^2),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(h*(a*g*x + b*f*x) + a*f*h
+ b*g*h*x^2), x)
```

Reduce [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx = \text{too large to display}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))
,x)
```

output

```
(4*int(log(((a + b*x)**n*e)/(c + d*x)**n)**3/(a*c*f + a*c*g*x + a*d*f*x +
a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**2*b**3*
c*d*g**2*n - 4*int(log(((a + b*x)**n*e)/(c + d*x)**n)**3/(a*c*f + a*c*g*x
+ a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x
)*a**2*b**3*d**2*f*g*n - 4*int(log(((a + b*x)**n*e)/(c + d*x)**n)**3/(a*c*
f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b
*d*g*x**3),x)*a*b**4*c**2*g**2*n + 4*int(log(((a + b*x)**n*e)/(c + d*x)**n
)**3/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*
f*x**2 + b*d*g*x**3),x)*a*b**4*d**2*f**2*n + 4*int(log(((a + b*x)**n*e)/(c
 + d*x)**n)**3/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x
**2 + b*d*f*x**2 + b*d*g*x**3),x)*b**5*c**2*f*g*n - 4*int(log(((a + b*x)**
n*e)/(c + d*x)**n)**3/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x +
b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*b**5*c*d*f**2*n + 12*int(log(((a
+ b*x)**n*e)/(c + d*x)**n)**2/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*
c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**3*b**2*c*d*g**2*n - 12
*int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(a*c*f + a*c*g*x + a*d*f*x + a*
d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**3*b**2*d*
*2*f*g*n - 12*int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(a*c*f + a*c*g*x +
 a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)
*a**2*b**3*c**2*g**2*n + 12*int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(...
```

3.260 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{afh+bghx^2+h(bfx+agx)}\,dx$

Optimal result . . . 2728
Mathematica [B] (verified) . . . 2729
Rubi [A] (warning: unable to verify) . . . 2730
Maple [F] . . . 2732
Fricas [F] . . . 2733
Sympy [F(-1)] . . . 2733
Maxima [F] . . . 2733
Giac [F] . . . 2734
Mupad [F(-1)] . . . 2734
Reduce [F] . . . 2735

Optimal result

Integrand size = 51, antiderivative size = 203

$$\begin{aligned}
&\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{afh+bghx^2+h(bfx+agx)}\,dx \\
&= -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad + \frac{2Bn\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad + \frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}
\end{aligned}$$

output

```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b
*x+a))/(-a*g+b*f)/h+2*B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,(-a*
g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+2*B^2*n^2*polylog(3,(-a*g+
b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1415 vs. $2(203) = 406$.

Time = 1.09 (sec) , antiderivative size = 1415, normalized size of antiderivative = 6.97

$$\int \frac{(A + B \log (e(a+bx)^n (c+dx)^{-n}))^2}{afh + bghx^2 + h(bfx + agx)}\, dx = \text{Too large to display}$$

input

```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a*f*h + b*g*h*x^2 +
h*(b*f*x + a*g*x)),x]
```

output

```
(3*Log[a + b*x]*(A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b
*x)^n)/(c + d*x)^n]))^2 - 3*(A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + L
og[(e*(a + b*x)^n)/(c + d*x)^n]))^2*Log[f + g*x] + 3*B*n*(A + B*(-(n*Log[a
 + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]))*(Log[a + b*
x]^2 - 2*(Log[a + b*x]*Log[(b*(f + g*x))/(b*f - a*g)] + PolyLog[2, (g*(a +
 b*x))/(-(b*f) + a*g)])) - 6*A*B*n*(Log[c + d*x]*(Log[(d*(a + b*x))/(-(b*c
) + a*d)] - Log[(d*(f + g*x))/(d*f - c*g)]) + PolyLog[2, (b*(c + d*x))/(b*
c - a*d)] - PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)]) + 6*B^2*n*(n*Log[a +
 b*x] - n*Log[c + d*x] - Log[(e*(a + b*x)^n)/(c + d*x)^n])*(Log[c + d*x]*(
Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[(d*(f + g*x))/(d*f - c*g)]) + Poly
Log[2, (b*(c + d*x))/(b*c - a*d)] - PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g
)]) + B^2*n^2*(Log[a + b*x]^2*(Log[a + b*x] - 3*Log[(b*(f + g*x))/(b*f - a
*g)]) - 6*Log[a + b*x]*PolyLog[2, (g*(a + b*x))/(-(b*f) + a*g)] + 6*PolyLo
g[3, (g*(a + b*x))/(-(b*f) + a*g)]) + 3*B^2*n^2*(Log[(d*(a + b*x))/(-(b*c)
 + a*d)]*Log[c + d*x]^2 - Log[c + d*x]^2*Log[(d*(f + g*x))/(d*f - c*g)] +
2*Log[c + d*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*Log[c + d*x]*Poly
Log[2, (g*(c + d*x))/(-(d*f) + c*g)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a
*d)] + 2*PolyLog[3, (g*(c + d*x))/(-(d*f) + c*g)]) - 6*B^2*n^2*((Log[a + b
*x]^2*(Log[c + d*x] - Log[(b*(c + d*x))/(b*c - a*d)]))/2 - Log[a + b*x]*Lo
g[c + d*x]*Log[(b*(f + g*x))/(b*f - a*g)] - (Log[(g*(c + d*x))/(-(d*f) ...
```

Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2973, 2976, 2026, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{h(agx+bfx)+afh+bghx^2}\,dx$$

$$\downarrow 2973$$

$$\int \frac{\left(B \log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{h(agx+bfx)+afh+bghx^2}\,dx$$

$$\downarrow 2976$$

$$(bc-ad)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\frac{(bc-ad)(bf-ag)h(a+bx)}{c+dx}-\frac{(bc-ad)(df-cg)h(a+bx)^2}{(c+dx)^2}}\,d\frac{a+bx}{c+dx}$$

$$\downarrow 2026$$

$$(bc-ad)\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left((bc-ad)(bf-ag)h-\frac{(bc-ad)(df-cg)h(a+bx)}{c+dx}\right)}\,d\frac{a+bx}{c+dx}$$

$$\downarrow 2779$$

$$(bc-ad)\left(\frac{2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}\,d\frac{a+bx}{c+dx}}{h(bc-ad)(bf-ag)}-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{h(bc-ad)(bf-ag)}$$

$$\downarrow 2821$$

$$(bc-ad)\left(\frac{2Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int \frac{(c+dx)\,\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}\,d\frac{a+bx}{c+dx}\right)}{h(bc-ad)(bf-ag)}-$$

$$\downarrow 7143$$

$$(bc - ad)\left(\frac{2Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+Bn\,\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\right)}{h(bc-ad)(bf-ag)} - \frac{\log\left(1-\right.}{}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a*f*h + b*g*h*x^2 + h*(b*f
*x + a*g*x)),x]
```

output
```
(b*c - a*d)*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - ((b*f - a
*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])/((b*c - a*d)*(b*f - a*g)*h)) + (2
*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, ((b*f - a*g)*(c +
d*x))/((d*f - c*g)*(a + b*x))] + B*n*PolyLog[3, ((b*f - a*g)*(c + d*x))/((
d*f - c*g)*(a + b*x))]))/((b*c - a*d)*(b*f - a*g)*h))
```

Defintions of rubi rules used

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))  Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)  Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2973

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 2976

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)^(2*m + 1)*(f*g - 2*c*
f*h + b*d*f^2/(b*c - a*d)^2)*(a + b*x)^(p + 1)*((A + B*Log[e*(a + b*x)^n/(c
 + d*x)^n])^p/(b*(b*c - a*d)^(2*(m + 1))*(p + 1))), x] - Simp[B*n*p*((b*c -
 a*d)^(2*m + 1)*(f*g - 2*c*f*h + b*d*f^2/(b*c - a*d)^2)/(b*(p + 1))), Int[(a
 + b*x)^(p + 1)*((A + B*Log[e*(a + b*x)^n/(c + d*x)^n])^(p - 1)/(c + d*x)^(2
*(m + 1))), x], x] - Simp[B*n*p*((b*c - a*d)^(2*m + 1)*(f*g - 2*c*f*h + b*d*
f^2/(b*c - a*d)^2)/(b*(p + 1))), Int[(a + b*x)^(p + 1)*((A + B*Log[e*(a + b*
x)^n/(c + d*x)^n])^(p - 1)/(c + d*x)^(2*(m + 1))), x], x] + Simp[(b*c - a*d)
^(2*m + 1)*(f*g - 2*c*f*h + b*d*f^2/(b*c - a*d)^2)*(a + b*x)^(p + 1)*((A + B
*Log[e*(a + b*x)^n/(c + d*x)^n])^p/(b*(b*c - a*d)^(2*(m + 1))*(p + 1))), x]
 /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && NeQ[b*c - a*d
, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(A + B \ln\left(e(bx+a)^n (dx+c)^{-n}\right)\right)^2}{afh + bgh\,x^2 + h\,(agx + bfx)}\,dx$$

input

```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x)),
x)
```

output

```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x)),
x)
```

Fricas [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{afh+bghx^2+h(bfx+agx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{bghx^2+afh+(bfx+agx)h}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(a*f*h+b*g*h*x^2+h*(a*g*x+b
*f*x)),x, algorithm="fricas")
```

output
```
integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(
d*x + c)^n) + A^2)/(b*g*h*x^2 + a*f*h + (b*f + a*g)*h*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{afh+bghx^2+h(bfx+agx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(a*f*h+b*g*h*x**2+h*(a*g*
x+b*f*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{afh+bghx^2+h(bfx+agx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{bghx^2+afh+(bfx+agx)h}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(a*f*h+b*g*h*x^2+h*(a*g*x+b
*f*x)),x, algorithm="maxima")
```

output
```
A^2*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) + integr
ate((B^2*log((b*x + a)^n)^2 + B^2*log((d*x + c)^n)^2 + B^2*log(e)^2 + 2*A*
B*log(e) + 2*(B^2*log(e) + A*B)*log((b*x + a)^n) - 2*(B^2*log((b*x + a)^n)
 + B^2*log(e) + A*B)*log((d*x + c)^n))/(b*g*h*x^2 + a*f*h + (b*f*h + a*g*h
)*x), x)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{afh+bghx^2+h(bfx+agx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{bghx^2+afh+(bfx+agx)h}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(a*f*h+b*g*h*x^2+h*(a*g*x+b
*f*x)),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*g*h*x^2 + a*f*h + (b
*f*x + a*g*x)*h), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{afh+bghx^2+h(bfx+agx)}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{h\,(a\,g\,x+b\,f\,x)+a\,f\,h+b\,g\,h\,x^2}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(h*(a*g*x + b*f*x) + a*f*h
+ b*g*h*x^2),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(h*(a*g*x + b*f*x) + a*f*h
+ b*g*h*x^2), x)
```

Reduce [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{afh + bghx^2 + h(bfx + agx)}\,dx = \text{Too large to display}$$

input

```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))
,x)
```

output

```
(3*int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(a*c*f + a*c*g*x + a*d*f*x +
a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**2*b**2*
c*d*g**2*n - 3*int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(a*c*f + a*c*g*x
+ a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x
)*a**2*b**2*d**2*f*g*n - 3*int(log(((a + b*x)**n*e)/(c + d*x)**n)**2/(a*c*
f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b
*d*g*x**3),x)*a*b**3*c**2*g**2*n + 3*int(log(((a + b*x)**n*e)/(c + d*x)**n
)**2/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*
f*x**2 + b*d*g*x**3),x)*a*b**3*d**2*f**2*n + 3*int(log(((a + b*x)**n*e)/(c
 + d*x)**n)**2/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x
**2 + b*d*f*x**2 + b*d*g*x**3),x)*b**4*c**2*f*g*n - 3*int(log(((a + b*x)**
n*e)/(c + d*x)**n)**2/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x +
b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*b**4*c*d*f**2*n + 6*int(log(((a +
 b*x)**n*e)/(c + d*x)**n)/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*
x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**3*b*c*d*g**2*n - 6*int(log
(((a + b*x)**n*e)/(c + d*x)**n)/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 +
b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**3*b*d**2*f*g*n - 6*i
nt(log(((a + b*x)**n*e)/(c + d*x)**n)/(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x
**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**2*b**2*c**2*g*
*2*n + 6*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a*c*f + a*c*g*x + a*d*...
```

3.261 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh+bghx^2+h(bfx+agx)}\,dx$

Optimal result 2736
Mathematica [B] (verified) 2737
Rubi [A] (warning: unable to verify) 2737
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Optimal result

Integrand size = 49, antiderivative size = 123

$$\begin{aligned}&\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh+bghx^2+h(bfx+agx)}\,dx\\&=-\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}\\&\quad+\frac{Bn\,\mathrm{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}\end{aligned}$$

output

```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x
+a))/(-a*g+b*f)/h+B*n*polylog(2,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a
*g+b*f)/h
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 303 vs. $2(123) = 246$.

Time $= 0.34$ (sec) , antiderivative size $= 303$, normalized size of antiderivative $= 2.46$

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh + bghx^2 + h(bfx + agx)}\,dx =$$
$$-\frac{-2A\log(a+bx) + Bn\log^2(a+bx) - 2Bn\log(a+bx)\log(c+dx) + 2Bn\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log(c+dx) -$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a*f*h + b*g*h*x^2 + h*
(b*f*x + a*g*x)),x]
```

output
```
-((-2*A*Log[a + b*x] + B*n*Log[a + b*x]^2 - 2*B*n*Log[a + b*x]*Log[c + d*x
] + 2*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 2*B*Log[a + b*x
]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*A*Log[f + g*x] - 2*B*n*Log[a + b*x]
*Log[f + g*x] + 2*B*n*Log[c + d*x]*Log[f + g*x] + 2*B*Log[(e*(a + b*x)^n)/
(c + d*x)^n]*Log[f + g*x] + 2*B*n*Log[a + b*x]*Log[(b*(f + g*x))/(b*f - a*
g)] - 2*B*n*Log[c + d*x]*Log[(d*(f + g*x))/(d*f - c*g)] + 2*B*n*PolyLog[2,
 (g*(a + b*x))/(-(b*f) + a*g)] + 2*B*n*PolyLog[2, (b*(c + d*x))/(b*c - a*d
)] - 2*B*n*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)])/((2*b*f - 2*a*g)*h))
```

Rubi [A] (warning: unable to verify)

Time $= 0.67$ (sec) , antiderivative size $= 151$, normalized size of antiderivative $= 1.23$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used $= \{2973, 2976, 2026, 2779, 2838\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right) + A}{h(agx + bfx) + afh + bghx^2}\,dx$$
$$\downarrow \text{2973}$$
$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right) + A}{h(agx + bfx) + afh + bghx^2}dx$$

$$\downarrow 2976$$

$$(bc-ad)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\frac{(bc-ad)(bf-ag)h(a+bx)}{c+dx}-\frac{(bc-ad)(df-cg)h(a+bx)^2}{(c+dx)^2}}d\frac{a+bx}{c+dx}$$

$$\downarrow 2026$$

$$(bc-ad)\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left((bc-ad)(bf-ag)h-\frac{(bc-ad)(df-cg)h(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\downarrow 2779$$

$$ad)\left(\frac{Bn\int \frac{(c+dx)\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{h(bc-ad)(bf-ag)}-\frac{(bc-\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{h(bc-ad)(bf-ag)}\right)$$

$$\downarrow 2838$$

$$ad)\left(\frac{Bn\,\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{h(bc-ad)(bf-ag)}-\frac{(bc-\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{h(bc-ad)(bf-ag)}\right)$$

input

```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a*f*h + b*g*h*x^2 + h*(b*f*x
 + a*g*x)),x]
```

output

```
(b*c - a*d)*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((b*f - a*g
)*(c + d*x))/((d*f - c*g)*(a + b*x))])/((b*c - a*d)*(b*f - a*g)*h)) + (B*n
*PolyLog[2, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])/((b*c - a*d)
*(b*f - a*g)*h))
```

Defintions of rubi rules used

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 2976
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)   Subst[Int[(b^2*f -
a*b*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^
2*h)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/
(c + d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] &
& NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 32.82 (sec) , antiderivative size = 1447, normalized size of antiderivative = 11.76

method	result	size
risch	Expression too large to display	1447

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x)),x,
method=_RETURNVERBOSE)
```

output
```
1/h/(a*g-b*f)*ln(g*x+f)*A-1/h/(a*g-b*f)*ln(b*x+a)*A-1/h*B*n/(a*g-b*f)*ln(g
*x+f)*ln((b*(g*x+f)+a*g-b*f)/(a*g-b*f))+1/h*B*n/(a*g-b*f)*ln(g*x+f)*ln((d*
(g*x+f)+c*g-d*f)/(c*g-d*f))-1/h*B*ln((b*x+a)^n)/(a*g-b*f)*ln(b*x+a)-1/h*B*
n/(a*g-b*f)*dilog((b*(g*x+f)+a*g-b*f)/(a*g-b*f))+1/2/h*B*n/(a*g-b*f)*ln(b*
x+a)^2+1/h*B/(a*g-b*f)*ln(g*x+f)*ln((b*x+a)^n)-1/h*B*n/(a*g-b*f)*ln(b*x+a)
*ln((-d*a+b*c+d*(b*x+a))/(-a*d+b*c))+1/h/(a*g-b*f)*ln(g*x+f)*B*ln(e)-1/h/(
a*g-b*f)*ln(b*x+a)*B*ln(e)-1/h*B/(a*g-b*f)*ln(g*x+f)*ln((d*x+c)^n)+1/h*B*l
n((d*x+c)^n)/(a*g-b*f)*ln(b*x+a)+1/h*B*n/(a*g-b*f)*dilog((d*(g*x+f)+c*g-d*
f)/(c*g-d*f))-1/h*B*n/(a*g-b*f)*dilog((-d*a+b*c+d*(b*x+a))/(-a*d+b*c))-1/2
*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I/((d*x+c)^n)*(b*x+a)^n)*csgn(I*e*(b*x+
a)^n/((d*x+c)^n))*csgn(I*e)+1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I/((d*x+
c)^n))*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n)*(b*x+a)^n)-1/2*I/h/(a*g-b*f)*l
n(g*x+f)*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n)*(b*
x+a)^n)+1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I/((d*x+c)^n)*(b*x+a)^n)*csg
n(I*e*(b*x+a)^n/((d*x+c)^n))*csgn(I*e)+1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*cs
gn(I/((d*x+c)^n))*csgn(I/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I/h/(a*g-b*f)*ln(g*x
+f)*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I/h/(a*g-b*
f)*ln(g*x+f)*B*Pi*csgn(I/((d*x+c)^n)*(b*x+a)^n)*csgn(I*e*(b*x+a)^n/((d*x+c
)^n))^2+1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I*e*(b*x+a)^n/((d*x+c)^n))^2
*csgn(I*e)-1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I/((d*x+c)^n))*csgn(I/...
```

Fricas [F]

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh + bghx^2 + h(bfx+agx)}\,dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{bghx^2 + afh + (bfx+agx)h}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f
*x)),x, algorithm="fricas")
```

output
```
integral((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*g*h*x^2 + a*f*h + (b*f
+ a*g)*h*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \log\left(e(a+bx)^n (c+dx)^{-n}\right)}{afh + bghx^2 + h(bfx + agx)}\, dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(a*f*h+b*g*h*x**2+h*(a*g*x+b
*f*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B \log\left(e(a+bx)^n (c+dx)^{-n}\right)}{afh + bghx^2 + h(bfx + agx)}\, dx = \int \frac{B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{bghx^2 + afh + (bfx + agx)h}\, dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f
*x)),x, algorithm="maxima")
```

output
```
A*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) - B*integr
ate(-(log((b*x + a)^n) - log((d*x + c)^n) + log(e))/(b*g*h*x^2 + a*f*h + (
b*f*h + a*g*h)*x), x)
```

Giac [F]

$$\int \frac{A + B \log\left(e(a+bx)^n (c+dx)^{-n}\right)}{afh + bghx^2 + h(bfx + agx)}\, dx = \int \frac{B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{bghx^2 + afh + (bfx + agx)h}\, dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f
*x)),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*g*h*x^2 + a*f*h + (b*f
*x + a*g*x)*h), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh+bghx^2+h(bfx+agx)}\,dx = \int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{h\,(a\,g\,x+b\,f\,x)+a\,f\,h+b\,g\,h\,x^2}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(h*(a*g*x + b*f*x) + a*f*h +
b*g*h*x^2),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(h*(a*g*x + b*f*x) + a*f*h +
b*g*h*x^2), x)
```

Reduce [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh+bghx^2+h(bfx+agx)}\,dx$$
$$= \frac{2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{bdg\,x^3+adg\,x^2+bcg\,x^2+bdf\,x^2+acgx+adfx+bcfx+acf}dx\right)a^2bcd\,g^2n - 2\left(\int \frac{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{bdg\,x^3+adg\,x^2+bcg\,x^2+bdf\,x^2+acgx+adfx+bcf}\right.}{}$$

input
```
int((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x)),x
)
```

output

```
(2*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a*c*f + a*c*g*x + a*d*f*x + a*d
*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**2*b*c*d*g*
*2*n - 2*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a*c*f + a*c*g*x + a*d*f*x
 + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a**2*b*
d**2*f*g*n - 2*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a*c*f + a*c*g*x + a
*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**3),x)*a
*b**2*c**2*g**2*n + 2*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a*c*f + a*c*
g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*d*g*x**
3),x)*a*b**2*d**2*f**2*n + 2*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a*c*f
 + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2 + b*
d*g*x**3),x)*b**3*c**2*f*g*n - 2*int(log(((a + b*x)**n*e)/(c + d*x)**n)/(a
*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2
+ b*d*g*x**3),x)*b**3*c*d*f**2*n - 2*log(a + b*x)*a**2*d*g*n + 2*log(a + b
*x)*a*b*c*g*n + 2*log(f + g*x)*a**2*d*g*n - 2*log(f + g*x)*a*b*c*g*n - log
(((a + b*x)**n*e)/(c + d*x)**n)**2*a*b*d*g + log(((a + b*x)**n*e)/(c + d*x
)**n)**2*b**2*d*f)/(2*g*h*n*(a**2*d*g - a*b*c*g - a*b*d*f + b**2*c*f))
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 2744
4.2 Links to plain text integration problems used in this report for each CAS . 2762

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral',
         'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
         'weierstrassPPrime','weierstrassSigma']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if  not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
          return 1
        else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
      return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
   if result.has(I):
      if optimal.has(I): #both result and optimal complex
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
         else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
      else: #result contains complex but optimal is not
         grade = "C"
         grade_annotation ="Result contains complex when optimal does not."
   else: # result do not contain complex, this assumes optimal do not as well
      if leaf_count_result <= 2*leaf_count_optimal:
         grade = "A"
         grade_annotation ="none"
      else:
         grade = "B"
         grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
   grade = "C"
   grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file